

lecture  
seven

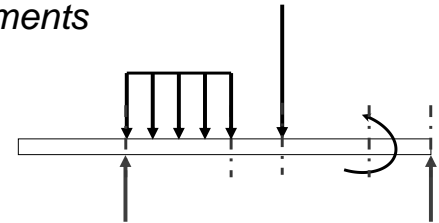
shear & bending  
moment diagrams



Forum, Pompeii

Equilibrium Method

- important places
  - supports
  - concentrated loads
  - start and end of distributed loads
  - concentrated moments
- free ends
  - zero forces



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Semigraphical Method

- by knowing
  - area under loading curve = change in V
  - area under shear curve = change in M
  - concentrated forces cause “jump” in V
  - concentrated moments cause “jump” in M

$$V_D - V_C = - \int_{x_C}^{x_D} w dx \quad M_D - M_C = \int_{x_C}^{x_D} V dx$$

Semigraphical M

- relationships

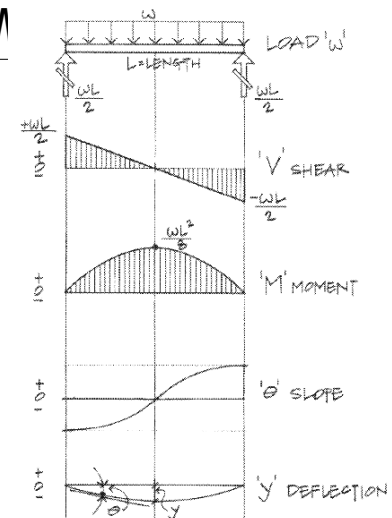


Figure 7.11 Relationship of load, shear, moment, slope, and deflection diagrams.

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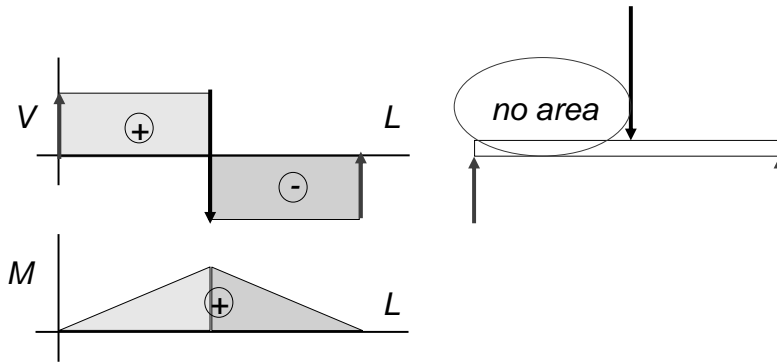
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## Semigraphical Method

- $M_{max}$  occurs where  $V = 0$  (calculus)



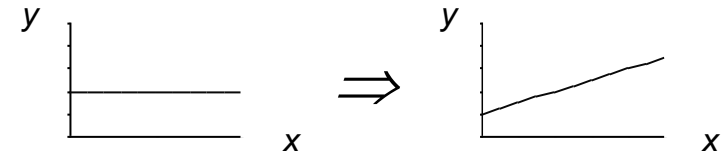
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## Curve Relationships

- integration of functions
- line with 0 slope, integrates to sloped



- ex: load to shear, shear to moment

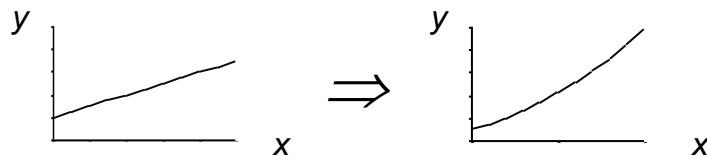
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## Curve Relationships

- line with slope, integrates to parabola



- ex: load to shear, shear to moment

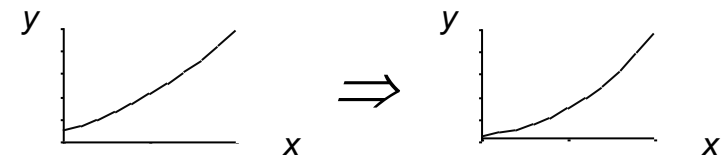
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## Curve Relationships

- parabola, integrates to 3<sup>rd</sup> order curve



- ex: load to shear, shear to moment

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## Basic Procedure

1. Find reaction forces & moments  
Plot axes, underneath beam load diagram

V:

2. Starting at left
3. Shear is 0 at free ends
4. Shear jumps with concentrated load
5. Shear changes with area under load

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## Basic Procedure

M:

6. Starting at left
7. Moment is 0 at free ends
8. Moment jumps with moment
9. Moment changes with area under V
10. Maximum moment is where shear = 0!  
(locate where  $V = 0$ )

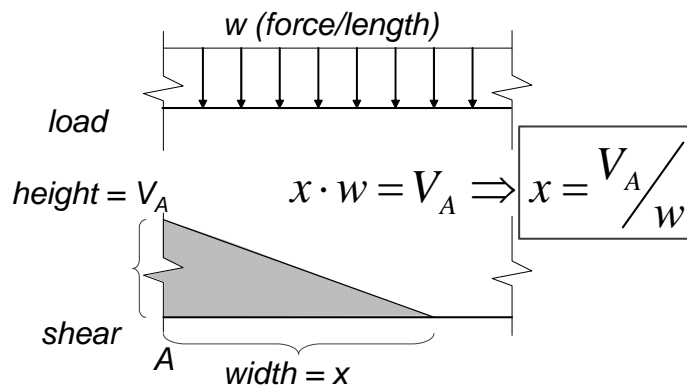
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## Triangle Geometry

- slope of V is  $w$  ( $-w:1$ )



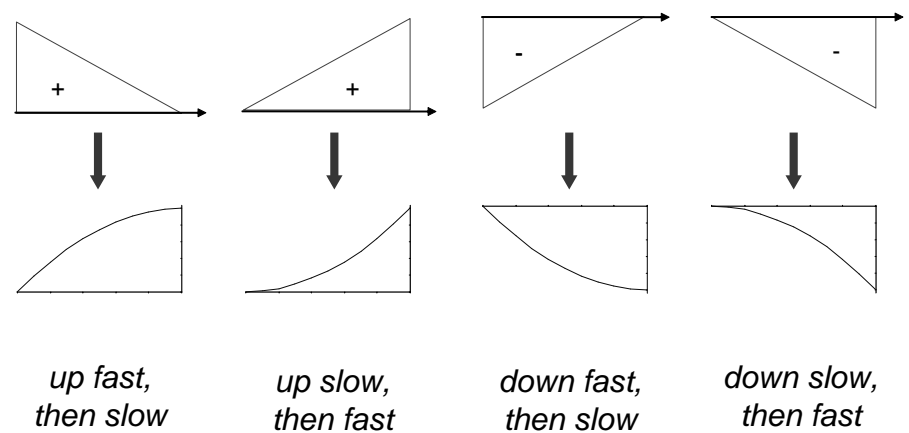
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## Parabolic Shapes

- cases



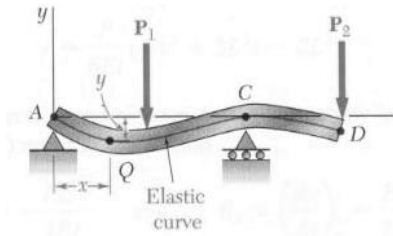
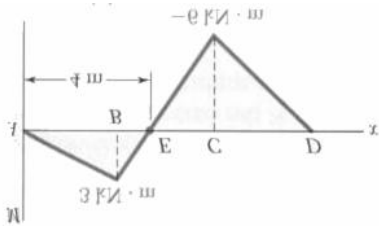
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## Deflected Shape & M(x)

- M(x) gives shape indication
- boundary conditions must be met



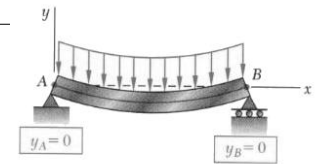
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## Boundary Conditions

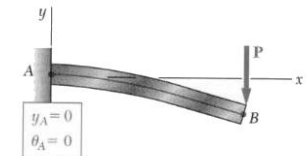
- at pins, rollers, fixed supports:  $y = 0$
- at fixed supports:  $\theta = 0$
- at inflection points from symmetry:  $\theta = 0$
- $y_{max}$  at  $\frac{dy}{dx} = 0$



(a) Simply supported beam



(b) Overhanging beam



(c) Cantilever beam

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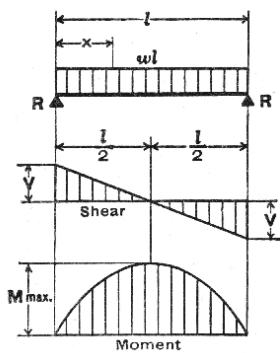
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## Tabulated Beam Formulas

- how to read charts

### 1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load	...	=	$wl$
$R = V$	...	=	$\frac{wl}{2}$
$V_x$	...	=	$w \left( \frac{l}{2} - x \right)$
$M_{max.}$ (at center)	...	=	$\frac{wl^2}{8}$
$M_x$	...	=	$\frac{wx}{2} (l - x)$
$\Delta_{max.}$ (at center)	...	=	$\frac{5wl^4}{384EI}$
$\Delta_x$	...	=	$\frac{wx}{24EI} (l^3 - 2lx^2 + x^3)$

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