Examples: Reinforced Concrete

Example 1

A simply supported beam 20 ft long carries a service dead load of 300 lb/ft and a live load of 500 lb/ft. Design an appropriate beam (for flexure only). Use grade 40 steel and concrete strength of 5000 psi.

Maximum Reinforcement Ratio ρ for Singly Reinforced Rectangular Beams (tensile strain = 0.005)

f_y	$f'_c = 3000$ psi, $\beta_1 = 0.85$	$f'_c = 3500$ psi, $\beta_1 = 0.85$	$f'_c = 4000$ psi, $\beta_1 = 0.85$	$f'_c = 5000$ psi, $\beta_1 = 0.80$	$f'_c = 6000$ psi, $\beta_1 = 0.75$
40,000 psi	0.0203	0.0237	0.0271	0.0319	0.0359
50,000 psi	0.0163	0.0190	0.0217	0.0255	0.0287
60,000 psi	0.0135	0.0158	0.0181	0.0213	0.0239
f_y	$f'_c = 20$ MPa, $\beta_1 = 0.85$	$f'_c = 25$ MPa, $\beta_1 = 0.85$	$f'_c = 30$ MPa, $\beta_1 = 0.85$	$f'_c = 35$ MPa, $\beta_1 = 0.81$	$f'_c = 40$ MPa, $\beta_1 = 0.77$
300 MPa	0.0181	0.0226	0.0271	0.0301	0.0327
350 MPa	0.0155	0.0194	0.0232	0.0258	0.0281
400 MPa	0.0135	0.0169	0.0203	0.0226	0.0245
500 MPa	0.0108	0.0135	0.0163	0.0181	0.0196

SOLUTION:

Find the design moment, M_u, from the factored load combination of 1.2D + 1.6L. It is good practice to guess a beam size to include self weight in the dead load, because "service" means dead load of everything except the beam itself.

Guess a size of 10 in x 12 in. Self weight for normal weight concrete is the density of 150 lb/ft^3 multiplied by the cross section area: self weight = $150\, {}^{lb}\!/\!_{ft^3}(10in)(12in)\cdot(\frac{1ft}{12in})^2$ = 125 lb/ft

w_u = 1.2(300 lb/ft + 125 lb/ft) + 1.6(500 lb/ft) = 1310 lb/ft^2

The maximum moment for a simply supported beam is $\frac{wl^2}{8}$:

$M_u = \frac{w_u l^2}{8} = \frac{1310\, {}^{lb}\!/\!_{ft}(20ft)^2}{8}$ 65,500 lb-ft

M_n required = M_u/ϕ = $\frac{65{,}500^{lb-ft}}{0.9}$ = 72,778 lb-ft

To use the design chart aid, find $R_n = \frac{M_n}{bd^2}$, estimating that d is about 1.75 inches less than h:

d = 12in – 1.75 in = 10.25 in

$R_n = \frac{72{,}778^{lb-ft}}{(10in)(10.25in)^2}\cdot(12\, {}^{in}\!/\!_{ft})$ = 831 psi

Table 3.7.1
Total Areas for Various Numbers of Reinforcing Bars

Bar Size	Nominal Diameter (in.)	Weight (lb/ft)	Number of Bars 1	2	3	4	5	6	7	8	9	10
#3	0.375	0.376	0.11	0.22	0.33	0.44	0.55	0.66	0.77	0.88	0.99	1.10
#4	0.500	0.668	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
#5	0.625	1.043	0.31	0.62	0.93	1.24	1.55	1.86	2.17	2.48	2.79	3.10
#6	0.750	1.502	0.44	0.88	1.32	1.76	2.20	2.64	3.08	3.52	3.96	4.40
#7	0.875	2.044	0.60	1.20	1.80	2.40	3.00	3.60	4.20	4.80	5.40	6.00
#8	1.000	2.670	0.79	1.58	2.37	3.16	3.95	4.74	5.53	6.32	7.11	7.90
#9	1.128	3.400	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
#10	1.270	4.303	1.27	2.54	3.81	5.08	6.35	7.62	8.89	10.16	11.43	12.70
#11	1.410	5.313	1.56	3.12	4.68	6.24	7.80	9.36	10.92	12.48	14.04	15.60
#14*	1.693	7.65	2.25	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50
#18*	2.257	13.60	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00

* #14 and #18 bars are used primarily as column reinforcement and are rarely used in beams.

ρ corresponds to approximately 0.023 (which is less than that for 0.005 strain of 0.0319) , so the estimated area required, A_s, can be found:

$A_s = \rho bd$ = (0.023)(10in)(10.25in) = 2.36 in^2

The number of bars for this area can be found from handy charts.

(Whether the number of bars actually fit for the width with cover and space between bars <u>must also be considered</u>. If you are at ρ_{max} <u>do not</u> choose an area bigger than the maximum!)

Try A_s = 2.37 in^2 from 3#8 bars

d = 12 in – 1.5 in (cover) – ½ (8/8in diameter bar) = 10 in

Check ρ = 2.37 in^2/(10 in)(10 in) = 0.0237 which is less than $\rho_{max-0.005}$ = 0.0319 OK (We cannot have an over reinforced beam!!)

Find the moment capacity of the beam as designed, ϕM_n

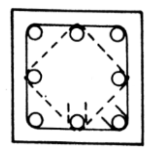
$a = A_s f_y / 0.85 f'_c b$ = 2.37 in^2 (40 ksi)/[0.85(5 ksi)10 in] = 2.23 in

$\phi M_n = \phi A_s f_y (d-a/2) = 0.9(2.37 in^2)(40 ksi)(10 in - \frac{2.23 in}{2}) \cdot (\frac{1}{12 \, ^{in}/_{ft}})$ = 63.2 k-ft $\not>$ 65.5 k-ft needed (not OK)

So, we can increase d to 13 in, and ϕM_n = 70.3 k-ft (OK). Or increase A_s to 2 # 10's (2.54 in^2), for a = 2.39 in and ϕM_n of 67.1 k-ft (OK). *Don't exceed ρ_{max} or $\rho_{max-0.005}$ if you want to use ϕ=0.9*

Example 2

Determine the capacity of a 16" x 16" column with 8- #10 bars, tied. Grade 40 steel and 4000 psi concrete.



SOLUTION:

Find ϕP_n, with ϕ=0.65 and $P_n = 0.80 P_o$ for tied columns and

$$P_o = 0.85 f'_c (A_g - A_{st}) + f_y A_{st}$$

Steel area (found from reinforcing bar table for the bar size):

A_{st} = 8 bars × (1.27 in^2) = 10.16 in^2

Concrete area (gross):

A_g = 16 in × 16 in = 256 in^2

ASTM STANDARD REINFORCING BARS

Bar size, no.	Nominal diameter, in.	Nominal area, in.^{2}	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.650
18	2.257	4.00	13.600

Grade 40 reinforcement has f_y = 40,000 psi and f'_c = 4000psi

ϕP_n = (0.65)(0.80)[0.85(4000 psi)(256 in^2 – 10.16 in^2) + (40,000 psi)(10.16 in^2)] = 646,026 lb = 646 kips