Examples: Beams (V, M, Stresses and Design)

Example 1

Example Problem 9.5: Section Modulus (Figures 9.26 to 9.28)

Two C10 × 15.3 steel channels are placed back to back to form a 10"-deep beam. Determine the permissible P if $F_b = 30$ ksi. Assume A572 grade 50 steel.

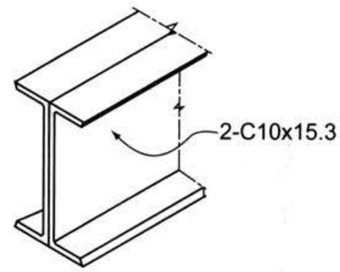


Figure 9.26 Two steel channels.

Solution:

$$I_x = 67.4 \text{ in.}^4 \times 2 = 134.8 \text{ in.}^4$$

$$M_{max} = \tfrac{1}{2}(5)(5) + (P/2)(5)$$

$$M_{max} = 12.5 + 2.5P$$

$$= (12.5 \text{ k-ft.} + 2.5P) \times (12 \text{ in./ft.})$$

$$f = \frac{Mc}{I} = \frac{M}{S}; \quad \therefore M = F_b \times S_x$$

$$S_x = 2 \times 13.5 \text{ in.}^3 = 27 \text{ in.}^3$$

Equating both M_{max} equations:

$$M = (30 \text{ k/in.}^2) \times (27 \text{ in.}^3) = 810 \text{ k-in.}$$

$$(12.5 \text{ k-ft.} + 2.5P)(12 \text{ in./ft.}) = 810 \text{ k-in.}$$

Dividing both sides of the equation by 12 in./ft.:

$$(12.5 \text{ k-ft.}) + (2.5 \text{ ft.})P + 67.5 \text{ k-ft.}$$

$$2.5P = 55 \text{ k}$$

$$\therefore P = 22 \text{ k}$$

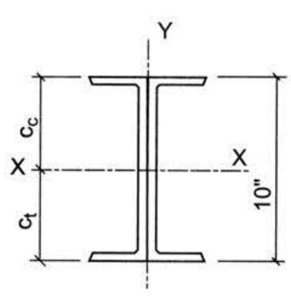


Figure 9.27 Beam cross-section.

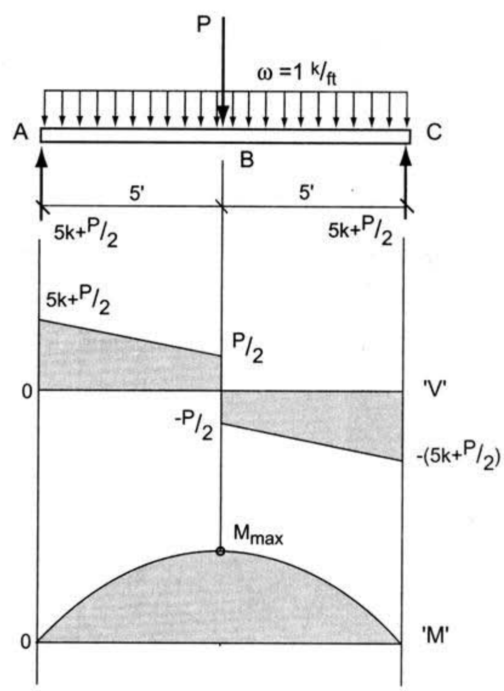


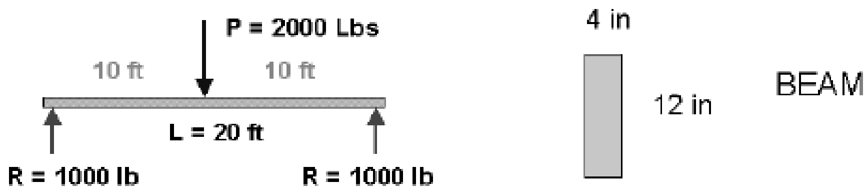
Figure 9.28 Load, V, and M diagrams.

Example 2 **From eStructures v1.1, Schodek and Pollalis, 2000 Harvard College**

Beam Analysis STEP 1

BEAM ANALYSIS

Determine the bending and shear stresses in the timber beam shown. Also determine the deflections present. Is the beam adequately sized? Assume that the allowable bending stresses is $F_{ballowable}$ = 1500 lbs/in^2, the allowable shear stress is $F_{vallowable}$ = 150 lbs/in^2, and the allowable deflection is L /360. Also assume that the allowable stress in bearing is f_{bg} = 400 lbs/in^2 and E = 1.6 x 10^6 lbs/in^2

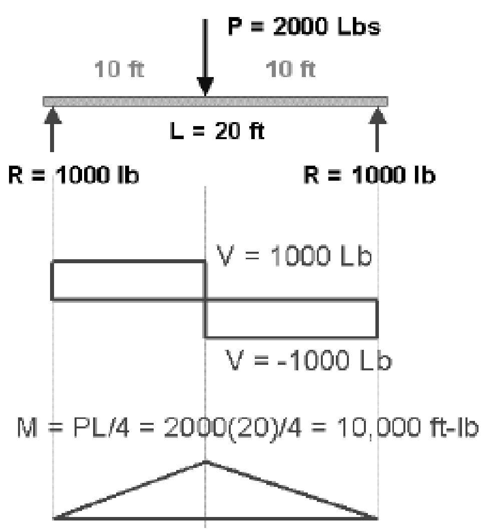


CHECK BENDING, SHEAR, BEARING STRESSES AND DEFLECTIONS

Reference: eStructures v1.1, Shodek & Pollalis, 2000
Simple Beams, Beam Analysis

Beam Analysis STEP 2

DRAW SHEAR AND MOMENT DIAGRAMS



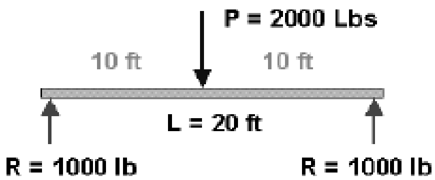
SHEAR AND MOMENT DIAGRAMS:

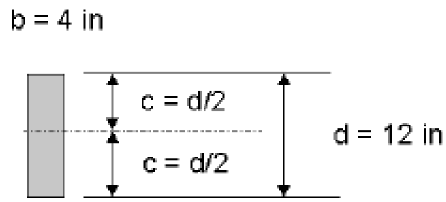
Maximum Shear Force:
= 1000 lbs
Maximum Bending Moment:
= 10,000 ft-lbs = 120, 000 in-lbs

Example 2 (continued)

Beam Analysis STEP 3

DETERMINE BEAM PROPERTIES





MOMENT OF INERTIA:

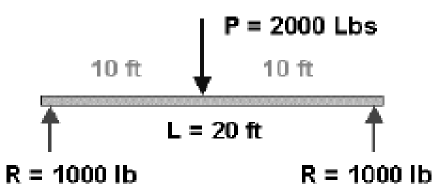
$I = bd^3/12 = (4)(12)^3/12 = 576 \text{ in}^4$

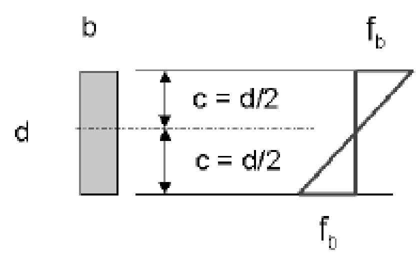
SECTION MODULUS:

$S = I / c = 576 / (12/2) = 96 \text{ in}^3$

Beam Analysis STEP 4

BENDING STRESSES



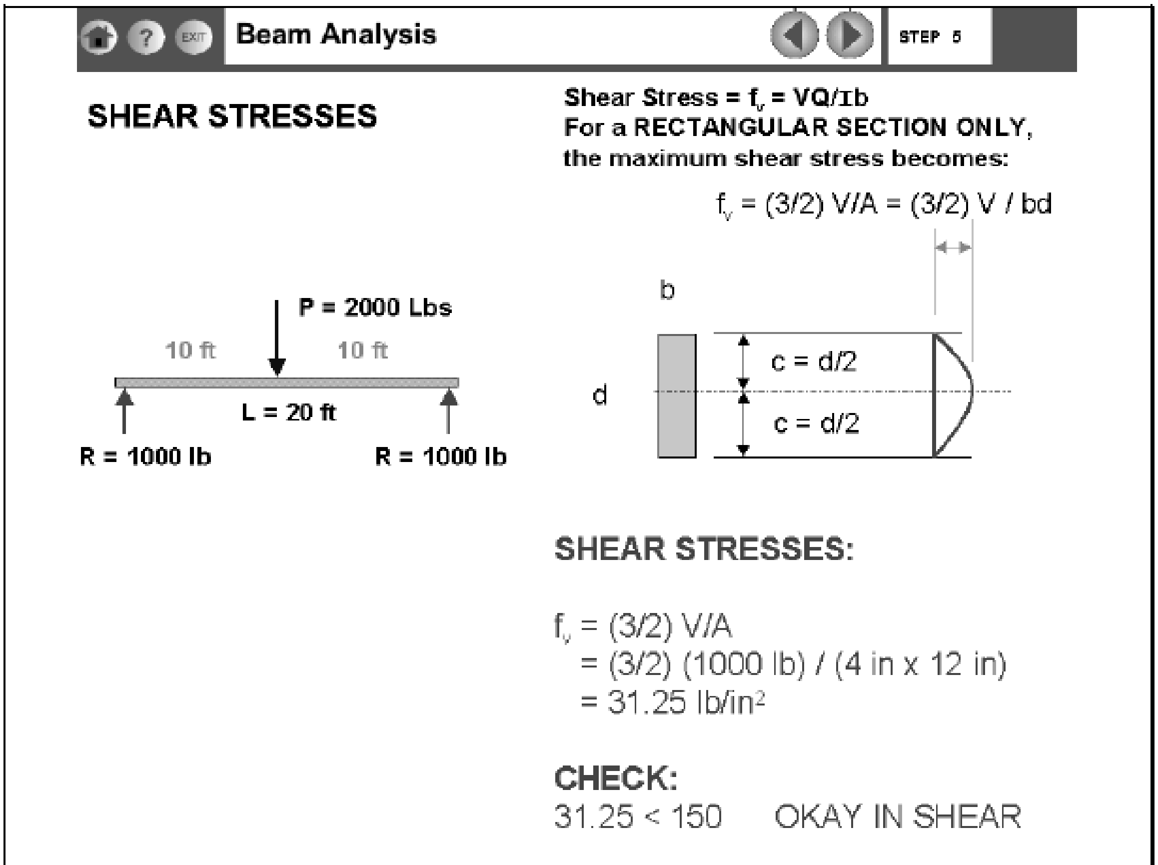


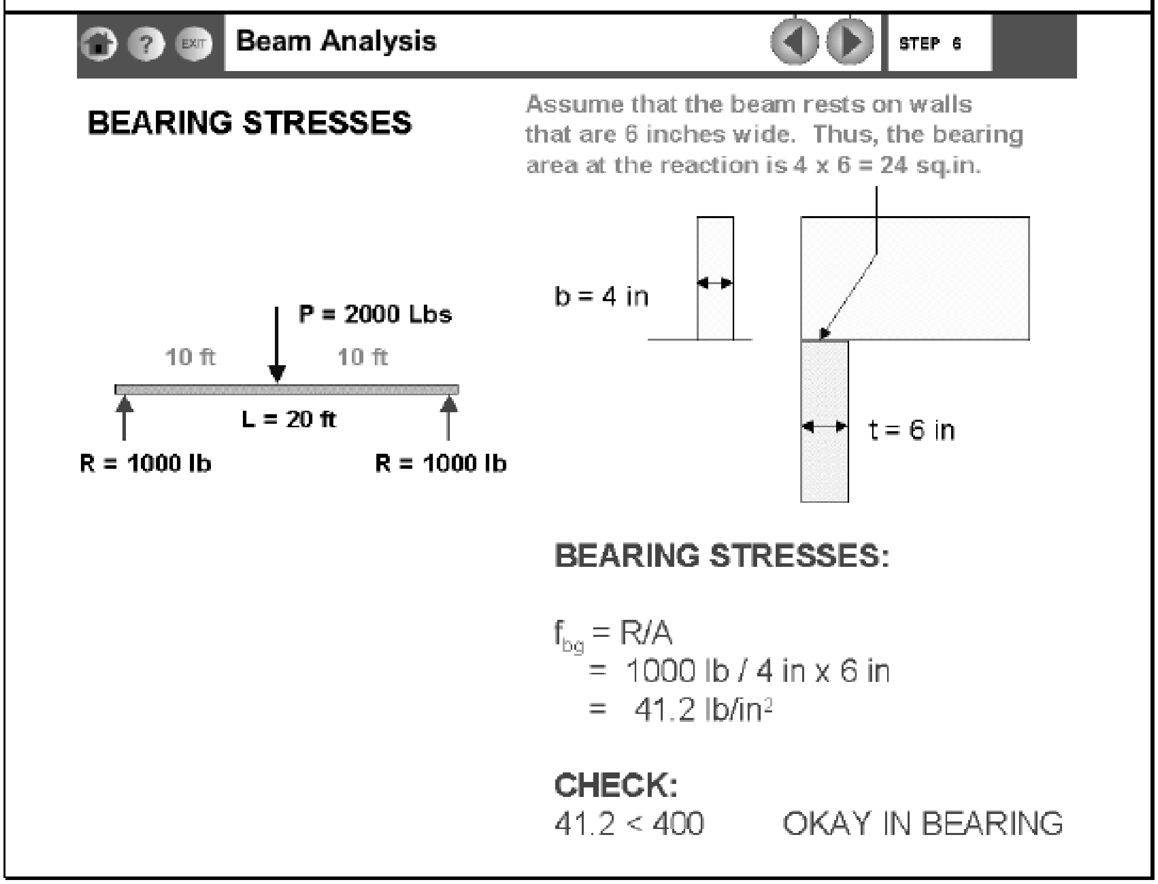
BENDING STRESSES:

$f_b = M / S = (120{,}000 \text{ in-lb}) / 96 \text{ in}^3$
$= 1250 \text{ lb/in}^2$

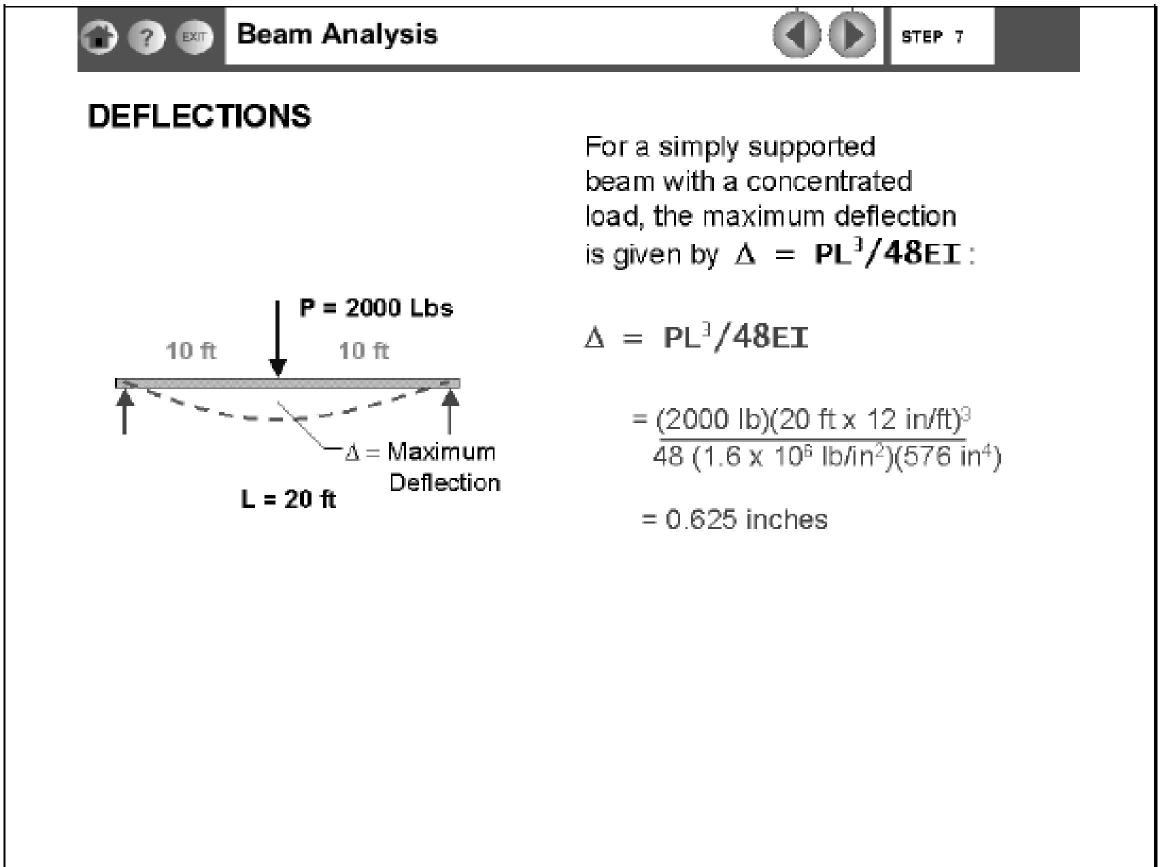
CHECK:
1250 < 1500 OKAY IN BENDING

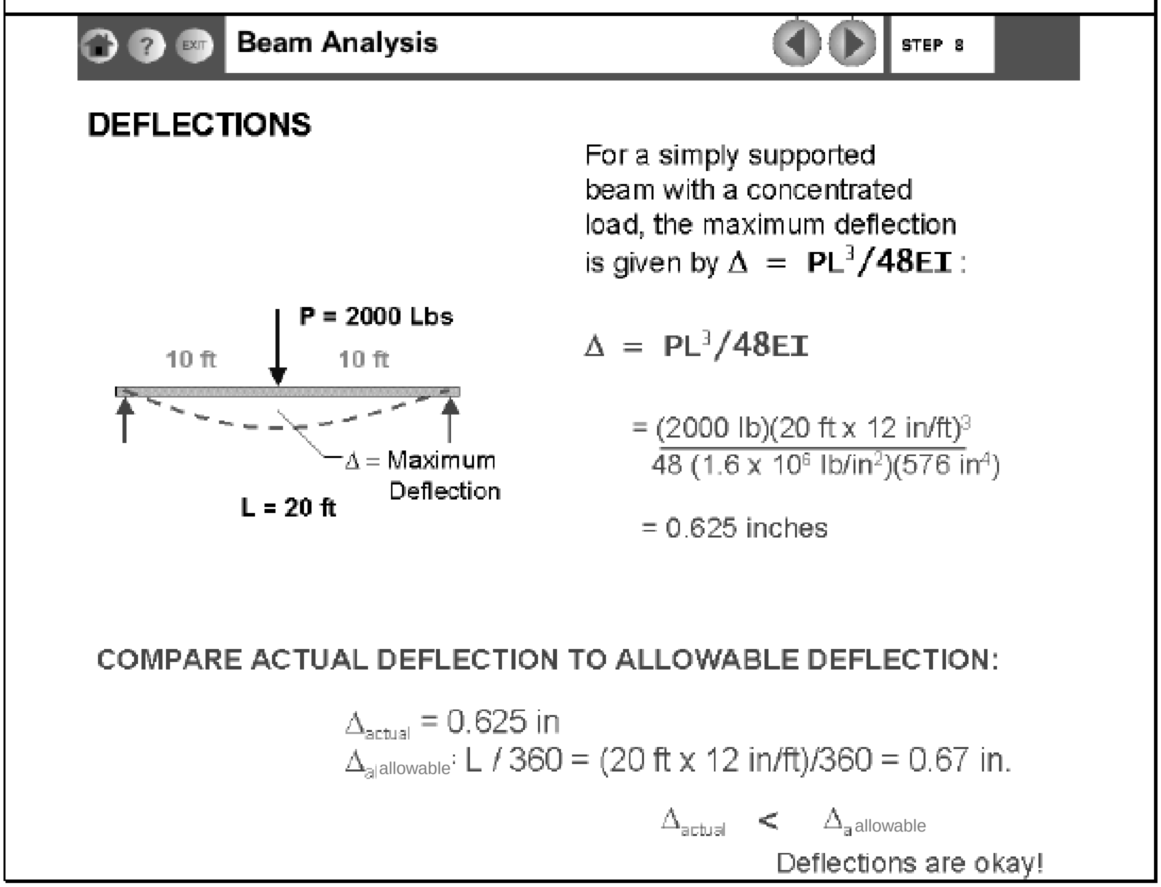
Example 2 (continued)





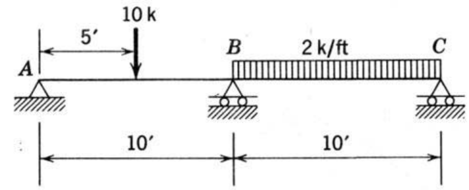
Example 2 (continued)





Example 3

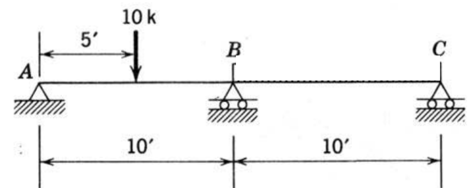
Using an "approximate" method of analysis (specifically beam diagrams and formulas with superpositioning), find reactions, shears, and moments present in the structure. Verify the solution using a computer-based structural analysis program (Multiframe4D).



SOLUTION:

The load cases can be divided into the two shown which correspond to beam diagrams 30 and 29 (mirrored).

Because the maximum moments **do not** occur at the same place, find the reactions to add up and construct the V & M diagrams. The moment diagram should look like the two diagrams (with one flipped) "added" together:



and

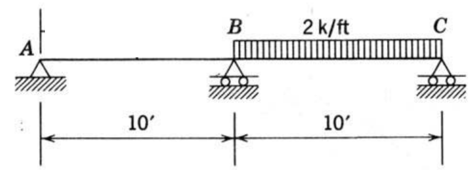


Diagram 30:

$$R_1 = \frac{13}{32}P = \frac{13}{32}(10k) = 4.06k \qquad R_2 = \frac{11}{16}P = \frac{11}{16}(10k) = 6.875k$$

$$R_3 = -\frac{3}{32}P = -\frac{3}{32}(10k) = -0.9375k$$

Diagram 29:

$$R_1(was\ R_3) = -\frac{1}{16}wl = -\frac{1}{16}(2\,{}^{k}\!/_{ft})10\,ft = -1.25k \qquad R_2 = \frac{5}{8}wl = \frac{5}{8}(2\,{}^{k}\!/_{ft})10\,ft = 12.5k$$

$$R_3(was\ R_1) = \frac{7}{16}wl = \frac{7}{16}(2\,{}^{k}\!/_{ft})10\,ft = 8.75k$$

Reaction sums:

R_1 = 4.06 + -1.25 = 2.81 k R_2 = 6.875+12.5=19.375k R_3 = -0.9375+8.75=7.8125k

Shear calculations:

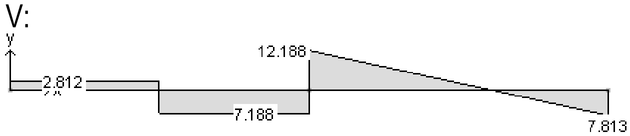
V_A = 0 and 2.81k $V_{at\ 5ft}$ = 2.81k and 2.81-10=-7.19k V_B = -7.19k and -7.19+19.375=12.185k
V_C = 12.185-2k/ft(10ft)=-7.8125 and -7.815+7.815=0k

Moment shapes:

M_A = 0 $M_{at\ 5ft}$ =0+2.81k(5ft) = 14.05k-ft M_B = 14.05-7.19k(5ft) = -21.9k-ft
location of cross over = 12.185k/(2k/ft) = 6.0925ft: $M_{at\ 6.1\ ft\ from\ B}$ = -21.9+12.185k(6.0925ft)/2 = 15.218 k-ft
M_C = 15.218 – 7.8125k(3.9075ft)/2 = 0

MULTIFRAME:

V:



M: