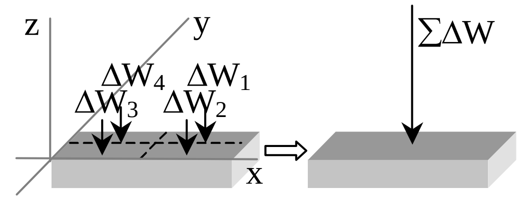
ELEMENTS OF ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN
ARCH 614
DR. ANNE NICHOLS
SPRING 2013

lecture
eight

beam sections - geometric properties

Center of Gravity

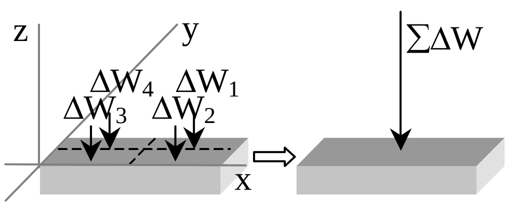
- location of equivalent weight
- determined with calculus



- sum element weights $W = \int dW$

Center of Gravity

- "average" x & y from moment



$$\sum M_y = \sum_{i=1}^{n} x_i \Delta W_i = \bar{x}\boldsymbol{W} \Rightarrow \quad \bar{x} = \frac{\sum(x\Delta W)}{\boldsymbol{W}}$$

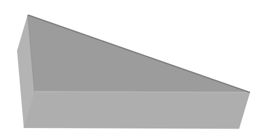
"bar" means average

$$\sum M_x = \sum_{i=1}^{n} y_i \Delta W_i = \bar{y}\boldsymbol{W} \Rightarrow \quad \bar{y} = \frac{\sum(y\Delta W)}{\boldsymbol{W}}$$

Centroid

- "average" x & y of an area
- for a volume of constant thickness
  - $\Delta W = \gamma t \Delta A$ where γ is weight/volume
  - center of gravity = centroid of area

$$\bar{x} = \frac{\sum(x\Delta A)}{\boldsymbol{A}}$$



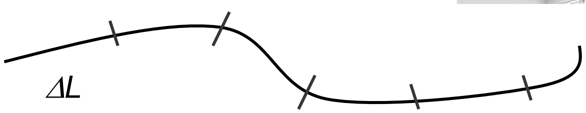
$$\bar{y} = \frac{\sum(y\Delta A)}{\boldsymbol{A}}$$

Centroid

- *for a line, sum up length*

$$\bar{x} = \frac{\sum(x\Delta L)}{\boldsymbol{L}}$$

$$\bar{y} = \frac{\sum(y\Delta L)}{\boldsymbol{L}}$$

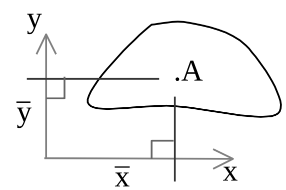


1^{st} Moment Area

- *math concept*
- *the moment of an <u>area</u> about an axis*

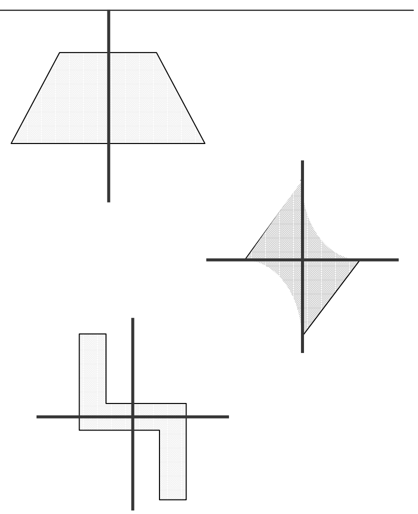
$$Q_x = \bar{y}A$$

$$Q_y = \bar{x}A$$



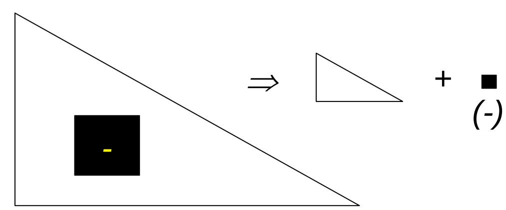
Symmetric Areas

- *symmetric about an axis*
- *symmetric about a center point*
- *mirrored symmetry*



Composite Areas

- *made up of basic shapes*
- *areas can be <u>negative</u>*
- *(centroids can be negative for any area)*



Basic Procedure

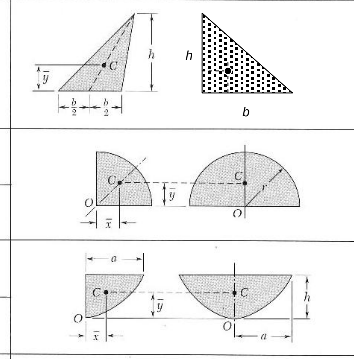
1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate $\hat{x}$ and $\hat{y}$

Component	Area	$\bar{x}$	$\bar{x}A$	$\bar{y}$	$\bar{y}A$
Σ					

Area Centroids

- Figure A.1 – pg 598

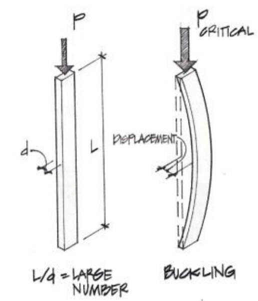
Centroids of Common Shapes of Areas and Lines

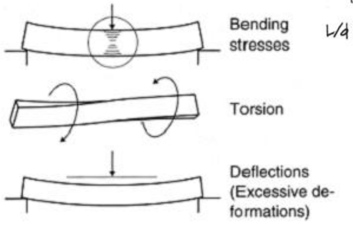


Shape		$\bar{x}$	$\bar{y}$
Triangular area		$\frac{b}{3}$ right triangle only	$\frac{h}{3}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Semicircular area		0	$\frac{4r}{3\pi}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$
Parabolic area		0	$\frac{3h}{5}$

Moments of Inertia

- 2nd moment area
  - math concept
  - area x (distance)^2
- need for behavior of
  - beams
  - columns





Transverse Loadings

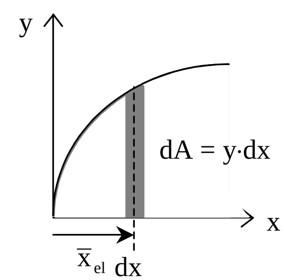
Moment of Inertia

- about any reference axis
- can be negative

$$I_y = \sum x_i^2 \Delta A = \int x^2 dA$$

$$I_x = \sum y_i^2 \Delta A = \int y^2 dA$$

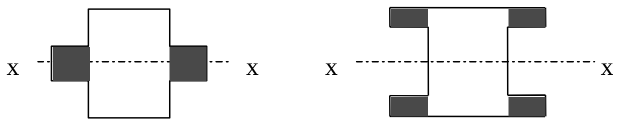
$$(or\ I_{x-x} = \sum z^2 a)$$



- resistance to bending and buckling

Moment of Inertia

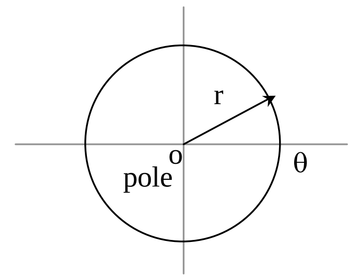
- same area moved away a distance
  - larger I



Polar Moment of Inertia

- for roundish shapes
- uses polar coordinates (r and θ)
- resistance to twisting

$$J_o = \int r^2 dA$$



Radius of Gyration

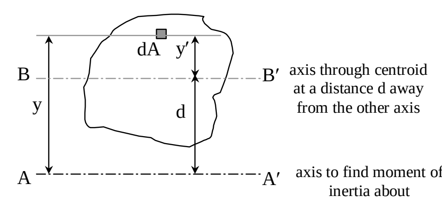
- measure of inertia with respect to area

$$r_x = \sqrt{\frac{I_x}{A}}$$

Parallel Axis Theorem

- can find composite I once composite centroid is known (basic shapes)

$$I = I_o + Az^2$$
$$= \bar{I}_x + Ad_y^{\ 2}$$



$$I = \sum \bar{I} + \sum Ad^2$$

$$\boxed{\bar{I} = I - Ad^2}$$

Basic Procedure

1. *Draw reference origin (if not given)*
2. *Divide into basic shapes (+/-)*
3. *Label shapes*
4. *Draw table with A,* $\bar{x}$ $\bar{x}A$ $\bar{y}$ $\bar{y}A$ $\bar{I}$*'s, d's, and Ad²'s*
5. *Fill in table and get* $\hat{x}$ *and* $\hat{y}$ *for composite*
6. *Sum necessary columns*
7. *Sum* I*'s and Ad²'s*

$$(d_x = \hat{x} - \bar{x})$$
$$(d_y = \hat{y} - \bar{y})$$

Area Moments of Inertia

- *Figure A.11 – pg. 611: (bars refer to centroid)*
  - *x, y*
  - *x', y'*
  - *C*