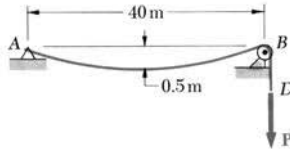


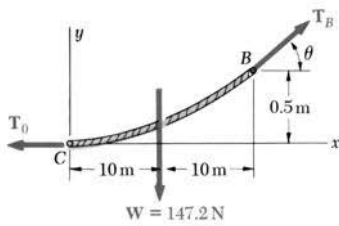
**Examples:
Cables and Arches**

Example 1



SAMPLE PROBLEM 7.9

A light cable is attached to a support at A, passes over a small pulley at B, and supports a load P. Knowing that the sag of the cable is 0.5 m and that the mass per unit length of the cable is 0.75 kg/m, determine (a) the magnitude of the load P, (b) the slope of the cable at B, and (c) the total length of the cable from A to B. Since the ratio of the sag to the span is small, assume the cable to be parabolic. Also, neglect the weight of the portion of cable from B to D.



a. Load P. We denote by C the lowest point of the cable and draw the free-body diagram of the portion CB of cable. Assuming the load to be uniformly distributed along the horizontal, we write

$$w = (0.75 \text{ kg/m})(9.81 \text{ m/s}^2) = 7.36 \text{ N/m}$$

The total load for the portion CB of the cable is

$$W = wx_B = (7.36 \text{ N/m})(20 \text{ m}) = 147.2 \text{ N}$$

and is applied halfway between C and B. Summing moments about B, we write

$$+\uparrow \Sigma M_B = 0: \quad (147.2 \text{ N})(10 \text{ m}) - T_0(0.5 \text{ m}) = 0 \quad T_0 = 2944 \text{ N}$$

From the force triangle we obtain

$$\begin{aligned} T_B &= \sqrt{T_0^2 + W^2} \\ &= \sqrt{(2944 \text{ N})^2 + (147.2 \text{ N})^2} = 2948 \text{ N} \end{aligned}$$

Since the tension on each side of the pulley is the same, we find

$$P = T_B = 2948 \text{ N} \quad \blacktriangleleft$$

b. Slope of Cable at B. We also obtain from the force triangle

$$\tan \theta = \frac{W}{T_0} = \frac{147.2 \text{ N}}{2944 \text{ N}} = 0.05$$

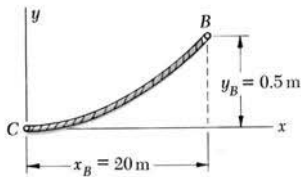
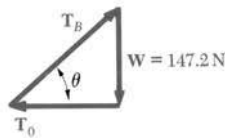
$$\theta = 2.9^\circ \quad \blacktriangleleft$$

c. Length of Cable. Applying Eq. (7.10) between C and B, we write

$$\begin{aligned} s_B &= x_B \left[1 + \frac{2}{3} \left(\frac{y_B}{x_B} \right)^2 + \dots \right] \\ &= (20 \text{ m}) \left[1 + \frac{2}{3} \left(\frac{0.5 \text{ m}}{20 \text{ m}} \right)^2 + \dots \right] = 20.00833 \text{ m} \end{aligned}$$

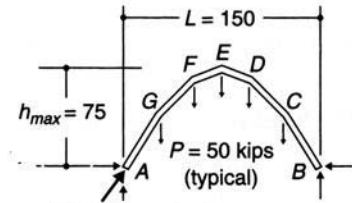
The total length of the cable between A and B is twice this value,

$$\text{Length} = 2s_B = 40.0167 \text{ m} \quad \blacktriangleleft$$



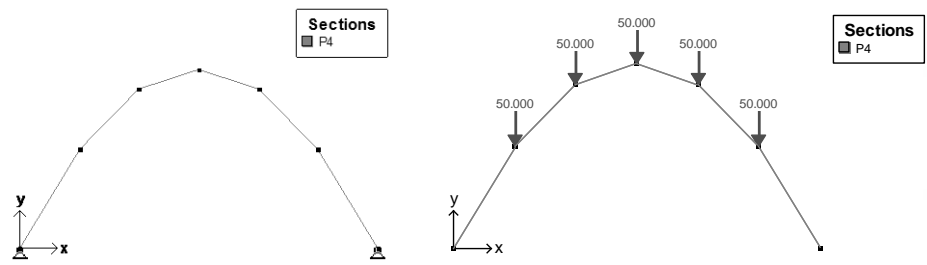
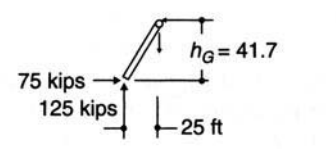
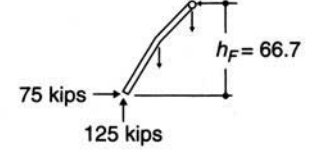
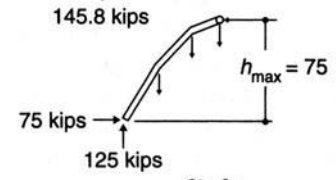
Example 2 (Figure 5.19(b))

Using Multiframe, verify the axial force, shear and bending moment for the funicular shape with $P = 50,000$ lb, $L = 150$ ft, and $h_{max} = 75$ ft.



Joint Coordinates (ft)

Joint	Label	x	y	z	Type
1		0.000	0.000	0.000	Rigi
2		25.000	41.700	0.000	Rigi
3		50.000	66.700	0.000	Rigi
4		75.000	75.000	0.000	Rigi
5		100.000	66.700	0.000	Rigi
6		125.000	41.700	0.000	Rigi
7		150.000	0.000	0.000	Rigi



Load Case 1

Joint Reactions (kip,lb-ft)

Joint Label	Load Case	Rx'	Ry'	Rz'
1	Load Case 1	74.967	125.000	0.000
2	Load Case 1	-0.000	-0.000	0.000
3	Load Case 1	0.000	0.000	0.000
4	Load Case 1	-0.000	0.000	0.000
5	Load Case 1	0.000	-0.000	0.000
6	Load Case 1	-0.000	0.000	0.000
7	Load Case 1	-74.967	125.000	0.000

