Reinforced Concrete Design

Notation:

a = depth of the effective compression block in a concrete beam
A = name for area
A_g = gross area, equal to the total area ignoring any reinforcement
A_s = area of steel reinforcement in concrete beam design
A'_s = area of steel compression reinforcement in concrete beam design
A_{st} = area of steel reinforcement in concrete column design
A_v = area of concrete shear stirrup reinforcement
ACI = American Concrete Institute
b = width, often cross-sectional
b_E = effective width of the flange of a concrete T beam cross section
b_f = width of the flange
b_w = width of the stem (web) of a concrete T beam cross section
cc = shorthand for clear cover
C = name for centroid
= name for a compression force
C_c = compressive force in the compression steel in a doubly reinforced concrete beam
C_s = compressive force in the concrete of a doubly reinforced concrete beam
d = effective depth from the top of a reinforced concrete beam to the centroid of the tensile steel
d' = effective depth from the top of a reinforced concrete beam to the centroid of the compression steel
d_b = bar diameter of a reinforcing bar
D = shorthand for dead load
DL = shorthand for dead load
E = modulus of elasticity or Young's modulus
= shorthand for earthquake load
E_c = modulus of elasticity of concrete
E_s = modulus of elasticity of steel
f = symbol for stress
f_c = compressive stress
f'_c = concrete design compressive stress
f_s = stress in the steel reinforcement for concrete design
f'_s = compressive stress in the compression reinforcement for concrete beam design
f_y = yield stress or strength
F = shorthand for fluid load
F_y = yield strength
G = relative stiffness of columns to beams in a rigid connection, as is Ψ
h = cross-section depth
H = shorthand for lateral pressure load
h_f = depth of a flange in a T section
$I_{transformed}$ = moment of inertia of a multi-material section transformed to one material
k = effective length factor for columns
ℓ_b = length of beam in rigid joint
ℓ_c = length of column in rigid joint
l_d = development length for reinforcing steel
l_{dh} = development length for hooks
l_n = clear span from face of support to face of support in concrete design
L = name for length or span length, as is l
= shorthand for live load
L_r = shorthand for live roof load
LL = shorthand for live load
M_n = nominal flexure strength with the steel reinforcement at the yield stress and concrete at the concrete design strength for reinforced concrete beam design
M_u = maximum moment from factored loads for LRFD beam design
n = modulus of elasticity transformation coefficient for steel to concrete
$n.a.$ = shorthand for neutral axis (N.A.)
pH = chemical alkalinity
P = name for load or axial force vector

P_o = maximum axial force with no concurrent bending moment in a reinforced concrete column

P_n = nominal column load capacity in concrete design

P_u = factored column load calculated from load factors in concrete design

R = shorthand for rain or ice load

R_n = concrete beam design ratio = M_u/bd^2

s = spacing of stirrups in reinforced concrete beams

S = shorthand for snow load

t = name for thickness

T = name for a tension force
= shorthand for thermal load

U = factored design value

V_c = shear force capacity in concrete

V_s = shear force capacity in steel shear stirrups

V_u = shear at a distance of *d* away from the face of support for reinforced concrete beam design

w_c = unit weight of concrete

w_{DL} = load per unit length on a beam from dead load

w_{LL} = load per unit length on a beam from live load

$w_{self\ wt}$ = name for distributed load from self weight of member

w_u = load per unit length on a beam from load factors

W = shorthand for wind load

x = horizontal distance
= distance from the top to the neutral axis of a concrete beam

y = vertical distance

β_1 = coefficient for determining stress block height, *a*, based on concrete strength, f'_c

Δ = elastic beam deflection

ε = strain

ϕ = resistance factor

ϕ_c = resistance factor for compression

γ = density or unit weight

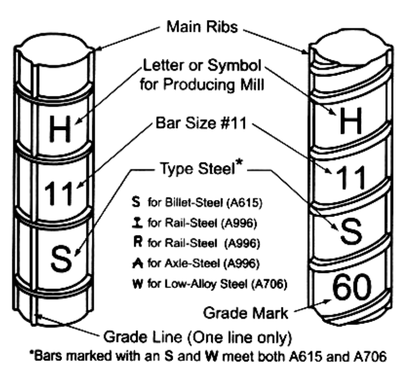
ρ = radius of curvature in beam deflection relationships
= reinforcement ratio in concrete beam design = A_s/bd

$\rho_{balanced}$ = balanced reinforcement ratio in concrete beam design

υ_c = shear strength in concrete design

Reinforced Concrete Design

Structural design standards for reinforced concrete are established by the *Building Code and Commentary (ACI 318-11)* published by the American Concrete Institute International, and uses ultimate strength design.



GRADE 60

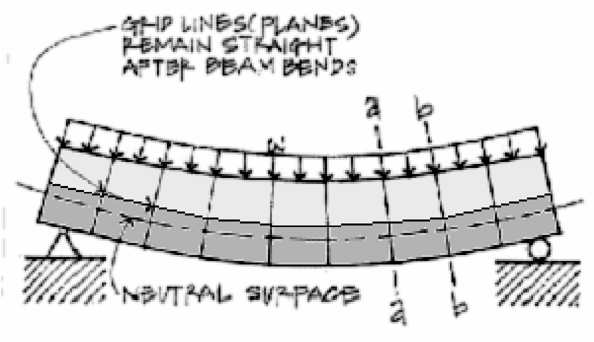
<u>Materials</u>

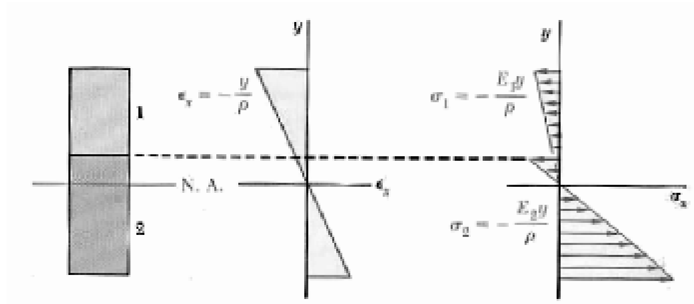
f'_c = concrete compressive design strength at 28 days (units of psi when used in equations)

Deformed reinforcing bars come in grades 40, 60 & 75 (for 40 ksi, 60 ksi and 75 ksi yield strengths). Sizes are given as # of 1/8" up to #8 bars. For #9 and larger, the number is a nominal size (while the actual size is larger).

Reinforced concrete is a composite material, and the average density is considered to be *150 lb/ft*^{3}. It has the properties that it will creep (deformation with long term load) and shrink (a result of hydration) that must be considered.

Plane sections of composite materials can still be assumed to be plane (strain is linear), *but* the stress distribution *is not* the same in both materials because the *modulus of elasticity* is different. (f=E·ε)





$$f_1 = E_1\varepsilon = -\frac{E_1 y}{\rho} \qquad f_2 = E_2\varepsilon = -\frac{E_2 y}{\rho}$$

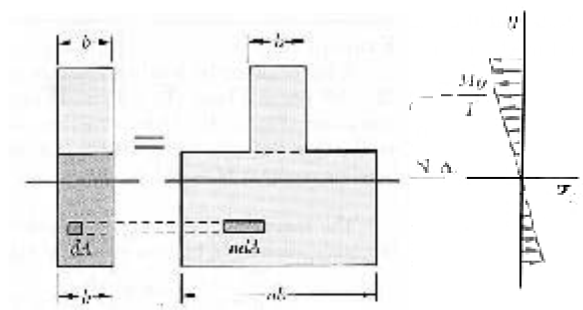
In order to determine the stress, we can define *n* as the ratio of the elastic moduli:

$$n = \frac{E_2}{E_1}$$

n is used to <u>transform</u> the <u>width</u> of the second material such that it sees the equivalent element stress.

Transformed Section y and I

In order to determine stresses in all types of material in the beam, we transform the materials into a single material, and calculate the location of the neutral axis and modulus of inertia for that material.



ex: When material 1 above is concrete and material 2 is steel:

to transform steel into concrete $n = \frac{E_2}{E_1} = \frac{E_{steel}}{E_{concrete}}$

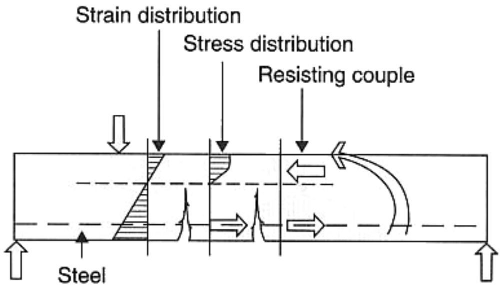
to find the neutral axis of the equivalent concrete member we transform the width of the steel by multiplying by *n*

to find the moment of inertia of the equivalent concrete member, $I_{transformed}$, use the new geometry resulting from transforming the width of the steel

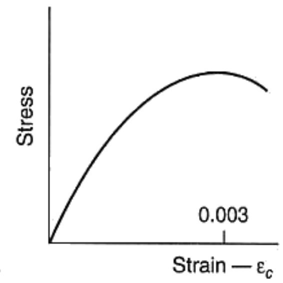
concrete stress: $f_{concrete} = -\frac{My}{I_{transformed}}$

steel stress: $f_{steel} = -\frac{Myn}{I_{transformed}}$

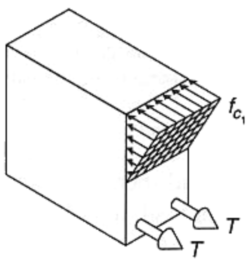
Reinforced Concrete Beam Members



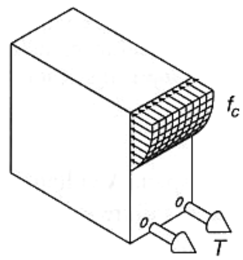
Stresses in the concrete above the neutral axis are compressive and nonlinearly distributed. In the tension zone below the neutral axis, the concrete is assumed to be cracked and the tensile force present to be taken up by reinforcing steel.



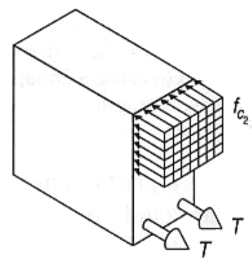
Typical stress-strain curve for concrete.



Working stress analysis. (Concrete stress distribution is assumed to be linear. Service loads are used in calculations.)



Actual stress distribution near ultimate strength (nonlinear).



Ultimate strength analysis. (A rectangular stress block is used to idealize the actual stress distribution. Calculations are based on ultimate loads and failure stresses.)

Ultimate Strength Design for Beams

The ultimate strength design method is similar to LRFD. There is a *nominal* strength that is reduced by a factor ϕ which must exceed the factored design stress. For beams, the concrete only works in compression over a rectangular "stress" block above the n.a. from elastic calculation, and the steel is exposed and reaches the yield stress, F_y

For stress analysis in reinforced concrete beams

- the steel is transformed to concrete
- any concrete in tension is assumed to be cracked and to have no strength
- the steel can be in tension, and is placed in the bottom of a beam that has positive bending moment

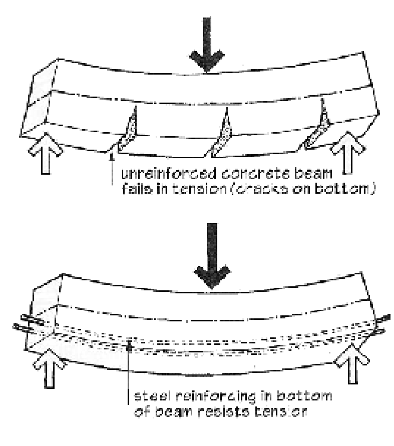
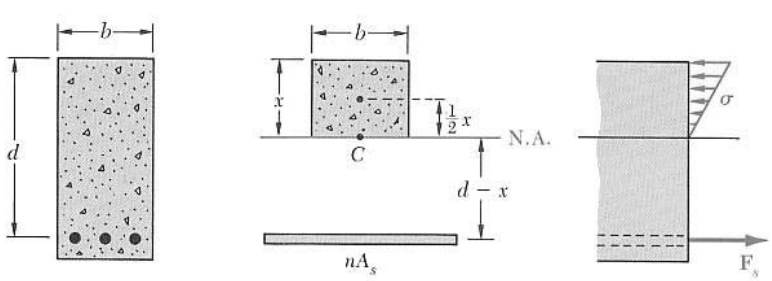


Figure 8.5: Bending in a concrete beam without and with steel reinforcing.



The neutral axis is where there is no stress and no strain. The concrete above the n.a. is in compression. The concrete below the n.a. is considered ineffective. The steel below the n.a. is in tension.

Because the n.a. is defined by the moment areas, we can solve for x knowing that d is the distance from the top of the concrete section to the centroid of the steel:

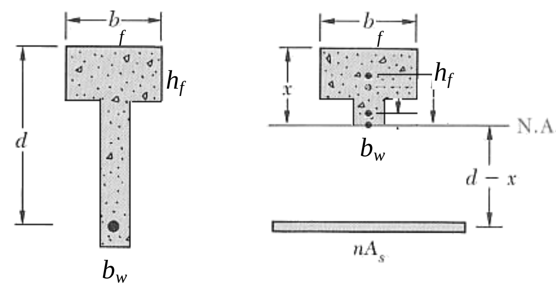
$$bx \cdot \frac{x}{2} - nA_s(d-x) = 0$$

x can be solved for when the equation is rearranged into the generic format with a, b & c in the binomial equation: $ax^2 + bx + c = 0$ by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

T-sections

If the n.a. is *above* the bottom of a flange in a T section, x is found as for a rectangular section.

If the n.a. is *below* the bottom of a flange in a T section, x is found by including the flange and the stem of the web (b_w) in the moment area calculation:



$$b_f h_f \left(x - \frac{h_f}{2}\right) + (x - h_f)b_w \frac{(x - h_f)}{2} - nA_s(d - x) = 0$$

Load Combinations - (Alternative values allowed)

1.4*D*

1.2*D* + 1.6*L* + 0.5(L_r or *S* or *R*)

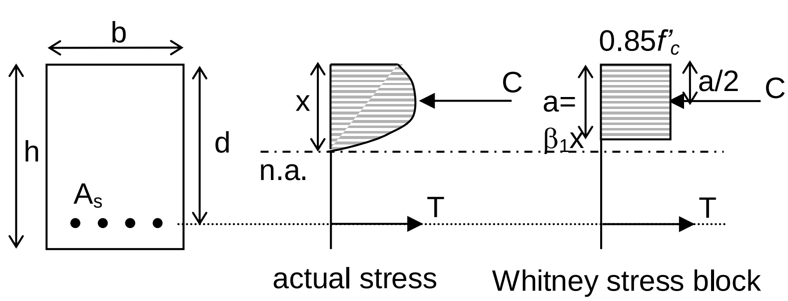
1.2*D* + 1.6(L_r or *S* or *R*) + (1.0*L* or 0.5*W*)

1.2*D* + 1.0*W* + 1.0*L* + 0.5(L_r or *S* or *R*)

1.2*D* + 1.0*E* + 1.0*L* + 0.2*S*

0.9*D* + 1.0*W*

0.9*D* + 1.0*E*



Internal Equilibrium

ASTM STANDARD REINFORCING BARS

Bar size, no.	Nominal diameter, in.	Nominal area, in.^{2}	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.650
18	2.257	4.00	13.600

C = compression in concrete = stress x area = $0.85 f'_c ba$
T = tension in steel = stress x area = $A_s f_y$

$C = T$ and $M_n = T(d-a/2)$

where f'_c = concrete compression strength
a = height of stress block
b = width of stress block
f_y = steel yield strength
A_s = area of steel reinforcement
d = effective depth of section
(depth to n.a. of reinforcement)

With C=T, $A_s f_y = 0.85 f'_c ba$ so *a* can be determined with $a = \dfrac{A_s f_y}{0.85 f'_c b}$

Criteria for Beam Design

For flexure design:

$M_u \leq \phi M_n$ $\phi = 0.9$ *for flexure (when the section is* <u>*tension*</u> *controlled)*
so, M_u *can be set* $= \phi M_n = \phi T(d-a/2) = \phi A_s f_y (d-a/2)$

Reinforcement Ratio

The amount of steel reinforcement is *limited.* Too much reinforcement, or *over-reinforced* will not allow the steel to yield before the concrete crushes and there is a sudden failure. A beam with the proper amount of steel to allow it to yield at failure is said to be *under reinforced*.

The reinforcement ratio is just a fraction: $\rho = \dfrac{A_s}{bd}$ (or p) and must be less than a value determined with a concrete strain of 0.003 and tensile strain of 0.004 (minimum). When the strain in the reinforcement is 0.005 or greater, the section is **tension controlled**. (For smaller strains the resistance factor reduces to 0.65 – see tied columns - because the stress is less than the yield stress in the steel.) Previous codes limited the amount to $0.75\rho_{balanced}$ where $\rho_{balanced}$ was determined from the amount of steel that would make the concrete start to crush at the exact same time that the steel would yield based on strain.

Flexure Design of Reinforcement

One method is to "wisely" estimate a height of the stress block, *a*, and solve for A_s, and calculate a new value for *a* using M_u.

1. guess *a* (less than n.a.)
2. $A_s = \dfrac{0.85 f'_c ba}{f_y}$
3. solve for *a* from

   $M_u = \phi A_s f_y\,(d\text{-}a/2)$:

   $$a = 2\left(d - \frac{M_u}{\phi A_s f_y}\right)$$

4. repeat from 2. until *a* found from step 3 matches *a* used in step 2.

Maximum Reinforcement Ratio ρ for Singly Reinforced Rectangular Beams (tensile strain = 0.005) for which ϕ is permitted to be 0.9

f_y	$f'_c = 3000$ psi, $\beta_1 = 0.85$	$f'_c = 3500$ psi, $\beta_1 = 0.85$	$f'_c = 4000$ psi, $\beta_1 = 0.85$	$f'_c = 5000$ psi, $\beta_1 = 0.80$	$f'_c = 6000$ psi, $\beta_1 = 0.75$
40,000 psi	0.0203	0.0237	0.0271	0.0319	0.0359
50,000 psi	0.0163	0.0190	0.0217	0.0255	0.0287
60,000 psi	0.0135	0.0158	0.0181	0.0213	0.0239
f_y	$f'_c = 20$ MPa, $\beta_1 = 0.85$	$f'_c = 25$ MPa, $\beta_1 = 0.85$	$f'_c = 30$ MPa, $\beta_1 = 0.85$	$f'_c = 35$ MPa, $\beta_1 = 0.81$	$f'_c = 40$ MPa, $\beta_1 = 0.77$
300 MPa	0.0181	0.0226	0.0271	0.0301	0.0327
350 MPa	0.0155	0.0194	0.0232	0.0258	0.0281
400 MPa	0.0135	0.0169	0.0203	0.0226	0.0245
500 MPa	0.0108	0.0135	0.0163	0.0181	0.0196

Design Chart Method:

1. calculate $R_n = \dfrac{M_n}{bd^2}$
2. find curve for f'_c and f_y to get ρ
3. calculate A_s and *a*

   $A_s = \rho bd$ *and* $a = \dfrac{A_s f_y}{0.85 f'_c b}$

Any method can simplify the size of d using h = 1.1d

from Reinforced Concrete, 7th,
Wang, Salmon, Pincheira, Wiley & Sons, 2007

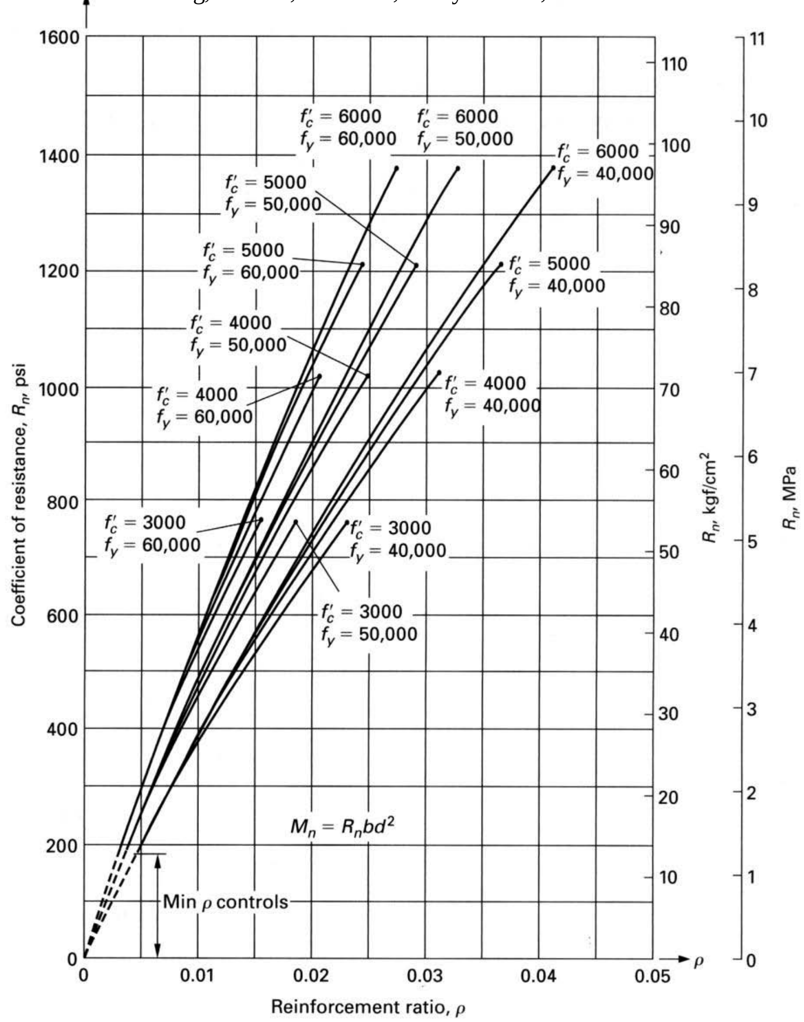


Figure 3.8.1 Strength curves (R_n vs ρ) for singly reinforced rectangular sections. Upper limit of curves is at ρ_{max}. (tensile strain of 0.004)

Maximum Reinforcement

Based on the limiting strain of 0.005 in the steel, *x(or c) = 0.375d* so

$a = \beta_1(0.375d)$ to find $A_{s\text{-}max}$

(β_1 is shown in the table above)

Minimum Reinforcement

Minimum reinforcement is provided even if the concrete can resist the tension. This is a means to control cracking.

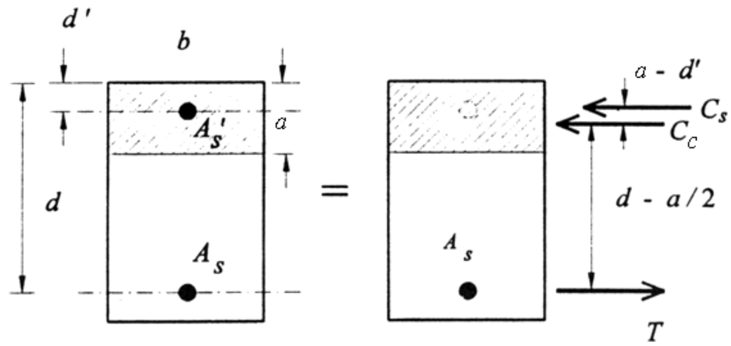
Minimum required: $A_s = \dfrac{3\sqrt{f'_c}}{f_y}(b_w d)$

but not less than: $A_s = \dfrac{200}{f_y}(b_w d)$

where f'_c is in psi. This can be translated to $\rho_{min} = \dfrac{3\sqrt{f'_c}}{f_y}$ but not less than $\dfrac{200}{f_y}$

Compression Reinforcement

If a section is *doubly reinforced*, it means there is steel in the beam seeing compression. The force in the compression steel at yield is equal to stress x area, $C_s = A_c \cdot F_y$. The total compression that balances the tension is now: $T = C_c + C_s$. And the moment taken about the centroid of the compression stress is $M_n = T(d-a/2)+C_s(a-d')$



where A_s' is the area of compression reinforcement, and d' is the effective depth to the centroid of the compression reinforcement

T-sections (pan joists)

T beams have an effective width, b_E, that sees compression stress in a wide flange beam or joist in a slab system.

For *interior* T-sections, b_E is the smallest of

L/4, b_w + 16t, or center to center of beams

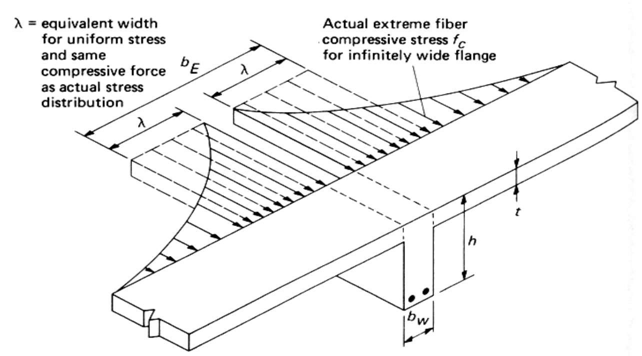


Figure 9.3.1 Actual and equivalent stress distribution over flange width.

For *exterior* T-sections, b_E is the smallest of

b_w + L/12, b_w + 6t, or b_w + ½(clear distance to next beam)

When the **web** is in tension the minimum reinforcement required is the same as for rectangular sections with the web width (b_w) in place of *b*.

When the **flange** is in tension (negative bending), the minimum reinforcement required is the greater value of $A_s = \frac{6\sqrt{f'_c}}{f_y}(b_w d)$ or $A_s = \frac{3\sqrt{f'_c}}{f_y}(b_f d)$

where f'_c is in psi, b_w is the beam width, and b_f is the effective flange width

Cover for Reinforcement

Cover of concrete over/under the reinforcement must be provided to protect the steel from corrosion. For indoor exposure, 3/4 inch is required for slabs, 1.5 inch is typical for beams, and for concrete cast against soil, 3 inches is typical.

Bar Spacing

Minimum bar spacings are specified to allow proper consolidation of concrete around the reinforcement.

Slabs

One way slabs can be designed as "one unit"-wide beams. Because they are thin, control of deflections is important, and minimum depths are specified, as is minimum reinforcement for shrinkage and crack control when not in flexure. Reinforcement is commonly small diameter bars and welded wire fabric. Maximum spacing between bars is also specified for shrinkage and crack control as five times the slab thickness not exceeding 18". For required flexure reinforcement spacing the limit is three times the slab thickness not exceeding 18".

TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE COMPUTED

	Minimum thickness, *h*			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

Notes:
Values given shall be used directly for members with normalweight concrete and Grade 60 reinforcement. For other conditions, the values shall be modified as follows:
a) For lightweight concrete having equilibrium density, w_c, in the range of 90 to 115 lb/ft^3, the values shall be multiplied by $(1.65 - 0.005w_c)$ but not less than 1.09.
b) For f_y other than 60,000 psi, the values shall be multiplied by $(0.4 + f_y/100{,}000)$.

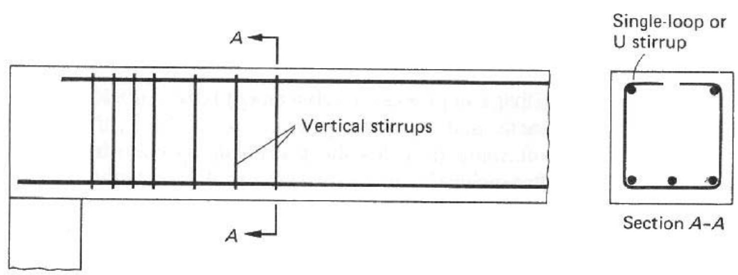
Shrinkage and temperature reinforcement (and minimum for flexure reinforcement):

Minimum for slabs with grade 40 or 50 bars: $\rho = \frac{A_s}{bt} = 0.002$ *or* $A_{s\text{-}min} = 0.002bt$

Minimum for slabs with grade 60 bars: $\rho = \frac{A_s}{bt} = 0.0018$ *or* $A_{s\text{-}min} = 0.0018bt$

Shear Behavior

Horizontal shear stresses occur along with bending stresses to cause tensile stresses where the concrete cracks. Vertical reinforcement is required to bridge the cracks which are called *shear stirrups*.



The maximum shear for design, V_u is the value at a distance of *d* from the face of the support.

Nominal Shear Strength

The shear force that can be resisted is the shear stress × cross section area: $V_c = \nu_c \times b_w d$

The shear stress for beams (one way) $\nu_c = 2\sqrt{f'_c}$ so $\phi V_c = \phi 2\sqrt{f'_c}\, b_w d$

where b_w = the beam width or the minimum width of the stem.

One-way joists are allowed an increase of 10% V_c if the joists are closely spaced.

Stirrups are necessary for strength (as well as crack control): $V_s = \frac{A_v f_y d}{s}$

where A_v = area of all vertical legs of stirrup
s = spacing of stirrups
d = effective depth

For shear design:

$$V_U \le \phi V_C + \phi V_S \quad \phi = 0.75 \text{ for shear}$$

Spacing Requirements

Stirrups are required when V_u is greater than $\frac{\phi V_c}{2}$

Table 3-8 ACI Provisions for Shear Design*

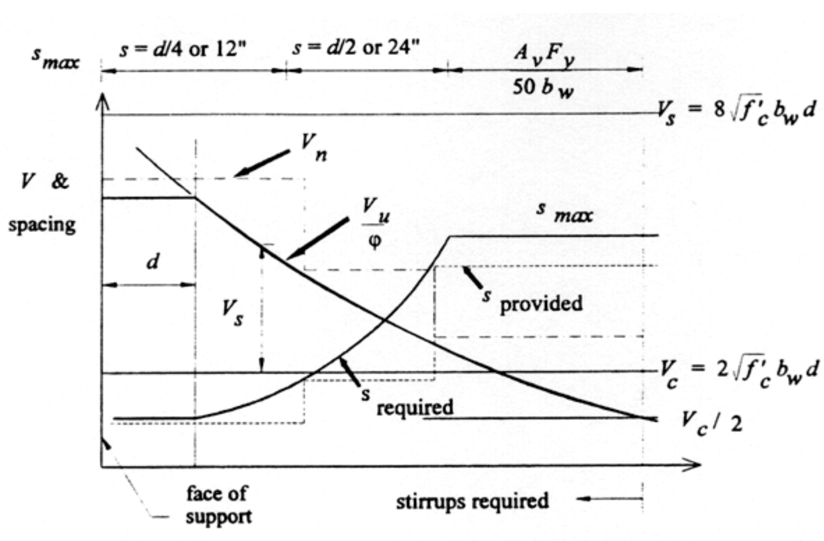
		$V_u \le \frac{\phi V_c}{2}$	$\phi V_c \ge V_u > \frac{\phi V_c}{2}$	$V_u > \phi V_c$
Required area of stirrups, A_v**		none	$\frac{50 b_w s}{f_y}$	$\frac{(V_u - \phi V_c)s}{\phi f_y d}$
Stirrup spacing, s	Required	—	$\frac{A_v f_y}{50 b_w}$	$\frac{\phi A_v f_y d}{V_u - \phi V_c}$
	Recommended Minimum†	—	—	4 in.
	Maximum†† (ACI 11.5.4)	—	$\frac{d}{2}$ or 24 in.	$\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \le \phi 4\sqrt{f'_c}\, b_w d$
				$\frac{d}{4}$ or 12 in. for $(V_u - \phi V_c) > \phi 4\sqrt{f'_c}\, b_w d$

*Members subjected to shear and flexure only; $\phi V_c = \phi 2\sqrt{f'_c}\, b_w d$, $\phi = 0.75$ (ACI 11.3.1.1)
**$A_v = 2 \times A_b$ for U stirrups; $f_y \le 60$ ksi (ACI 11.5.2)
†A practical limit for minimum spacing is d/4
††Maximum spacing based on minimum shear reinforcement (= $A_v f_y/50 b_w$) must also be considered (ACI 11.5.5.3).

Economical spacing of stirrups is considered to be greater than d/4. Common spacings of d/4, d/3 and d/2 are used to determine the values of ϕV_s at which the spacings can be increased.

$$\phi V_s = \frac{\phi A_v f_y d}{s}$$

This figure shows the size of V_n provided by $V_c + V_s$ (long dashes) exceeds V_u/ϕ in a step-wise function, while the spacing provided (short dashes) is at or less than the required s (limited by the maximum allowed). *(Note that the maximum shear permitted from the stirrups is* $8\sqrt{f'_c}\, b_w d$



The minimum recommended spacing for the first stirrup is 2 inches from the face of the support.

Torsional Shear Reinforcement

On occasion beam members will see twist along the access caused by an eccentric shape supporting a load, like on an L-shaped spandrel (edge) beam. The torsion results in shearing stresses, and closed stirrups may be needed to resist the stress that the concrete cannot resist.

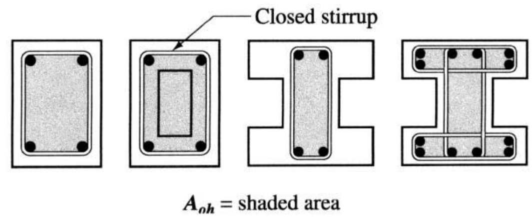


Fig. R11.6.3.6(b)—Definition of $\mathbf{A_{oh}}$

Development Length for Reinforcement

Because the design is based on the reinforcement attaining the yield stress, the reinforcement needs to be properly bonded to the concrete for a finite length (*both sides*) so it won't slip. This is referred to as the development length, l_d. Providing sufficient length to anchor bars that need to reach the yield stress near the end of connections are also specified by hook lengths. *Detailing reinforcement is a tedious job.* Splices are also necessary to extend the length of reinforcement that come in standard lengths. The equations are not provided here.

Development Length in Tension

With the proper bar to bar spacing and cover, the common development length equations are:

#6 bars and smaller: $l_d = \dfrac{d_b F_y}{25\sqrt{f'_c}}$ or 12 in. minimum

#7 bars and larger: $l_d = \dfrac{d_b F_y}{20\sqrt{f'_c}}$ or 12 in. minimum

Development Length in Compression

$$l_d = \frac{0.02 d_b F_y}{\sqrt{f'_c}} \le 0.0003 d_b F_y$$

Hook Bends and Extensions

The minimum hook length is $l_{dh} = \dfrac{1200 d_b}{\sqrt{f'_c}}$

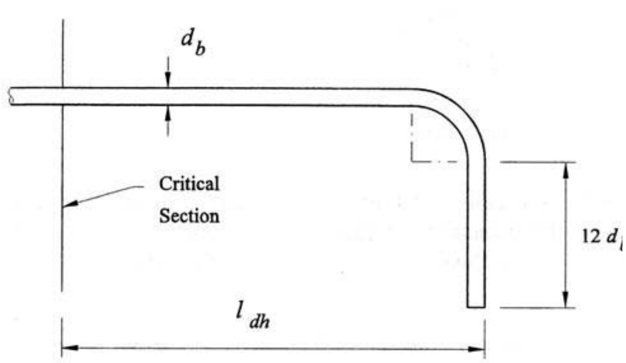


Figure 9-17: Minimum requirements for 90° bar hooks.

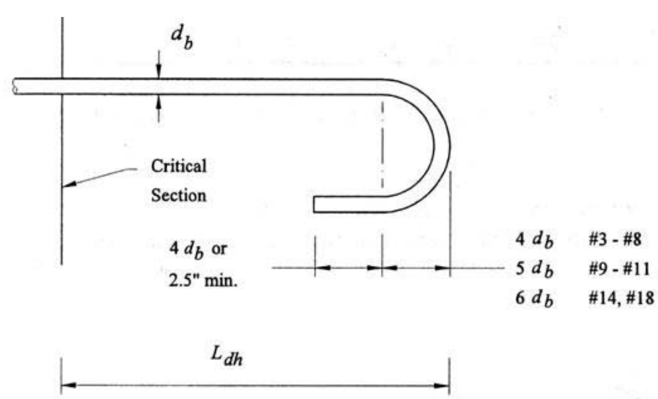


Figure 9-18: Minimum requirements for 180° bar hooks.

Modulus of Elasticity & Deflection

E_c for deflection calculations can be used with the transformed section modulus <u>in the elastic range</u>. After that, the cracked section modulus is calculated and E_c is adjusted.

Code values:

$$E_c = 57{,}000\sqrt{f'_c}\ \textit{(normal weight)} \qquad E_c = w_c^{1.5}33\sqrt{f'_c}\text{, } w_c = 90\ lb/ft^3 - 160\ lb/ft^3$$

Deflections of beams and one-way slabs need not be computed if the overall member thickness meets the minimum specified by the code, and are shown in Table 9.5(a) (see *Slabs*).

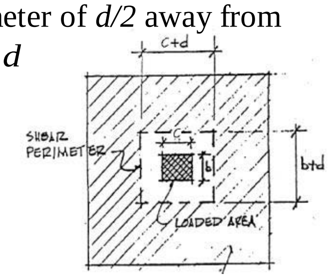
Criteria for Flat Slab & Plate System Design

Systems with slabs and supporting beams, joists or columns typically have multiple bays. The horizontal elements can act as one-way or two-way systems. Most often the flexure resisting elements are continuous, having positive and negative bending moments. These moment and shear values can be found using beam tables, or from code specified approximate design factors. Flat slab two-way systems have drop panels (for shear), while flat plates do not.

Two way shear at columns is resisted by the thickness of the slab at a perimeter of *d/2* away from the face of the support by the shear stress × cross section area: $V_c = \nu_c \times b_o d$

The shear stress (two way) $\nu_c = 4\sqrt{f'_c}$ so $\phi V_c = \phi 4\sqrt{f'_c}\, b_o d$

where b_o = perimeter length.



Criteria for Column Design

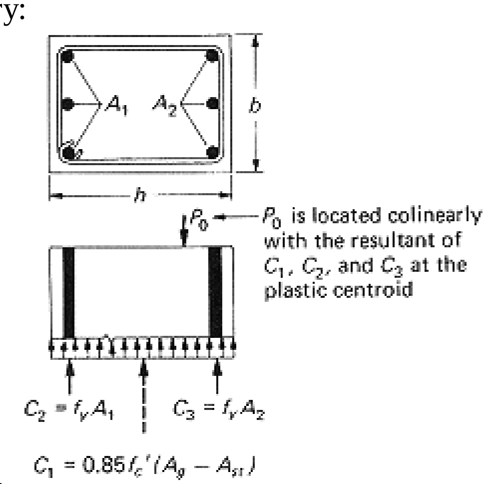
(American Concrete Institute) ACI 318-11 Code and Commentary:

$P_u \le \phi_c P_n$ where

P_u is a <u>factored load</u>

ϕ is a <u>resistance factor</u>

P_n is the <u>nominal load capacity (strength)</u>



Load combinations, ex:
- 1.4D (D is dead load)
- 1.2D + 1.6L (L is live load)
- 1.2D + 1.6L + 0.5W (W is wind load)
- 0.90D + 1.0W

For compression, ϕ_c = 0.75 and $P_n = 0.85P_o$ for spirally reinforced, ϕ_c = 0.65 $P_n = 0.8P_o$ for tied columns where $P_o = 0.85f'_c(A_g - A_{st}) + f_y A_{st}$ and P_o is the name of the maximum axial force with no concurrent bending moment.

Columns which have reinforcement ratios, $\rho_g = A_{st}/A_g$, in the range of 1% to 2% will usually be the most economical, with 1% as a minimum and 8% as a maximum by code.

Bars are symmetrically placed, typically.

Columns with Bending (Beam-Columns)

Concrete columns rarely see only axial force and must be designed for the combined effects of axial load and bending moment. The *interaction* diagram shows the reduction in axial load a column can carry with a bending moment.

Design aids commonly present the interaction diagrams in the form of load vs. equivalent eccentricity for standard column sizes and bars used.

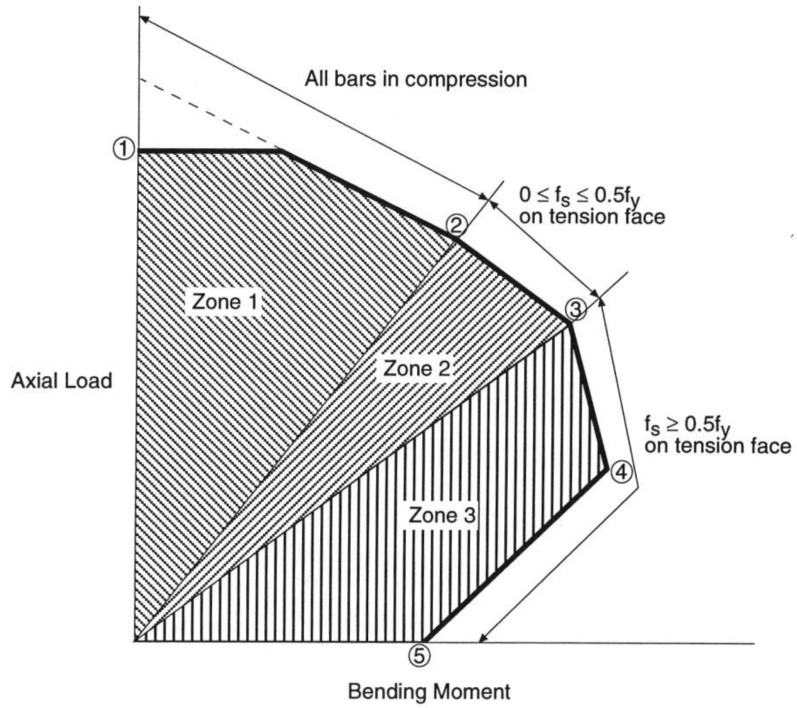


Figure 5-3 Transition Stages on Interaction Diagram

Eccentric Design

The strength interaction diagram is dependant upon the strain developed in the steel reinforcement.

If the strain in the steel is less than the yield stress, the section is said to be *compression controlled.*

Below the *transition zone*, where the steel starts to yield, and when the net tensile strain in the reinforcement exceeds 0.005 the section is said to be *tension controlled.* This is a ductile condition and is preferred.

Rigid Frames

Monolithically cast frames with beams and column elements will have members with shear, bending and axial loads. Because the joints can rotate, the effective length must be determined from methods like that presented in the handout on Rigid Frames. The charts for evaluating k for non-sway and sway frames can be found in the ACI code.

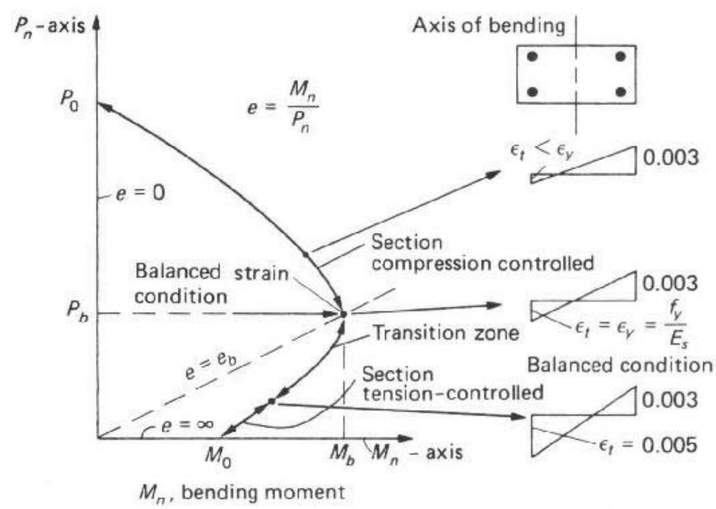
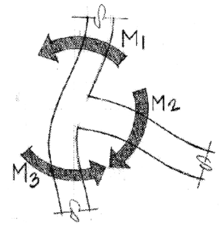


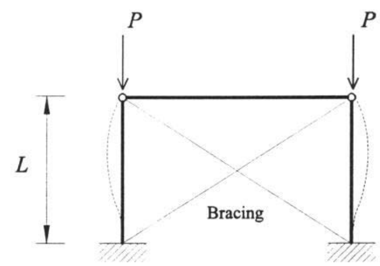
Figure 13.6.1 Typical strength interaction diagram for axial compression and bending moment about one axis. Transition zone is where $\epsilon_y \le \epsilon_t \le 0.005$.

Frame Columns

Because joints can rotate in frames, the effective length of the column in a frame is harder to determine. The stiffness (EI/L) of each member in a joint determines how rigid or flexible it is. To find k, the relative stiffness, G or Ψ, must be found for both ends, plotted on the alignment charts, and connected by a line for braced and unbraced fames.

$$G = \Psi = \frac{\Sigma EI/l_c}{\Sigma EI/l_b}$$



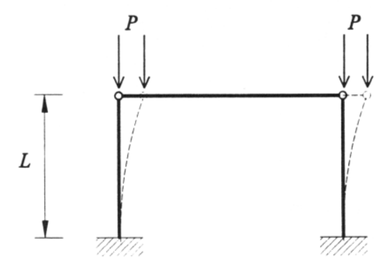


Braced – non-sway frame

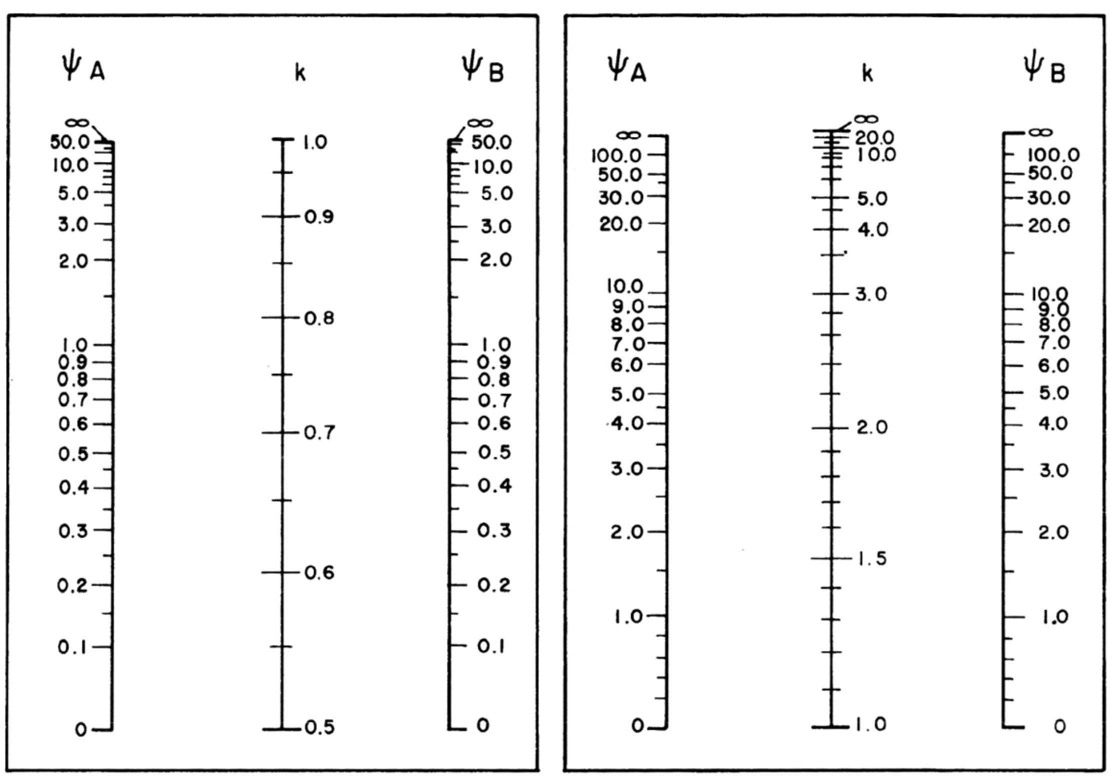
where

E = modulus of elasticity for a member
I = moment of inertia of for a member
l_c = length of the column from center to center
l_b = length of the beam from center to center

- For pinned connections we typically use a value of 10 for Ψ.
- For fixed connections we typically use a value of 1 for Ψ.



Unbraced – sway frame



(a) Nonsway Frames

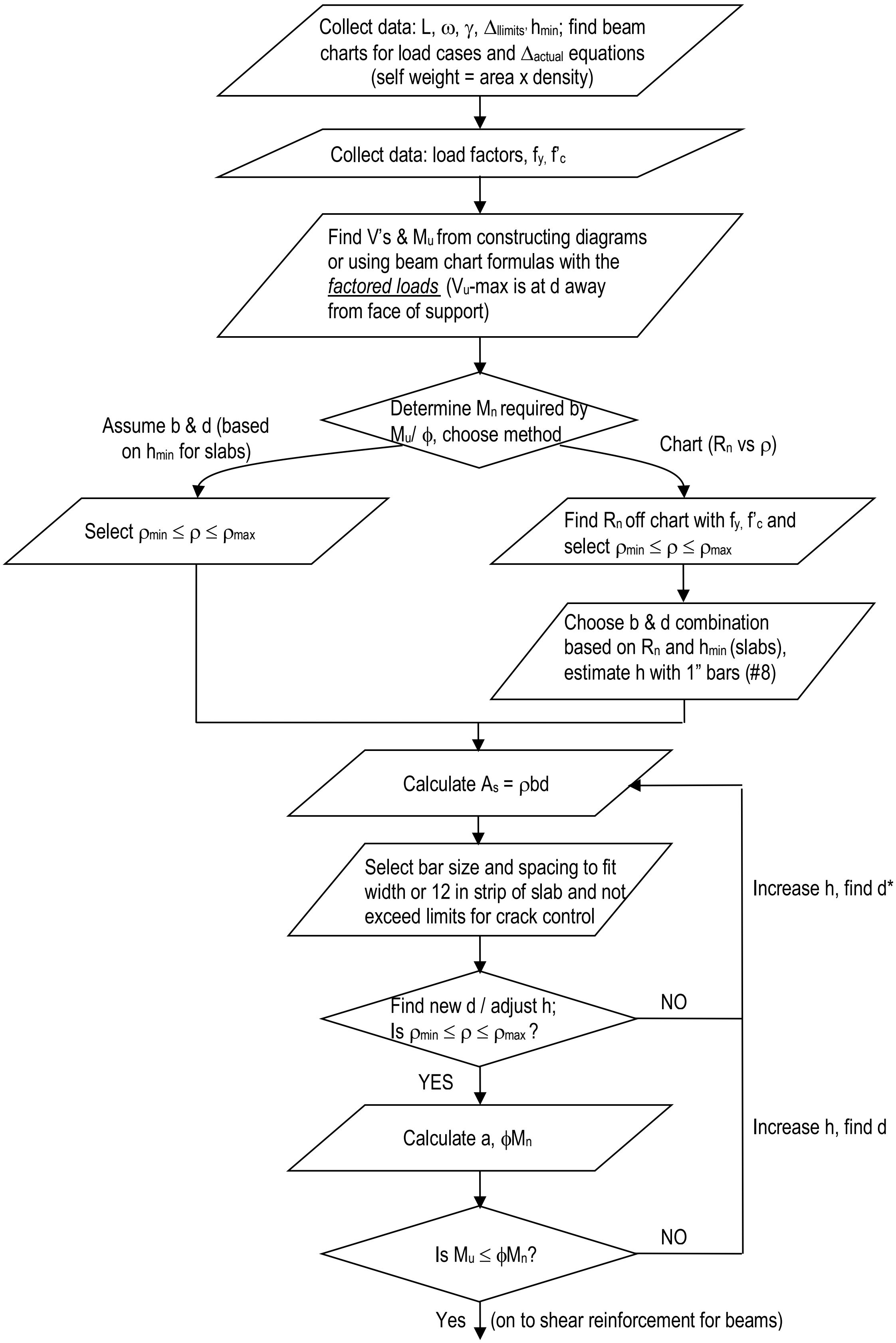
(b) Sway Frames

Factored Moment Resistance of Concrete Beams, ϕM_n (k-ft) with f'_c = 4 ksi, f_y = 60 ksi[a]

	Approximate Values for a/d		
	0.1	0.2	0.3
	Approximate Values for ρ		
b x d (in)	0.0057	0.01133	0.017
10 x 14	2 #6	2 #8	3 #8
	53	90	127
10 x 18	3 #5	2 #9	3 #9
	72	146	207
10 x 22	2 #7	3 #8	(3 #10)
	113	211	321
12 x 16	2 #7	3 #8	4 #8
	82	154	193
12 x 20	2 #8	3 #9	4 #9
	135	243	306
12 x 24	2 #8	3 #9	(4 #10)
	162	292	466
15 x 20	3 #7	4 #8	5 #9
	154	256	383
15 x 25	3 #8	4 #9	4 #11
	253	405	597
15 x 30	3 #8	5 #9	(5 #11)
	304	608	895
18 x 24	3 #8	5 #9	6 #10
	243	486	700
18 x 30	3 #9	6 #9	(6 #11)
	385	729	1074
18 x 36	3 #10	6 #10	(7 #11)
	586	1111	1504
20 x 30	3 # 10	7 # 9	6 # 11
	489	851	1074
20 x 35	4 #9	5 #11	(7 #11)
	599	1106	1462
20 x 40	6 #8	6 #11	(9 #11)
	811	1516	2148
24 x 32	6 #8	7 #10	(8 #11)
	648	1152	1528
24 x 40	6 #9	7 #11	(10 #11)
	1026	1769	2387
24 x 48	5 #10	(8 #11)	(13 #11)
	1303	2426	3723

[a]Table yields values of factored moment resistance in kip-ft with reinforcement indicated. Reinforcement choices shown in parentheses require greater width of beam or use of two stack layers of bars. (*Adapted and corrected from Simplified Engineering for Architects and Builders, 11th ed, Ambrose and Tripeny, 2010.*

Beam / One-Way Slab Design Flow Chart



Beam / One-Way Slab Design Flow Chart - continued