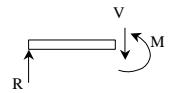
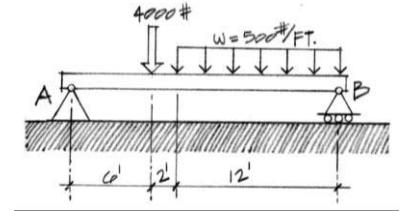




# beam introduction & internal forces

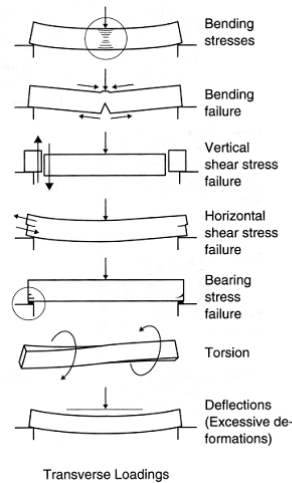
## Beams

- span horizontally
  - floors
  - bridges
  - roofs
- loaded transversely by gravity loads
- may have internal axial force
- will have internal shear force
- will have internal moment (bending)



## Beams

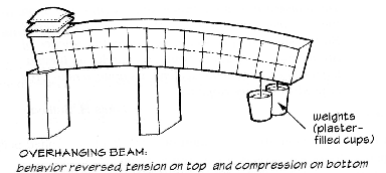
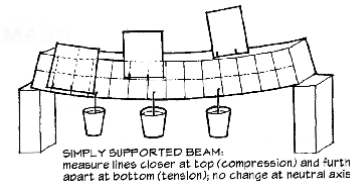
- transverse loading
- sees:
  - bending
  - shear
  - deflection
  - torsion
  - bearing
- behavior depends on cross section shape



Transverse Loadings

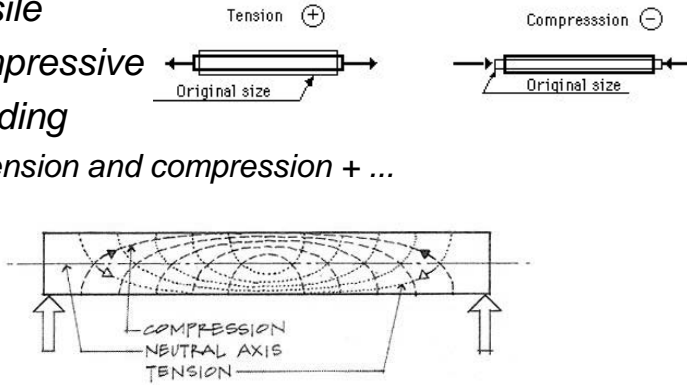
## Beams

- bending
  - bowing of beam with loads
  - one edge surface stretches
  - other edge surface squishes



# Beam Stresses

- stress = relative force over an area
  - tensile
  - compressive
  - bending
    - tension and compression + ...

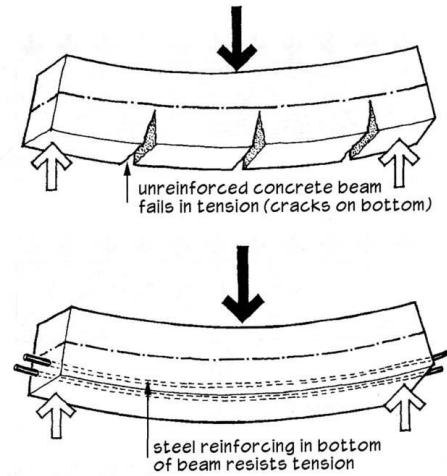


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# Beam Stresses



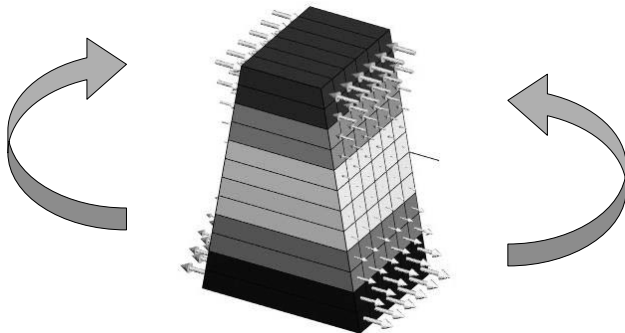
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# Beam Stresses

- tension and compression
  - causes moments



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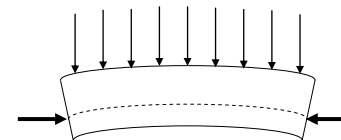
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# Beam Stresses

- prestress or post-tensioning
  - put stresses in tension area to “pre-compress”



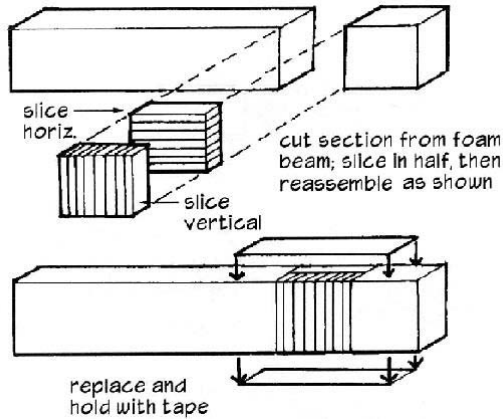
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# Beam Stresses

- shear – horizontal & vertical



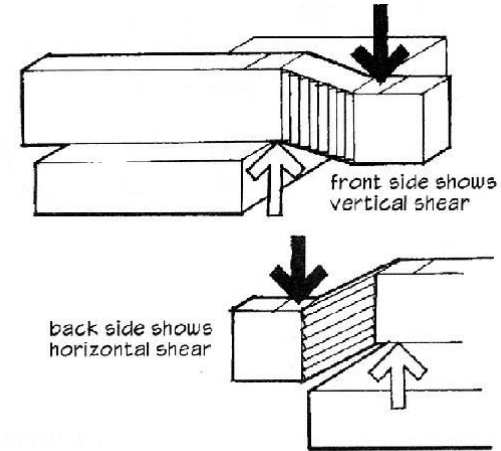
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# Beam Stresses

- shear – horizontal & vertical



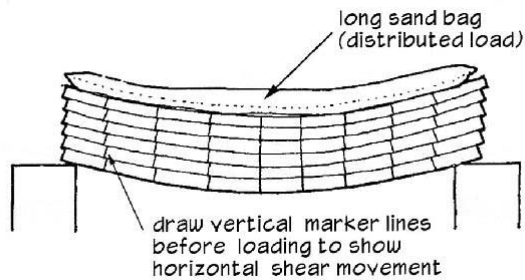
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# Beam Stresses

- shear – horizontal



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# Beam Deflections

- depends on
  - load
  - section
  - material

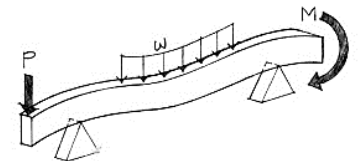
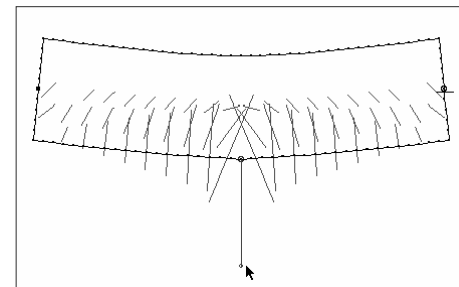


Figure 5.4 Bending (flexural) loads on a beam.



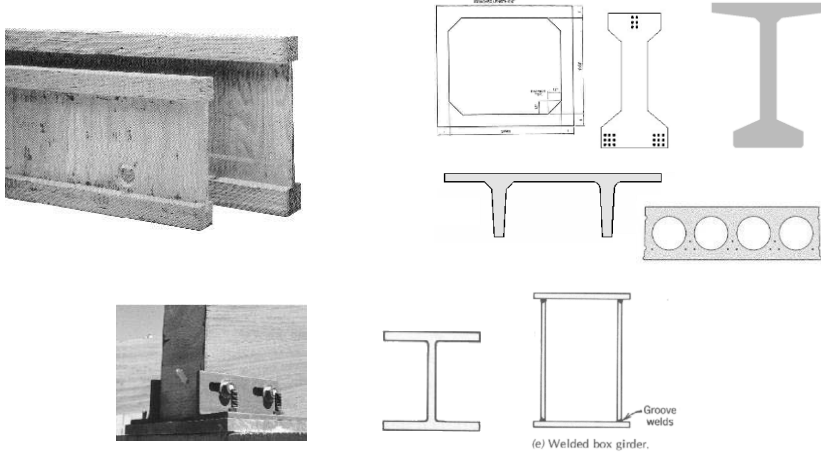
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# Beam Deflections

- “moment of inertia”



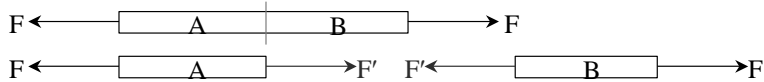
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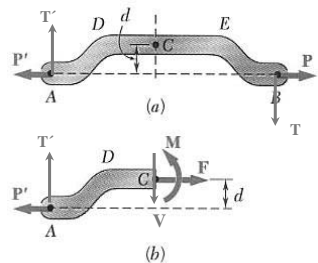
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# Internal Forces

- trusses
  - axial only, (compression & tension)



- in general
  - axial force
  - shear force,  $V$
  - bending moment,  $M$



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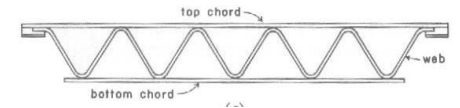
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# Beam Styles

- vierendeel
- open web joists
- manufactured



<http://nisee.berkeley.edu/godden>



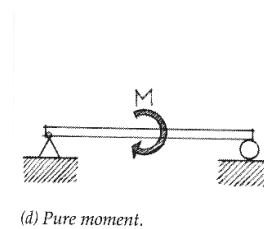
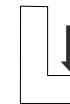
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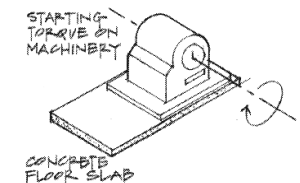
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# Beam Loading

- concentrated force
- concentrated moment
  - spandrel beams



(d) Pure moment.



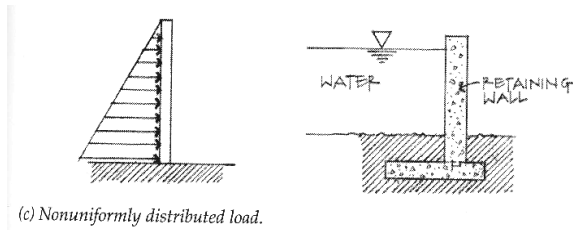
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## Beam Loading

- *uniformly distributed load (line load)*
- *non-uniformly distributed load*
  - hydrostatic pressure =  $\gamma h$
  - wind loads



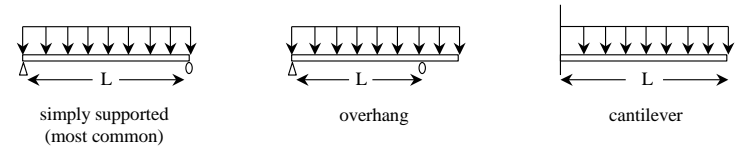
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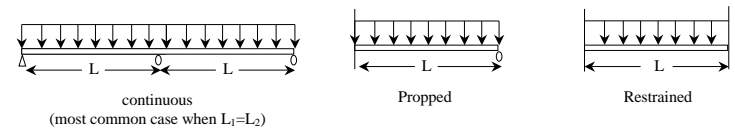
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## Beam Supports

- *statically determinate*



- *statically indeterminate*



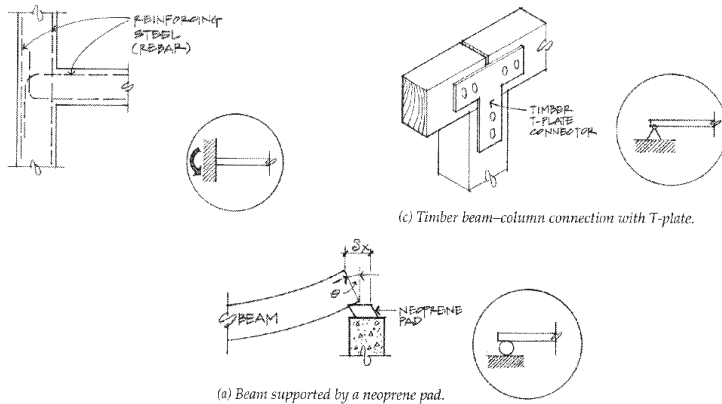
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## Beam Supports

- *in the real world, modeled type*



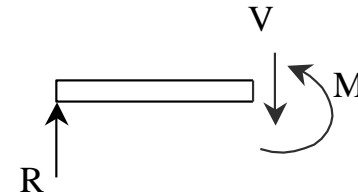
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## Internal Forces in Beams

- *like method of sections / joints*
  - no axial forces
- *section must be in equilibrium*
- *want to know where biggest internal forces and moments are for designing*



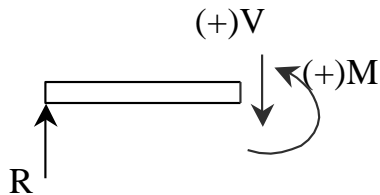
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## V & M Diagrams

- tool to locate  $V_{max}$  and  $M_{max}$
- necessary for designing
- $M_{max}$  occurs when  $V = 0$



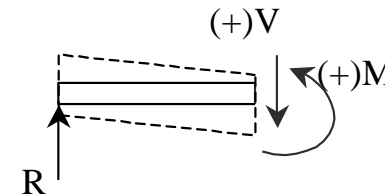
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## Sign Convention

- shear force,  $V$ :
  - cut section to LEFT
  - if  $\sum F_y$  is positive by statics,  $V$  acts down and is POSITIVE
  - beam has to resist shearing apart by  $V$

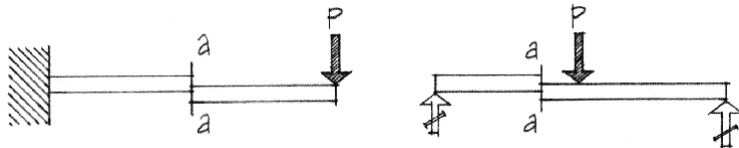


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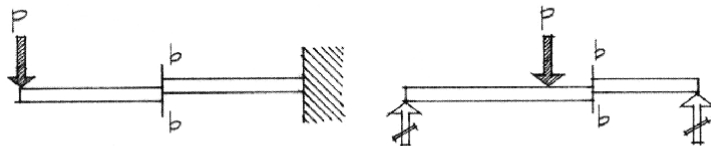
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## Shear Sign Convention



(+) Shear.

(+) Shear.



(-) Shear.

(-) Shear.

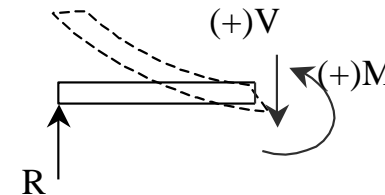
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## Sign Convention

- bending moment,  $M$ :
  - cut section to LEFT
  - if  $\sum M_{cut}$  is clockwise,  $M$  acts ccw and is POSITIVE – flexes into a “smiley” beam has to resist bending apart by  $M$

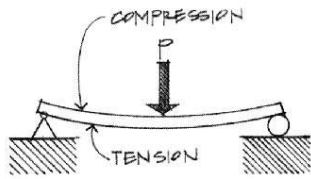


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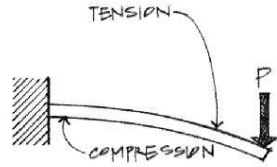
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## Bending Moment Sign Convention



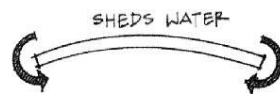
(+) Moment.



(-) Moment.



(+) Moment.



(-) Moment.

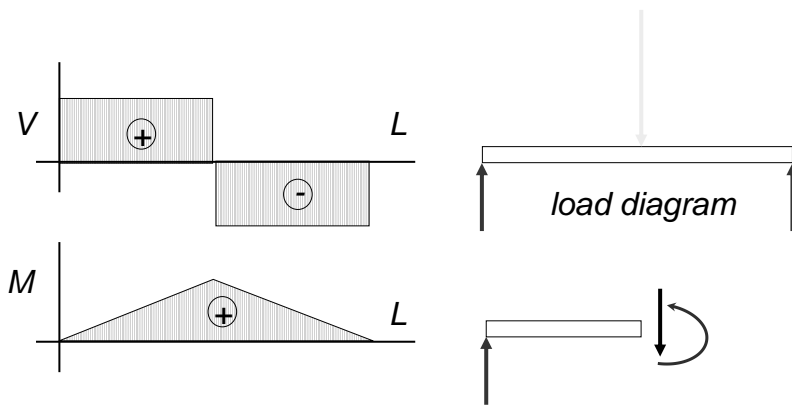
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## Constructing V & M Diagrams

- along the beam length, plot V, plot M

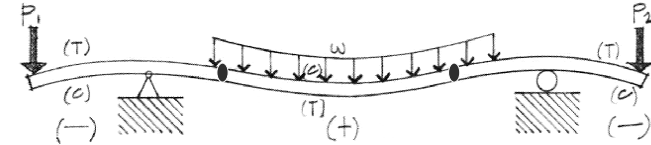


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## Deflected Shape



- positive bending moment
  - tension in bottom, compression in top
- negative bending moment
  - tension in top, compression in bottom
- zero bending moment
  - inflection point

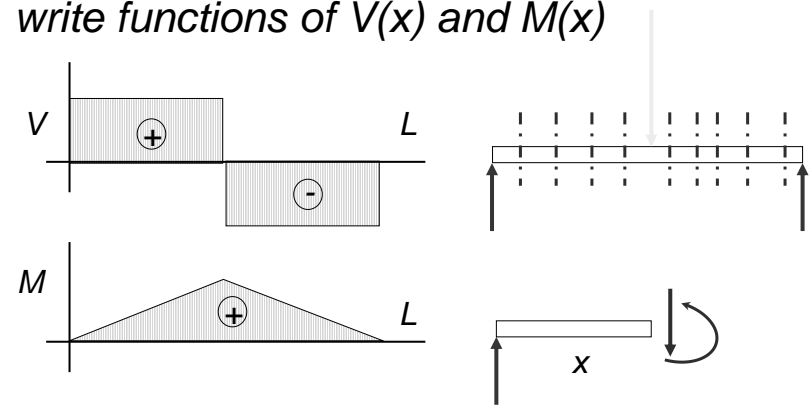
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## Mathematical Method

- cut sections with  $x$  as width
- write functions of  $V(x)$  and  $M(x)$



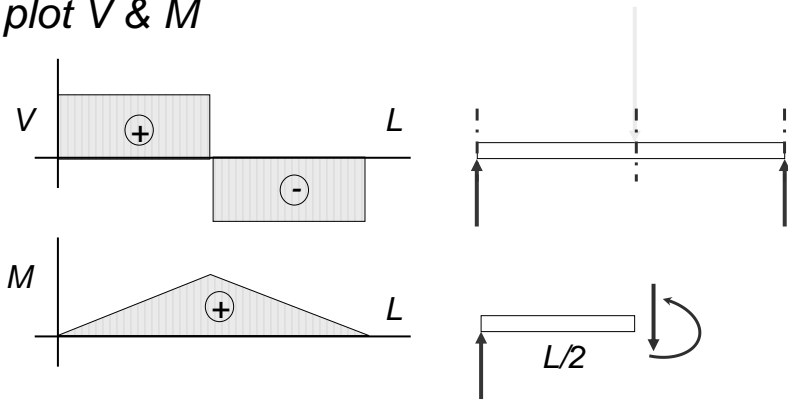
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## Equilibrium Method

- cut sections at important places
- plot V & M



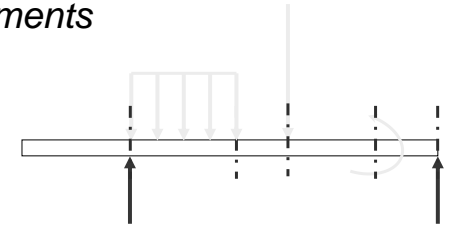
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## Equilibrium Method

- important places
  - supports
  - concentrated loads
  - start and end of distributed loads
  - concentrated moments
- free ends
  - zero forces



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## Equilibrium Met.

- relationships

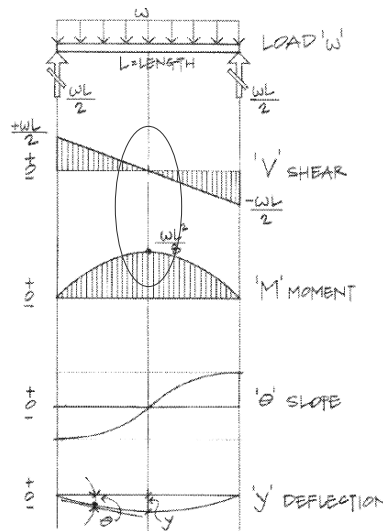


Figure 7.11 Relationship of load, shear, moment, slope, and deflection diagrams.

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## Basic Procedure

1. Find reaction forces & moments  
Plot axes, underneath beam load diagram
2. Starting at left
3. Shear is 0 at free ends
4. Shear has 2 values at point loads
5. Sum vertical forces at each section

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## Basic Procedure

M:

6. Starting at left
7. Moment is 0 at free ends
8. Moment has 2 values at moments
9. Sum moments at each section
10. Maximum moment is where shear = 0!

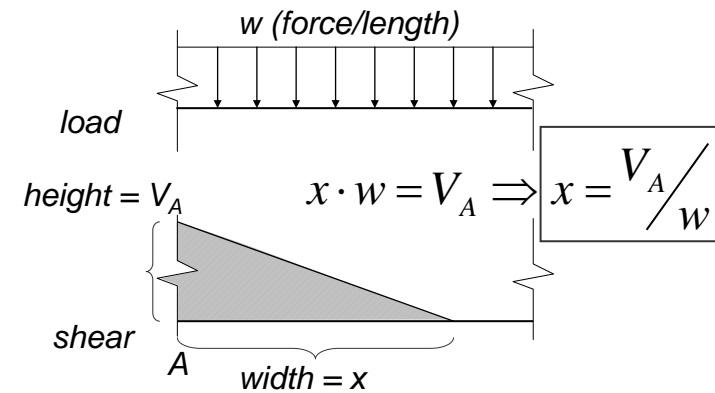
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## Shear Through Zero

- slope of  $V$  is  $w$  ( $-w:1$ )



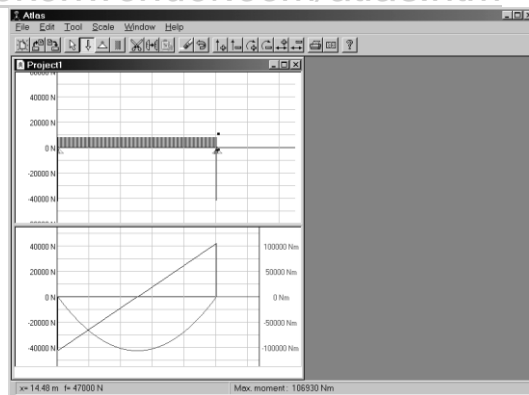
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## Tools

- software & spreadsheets help
- <http://www.rekenwonder.com/atlas.htm>



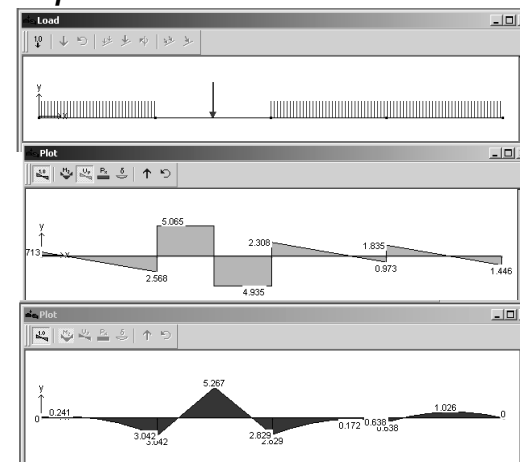
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## Tools – Multiframe

- in computer lab



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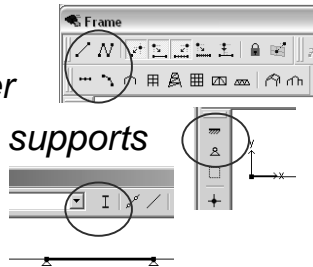
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## Tools – Multiframe

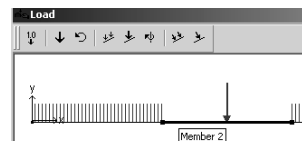
- *frame window*

- *define beam member*
- *select points, assign supports*
- *select members, assign section*



- *load window*

- *select point or member, add point or distributed loads*



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## Tools – Multiframe

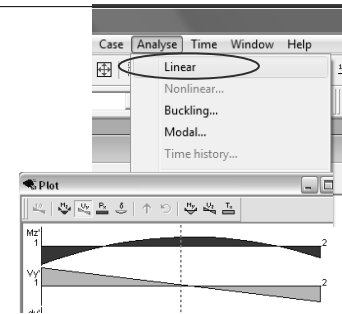
- *to run analysis choose*

- *Analyze menu*

- *Linear*

- *plot*

- *choose options*
- *double click (all)*



- *results*

- *choose options*

Joint	Label	Rx' kip	Ry' kip	Mz' kip-ft
1		0.000	-0.000	0.000
2		0.000	9.250	0.000
3		0.000	6.102	0.000
4		0.000	3.093	0.000
5		0.000	1.398	-0.000
Total (Global)		Rx=0.000	Ry=19.843	

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