ELEMENTS OF ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN

ARCH 614

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SPRING 2013

lecture
four

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mechanics of materials

Mechanics of Materials

- MECHANICS
- MATERIALS

Mechanics of Materials

- external loads and their effect on deformable bodies
- use it to answer question if structure meets requirements of
  - stability and equilibrium
  - strength and stiffness
- other principle building requirements
  - economy, functionality and aesthetics

Knowledge Required

- material properties
- member cross sections
- ability of a material to resist breaking
- structural elements that resist excessive
  - deflection
  - deformation

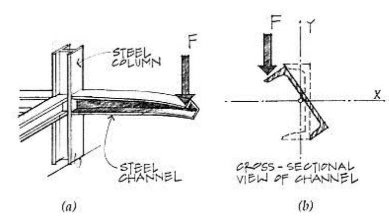


Figure 2.34 An example of torsion on a cantilever beam.

Problem Solving

1. STATICS:

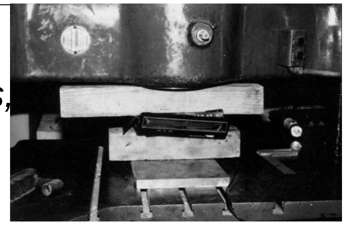
   equilibrium of external forces, internal forces, <u>stresses</u>

2. GEOMETRY:

   cross section properties, deformations and conditions of geometric fit, <u>strains</u>

3. MATERIAL PROPERTIES:

   <u>stress-strain relationship</u> for each material obtained from testing



Stress

- stress is a term for the <u>intensity</u> of a force, like a pressure
- internal <u>or</u> applied
- force per unit area

$$stress = f = \frac{P}{A}$$

Design

- materials have a critical stress value where they could break or yield
  - ultimate stress
  - yield stress
  - compressive stress
  - fatigue strength
  - (creep & temperature)

  (ultimate stress, yield stress, compressive stress, fatigue strength: acceptance vs. failure)

Design (cont)

- we'd like

$$f_{actual} << F_{allowable}$$

- stress distribution may vary: <u>average</u>
- uniform distribution exists IF the member is loaded axially (concentric)

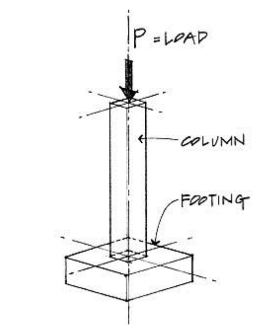
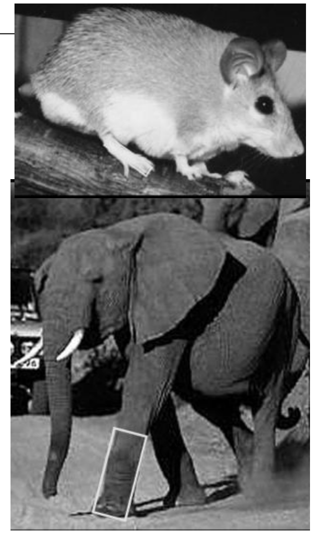


Figure 5.3 Centric loads.

Scale Effect

- model scale
  - material weights by volume, small section areas
- structural scale
  - much more material weight, bigger section areas
- scale for strength is not proportional: $\frac{\gamma L^3}{L^2} = \gamma L$



Normal Stress

- normal stress is normal to the cross section
  - stressed area is perpendicular to the load

$$f_{t\ or\ c} \; (\sigma) = \frac{P}{A}$$

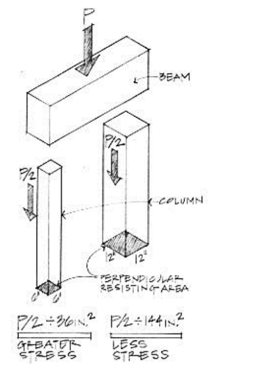


Figure 5.7 Two columns with the same load, different stress.

Shear Stress

- stress parallel to a surface

$$f_v \; (\tau_{ave}) = \frac{P}{A} = \frac{P}{td}$$

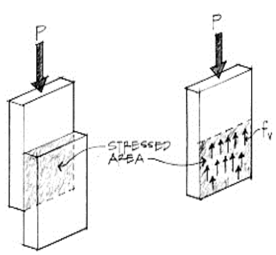


Figure 5.10 Shear stress between two glued blocks.

Bearing Stress

- stress on a surface by contact in compression

$$f_p \; (\sigma) = \frac{P}{A} = \frac{P}{td}$$

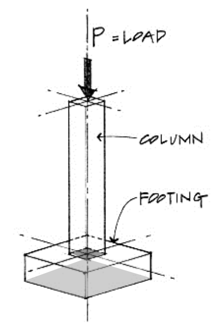


Figure 5.3 Centric loads.

Bending Stress

- *normal stress caused by bending*

$$f_b = \frac{Mc}{I} = \frac{M}{S}$$
$$(\sigma)$$

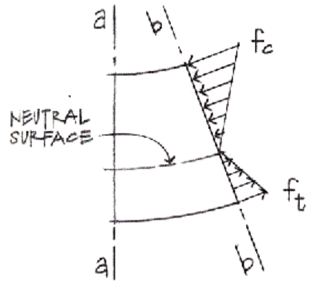
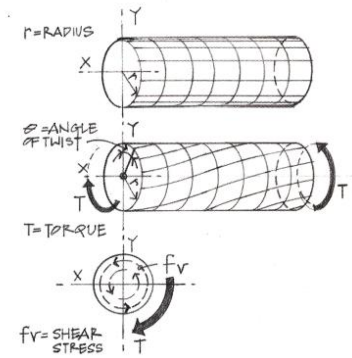


Figure 8.8 Bending stresses on section b-b.

Torsional Stress

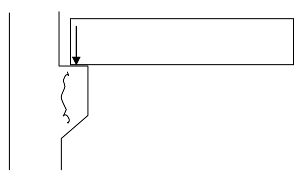
- *shear stress caused by twisting*

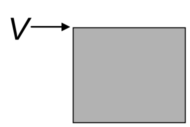
$$f_v = \frac{T\rho}{J}$$
$$(\tau)$$



Structures and Shear

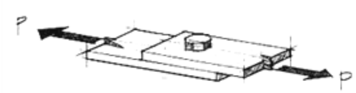
- *what structural elements see shear?*
  - *beams*
  - *bolts* } *connections*
  - *splices* } *connections*
  - *slabs*
  - *footings*
  - *walls*
    - *wind*
    - *seismic loads*



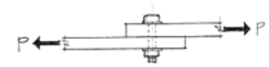


Bolts

- *connected members in tension cause shear stress*

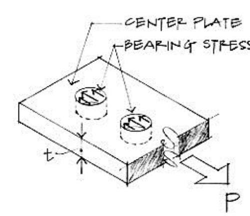


(a) Two steel plates bolted using one bolt.



(b) Elevation showing the bolt in

- *connected members in compression cause bearing stress*



Bearing stress on plate.

Single Shear

- seen when 2 members are connected

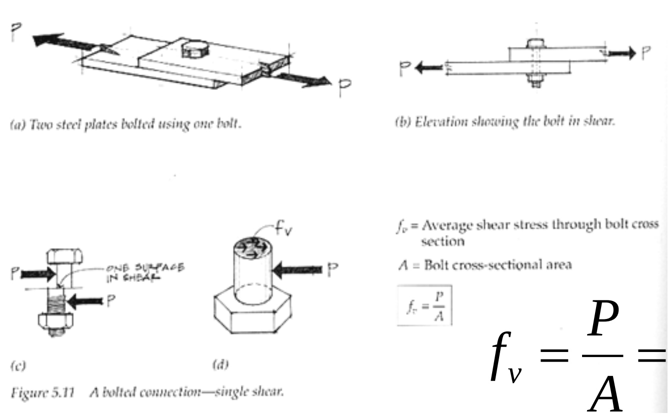


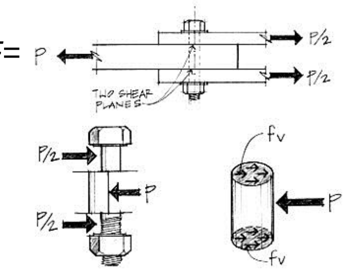
Figure 5.11 A bolted connection—single shear.

$$f_v = \frac{P}{A} = \frac{P}{\pi \, d^2/4}$$

Double Shear

- seen when 3 members are connected
- <u>two</u> areas

F= P



Free-body diagram of middle section of the bolt in shear.
Figure 5.12 A bolted connection in double shear.

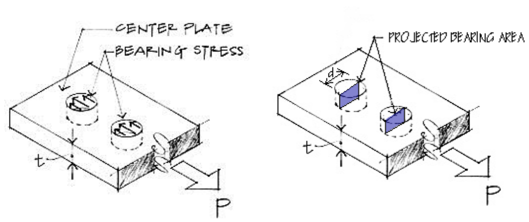
$f_v = \frac{P}{2A}$

(two shear planes)

$$f_v = \frac{P}{2A} = \frac{P/2}{A} = \frac{P/2}{\pi \, d^2/4}$$

Bolt Bearing Stress

- compression & contact
- projected area

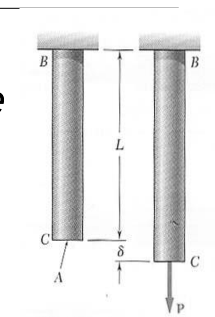


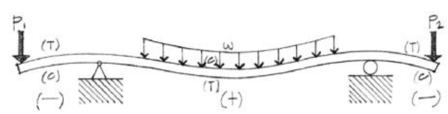
Bearing stress on plate.

$$f_p = \frac{P}{A_{projected}} = \frac{P}{td}$$

Strain

- materials deform
- axially loaded materials change length
- bending materials deflect





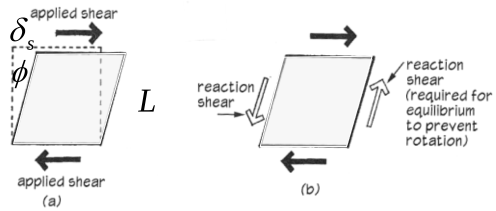
- STRAIN:
  - change in length over length

$$strain = \varepsilon = \frac{\Delta L}{L}$$

(S)

Shearing Strain

- *deformations with shear*
- *parallelogram*
- *change in angles*
- *stress:* τ
- *strain:* γ
  - *unitless (radians)*

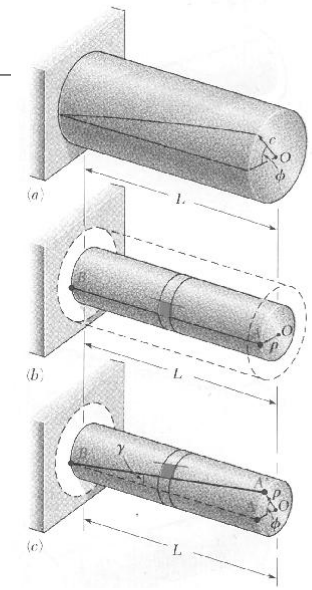


$$\gamma = \frac{\delta_s}{L} = \tan\phi \cong \phi$$

Shearing Strain

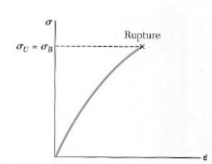
- *deformations with torsion*
- *twist*
- *change in angle of line*
- *stress:* τ
- *strain:* γ
  - *unitless (radians)*

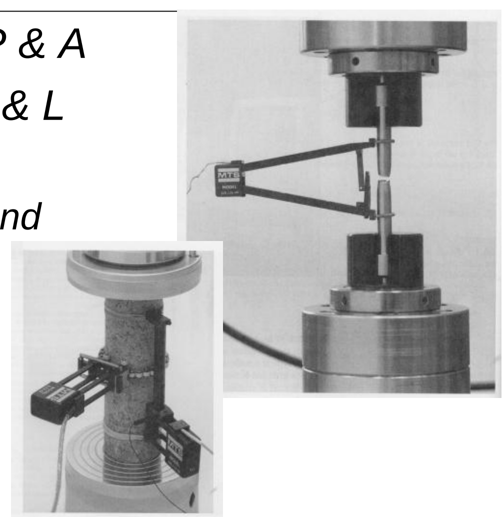
$$\gamma = \frac{\rho\phi}{L}$$



Load and Deformation

- *for stress, need P & A*
- *for strain, need* δ *& L*
  - *how?*
  - *TEST with load and measure*
  - *plot P/A vs.* ε





Material Behavior

- *every material has its own response*
  - *10,000 psi*
  - *L = 10 in*
  - *Douglas fir vs. steel?*

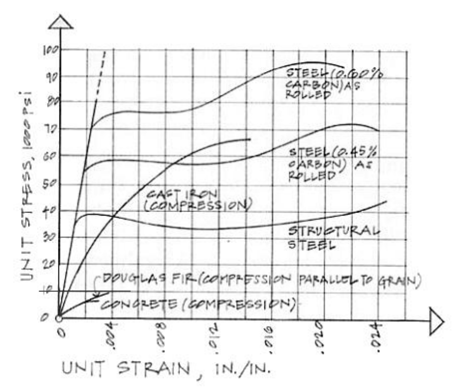


Figure 5.20 Stress-strain diagram for various materials.

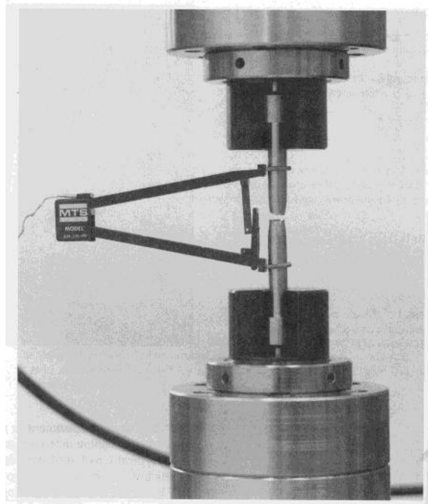
Behavior Types

- ductile - “necking”
- true stress

$$f = \frac{P}{A}$$

- engineering stress
  - (simplified)

$$f = \frac{P}{A_o}$$



Behavior Types

- brittle

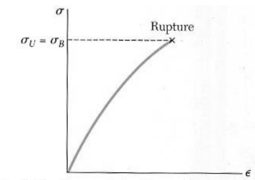


Fig. 2.11 Stress-strain diagram for a typical brittle material.

- semi-brittle

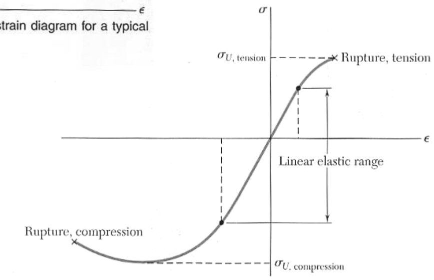


Fig. 2.14 Stress-strain diagram for concrete.

Stress to Strain

- important to us in f-ε diagrams:
  - straight section
  - LINEAR-ELASTIC
  - recovers shape (no permanent deformation)

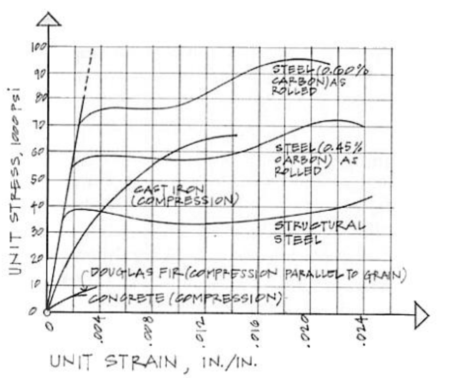
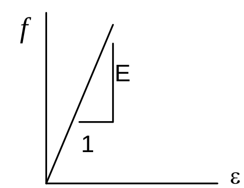


Figure 5.20 Stress-strain diagram for various materials.

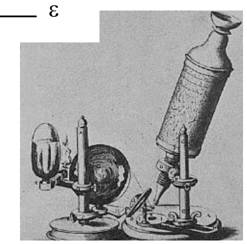
Hooke’s Law

- straight line has constant slope
- Hooke’s Law

$$f = E \cdot \varepsilon$$



- E
  - Modulus of elasticity
  - Young’s modulus
  - units just like stress



Stiffness

- ability to resist strain
- steels
  - same E
  - different <u>yield points</u>
  - different <u>ultimate strength</u>

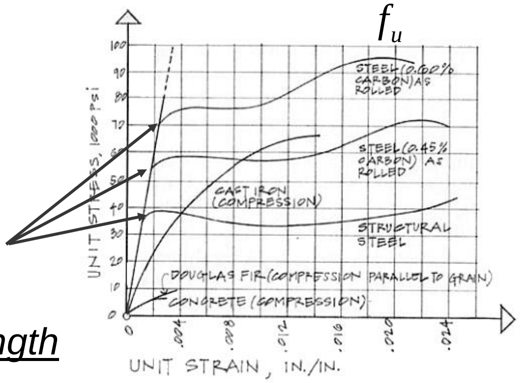
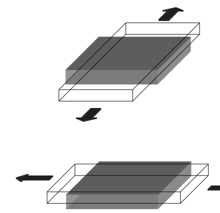
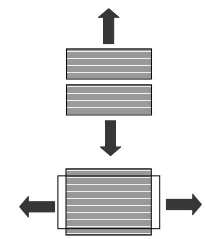


Figure 5.20 Stress-strain diagram for various materials.

Isotropy & Anisotropy

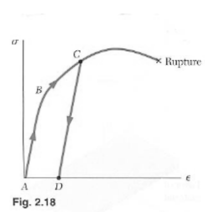
- ISOTROPIC
  - materials with E same at any direction of loading
  - ex. steel
- ANISOTROPIC
  - materials with different E at any direction of loading
  - ex. wood is orthotropic

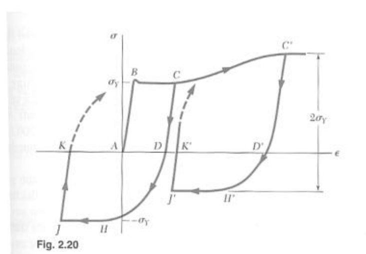




Elastic, Plastic, Fatigue

- elastic springs back
- plastic has permanent deformation
- fatigue caused by reversed loading cycles





Plastic Behavior

- ductile

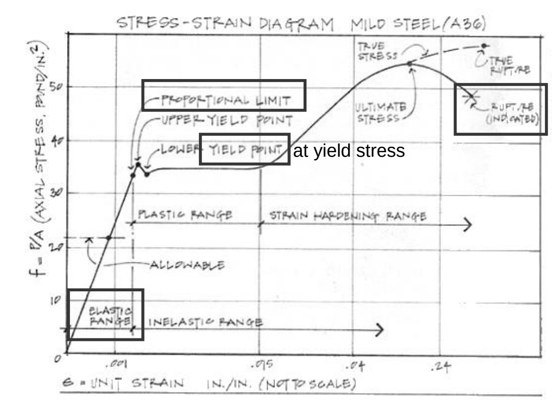


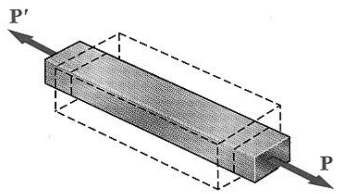
Figure 5.22 Stress-strain diagram for mild steel (A36) with key points highlighted.

Lateral Strain

- or "what happens to the cross section with axial stress"

$$\varepsilon_x = \frac{f_x}{E}$$

$$f_y = f_z = 0$$



- strain in lateral direction
  - negative
  - equal for isometric materials

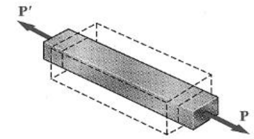
$$\varepsilon_y = \varepsilon_z$$

Poisson's Ratio

- constant relationship between longitudinal strain and lateral strain

$$\mu = -\frac{lateral\ strain}{axial\ strain} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

$$\varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E}$$



- sign! $0 < \mu < 0.5$

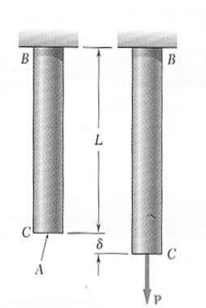
Calculating Strain

- from Hooke's law

$$f = E \cdot \varepsilon$$

- substitute

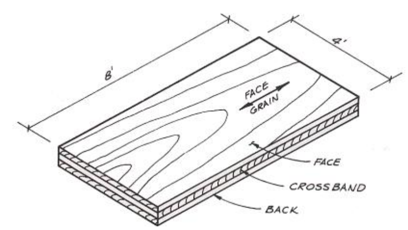
$$\frac{P}{A} = E \cdot \frac{\delta}{L}$$

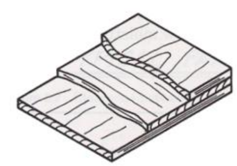


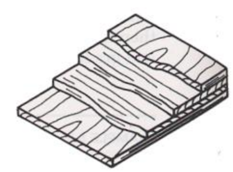
- get ⇒ $\delta = \frac{PL}{AE}$ (ΔL)

Orthotropic Materials

- non-isometric
- directional values of E and μ
- ex:
  - plywood
  - laminates
  - polymer composites

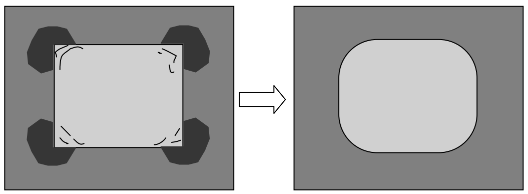






Stress Concentrations

- why we use f_{ave}
- increase in stress at changes in geometry
  - sharp notches
  - holes
  - corners



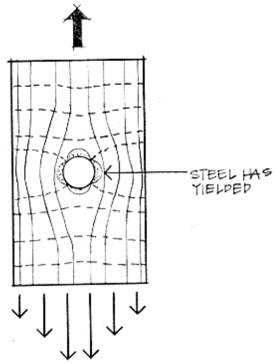
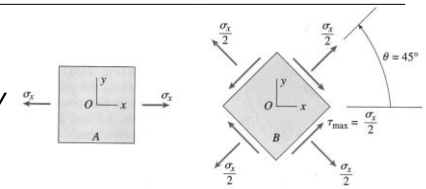


Figure 5.35 Stress trajectories around a hole.

Maximum Stresses

- if we need to know where max f and f_v happen:



$$\theta = 0° \rightarrow cos\,\theta = 1 \qquad f_{max} = \frac{P}{A_o}$$

$$\theta = 45° \rightarrow cos\,\theta = sin\,\theta = \sqrt{0.5}$$

$$f_{v-max} = \frac{P}{2A_o} = \frac{f_{max}}{2}$$

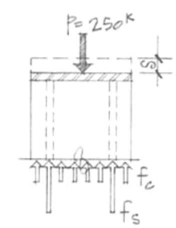
Maximum Stresses

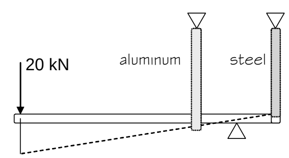
FIG. 2-37 Shear failure along a 45° plane of a wood block loaded in compression

FIG. 2-38 Slip bands (or Lüders' bands) in a polished steel specimen loaded in tension

Deformation Relationships

- physical movement
  - axially (same or zero)
  - rotations from axial changes

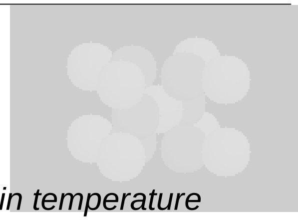


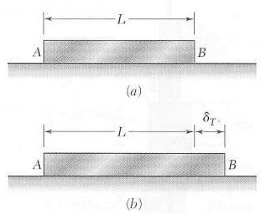


- $\delta = \frac{PL}{AE}$ relates δ to P

Deformations from Temperature

- *atomic chemistry reacts to changes in energy*
- *solid materials*
  - *can contract with decrease in temperature*
  - *can expand with increase in temperature*
- *linear change can be measured per degree*



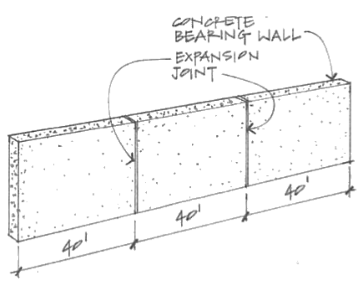


Thermal Deformation

- α *- the rate of strain per degree*
- *UNITS :* $/°F$, $/°C$
- *length change:* $\delta_T = \alpha(\Delta T)L$
- *thermal strain:* $\varepsilon_T = \alpha(\Delta T)$
  - *<u>no stress</u> when movement allowed*

Coefficients of Thermal Expansion

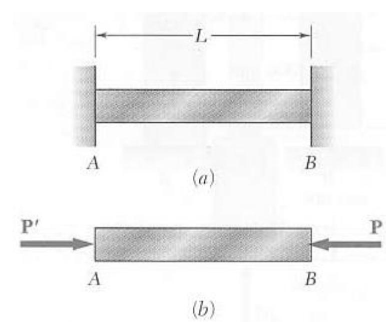
Material	Coefficients (α) [in./in./°F]
Wood	3.0×10^{-6}
Glass	4.4×10^{-6}
Concrete	5.5×10^{-6}
Cast Iron	5.9×10^{-6}
Steel	6.5×10^{-6}
Wrought Iron	6.7×10^{-6}
Copper	9.3×10^{-6}
Bronze	10.1×10^{-6}
Brass	10.4×10^{-6}
Aluminum	12.8×10^{-6}



Stresses and Thermal Strains

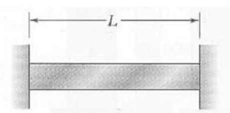
- *if thermal movement is restrained <u>stresses</u> are induced*

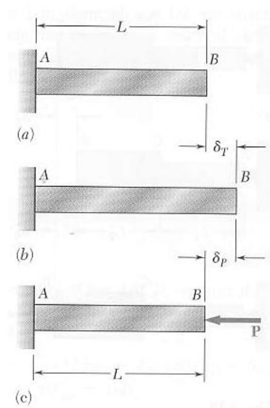
1. *bar pushes on supports*
2. *support pushes back*
3. *reaction causes internal stress* $f = \frac{P}{A} = \frac{\delta}{L}E$



Superposition Method

– can remove a support to make it look determinant
– replace the support with a reaction
– enforce the geometry constraint





Superposition Method

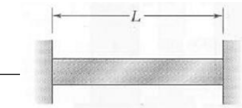
– total length change restrained to <u>zero</u>

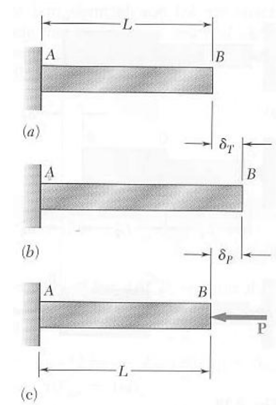
constraint: $\delta_P + \delta_T = 0$

$$\delta_p = -\frac{PL}{AE} \qquad \delta_T = \alpha(\Delta T)L$$

sub: $$-\frac{PL}{AE} + \alpha(\Delta T)L = 0$$

$$f = -\frac{P}{A} = -\alpha(\Delta T)E$$



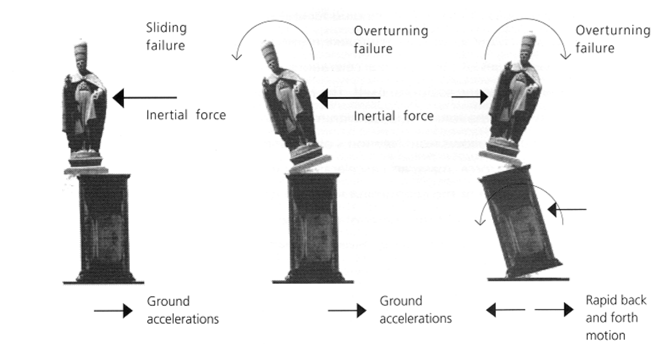


Dynamics

- kinematics
  - time, velocity, acceleration
  - linear motion $s(t) = v(0)t + \frac{1}{2}at^2$
  - angular rotation
- kinetics
  - forces causing motion
    W = m·g
  - work
  - conservation of energy

Dynamic Response

Statue in front of the cathedral of Palermo, Sicily



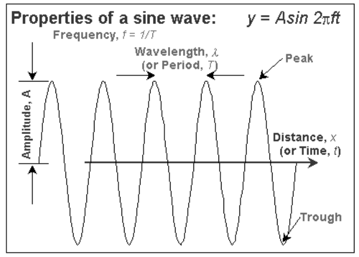
Lateral ground motions associated with earthquakes cause inertial forces to develop that are dependent on the weight of the structure. Sliding failures can occur.

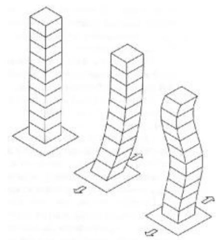
The lateral ground motions can also cause a sculpture to overturn. The magnitude of the overturning effect depends on the weight of the sculpture and its height above the ground.

Back and forth ground motions can cause different parts of the sculpture to move in different directions. Overturning or cracking of elements can consequently occur.

Dynamic Response

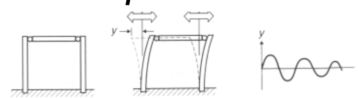
- *period of vibration or frequency*
  - *wave*
  - *sway/time period*
- *damping*
  - *reduction in sway*
- *resonance*
  - *amplification of sway*



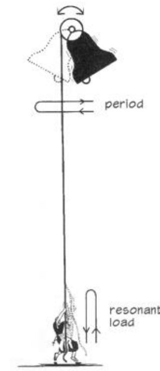


Frequency and Period

- *natural period of vibration*



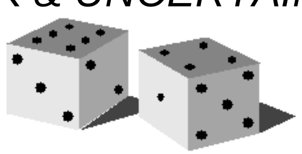
  - *avoid resonance*
  - *hard to predict seismic period*
  - *affected by soil*
  - *short period*
    - *high stiffness*
  - *long period*
    - *low stiffness*



"To ring the bell, the sexton must pull on the downswing of the bell in time with the natural frequency of the bell."

Design of Members

- *beyond allowable stress...*
- *materials aren't uniform 100% of the time*
  - *ultimate strength or capacity to failure may be different and some strengths hard to test for*
- *RISK & UNCERTAINTY*



$$f_u = \frac{P_u}{A}$$

Factor of Safety

- *accommodate uncertainty with a safety factor:*

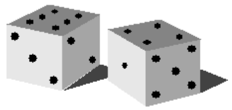
$$\text{allowable load} = \frac{\text{ultimate load}}{F.S}$$

- *with linear relation between load and stress:*

$$F.S = \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

Load and Resistance Factor Design

- *loads on structures are*
  - *not constant*
  - *can be more influential on failure*
  - *happen more or less often*
  - *UNCERTAINTY*



$$R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n$$

ϕ - *resistance factor*

γ - *load factor for (D)ead & (L)ive load*