ELEMENTS OF ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN

ARCH 614

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SPRING 2013

lecture
nine

beams:
bending and shear stress

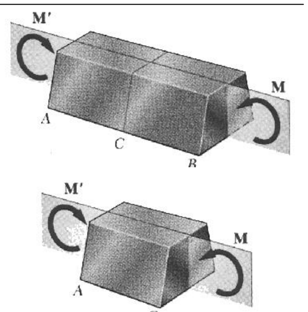
Beam Bending

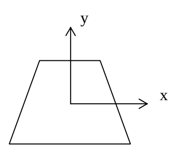
- Galileo
  - relationship between stress and depth^2
- can see
  - top squishing
  - bottom stretching

- what are the stress across the section?

Pure Bending

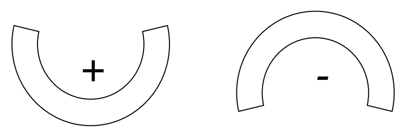
- bending only
- no shear
- axial normal stresses from bending can be found in
  - homogeneous materials
  - plane of symmetry
  - follow Hooke's law

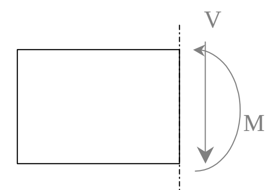




Bending Moments

- sign convention:





- size of maximum internal moment will govern our design of the section

Normal Stresses

- geometric fit
  - plane sections remain plane
  - stress varies linearly

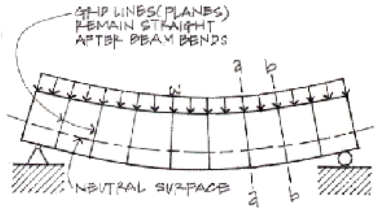


Figure 8.5(b) Beam bending under load.

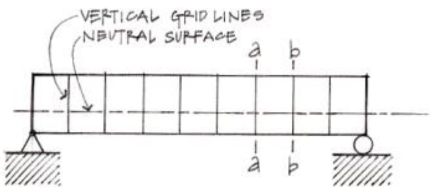
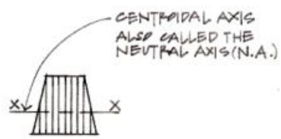


Figure 8.5(a) Beam elevation before loading.



Beam cross section.

Neutral Axis

- stresses vary linearly
- zero stress occurs at the centroid
- <u>neutral axis</u> is line of centroids (n.a.)

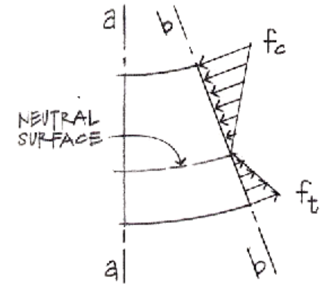


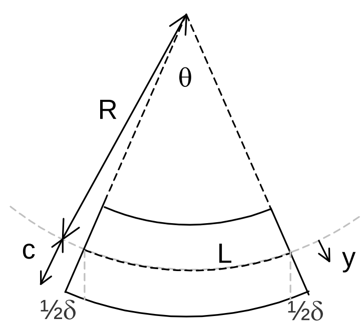
Figure 8.8 Bending stresses on section b-b.

Derivation of Stress from Strain

- pure bending = arc shape

$$L = R\theta$$

$$L_{outside} = (R + y)\theta$$



$$\varepsilon = \frac{\delta}{L} = \frac{L_{outside} - L}{L} = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

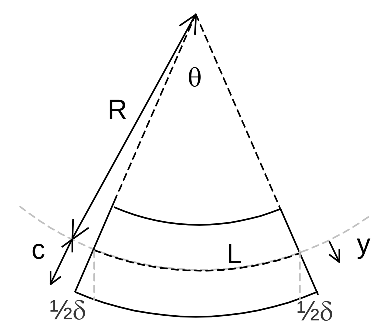
Derivation of Stress

- zero stress at n.a.

$$f = E\varepsilon = \frac{Ey}{R}$$

$$f_{\max} = \frac{Ec}{R}$$

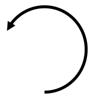
$$f = \frac{y}{c} f_{\max}$$



Bending Moment

- resultant moment from stresses = bending moment!

$$M = \Sigma f y \Delta A$$



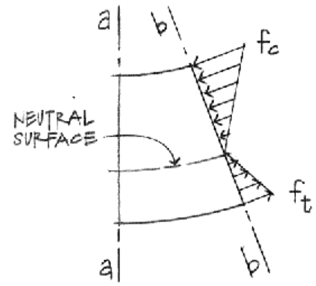


Figure 8.8 Bending stresses on section b-b.

$$= \Sigma \frac{y f_{max}}{c} y \Delta A = \frac{f_{max}}{c} \underline{\Sigma y^2 \Delta A} = \frac{f_{max}}{c} I = f_{max} S$$

Bending Stress Relations

$$\frac{1}{R} = \frac{M}{EI}$$

curvature

$$f_b = \frac{My}{I}$$

general bending stress

$$S = \frac{I}{c}$$

section modulus

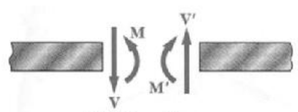
$$f_b = \frac{M}{S}$$

maximum bending stress

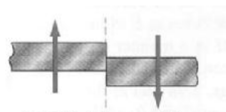
$$S_{required} \geq \frac{M}{F_b}$$

required section modulus for design

Transverse Loading and Shear



(a) Internal forces (positive shear and positive bending moment)



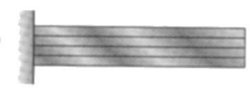
(b) Effect of external forces (positive shear)

- perpendicular loading
- internal shear
- along with bending moment

Bending vs. Shear in Design

- bending stresses dominate

(a)



- shear stresses exist horizontally with shear

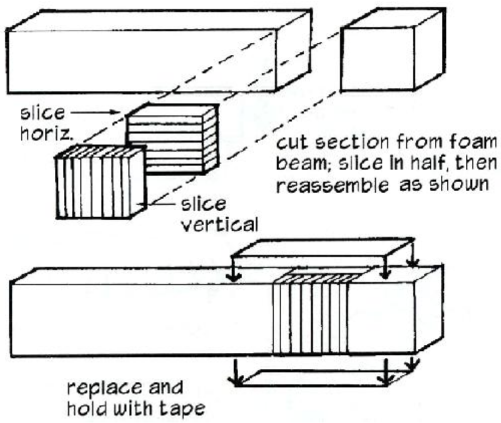
(b)

- <u>no shear stresses</u> with pure bending

(c)

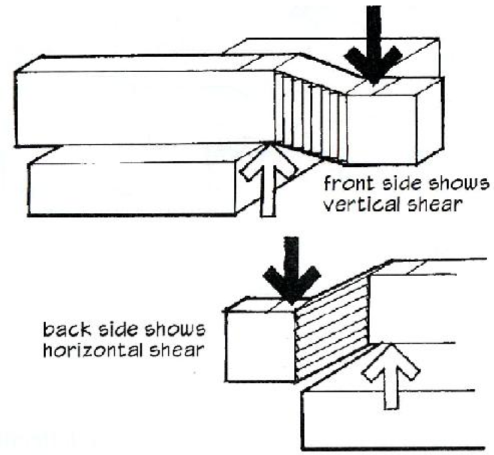
Shear Stresses

- *horizontal & vertical*



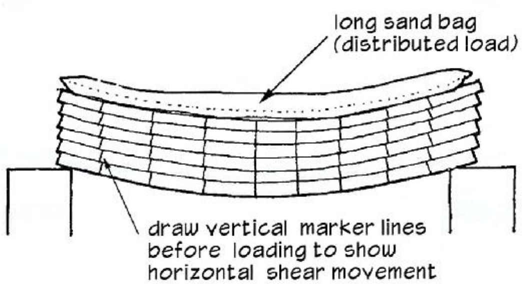
Shear Stresses

- *horizontal & vertical*



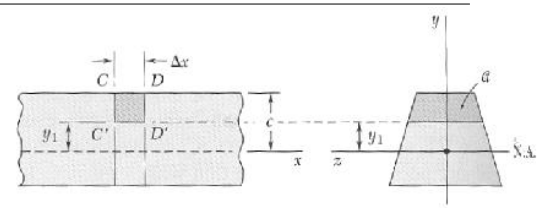
Beam Stresses

- *horizontal with bending*

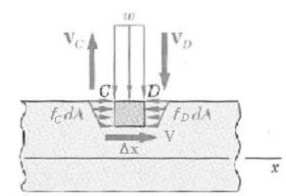


Equilibrium

- *horizontal force V needed*



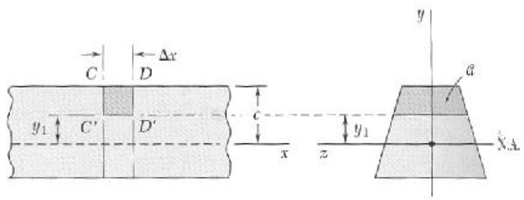
$$V_{longitudinal} = \frac{V_T Q}{I} \Delta x$$



- *Q is a moment area*

Moment of Area

- *Q is a moment area with respect to the n.a. of area above or below the horizontal*
- Q_{max} *at y=0 (neutral axis)*



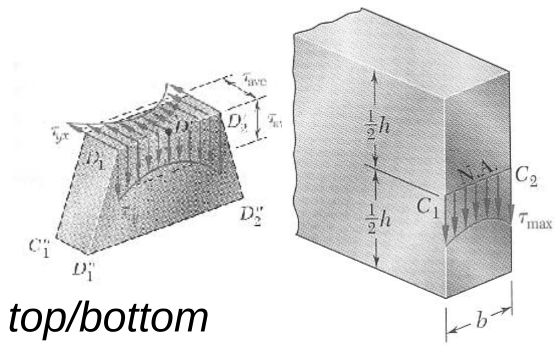
- *q is shear flow:*

$$q = \frac{V_{longitudinal}}{\Delta x} = \frac{V_T Q}{I}$$

Shearing Stresses

$$f_v = \frac{V}{\Delta A} = \frac{V}{b \cdot \Delta x}$$

$$f_{v-ave} = \frac{VQ}{Ib}$$

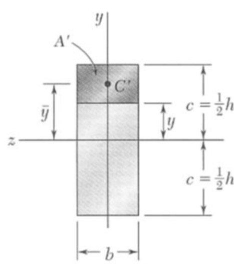


- f_{v-ave} *= 0 on the top/bottom*
- *b min may not be with Q max*
- *with h/4 ≥b,* $f_{v\text{-}max} \le$ *1.008* $f_{v\text{-}ave}$

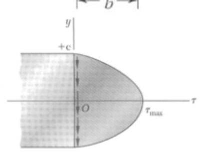
Rectangular Sections

$$I = \frac{bh^3}{12} \qquad Q = A\bar{y} = \frac{bh^2}{8}$$

$$f_v = \frac{VQ}{Ib} = \frac{3V}{2A}$$

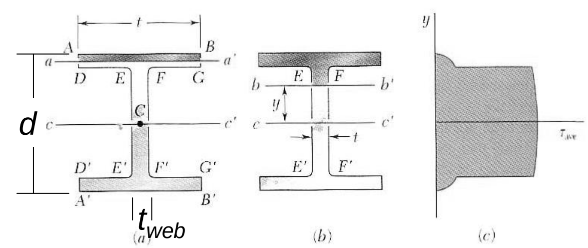


- $f_{v\text{-}max}$ *occurs at n.a.*



Steel Beam Webs

- *W and S sections*
  - *b varies*



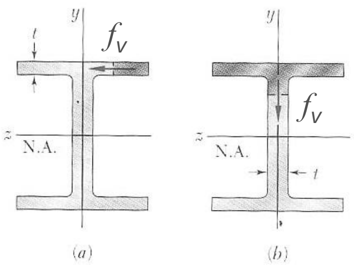
  - *stress in flange negligible*
  - *presume constant stress in web*

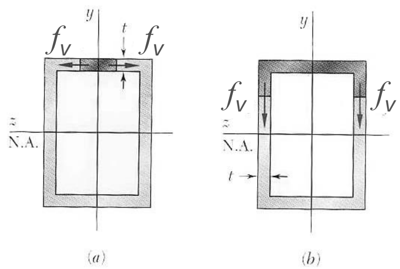
$$f_{v-\max} = \frac{3V}{2A} \approx \frac{V}{A_{web}}$$

Shear Flow

- loads applied in plane of symmetry
- cut made perpendicular

$$q = \frac{VQ}{I}$$

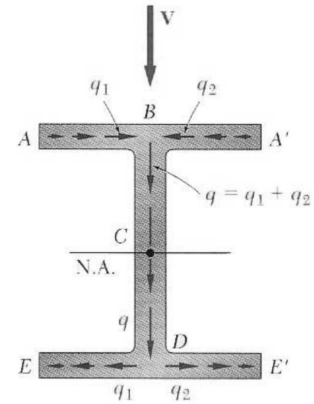




Shear Flow Quantity

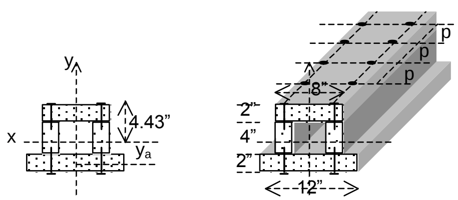
- sketch from Q

$$q = \frac{VQ}{I}$$



Connectors Resisting Shear

- plates with
  - nails
  - rivets
  - bolts
- splices



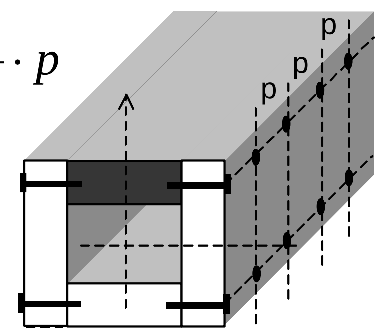
$$\frac{V_{longitudinal}}{p} = \frac{VQ}{I}$$

$$nF_{connector} \geq \frac{VQ_{connected\ area}}{I} \cdot p$$

Vertical Connectors

- isolate an area with vertical interfaces

$$nF_{connector} \geq \frac{VQ_{connected\ area}}{I} \cdot p$$



Unsymmetrical Shear or Section

- member can bend and twist
  - not symmetric
  - shear not in that plane
- shear center
  - moments balance