ELEMENTS OF ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN

ARCH 614

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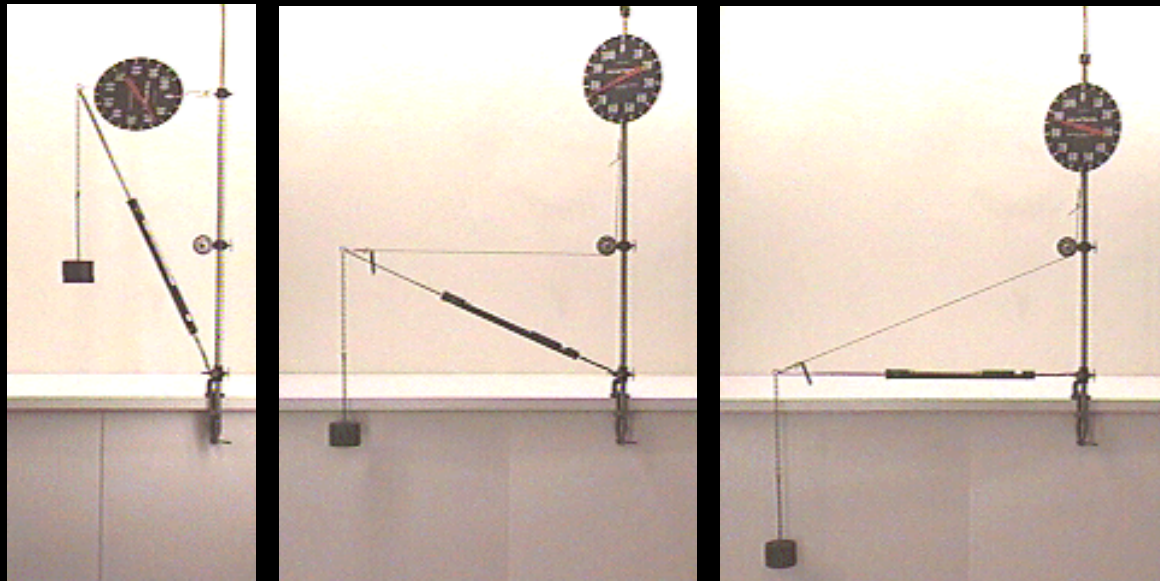SPRING 2013

lecture
five

moments

Moments

- *forces have the tendency to make a body rotate about an axis*

http://www.physics.umd.edu

– *same translation but different rotation*

Moments

(a) Unloaded. (b) Loaded.

Figure 2.33 Moment on a cantilever beam.

STEEL COLUMN

F

STEEL CHANNEL

(a)

F Y

X

CROSS-SECTIONAL VIEW OF CHANNEL
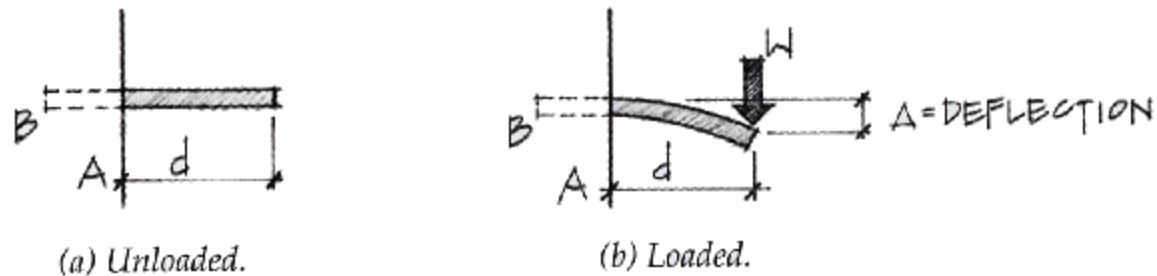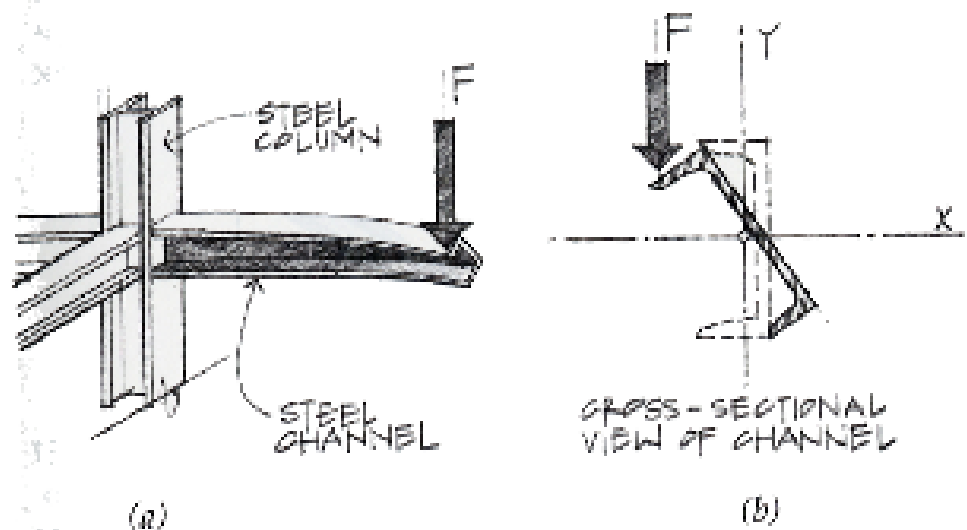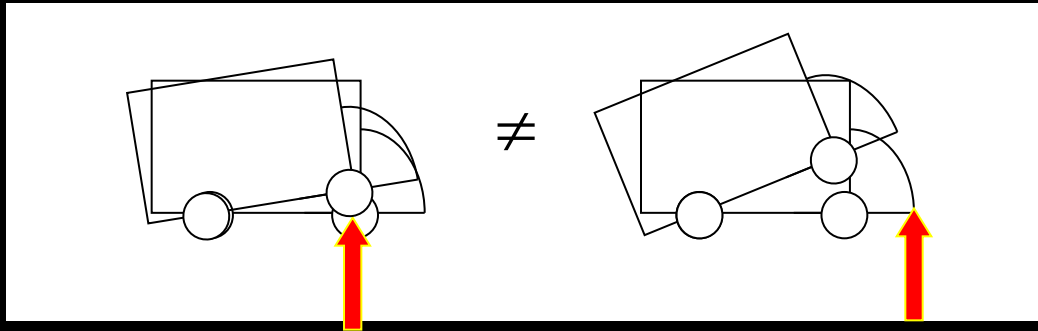
(b)

Figure 2.34 An example of torsion on a cantilever beam.

Moments

- *a force acting at a different point causes a different moment:*

Moments

- *defined by magnitude and direction*
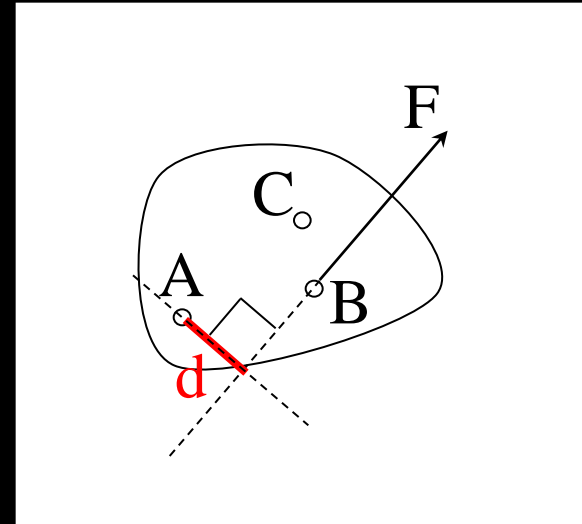
- *units: N·m, k·ft*

- *direction:*

  + cw (!)

  - ccw

- *value found from F and ⊥ distance*

$$M = F \cdot d$$

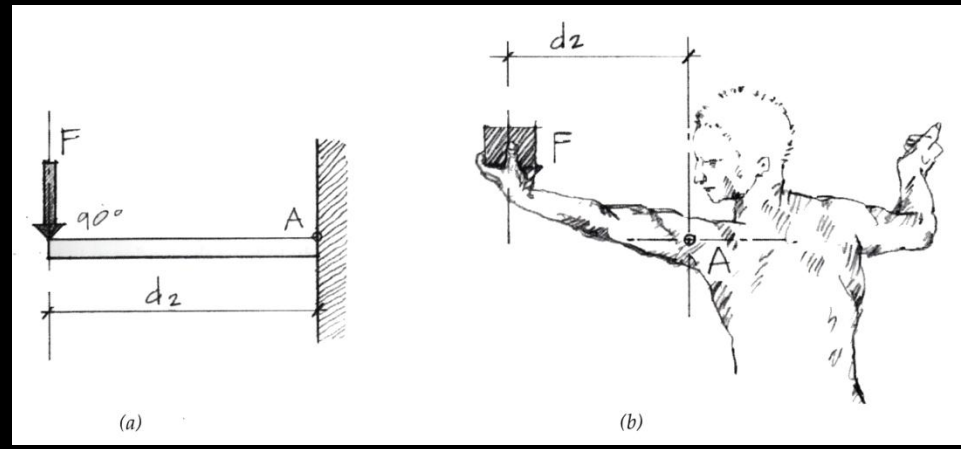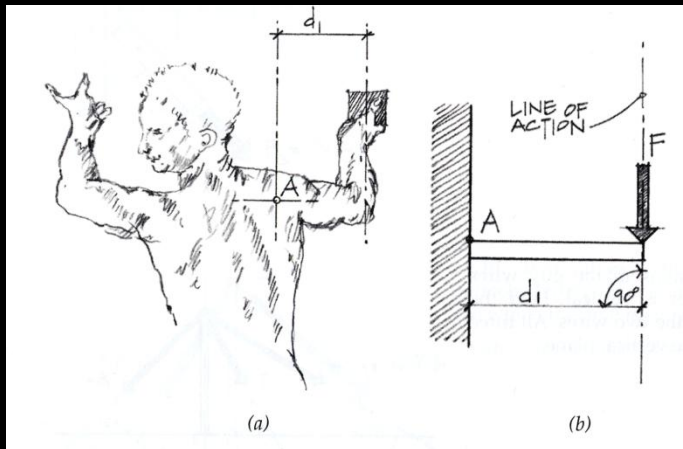- *d also called "lever" or "moment" arm*
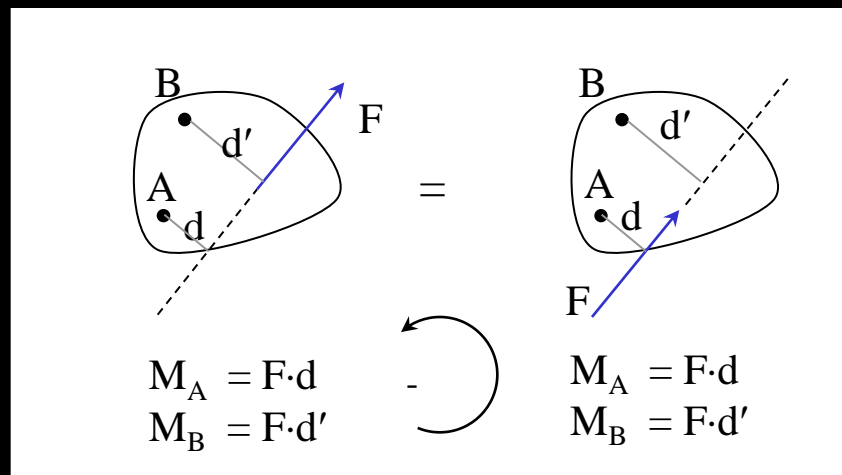
Moments

- *with same F:*

$$M_A = F \cdot d_1 < M_A = F \cdot d_2$$

(bigger)

Moments

- *additive with sign convention*

- *can still move the force*
  along the line of action

$M_A = F{\cdot}d$
$M_B = F{\cdot}d'$
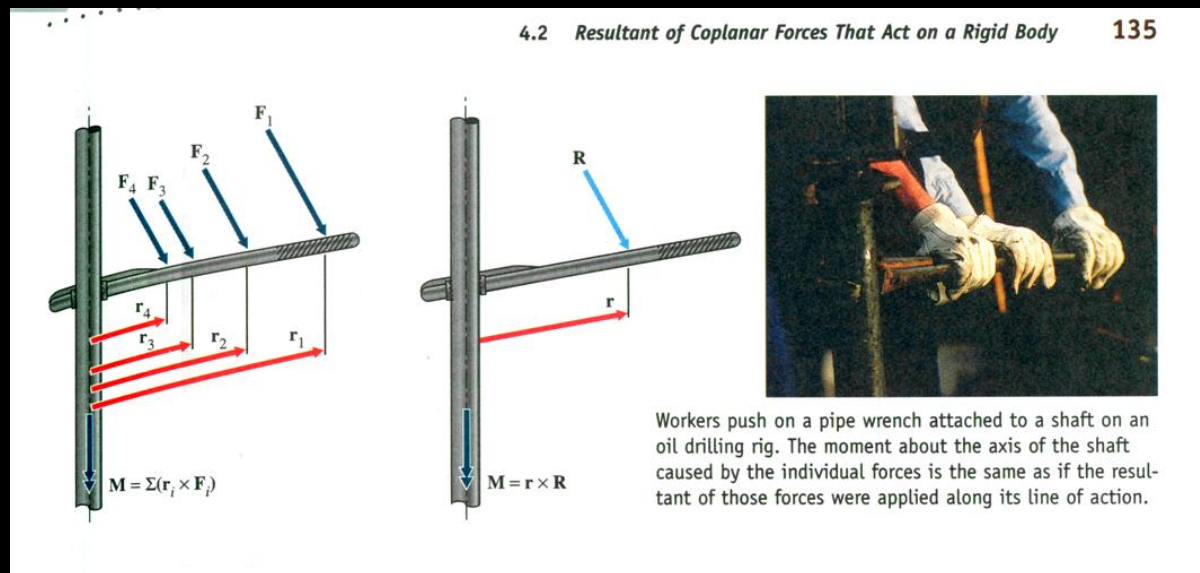
$M_A = F{\cdot}d$
$M_B = F{\cdot}d'$

Moments

- *Varignon's Theorem*
  - *resolve a force into components at a point and finding perpendicular distances*
  - *calculate sum of moments*
  - *equivalent to original moment*
- *makes life easier!*
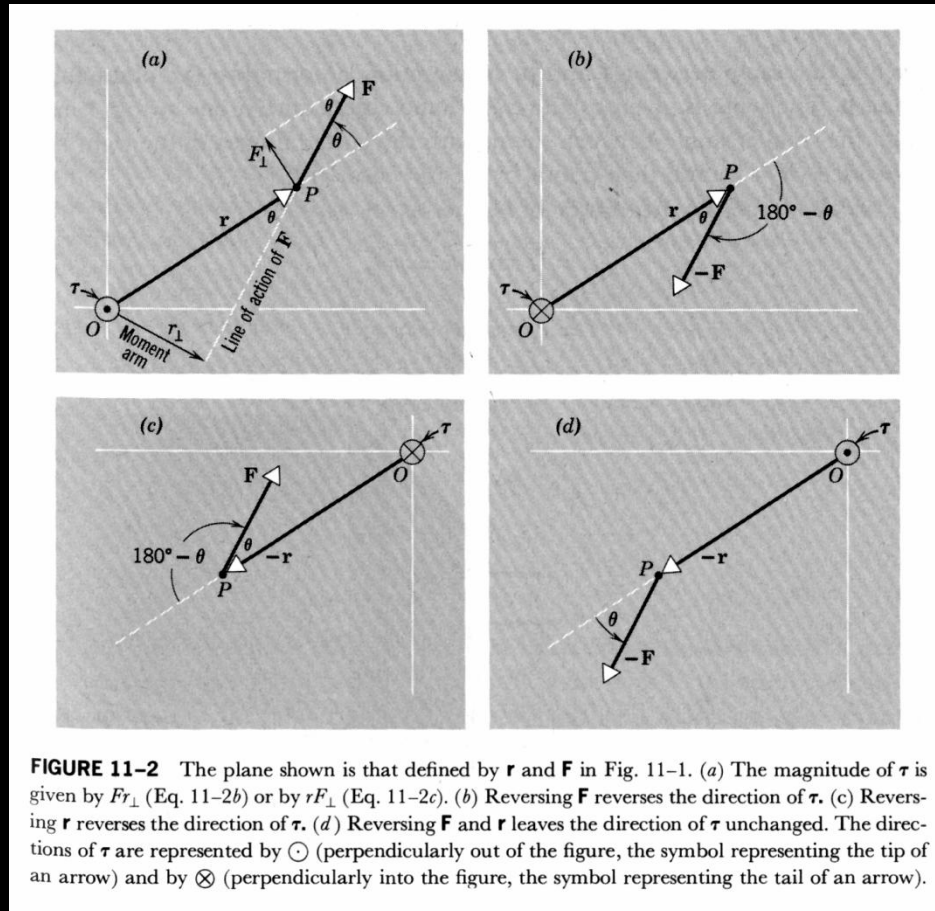 - *geometry*
  - *when component runs through point, d=0*

Moments of a Force

- *moments of a force*
  - *introduced in Physics as "Torque Acting on a Particle"*
 - *and used to satisfy rotational equilibrium*

4.2 **Resultant of Coplanar Forces That Act on a Rigid Body** 135

$M = \Sigma(r_i \times F_i)$

$M = r \times R$

Workers push on a pipe wrench attached to a shaft on an oil drilling rig. The moment about the axis of the shaft caused by the individual forces is the same as if the resultant of those forces were applied along its line of action.

Physics and Moments of a Force

- *my Physics book (right hand rule):*

FIGURE 11-2 The plane shown is that defined by **r** and **F** in Fig. 11-1. (*a*) The magnitude of τ is given by $Fr_\perp$ (Eq. 11-2*b*) or by $rF_\perp$ (Eq. 11-2*c*). (*b*) Reversing **F** reverses the direction of τ. (*c*) Reversing **r** reverses the direction of τ. (*d*) Reversing **F** and **r** leaves the direction of τ unchanged. The directions of τ are represented by $\odot$ (perpendicularly out of the figure, the symbol representing the tip of an arrow) and by $\otimes$ (perpendicularly into the figure, the symbol representing the tail of an arrow).

Moment Couples

- *2 forces*
  - *same size*
  - *opposite direction*
  - *distance d apart*
  - *cw or ccw*

$$M = F \cdot d$$
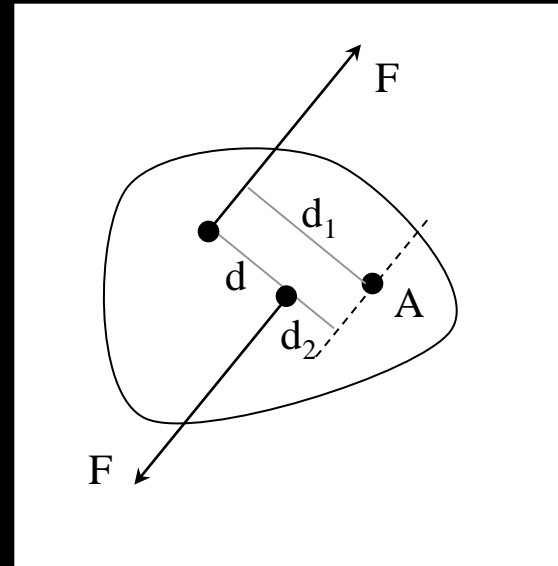
  - *not dependant on point of application*
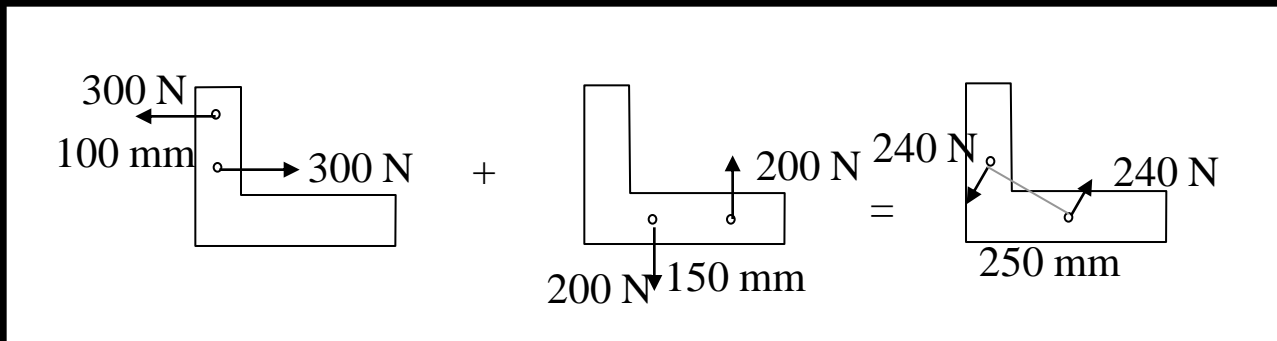
$$M = F \cdot d_1 - F \cdot d_2$$

Moment Couples

- ## equivalent couples
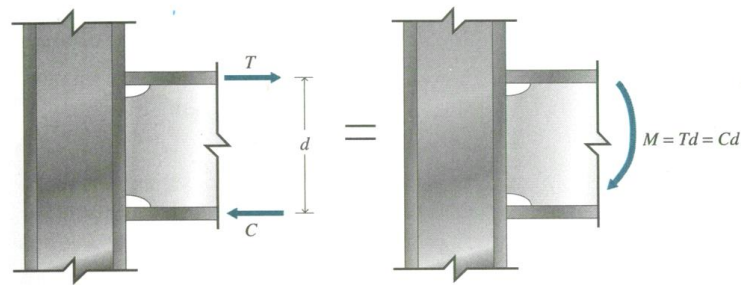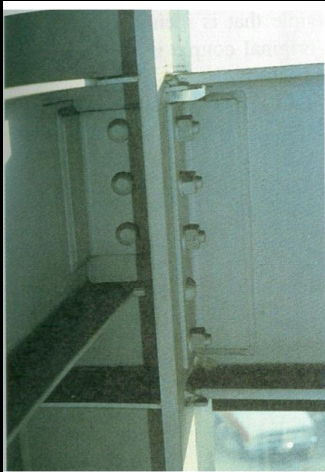 - ### same magnitude and direction
  - ### F & d may be different
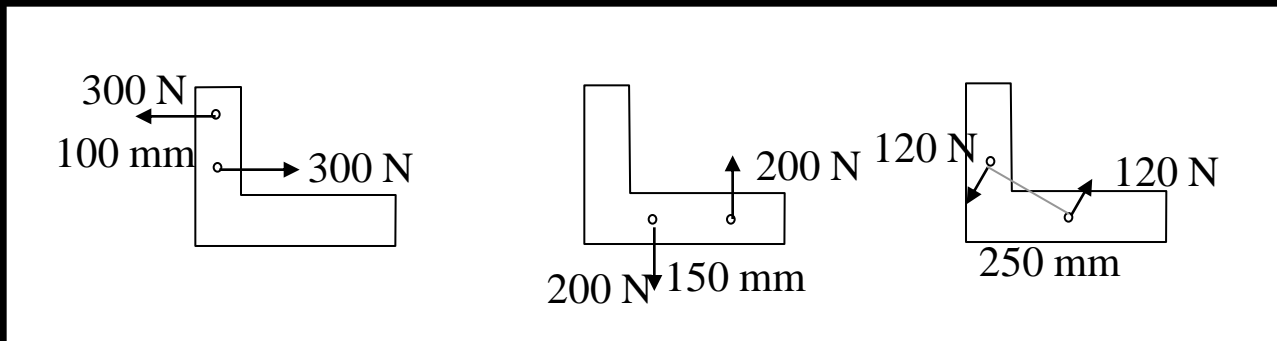
Moment Couples

- *added just like moments caused by one force*

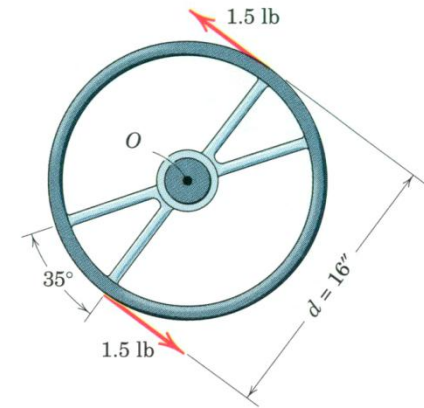- *can replace two couples with a single couple*

Moment Couples

- *moment couples in structures*

The flanges of a steel beam are welded to the flange of a column. Equal and opposite forces T and C in the beam flanges form a couple with moment M that is transferred into the column.

$M = Td = Cd$

1.5 lb

O

35°

$d = 16"$

1.5 lb

Equivalent Force Systems

- *two forces at a point is equivalent to the resultant at a point*

- *resultant is equivalent to two components at a point*

- *resultant of equal & opposite forces at a point is zero*

- *put equal & opposite forces at a point (sum to 0)*

- *transmission of a force along action line*

Force-Moment Systems

- *single force causing a moment can be replaced by the same force at a different point by providing the moment that force caused*
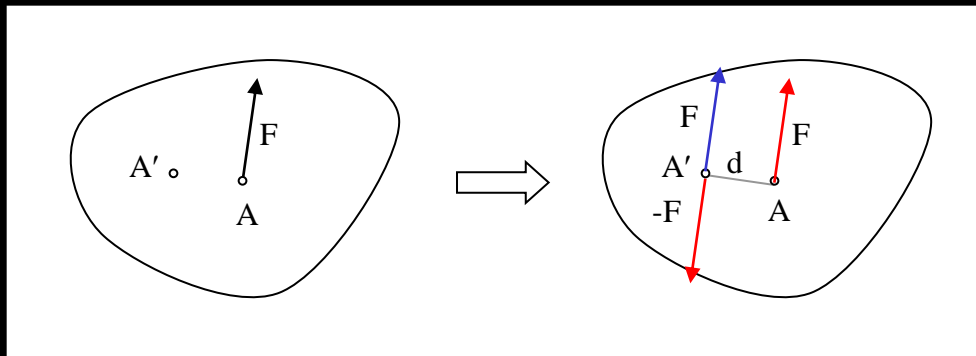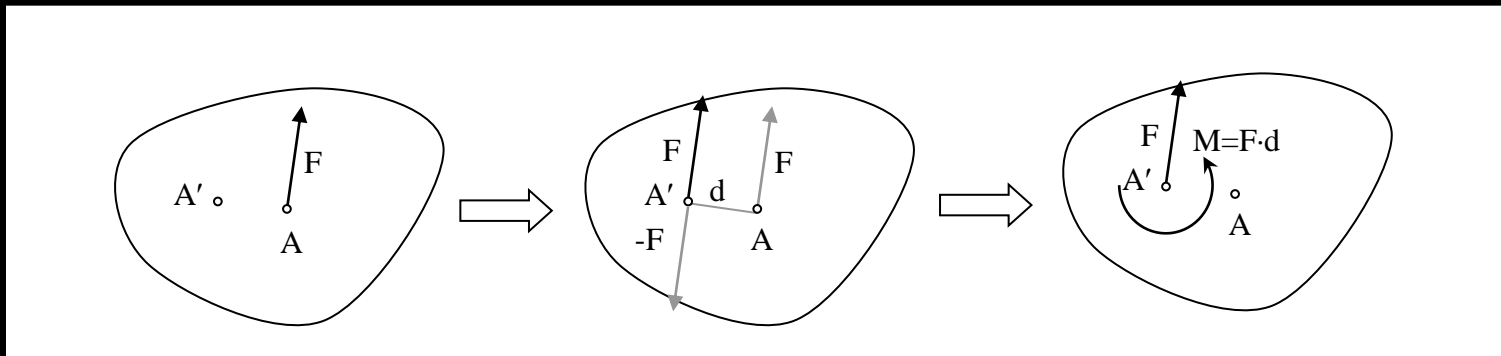
- *moments are shown as arched arrows*
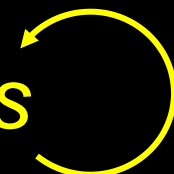
Force-Moment Systems

- *a force-moment pair can be replaced by a force at another point causing the original moment*
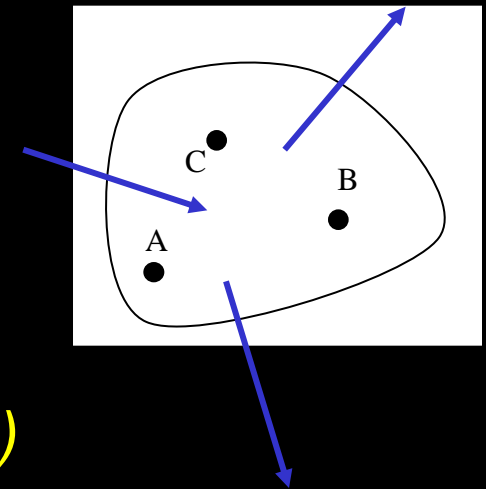
Parallel Force Systems

- *forces are in the same direction*

- *can find resultant force*

- *need to find <u>location</u> for equivalent moments*

$$A \cdot a$$

$$B \cdot b$$

$$(A+B) \cdot x$$
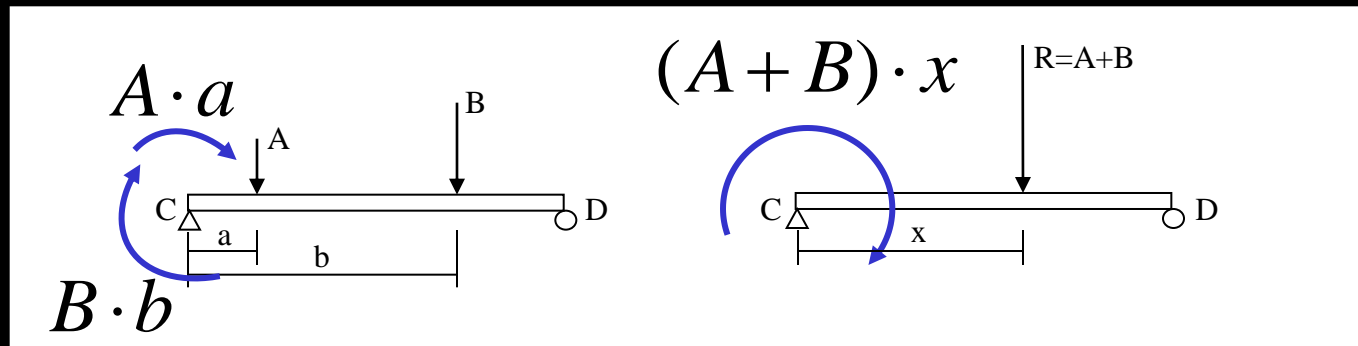
Equilibrium

- *rigid body*
  - *doesn't deform*
  - *coplanar force systems*

- *static:*

$$R_x = \sum F_x = 0 \quad \text{(Σ H)}$$

$$R_y = \sum F_y = 0 \quad \text{(Σ V)}$$

$$\boxed{M = \sum M = 0}$$

Free Body Diagram

- *FBD (sketch)*

- *tool to see all forces on a body or a point including*
  - *external forces*
  - *weights*
  - *force reactions*
  - *external moments*
  - *moment reactions*
  - *internal forces*
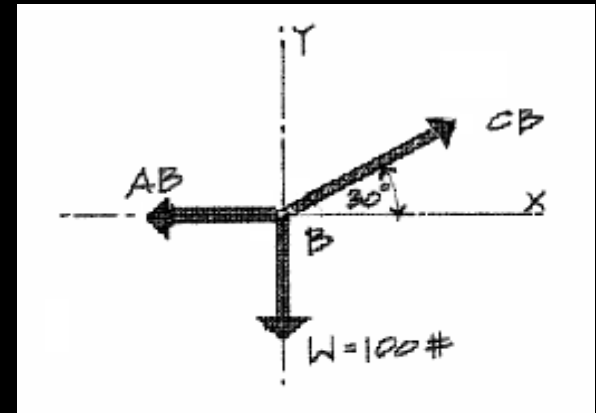
Free Body Diagram

- *determine body*
- *FREE it from:*
  - *ground*
  - *supports & connections*
- *draw all external forces acting ON the body*
  - *reactions*
  - *applied forces*
  - *gravity*
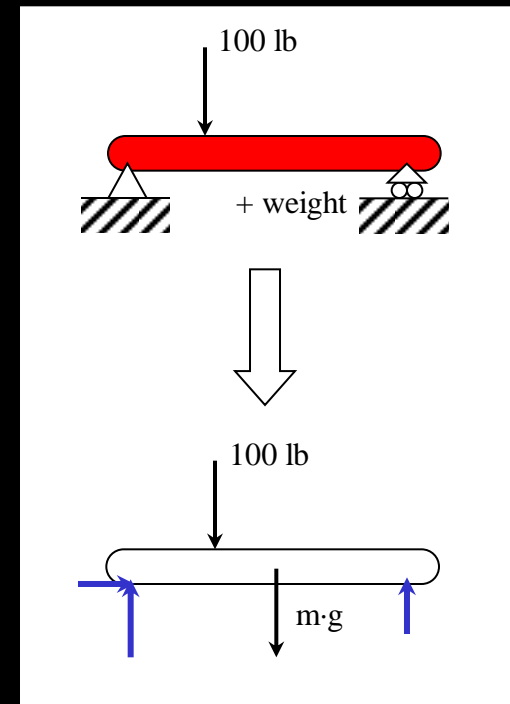
Free Body Diagram

- *sketch FBD with relevant geometry*

- *resolve each force into components*
  - *known & unknown <u>angles</u> – name them*
  - *known & unknown <u>forces</u> – name them*
  - *known & unknown <u>moments</u> – name them*

- *are any forces related to other forces?*

- *for the unknowns*

- *write only as many equilibrium equations as needed*

- *solve up to 3 equations*

Free Body Diagram

- *solve equations*
  - *most times 1 unknown easily solved*
  - *plug into other equation(s)*

- *common to have unknowns of*
  - *force magnitudes*
  - *force angles*
  - *moment magnitudes*

Reactions on Rigid Bodies

- *result of applying force*

- *unknown size*

- *connection or support type*
  - *known direction*
  - *related to motion prevented*
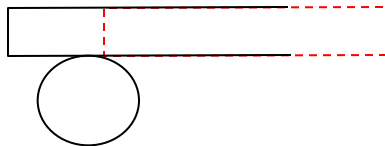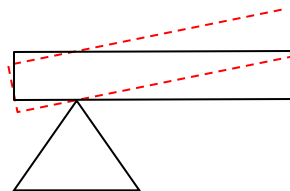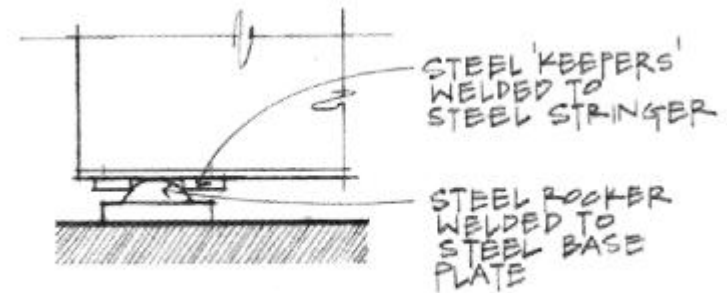

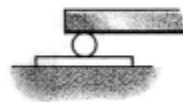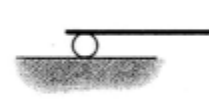*no vertical motion*   *no translation*   *no translation no rotation*
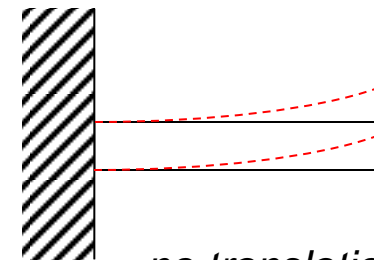
Supports and Connections

Supports and Connections

STEEL COLUMN/
BEAM
CONNECTION,
BOLTED.

smooth pin or hinge

Pin

CAST-IN-PLACE
CONCRETE
COLUMN/BEAM
CONNECTION.

NOTE:
OVERLAPPING
STEEL
REINFORCEMENT
MAKES ASSEMBLY
MONOLITHIC.

fixed support

F_y

F_x

M

F_y

F_x

Fixed
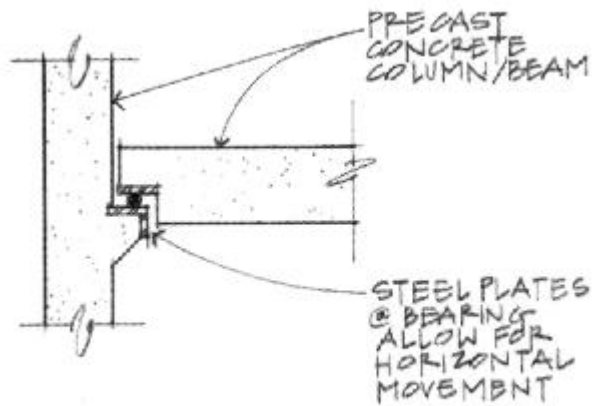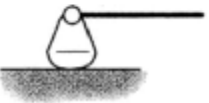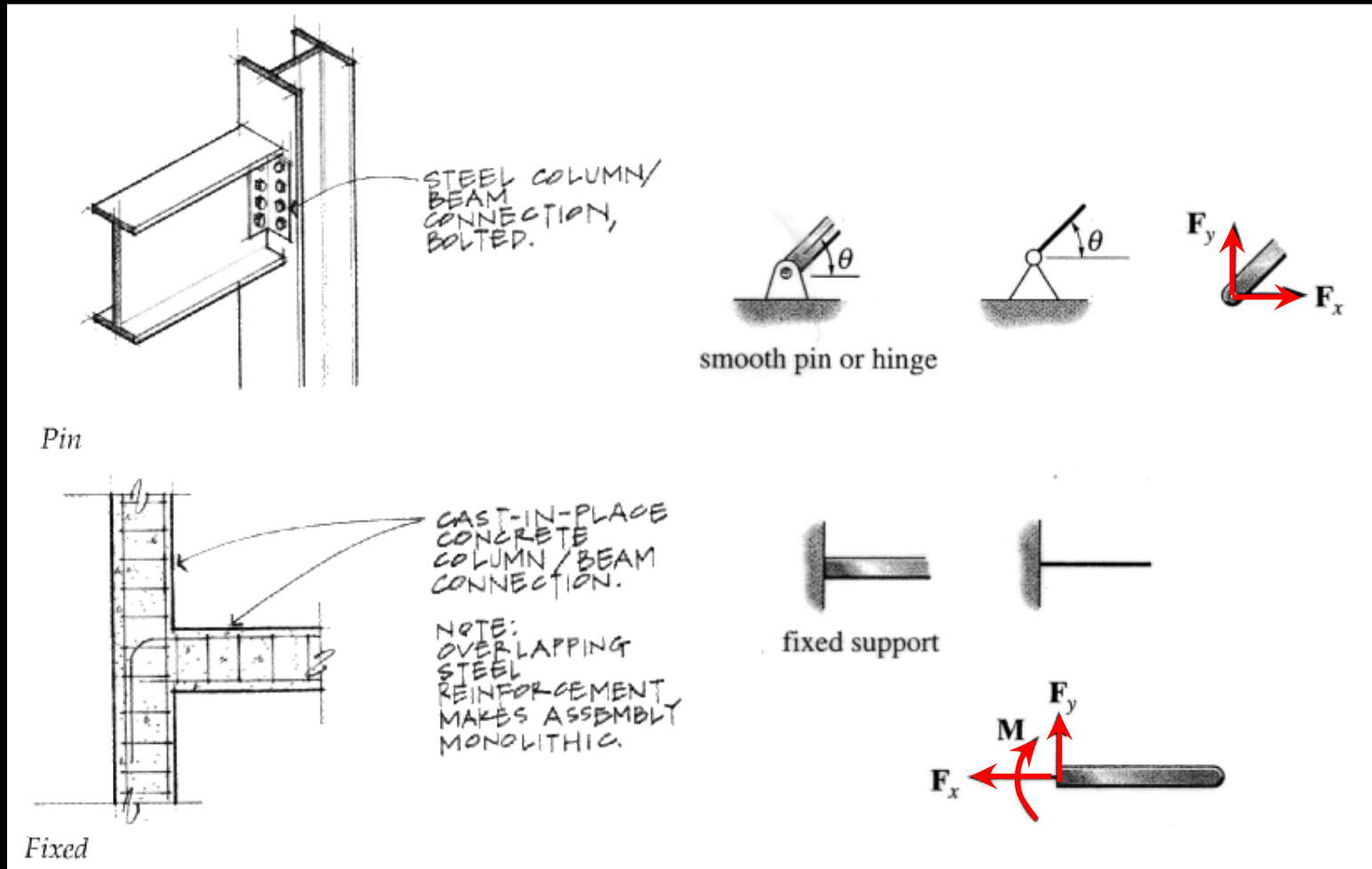
Moment Equations

- *sum moments at intersection where the most forces intersect*

- *multiple moment equations may not be useful*

- *combos:*

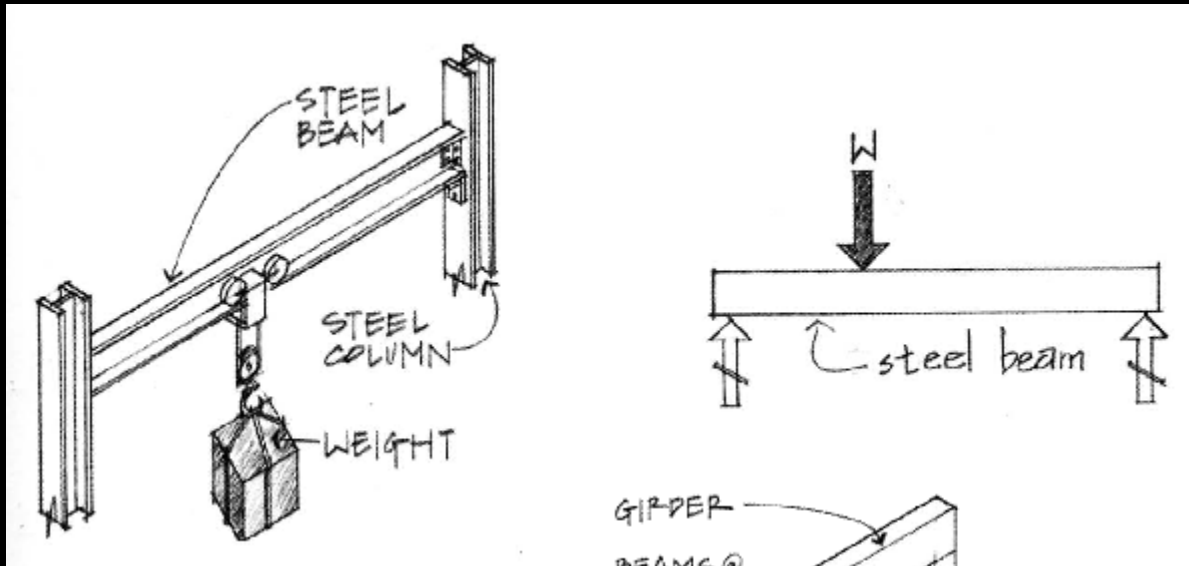$$\sum F_x = 0 \qquad\qquad \sum F = 0 \qquad\qquad \sum M_1 = 0$$

$$\sum F_y = 0 \qquad\qquad \sum M_1 = 0 \qquad\qquad \sum M_2 = 0$$

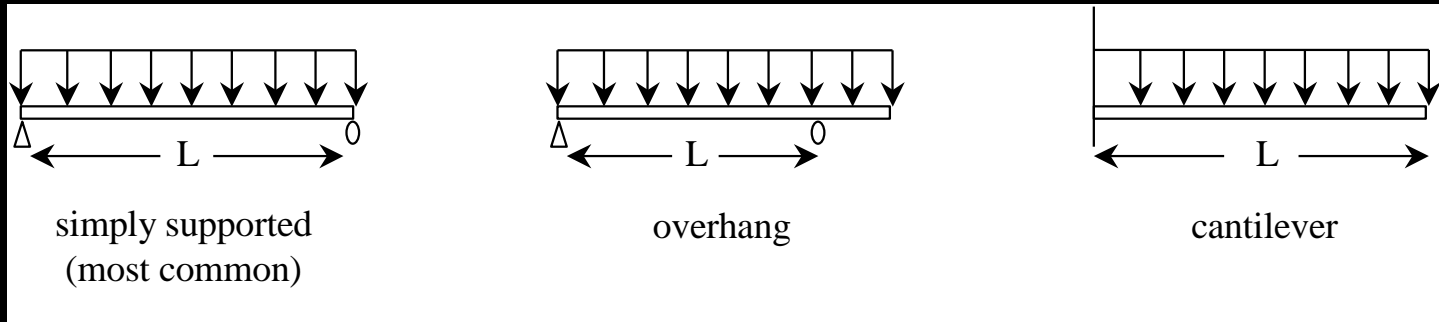$$\sum M_1 = 0 \qquad\qquad \sum M_2 = 0 \qquad\qquad \sum M_3 = 0$$

Concentrated Loads
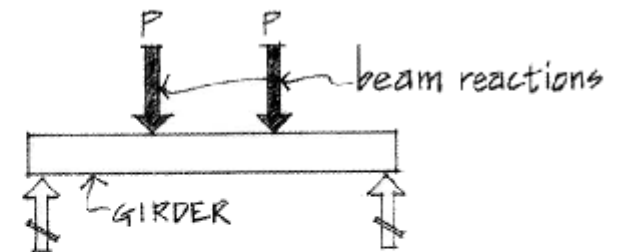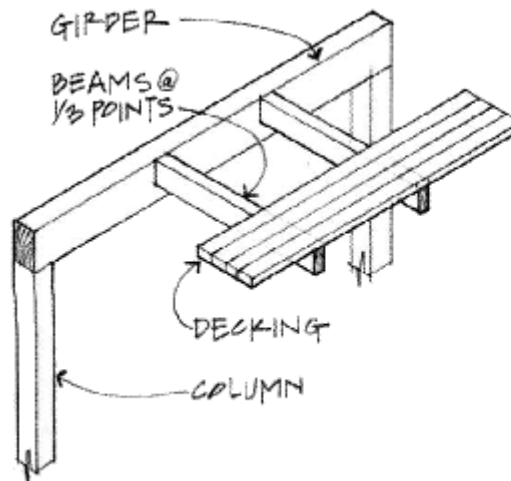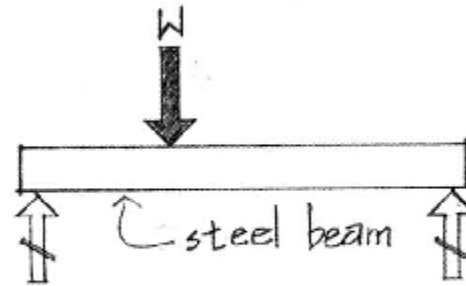
Distributed Loads
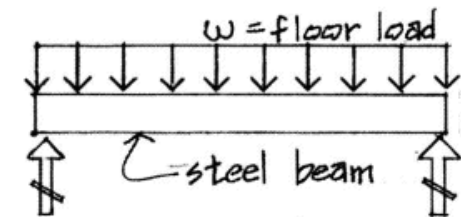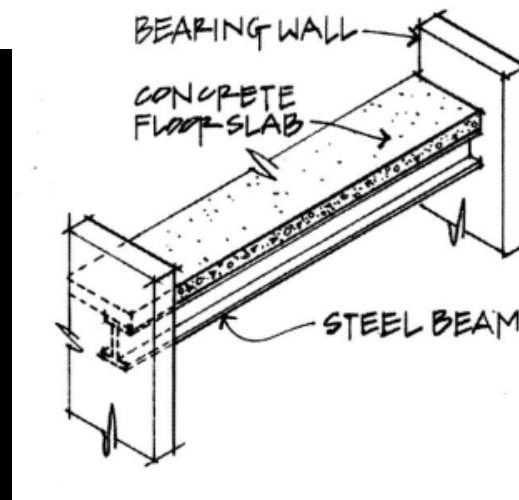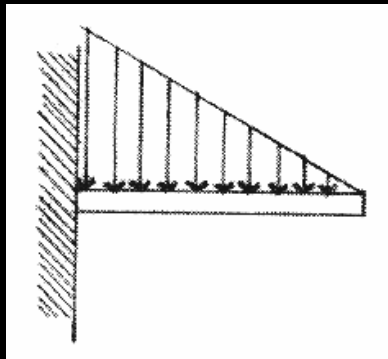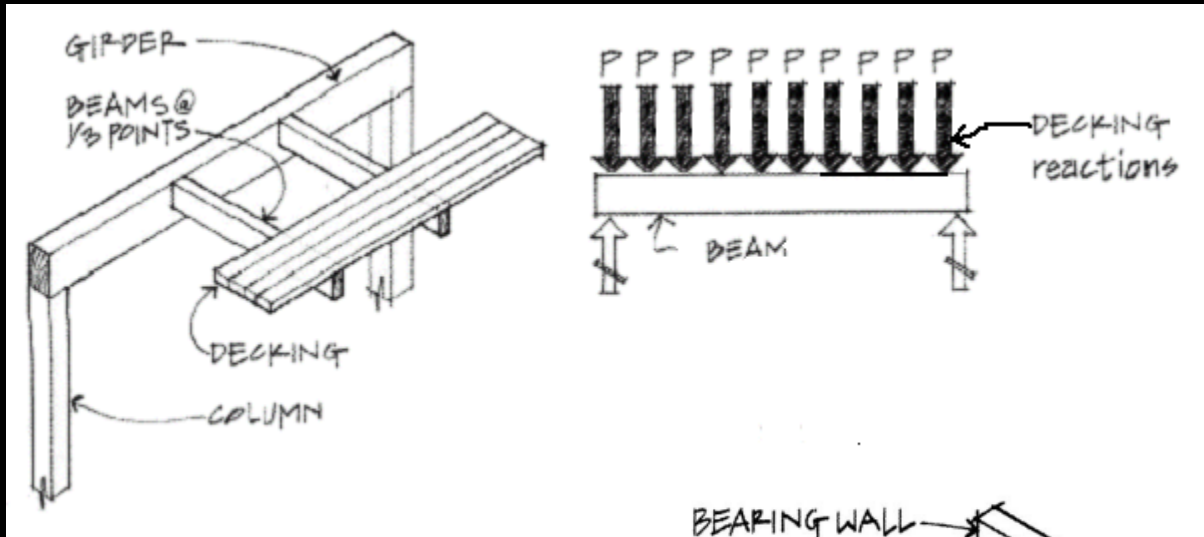
Beam Supports

- ## *statically determinate*

simply supported
(most common)

overhang

cantilever

- ## *statically indeterminate*

continuous
(most common case when $L_1 = L_2$)

Propped

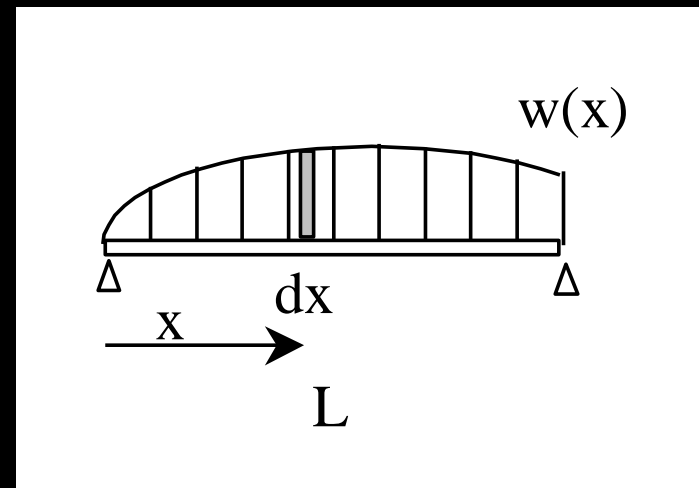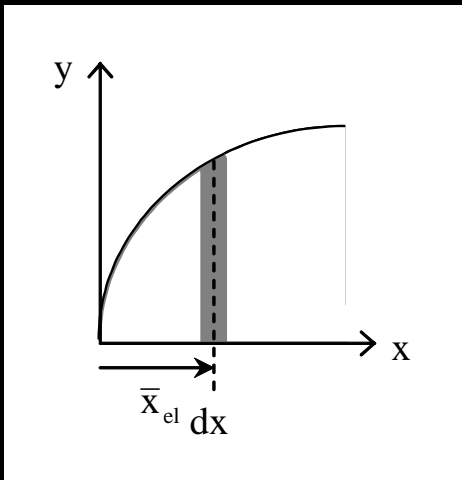Restrained
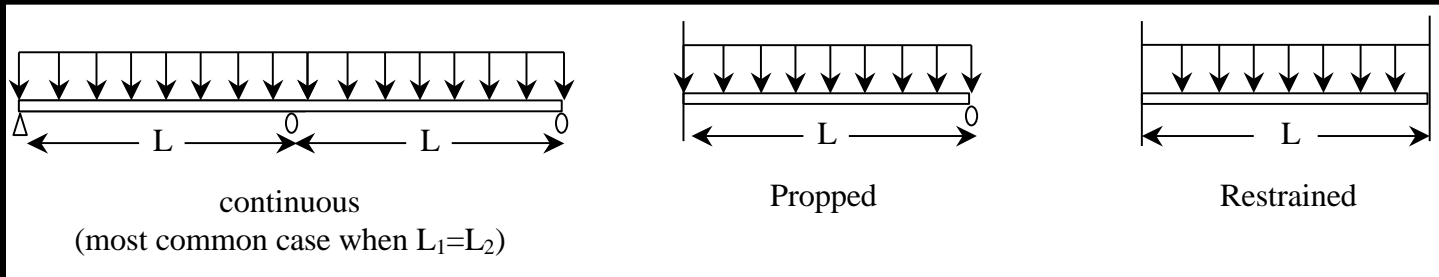
Equivalent Force Systems

- *replace forces by resultant*

- *place resultant where M = 0*

- *using <u>calculus</u> and area centroids*

$$W = \int_0^L w\,dx = \int dA_{loading} = A_{loading}$$

Load Areas

- *area is width x "height" of load*
- *w is load per unit length*
- *W is total load*