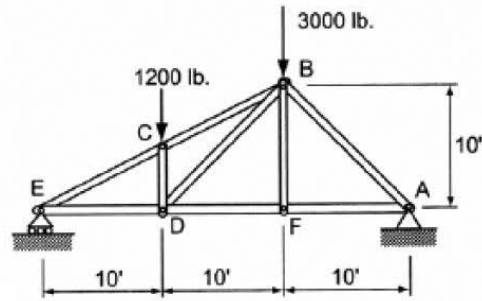


Examples: Trusses and Columns

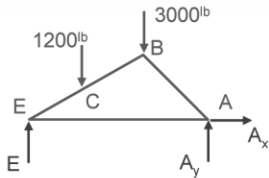
Example 1

Example Problem 4.1 (Method of Joints)

An asymmetrical roof truss, shown in Figure 4.4, supports two vertical roof loads. Determine the support reactions at each end, then, using the method of joints, solve for all member forces. Summarize the results of all member forces on a FBD (this diagram is referred to as a *force summation diagram*).



1. FBD

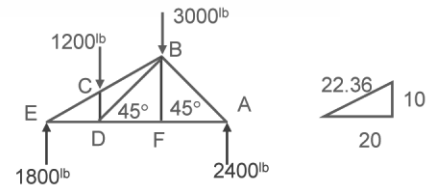


2. solve for support forces

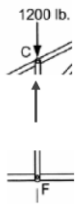
$$\sum F_x = A_x = 0$$

$$\sum M_A = 3000^{lb} \cdot 10^{ft} + 1200^{lb} \cdot 20^{ft} - E \cdot 30^{ft} = 0 \quad E = \frac{54000^{lb-ft}}{30^{ft}} = 1800^{lb}$$

$$\sum F_y = 1800^{lb} - 1200^{lb} - 3000^{lb} + A_y = 0 \quad A_y = 2400^{lb}$$



3. look for special cases:



C, so $CE = BC$ and $CD = -1200^{lb}$

F, so $DF = AF$ and $BF = 0$

4. choose a joint with 2 or less unknowns: E or A will work (C won't)

E:

$$\sum F_y = 1800^{lb} + EC \left(\frac{10}{22.36} \right) = 0$$

$$EC = -1800^{lb} \left(\frac{22.36}{10} \right) = -4025^{lb} = BC$$

$$\sum F_x = ED + (-4025^{lb}) \left(\frac{20}{22.36} \right) = 0 \quad ED = 4025^{lb} \left(\frac{20}{22.36} \right) = 3600^{lb}$$

need BD, AB, (AF or DF) which leaves joints B, D & A (F won't work)

6. last joint needs only one equation

A:

$$\sum F_y = 2400^{lb} + AB \sin 45 = 0$$

$$AB = \frac{-2400^{lb}}{\sin 45} = -3394^{lb}$$

$$(\sum F_x = -2400^{lb} - (-3394^{lb}) \cos 45 = 0) \checkmark$$

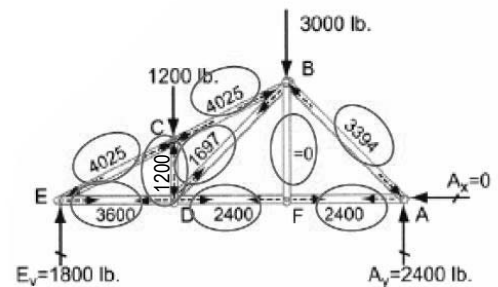
5. choose a joint with 2 or less unknowns: B, D or A will work (F won't)

D:

$$\sum F_y = -1200^{lb} + BD \sin 45 = 0 \quad BD = \frac{1200^{lb}}{\sin 45} = 1697^{lb}$$

$$\sum F_x = -3600^{lb} + DF + (1697^{lb}) \cos 45 = 0$$

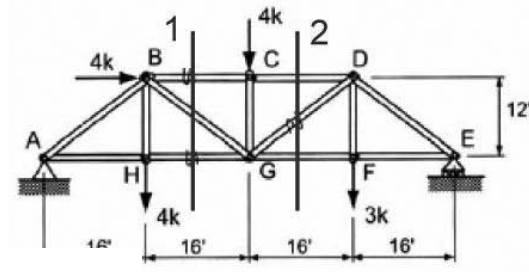
$$DF = 2400^{lb} = AF$$



Example 2

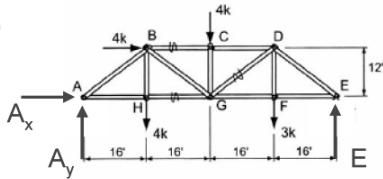
Example Problem 4.3 (Method of Sections)

A 64-foot parallel chord truss (Figure 4.30) supports horizontal and vertical loads as shown. Using the method of sections, determine the member forces BC, HG, and GD.



1. look for sections

2. FBD



3. solve for support forces

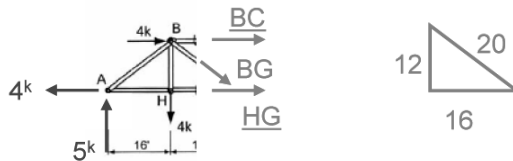
$$\sum F_x = A_x + 4^k = 0 \quad A_x = \boxed{-4^k}$$

$$\sum F_y = A_y - 4^k - 4^k - 3^k + E = 0$$

$$\sum M_A = -4^k \cdot 12^{\text{ft}} - 4^k \cdot 16^{\text{ft}} - 4^k \cdot 32^{\text{ft}} - 3^k \cdot 48^{\text{ft}} + E \cdot 64^{\text{ft}} = 0$$

$$E = \frac{384^{\text{k-ft}}}{64^{\text{ft}}} = \boxed{6^k} \quad \text{and sub: } A_y = \boxed{5^k}$$

4. draw section



5. look for intersection for summing moments (B or G)

6. write equilibrium equations

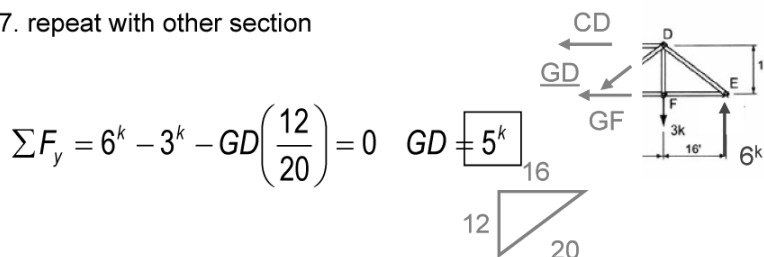
$$\sum M_B = HG \cdot 12^{\text{ft}} - 5^k \cdot 16^{\text{ft}} - 4^k \cdot 12^{\text{ft}} = 0 \quad HG = \frac{128^{\text{k-ft}}}{12^{\text{ft}}} = \boxed{10.67^k}$$

$$\sum M_G = 4^k \cdot 16^{\text{ft}} - 5^k \cdot 32^{\text{ft}} - 4^k \cdot 12^{\text{ft}} - BC \cdot 12^{\text{ft}} = 0$$

$$BC = \frac{144^{\text{k-ft}}}{-12^{\text{ft}}} = \boxed{-12^k}$$

$$(\sum F_y = 5^k - 4^k - BG \left(\frac{12}{20}\right) = 0 \quad BG = -1.67^k)$$

7. repeat with other section



$$\sum F_y = 6^k - 3^k - GD \left(\frac{12}{20}\right) = 0 \quad GD = \boxed{5^k}$$

Example 3 From eStructures v1.1, Schodek and Pollalis, 2000 Harvard College

Braced Column
STEP 1

COLUMNS

$L_y = L_x / 2$

Bracing Level

$L_y = L_x / 2$

MEMBER

$b = 2 \text{ in.}$

$d = 3 \text{ in.}$

$L_x = 12 \text{ ft} = 144 \text{ in}$

TIMBER

Modulus of Elasticity
 $E_T = 1.6 \times 10^6 \text{ lb/in}^2$

Crushing Stress
 $F_C = 2400 \text{ lb/in}^2$

BRACED COLUMNS

Two buckling modes must be checked

Braced Column
STEP 2

COLUMNS

$L_y = L_x / 2$

Bracing Level

$L_y = L_x / 2$

MEMBER

$b = 2 \text{ in.}$

$d = 3 \text{ in.}$

$L_x = 12 \text{ ft} = 144 \text{ in}$

TIMBER

Modulus of Elasticity
 $E_T = 1.6 \times 10^6 \text{ lb/in}^2$

Crushing Stress
 $F_C = 2400 \text{ lb/in}^2$

OUT OF PLANE BUCKLING: P_{CRx}

Length = Overall Physical Length = L_x

Moment of Inertia: I_x

$$I_x = \frac{bd^3}{12} = \frac{2(3)^3}{12} = 4.5 \text{ in}^4$$

$$P_{CRx} = \frac{\pi^2 E I_x}{L_x^2} = \frac{\pi^2 (1.6 \times 10^6) (4.5)}{(144)^2} = 3,423 \text{ LBS}$$

Critical Buckling Stress

$$f_{CR} = \frac{P_{CR}}{A} = \frac{3423}{(2 \times 3)} = 570 \text{ LBS/in}^2$$

$f_{CR} < F_C \therefore$ Member Buckles

Example 3 (continued)

Braced Column
STEP 3

COLUMNS

$L_y = L_x / 2$

Bracing Level

$L_y = L_x / 2$

MEMBER

$b = 2 \text{ in.}$

$d = 3 \text{ in.}$

$L_x = 12 \text{ ft} = 144 \text{ in}$

TIMBER

Modulus of Elasticity
 $E_T = 1.6 \times 10^6 \text{ lb/in}^2$

Crushing Stress
 $F_C = 2400 \text{ lb/in}^2$

IN PLANE BUCKLING: P_{CR_y}

Length = Overall Physical Length = L_y

Moment of Inertia: I_y

$L_y = L_x / 2$

$L_y = L_x / 2$

$$I_y = \frac{db^3}{12} = \frac{3(2)^3}{12} = 2.0 \text{ in}^4$$

$$P_{cr_y} = \frac{\pi^2 E I_y}{L_y^2}$$

$$= \frac{\pi^2 E I_y}{(L_x/2)^2}$$

$$= \frac{\pi^2 (1.6 \times 10^6) (2)}{(144/2)^2}$$

$$= 6,086 \text{ lbs}$$

$$f_{cr} = \frac{P_{cr_y}}{A} = \frac{6086}{(2 \times 3)}$$

$$= 1014 \text{ lbs/in}^2 < F_C$$

∴ Member Buckles

Braced Column
STEP 4

COLUMNS

$L_y = L_x / 2$

Bracing Level

$L_y = L_x / 2$

MEMBER

$b = 2 \text{ in.}$

$d = 3 \text{ in.}$

$L_x = 12 \text{ ft} = 144 \text{ in}$

TIMBER

Modulus of Elasticity
 $E_T = 1.6 \times 10^6 \text{ lb/in}^2$

Crushing Stress
 $F_C = 2400 \text{ lb/in}^2$

CRITICAL BUCKLING IN OUT-OF-PLANE DIRECTION:

$P_x = 3,423 \text{ lbs}$

$L_y = L_x / 2$

$L_y = L_x / 2$

CRITICAL BUCKLING IN THE IN-PLANE DIRECTION:

$P_y = 6,086 \text{ lbs}$

Bracing Level

Example 3 (continued)

Braced Column
STEP 5

COLUMNS

$L_y = L_x / 2$

Bracing Level

$L_y = L_x / 2$

MEMBER

$b = 2 \text{ in.}$

$d = 3 \text{ in.}$

$L_x = 12 \text{ ft} = 144 \text{ in}$

TIMBER

Modulus of Elasticity
 $E_T = 1.6 \times 10^6 \text{ lb/in}^2$

Crushing Stress
 $F_C = 2400 \text{ lb/in}^2$

SINCE $P_x < P_y$, THE COLUMN ACTUALLY BUCKLES IN THE OUT-OF-PLANE DIRECTION.

Critical Buckling Load for Column:
= 3,423 lbs

Braced Column
STEP 6

COLUMNS

$L_y = 144 \text{ in.}$

MEMBER

$b = 2 \text{ in.}$

$d = 3 \text{ in.}$

$L_x = 12 \text{ ft} = 144 \text{ in}$

TIMBER

Modulus of Elasticity
 $E_T = 1.6 \times 10^6 \text{ lb/in}^2$

Crushing Stress
 $F_C = 2400 \text{ lb/in}^2$

NOTE THAT IF THE MID-HEIGHT BRACING WERE "REMOVED", THEN THE COLUMN WOULD BUCKLE AT A LOWER LOAD IN THE OTHER DIRECTION

$$P_{CR_y} = \frac{\pi^2 E I_y}{L_y^2}$$

$$= \frac{\pi^2 (1.6 \times 10^6) (2.0)}{(144)^2}$$

$$= 1,521 \text{ LBS}$$

$P_{CR_x} = 3,423 \text{ As Before}$

Since $P_{CR_y} < P_{CR_x}$

The Column buckles as shown at a load of 1,521 LBS