

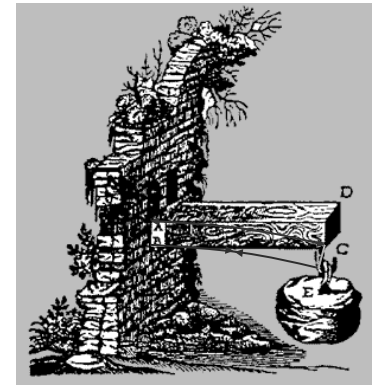
lecture  
nine

beams:  
bending and shear stress



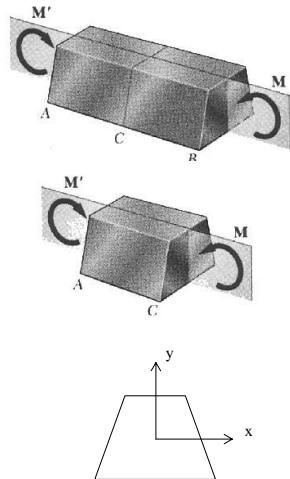
Beam Bending

- Galileo
  - relationship between stress and depth<sup>2</sup>
- can see
  - top squishing
  - bottom stretching
- what are the stress across the section?



Pure Bending

- bending only
- no shear
- axial normal stresses from bending can be found in
  - homogeneous materials
  - plane of symmetry
  - follow Hooke's law



Bending Moments

- sign convention:
  - concave up (+)
  - concave down (-)
  - shear force (V) and bending moment (M) diagram
- size of maximum internal moment will govern our design of the section

# Normal Stresses

- *geometric fit*
  - plane sections remain plane
  - stress varies linearly

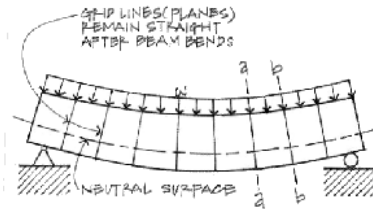


Figure 8.5(b) Beam bending under load.

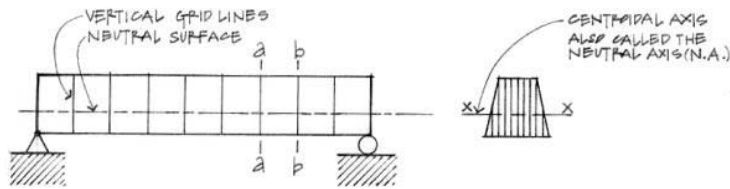


Figure 8.5(a) Beam elevation before loading.

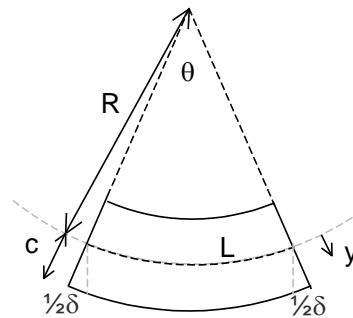
Beam cross section.

# Derivation of Stress from Strain

- pure bending = arc shape

$$L = R\theta$$

$$L_{outside} = (R + y)\theta$$



$$\epsilon = \frac{\delta}{L} = \frac{L_{outside} - L}{L} = \frac{(R + y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

# Neutral Axis

- stresses vary linearly
- zero stress occurs at the centroid
- neutral axis is line of centroids (n.a.)

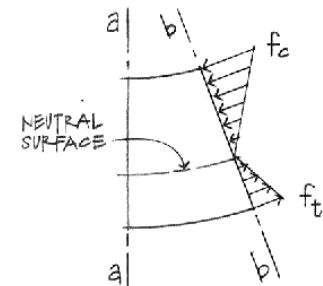


Figure 8.8 Bending stresses on section b-b.

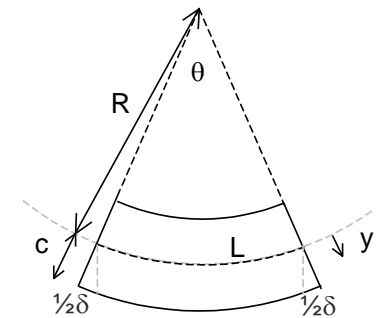
# Derivation of Stress

- zero stress at n.a.

$$f = E\epsilon = \frac{Ey}{R}$$

$$f_{max} = \frac{Ec}{R}$$

$$f = \frac{y}{c} f_{max}$$



## Bending Moment

- resultant moment from stresses = bending moment!

$$M = \sum fy\Delta A$$

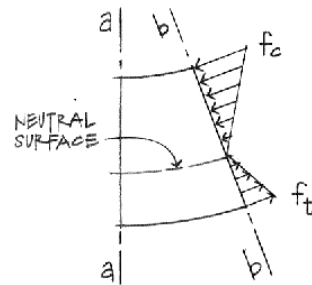


Figure 8.8 Bending stresses on section b-b.

$$= \sum \frac{yf_{max}}{c} y\Delta A = \frac{f_{max}}{c} \sum y^2 \Delta A = \frac{f_{max}}{c} I = f_{max} S$$

## Bending Stress Relations

$$\frac{1}{R} = \frac{M}{EI}$$

curvature

$$f_b = \frac{My}{I}$$

general bending stress

$$S = \frac{I}{c}$$

section modulus

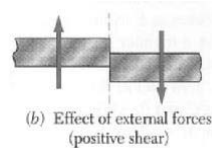
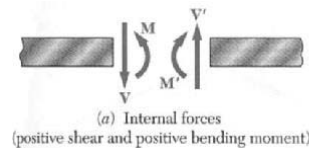
$$f_b = \frac{M}{S}$$

maximum bending stress

$$S_{required} \geq \frac{M}{F_b}$$

required section modulus for design

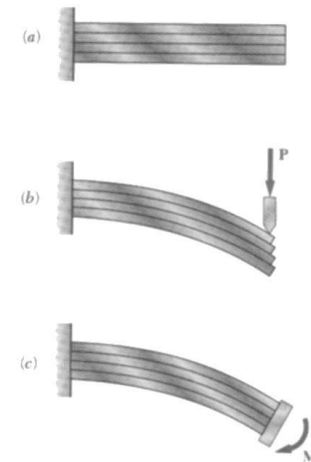
## Transverse Loading and Shear



- perpendicular loading
- internal shear
- along with bending moment

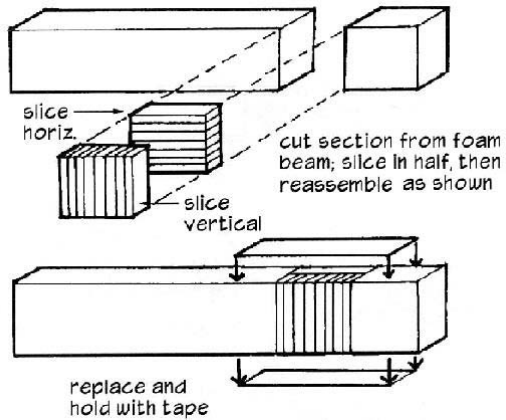
## Bending vs. Shear in Design

- bending stresses dominate
- shear stresses exist horizontally with shear
- no shear stresses with pure bending



# Shear Stresses

- horizontal & vertical



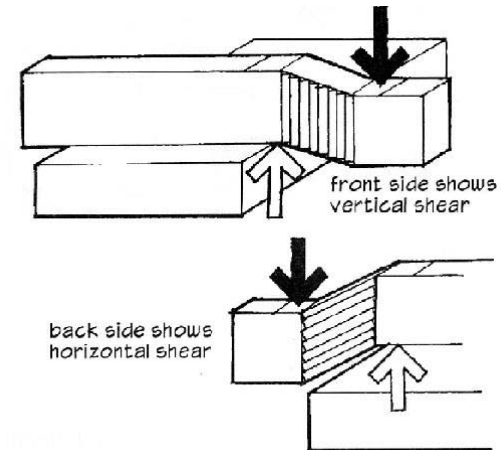
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# Shear Stresses

- horizontal & vertical



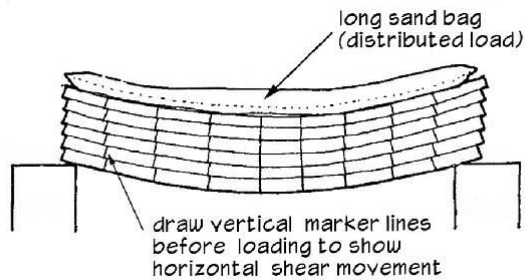
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# Beam Stresses

- horizontal with bending



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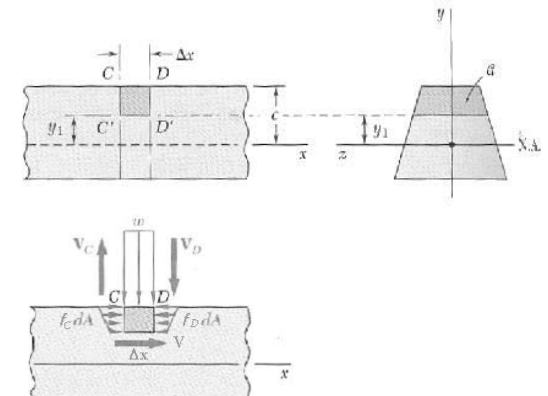
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# Equilibrium

- horizontal force  $V$  needed

$$V_{longitudinal} = \frac{V_T Q}{I} \Delta x$$



- $Q$  is a moment area

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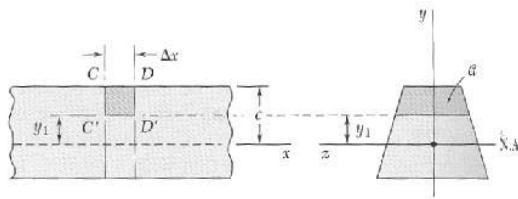
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## Moment of Area

- $Q$  is a moment area with respect to the n.a. of area above or below the horizontal

- $Q_{max}$  at  $y=0$  (neutral axis)



- $q$  is shear flow:

$$q = \frac{V_{longitudinal}}{\Delta x} = \frac{V_T Q}{I}$$

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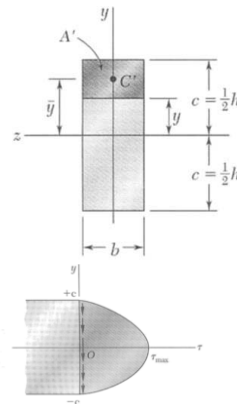
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## Rectangular Sections

$$I = \frac{bh^3}{12} \quad Q = A\bar{y} = \frac{bh^2}{8}$$

$$f_v = \frac{VQ}{Ib} = \frac{3V}{2A}$$

- $f_{v-max}$  occurs at n.a.



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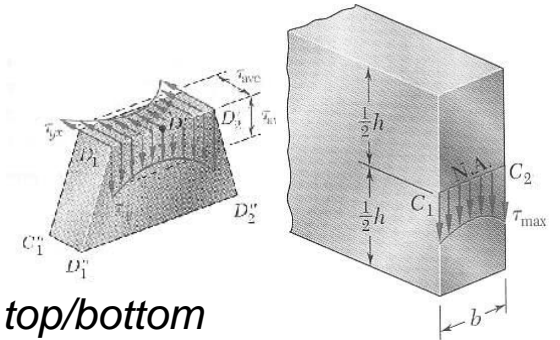
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## Shearing Stresses

$$f_v = \frac{V}{\Delta A} = \frac{V}{b \cdot \Delta x}$$

$$f_{v-ave} = \frac{VQ}{Ib}$$



- $f_{v-ave} = 0$  on the top/bottom
- $b$  min may not be with  $Q$  max
- with  $h/4 \geq b$ ,  $f_{v-max} \leq 1.008 f_{v-ave}$

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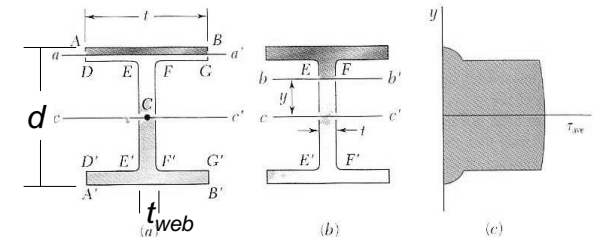
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## Steel Beam Webs

- $W$  and  $S$  sections

–  $b$  varies



– stress in flange negligible

– presume constant stress in web

$$f_{v-max} = \frac{3V}{2A} \approx \frac{V}{A_{web}}$$

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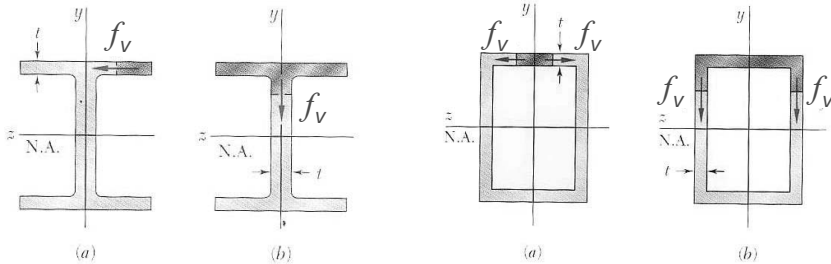
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## Shear Flow

- loads applied in plane of symmetry
- cut made perpendicular

$$q = \frac{VQ}{I}$$



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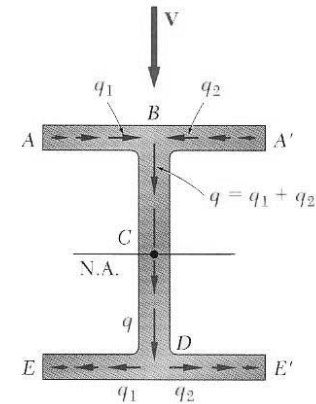
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## Shear Flow Quantity

- sketch from Q

$$q = \frac{VQ}{I}$$



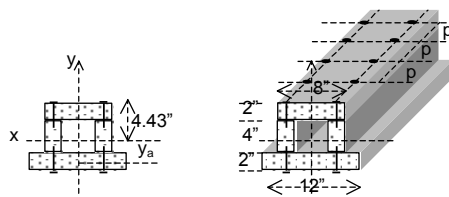
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## Connectors Resisting Shear

- plates with
  - nails
  - rivets
  - bolts
- splices



$$\frac{V_{longitudinal}}{p} = \frac{VQ}{I}$$

$$nF_{connector} \geq \frac{VQ_{connected\ area}}{I} \cdot p$$

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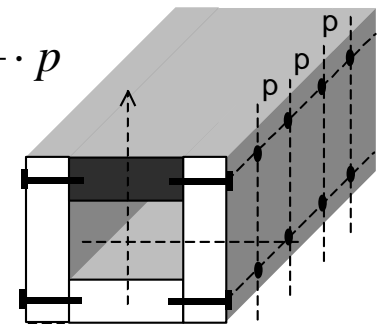
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## Vertical Connectors

- isolate an area with vertical interfaces

$$nF_{connector} \geq \frac{VQ_{connected\ area}}{I} \cdot p$$



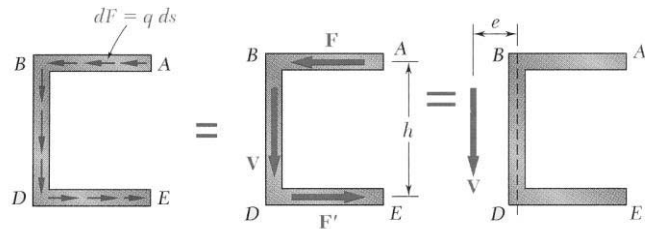
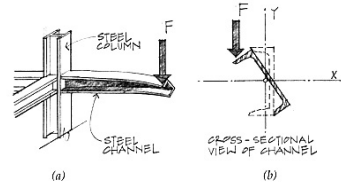
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# Unsymmetrical Shear or Section

- member can bend and twist
  - not symmetric
  - shear not in that plane
- shear center
  - moments balance



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