ELEMENTS OF ARCHITECTURAL STRUCTURES:

FORM, BEHAVIOR, AND DESIGN
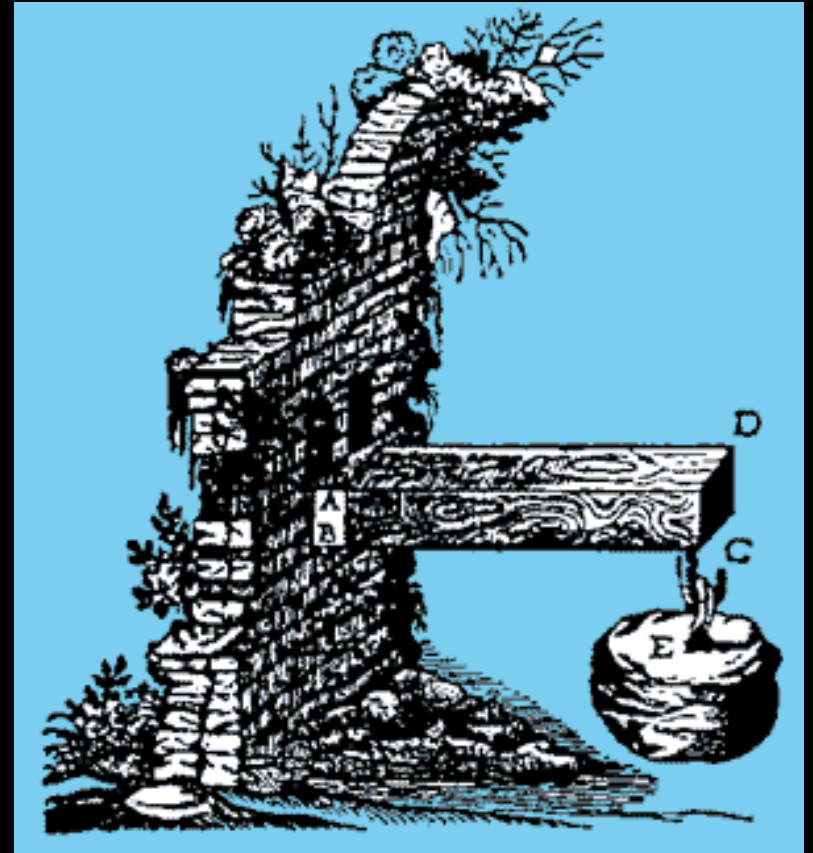
ARCH 614

DR. ANNE NICHOLS

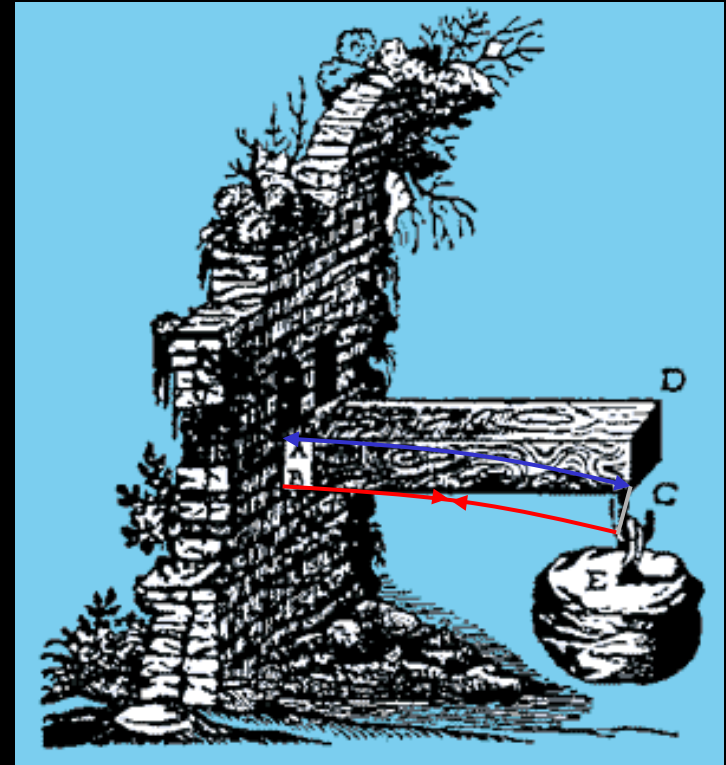SPRING 2013

lecture
nine

beams:
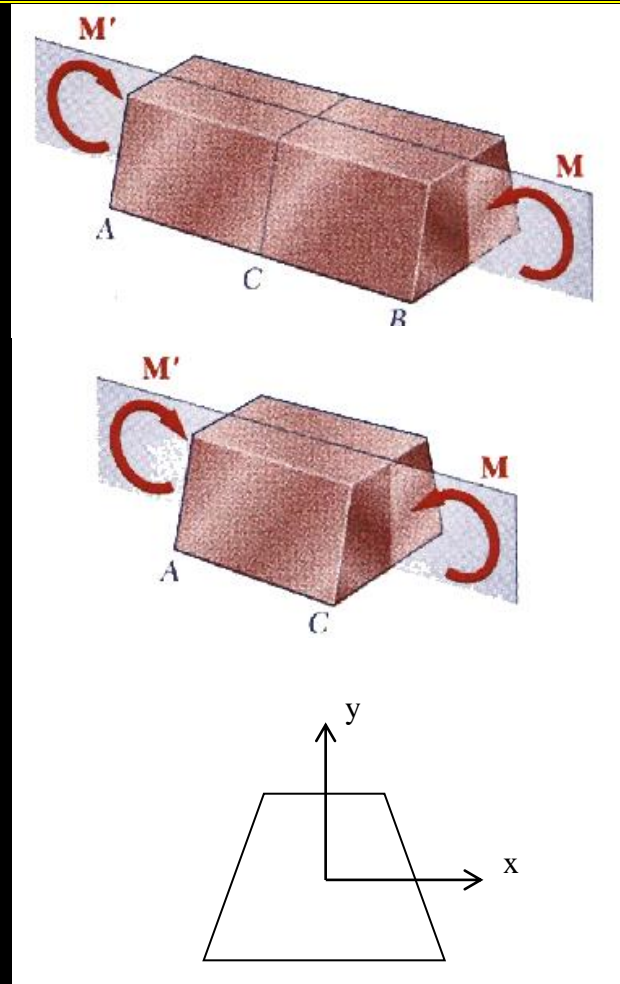# *bending and shear stress*

Beam Bending

- *Galileo*
  - *relationship between stress and depth^2*

- *can see*
  - *top squishing*
  - *bottom stretching*

- *what are the stress across the section?*
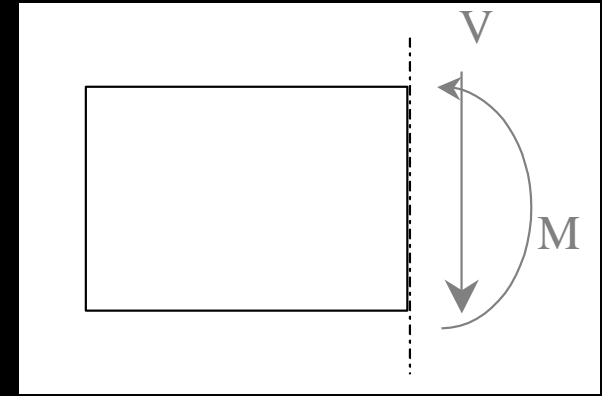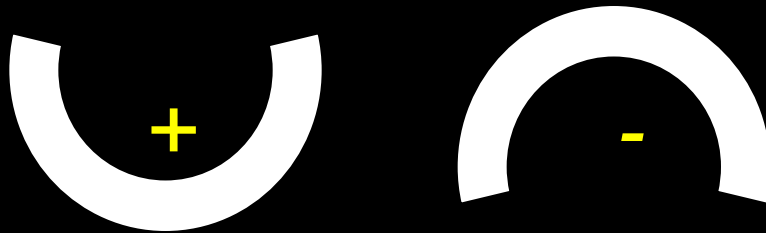
Pure Bending

- *bending only*

- *no shear*

- *axial normal stresses from bending can be found in*
  - *homogeneous materials*
  - *plane of symmetry*
  - *follow Hooke's law*

Bending Moments

- *sign convention:*

- *size of maximum internal moment will govern our design of the section*

Normal Stresses

- *geometric fit*
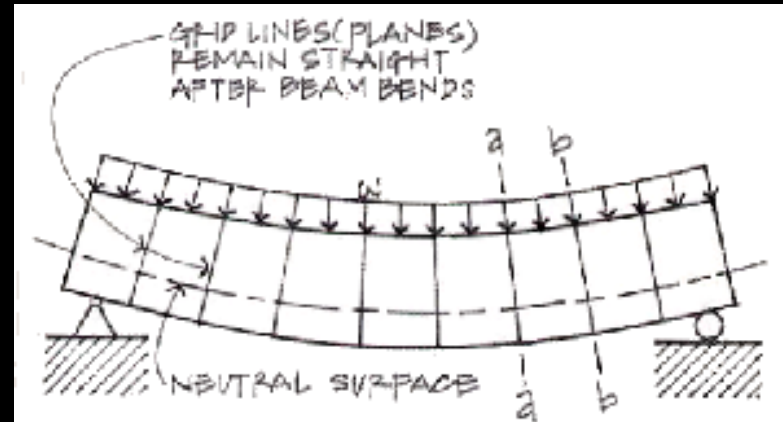 - *plane sections remain plane*
  - *stress varies linearly*

GRID LINES (PLANES) REMAIN STRAIGHT AFTER BEAM BENDS

NEUTRAL SURFACE

Figure 8.5(b)   Beam bending under load.

VERTICAL GRID LINES NEUTRAL SURFACE

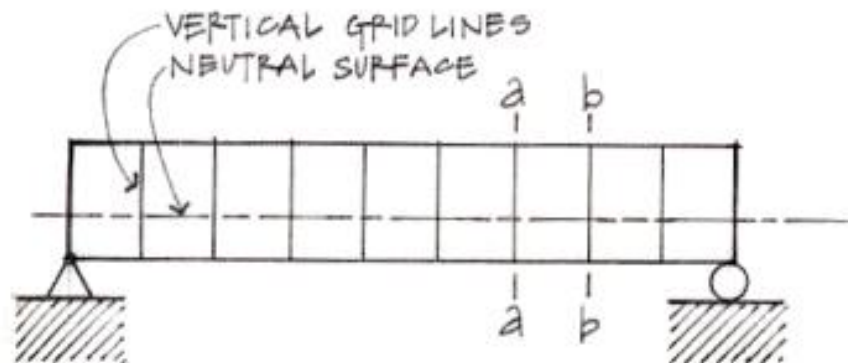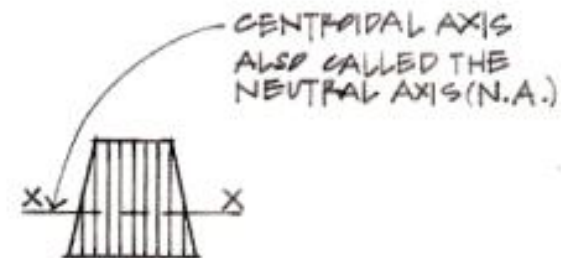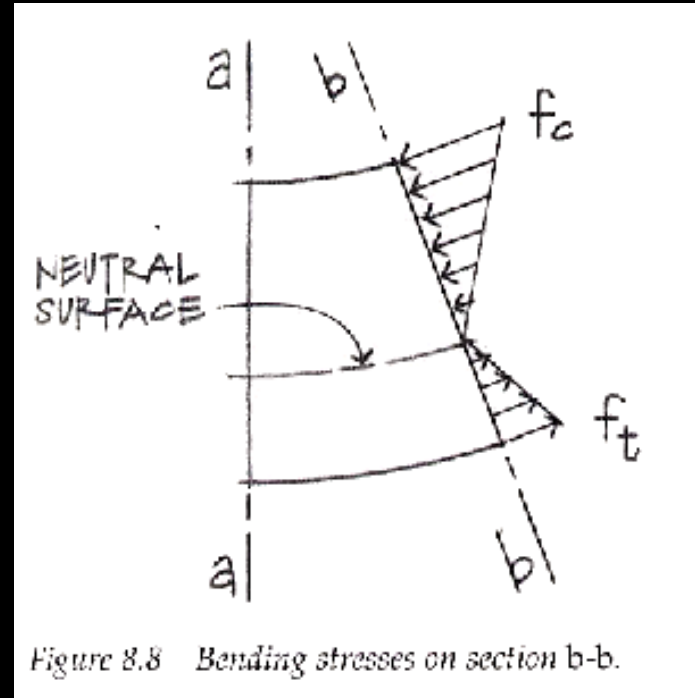CENTROIDAL AXIS ALSO CALLED THE NEUTRAL AXIS (N.A.)

Figure 8.5(a)   Beam elevation before loading.

Beam cross section.

Neutral Axis

- *stresses vary linearly*

- *zero stress occurs at the centroid*
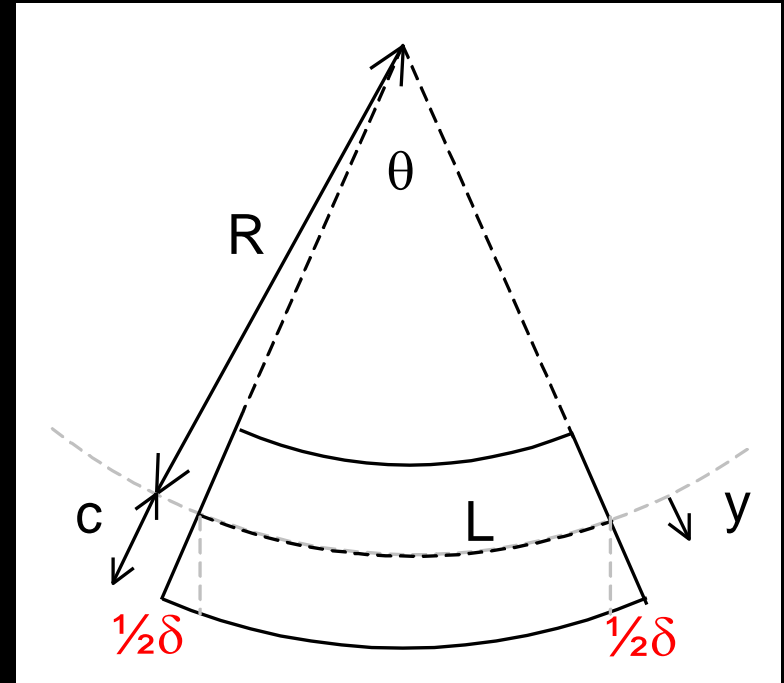
- *neutral axis is line of centroids (n.a.)*

Figure 8.8   Bending stresses on section b-b.

Derivation of Stress from Strain

- *pure bending =
  arc shape*

$$L = R\theta$$

$$L_{outside} = (R + y)\theta$$

$$\varepsilon = \frac{\delta}{L} = \frac{L_{outside} - L}{L} = \frac{(R + y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

Derivation of Stress

- *zero stress at n.a.*

$$f = E\varepsilon = \frac{Ey}{R}$$

$$f_{max} = \frac{Ec}{R}$$

$$f = \frac{y}{c}f_{max}$$

Bending Moment

- *resultant moment from stresses = bending moment!*
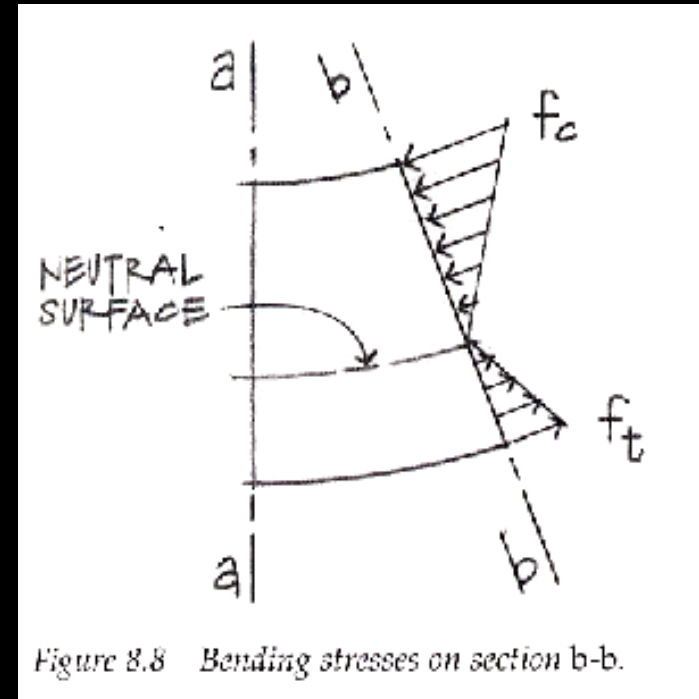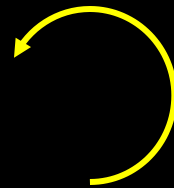
$$M = \Sigma f y \Delta A$$

Figure 8.8   Bending stresses on section b-b.

$$= \Sigma \frac{y f_{max}}{c} y \Delta A = \frac{f_{max}}{c} \underline{\Sigma y^2 \Delta A} = \frac{f_{max}}{c} I = f_{max} S$$

Bending Stress Relations

$$\frac{1}{R} = \frac{M}{EI}$$

curvature

$$f_b = \frac{My}{I}$$

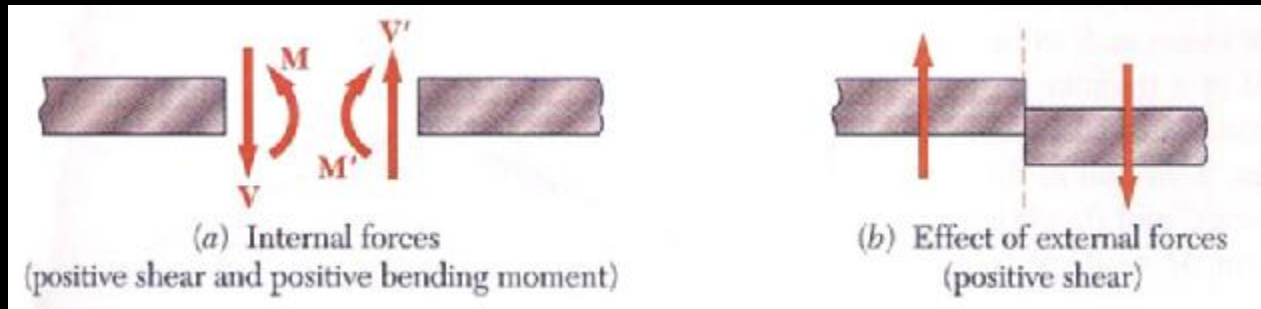*general bending stress*

$$S = \frac{I}{c}$$

section modulus

$$f_b = \frac{M}{S}$$

maximum bending stress

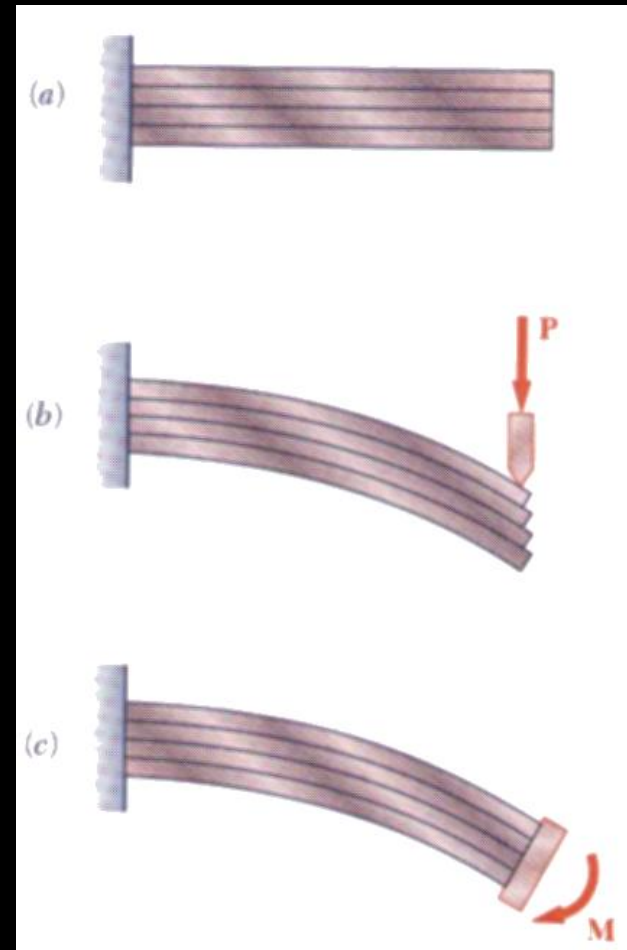$$S_{required} \geq \frac{M}{F_b}$$

required section modulus for design

Transverse Loading and Shear

(a) Internal forces
(positive shear and positive bending moment)

(b) Effect of external forces
(positive shear)

- *perpendicular loading*

- *internal shear*

- *along with bending moment*

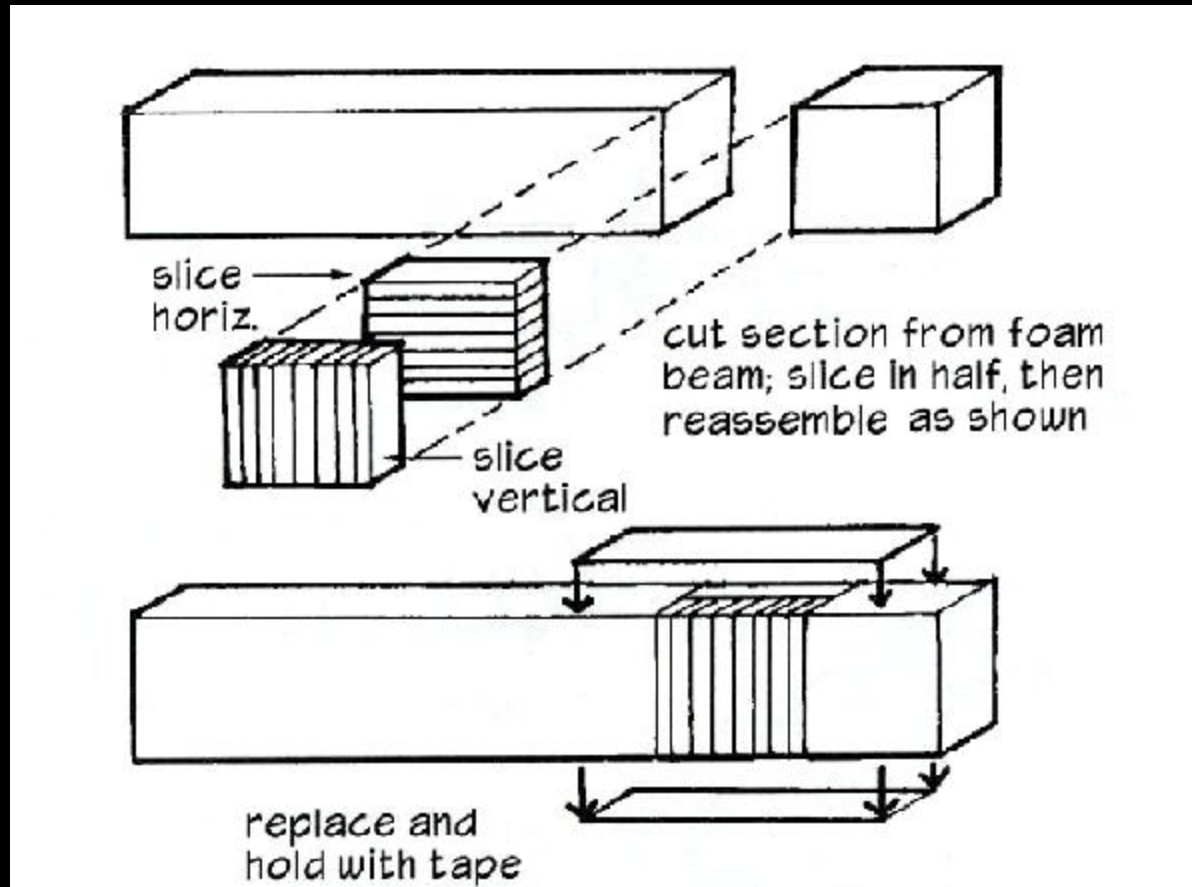# *Bending vs. Shear in Design*

- *bending stresses dominate*

- *shear stresses exist horizontally with shear*
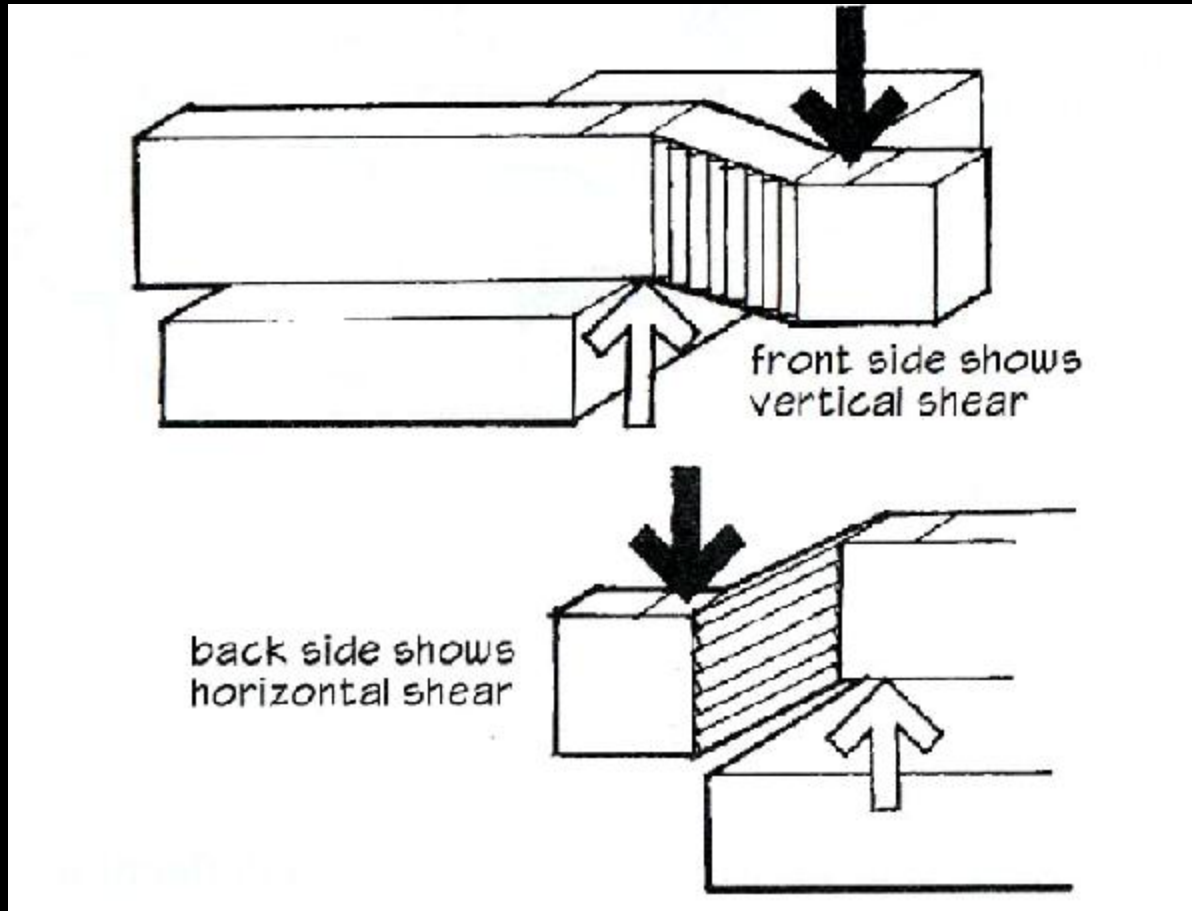
- *no shear stresses with pure bending*
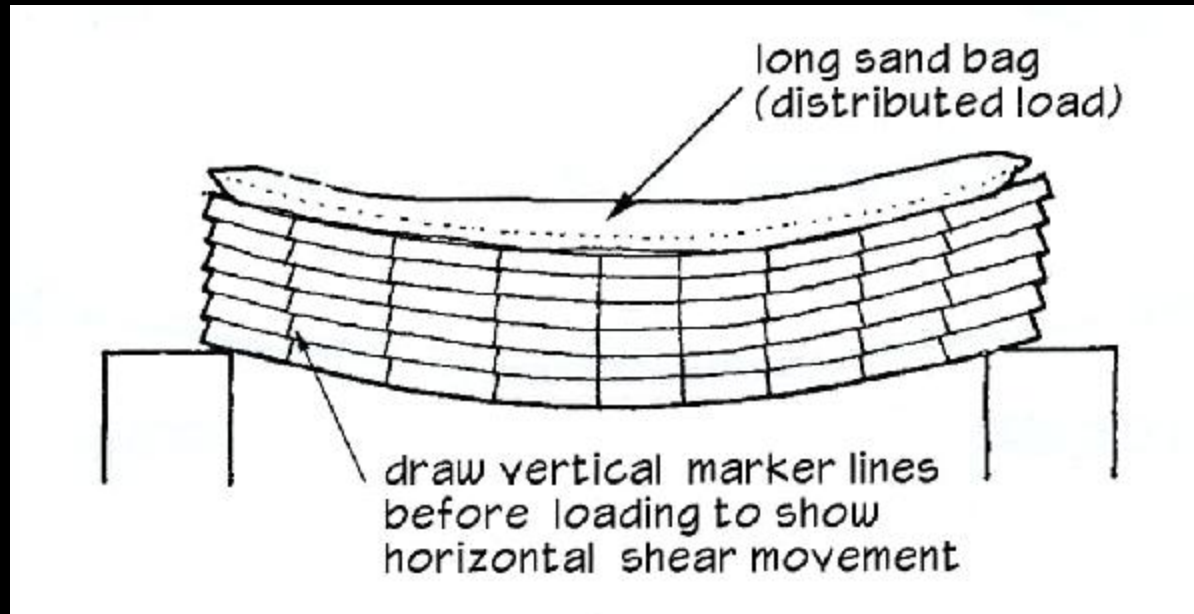
Shear Stresses

- *horizontal & vertical*

slice horiz.

slice vertical

cut section from foam beam; slice in half, then reassemble as shown

replace and hold with tape

Shear Stresses

- ## *horizontal & vertical*

front side shows
vertical shear

back side shows
horizontal shear

Beam Stresses

- *horizontal with bending*

long sand bag
(distributed load)

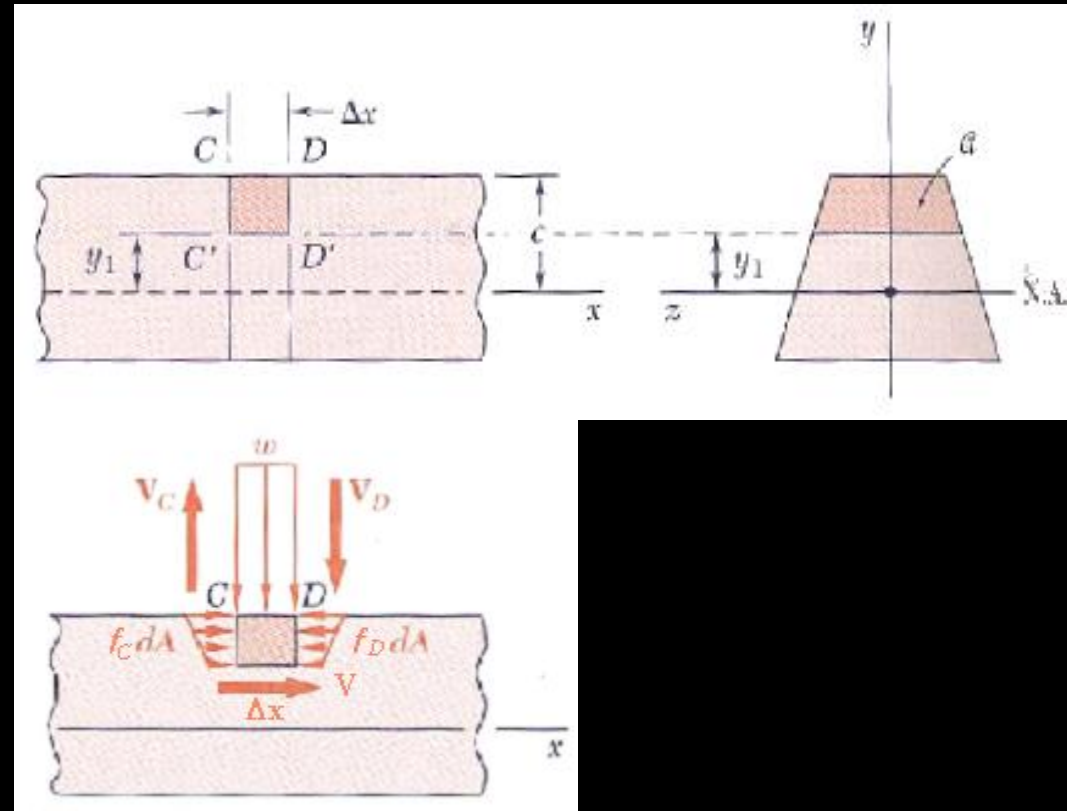draw vertical marker lines
before loading to show
horizontal shear movement

Equilibrium

- *horizontal force V needed*

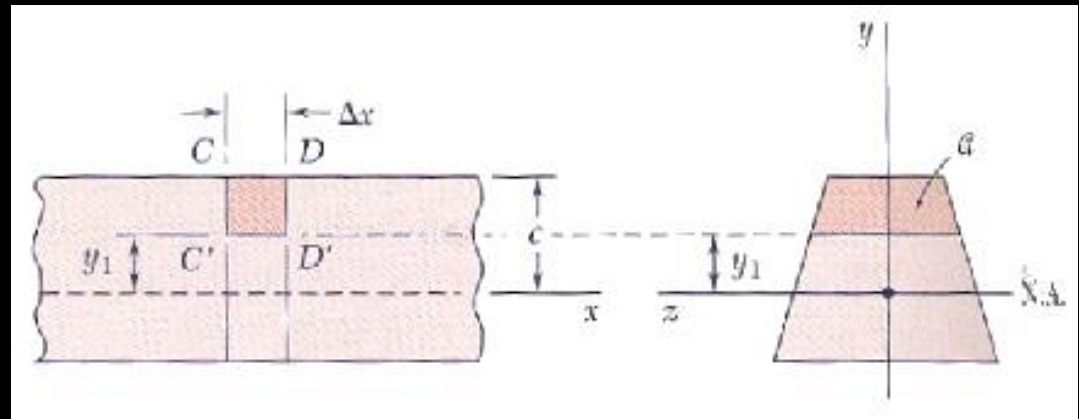$$V_{longitudinal} = \frac{V_T Q}{I} \Delta x$$

- *Q is a moment area*

Moment of Area

- *Q is a moment area with respect to the n.a. of area <u>above or below</u> the horizontal*
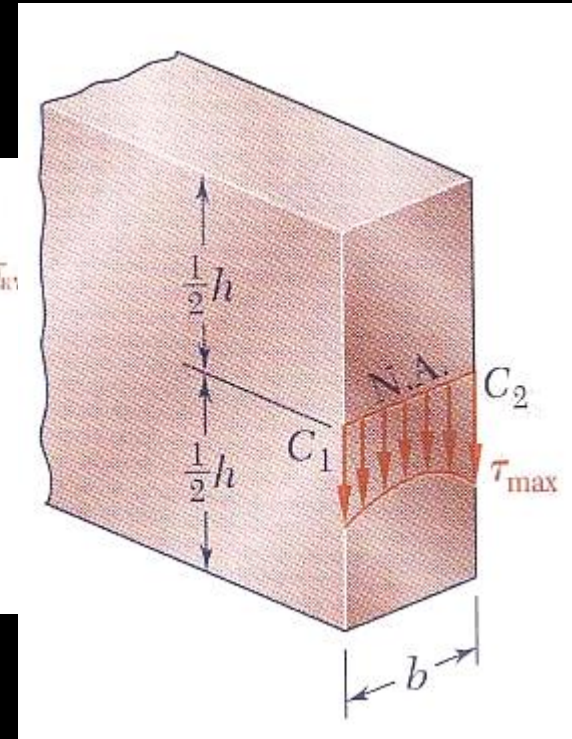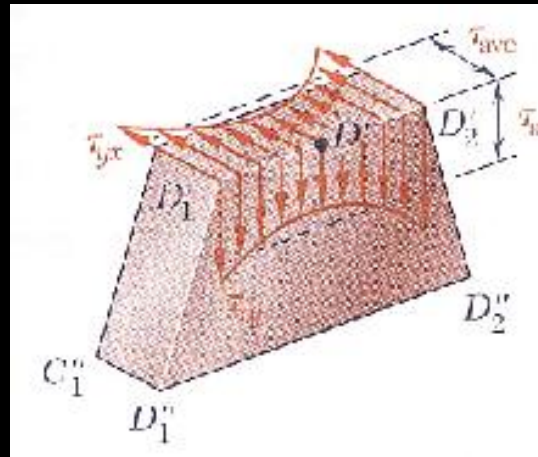
- *Q_{max} at y=0 (neutral axis)*

- *q is shear flow:*

$$q = \frac{V_{longitudinal}}{\Delta x} = \frac{V_T Q}{I}$$

Shearing Stresses

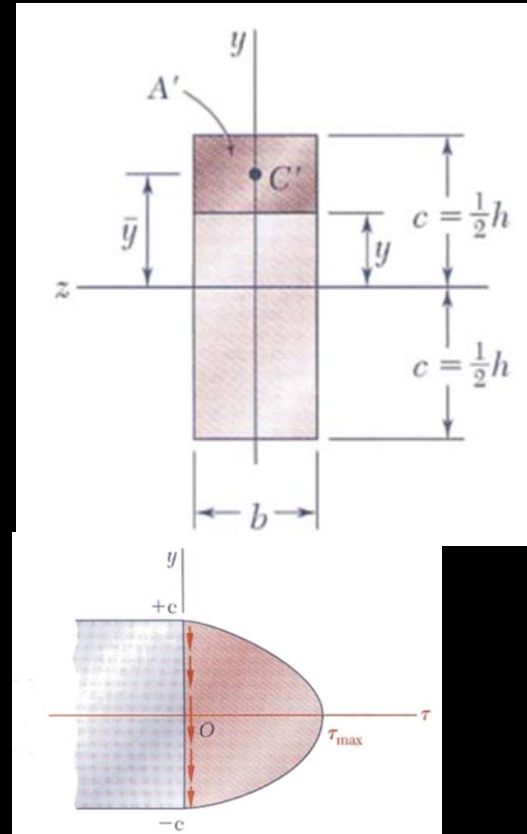$$f_v = \frac{V}{\Delta A} = \frac{V}{b \cdot \Delta x}$$

$$f_{v-ave} = \frac{VQ}{Ib}$$

- $f_{v-ave} = 0$ on the top/bottom

- *b min may not be with Q max*

- *with h/4 $\geq$ b, $f_{v-max} \leq 1.008 f_{v-ave}$*

Rectangular Sections
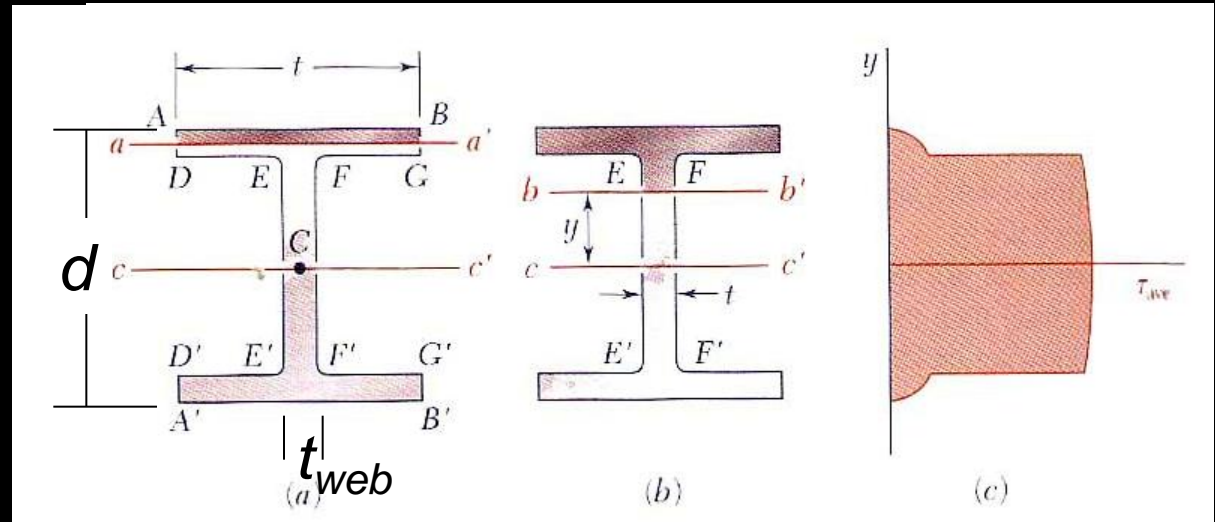
$$I = \frac{bh^3}{12} \qquad Q = A\bar{y} = \frac{bh^2}{8}$$

$$f_v = \frac{VQ}{Ib} = \frac{3V}{2A}$$

- $f_{v\text{-}max}$ *occurs at <u>n.a.</u>*

Steel Beam Webs

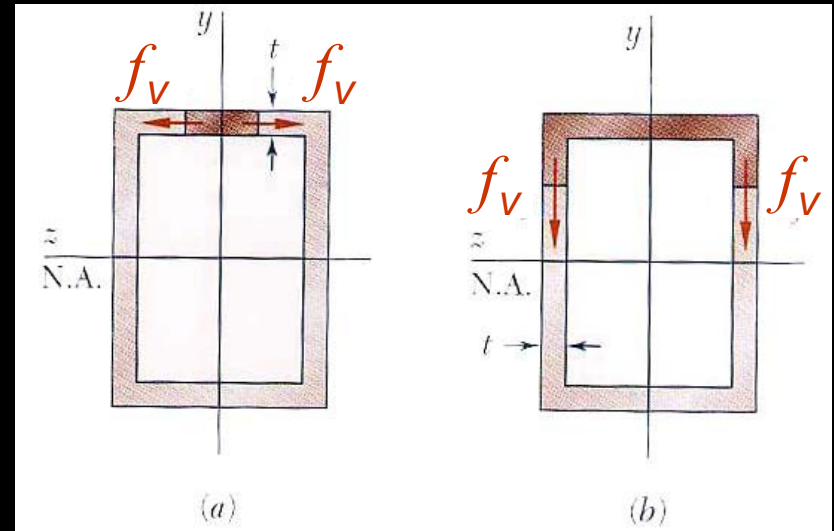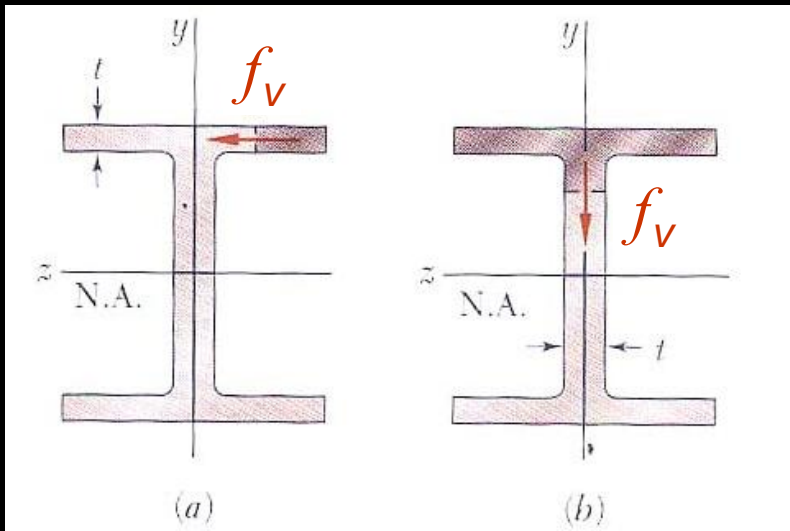- ## *W and S sections*
  - ### *b varies*

  - ### *stress in flange negligible*
  - ### *presume constant stress in web*

$$f_{v-\max} = \frac{3V}{2A} \approx \frac{V}{A_{web}}$$

Shear Flow

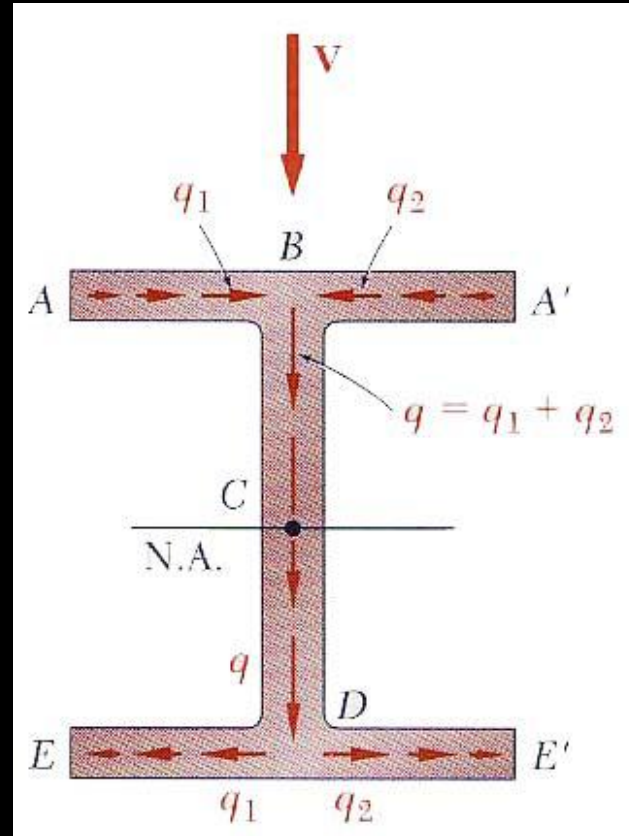- *loads applied in plane of symmetry*
- *cut made perpendicular*
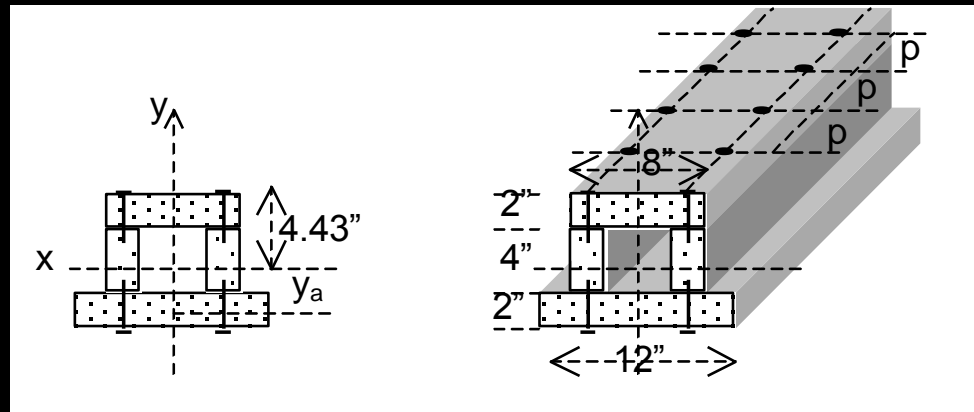
$$q = \frac{VQ}{I}$$

Shear Flow Quantity

- *sketch from Q*

$$q = \frac{VQ}{I}$$

Connectors Resisting Shear

- *plates with*
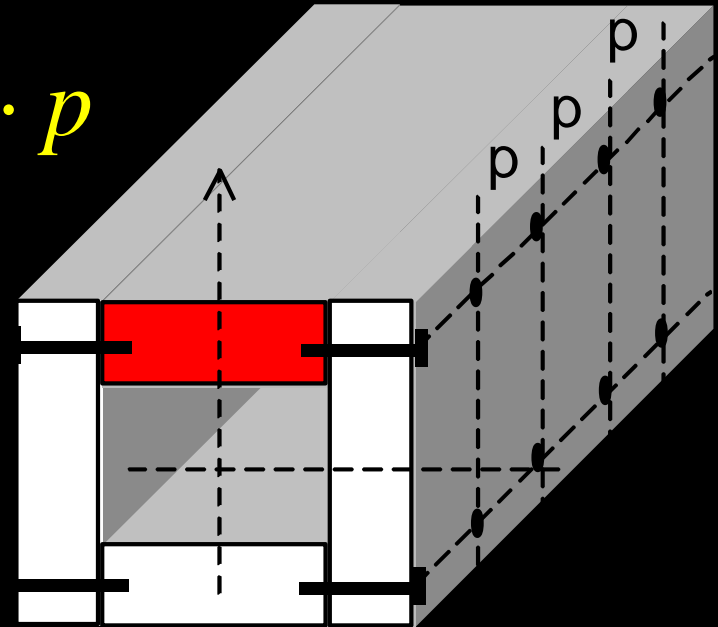 - *nails*
  - *rivets*
  - *bolts*
- *splices*

$$\frac{V_{longitudinal}}{p} = \frac{VQ}{I}$$

$$nF_{connector} \geq \frac{VQ_{connected\,area}}{I} \cdot p$$

Vertical Connectors

- *isolate an area with vertical interfaces*

$$nF_{connector} \geq \frac{VQ_{connected\ area}}{I} \cdot p$$

Unsymmetrical Shear or Section

- *member can bend and twist*
  - *not symmetric*
  - *shear not in that plane*

- *shear center*

  - *moments balance*