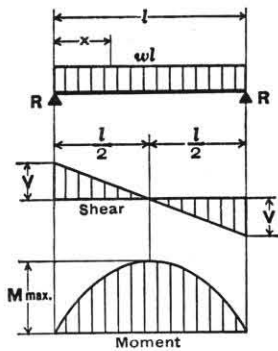
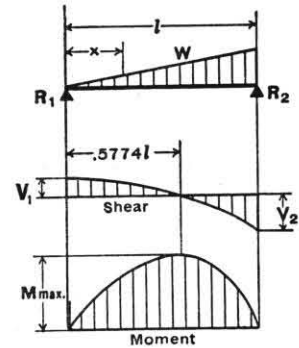


1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



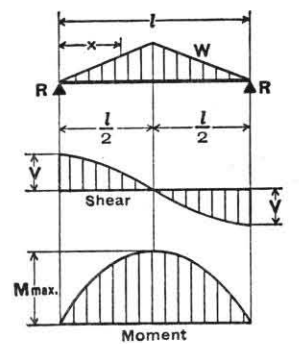
Total Equiv. Uniform Load . . . = wl
 $R = V$ = $\frac{wl}{2}$
 V_x = $w \left(\frac{l}{2} - x \right)$
 $M_{max.}$ (at center) = $\frac{wl^2}{8}$
 M_x = $\frac{wx}{2} (l-x)$
 $\Delta_{max.}$ (at center) = $\frac{5wl^4}{384EI}$
 Δ_x = $\frac{wx}{24EI} (l^3 - 2lx^2 + x^3)$

2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END



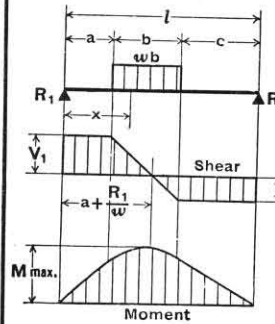
Total Equiv. Uniform Load . . . = $\frac{16W}{9\sqrt{3}} = 1.0264W$
 $R_1 = V_1$ = $\frac{W}{3}$ $W = \frac{wl}{2}$
 $R_2 = V_2$ max. = $\frac{2W}{3}$
 V_x = $\frac{W}{3} - \frac{Wx^2}{l^2}$
 $M_{max.}$ (at $x = \frac{l}{\sqrt{3}} = .5774l$) . . . = $\frac{2Wl}{9\sqrt{3}} = .1283Wl$
 M_x = $\frac{Wx}{3l^2} (l^2 - x^2)$
 $\Delta_{max.}$ (at $x = l\sqrt{1 - \sqrt{\frac{8}{15}}} = .5193l$) = $.01304 \frac{Wl^3}{EI}$
 Δ_x = $\frac{Wx}{180EI l^2} (3x^4 - 10l^2x^2 + 7l^4)$

3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER



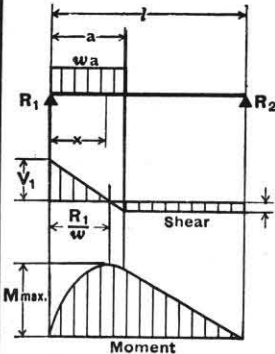
Total Equiv. Uniform Load . . . = $\frac{4W}{3}$ $W = \frac{wl}{2}$
 $R = V$ = $\frac{W}{2}$
 V_x (when $x < \frac{l}{2}$) = $\frac{W}{2l^2} (l^2 - 4x^2)$
 $M_{max.}$ (at center) = $\frac{Wl}{6}$
 M_x (when $x < \frac{l}{2}$) = $Wx \left(\frac{1}{2} - \frac{2x^2}{3l^2} \right)$
 $\Delta_{max.}$ (at center) = $\frac{Wl^3}{60EI}$
 Δ_x (when $x < \frac{l}{2}$) = $\frac{Wx}{480EI l^2} (5l^2 - 4x^2)^2$

4. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED



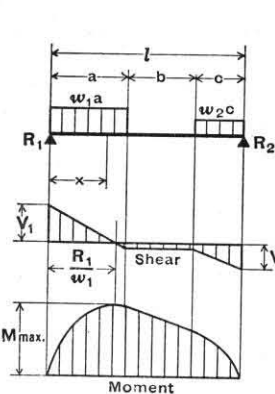
$R_1 = V_1$ (max. when $a < c$) . . . = $\frac{wb}{2l} (2c + b)$
 $R_2 = V_2$ (max. when $a > c$) . . . = $\frac{wb}{2l} (2a + b)$
 V_x (when $x > a$ and $< (a + b)$) . = $R_1 - w(x - a)$
 $M_{max.}$ (at $x = a + \frac{R_1}{w}$) = $R_1 \left(a + \frac{R_1}{2w} \right)$
 M_x (when $x < a$) = R_1x
 M_x (when $x > a$ and $< (a + b)$) . = $R_1x - \frac{w}{2} (x - a)^2$
 M_x (when $x > (a + b)$) = $R_2(l - x)$

5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END



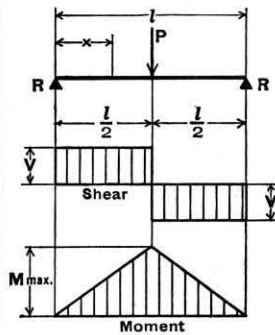
$R_1 = V_1$ max. = $\frac{wa}{2l} (2l - a)$
 $R_2 = V_2$ = $\frac{wa^2}{2l}$
 V_x (when $x < a$) = $R_1 - wx$
 $M_{max.}$ (at $x = \frac{R_1}{w}$) = $\frac{R_1^2}{2w}$
 M_x (when $x < a$) = $R_1x - \frac{wx^2}{2}$
 M_x (when $x > a$) = $R_2(l - x)$
 Δ_x (when $x < a$) = $\frac{wx}{24EI l} (a^2(2l - a)^2 - 2ax^2(2l - a) + lx^3)$
 Δ_x (when $x > a$) = $\frac{wa^2(l - x)}{24EI l} (4xl - 2x^2 - a^2)$

6. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END



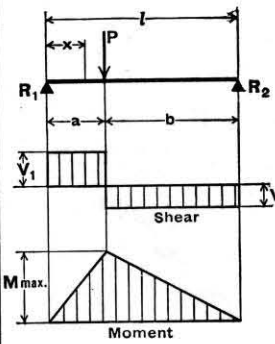
$R_1 = V_1$ = $\frac{w_1a(2l - a) + w_2c^2}{2l}$
 $R_2 = V_2$ = $\frac{w_2c(2l - c) + w_1a^2}{2l}$
 V_x (when $x < a$) = $R_1 - w_1x$
 V_x (when $x > a$ and $< (a + b)$) . = $R_1 - w_1a$
 V_x (when $x > (a + b)$) = $R_2 - w_2(l - x)$
 $M_{max.}$ (at $x = \frac{R_1}{w_1}$ when $R_1 < w_1a$) . = $\frac{R_1^2}{2w_1}$
 $M_{max.}$ (at $x = l - \frac{R_2}{w_2}$ when $R_2 < w_2c$) = $\frac{R_2^2}{2w_2}$
 M_x (when $x < a$) = $R_1x - \frac{w_1x^2}{2}$
 M_x (when $x > a$ and $< (a + b)$) . = $R_1x - \frac{w_1a}{2} (2x - a)$
 M_x (when $x > (a + b)$) = $R_2(l - x) - \frac{w_2(l - x)^2}{2}$

7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER



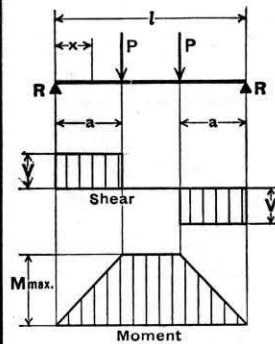
Total Equiv. Uniform Load = $2P$
 $R = V$ = $\frac{P}{2}$
 M max. (at point of load) = $\frac{Pl}{4}$
 M_x (when $x < \frac{l}{2}$) = $\frac{Px}{2}$
 Δ max. (at point of load) = $\frac{Pl^3}{48EI}$
 Δ_x (when $x < \frac{l}{2}$) = $\frac{Px}{48EI} (3l^2 - 4x^2)$

8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT



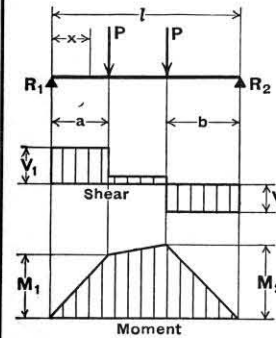
Total Equiv. Uniform Load = $\frac{8Pab}{l^2}$
 $R_1 = V_1$ (max. when $a < b$) = $\frac{Pb}{l}$
 $R_2 = V_2$ (max. when $a > b$) = $\frac{Pa}{l}$
 M max. (at point of load) = $\frac{Pab}{l}$
 M_x (when $x < a$) = $\frac{Pbx}{l}$
 Δ max. (at $x = \sqrt{\frac{a(a+2b)}{3}}$ when $a > b$) = $\frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EI l}$
 Δ_a (at point of load) = $\frac{Pa^2b^2}{3EI l}$
 Δ_x (when $x < a$) = $\frac{Pbx}{6EI l} (l^2 - b^2 - x^2)$

9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED



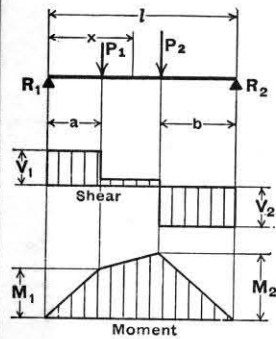
Total Equiv. Uniform Load = $\frac{8Pa}{l}$
 $R = V$ = P
 M max. (between loads) = Pa
 M_x (when $x < a$) = Px
 Δ max. (at center) = $\frac{Pa}{24EI} (3l^2 - 4a^2)$
 Δ_x (when $x < a$) = $\frac{Px}{6EI} (3la - 3a^2 - x^2)$
 Δ_x (when $x > a$ and $< (l-a)$) = $\frac{Pa}{6EI} (3lx - 3x^2 - a^2)$

10. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



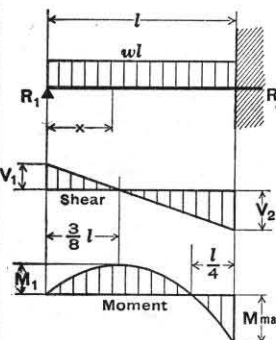
$R_1 = V_1$ (max. when $a < b$) = $\frac{P}{l} (l-a+b)$
 $R_2 = V_2$ (max. when $a > b$) = $\frac{P}{l} (l-b+a)$
 V_x (when $x > a$ and $< (l-b)$) = $\frac{P}{l} (b-a)$
 M_1 (max. when $a > b$) = R_1a
 M_2 (max. when $a < b$) = R_2b
 M_x (when $x < a$) = R_1x
 M_x (when $x > a$ and $< (l-b)$) = $R_1x - P(x-a)$

11. SIMPLE BEAM—TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



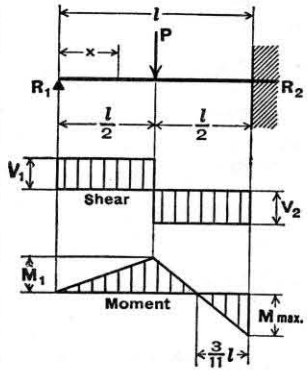
$R_1 = V_1$ = $\frac{P_1(l-a) + P_2b}{l}$
 $R_2 = V_2$ = $\frac{P_1a + P_2(l-b)}{l}$
 V_x (when $x > a$ and $< (l-b)$) = $R_1 - P_1$
 M_1 (max. when $R_1 < P_1$) = R_1a
 M_2 (max. when $R_2 < P_2$) = R_2b
 M_x (when $x < a$) = R_1x
 M_x (when $x > a$ and $< (l-b)$) = $R_1x - P_1(x-a)$

12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—UNIFORMLY DISTRIBUTED LOAD



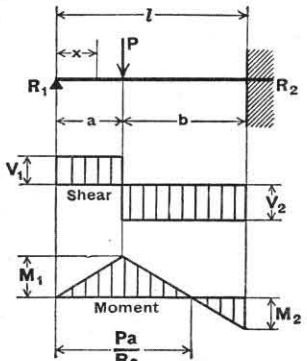
Total Equiv. Uniform Load = wl
 $R_1 = V_1$ = $\frac{3wl}{8}$
 $R_2 = V_2$ max. = $\frac{5wl}{8}$
 V_x = $R_1 - wx$
 M max. = $\frac{wl^2}{8}$
 M_1 (at $x = \frac{3}{8}l$) = $\frac{9}{128}wl^2$
 M_x = $R_1x - \frac{wx^2}{2}$
 Δ max. (at $x = \frac{l}{16} (1 + \sqrt{33}) = .4215l$) = $\frac{wl^4}{185EI}$
 Δ_x = $\frac{wx}{48EI} (l^3 - 3lx^2 + 2x^3)$

13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—
CONCENTRATED LOAD AT CENTER



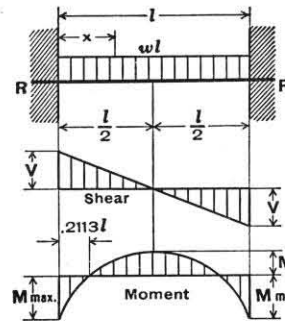
Total Equiv. Uniform Load = $\frac{3P}{2}$
 $R_1 = V_1$ = $\frac{5P}{16}$
 $R_2 = V_2$ max. = $\frac{11P}{16}$
 M max. (at fixed end) = $\frac{3Pl}{16}$
 M_1 (at point of load) = $\frac{5Pl}{32}$
 M_x (when $x < \frac{l}{2}$) = $\frac{5Px}{16}$
 M_x (when $x > \frac{l}{2}$) = $P \left(\frac{l}{2} - \frac{11x}{16} \right)$
 Δ max. (at $x = l \sqrt{\frac{1}{5}} = .4472l$) = $\frac{Pl^3}{48EI \sqrt{5}} = .009317 \frac{Pl^3}{EI}$
 Δ_x (at point of load) = $\frac{7Pl^3}{768EI}$
 Δ_x (when $x < \frac{l}{2}$) = $\frac{Px}{96EI} (3l^2 - 5x^2)$
 Δ_x (when $x > \frac{l}{2}$) = $\frac{P}{96EI} (x-l)^2 (11x - 2l)$

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—
CONCENTRATED LOAD AT ANY POINT



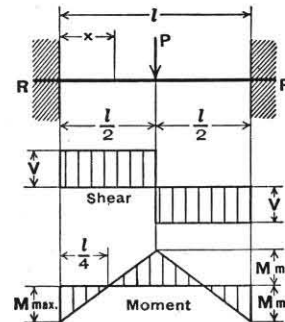
$R_1 = V_1$ = $\frac{Pb^2}{2l^3} (a + 2l)$
 $R_2 = V_2$ = $\frac{Pa}{2l^3} (3l^2 - a^2)$
 M_1 (at point of load) = $R_1 a$
 M_2 (at fixed end) = $\frac{Pab}{2l^2} (a + l)$
 M_x (when $x < a$) = $R_1 x$
 M_x (when $x > a$) = $R_1 x - P(x - a)$
 Δ max. (when $a < .414l$ at $x = l \frac{l^2 + a^2}{3l^2 - a^2}$) = $\frac{Pa}{3EI} \frac{(l^2 - a^2)^3}{(3l^2 - a^2)^2}$
 Δ max. (when $a > .414l$ at $x = l \sqrt{\frac{a}{2l+a}}$) = $\frac{Pab^2}{6EI} \sqrt{\frac{a}{2l+a}}$
 Δ_a (at point of load) = $\frac{Pa^2 b^3}{12EI l^3} (3l + a)$
 Δ_x (when $x < a$) = $\frac{Pb^2 x}{12EI l^3} (3a l^2 - 2l x^2 - a x^2)$
 Δ_x (when $x > a$) = $\frac{Pa}{12EI l^3} (l-x)^2 (3l^2 x - a^2 x - 2a^2 l)$

15. BEAM FIXED AT BOTH ENDS—UNIFORMLY DISTRIBUTED
LOADS



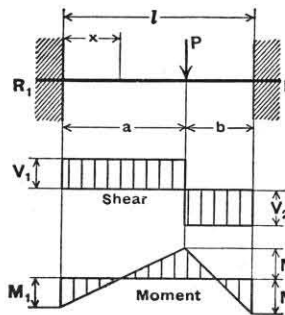
Total Equiv. Uniform Load = $\frac{2wl}{3}$
 $R = V$ = $\frac{wl}{2}$
 V_x = $w \left(\frac{l}{2} - x \right)$
 M max. (at ends) = $\frac{wl^2}{12}$
 M_1 (at center) = $\frac{wl^2}{24}$
 M_x = $\frac{w}{12} (6lx - l^2 - 6x^2)$
 Δ max. (at center) = $\frac{wl^4}{384EI}$
 Δ_x = $\frac{wx^2}{24EI} (l-x)^2$

16. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT
CENTER



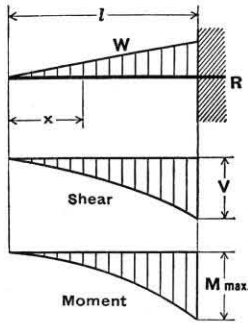
Total Equiv. Uniform Load = P
 $R = V$ = $\frac{P}{2}$
 M max. (at center and ends) = $\frac{Pl}{8}$
 M_x (when $x < \frac{l}{2}$) = $\frac{P}{8} (4x - l)$
 Δ max. (at center) = $\frac{Pl^3}{192EI}$
 Δ_x (when $x < \frac{l}{2}$) = $\frac{Px^2}{48EI} (3l - 4x)$

17. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT
ANY POINT



$R_1 = V_1$ (max. when $a < b$) = $\frac{Pb^2}{l^3} (3a + b)$
 $R_2 = V_2$ (max. when $a > b$) = $\frac{Pa^2}{l^3} (a + 3b)$
 M_1 (max. when $a < b$) = $\frac{Pab^2}{l^2}$
 M_2 (max. when $a > b$) = $\frac{Pa^2 b}{l^2}$
 M_a (at point of load) = $\frac{2Pa^2 b^2}{l^3}$
 M_x (when $x < a$) = $R_1 x - \frac{Pab^2}{l^2}$
 Δ max. (when $a > b$ at $x = \frac{2al}{3a+b}$) = $\frac{2Pa^3 b^2}{3EI (3a+b)^2}$
 Δ_a (at point of load) = $\frac{Pa^3 b^3}{3EI l^3}$
 Δ_x (when $x < a$) = $\frac{Pb^2 x^2}{6EI l^3} (3al - 3ax - bx)$

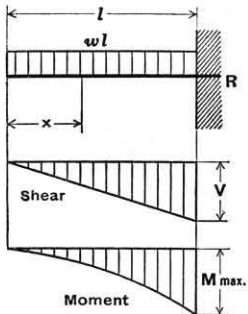
18. CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END



Total Equiv. Uniform Load = $\frac{8}{3} W$
 $R = V$ = W
 V_x = $W \frac{x^2}{l^2}$
 M max. (at fixed end) = $\frac{Wl}{3}$
 M_x = $\frac{Wx^3}{3l^2}$
 Δ max. (at free end) = $\frac{Wl^3}{15EI}$
 Δ_x = $\frac{W}{60EI l^2} (x^5 - 5l^4x + 4l^5)$

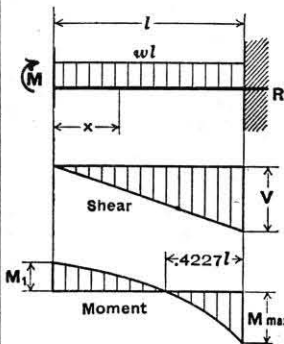
$W = \frac{wl}{2}$

19. CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD



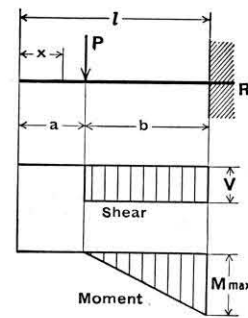
Total Equiv. Uniform Load = $4wl$
 $R = V$ = wl
 V_x = wx
 M max. (at fixed end) = $\frac{wl^2}{2}$
 M_x = $\frac{wx^2}{2}$
 Δ max. (at free end) = $\frac{wl^4}{8EI}$
 Δ_x = $\frac{w}{24EI} (x^4 - 4l^3x + 3l^4)$

20. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—UNIFORMLY DISTRIBUTED LOAD



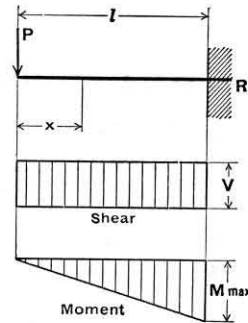
Total Equiv. Uniform Load = $\frac{8}{3} wl$
 $R = V$ = wl
 V_x = wx
 M max. (at fixed end) = $\frac{wl^2}{3}$
 M_1 (at deflected end) = $\frac{wl^2}{6}$
 M_x = $\frac{w}{6} (l^2 - 3x^2)$
 Δ max. (at deflected end) = $\frac{wl^4}{24EI}$
 Δ_x = $\frac{w(l^2 - x^2)^2}{24EI}$

21. CANTILEVER BEAM—CONCENTRATED LOAD AT ANY POINT



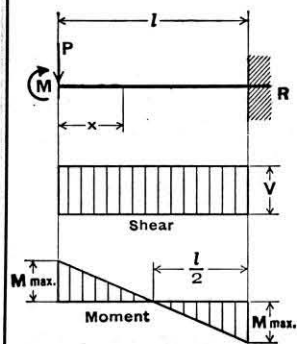
Total Equiv. Uniform Load = $\frac{8Pb}{l}$
 $R = V$ = P
 M max. (at fixed end) = Pb
 M_x (when $x > a$) = $P(x - a)$
 Δ max. (at free end) = $\frac{Pb^2}{6EI} (3l - b)$
 Δa (at point of load) = $\frac{Pb^3}{3EI}$
 Δ_x (when $x < a$) = $\frac{Pb^2}{6EI} (3l - 3x - b)$
 Δ_x (when $x > a$) = $\frac{P(l - x)^2}{6EI} (3b - l + x)$

22. CANTILEVER BEAM—CONCENTRATED LOAD AT FREE END



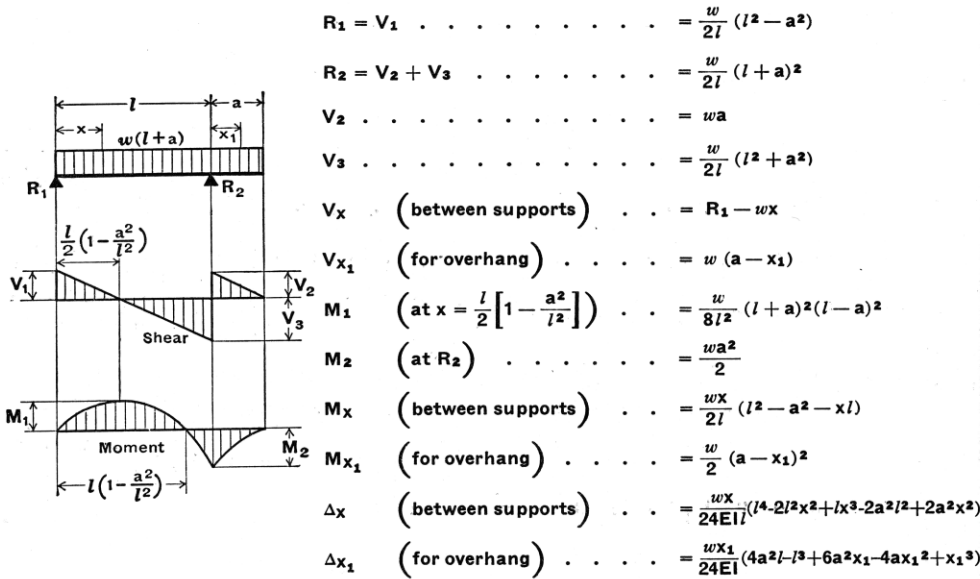
Total Equiv. Uniform Load = $8P$
 $R = V$ = P
 M max. (at fixed end) = Pl
 M_x = Px
 Δ max. (at free end) = $\frac{Pl^3}{3EI}$
 Δ_x = $\frac{P}{6EI} (2l^3 - 3l^2x + x^3)$

23. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—CONCENTRATED LOAD AT DEFLECTED END

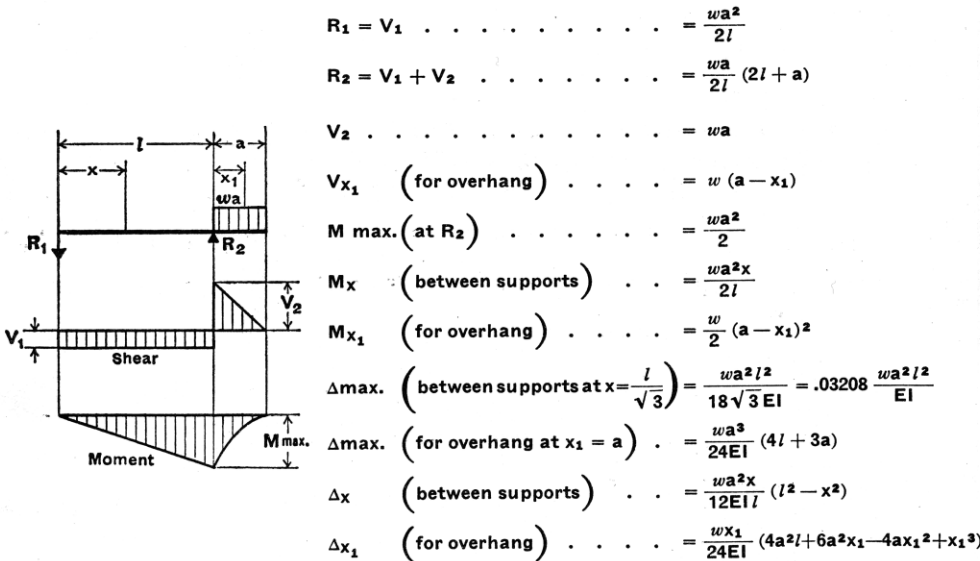


Total Equiv. Uniform Load = $4P$
 $R = V$ = P
 M max. (at both ends) = $\frac{Pl}{2}$
 M_x = $P(\frac{l}{2} - x)$
 Δ max. (at deflected end) = $\frac{Pl^3}{12EI}$
 Δ_x = $\frac{P(l - x)^2}{12EI} (l + 2x)$

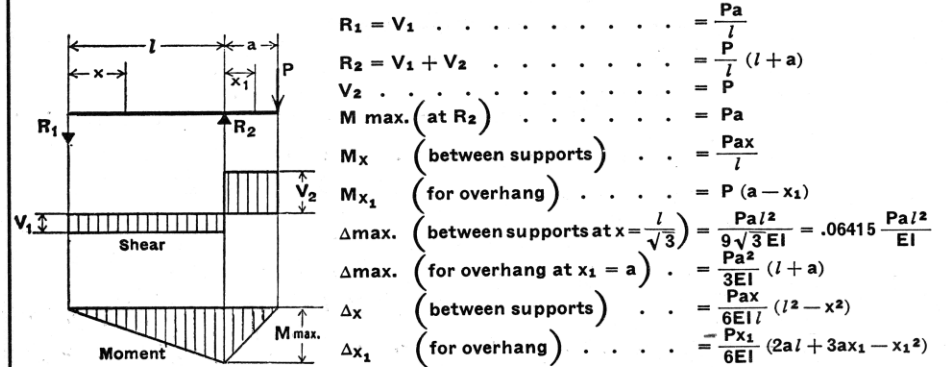
24. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD



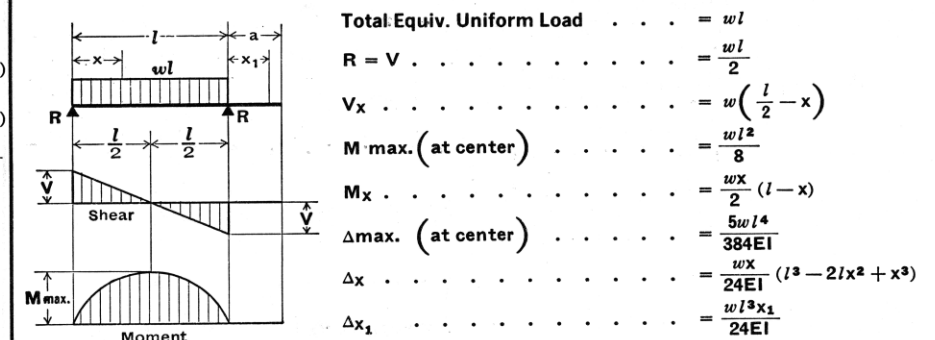
25. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD ON OVERHANG



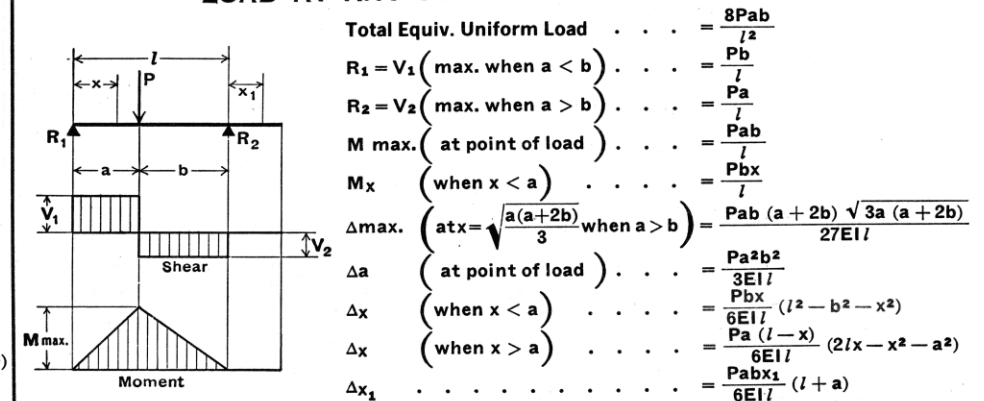
26. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT END OF OVERHANG



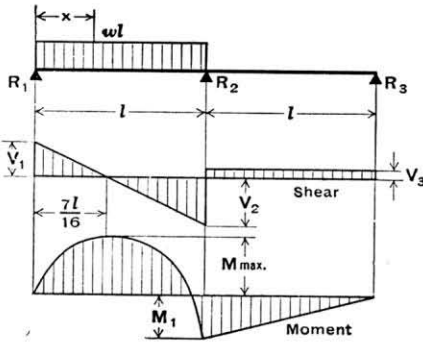
27. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD BETWEEN SUPPORTS



28. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT ANY POINT BETWEEN SUPPORTS

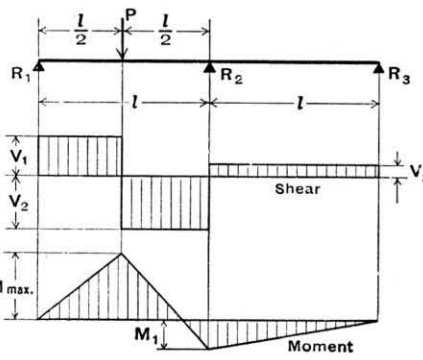


29. CONTINUOUS BEAM—TWO EQUAL SPANS—UNIFORM LOAD ON ONE SPAN



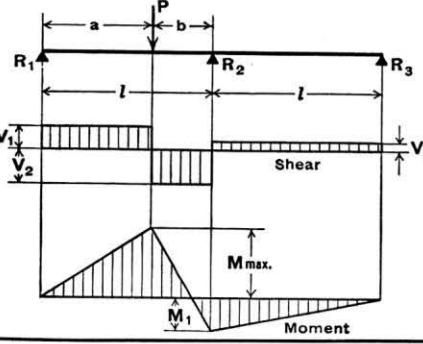
Total Equiv. Uniform Load = $\frac{49}{64} wl$
 $R_1 = V_1 = \frac{7}{16} wl$
 $R_2 = V_2 + V_3 = \frac{5}{8} wl$
 $R_3 = V_3 = -\frac{1}{16} wl$
 $V_2 = \frac{9}{16} wl$
 $M_{max.} \text{ (at } x = \frac{7}{16} l \text{)} = \frac{49}{512} wl^2$
 $M_1 \text{ (at support } R_2 \text{)} = \frac{1}{16} wl^2$
 $M_x \text{ (when } x < l \text{)} = \frac{wx}{16} (7l - 8x)$
 $\Delta_{Max.} \text{ (0.472 } l \text{ from } R_1 \text{)} = 0.0092 wl^4/EI$

30. CONTINUOUS BEAM—TWO EQUAL SPANS—CONCENTRATED LOAD AT CENTER OF ONE SPAN



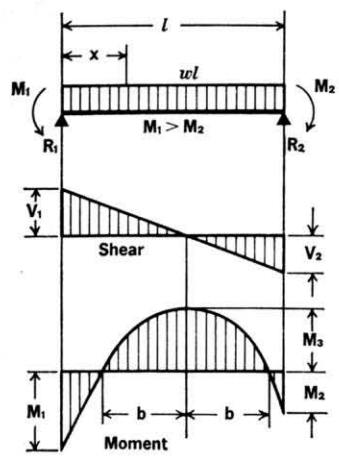
Total Equiv. Uniform Load = $\frac{13}{8} P$
 $R_1 = V_1 = \frac{13}{32} P$
 $R_2 = V_2 + V_3 = \frac{11}{16} P$
 $R_3 = V_3 = -\frac{3}{32} P$
 $V_2 = \frac{19}{32} P$
 $M_{max.} \text{ (at point of load)} = \frac{13}{64} Pl$
 $M_1 \text{ (at support } R_2 \text{)} = \frac{3}{32} Pl$
 $\Delta_{Max.} \text{ (0.480 } l \text{ from } R_1 \text{)} = 0.015 P l^3/EI$

31. CONTINUOUS BEAM—TWO EQUAL SPANS—CONCENTRATED LOAD AT ANY POINT



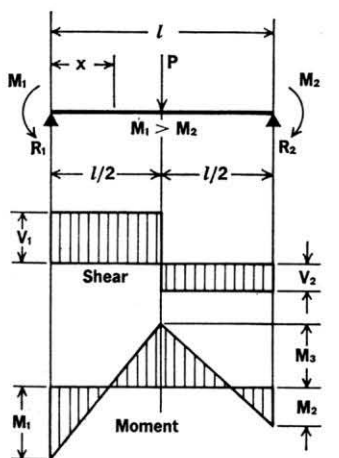
$R_1 = V_1 = \frac{Pb}{4l^3} (4l^2 - a(l+a))$
 $R_2 = V_2 + V_3 = \frac{Pa}{2l^3} (2l^2 + b(l+a))$
 $R_3 = V_3 = -\frac{Pab}{4l^3} (l+a)$
 $V_2 = \frac{Pa}{4l^3} (4l^2 + b(l+a))$
 $M_{max.} \text{ (at point of load)} = \frac{Pab}{4l^3} (4l^2 - a(l+a))$
 $M_1 \text{ (at support } R_2 \text{)} = \frac{Pab}{4l^2} (l+a)$

32. BEAM—UNIFORMLY DISTRIBUTED LOAD AND VARIABLE END MOMENTS



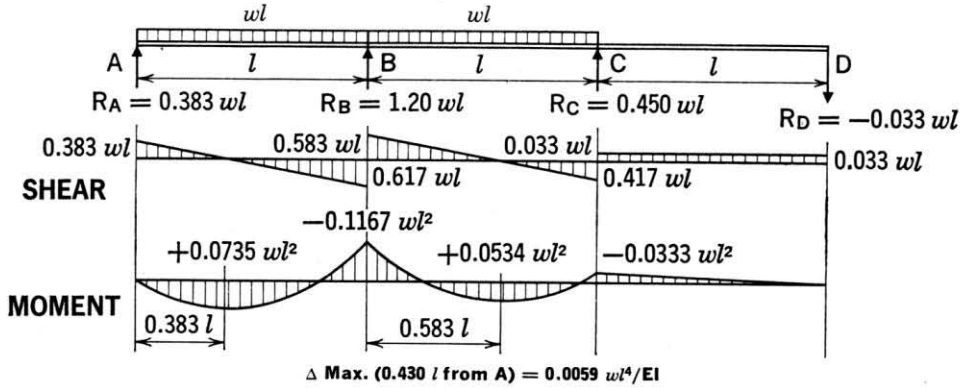
$R_1 = V_1 = \frac{wl}{2} + \frac{M_1 - M_2}{l}$
 $R_2 = V_2 = \frac{wl}{2} - \frac{M_1 - M_2}{l}$
 $V_x = w \left(\frac{l}{2} - x \right) + \frac{M_1 - M_2}{l}$
 $M_3 \text{ (at } x = \frac{l}{2} + \frac{M_1 - M_2}{wl} \text{)} = \frac{wl^2}{8} - \frac{M_1 + M_2}{2} + \frac{(M_1 - M_2)^2}{2wl^2}$
 $M_x = \frac{wx}{2} (l - x) + \left(\frac{M_1 - M_2}{l} \right) x - M_1$
 $b \text{ (To locate inflection points)} = \sqrt{\frac{l^2}{4} - \left(\frac{M_1 + M_2}{w} \right) + \left(\frac{M_1 - M_2}{wl} \right)^2}$
 $\Delta_x = \frac{wx}{24EI} \left[x^3 - \left(2l + \frac{4M_1}{wl} - \frac{4M_2}{wl} \right) x^2 + \frac{12M_1}{w} x + l^3 - \frac{8M_1l}{w} - \frac{4M_2l}{w} \right]$

33. BEAM—CONCENTRATED LOAD AT CENTER AND VARIABLE END MOMENTS

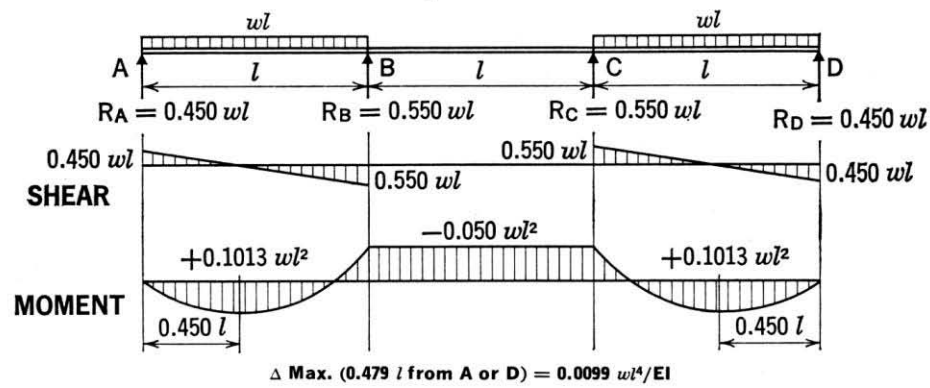


$R_1 = V_1 = \frac{P}{2} + \frac{M_1 - M_2}{l}$
 $R_2 = V_2 = \frac{P}{2} - \frac{M_1 - M_2}{l}$
 $M_3 \text{ (At center)} = \frac{Pl}{4} - \frac{M_1 + M_2}{2}$
 $M_x \text{ (When } x < \frac{l}{2} \text{)} = \left(\frac{P}{2} + \frac{M_1 - M_2}{l} \right) x - M_1$
 $M_x \text{ (When } x > \frac{l}{2} \text{)} = \frac{P}{2} (l - x) + \frac{(M_1 - M_2)x}{l} - M_1$
 $\Delta_x \text{ (When } x < \frac{l}{2} \text{)} = \frac{Px}{48EI} \left(3l^2 - 4x^2 - \frac{8(l-x)}{Pl} [M_1(2l-x) + M_2(l+x)] \right)$

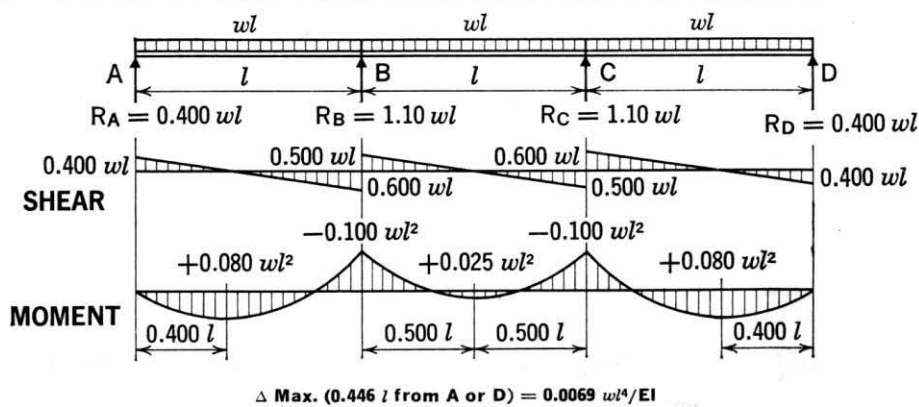
34. CONTINUOUS BEAM—THREE EQUAL SPANS—ONE END SPAN UNLOADED



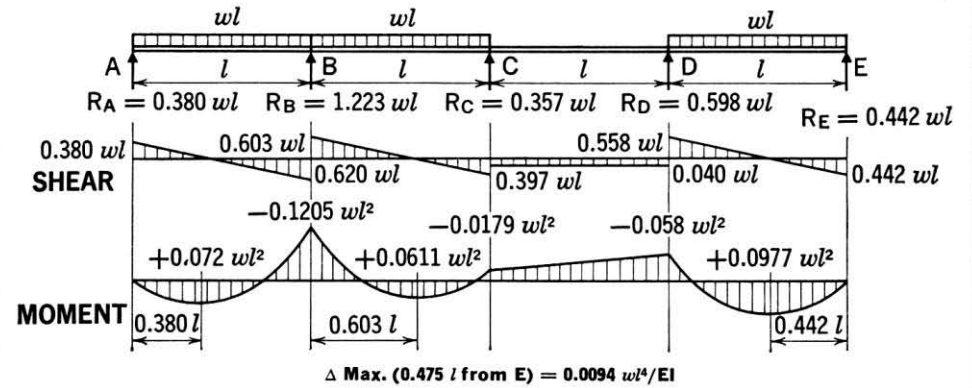
35. CONTINUOUS BEAM—THREE EQUAL SPANS—END SPANS LOADED



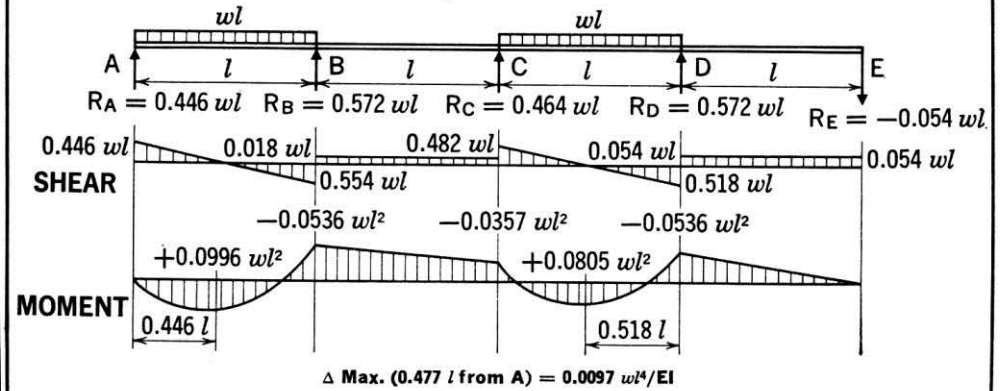
36. CONTINUOUS BEAM—THREE EQUAL SPANS—ALL SPANS LOADED



37. CONTINUOUS BEAM—FOUR EQUAL SPANS—THIRD SPAN UNLOADED



38. CONTINUOUS BEAM—FOUR EQUAL SPANS—LOAD FIRST AND THIRD SPANS



39. CONTINUOUS BEAM—FOUR EQUAL SPANS—ALL SPANS LOADED

