Examples:
Reinforced Concrete

Example 1

A simply supported beam 20 ft long carries a service dead load of 300 lb/ft and a live load of 500 lb/ft. Design an appropriate beam (for flexure only). Use grade 40 steel and concrete strength of 5000 psi.

SOLUTION:
Find the design moment, M_u, from the factored load combination of 1.2D + 1.6L. It is good practice to guess a beam size to include self weight in the dead load, because "service" means dead load of everything except the beam itself.

Guess a size of 10 in x 12 in. Self weight for normal weight concrete is the density of 150 lb/ft^3 multiplied by the cross section area: self weight = $150\, {}^{lb}\!/_{ft^3}\,(10in)(12in)\cdot(\frac{1ft}{12in})^2$ = 125 lb/ft

w_u = 1.2(300 lb/ft + 125 lb/ft) + 1.6(500 lb/ft) = 1310 lb/ft^2

The maximum moment for a simply supported beam is $\frac{wl^2}{8}$:

$M_u = \frac{w_u l^2}{8} = \frac{1310\, {}^{lb}\!/_{ft}\,(20\,ft)^2}{8}$ 65,500 lb-ft

M_n required = M_u/ϕ = $\frac{65{,}500^{lb-ft}}{0.9}$ = 72,778 lb-ft

To use the design chart aid, find $R_n = \frac{M_n}{bd^2}$, estimating that d is about 1.75 inches less than h:

Table 3.7.1
Total Areas for Various Numbers of Reinforcing Bars

Bar Size	Nominal Diameter (in.)	Weight (lb/ft)	Number of Bars 1	2	3	4	5	6	7	8	9	10
#3	0.375	0.376	0.11	0.22	0.33	0.44	0.55	0.66	0.77	0.88	0.99	1.10
#4	0.500	0.668	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
#5	0.625	1.043	0.31	0.62	0.93	1.24	1.55	1.86	2.17	2.48	2.79	3.10
#6	0.750	1.502	0.44	0.88	1.32	1.76	2.20	2.64	3.08	3.52	3.96	4.40
#7	0.875	2.044	0.60	1.20	1.80	2.40	3.00	3.60	4.20	4.80	5.40	6.00
#8	1.000	2.670	0.79	1.58	2.37	3.16	3.95	4.74	5.53	6.32	7.11	7.90
#9	1.128	3.400	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
#10	1.270	4.303	1.27	2.54	3.81	5.08	6.35	7.62	8.89	10.16	11.43	12.70
#11	1.410	5.313	1.56	3.12	4.68	6.24	7.80	9.36	10.92	12.48	14.04	15.60
#14[a]	1.693	7.65	2.25	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50
#18[a]	2.257	13.60	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00

[a] #14 and #18 bars are used primarily as column reinforcement and are rarely used in beams.

d = 12in – 1.75 in = 10.25 in

$R_n = \frac{72{,}778^{lb-ft}}{(10in)(10.25in)^2}\cdot(12\, {}^{in}\!/_{ft})$ = 831 psi

ρ corresponds to approximately 0.023, so the estimated area required, A_s, can be found:

$A_s = \rho bd$ = (0.023)(10in)(10.25in) = 2.36 in^2

The number of bars for this area can be found from handy charts.

(Whether the number of bars actually fit for the width with cover and space between bars must also be considered.)

Try A_s = 2.37 in^2 from 3#8 bars

d = 12 in – 1.5 in (cover) – ½ (8/8in diameter bar) = 10 in

Find the moment capacity of the beam as designed, ϕM_n

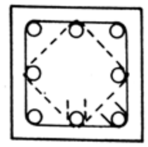
$a = A_s f_y / 0.85 f'_c b$ = 2.37 in^2 (40 ksi)/[0.85(5 ksi)10 in] = 2.23 in

$\phi M_n = \phi A_s f_y (d - a/2) = 0.9(2.37in^2)(40ksi)(10in - \frac{2.23in}{2})\cdot(\frac{1}{12\, {}^{in}\!/_{ft}}) =$ 63.2 k-ft $\ngtr$ 64 k-ft needed (not OK)

So, we can increase d to 13 in, and ϕM_n = 70.3 k-ft (OK). Or increase A_s to 2 # 10's (2.54 in^2), for a = 2.39 in and ϕM_n of 67.1 k-ft (OK).

<u>Example 2</u>

Determine the capacity of a 16" x 16" column with 8- #10 bars, tied. Grade 40 steel and 4000 psi concrete.



SOLUTION:

Find ϕP_n, with ϕ=0.65 and $P_n = 0.80P_o$ for tied columns and

$$P_o = 0.85 f_c'(A_g - A_{st}) + f_y A_{st}$$

Steel area (found from reinforcing bar table for the bar size):

A_{st} = 8 bars × (1.27 in^2) = 10.16 in^2

Concrete area (gross):

A_g = 16 in × 16 in = 256 in^2

ASTM STANDARD REINFORCING BARS

Bar size, no.	Nominal diameter, in.	Nominal area, in.^{2}	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.650
18	2.257	4.00	13.600

Grade 40 reinforcement has f_y = 40,000 psi and f_c' = 4000psi

ϕP_n = (0.65)(0.80)[0.85(4000 psi)(256 in^2 – 10.16 in^2) + (40,000 psi)(10.16 in^2)] = 646,026 lb = 646 kips