

## Case Study in Reinforced Concrete

adapted from Simplified Design of Concrete Structures, James Ambrose, 7<sup>th</sup> ed.

### Building description

The building is a three-story office building intended for speculative rental. Figure 17.37 presents a full-building section and a plan of the upper floor. The exterior walls are permanent. The design is a rigid perimeter frame to resist lateral loads.

### Loads (UBC 1994)

#### *Live Loads:*

Roof:

20 lb/ft<sup>2</sup>

Floors:

Office areas: 50 lb/ft<sup>2</sup> (2.39 kPa)

Corridor and lobby: 100 lb/ft<sup>2</sup> (4.79 kPa)

Partitions: 20 lb/ft<sup>2</sup> (0.96 kPa)

*Wind:* map speed of 80 mph (190 km/h);  
exposure B

#### *Assumed Construction Loads:*

Floor finish: 5 lb/ft<sup>2</sup> (0.24 kPa)

Ceilings, lights, ducts: 15 lb/ft<sup>2</sup> (0.72 kPa)

Walls (average surface weight):

Interior, permanent: 10 lb/ft<sup>2</sup> (0.48 kPa)

Exterior curtain wall: 15 lb/ft<sup>2</sup> (0.72 kPa)

### Materials

Use  $f'_c = 3000$  psi (20.7 MPa) and  
grade 60 reinforcement ( $f_y = 60$  ksi or 414 MPa).

### Structural Elements/Plan

**Case 1** is shown in Figure 17.44 and consists of a flat plate supported on interior beams, which in turn, are supported on girders supported by columns. We will examine the slab, and a four-span interior beam.

**Case 2** will consider the bays with flat slabs, no interior beams with drop panels at the columns and an exterior rigid frame with spandrel (edge) beams. An example of an edge bay is shown to the right. We will examine the slab and the drop panels.

For both cases, we will examine the exterior frames in the 3-bay direction.

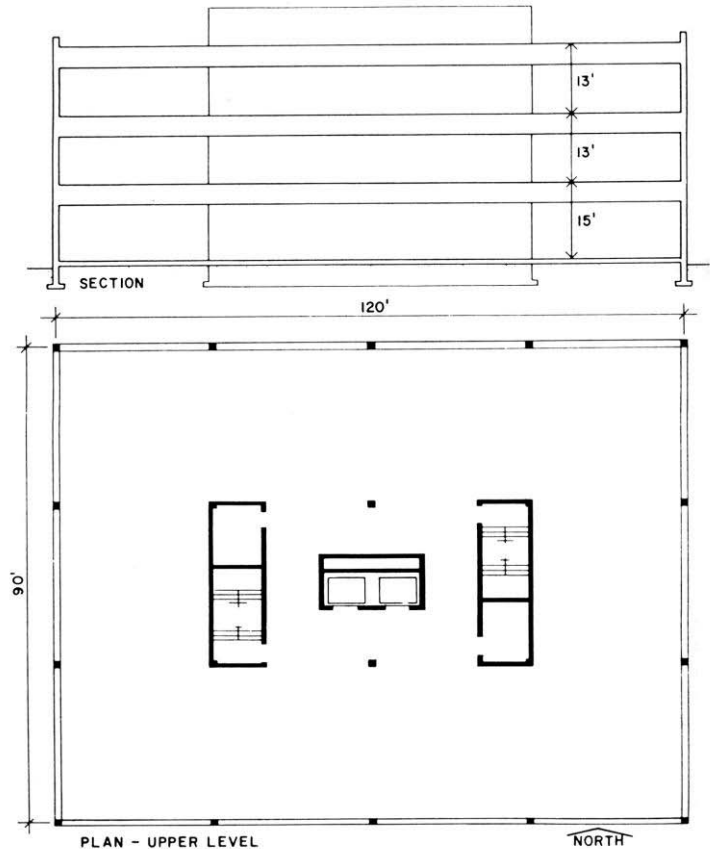
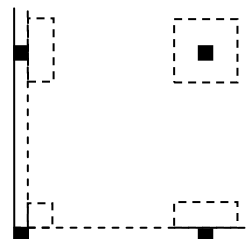


FIGURE 17.37 Building Five: General form.



**Case 1:**

Slab:

The slabs are effectively 10 ft x 30 ft, with an aspect ratio of 3, making them one-way slabs. Minimum depths (by ACI) are a function of the span. Using Table 3-1 for one way slabs the minimum is  $\frac{l_n}{24}$  with 5 inches minimum for fire rating. We'll presume the interior beams are 12" wide, so

$$l_n = 10 \text{ ft} - 1 \text{ ft} = 9 \text{ ft}$$

$$\text{minimum } t \text{ (or } h) = \frac{9 \text{ ft} \cdot 12 \text{ in/ft}}{24} = 4.5 \text{ in}$$

Use 5 in.

$$\text{dead load from slab} = \frac{150 \text{ lb/ft}^3 \cdot 5 \text{ in}}{12 \text{ in/ft}} = 62.5 \text{ lb/ft}^2$$

total dead load = (5 + 15 + 62.5) lb/ft<sup>2</sup> + 2" of lightweight concrete topping with weight of 18 lb/ft<sup>2</sup> (0.68 KPa) (presuming interior wall weight is over beams & girders)

$$\text{dead load} = 100.5 \text{ lb/ft}^2$$

$$\text{live load (worst case in corridor)} = 100 \text{ lb/ft}^2$$

total factored distributed load (ASCE-7) of 1.2D+1.6L:

$$w_u' = 1.2(100.5 \text{ lb/ft}^2) + 1.6(100 \text{ lb/ft}^2) = 280.6 \text{ lb/ft}^2$$

**Maximum Positive Moments** from Figure 2-3, end span (integral with support) for a 1 ft wide strip:

$$M_u \text{ (positive)} = \frac{w_u \ell_n^2}{14} = \frac{w_u' \cdot 1 \text{ ft} \cdot \ell_n^2}{14} = \frac{(280.6 \text{ lb/ft}^2)(1 \text{ ft})(9 \text{ ft})^2}{14} \cdot \frac{1 \text{ k}}{1000 \text{ lb}} = 1.62 \text{ k-ft}$$

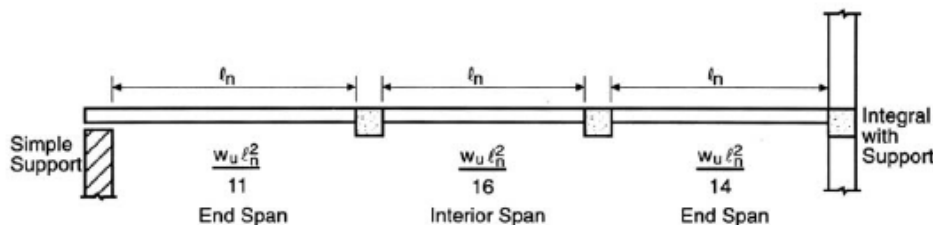


Figure 2-3 Positive Moments—All Cases

Member	Minimum thickness, <i>h</i>			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.				
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

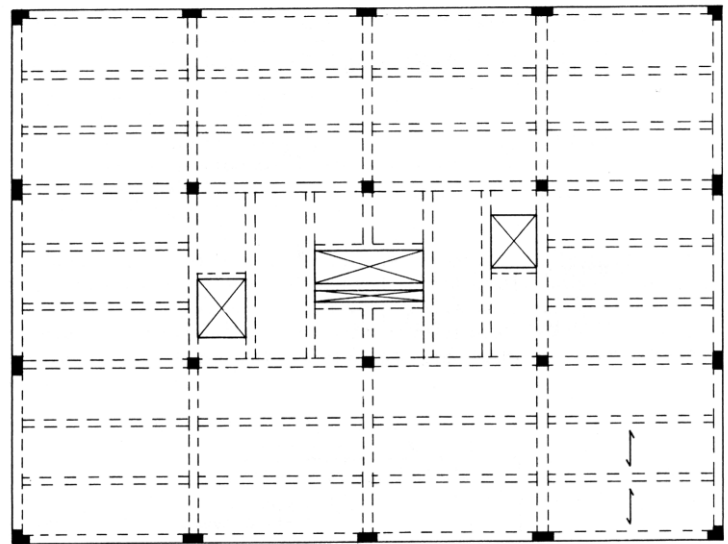


FIGURE 17.44 Building Five: Framing plan for the concrete structure for the upper floor.

**Maximum Negative Moments** from Figure 2-5, end span (integral with support) for a **1 ft wide strip**:

$$M_{u(-negative)} = \frac{w_u \ell_n^2}{12} = \frac{w_u \cdot 1ft \cdot \ell_n^2}{12} = \frac{(280.6^{lb/ft^2})(1ft)(9ft)^2}{12} \cdot \frac{1k}{1000lb} = 1.89 \text{ k-ft}$$

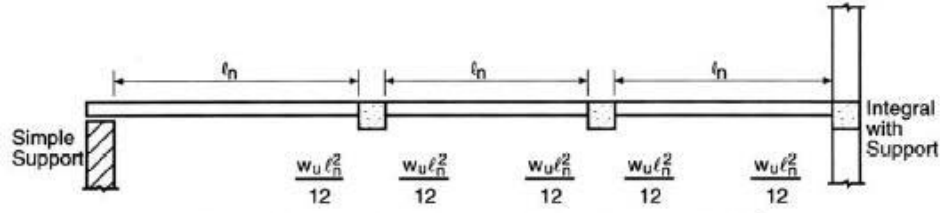


Figure 2-5 Negative Moments—Slabs with spans  $\leq 10$  ft

The design aid (Figure 3.8.1) can be used to find the reinforcement ratio,  $\rho$ , knowing  $R_n = M_n/bd^2$  with  $M_n = M_u/\phi_f$ , where  $\phi_f = 0.9$ . We can presume the effective depth to the centroid of the reinforcement,  $d$ , is 1.25" less than the slab thickness (with 3/4" cover and 1/2 of a bar diameter for a #8 (1") bar) = 3.75".

$$R_n = \frac{1.89^{k-ft}}{(0.9)(12^{in})(3.75^{in})^2} \cdot 12^{in/ft} \cdot 1000^{lb/k} = 149.3 \text{ psi}$$

so  $\rho$  for  $f'_c = 3000$  psi and  $f_y = 60,000$  psi is the minimum. For slabs,  $A_s$  minimum is  $0.0018bt$  for grade 60 steel.

$$A_s = 0.0018(12in)(5in) = 0.108 \text{ in}^2/\text{ft}$$

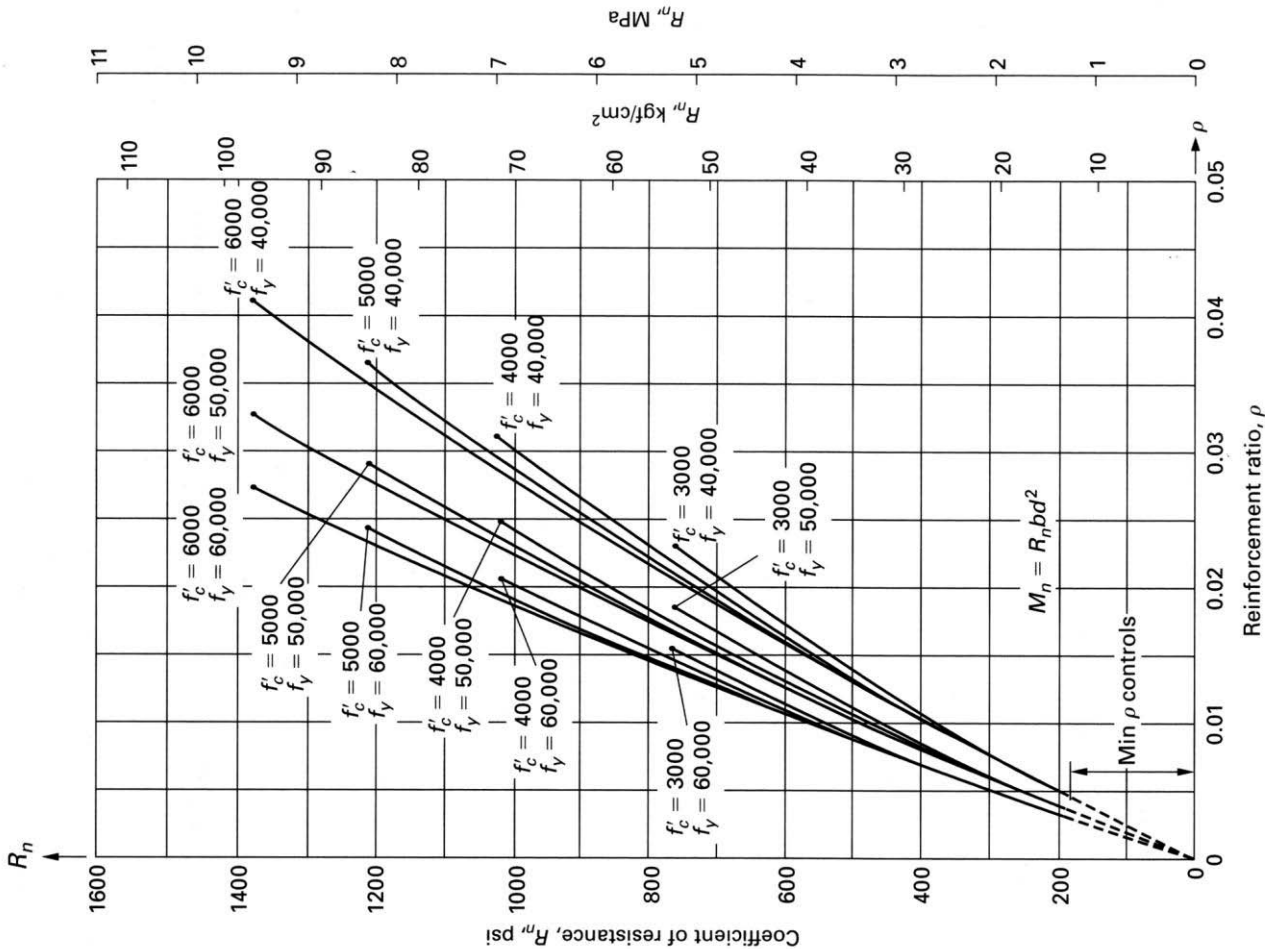


Figure 3.8.1 Strength curves ( $R_n$  vs  $\rho$ ) for singly reinforced rectangular sections. Upper limit of curves is at  $\rho_{max}$ .

Pick bars and spacing off Table 3-7. Use #3 bars @ 12 in ( $A_s = 0.11 \text{ in}^2$ ).

Table 3-7 Areas of Bars per Foot Width of Slab— $A_s$  ( $\text{in}^2/\text{ft}$ )

Bar size	Bar spacing (in.)												
	6	7	8	9	10	11	12	13	14	15	16	17	18
#3	0.22	0.19	0.17	0.15	0.13	0.12	0.11	0.10	0.09	0.09	0.08	0.08	0.07
#4	0.40	0.34	0.30	0.27	0.24	0.22	0.20	0.18	0.17	0.16	0.15	0.14	0.13
#5	0.62	0.53	0.46	0.41	0.37	0.34	0.31	0.29	0.27	0.25	0.23	0.22	0.21
#6	0.88	0.75	0.66	0.59	0.53	0.48	0.44	0.41	0.38	0.35	0.33	0.31	0.29
#7	1.20	1.03	0.90	0.80	0.72	0.65	0.60	0.55	0.51	0.48	0.45	0.42	0.40
#8	1.58	1.35	1.18	1.05	0.95	0.86	0.79	0.73	0.68	0.63	0.59	0.56	0.53
#9	2.00	1.71	1.50	1.33	1.20	1.09	1.00	0.92	0.86	0.80	0.75	0.71	0.67
#10	2.54	2.18	1.91	1.69	1.52	1.39	1.27	1.17	1.09	1.02	0.95	0.90	0.85
#11	3.12	2.67	2.34	2.08	1.87	1.70	1.56	1.44	1.34	1.25	1.17	1.10	1.04

Check the moment capacity.  $d$  is actually  $5 \text{ in} - 0.75 \text{ in (cover)} - \frac{1}{2} (3/8 \text{ in bar diameter}) = 4.06 \text{ in}$

$$a = A_s f_y / 0.85 f'_c b = 0.11 \text{ in}^2 (60 \text{ ksi}) / [0.85 (3 \text{ ksi}) 12 \text{ in}] = 0.22 \text{ in}$$

$$\phi M_n = \phi A_s f_y (d - a/2) = 0.9 (0.11 \text{ in}^2) (60 \text{ ksi}) (4.06 \text{ in} - \frac{0.22 \text{ in}}{2}) \cdot (\frac{1}{12 \text{ in/ft}}) = 1.96 \text{ k-ft} > 1.89 \text{ k-ft needed}$$

(OK)

**Maximum Shear:** Figure 2-7 shows end shear that is  $w_u l_n / 2$  except at the end span on the interior column which sees a little more and you must design for 15% increase:

$$V_{u-\text{max}} = 1.15 w_u l_n / 2 = \frac{1.15 (280.6 \text{ lb/ft}^2) (.1 \text{ ft}) (9 \text{ ft})}{2} = 1452 \text{ lb (for a 1 ft strip)}$$

$$V_u \text{ at } d \text{ away from the support} = V_{u-\text{max}} - w(d) = 1452 \text{ lb} - \frac{(280.6 \text{ lb/ft}^2) (.1 \text{ ft}) (4.06 \text{ in})}{12 \text{ in/ft}} = 1357 \text{ lb}$$

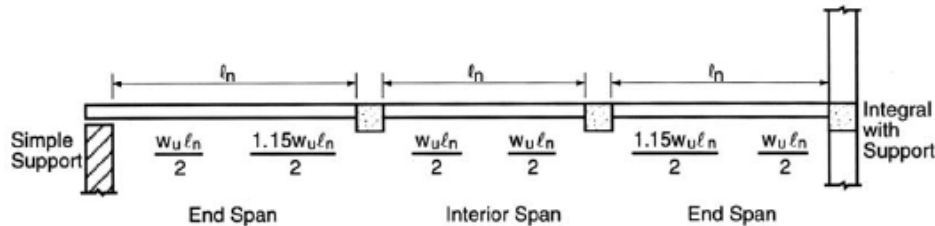


Figure 2-7 End Shears—All Cases

Check the one way shear capacity:  $\phi_v V_c = \phi_v 2 \sqrt{f'_c} b d$  ( $\phi_v = 0.75$ ):

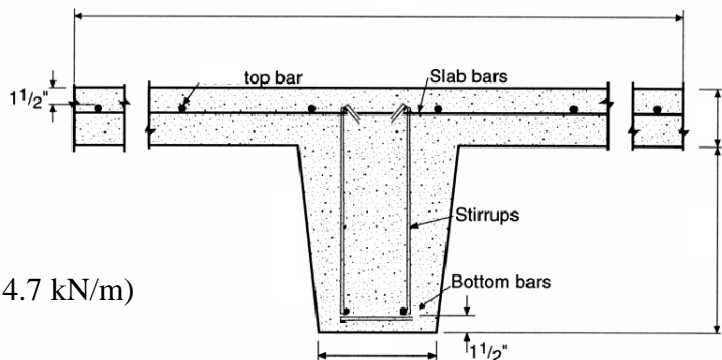
$$\phi_v V_c = 0.75 (2) \sqrt{3000 \text{ psi}} (12 \text{ in}) (4.06 \text{ in}) = 4003 \text{ lb}$$

Is  $V_u$  (needed)  $<$   $\phi_v V_c$  (capacity)? YES:  $1357 \text{ lb} \leq 4003 \text{ lb}$ , so we don't need to make the slab thicker.

Interior Beam (effectively a T-beam):

Tributary width = 10 ft for an interior beam.

$$\text{dead load} = (100.5 \text{ lb/ft}^2) (10 \text{ ft}) = 1005 \text{ lb/ft (14.7 kN/m)}$$



Reduction of live load is allowed, with an influence area,  $A_I$ , of 2 panels beside an interior beam, assuming the girder is 12" wide. The live load is 100 lb/ft<sup>2</sup>:

$$L = L_o \left( 0.25 + \frac{15}{\sqrt{A_I}} \right) = 100 \frac{\text{lb}}{\text{ft}^2} \left( 0.25 + \frac{15}{\sqrt{(30\text{ft} - 1\text{ft})(2 \times 10\text{ft})}} \right) = 87.3 \text{ lb/ft}^2$$

(Reduction Multiplier = 0.873)

$$\text{live load} = 87.3 \text{ lb/ft}^2 (10\text{ft}) = 873 \text{ lb/ft} \quad (12.7 \text{ kN/m})$$

Estimating a 12" wide x 24" deep beam means the additional dead load from self weight ( $w = \gamma \cdot A$  in units of load/length) can be included. The top 5 inches of slab has already been included in the dead load:

$$\text{dead load from self weight} = 150 \frac{\text{lb}}{\text{ft}^3} (12\text{in wide})(24 - 5\text{in deep}) \cdot \left( \frac{1\text{ft}}{12\text{in}} \right)^2 = 237.5 \text{ lb/ft} \quad (3.46 \text{ kN/m})$$

$$w_u = 1.2(1005 \text{ lb/ft} + 237.5 \text{ lb/ft}) + 1.6(873 \text{ lb/ft}) = 2888 \text{ lb/ft} \quad (4.30 \text{ kN/m})$$

The effective width,  $b_E$ , of the T part is the smaller of  $\frac{\ell_n}{4}$ ,  $b_w + 16t$ , or center-center spacing

$$b_E = \text{minimum} \{ 29 \text{ ft} / 4 = 7.25 \text{ ft} = 87 \text{ in}, 12 \text{ in} + 16 \times 5 \text{ in} = 92 \text{ in}, 10 \text{ ft} = 120 \text{ in} \} = 87 \text{ in}$$

The clear span for the beam is

$$\ell_n = 30 \text{ ft} - 1 \text{ ft} = 29 \text{ ft}$$

**Maximum Positive Moments** from Figure 2-3, end span (integral with support):

$$M_u \text{ (positive)} = \frac{w_u \ell_n^2}{14} = \frac{2888 \text{ lb/ft} (29\text{ft})^2}{14} \cdot \frac{1\text{k}}{1000\text{lb}} = 173.5 \text{ k-ft}$$

**Maximum Negative Moments** from Figure 2-4, end span (integral with support):

$$M_u \text{ (negative)} = \frac{w_u \ell_n^2}{10} = \frac{2888 \text{ lb/ft} (29\text{ft})^2}{10} \cdot \frac{1\text{k}}{1000\text{lb}} = 242.9 \text{ k-ft}$$

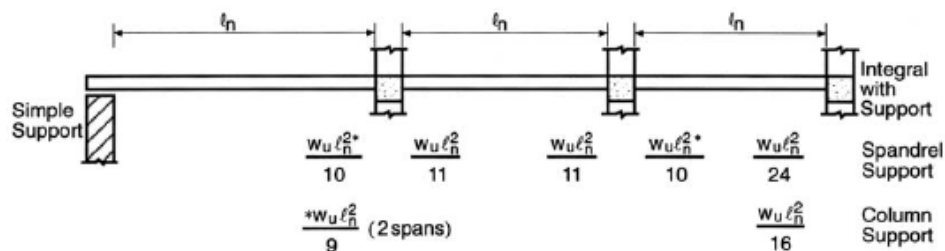


Figure 2-4 Negative Moments—Beams and Slabs

Figure 3.8.1 can be used to find an approximate  $\rho$  for top reinforcement if  $R_n = M_n/bd^2$  and we set  $M_n = M_u/\phi_f$ . We can presume the effective depth is 2.5" less than the 24" depth (for 1.5" cover and 1/2 bar diameter for a #10 (10/8)" bar + #3 stirrups (3/8" more)), so  $d = 21.5"$ .

$$R_n = \frac{1000^{lb/k} \cdot 242.9^{k-ft}}{0.9 \cdot (12^{in})(21.5^{in})^2} \cdot 12^{in/ft} = 584 \text{ psi}$$

so  $\rho$  for  $f'_c = 3000$  psi and  $f_y = 60,000$  psi is about 0.011

Then we pick bars and spacing off Table 3-7 to fit in the effective flange width in the slab.

For bottom reinforcement (positive moment) the effective flange is so wide at 87 in, that it resists a lot of compression, and needs very little steel to stay under-reinforced ( $a$  is between 0.6" and 0.5"). We'd put in bottom bars at the minimum reinforcement allowed and for tying the stirrups to.

**Maximum Shear:**  $V_{max} = w_u l/2$  normally, but the end span sees a little more and you must design for 15% increase. But for beams, we can use the lower value of  $V$  that is a distance of  $d$  from the face of the support

$$V_{u\text{-design}} = 1.15w_u l_n/2 - w_u d = \frac{1.15(2888^{lb/ft})(29^{ft})}{2} - \frac{2888^{lb/ft}(21.5^{in})}{12^{in/ft}} = 42,983 \text{ lb} = 43.0 \text{ k}$$

Check the one way shear capacity =  $\phi_v V_c = \phi_v 2 \sqrt{f'_c} bd$ , where  $\phi_v = 0.75$

$$\phi_v V_c = 0.75(2)\sqrt{3000} \text{ psi}(12^{in})(21.5^{in}) = 21,197 \text{ lb} = 21.2 \text{ k}$$

Is  $V_u$  (needed) <  $\phi_v V_c$  (capacity)?

NO: 43.0 k is greater than 21.2 k, so stirrups are needed

$$\phi_v V_s = V_u - \phi_v V_c = 43.0 \text{ k} - 21.2 \text{ k} = 21.8 \text{ k} \text{ (max needed)}$$

Using #3 bars (typical) with two legs means  $A_v = 2(0.11 \text{ in}^2) = 0.22 \text{ in}^2$ .

To determine required spacing, use Table 3-8. For  $d = 21.5"$  and  $\phi_v V_s \leq \phi_v 4 \sqrt{f'_c} bd$  (where  $\phi_v 4 \sqrt{f'_c} bd = 2\phi_v V_c = 2(21.2 \text{ k}) = 42.4 \text{ k}$ ), the maximum spacing is  $d/2 = 10.75 \text{ in.}$  or 24".

$$S_{\text{required}} = \frac{\phi A_v f_y d}{V_u - \phi V_c} = \frac{\phi A_v f_y d}{\phi V_s} = \frac{0.75 \cdot 0.22 \text{ in}^2 \cdot 60 \text{ ksi} \cdot 21.5 \text{ in}}{21.8 \text{ k}} = 9.75 \text{ in.}, \text{ so use } 9 \text{ in.}$$

We would try to increase the spacing as the shear decreases, but it is a tedious job. We need stirrups anywhere that  $V_u > \phi_v V_c/2$ . One recommended intermediate spacing is  $d/3$ .

Table 3-8 ACI Provisions for Shear Design\*

		$V_u \leq \frac{\phi V_c}{2}$	$\phi V_c \geq V_u > \frac{\phi V_c}{2}$	$V_u > \phi V_c$
Required area of stirrups, $A_v$ **		none	$\frac{50b_w s}{f_y}$	$\frac{(V_u - \phi V_c)s}{\phi f_y d}$
Stirrup spacing, s	Required	—	$\frac{A_v f_y}{50b_w}$	$\frac{\phi A_v f_y d}{V_u - \phi V_c}$
	Recommended Minimum†	—	—	4 in.
	Maximum†† (ACI 11.5.4)	—	$\frac{d}{2}$ or 24 in.	$\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \leq \phi 4\sqrt{f'_c} b_w d$ $\frac{d}{4}$ or 12 in. for $(V_u - \phi V_c) > \phi 4\sqrt{f'_c} b_w d$

\*Members subjected to shear and flexure only;  $\phi V_c = \phi 2\sqrt{f'_c} b_w d$ ,  $\phi = 0.75$  (ACI 11.3.1.1)

\*\* $A_v = 2 \times A_b$  for U stirrups;  $f_y \leq 60$  ksi (ACI 11.5.2)

†A practical limit for minimum spacing is  $d/4$

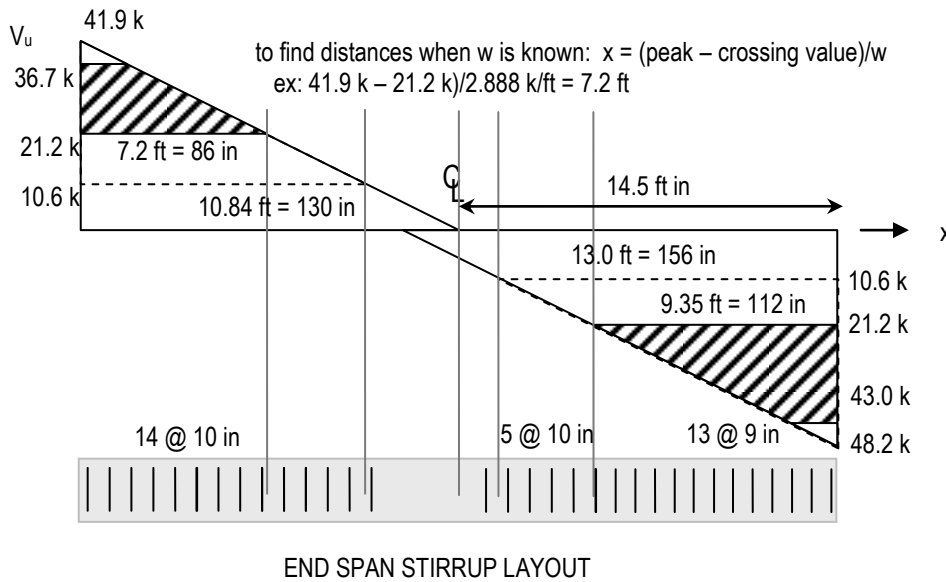
††Maximum spacing based on minimum shear reinforcement ( $= A_v f_y / 50b_w$ ) must also be considered (ACI 11.5.5.3).

The required spacing where stirrups are needed for crack control ( $\phi V_c \geq V_u > \frac{1}{2}\phi V_c$ ) is

$$s_{\text{required}} = \frac{A_v f_y}{50b_w} = \frac{0.22 \text{ in}^2 (60,000 \text{ psi})}{50(12 \text{ in})} = 22 \text{ in}$$

and the maximum spacing is  $d/2 = 10.75$  in. or 24". Use 10 in.

A recommended minimum spacing for the first stirrup is 2 in. from the face of the support. A distance of one half the spacing near the support is often used.



Spandrel Girders:

Because there is a concentrated load on the girder, the approximate analysis can't technically be used. If we converted the maximum moment (at midspan) to an equivalent distributed load by setting it equal to  $w_u l^2 / 8$  we would then use:

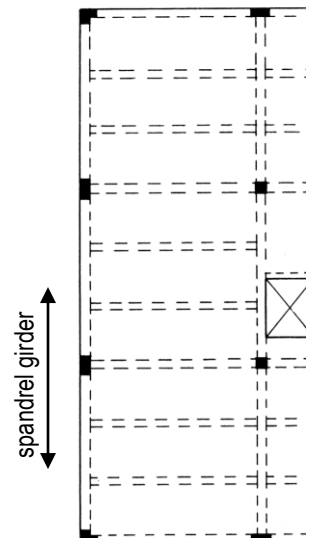


FIGURE 17.44 Building F floor

**Maximum Positive Moments** from Figure 2-3, end span (integral with support):

$$M_{u+} = \frac{w_u \ell_n^2}{14}$$

**Maximum Negative Moments** from Figure 2-4, end span (column support):

$$M_{u-} = \frac{w_u \ell_n^2}{10} \text{ (with } \frac{w_u \ell_n^2}{16} \text{ at end)}$$

Column:

An exterior or corner column will see axial load and bending moment. We'd use interaction charts for  $P_u$  and  $M_u$  for standard sizes to determine the required area of steel. An interior column sees very little bending. The axial loads come from gravity. The factored load combination is  $1.2D+1.6L + 0.5L_r$ .

The girder weight, presuming 1' x 4' girder at  $150 \text{ lb/ft}^3 = 600 \text{ lb/ft}$

Top story: presuming  $20 \text{ lb/ft}^2$  roof live load, the total load for an interior column (tributary area of  $30' \times 30'$ ) is:

DL <sub>roof*</sub> :	$1.2 \times 100.5 \text{ lb/ft}^2 \times 30 \text{ ft} \times 30 \text{ ft}$	= 108.5 k
	* assuming the same live load and materials as the floors	
DL <sub>beam</sub>	$1.2 \times 237.5 \text{ lb/ft} \times 30 \text{ ft} \times 3 \text{ beams}$	= 25.6 k
DL <sub>girder</sub>	$1.2 \times 600 \text{ lb/ft} \times 30 \text{ ft}$	= 21.6 k
LL <sub>r</sub> :	$0.5 \times 20 \text{ lb/ft}^2 \times 30 \text{ ft} \times 30 \text{ ft}$	= 9.0 k
Total		= 164.7 k

Lower stories:

DL <sub>floor</sub> :	$1.2 \times 100.5 \text{ lb/ft}^2 \times 30 \text{ ft} \times 30 \text{ ft}$	= 108.5 k
DL <sub>beam</sub>	$1.2 \times 237.5 \text{ lb/ft} \times 30 \text{ ft} \times 3 \text{ beams}$	= 25.6 k
DL <sub>girder</sub>	$1.2 \times 600 \text{ lb/ft} \times 30 \text{ ft}$	= 21.6 k
LL <sub>floor</sub> :	$1.6 \times (0.873) \times 100 \text{ lb/ft}^2 \times 30 \text{ ft} \times 30 \text{ ft}$	= 125.7 k (includes reduction)
Total		= 281.4 k

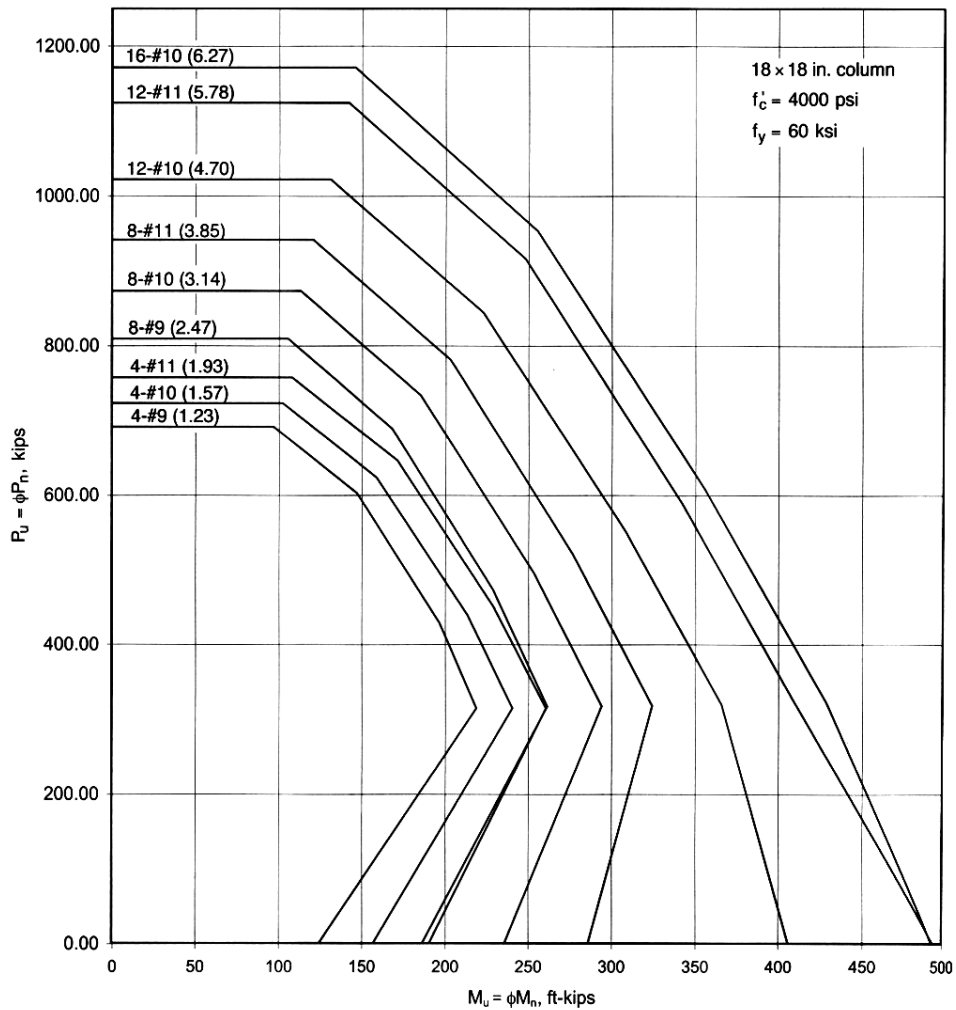
2<sup>nd</sup> floor column sees  $P_u = 164.7 + 281.4 = 446.1 \text{ k}$

1<sup>st</sup> floor column sees  $P_u = 446.1 + 281.4 = 727.5 \text{ k}$

Look at the example interaction diagram for an 18" x 18" column (Figure 5-20 – ACI 318-02) using  $f'_c = 4000 \text{ psi}$  and  $f_y = 60,000 \text{ psi}$  for the first floor having  $P_u = 727.5 \text{ k}$ , and  $M_u$  to the column being approximately 10% of the beam negative moment =  $0.1 \times 242.9 \text{ k-ft} = 24.3 \text{ k-ft}$ : (See maximum negative moment calculation for an interior beam.) The chart indicates the capacity for the reinforcement amounts shown by the solid lines.

For  $P_u = 727.5 \text{ k}$  and  $M_u = 24.3 \text{ k-ft}$ , the point plots below the line marked 4-#10 (1.57% area of steel to an 18 in x 18 in area).





Lateral Force Design:

The wind loads from the wind speed, elevation, and exposure we'll accept as shown in Figure 17.42 given on the left in psf. The wind is acting on the long side of the building. The perimeter frame resists the lateral loads, so there are two with a tributary width of  $\frac{1}{2} [(30\text{ft}) \times (4 \text{ bays}) + 2\text{ft}]$  for beam widths and cladding] =  $122\text{ft}/2 = 61\text{ft}$

The factored combinations with dead and wind load are:

$$1.2D + 1.6L_r + 0.5W$$

$$1.2D + 1.0W + L + 0.5L_r$$

The tributary height for each floor is half the distance to the next floor (top and bottom):

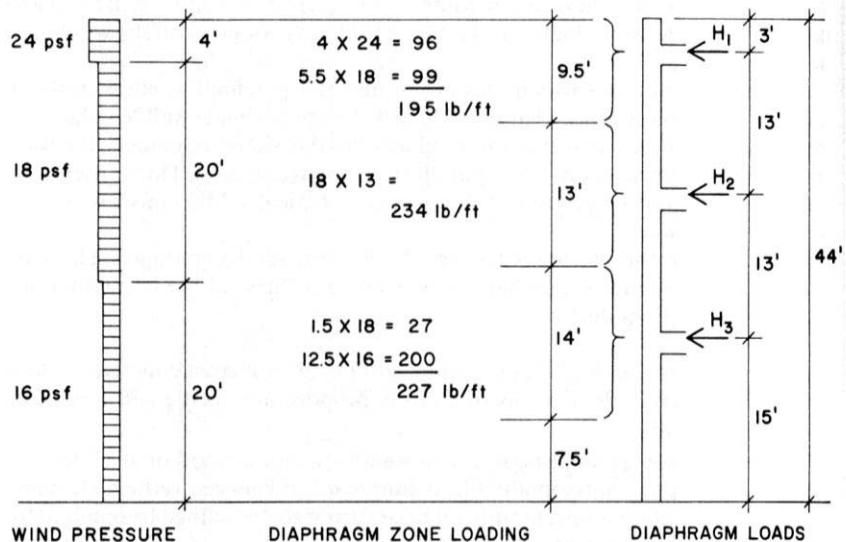


FIGURE 17.42 Building Five: How wind loads affect the lateral bracing system.

Exterior frame (bent) loads:

$$H_1 = 195^{lb/ft} (61^{ft}) = 11,895 \text{ lb} = 11.9 \text{ k/bent}$$

$$H_2 = \frac{234^{lb/ft} (61^{ft})}{1000^{lb/k}} = 14.3 \text{ k/bent}$$

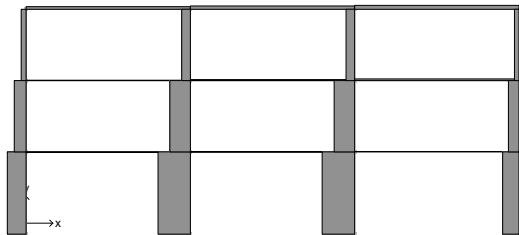
$$H_3 = \frac{227^{lb/ft} (61^{ft})}{1000^{lb/k}} = 13.8 \text{ k/bent}$$

Using Multiframe4D, the axial force, shear and bending moment diagrams can be determined using the load combinations, and the largest moments, shear and axial forces for each member determined.

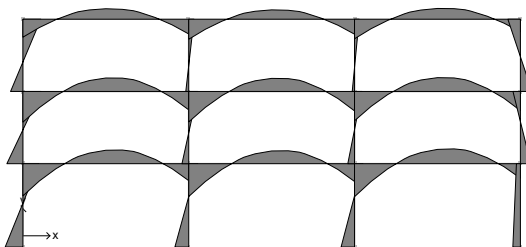
M = 218.8 k-ft V = 45.4 k P = 26.6 k	M = 237.8 k-ft V = 45.5 k P = 28.9 k	M = 236.3 k-ft V = 46.6 k P = 30.8 k	M = 183.9 k-ft V = 24.8 k P = 53.5 k
M = 203.7 k-ft V = 26.6 k P = 54.7 k	M = 39.3 k-ft V = 4.7 k P = 91.0 k	M = 32.4 k-ft V = 3.7 k P = 91.0 k	M = 142.7 k-ft V = 21.2 k P = 120.3 k
M = 332.4 -ft V = 60.5 k P = 1.9 k	M = 70.2 k-ft V = 9.2 k P = 215.5 k	M = 330.9 k-ft V = 60.0 k P = 6.4 k	M = 317.9 k-ft V = 59.7 k P = 10.4 k
M = 190.8 k-ft V = 28.0 k P = 121.6 k	M = 338.5 k-ft V = 60.9 k P = 6.5 k	M = 349.5 k-ft V = 61.0 k P = 4.1 k	M = 347.9 k-ft V = 62.0 k P = 1.6 k
M = 165.1 k-ft V = 21.6 k P = 192.8 k	M = 104.4 k-ft V = 11.6 k P = 340.6 k	M = 95.5 k-ft V = 10.3 k P = 341.7 k	M = 102.6 k-ft V = 8.5 k P = 185.7 k

(This is the summary diagram of force, shear and moment magnitudes refer to the maximum values in the column or beams, with the maximum moment in the beams being negative over the supports, and the maximum moment in the columns being at an end.)

*Axial force diagram:*

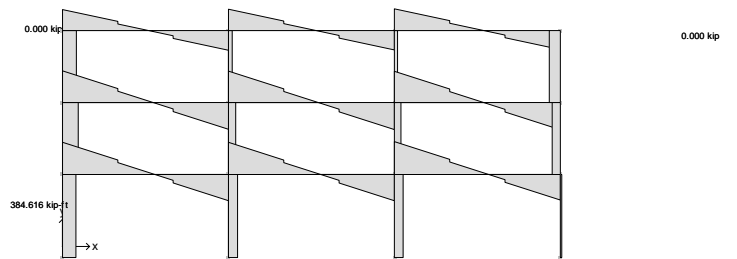


*Bending moment diagram:*

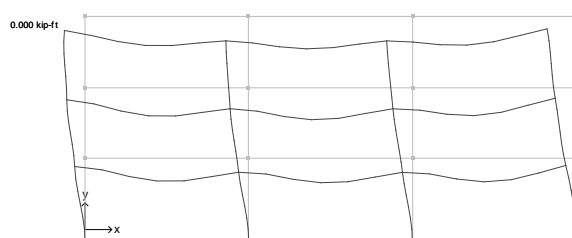


342.078 kip 64.549 kip

*Shear diagram:*



*Displacement:*



*Beam-Column loads for design:*

The bottom exterior columns see the largest bending moment on the lee-ward side (left):

$$P_u = 192.8 \text{ k and } M_u = 165.1 \text{ k-ft (with large axial load)}$$

The interior columns see the largest axial forces:

$$P_u = 341.7 \text{ k and } M_u = 95.5 \text{ k-ft and } P_u = 340.6 \text{ k and } M_u = 104.4 \text{ k-ft}$$

Refer to an interaction diagram for column reinforcement and sizing.

**Case 2**

Slab:

The slabs are effectively 30 ft x 30 ft, making them two-way slabs. Minimum thicknesses (by ACI) are a function of the span. Using Table 4-1 for two way slabs, the minimum is the larger of  $l_n/36$  or 4 inches. Presuming the columns are 18" wide,  $l_n = 30 \text{ ft} - (18 \text{ in}) / (12 \text{ ft/in}) = 28.5 \text{ ft}$ ,

$$h = l_n/36 = (28.5 \times 12) / 36 = 9.5 \text{ in}$$

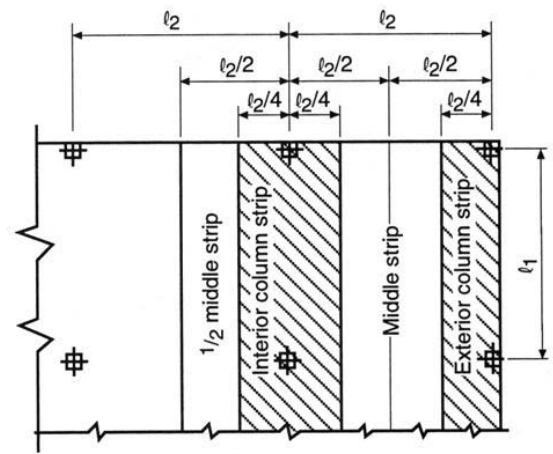
Table 4-1 Minimum Thickness for Two-Way Slab Systems

Two-Way Slab System	$\alpha_m$	$\beta$	Minimum h
Flat Plate	—	$\leq 2$	$l_n/30$
Flat Plate with Spandrel Beams <sup>1</sup>	[Min. h = 5 in.]	$\leq 2$	$l_n/33$
Flat Slab <sup>2</sup>	—	$\leq 2$	$l_n/33$
Flat Slab <sup>2</sup> with Spandrel Beams <sup>1</sup>	[Min. h = 4 in.]	$\leq 2$	$l_n/36$
Two-Way Beam-Supported Slab <sup>3</sup>	$\leq 0.2$	$\leq 2$	$l_n/30$
	1.0	1	$l_n/33$
	$\geq 2.0$	2	$l_n/36$
		1	$l_n/37$
Two-Way Beam-Supported Slab <sup>1,3</sup>	$\leq 0.2$	$\leq 2$	$l_n/33$
	1.0	1	$l_n/36$
	$\geq 2.0$	2	$l_n/40$
		1	$l_n/41$
		2	$l_n/49$

<sup>1</sup>Spandrel beam-to-slab stiffness ratio  $\alpha \geq 0.8$  (ACI 9.5.3.3)

<sup>2</sup>Drop panel length  $\geq \ell/3$ , depth  $\geq 1.25h$  (ACI 13.4.7)

<sup>3</sup>Min. h = 5 in. for  $\alpha_m \leq 2.0$ ; min. h = 3.5 in. for  $\alpha_m > 2.0$  (ACI 9.5.3.3)



(a) Column strip for  $l_2 \leq l_1$

The table also says the drop panel needs to be  $l/3$  long =  $28.5 \text{ ft} / 3 = 9.5 \text{ ft}$ , and that the minimum depth must be  $1.25h = 1.25(9.5 \text{ in}) = 12 \text{ in}$ .

For the strips,  $l_2 = 30 \text{ ft}$ , so the interior column strip will be  $30 \text{ ft} / 4 + 30 \text{ ft} / 4 = 15 \text{ ft}$ , and the middle strip will be the remaining 15 ft.

$$\text{dead load from slab} = \frac{150^{lb/ft^3} \cdot 9.5^{in}}{12^{in/ft}} = 118.75 \text{ lb/ft}^2$$

total dead load = 5 + 15 + 118.75 lb/ft<sup>2</sup> + 2" of lightweight concrete topping @ 18 lb/ft<sup>2</sup> (0.68 KPa)  
(presuming interior wall weight is over beams & girders)

$$\text{total dead load} = 156.75 \text{ lb/ft}^2$$

live load with reduction, where the influence area,  $A_I$ , for two way slabs is one panel:

$$L = L_o \left( 0.25 + \frac{15}{\sqrt{A_I}} \right) = 100 \text{ lb/ft}^2 \left( 0.25 + \frac{15}{\sqrt{(30\text{ft})(30\text{ft})}} \right) = 75 \text{ lb/ft}^2$$

total factored distributed load:

$$w_u = 1.2(156.75 \text{ lb/ft}^2) + 1.6(75 \text{ lb/ft}^2) = 308.1 \text{ lb/ft}^2$$

total panel moment to distribute:

$$M_o = \frac{w_u l_2 l_n^2}{8} = \frac{308.1 \text{ lb/ft}^2 (30\text{ft})(28.5\text{ft})^2}{8} \cdot \frac{1\text{k}}{1000\text{lb}} = 938.4 \text{ k-ft}$$

*Column strip, end span:*

**Maximum Positive Moments** from Table 4-3, (flat slab with spandrel beams):

$$M_{u+} = 0.30M_o = 0.30 \cdot (938.4 \text{ k-ft}) = 281.5 \text{ k-ft}$$

**Maximum Negative Moments** from Table 4-3, (flat slab with spandrel beams):

$$M_{u-} = 0.53M_o = 0.53 \cdot (938.4 \text{ k-ft}) = 497.4 \text{ k-ft}$$

Table 4-3 Flat Plate or Flat Slab with Spandrel Beams

Slab Moments	End Span			Interior Span	
	1 Exterior Negative	2 Positive	3 First Interior Negative	4 Positive	5 Interior Negative
Total Moment	0.30 $M_o$	0.50 $M_o$	0.70 $M_o$	0.35 $M_o$	0.65 $M_o$
Column Strip	0.23 $M_o$	0.30 $M_o$	0.53 $M_o$	0.21 $M_o$	0.49 $M_o$
Middle Strip	0.07 $M_o$	0.20 $M_o$	0.17 $M_o$	0.14 $M_o$	0.16 $M_o$

Notes: (1) All negative moments are at face of support.

(2) Torsional stiffness of spandrel beams  $\beta_t \geq 2.5$ . For values of  $\beta_t$  less than 2.5, exterior negative column strip moment increases to  $(0.30 - 0.03\beta_t) M_o$ .

*Middle strip, end span:*

**Maximum Positive Moments** from Table 4-3, (flat slab with spandrel beams):

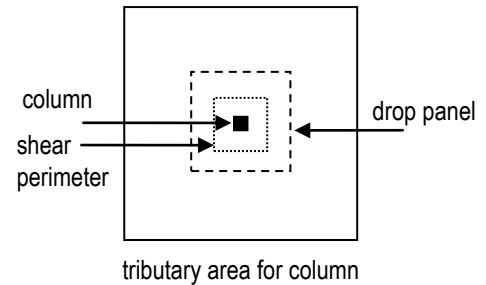
$$M_{u+} = 0.20M_o = 0.20 \cdot (938.4 \text{ k-ft}) = 187.9 \text{ k-ft}$$

**Maximum Negative Moments** from Table 4-3, (flat slab with spandrel beams):

$$M_{u-} = 0.17M_o = 0.17 \cdot (938.4 \text{ k-ft}) = 159.5 \text{ k-ft}$$

Design as for the slab in Case 1, but provide steel *in both directions* distributing the reinforcing needed by strips.

Shear around columns: The shear is critical at a distance  $d/2$  away from the column face. If the drop panel depth is 12 inches, the minimum  $d$  with two layers of 1" diameter bars would be  $12'' - 3/4''$  (cover)  $- (1'') - 1/2(1'')$  = about 9.75 in (to the top steel).



The shear resistance is  $\phi_v V_c = \phi_v 4 \sqrt{f'_c} b_o d$ ,  $\phi_v = 0.75$  where  $b_o$  is the perimeter length.

The design shear value is the distributed load over the tributary area *outside* the shear perimeter,  $V_u = w_u (\text{tributary area} - b_1 \times b_2)$  where  $b$ 's are the column width plus  $d/2$  each side.

$$b_1 = b_2 = 18'' + 9.75''/2 + 9.75''/2 = 27.75 \text{ in}$$

$$b_1 \times b_2 = (27.75 \text{ in})^2 \cdot \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)^2 = 5.35 \text{ ft}^2$$

$$V_u = (284.7 \text{ lb/ft}^2)(30 \text{ ft} \cdot 30 \text{ ft} - 4.97 \text{ ft}^2) \cdot \frac{1 \text{ k}}{1000 \text{ lb}} = 254.7 \text{ k}$$

Shear capacity:

$$b_o = 2(b_1) + 2(b_2) = 4(27.75 \text{ in}) = 111 \text{ in}$$

$$\phi_v V_c = 0.75 \cdot 4 \cdot \sqrt{3000} \text{ psi} \cdot 111 \text{ in} \cdot 9.75 \text{ in} = 177,832 \text{ lb} = 177.8 \text{ ksi} < V_u!$$

The shear capacity is not large enough. The options are to provide shear heads or a deeper drop panel, or change concrete strength, or even a different system selection...

There also is some transfer by the moment across the column into shear.

Deflections:

Elastic calculations for deflections require that the steel be turned into an equivalent concrete material using  $n = \frac{E_s}{E_c}$ .  $E_c$  can be measured or calculated with respect to concrete strength.

For normal weight concrete (150 lb/ft<sup>3</sup>):  $E_c = 57,000 \sqrt{f'_c}$

$$E_c = 57,000 \sqrt{3000} \text{ psi} = 3,122,019 \text{ psi} = 3122 \text{ ksi}$$

$$n = 29,000 \text{ psi} / 3122 \text{ ksi} = 9.3$$

Deflection limits are given in Table 9.5(b)

**TABLE 9.5(b) — MAXIMUM PERMISSIBLE COMPUTED DEFLECTIONS**

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load $L$	$\leq 180^*$
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load $L$	$\leq 360$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load) <sup>†</sup>	$\leq 480^{\ddagger}$
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\leq 240^{\S}$

\* Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

† Long-term deflection shall be determined in accordance with 9.5.2.5 or 9.5.4.2, but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

‡ Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.

§ Limit shall not be greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.