Elements of Architectural Structures: Form, Behavior, and Design

ARCH 614

Dr. Anne Nichols

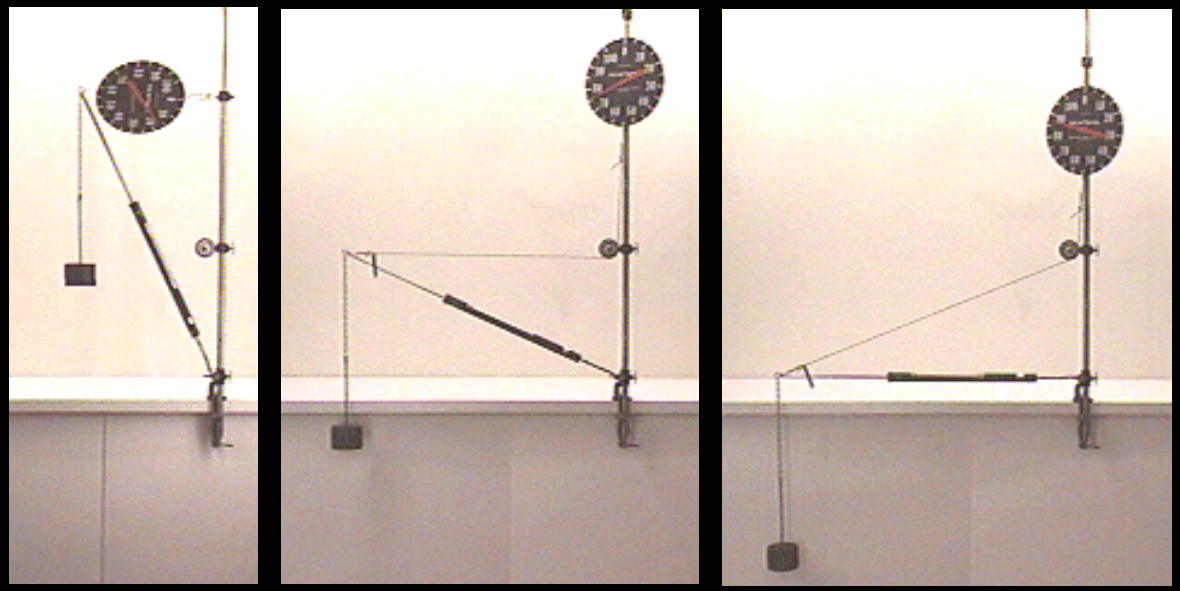
Spring 2014

lecture five

moments

Moments

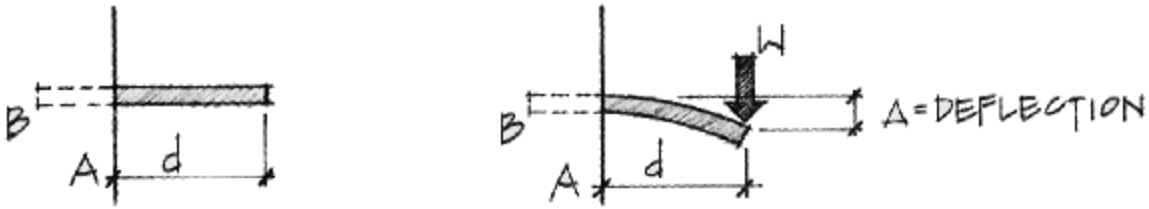
- *forces have the tendency to make a body rotate about an axis*



http://www.physics.umd.edu

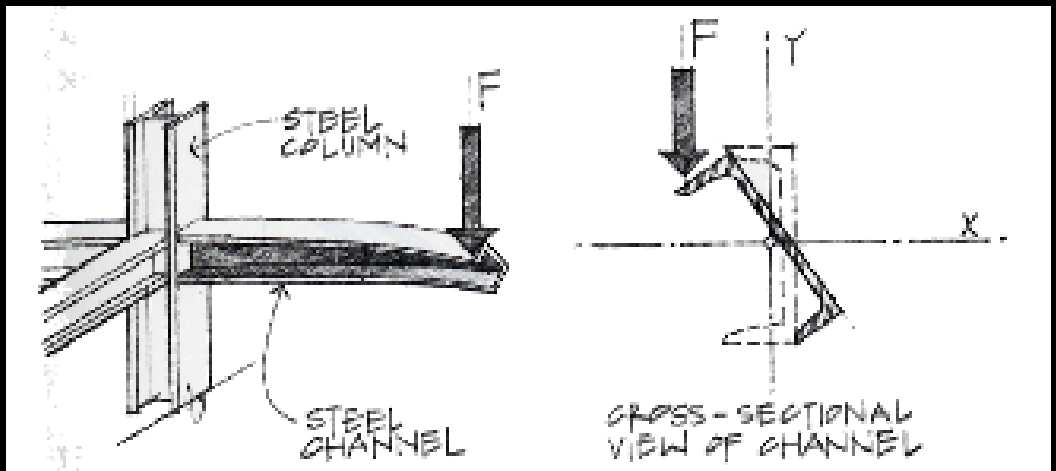
– same translation but different rotation

Moments



(a) Unloaded. (b) Loaded.

Figure 2.33 Moment on a cantilever beam.

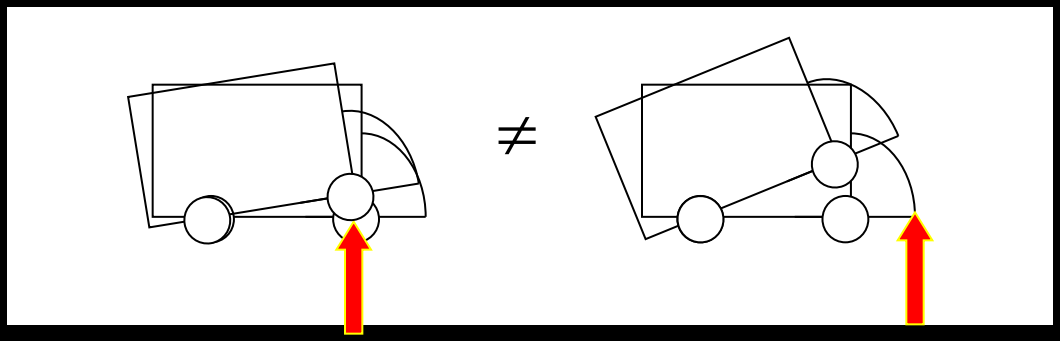


(a) (b)

Figure 2.34 An example of torsion on a cantilever beam.

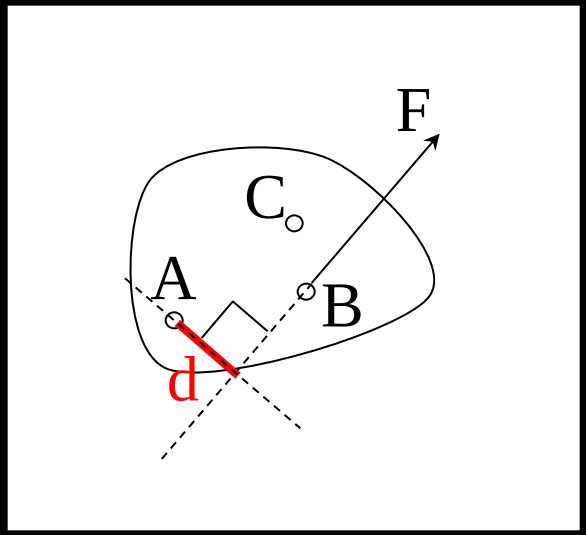
Moments

- *a force acting at a different point causes a different moment:*



Moments

- *defined by magnitude and direction*
- *units: N·m, k·ft*
- *direction:*
  - *+ cw (!)*
  - *- ccw*
- *value found from F and ⊥ distance*



$$M = F \cdot d$$

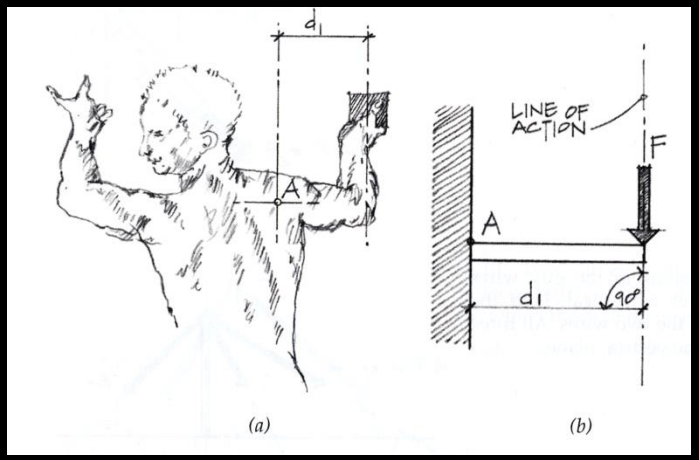
- *d also called "lever" or "moment" arm*

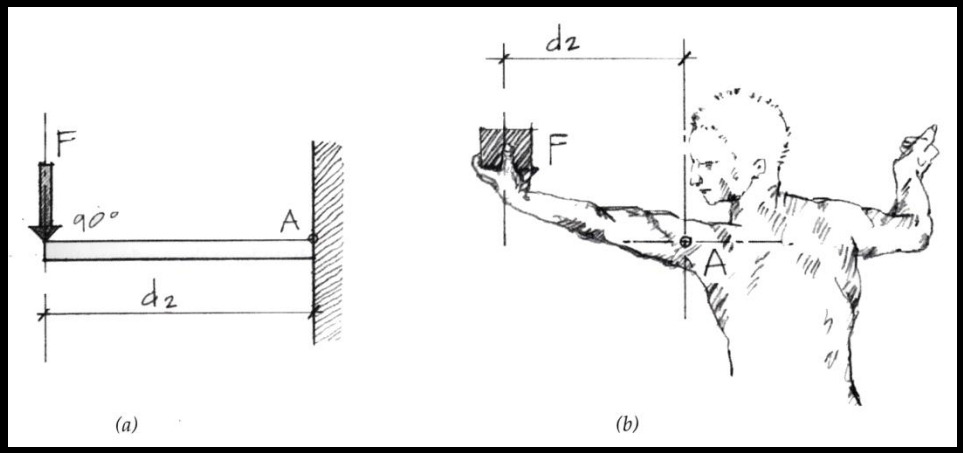
Moments

- *with same F:*

$$M_A = F \cdot d_1 < M_A = F \cdot d_2$$

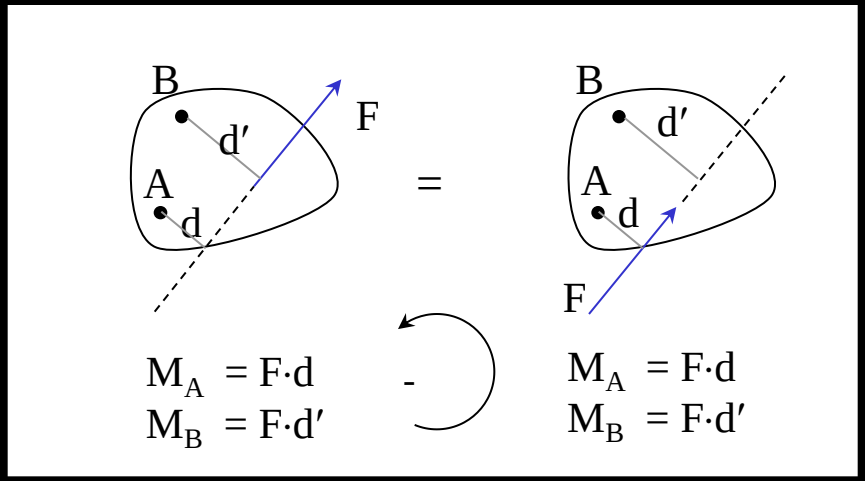
(bigger)





Moments

- *additive with sign convention*
- *can still move the force*
  <u>along the line of action</u>

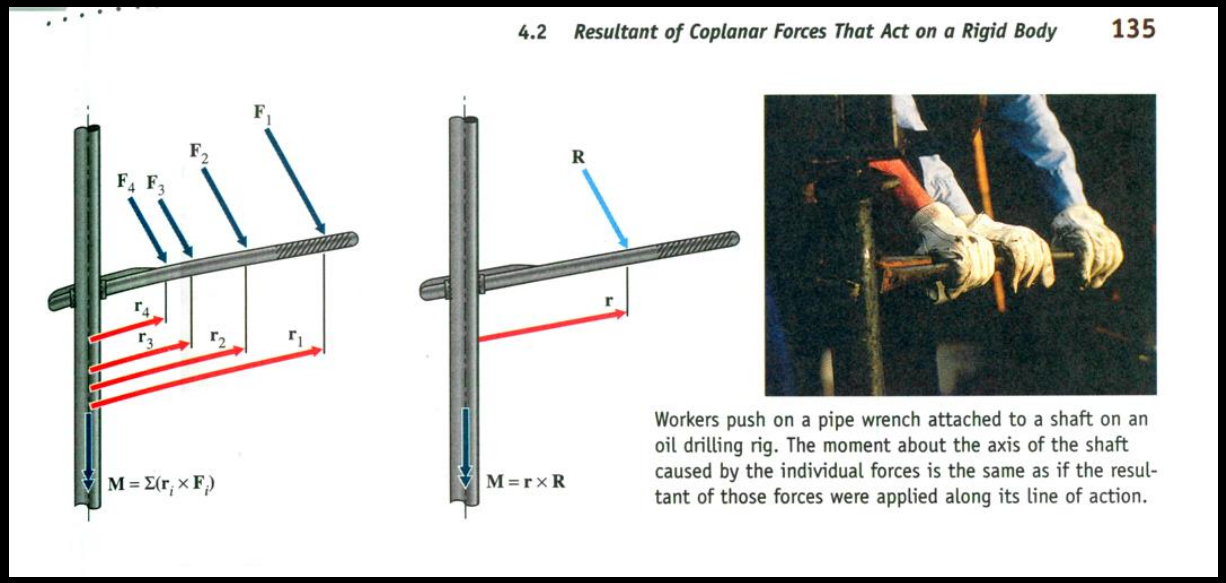


Moments

- *Varignon's Theorem*
  - *resolve a force into components at a point and finding perpendicular distances*
  - *calculate sum of moments*
  - *equivalent to original moment*
- *makes life easier!*
  - *geometry*
  - *when component runs through point, d=0*

Moments of a Force

- *moments of a force*
  - *introduced in Physics as "Torque Acting on a Particle"*
  - *and used to satisfy rotational equilibrium*



Physics and Moments of a Force

- *my Physics book (right hand rule):*

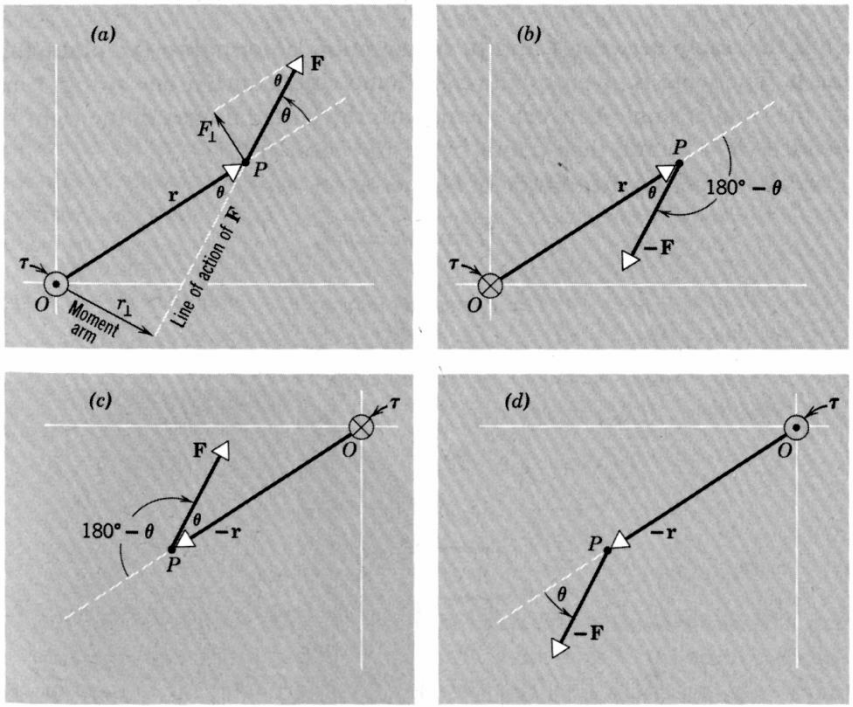
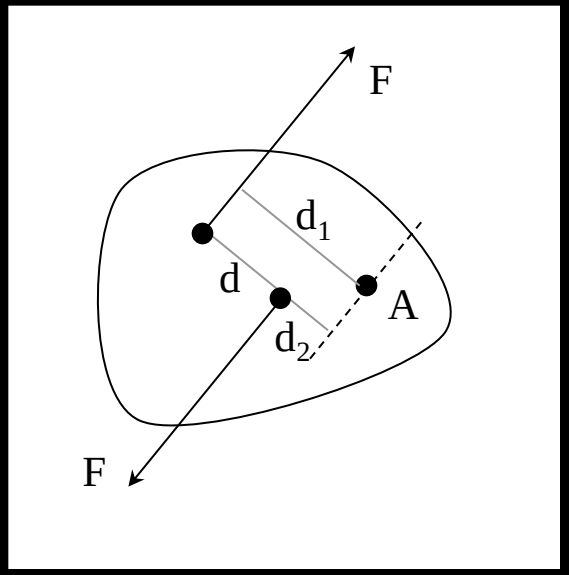


FIGURE 11–2 The plane shown is that defined by **r** and **F** in Fig. 11–1. (*a*) The magnitude of $\boldsymbol{\tau}$ is given by $Fr_\perp$ (Eq. 11–2*b*) or by $rF_\perp$ (Eq. 11–2*c*). (*b*) Reversing **F** reverses the direction of $\boldsymbol{\tau}$. (c) Reversing **r** reverses the direction of $\boldsymbol{\tau}$. (*d*) Reversing **F** and **r** leaves the direction of $\boldsymbol{\tau}$ unchanged. The directions of $\boldsymbol{\tau}$ are represented by $\odot$ (perpendicularly out of the figure, the symbol representing the tip of an arrow) and by $\otimes$ (perpendicularly into the figure, the symbol representing the tail of an arrow).

Moment Couples

- *2 forces*
  - *same size*
  - *opposite direction*
  - *distance d apart*
  - *cw or ccw*



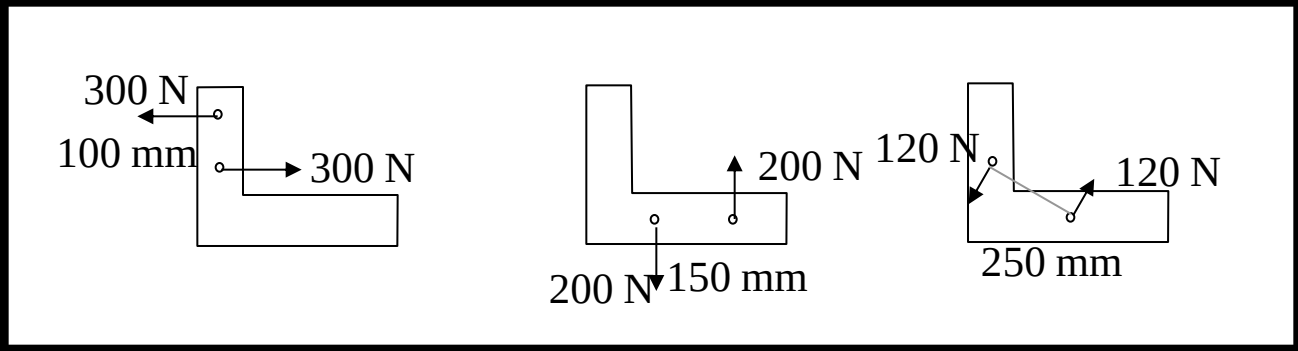
$$M = F \cdot d$$

  - *not dependant on point of application*

$$M = F \cdot d_1 - F \cdot d_2$$

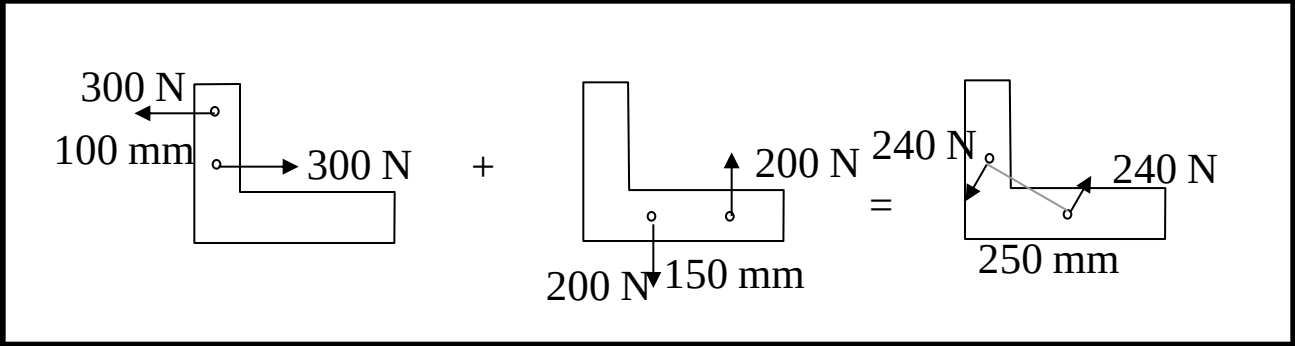
Moment Couples

- *equivalent couples*
  - *same magnitude and direction*
  - *F & d may be different*



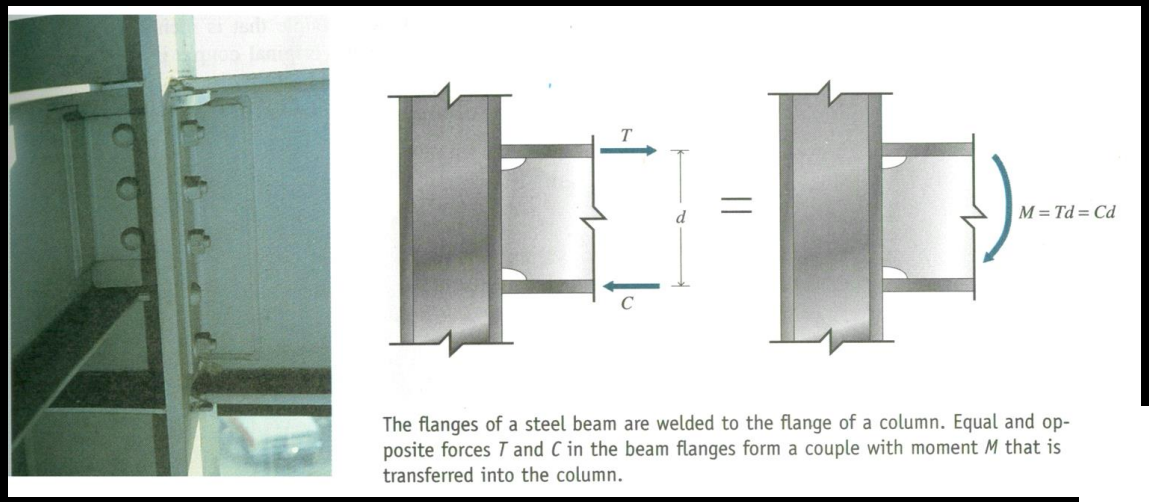
Moment Couples

- *added just like moments caused by one force*
- *can <u>replace</u> two couples with a single couple*

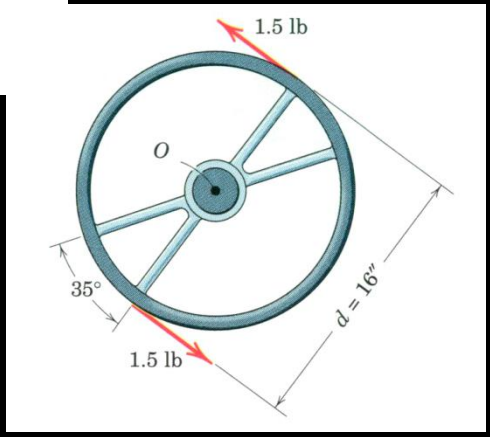


Moment Couples

- *moment couples in structures*



The flanges of a steel beam are welded to the flange of a column. Equal and opposite forces T and C in the beam flanges form a couple with moment M that is transferred into the column.

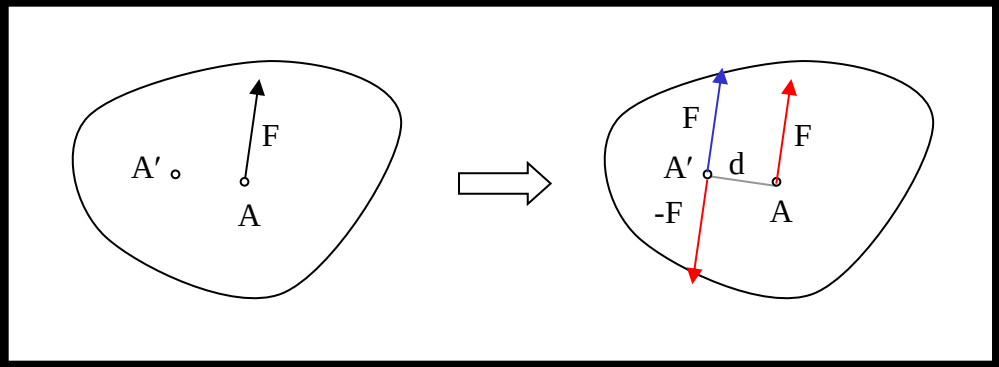


Equivalent Force Systems

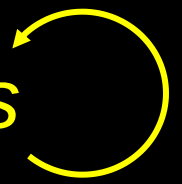
- *two forces at a point is equivalent to the resultant at a point*
- *resultant is equivalent to two components at a point*
- *resultant of equal & opposite forces at a point is zero*
- *put equal & opposite forces at a point (sum to 0)*
- *transmission of a force along action line*

Force-Moment Systems

- *single force causing a moment can be replaced by the same force at a different point by providing the moment that force caused*

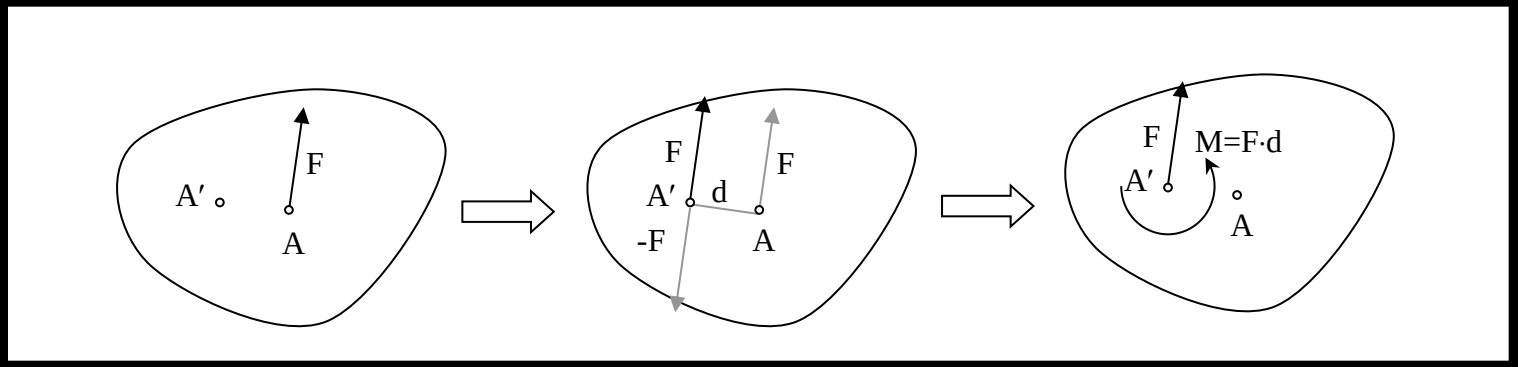


- *moments are shown as arched arrows*



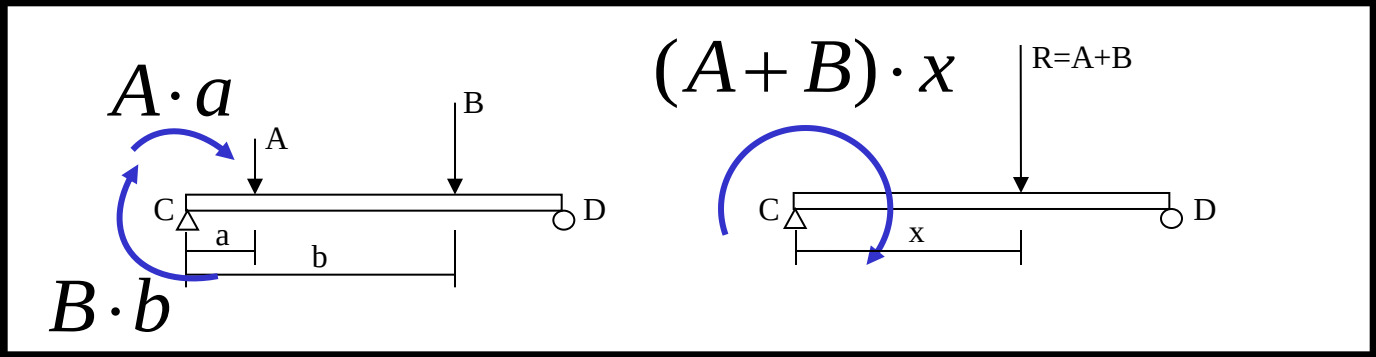
Force-Moment Systems

- *a force-moment pair can be replaced by a force at another point causing the original moment*



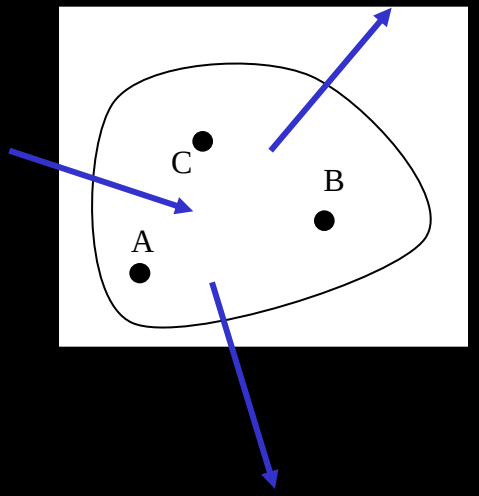
Parallel Force Systems

- *forces are in the same direction*
- *can find resultant force*
- *need to find location for equivalent moments*



Equilibrium

- *rigid body*
  - *doesn't deform*
  - *coplanar force systems*
- *static:*



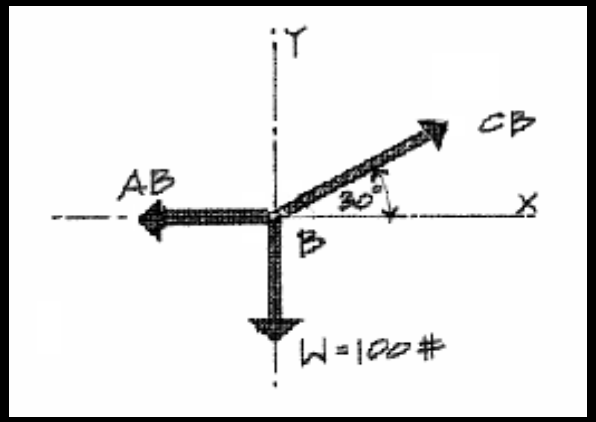
$$R_x = \sum F_x = 0 \quad (\Sigma H)$$

$$R_y = \sum F_y = 0 \quad (\Sigma V)$$

$$\boxed{M = \sum M = 0}$$

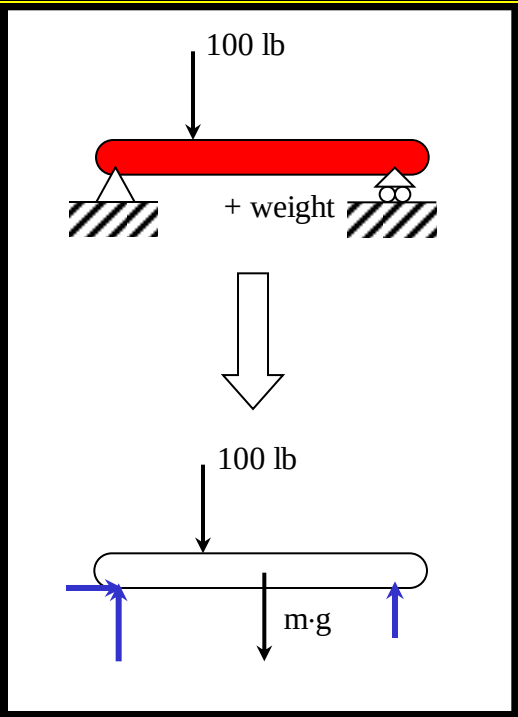
Free Body Diagram

- *FBD (sketch)*
- *tool to see all forces on a body or a point including*
  - *external forces*
  - *weights*
  - *force reactions*
  - *external moments*
  - *moment reactions*
  - *internal forces*



Free Body Diagram

- *determine body*
- *FREE it from:*
  - *ground*
  - *supports & connections*
- *draw all external forces acting ON the body*
  - *reactions*
  - *applied forces*
  - *gravity*



Free Body Diagram

- *sketch FBD with relevant geometry*
- *resolve each force into components*
  - *known & unknown angles – name them*
  - *known & unknown forces – name them*
  - *known & unknown moments – name them*
- *are any forces related to other forces?*
- *for the unknowns*
- *write only as many equilibrium equations as needed*
- *solve up to 3 equations*

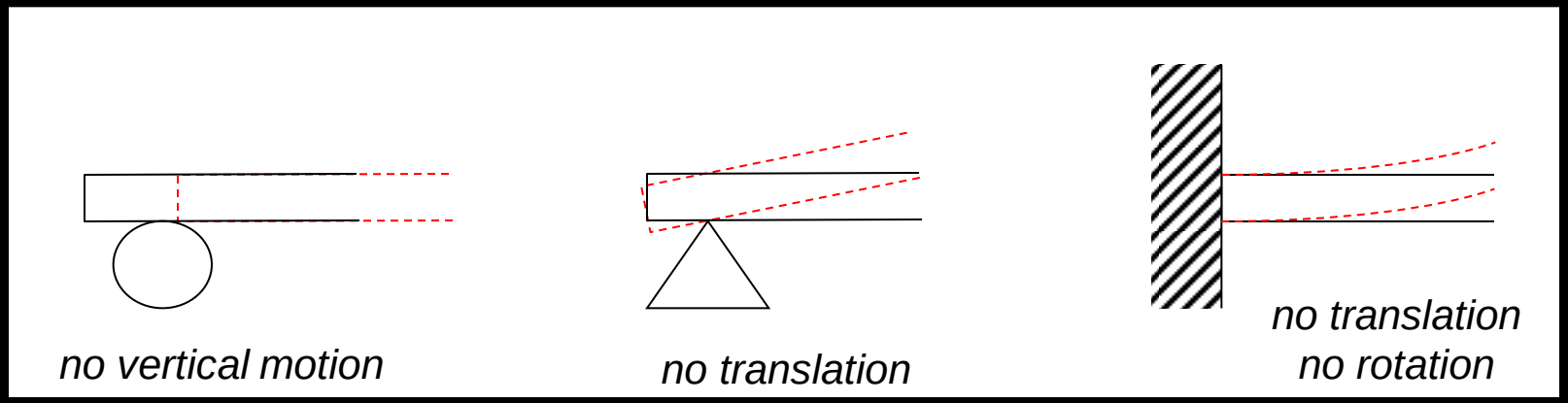
Free Body Diagram

- *solve equations*
  - *most times 1 unknown easily solved*
  - *plug into other equation(s)*

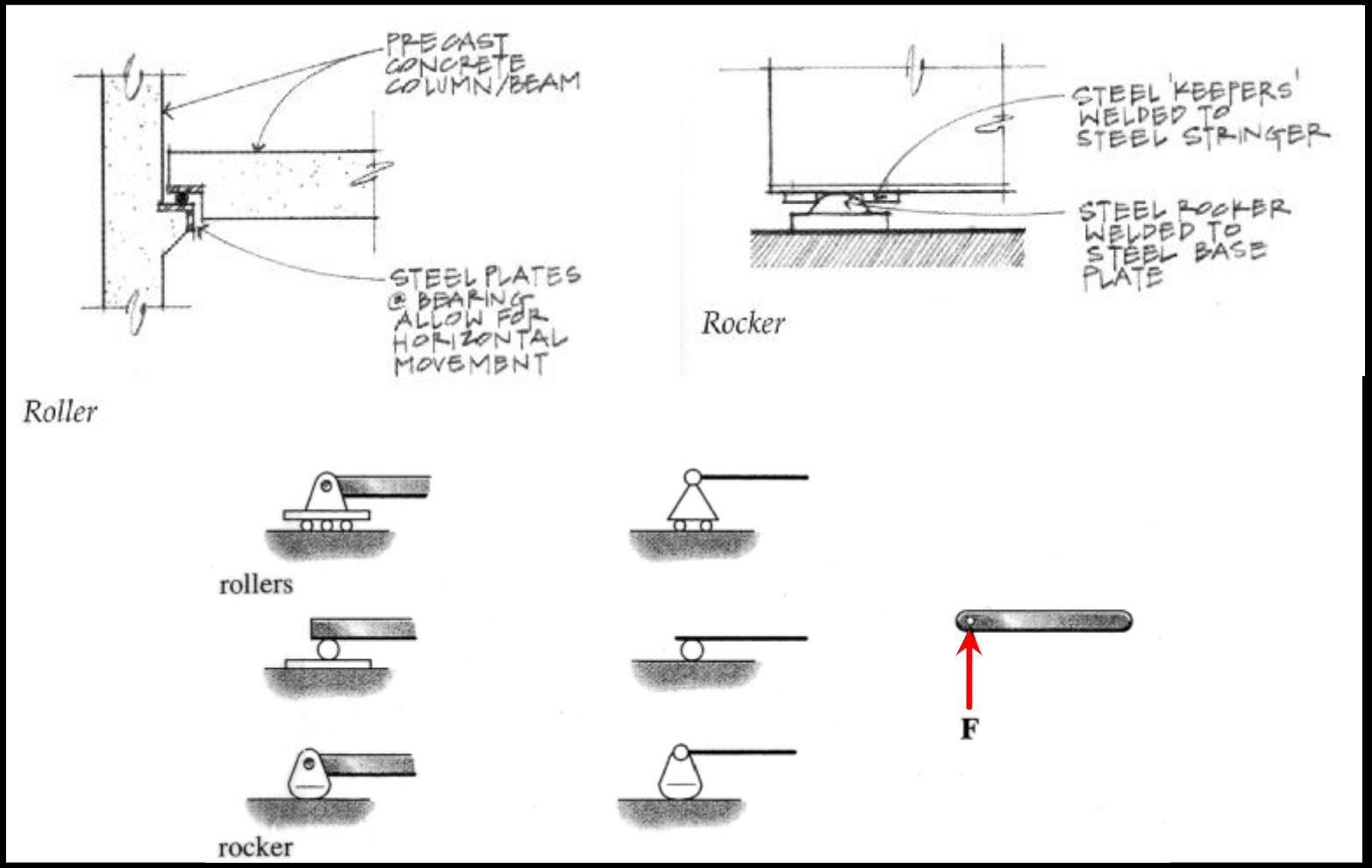
- *common to have unknowns of*
  - *force magnitudes*
  - *force angles*
  - *moment magnitudes*

Reactions on Rigid Bodies

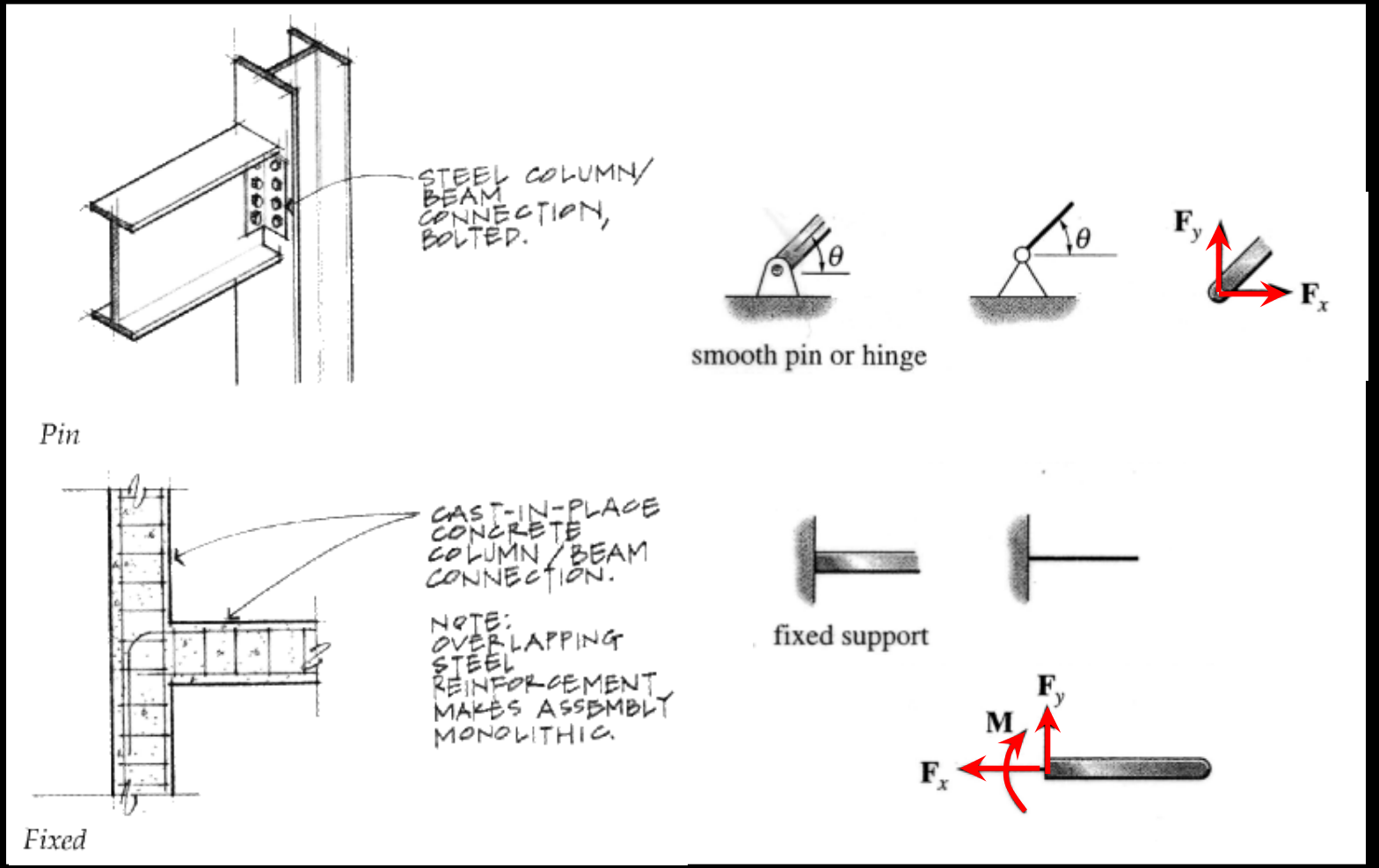
- *result of applying force*
- *unknown size*
- *connection or support type*
  - *known direction*
  - *related to motion prevented*



Supports and Connections



Supports and Connections

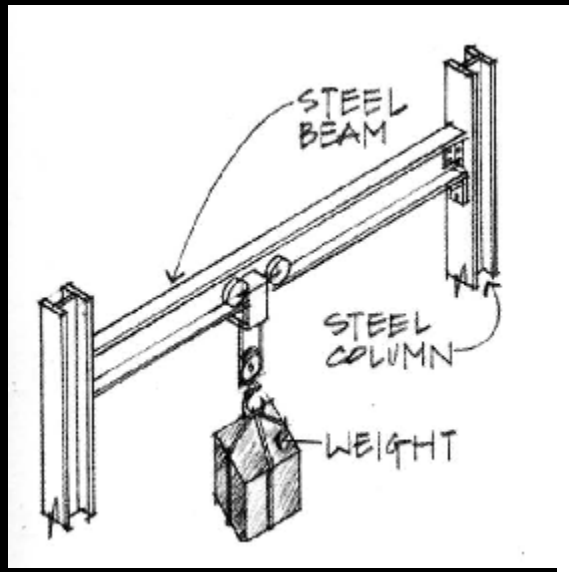


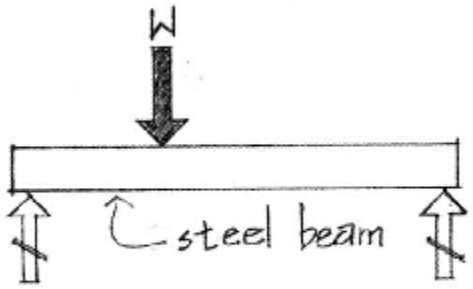
Moment Equations

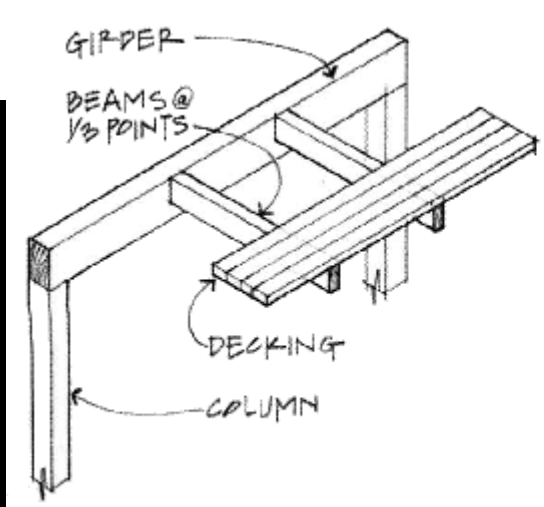
- *sum moments at intersection where the most forces intersect*
- *multiple moment equations may not be useful*
- *combos:*

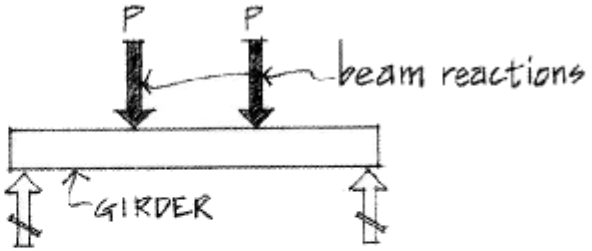
$$\sum F_x = 0 \qquad \sum F = 0 \qquad \sum M_1 = 0$$
$$\sum F_y = 0 \qquad \sum M_1 = 0 \qquad \sum M_2 = 0$$
$$\sum M_1 = 0 \qquad \sum M_2 = 0 \qquad \sum M_3 = 0$$

Concentrated Loads

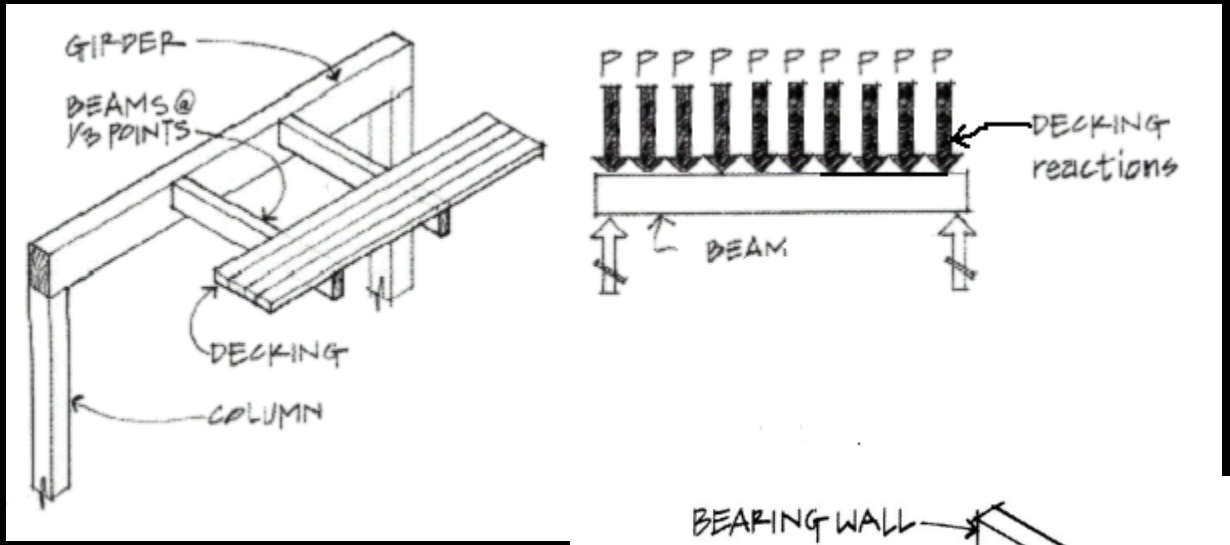


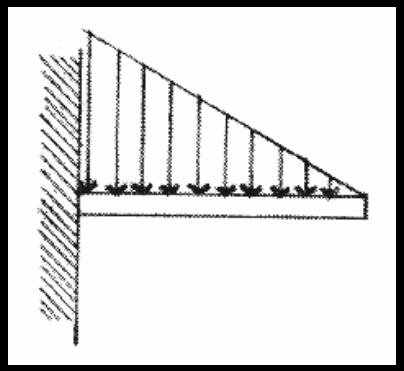


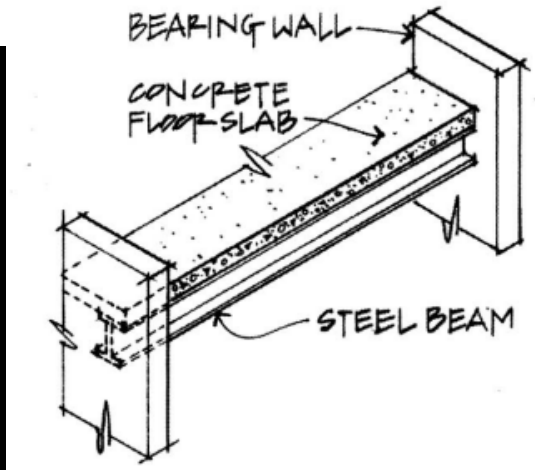


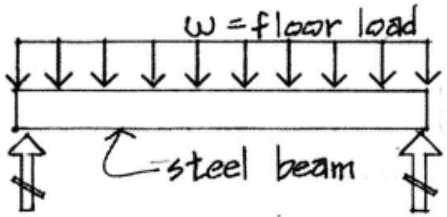


Distributed Loads



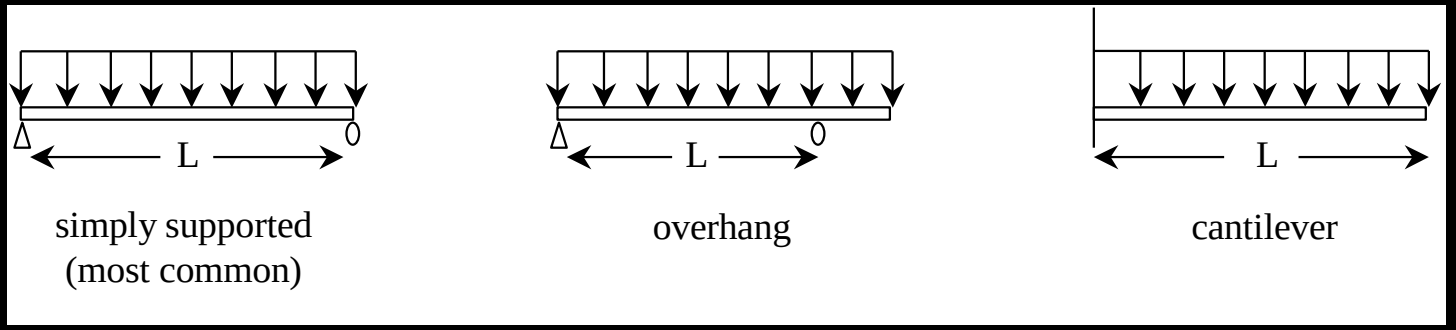




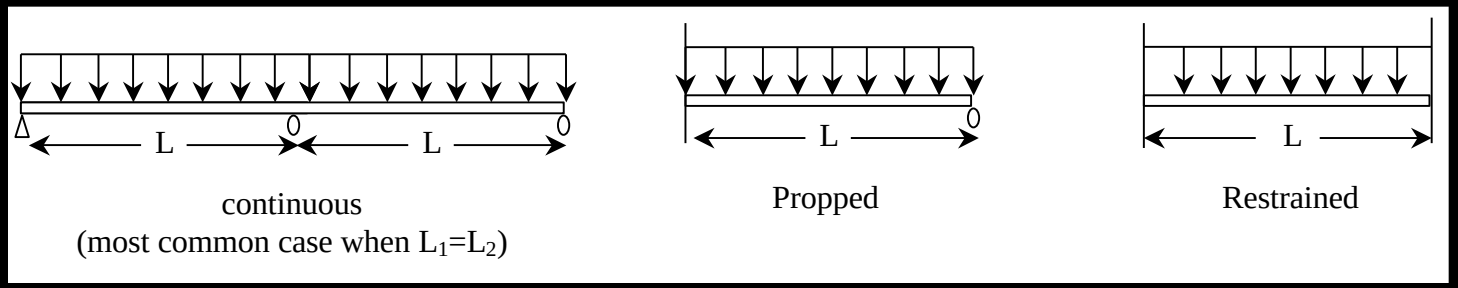


Beam Supports

- *statically determinate*



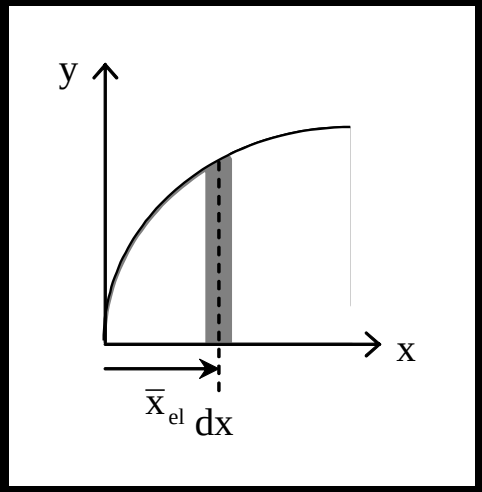
- *statically indeterminate*

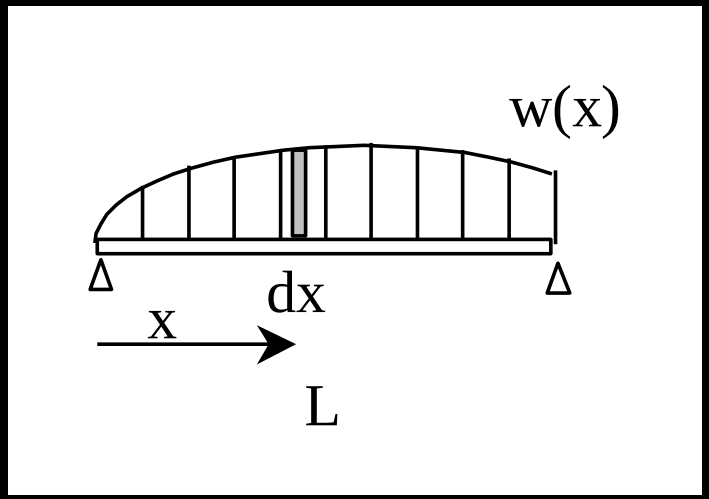


Equivalent Force Systems

- *replace forces by resultant*
- *place resultant where M = 0*
- *using <u>calculus</u> and area centroids*

$$W = \int_0^L w\,dx = \int dA_{loading} = A_{loading}$$





Load Areas

- area is width x "height" of load
- w is load per unit length
- W is total load