Examples: Timber

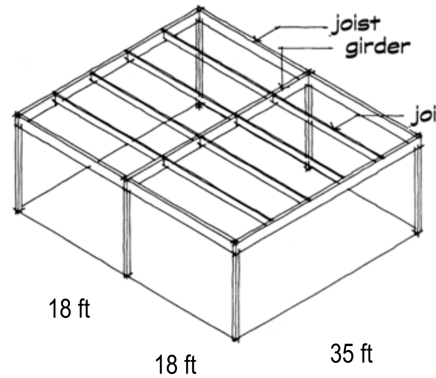
Example 1

Design a Flat Roof joist, 16 in. on center (o.c.) , 18 ft span with Douglas fir-larch No. 2. Snow load is 30 psf. Dead load (including ballast, roofing, sheathing, joists & ceiling) = 18.9 psf. C_r = 1.15 *for bending only*.

F_b = 875 psi; F_v = 95 psi; E = 1.6 x 10^6 psi

Also design the glulam girder supporting the joists if it spans 35 ft (simply supported) and F_b = 2400 psi.

Assume the density of the glulam timber is 32 lb/ft^3.



SOLUTION:

The load case that is most likely to govern the design is Dead + Live. Because the live load is from snow, C_D = 1.15:

$$\frac{18.9psf}{0.9} = 21psf < \frac{(18.9psf + 30psf)}{1.15} = 42.5$$

Joist

The distributed load for each joist needs to be found by multiplying the area load by the tributary width:

w = (30 lb/ft^2 + 18.9 lb/ft^2)(16in)(1ft/12in) = 65.2 lb/ft

$$M_{max} = \frac{wl^2}{8} = \frac{(65.2\,{}^{lb}/_{ft})(18\,ft)^2}{8} = 2641^{lb-ft}$$

Allowable stress is the tabulated stress multiplied by all applicable adjustment factors, which would be C_D and C_r:

$$F_b' = F_b C_D C_r = 875\,{}^{lb}/_{in^2}(1.15)(1.15) = 1157 \text{ lb/in}^2$$

$$S_{req'd} \geq \frac{M}{F_b'} = \frac{2641^{lb-ft}}{1157\,{}^{lb}/_{in^2}} \cdot (12\,{}^{in}/_{ft}) = 27.4in^3$$

Shear can quite often govern the design of timber beams:

$$V_{max} = \frac{wl}{2} = \frac{(65.2\,{}^{lb}/_{ft})(18\,ft)}{2} = 587^{lb}$$

Allowable stress is the tabulated stress multiplied by all applicable adjustment factors, which would be C_D *only:*

$$F_v' = F_v C_D = 95\,{}^{lb}/_{in^2}(1.15) = 109 \text{ lb/in}^2$$

Shear stress in a rectangular beam is found from 3V/2A:

$$A_{req'd} \geq \frac{3V}{2F_v'} = \frac{3(587^{lb})}{2(109\,{}^{lb}/_{in^2})} = 8.1in^2$$

SECTION PROPERTIES JOISTS AND BEAMS

Nominal Size In Inches b h	Surfaced Size In Inches For Design b h	Area (A) $A = bh$ (In²)	Section Modulus (S) $S = \frac{bh^2}{6}$ (In³)	Moment of Inertia (I) $I = \frac{bh^3}{12}$ (In⁴)	Board Feet Per Linear Foot of Piece
2 x 2	1.5 x 1.5	2.25	0.562	0.422	0.33
2 x 3	1.5 x 2.5	3.75	1.56	1.95	0.50
2 x 4	1.5 x 3.5	5.25	3.06	5.36	0.67
2 x 5	1.5 x 4.5	6.75	5.06	11.39	.83
2 x 6	1.5 x 5.5	8.25	7.56	20.80	1.00
2 x 8	1.5 x 7.25	10.88	13.14	47.63	1.33
2 x 10	1.5 x 9.25	13.88	21.39	98.93	1.67
2 x 12	1.5 x 11.25	16.88	31.64	177.98	2.00
2 x 14	1.5 x 13.25	19.88	43.89	290.78	2.33
3 x 3	2.5 x 2.5	6.25	2.60	3.26	0.75
3 x 4	2.5 x 3.5	8.75	5.10	8.93	1.00
3 x 5	2.5 x 4.5	11.25	8.44	18.98	1.25
3 x 6	2.5 x 5.5	13.75	12.60	34.66	1.50
3 x 8	2.5 x 7.25	18.12	21.90	79.39	2.00
3 x 10	2.5 x 9.25	23.12	35.65	164.89	2.50
3 x 12	2.5 x 11.25	28.12	52.73	296.63	3.00
3 x 14	2.5 x 13.25	33.12	73.15	484.63	3.50
3 x 16	2.5 x 15.25	38.12	96.90	738.87	4.00

Allowable deflection is not known, but $I_{req'd}$ could be determined from $\Delta = \frac{5wl^4}{384EI} \le \Delta_{limit}$ then $I_{req'd} \ge \frac{5wl^4}{384E\Delta_{limit}}$

From the section property table, a 2 x 12 satisfies $A_{req'd}$ and $I_{req'd}$. (bending governs)

Girder

The distributed load on the girder is the reaction of each joist over the 16 inch spacing plus the self weight of the girder.

Guessing a self weight of 40 lb/ft (≈ 32 lb/ft³x1ft²):

$$w = \frac{V}{spacing} + s.w. = \frac{587lb}{16in} \cdot \frac{12in}{ft} + 40\,{}^{lb}\!/\!_{ft} = 480\,{}^{lb}\!/\!_{ft}$$

$$M_{max} = \frac{wl^2}{8} = \frac{(480\,{}^{lb}\!/\!_{ft})(35\,ft)^2}{8} = 73{,}500^{lb-ft}$$

Allowable stress is the tabulated stress multiplied by all applicable adjustment factors, which would be C_F. The charts provided say that C_F has been included in the section modulus. If we didn't have a chart that included C_F and we don't know the depth, we could guess - say 18 inches:

$C_F = \left(\frac{12}{d}\right)^{1/9} = \left(\frac{12}{18}\right)^{1/9} = 0.956$ (< 1) which would need to be multiplied with all the other adjustment factors by F_b to find F'_b

$$S_{req'd} \ge \frac{M}{F'_b} = \frac{73{,}500^{lb-ft}}{2400\,{}^{lb}\!/\!_{in^2}} \cdot (12\,{}^{in}\!/\!_{ft}) = 367.5in^3$$

No information is available to evaluate shear or deflection. Based on that, try a 5 1/8 x 22.5. It has a smaller area than the 8 ¾ section with a big enough adjusted S. (Real S = 5.125x22.5²/6 = 432.42 in³, C_F = 0.932, $S_{adjusted}$ = 403.2 in³)

Check self weight: $=\gamma \cdot A = 32\,{}^{lb}\!/\!_{ft^3}\,(115.3in^2)\left(\frac{1ft}{12in}\right)^2 = 26\,{}^{lb}\!/\!_{ft}$ which is less than what was used.

We could try a smaller section, which would mean calculating a new self weight, then moment, then $S_{req'd}$ and comparing S_{actual} to $S_{req'd}$.

The lower self weight means a lower design moment, but the smaller C_F means a smaller allowed stress, so we might end up with the same section.

$w_{revised}$ = 480 lb/ft + (26-40 lb/ft), $M_{revised}$ = 71,356 lb-ft , $S_{req'd\ now}$ = 356.8 in³ and the 5 1/8 x 22.5 is the choice for bending.

Of course, we need to satisfy shear and deflection criteria as well.

DEPTH, d in.	AREA, A in.²	MODIFIED SECTION MODULUS, S_{CF} in.³	MOMENT OF INERTIA, I in.⁴
3⅛" WIDTH			
6.0	18.8	18.8	56
7.5	23.4	29.3	110
9.0	28.1	42.2	190
10.5	32.8	57.4	302
12.0	37.5	75.0	450
13.5	42.2	93.7	641
15.0	46.9	114.3	879
16.5	51.6	136.9	1,170
18.0	56.3	161.3	1,519
19.5	60.9	187.6	1,931
21.0	65.6	215.8	2,412
22.5	70.3	245.9	2,966
24.0	75.0	277.3	3,600
5⅛" WIDTH			
7.5	38.4	48.0	180
9.0	46.1	69.2	311
10.5	53.8	94.2	494
12.0	61.5	123.0	738
13.5	69.2	153.6	1,051
15.0	76.9	187.5	1,441
16.5	84.6	224.5	1,919
18.0	92.3	264.6	2,491
19.5	99.9	307.7	3,167
21.0	107.6	354.0	3,955
22.5	115.3	403.2	4,865
24.0	123.0	455.5	5,904
25.5	130.7	510.8	7,082
27.0	138.4	569.0	8,406
28.5	146.1	630.2	9,887
30.0	153.8	694.3	11,531
31.5	161.4	761.4	13,349
33.0	169.1	831.3	15,348
34.5	176.8	904.1	17,538
36.0	184.5	979.8	19,926
6¾" WIDTH			
12.0	81.0	162.0	972
13.5	91.1	202.4	1,384
15.0	101.3	246.9	1,898
16.5	111.4	295.6	2,527
18.0	121.5	348.4	3,280
19.5	131.6	405.3	4,171
21.0	141.8	466.2	5,209
22.5	151.9	531.1	6,407

Example 2

Example Problem 10.20:
Design of Wood Columns(Figure 10.66)

A 22'-tall glu-lam column is required to support a roof load (including snow) of 40 kips. Assuming $8\frac{3}{4}$" in one dimension (to match the beam width above), determine the minimum column size if the top and bottom are pin supported.

Select from the following sizes:

$8\frac{3}{4}" \times 9"$ ($A = 78.75$ in.^{2})

$8\frac{3}{4}" \times 10\frac{1}{2}"$ ($A = 91.88$ in.^{2})

$8\frac{3}{4}" \times 12"$ ($A = 105.00$ in.^{2})

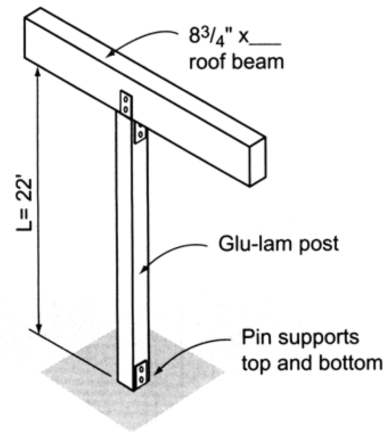


Figure 10.66 Glu-lam column design.

Solution:

Glu-lam column: ($F_c = 1650$ psi, $E = 1.8 \times 10^6$ psi)

Try $8\frac{3}{4}" \times 10\frac{1}{2}"$ ($A = 105.00$ in.^{2})

$$\frac{L_e}{d} = \frac{(22' \times 12\text{ in./ft.})}{8.75\text{ in.}}$$

$$= 30.2 < 50 \text{ (max. slenderness ratio)}$$

$$F_{cE} = \frac{0.418E}{(L_e/d)^2} = \frac{0.418(1.8 \times 10^6\text{ lb./in.}^2)}{(30.2)^2} = 825\text{ psi}$$

$$F_c^* \cong F_cC_D = (1650\text{ psi}) \times \underset{\text{(snow)}}{(1.15)} = 1900\text{ psi}$$

$$\frac{F_{cE}}{F_c^*} = \frac{825}{1900} = 0.43$$

From Appendix Table 14: $C_p = 0.403$

$$F_c' = F_c^*C_p = (1900\text{ lb./in.}^2) \times (0.403) = 765\text{ psi}$$

$$P_a = F_c' \times A = (765\text{ lb./in.}^2) \times (91.9\text{ in.}^2)$$

$$= 70{,}300\text{ lb.} > 40{,}000\text{ lb.}$$

Cycle again, trying a smaller, more economical section. Try $8\frac{3}{4}" \times 9"$ ($A = 78.8$ in.^{2})

Since the critical dimension is still $8\frac{3}{4}$", the values for F_{cE}, F_c^*, and F_c' all remain the same as in trial 1. The only change that affects the capability of the column is the available cross-sectional area.

$$\therefore P_a = F_c' \times A = (765\text{ lb./in.}^2) \times (78.8\text{ in.}^2)$$

$$= 60{,}300\text{ lb.}$$

$$P_a = 60.3\text{ k} > 40\text{ k}$$

Use $8\frac{3}{4}" \times 9"$ glu-lam section.