

lecture
eight



beam sections -
geometric properties

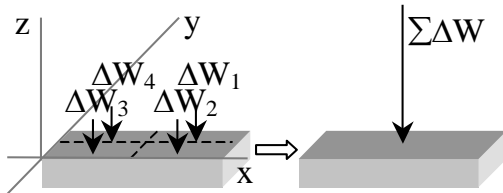
Sections 1
Lecture 8

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Center of Gravity

- “average” x & y from moment



$$\sum M_y = \sum_{i=1}^n x_i \Delta W_i = \bar{x} W \Rightarrow \bar{x} = \frac{\sum (x \Delta W)}{W}$$

“bar” means average

$$\sum M_x = \sum_{i=1}^n y_i \Delta W_i = \bar{y} W \Rightarrow \bar{y} = \frac{\sum (y \Delta W)}{W}$$

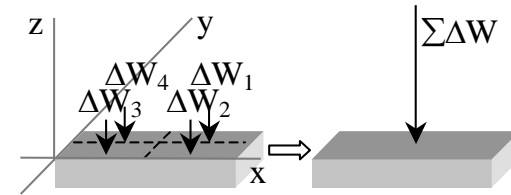
Centroids 7

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Center of Gravity

- location of equivalent weight
- determined with calculus



- sum element weights $W = \int dW$

Centroids 6

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Centroid

- “average” x & y of an area
- for a volume of constant thickness
 - $\Delta W = \gamma \Delta A$ where γ is weight/volume
 - center of gravity = centroid of area

$$\bar{x} = \frac{\sum (x \Delta A)}{A}$$

$$\bar{y} = \frac{\sum (y \Delta A)}{A}$$



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Centroid

- for a line, sum up length

$$\bar{x} = \frac{\sum(x\Delta L)}{L}$$

$$\bar{y} = \frac{\sum(y\Delta L)}{L}$$



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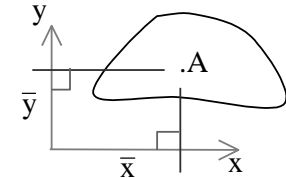
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1st Moment Area

- math concept
- the moment of an area about an axis

$$Q_x = \bar{y}A$$

$$Q_y = \bar{x}A$$



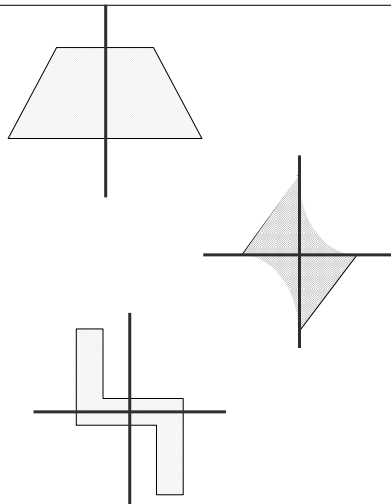
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Symmetric Areas

- symmetric about an axis
- symmetric about a center point
- mirrored symmetry



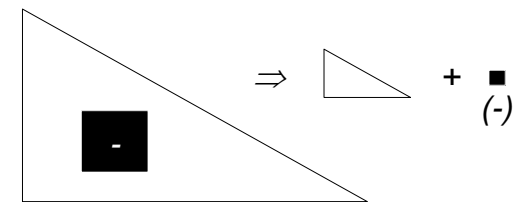
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Composite Areas

- made up of basic shapes
- areas can be negative
- (centroids can be negative for any area)



Centroids 12

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Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate \hat{x} and \hat{y}

Component	Area	\bar{x}	$\bar{x}A$	\bar{y}	$\bar{y}A$
Σ					

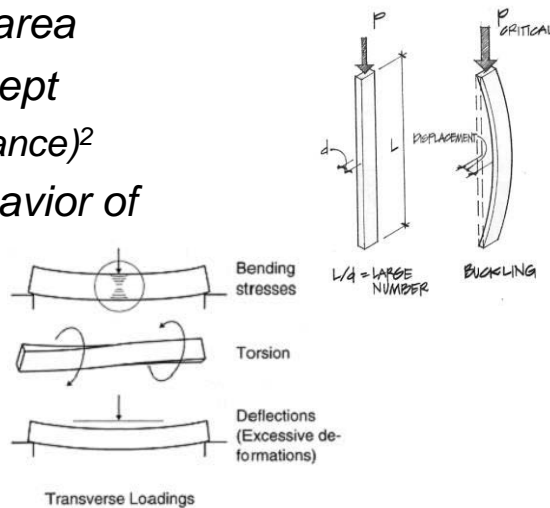
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Moments of Inertia

- 2nd moment area
 - math concept
 - area \times (distance)²
- need for behavior of
 - beams
 - columns



Moment of Inertia 4
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Area Centroids

- Figure A.1 – pg 598

Centroids of Common Shapes of Areas and Lines

Shape		\bar{x}	\bar{y}
Triangular area		$\frac{b}{3}$	$\frac{h}{3}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Semicircular area		0	$\frac{4r}{3\pi}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$
Parabolic area		0	$\frac{3h}{5}$

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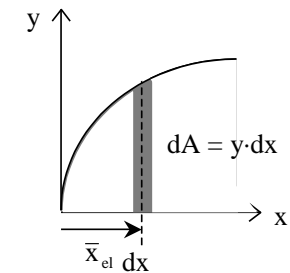
Moment of Inertia

- about any reference axis
- can be negative

$$I_y = \sum x_i^2 \Delta A = \int x^2 dA$$

$$I_x = \sum y_i^2 \Delta A = \int y^2 dA$$

(or $I_{x-x} = \sum z^2 a$)



- resistance to bending and buckling

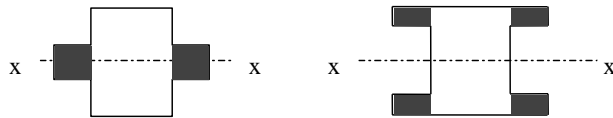
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Moment of Inertia

- same area moved away a distance
– larger I



Moment of Inertia 6
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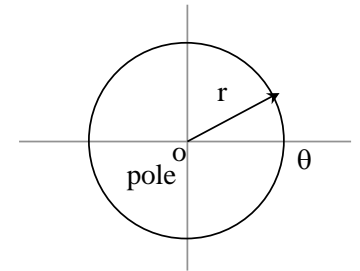
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Polar Moment of Inertia

- for roundish shapes
- uses polar coordinates (r and θ)
- resistance to twisting

$$J_o = \int r^2 dA$$



Moment of Inertia 7
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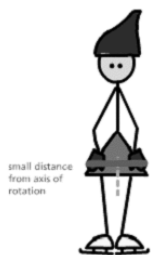
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Radius of Gyration

- measure of inertia with respect to area

$$r_x = \sqrt{\frac{I_x}{A}}$$



small distance
from axis of
rotation

When a figure skater changes position, he or she is redistributing his or her mass. Thus, every position has its own unique rotational inertia.



arms & hands
further
from axis
of rotation

The rotational inertia of the figure skater increases when her arms are raised because more of her mass is redistributed further from her axis of rotation.

Moment of Inertia 8
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Parallel Axis Theorem

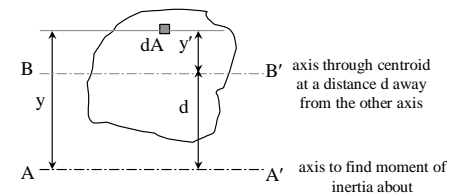
- can find composite I once composite centroid is known (basic shapes)

$$I = I_o + Az^2$$

$$= \bar{I}_x + Ad_y^2$$

$$I = \sum \bar{I} + \sum Ad^2$$

$$\bar{I} = I - Ad^2$$



axis through centroid
at a distance d away
from the other axis

axis to find moment of
inertia about

Moment of Inertia 9
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Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with A , \bar{x} , $\bar{x}A$, \bar{y} , $\bar{y}A$, \bar{I} 's, d 's, and Ad^2 's
5. Fill in table and get \hat{x} and \hat{y} for composite
6. Sum necessary columns
7. Sum I 's and Ad^2 's

$$\begin{aligned} (d_x &= \hat{x} - \bar{x}) \\ (d_y &= \hat{y} - \bar{y}) \end{aligned}$$

Moment of Inertia 8
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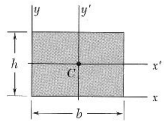
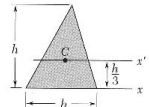
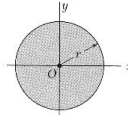
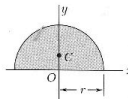
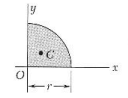
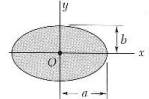
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Area Moments of Inertia

- Figure A.11 – pg. 611: (bars refer to centroid)

- x, y
- x', y'
- C

Rectangle		$\begin{aligned} \bar{I}_x &= \frac{1}{12}bh^3 \\ \bar{I}_y &= \frac{1}{12}b^3h \\ I_x &= \frac{1}{3}bh^3 \\ I_y &= \frac{1}{3}b^3h \\ J_C &= \frac{1}{12}bh(b^2 + h^2) \end{aligned}$
Triangle		$\begin{aligned} \bar{I}_x &= \frac{1}{36}bh^3 \\ \bar{I}_y &= \frac{1}{72}bh^3 \end{aligned}$
Circle		$\begin{aligned} \bar{I}_x &= \bar{I}_y = \frac{1}{4}\pi r^4 \\ J_O &= \frac{1}{2}\pi r^4 \end{aligned}$
Semicircle		$\begin{aligned} I_x &= I_y = \frac{1}{8}\pi r^4 \\ J_O &= \frac{1}{4}\pi r^4 \end{aligned}$
Quarter circle		$\begin{aligned} I_x &= I_y = \frac{1}{16}\pi r^4 \\ J_O &= \frac{1}{8}\pi r^4 \end{aligned}$
Ellipse		$\begin{aligned} \bar{I}_x &= \frac{1}{4}\pi ab^3 \\ \bar{I}_y &= \frac{1}{4}\pi a^3b \\ J_O &= \frac{1}{4}\pi ab(a^2 + b^2) \end{aligned}$

Moment of Inertia 11
Lecture 11

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