

lecture  
three

equilibrium  
and planar trusses



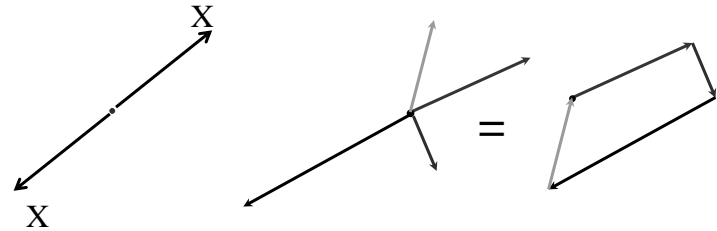
Equilibrium 1  
Lecture 3

Elements of Architectural Structures  
ARCH 614

S2009abn

Equilibrium

- balanced
- steady
- resultant of forces on a particle is 0



Equilibrium 2  
Lecture 5

Elements of Architectural Structures  
ARCH 614

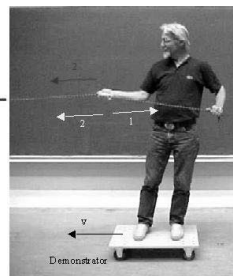
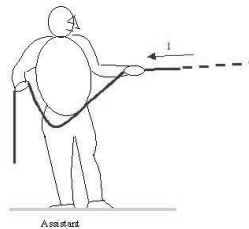
S2006abn

Equilibrium on a Point

- analytically

$$R_x = \sum F_x = 0$$

$$R_y = \sum F_y = 0$$



<http://www.physics.umd.edu>

- Newton convinces us it will stay at rest

Equilibrium 6

Elements of Architectural Structures  
ARCH 614

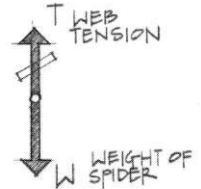
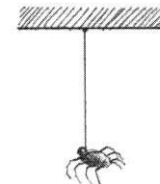
S2004abn

Equilibrium on a Point

- collinear force system

– ex: cables

$$\sum F_{in-line} = 0$$



$$\left[ R_x = \sum F_x = 0 \quad R_y = \sum F_y = 0 \right]$$

Equilibrium 7

Elements of Architectural Structures  
ARCH 614

S2004abn

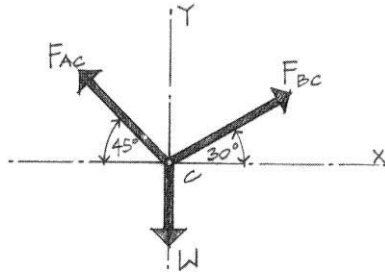
## Equilibrium on a Point

- concurrent force system

– ex: cables

$$R_x = \sum F_x = 0$$

$$R_y = \sum F_y = 0$$



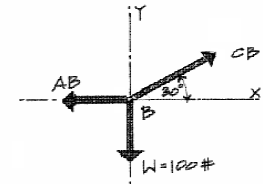
Equilibrium 8

Elements of Architectural Structures  
ARCH 614

S2004abn

## Free Body Diagram

- FBD (sketch)
- tool to see all forces on a body or a point including
  - external forces
  - weights
  - force reactions
  - internal forces



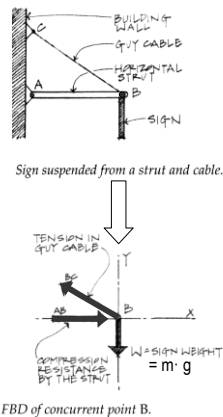
Equilibrium 10

Elements of Architectural Structures  
ARCH 614

S2004abn

## Free Body Diagram

- determine point
- FREE it from:
  - ground
  - supports & connections
- draw all external forces acting ON the body
  - reactions
  - (supporting forces)
  - applied forces
  - gravity



Equilibrium 11

Elements of Architectural Structures  
ARCH 614

S2004abn

## Free Body Diagram

- sketch FBD with relevant geometry
- resolve each force into components
  - known & unknown angles – name them
  - known & unknown forces – name them
- are any forces related to other forces?
- for the unknowns
- write only as many equilibrium equations as needed
- solve up to 2 equations

Equilibrium 12

Elements of Architectural Structures  
ARCH 614

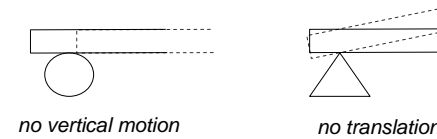
S2004abn

## Free Body Diagram

- solve equations
  - most times 1 unknown easily solved
  - plug into other equation(s)
- common to have unknowns of
  - force magnitudes
  - force angles

## Force Reactions

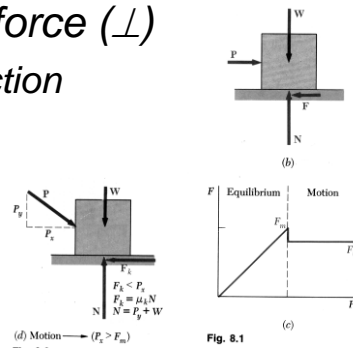
- result of applying force
- unknown size
- connection or support type
  - known direction
  - related to motion prevented



## Friction

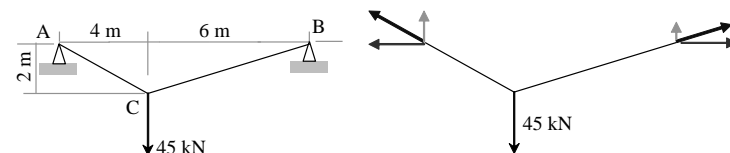
- resistance to movement
- contact surfaces determine  $\mu$
- proportion of normal force ( $\perp$ )
  - opposite to slide direction
  - static > kinetic

$$F = \mu N$$



## Cable Reactions

- equilibrium:
  - more reactions (4) than equations
  - but, we have slope relationships
  - X component the same everywhere



## Cable-Stayed Structures

- diagonal cables support horizontal spans
- typically symmetrical
- Patcenter, Rogers 1986



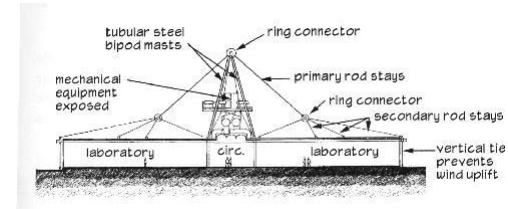
Equilibrium 35  
Lecture 5

Elements of Architectural Structures  
ARCH 614

www.columbia.edu S2005abn

## Patcenter, Rogers 1986

- column free space
- roof suspended
- solid steel ties
- steel frame supports masts



Equilibrium 36  
Lecture 5

Elements of Architectural Structures  
ARCH 614

S2005abn

## Patcenter, Rogers 1986

- dashes – cables pulling

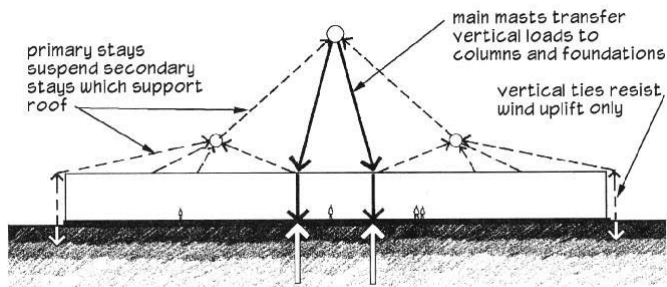


Figure 3.5: Patcenter, load path diagram.

Equilibrium 37  
Lecture 5

Elements of Architectural Structures  
ARCH 614

S2005abn

## Truss Structures

- ancient (?) wood  
– Romans 500 B.C.
- Renaissance revival
- 1800's analysis
- efficient



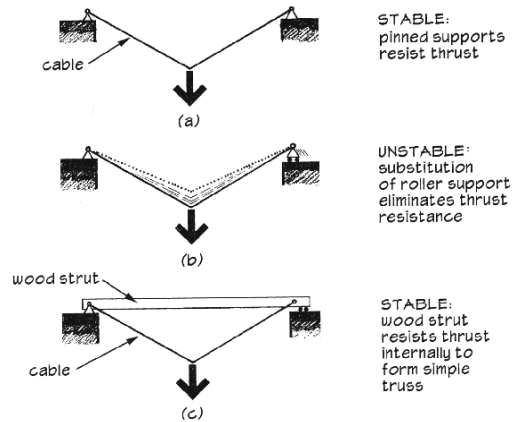
Trusses Intro10

Elements of Architectural Structures  
ARCH 614

S2004abn

# Truss Structures

– analogous to cables and struts



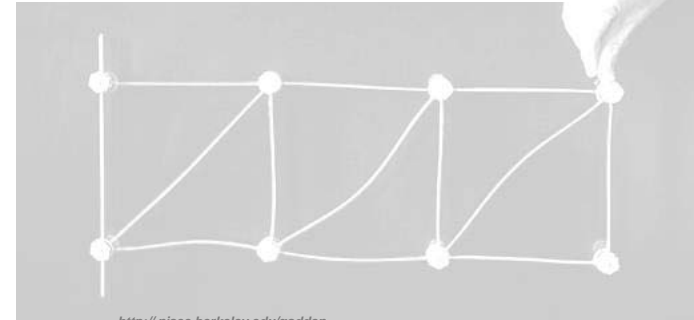
Trusses Intro11

Elements of Architectural Structures  
ARCH 614

S2004abn

# Truss Structures

- comprised of straight members
- geometry with triangles is stable
- loads applied only at pin joints



<http://nisee.berkeley.edu/godden>

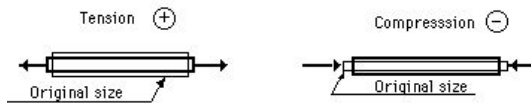
Trusses Intro12

Elements of Architectural Structures  
ARCH 614

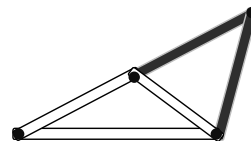
S2004abn

# Truss Structures

- 2 force members
  - forces in line, equal and opposite
  - compression
  - tension



- 3 members connected by 3 joints
- 2 more members need 1 more joint  $b = 2n - 3$



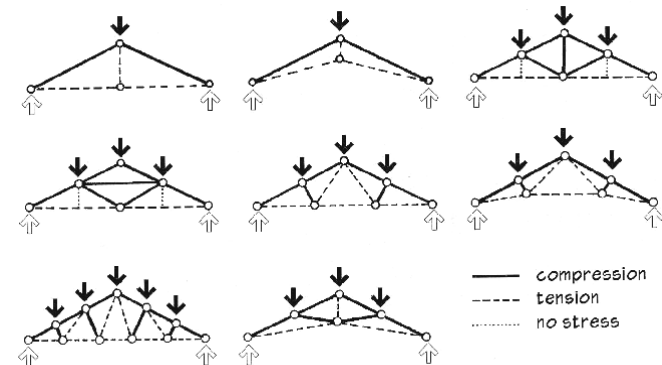
Trusses Intro13

Elements of Architectural Structures  
ARCH 614

S2004abn

# Truss Structures

- compression and tension



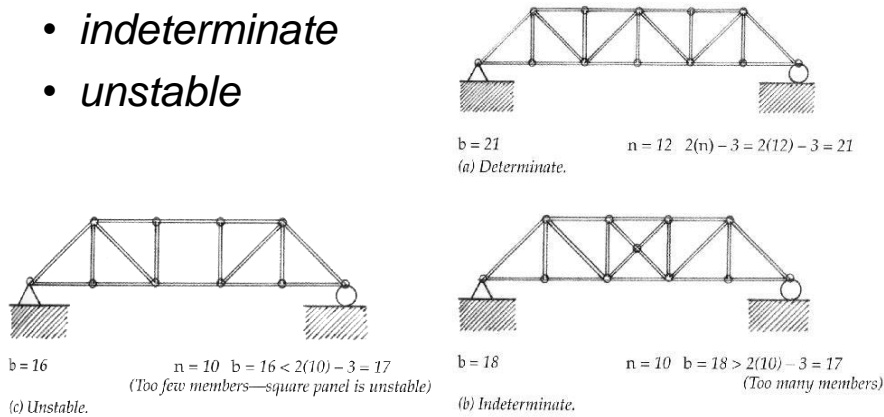
Trusses Intro14

Elements of Architectural Structures  
ARCH 614

S2004abn

# Truss Structures

- statically determinate
- indeterminate
- unstable



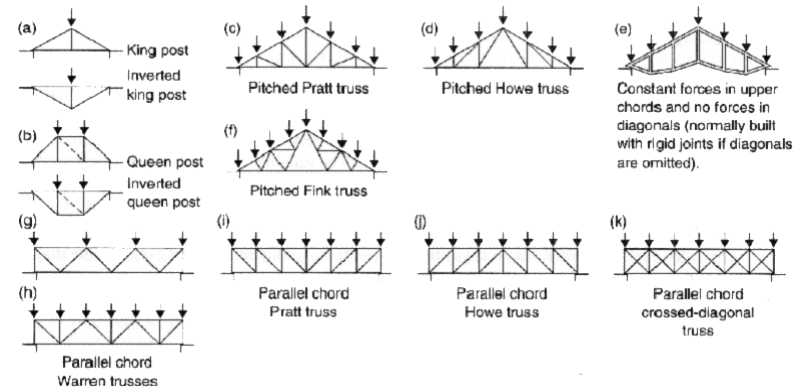
Trusses Intro15

Elements of Architectural Structures ARCH 614

S2004abn

# Trusses

- common designs



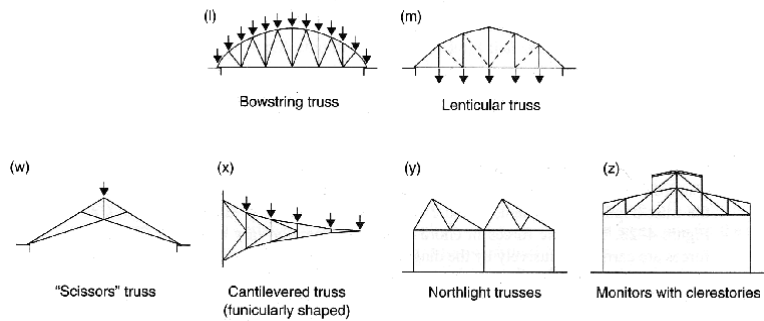
Trusses Intro16

Elements of Architectural Structures ARCH 614

S2004abn

# Trusses

- common designs



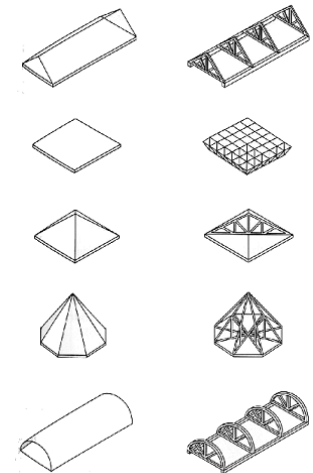
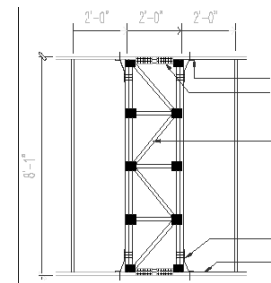
Trusses Intro17

Elements of Architectural Structures ARCH 614

S2004abn

# Trusses

- uses
  - roofs & canopies
  - long spans
  - lateral bracing



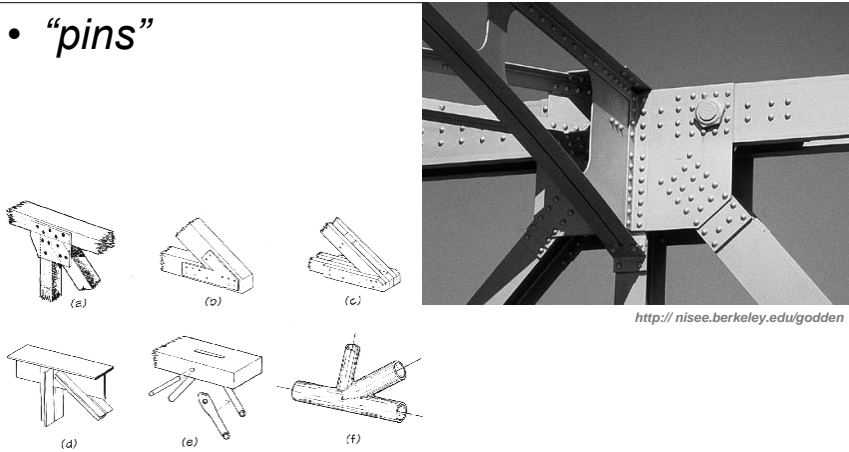
Trusses Intro18

Elements of Architectural Structures ARCH 614

S2004abn

# Truss Connections

- “pins”



<http://nisee.berkeley.edu/godden>

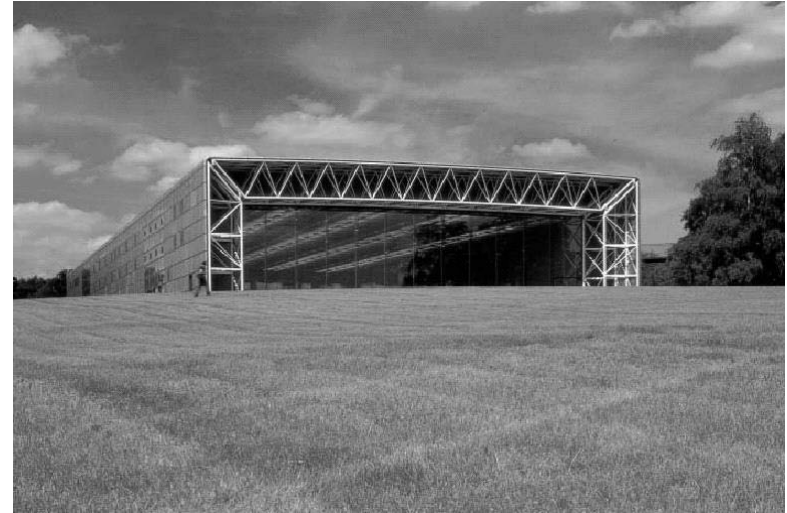
Figure 4.8: Truss joints.

Trusses Intro19

Elements of Architectural Structures  
ARCH 614

S2004abn

# Sainsbury Center, Foster 1978

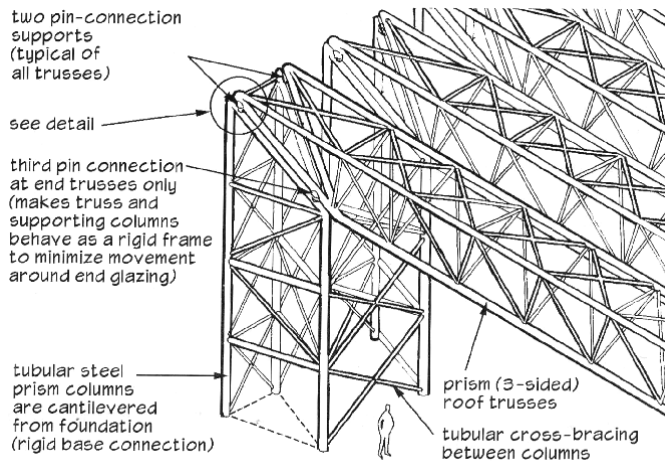


Trusses Intro20

Elements of Architectural Structures  
ARCH 614

S2004abn

# Sainsbury Center, Foster 1978



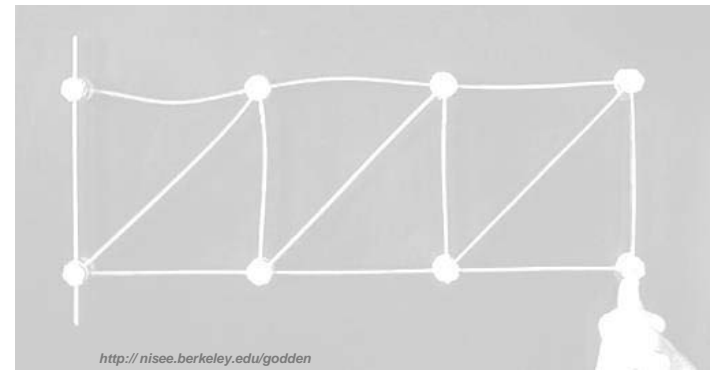
Trusses Intro21

Elements of Architectural Structures  
ARCH 614

S2004abn

# Truss Analysis

- visualize compression and tension from deformed shape



<http://nisee.berkeley.edu/godden>

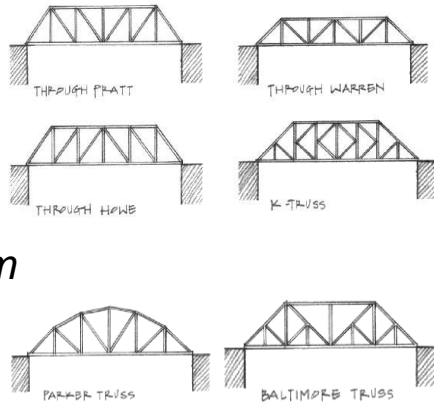
Trusses Intro 22  
Lecture 6

Elements of Architectural Structures  
ARCH 614

S2005abn

# Truss Analysis

- Method of Joints
- Graphical Methods
- Method of Sections
- all rely on equilibrium
  - of bodies
  - internal equilibrium



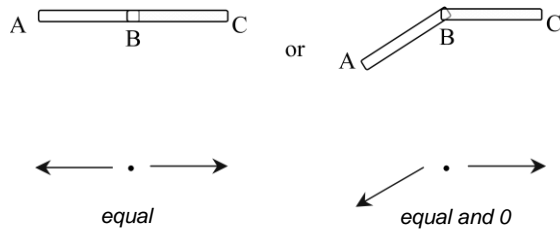
Trusses Intro22

Elements of Architectural Structures  
ARCH 614

S2004abn

# Joint Cases

- two bodies connected



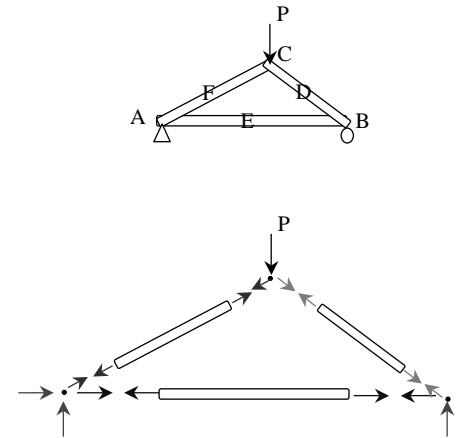
Trusses Intro 25  
Lecture 6

Elements of Architectural Structures  
ARCH 614

S2005abn

# Method of Joints

- isolate each joint
- enforce equilibrium in  $F_x$  and  $F_y$
- can find all forces
- long
- easy to mess up



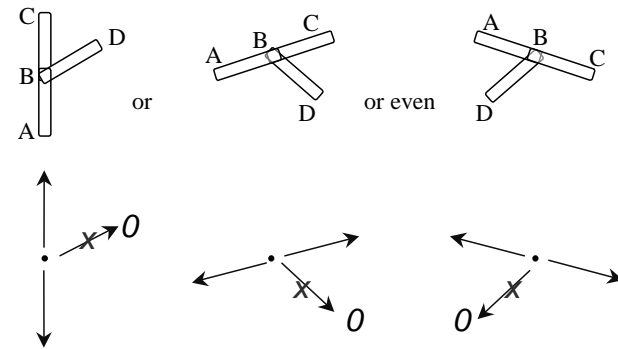
Trusses Intro23

Elements of Architectural Structures  
ARCH 614

S2004abn

# Joint Cases

- three bodies with two in line



Trusses Intro 26  
Lecture 6

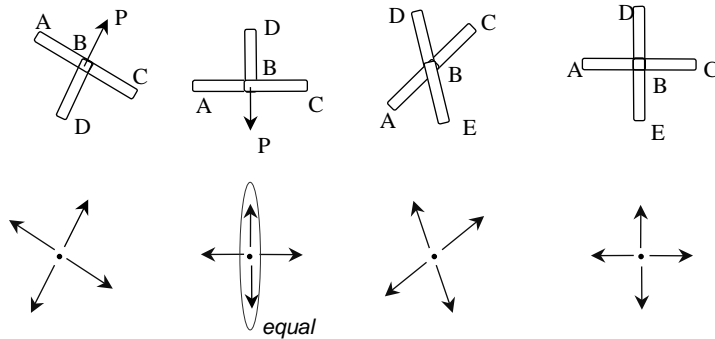
Elements of Architectural Structures  
ARCH 614

S2005abn



## Joint Cases

- crossed



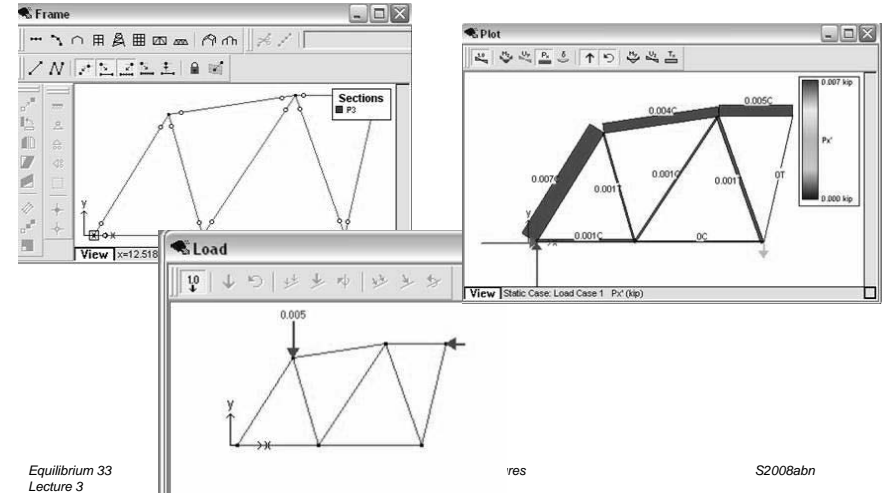
Trusses Intro 27  
Lecture 6

Elements of Architectural Structures  
ARCH 614

S2005abn

## Tools – Multifram

- in computer lab



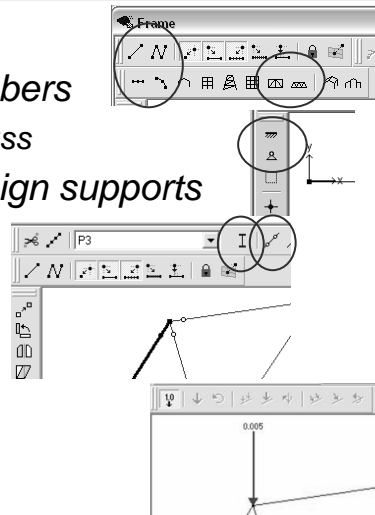
Equilibrium 33  
Lecture 3

yes

S2008abn

## Tools – Multifram

- frame window
  - define truss members
    - or pre-defined truss
  - select points, assign supports
  - select members, assign section & assign pin ends
- load window
  - select points, add point load



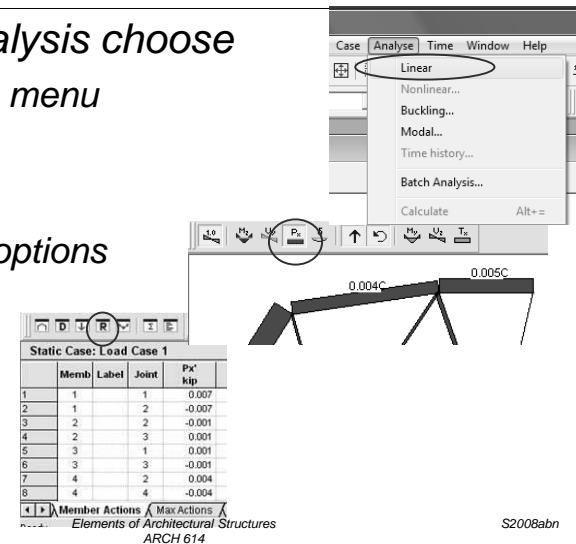
Equilibrium 34  
Lecture 3

Elements of Architectural Structures  
ARCH 614

S2008abn

## Tools – Multifram

- to run analysis choose
  - Analysis menu
    - Linear
- plot
  - choose options
- results
  - choose options



Equilibrium 35  
Lecture 3

Elements of Architectural Structures  
ARCH 614

S2008abn