

Examples: Beams (V, M, Stresses and Design)

Example 1

Example Problem 9.5: Section Modulus (Figures 9.26 to 9.28)

Two C10×15.3 steel channels are placed back to back to form a 10"-deep beam. Determine the permissible P if $F_b = 30$ ksi. Assume A572 grade 50 steel.

Solution:

$$I_x = 67.4 \text{ in.}^4 \times 2 = 134.8 \text{ in.}^4$$

$$M_{\max} = \frac{1}{2}(5)(5) + (P/2)(5)$$

$$M_{\max} = 12.5 + 2.5P$$

$$= (12.5 \text{ k-ft.} + 2.5P) \times (12 \text{ in./ft.})$$

$$f = \frac{Mc}{I} = \frac{M}{S}; \quad \therefore M = F_b \times S_x$$

$$S_x = 2 \times 13.5 \text{ in.}^3 = 27 \text{ in.}^3$$

Equating both M_{\max} equations:

$$M = (30 \text{ k/in.}^2) \times (27 \text{ in.}^3) = 810 \text{ k-in.}$$

$$(12.5 \text{ k-ft.} + 2.5P)(12 \text{ in./ft.}) = 810 \text{ k-in.}$$

Dividing both sides of the equation by 12 in./ft.:

$$(12.5 \text{ k-ft.}) + (2.5 \text{ ft.})P + 67.5 \text{ k-ft.}$$

$$2.5P = 55 \text{ k}$$

$$\therefore P = 22 \text{ k}$$

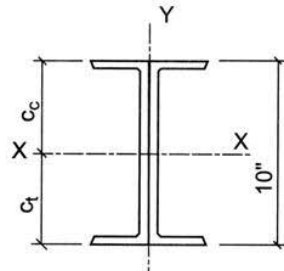


Figure 9.27 Beam cross-section.

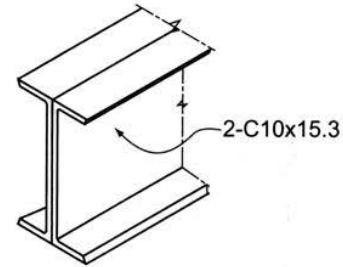


Figure 9.26 Two steel channels.

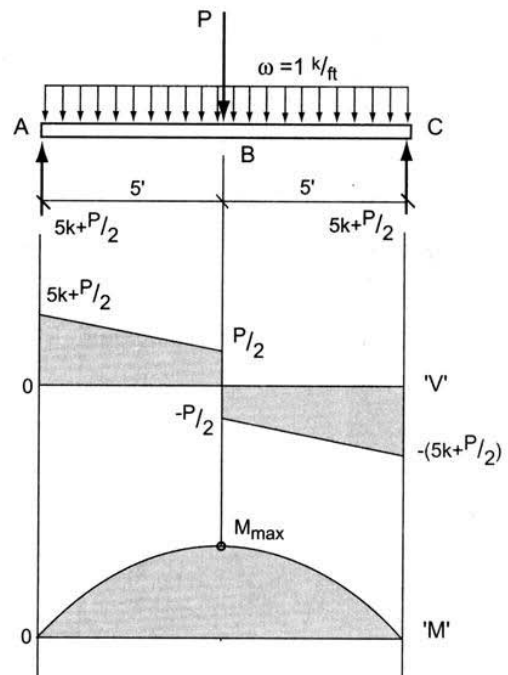



Figure 9.28 Load, V, and M diagrams.

Example 2 From eStructures v1.1, Schodek and Pollalis, 2000 Harvard College

Beam Analysis
STEP 1

BEAM ANALYSIS

Determine the bending and shear stresses in the timber beam shown. Also determine the deflections present. Is the beam adequately sized? Assume that the allowable bending stresses is $F_{b,allowable} = 1500 \text{ lbs/in}^2$, the allowable shear stress is $F_{v,allowable} = 150 \text{ lbs/in}^2$, and the allowable deflection is $L/360$. Also assume that the allowable stress in bearing is $f_{bg} = 400 \text{ lbs/in}^2$ and $E = 1.6 \times 10^6 \text{ lbs/in}^2$

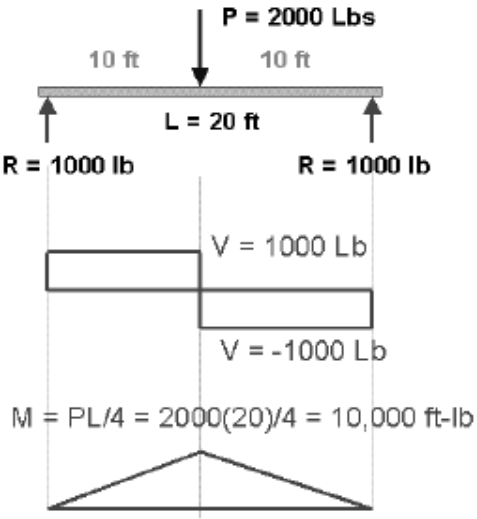


CHECK BENDING, SHEAR, BEARING STRESSES AND DEFLECTIONS

*Reference: eStructures v1.1, Shodek & Pollalis, 2000
Simple Beams, Beam Analysis*

Beam Analysis
STEP 2

DRAW SHEAR AND MOMENT DIAGRAMS



SHEAR AND MOMENT DIAGRAMS:

Maximum Shear Force:
= 1000 lbs

Maximum Bending Moment:
= 10,000 ft-lbs = 120,000 in-lbs

Example 2 (continued)

Beam Analysis
STEP 3

DETERMINE BEAM PROPERTIES

$P = 2000 \text{ Lbs}$
 $L = 20 \text{ ft}$
 $R = 1000 \text{ lb}$

$b = 4 \text{ in}$

$c = d/2$
 $d = 12 \text{ in}$

MOMENT OF INERTIA:

$$I = bd^3/12 = (4)(12)^3/12 = 576 \text{ in}^4$$

SECTION MODULUS:

$$S = I / c = 576 / (12/2) = 96 \text{ in}^3$$

Beam Analysis
STEP 4

BENDING STRESSES

$P = 2000 \text{ Lbs}$
 $L = 20 \text{ ft}$
 $R = 1000 \text{ lb}$

f_b
 $c = d/2$
 d
 f_b

BENDING STRESSES:

$$f_b = M / S = (120,000 \text{ in-lb}) / 96 \text{ in}^3$$

$$= 1250 \text{ lb/in}^2$$

CHECK:
1250 < 1500 OKAY IN BENDING

Example 2 (continued)

Beam Analysis STEP 5

SHEAR STRESSES

$P = 2000 \text{ Lbs}$
 $L = 20 \text{ ft}$
 $R = 1000 \text{ lb}$

Shear Stress = $f_v = VQ/Ib$
For a **RECTANGULAR SECTION ONLY**,
the maximum shear stress becomes:

$$f_v = (3/2) V/A = (3/2) V / bd$$

SHEAR STRESSES:

$$f_v = (3/2) V/A$$

$$= (3/2) (1000 \text{ lb}) / (4 \text{ in} \times 12 \text{ in})$$

$$= 31.25 \text{ lb/in}^2$$

CHECK:
31.25 < 150 OKAY IN SHEAR

Beam Analysis STEP 6

BEARING STRESSES

$P = 2000 \text{ Lbs}$
 $L = 20 \text{ ft}$
 $R = 1000 \text{ lb}$

Assume that the beam rests on walls that are 6 inches wide. Thus, the bearing area at the reaction is 4 x 6 = 24 sq.in.

BEARING STRESSES:

$$f_{bg} = R/A$$

$$= 1000 \text{ lb} / 4 \text{ in} \times 6 \text{ in}$$

$$= 41.2 \text{ lb/in}^2$$

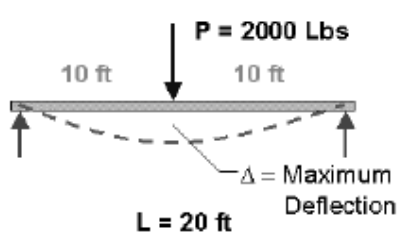
CHECK:
41.2 < 400 OKAY IN BEARING

Example 2 (continued)

Beam Analysis
STEP 7

DEFLECTIONS

For a simply supported beam with a concentrated load, the maximum deflection is given by $\Delta = PL^3/48EI$:



$L = 20 \text{ ft}$

$$\Delta = PL^3/48EI$$

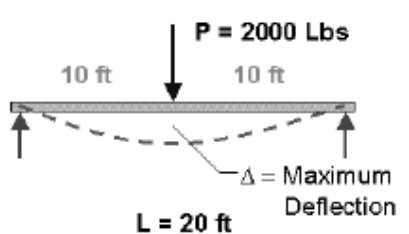
$$= \frac{(2000 \text{ lb})(20 \text{ ft} \times 12 \text{ in/ft})^3}{48 (1.6 \times 10^6 \text{ lb/in}^2)(576 \text{ in}^4)}$$

$$= 0.625 \text{ inches}$$

Beam Analysis
STEP 8

DEFLECTIONS

For a simply supported beam with a concentrated load, the maximum deflection is given by $\Delta = PL^3/48EI$:



$L = 20 \text{ ft}$

$$\Delta = PL^3/48EI$$

$$= \frac{(2000 \text{ lb})(20 \text{ ft} \times 12 \text{ in/ft})^3}{48 (1.6 \times 10^6 \text{ lb/in}^2)(576 \text{ in}^4)}$$

$$= 0.625 \text{ inches}$$

COMPARE ACTUAL DEFLECTION TO ALLOWABLE DEFLECTION:

$\Delta_{\text{actual}} = 0.625 \text{ in}$

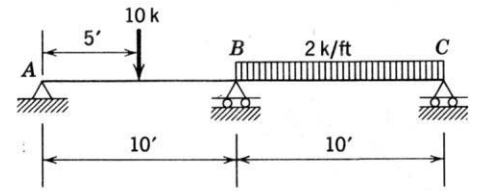
$\Delta_{\text{allowable}} = L / 360 = (20 \text{ ft} \times 12 \text{ in/ft}) / 360 = 0.67 \text{ in.}$

$\Delta_{\text{actual}} < \Delta_{\text{allowable}}$

Deflections are okay!

Example 3

Using an “approximate” method of analysis (specifically beam diagrams and formulas with superpositioning), find reactions, shears, and moments present in the structure. Verify the solution using a computer-based structural analysis program (Multiframe4D).



SOLUTION:

The load cases can be divided into the two shown which correspond to beam diagrams 30 and 29 (mirrored).

Because the maximum moments **do not** occur at the same place, find the reactions to add up and construct the V & M diagrams. The moment diagram should look like the two diagrams (with one flipped) “added” together:

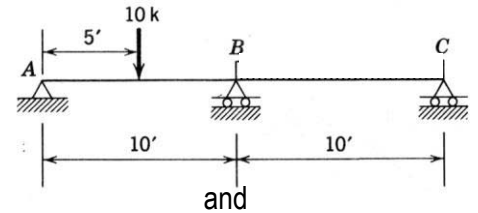


Diagram 30:

$$R_1 = \frac{13}{32} P = \frac{13}{32} (10k) = 4.06k \quad R_2 = \frac{11}{16} P = \frac{11}{16} (10k) = 6.875k$$

$$R_3 = -\frac{3}{32} P = -\frac{3}{32} (10k) = -0.9375k$$

Diagram 29:

$$R_1 (was R_3) = -\frac{1}{16} wl = -\frac{1}{16} (2 \frac{k}{ft}) 10ft = -1.25k \quad R_2 = \frac{5}{8} wl = \frac{5}{8} (2 \frac{k}{ft}) 10ft = 12.5k$$

$$R_3 (was R_1) = \frac{7}{16} wl = \frac{7}{16} (2 \frac{k}{ft}) 10ft = 8.75k$$

Reaction sums:

$$R_1 = 4.06 + -1.25 = 2.81k \quad R_2 = 6.875 + 12.5 = 19.375k \quad R_3 = -0.9375 + 8.75 = 7.8125k$$

Shear calculations:

$$V_A = 0 \text{ and } 2.81k \quad V_{at 5ft} = 2.81k \text{ and } 2.81 - 10 = -7.19k \quad V_B = -7.19k \text{ and } -7.19 + 19.375 = 12.185k$$

$$V_C = 12.185 - 2k/ft(10ft) = -7.8125 \text{ and } -7.815 + 7.815 = 0k$$

Moment shapes:

$$M_A = 0 \quad M_{at 5ft} = 0 + 2.81k(5ft) = 14.05k\text{-ft} \quad M_B = 14.05 - 7.19k(5ft) = -21.9k\text{-ft}$$

$$\text{location of cross over} = 12.185k / (2k/ft) = 6.0925ft: \quad M_{at 6.1ft \text{ from B}} = -21.9 + 12.185k(6.0925ft) / 2 = 15.218 \text{ k-ft}$$

$$M_C = 15.218 - 7.8125k(3.9075ft) / 2 = 0$$

MULTIFRAME:

