ELEMENTS OF ARCHITECTURAL STRUCTURES: FORM, BEHAVIOR, AND DESIGN

ARCH 614
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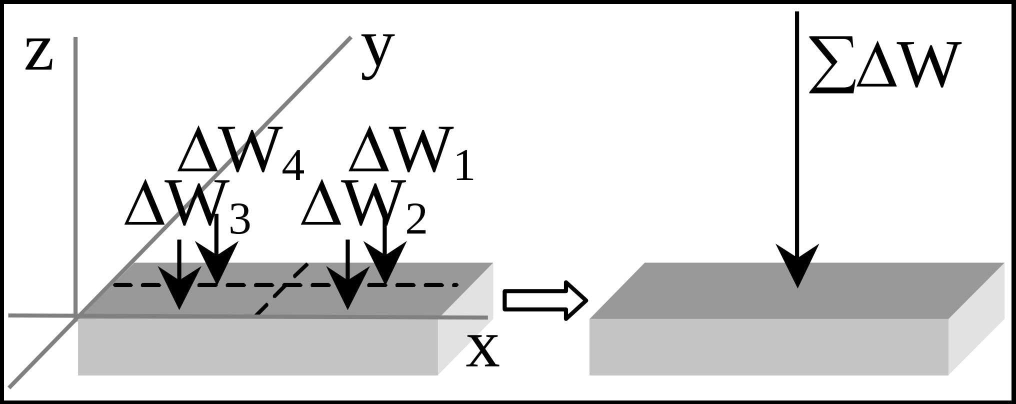
SPRING 2014

lecture
eight

beam sections - geometric properties

Center of Gravity

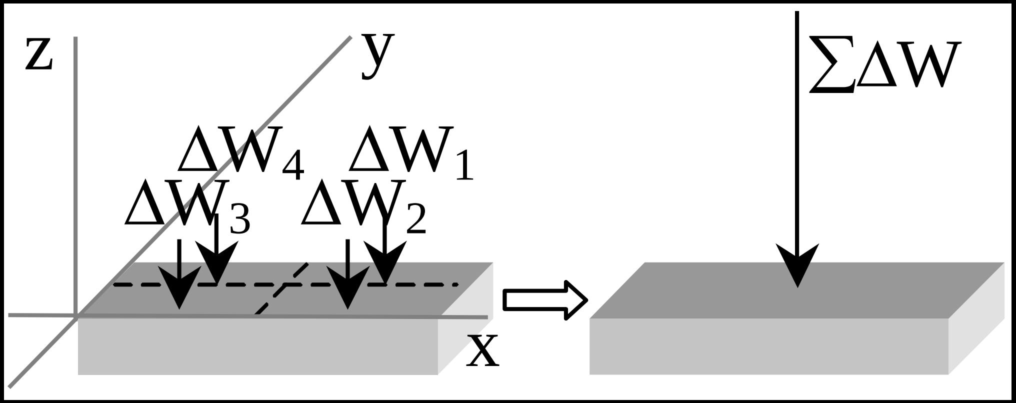
- *location of equivalent weight*
- *determined with calculus*



- *sum element weights* $$W = \int dW$$

Center of Gravity

- *"average" x & y from moment*



$$\sum M_y = \sum_{i=1}^{n} x_i \Delta W_i = \bar{x} W \Rightarrow \quad \bar{x} = \frac{\sum (x \Delta W)}{W}$$

"bar" means average

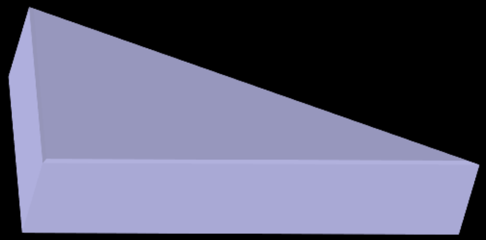
$$\sum M_x = \sum_{i=1}^{n} y_i \Delta W_i = \bar{y} W \Rightarrow \quad \bar{y} = \frac{\sum (y \Delta W)}{W}$$

Centroid

- *"average" x & y of an area*
- *for a volume of constant thickness*
  - $\Delta W = \gamma t \Delta A$ *where γ is weight/volume*
  - *center of gravity = centroid of area*

$$\bar{x} = \frac{\sum(x\Delta A)}{\boldsymbol{A}}$$

$$\bar{y} = \frac{\sum(y\Delta A)}{\boldsymbol{A}}$$

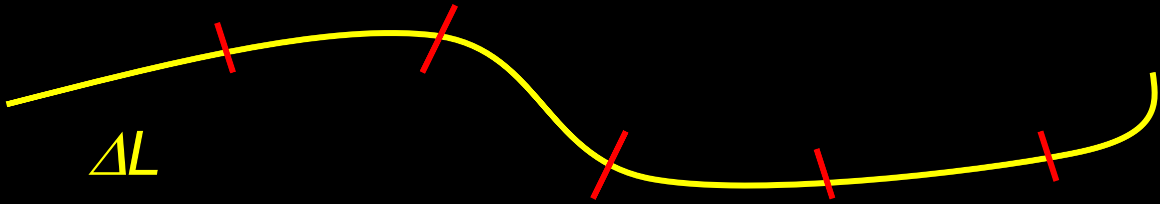


Centroid

- *for a line, sum up length*

$$\bar{x} = \frac{\sum(x\Delta L)}{\boldsymbol{L}}$$

$$\bar{y} = \frac{\sum(y\Delta L)}{\boldsymbol{L}}$$

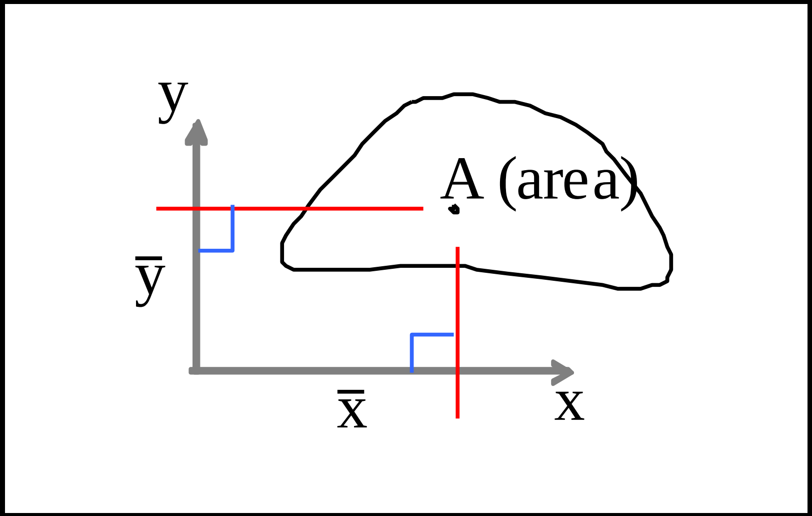


1st Moment Area

- *math concept*
- *the moment of an area about an axis*

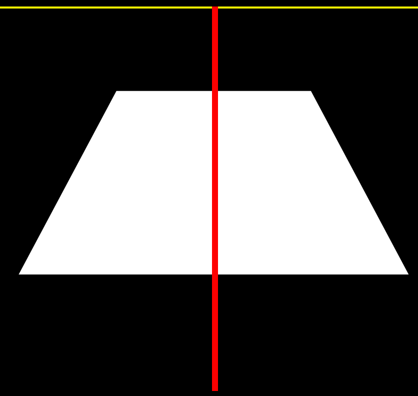
$$Q_x = \bar{y}A$$

$$Q_y = \bar{x}A$$

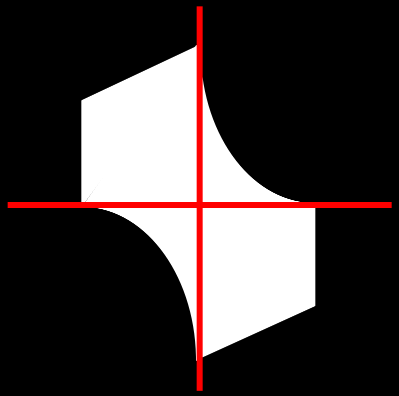


Symmetric Areas

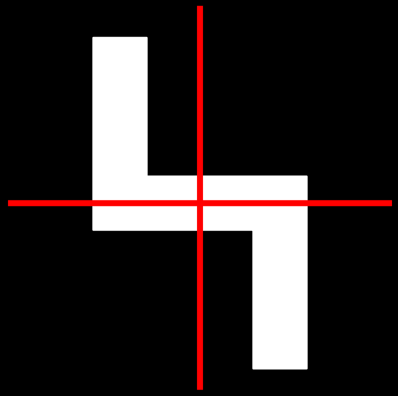
- *symmetric about an axis*



- *symmetric about a center point*

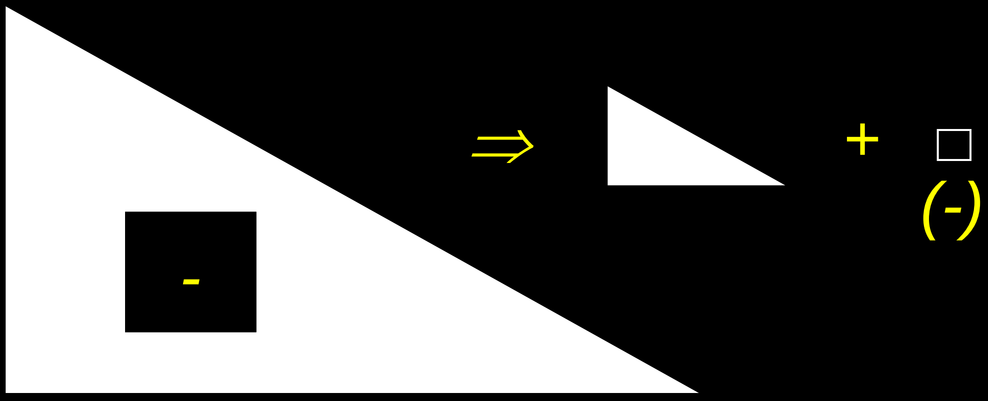


- *mirrored symmetry*



Composite Areas

- made up of basic shapes
- areas can be negative
- (centroids can be negative for any area)



Basic Procedure

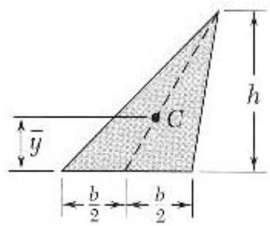
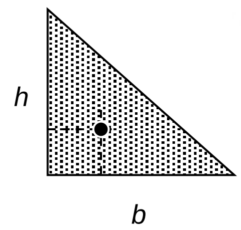
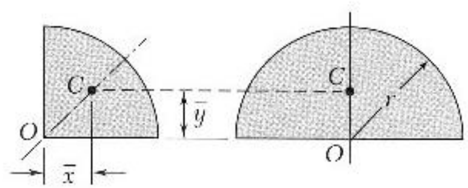
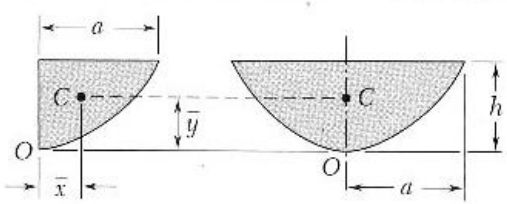
1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate $\hat{x}$ and $\hat{y}$

Component	Area	$\bar{x}$	$\bar{x}A$	$\bar{y}$	$\bar{y}A$
Σ					

Area Centroids

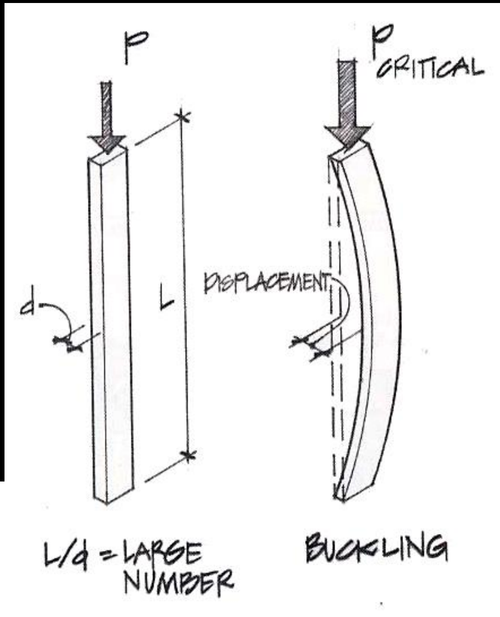
- *Figure A.1 – pg 598*

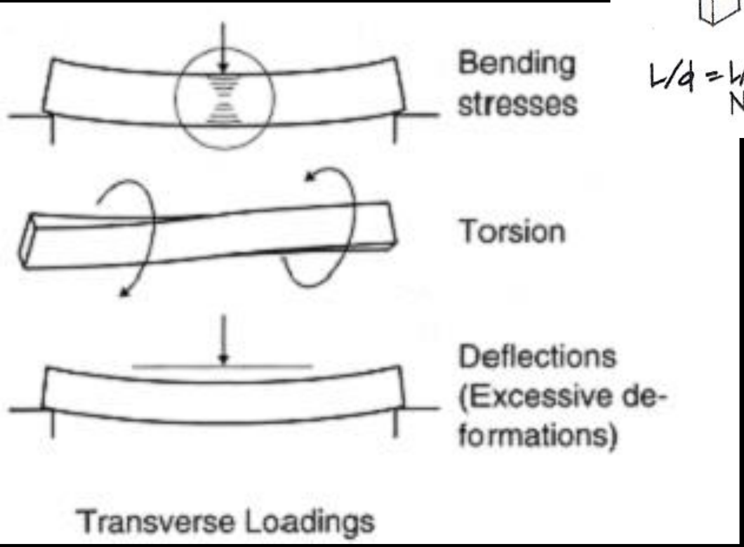
Centroids of Common Shapes of Areas and Lines

Shape		$\bar{x}$	$\bar{y}$
Triangular area	 	$\frac{b}{3}$ *right triangle only*	$\frac{h}{3}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Semicircular area		0	$\frac{4r}{3\pi}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$
Parabolic area		0	$\frac{3h}{5}$

Moments of Inertia

- *2^{nd} moment area*
  - *math concept*
  - *area x (distance)^2*
- *need for behavior of*
  - *beams*
  - *columns*





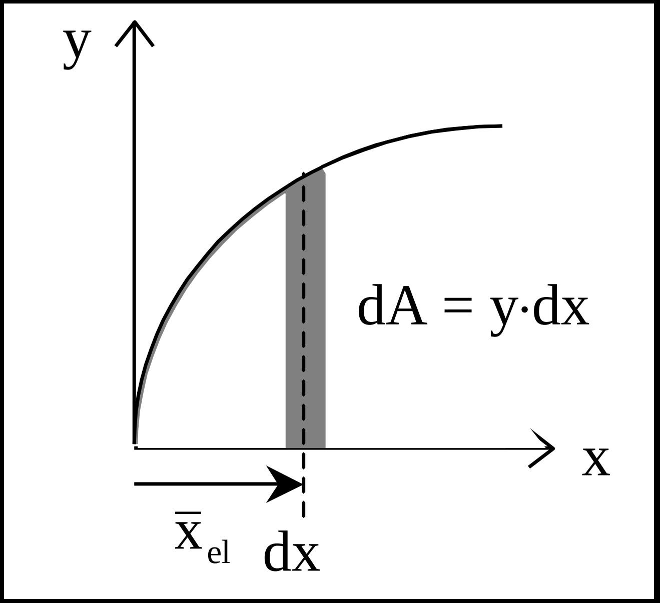
Moment of Inertia

- *about any reference axis*
- *can be negative*

$$I_y = \sum x_i^2 \Delta A = \int x^2 dA$$

$$I_x = \sum y_i^2 \Delta A = \int y^2 dA$$

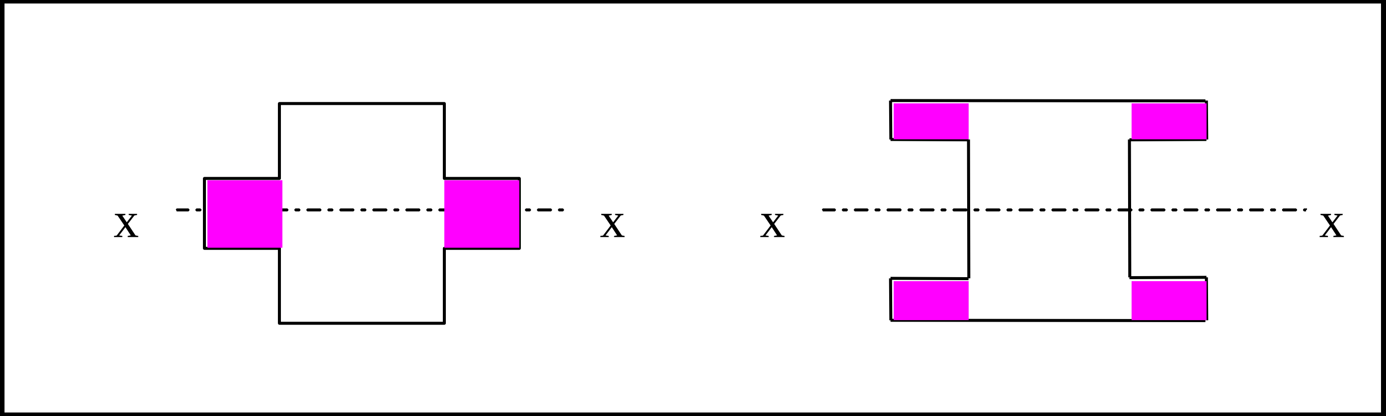
$$(or\ I_{x-x} = \sum z^2 a)$$



- *resistance to bending and buckling*

Moment of Inertia

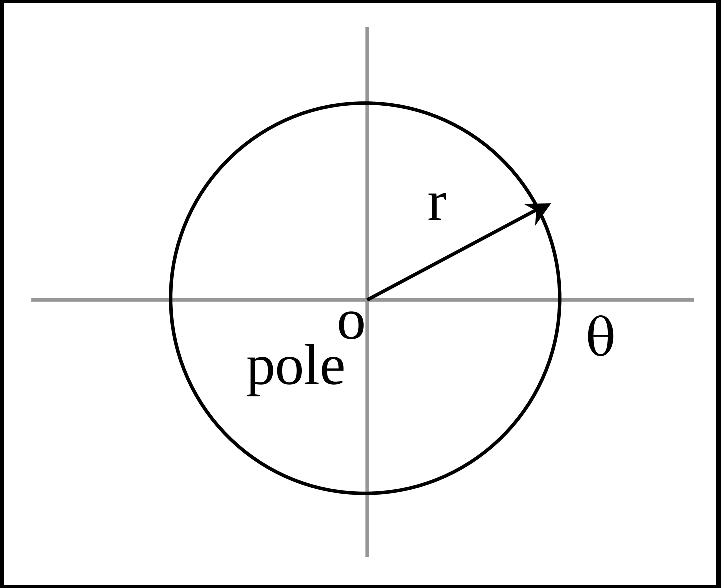
- same area moved away a distance
  - larger I



Polar Moment of Inertia

- *for roundish shapes*
- *uses polar coordinates (r and θ)*
- *resistance to twisting*

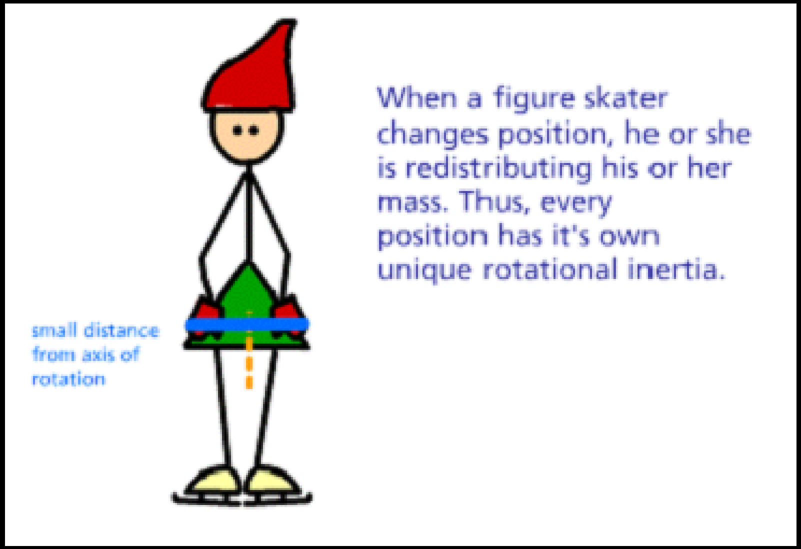
$$J_o = \int r^2 dA$$

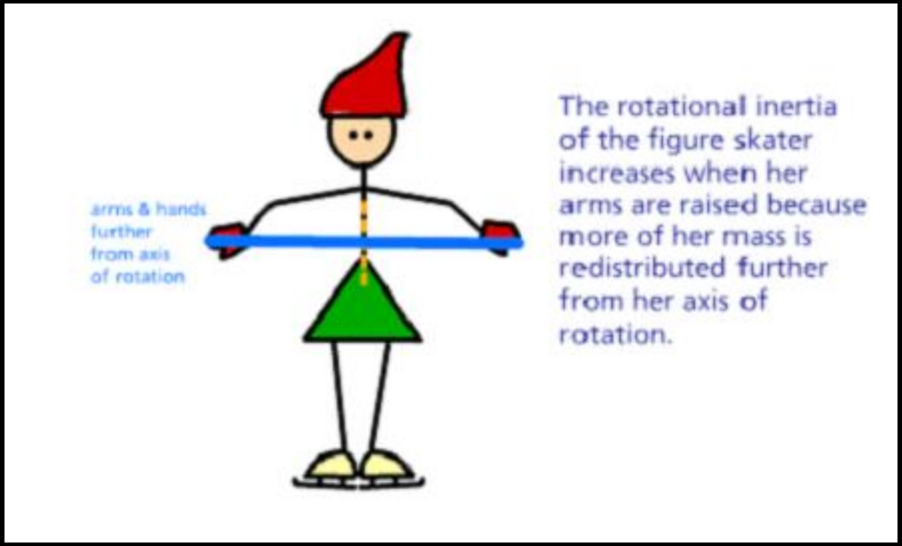


Radius of Gyration

- *measure of inertia with respect to area*

$$r_x = \sqrt{\frac{I_x}{A}}$$





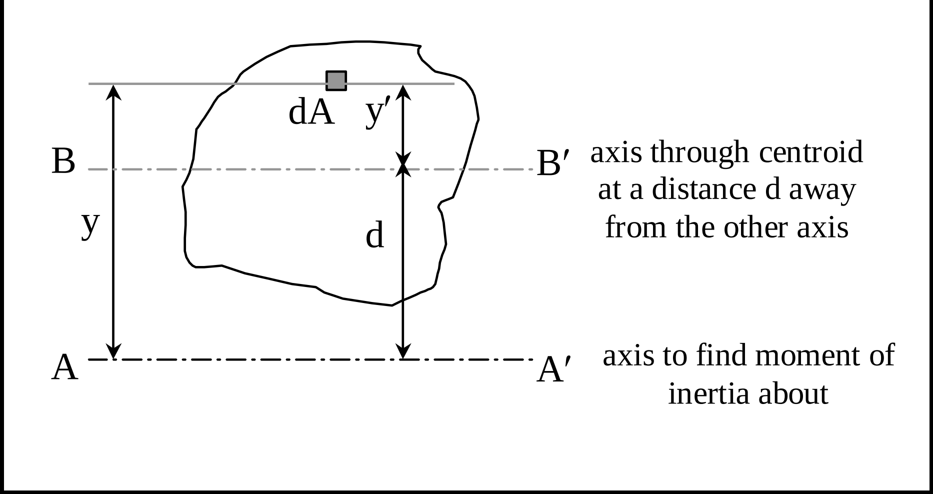
Parallel Axis Theorem

- can find composite I once composite centroid is known (basic shapes)

$$I = I_o + Az^2$$
$$= \bar{I}_x + Ad_y^{\,2}$$

$$I = \sum \bar{I} + \sum Ad^2$$

$$\boxed{\bar{I} = I - Ad^2}$$



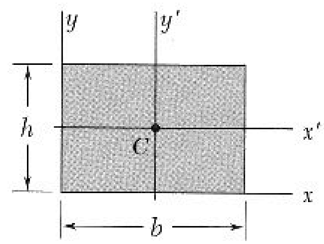
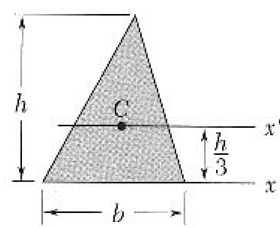
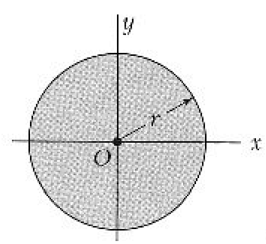
Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with A, $\bar{x}$, $\bar{x}A$, $\bar{y}$, $\bar{y}A$, $\bar{I}$'s, d's, and Ad²'s
5. Fill in table and get $\hat{x}$ and $\hat{y}$ for composite
6. Sum necessary columns
7. Sum $\bar{I}$'s and Ad²'s

$$(d_x = \hat{x} - \bar{x})$$
$$(d_y = \hat{y} - \bar{y})$$

Area Moments of Inertia

- *Figure A.11 – pg. 611: (bars refer to centroid)*
  - *x, y*
  - *x', y'*
  - *C*

Rectangle		$\boxed{\bar{I}_{x'} = \frac{1}{12}bh^3}$ $\boxed{\bar{I}_{y'} = \frac{1}{12}b^3h}$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$