

ELEMENTS OF ARCHITECTURAL STRUCTURES:

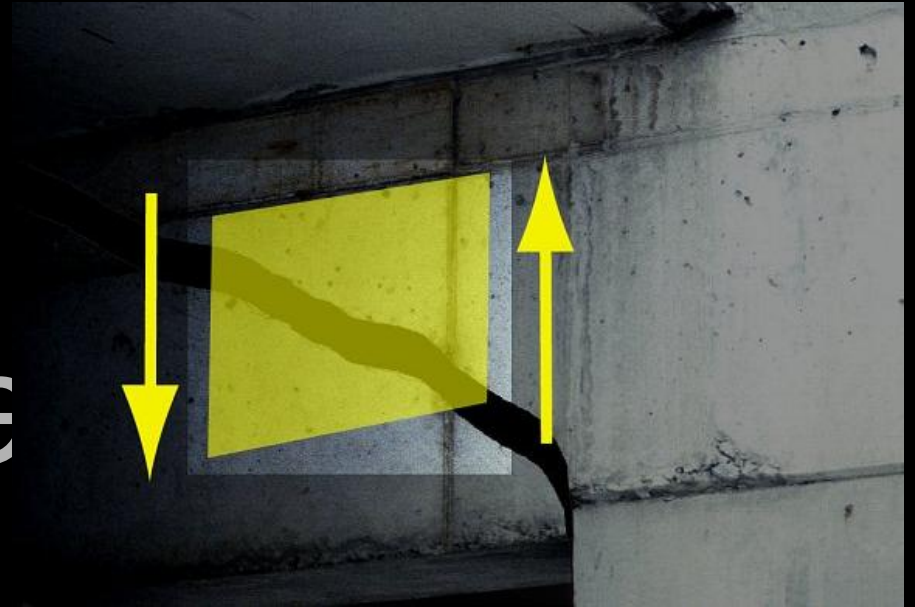
FORM, BEHAVIOR, AND DESIGN

ARCH 614

DR. ANNE NICHOLS

SPRING 2013

lecture
twenty three

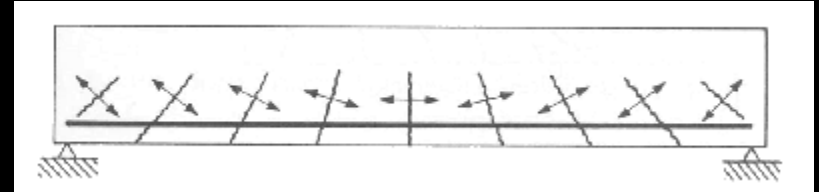


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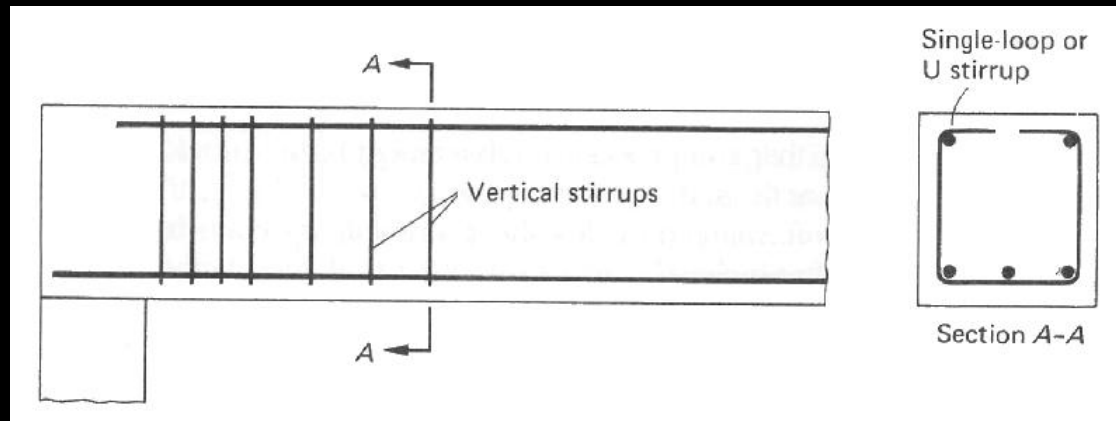
***concrete construction:
shear & deflection***

Shear in Concrete Beams

- *flexure combines with shear to form diagonal cracks*



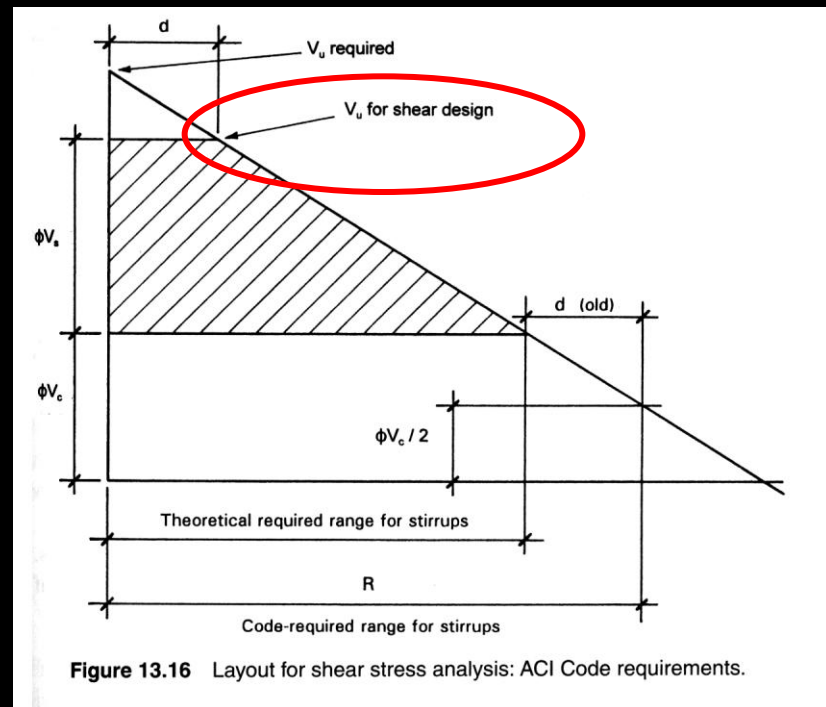
- *horizontal reinforcement doesn't help*
- *stirrups = vertical reinforcement*



ACI Shear Values

- V_u is at distance d from face of support
- shear capacity: $V_c = v_c \times b_w d$

– where b_w means thickness of web at n.a.



ACI Shear Values

- *shear stress (beams)*

- $v_c = 2\sqrt{f'_c}$

- $\phi V_c = \phi 2\sqrt{f'_c} b_w d$

$\phi = 0.75$ for shear
 f'_c is in psi

- *shear strength:*

$$V_u \leq \phi V_c + \phi V_s$$

 - V_s is strength from stirrup reinforcement

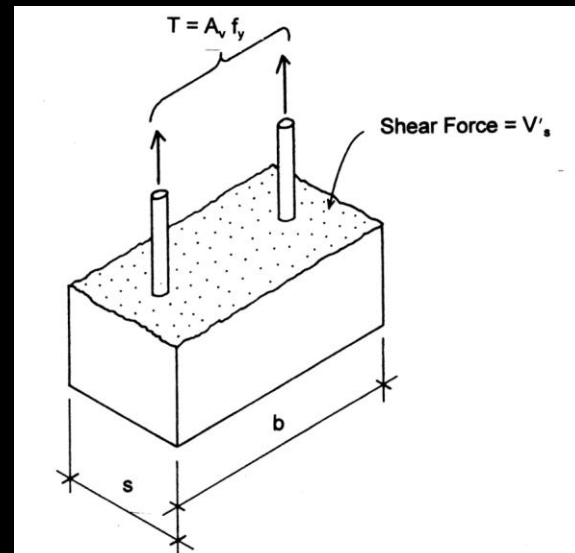


Figure 13.17 Consideration for spacing of a single stirrup.

Stirrup Reinforcement

- *shear capacity:*

$$V_s = \frac{A_v f_y d}{s}$$

- A_v = area in all legs of stirrups
 - s = spacing of stirrup
-
- *may need stirrups when concrete has enough strength!*

Required Stirrup Reinforcement

- spacing limits

Table 3-8 ACI Provisions for Shear Design*

		$V_u \leq \frac{\phi V_c}{2}$	$\phi V_c \geq V_u > \frac{\phi V_c}{2}$	$V_u > \phi V_c$
Required area of stirrups, A_v^{**}		none	$\frac{50b_w s}{f_y}$	$\frac{(V_u - \phi V_c)s}{\phi f_y d}$
Stirrup spacing, s	Required	—	$\frac{A_v f_y}{50b_w}$	$\frac{\phi A_v f_y d}{V_u - \phi V_c}$
	Recommended Minimum [†]	—	—	4 in.
	Maximum ^{††} (ACI 11.5.4)	—	$\frac{d}{2}$ or 24 in.	$\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \leq \phi 4\sqrt{f'_c} b_w d$ $\frac{d}{4}$ or 12 in. for $(V_u - \phi V_c) > \phi 4\sqrt{f'_c} b_w d$

*Members subjected to shear and flexure only; $\phi V_c = \phi 2 \sqrt{f'_c} b_w d$, $\phi = 0.85$ (ACI 11.3.1.1)

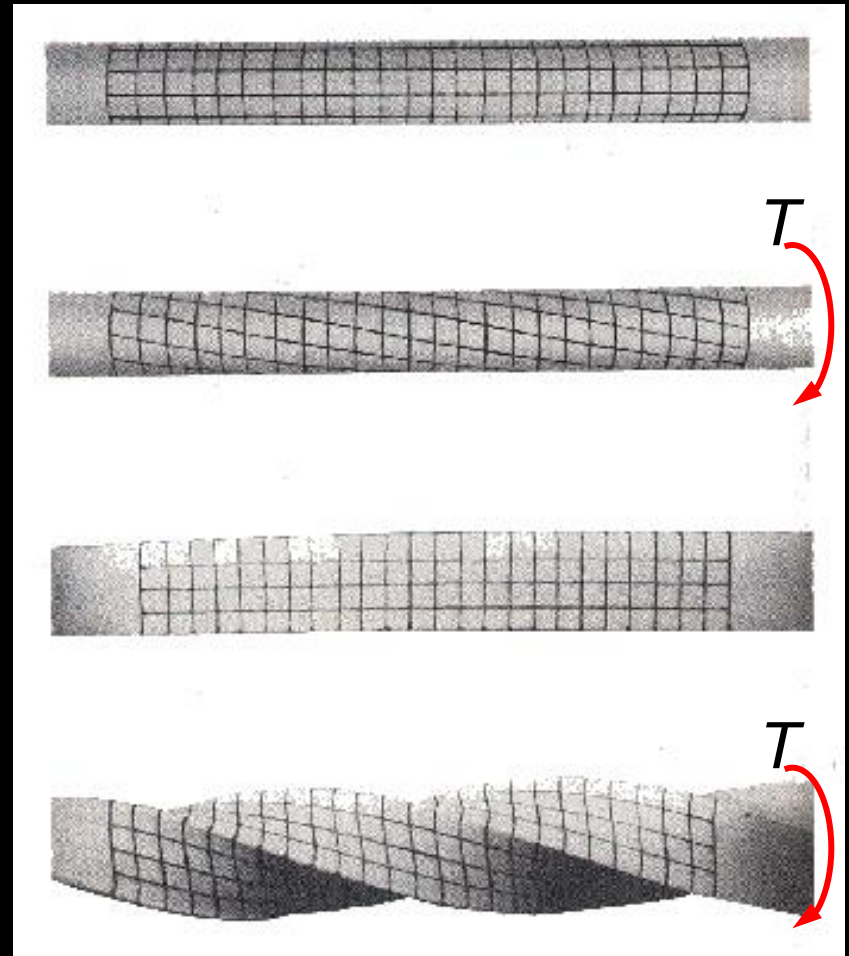
** $A_v = 2 \times A_b$ for U stirrups; $f_y \leq 60$ ksi (ACI 11.5.2)

†A practical limit for minimum spacing is $d/4$

††Maximum spacing based on minimum shear reinforcement ($= A_v f_y / 50b_w$) must also be considered (ACI 11.5.5.3).

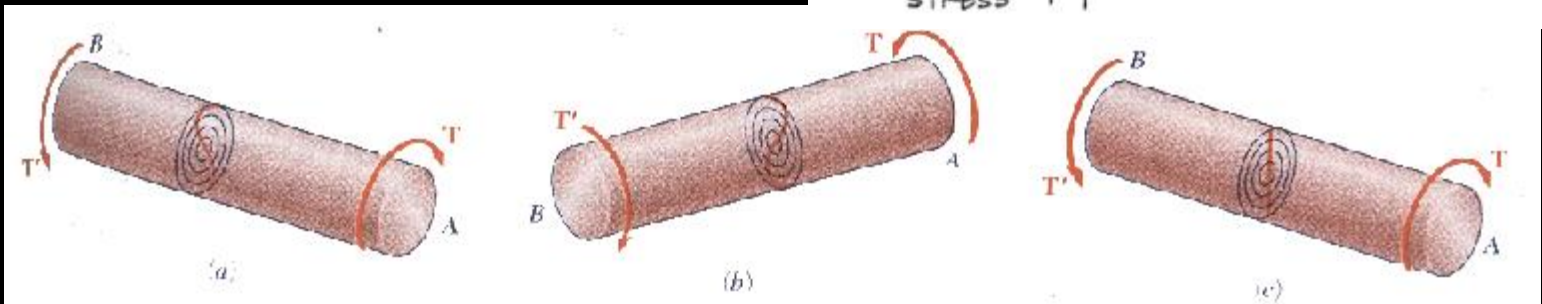
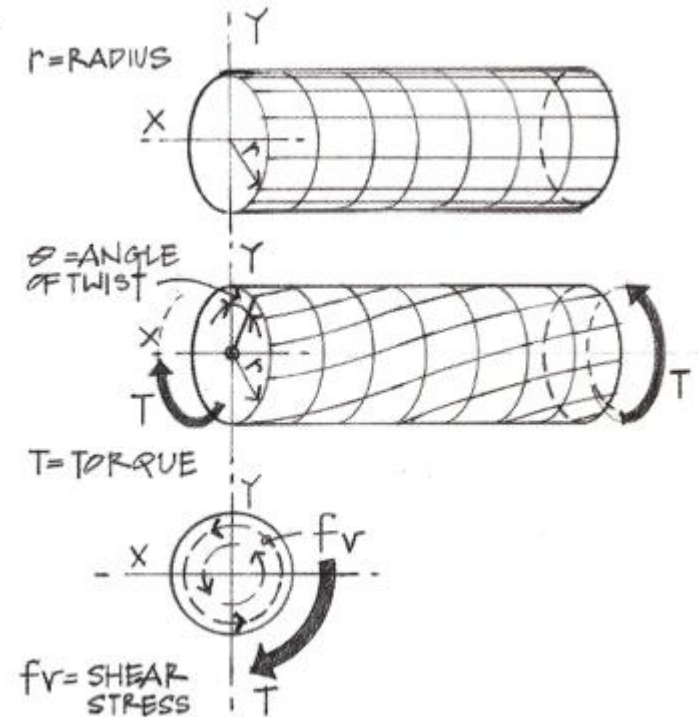
Torsional Stress & Strain

- *can see torsional stresses & twisting of axi-symmetrical cross sections*
 - *torque*
 - *remain plane*
 - *undistorted*
 - *rotates*
- *not true for square sections....*



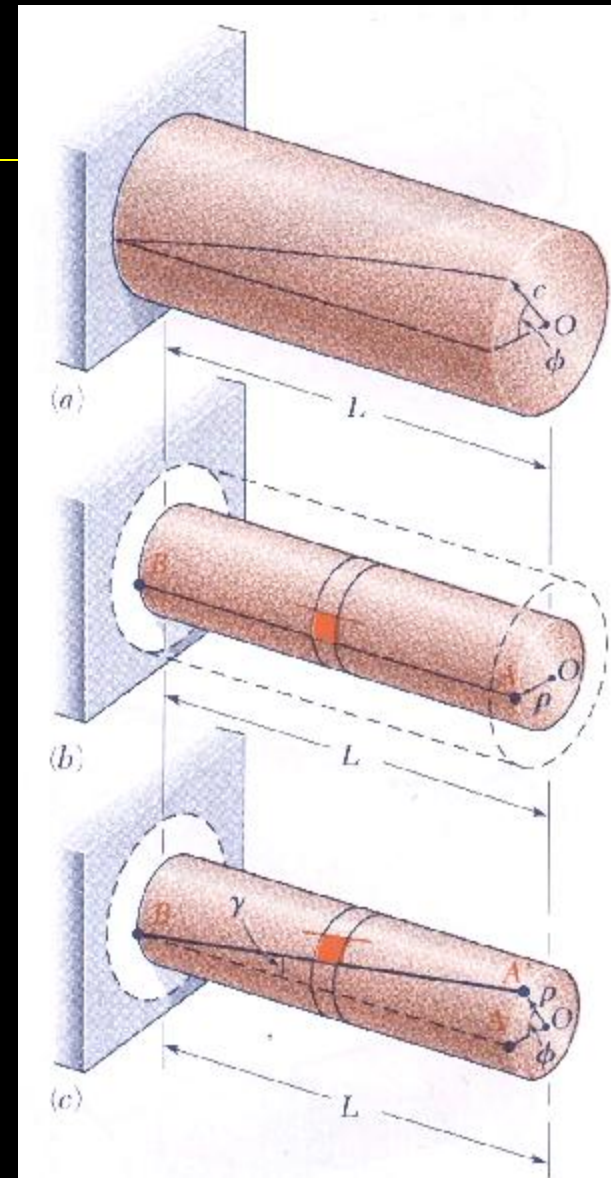
Shear Stress Distribution

- depend on the deformation
- $\phi =$ angle of twist
– measure
- can prove planar section doesn't distort



Shearing Strain

- related to ϕ
$$\gamma = \frac{\rho\phi}{L}$$
- ρ is the radial distance from the centroid to the point under strain
- shear strain varies linearly along the radius: γ_{max} is at outer diameter



Torsional Stress - Strain

- know $f_v = \tau = G \cdot \gamma$ and $\gamma = \frac{\rho\phi}{L}$
- so $\tau = G \cdot \frac{\rho\phi}{L}$
- where G is the Shear Modulus

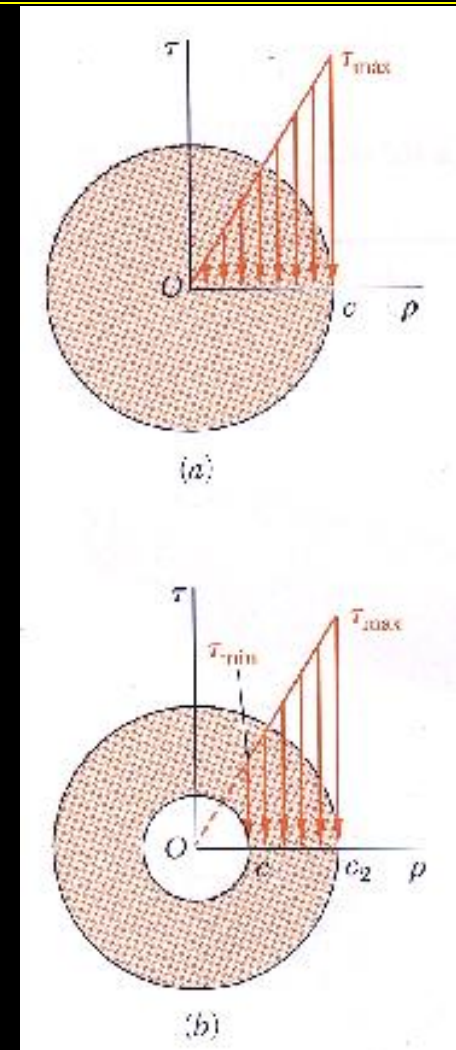
Torsional Stress - Strain

- from $T = \Sigma \tau(\rho) \Delta A$

- can derive $T = \frac{\tau J}{\rho}$

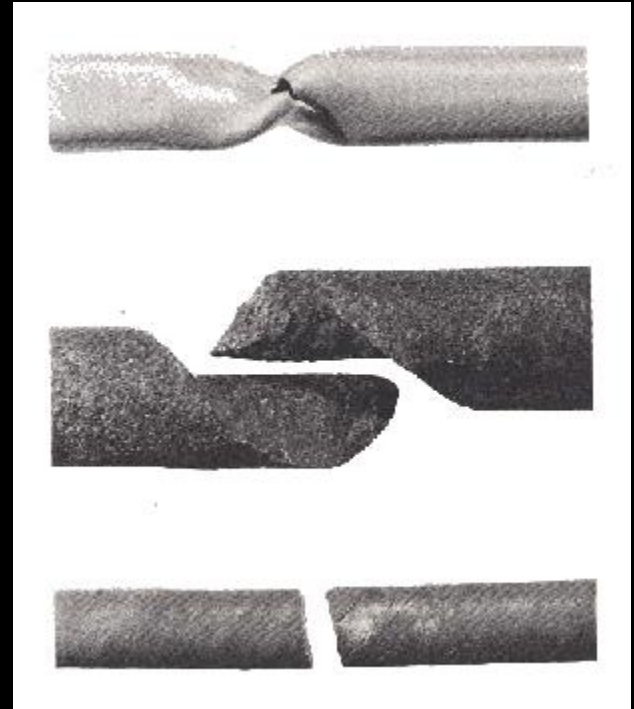
- where J is the polar moment of inertia

- elastic range $\tau = \frac{T\rho}{J}$



Shear Stress

- τ_{max} happens at outer diameter
- *combined shear and axial stresses*
 - *maximum shear stress at 45° “twisted” plane*



Shear Strain

- knowing $\tau = G \cdot \frac{\rho\phi}{L}$ and $\tau = \frac{T\rho}{J}$
- solve: $\phi = \frac{TL}{JG}$
- composite shafts: $\phi = \sum_i \frac{T_i L_i}{J_i G_i}$

Noncircular Shapes

- torsion depends on J
- plane sections don't remain plane
- τ_{max} is still at outer diameter

$$\tau_{max} = \frac{T}{c_1 ab^2} \quad \phi = \frac{TL}{c_2 ab^3 G}$$

– where a is longer side ($> b$)

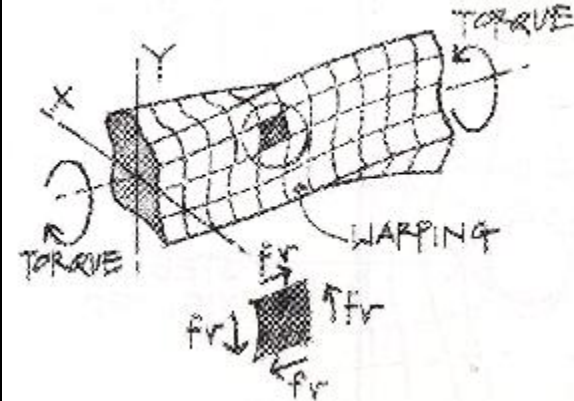
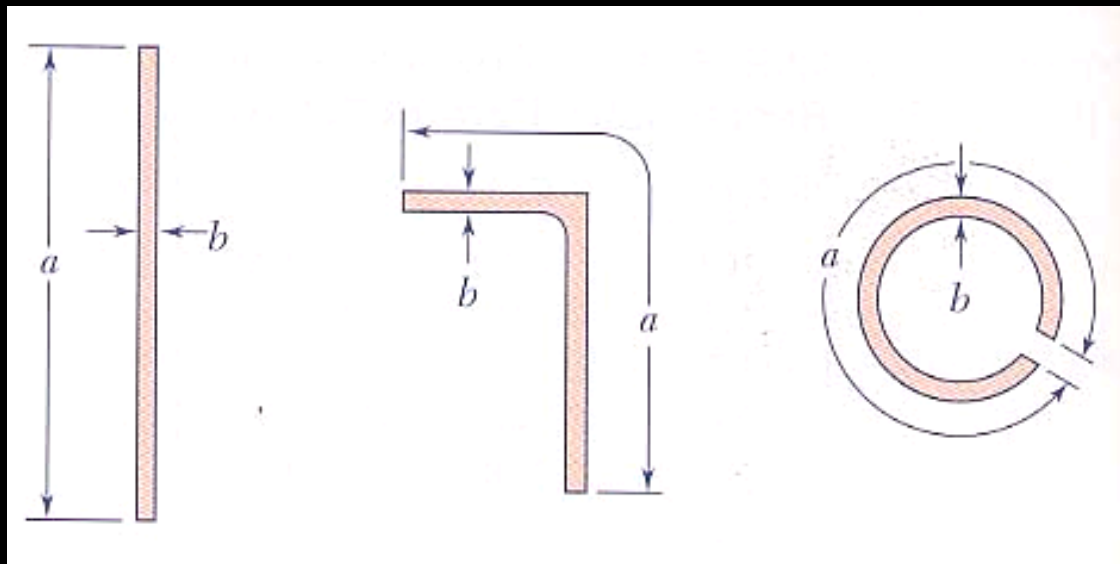


TABLE 3.1. Coefficients for Rectangular Bars in Torsion

a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

Open Thin-Walled Sections

- with very large a/b ratios:



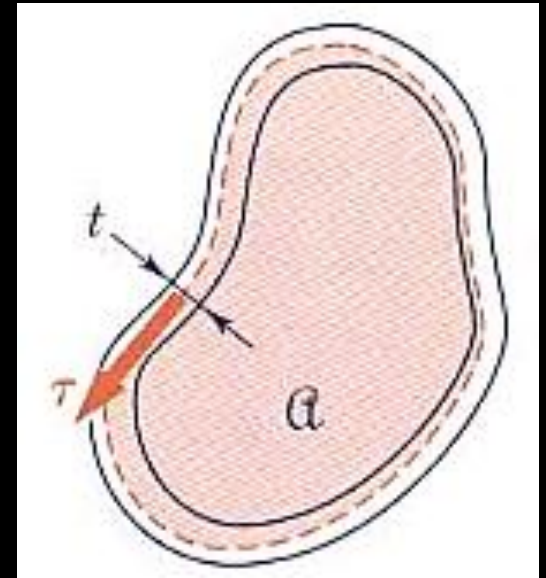
$$\tau_{\max} = \frac{T}{\frac{1}{3} ab^2} \quad \phi = \frac{TL}{\frac{1}{3} ab^3 G}$$

Shear Flow in Closed Sections

- q is the internal shear force/unit length

$$\tau = \frac{T}{2t\mathcal{A}}$$

$$\phi = \frac{TL}{4t\mathcal{A}^2} \sum_i \frac{s_i}{t_i}$$



- \mathcal{A} is the area bounded by the centerline
- s_i is the length segment, t_i is the thickness

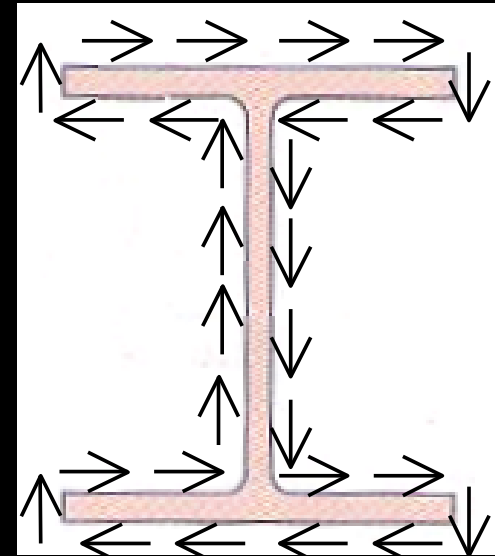
Shear Flow in Open Sections

- each segment has proportion of T with respect to torsional rigidity,

$$\tau_{\max} = \frac{T t_{\max}}{\frac{1}{3} \sum b_i t_i^3}$$

- total angle of twist:

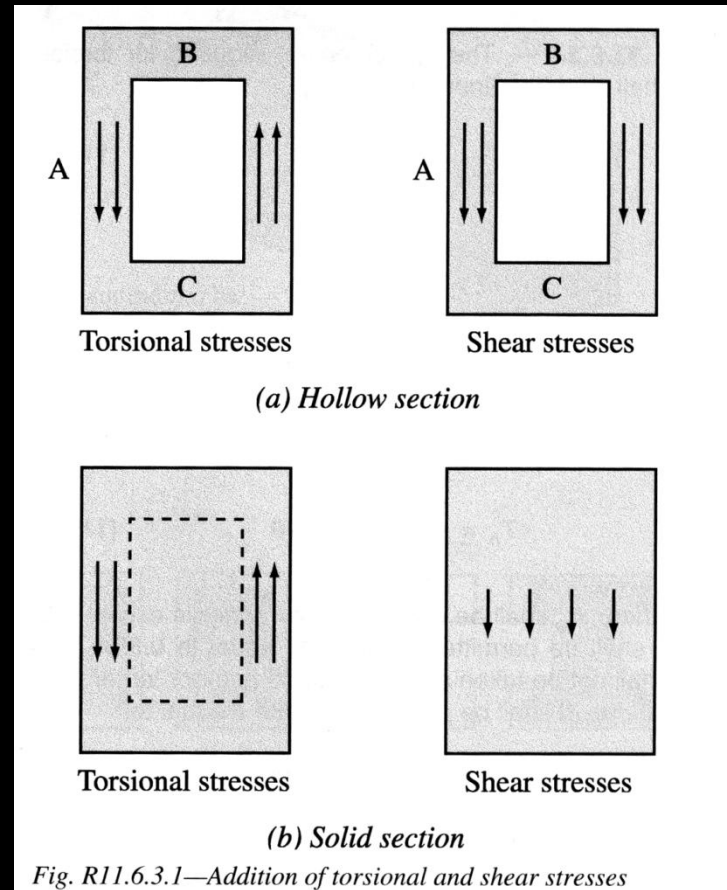
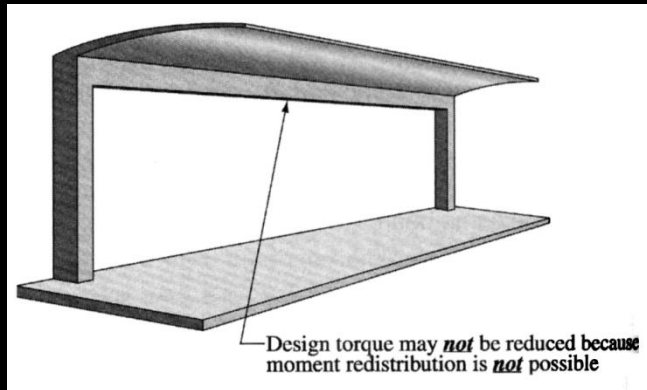
$$\phi = \frac{TL}{\frac{1}{3} G \sum b_i t_i^3}$$



- *I* beams - web is thicker, so τ_{\max} is in web

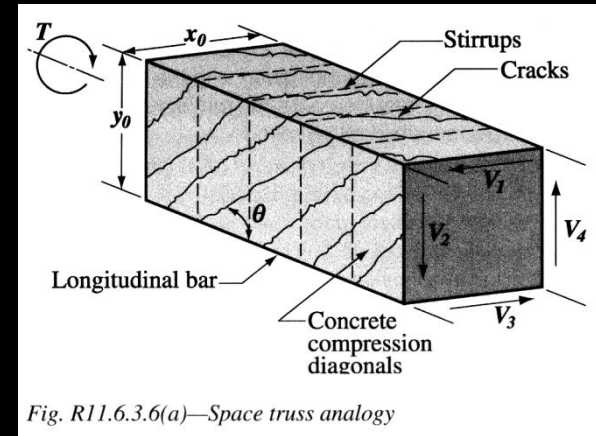
Torsional Shear Stress

- *twisting moment*
- and beam shear

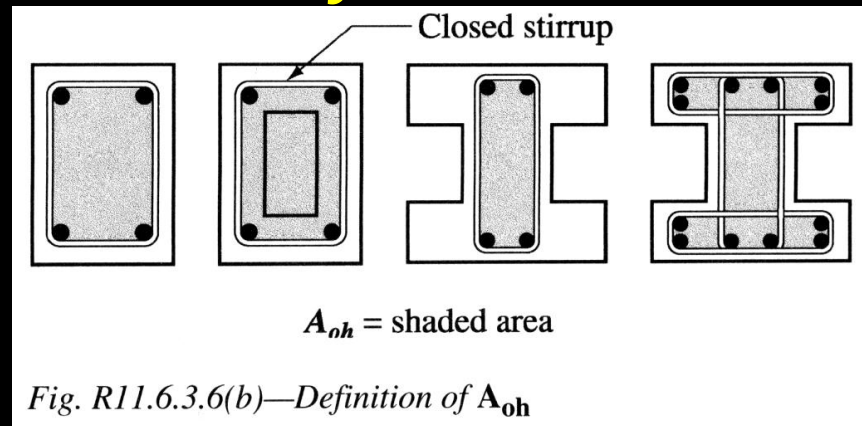


Torsional Shear Reinforcement

- *closed stirrups*
- *more longitudinal reinforcement*



- *area enclosed by shear flow*



Development Lengths

- required to allow steel to yield (f_y)
- standard hooks
 - moment at beam end

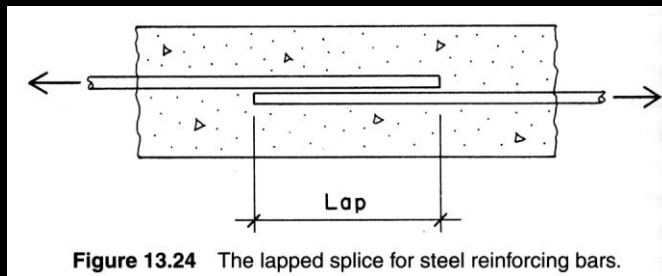
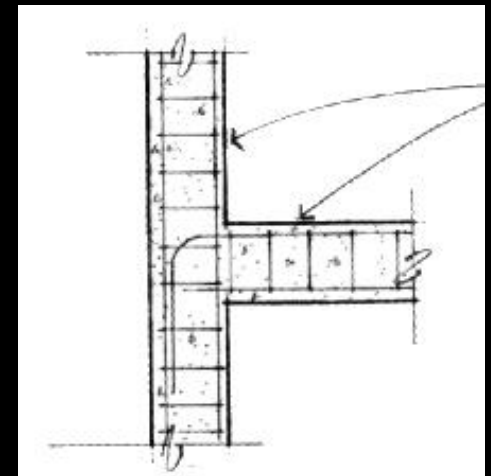
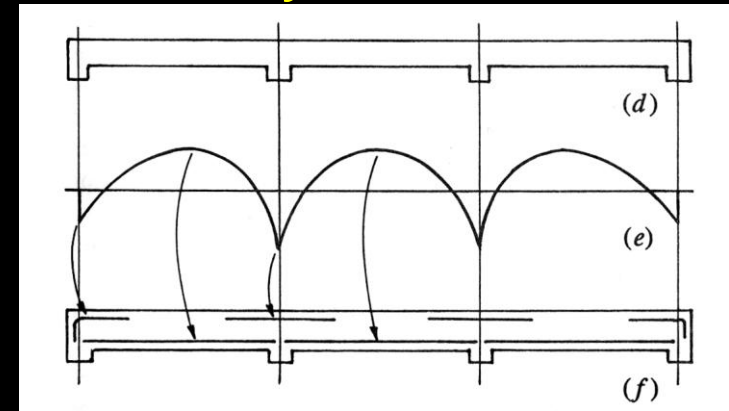


Figure 13.24 The lapped splice for steel reinforcing bars.

- splices
 - lapped
 - mechanical connectors



Development Lengths

- l_d , embedment required both sides
- proper cover, spacing:
 - No. 6 or smaller

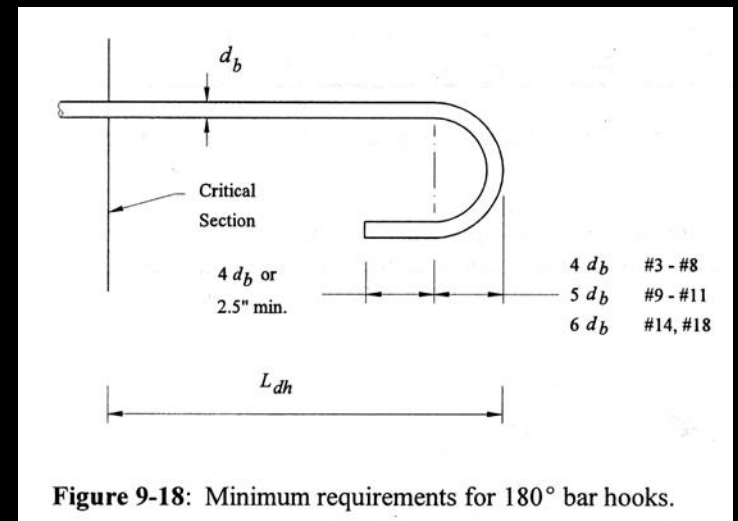
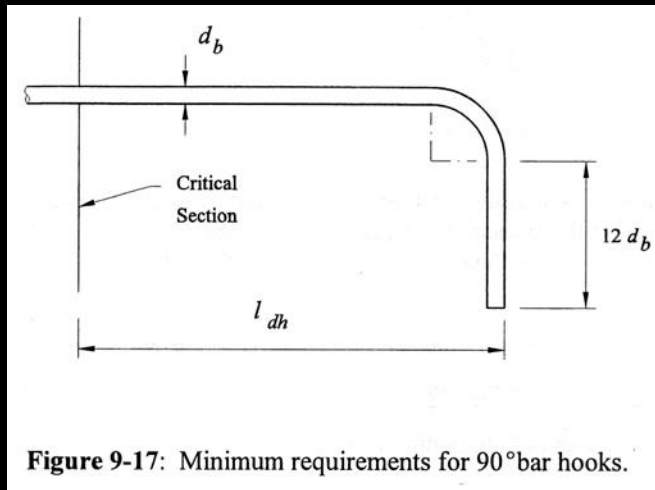
$$l_d = \frac{d_b F_y}{25 \sqrt{f'_c}} \quad \text{or 12 in. minimum}$$

- No. 7 or larger

$$l_d = \frac{d_b F_y}{20 \sqrt{f'_c}} \quad \text{or 12 in. minimum}$$

Development Lengths

- *hooks*
 - *bend and extension*



- *minimum*

$$l_{dh} = \frac{1200d_b}{\sqrt{f'_c}}$$

Development Lengths

- *bars in compression*

$$l_d = \frac{0.02d_b F_y}{\sqrt{f'_c}} \leq 0.0003d_b F_y$$

- *splices*
 - *tension minimum is function of l_d and splice classification*
 - *compression minimum*
 - *is function of d_b and F_y*

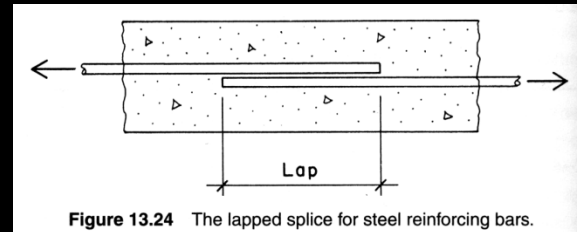


Figure 13.24 The lapped splice for steel reinforcing bars.

Concrete Deflections

- *elastic range*

- *I transformed*

- E_c (with f'_c in psi)

- *normal weight concrete (~ 145 lb/ft³)*

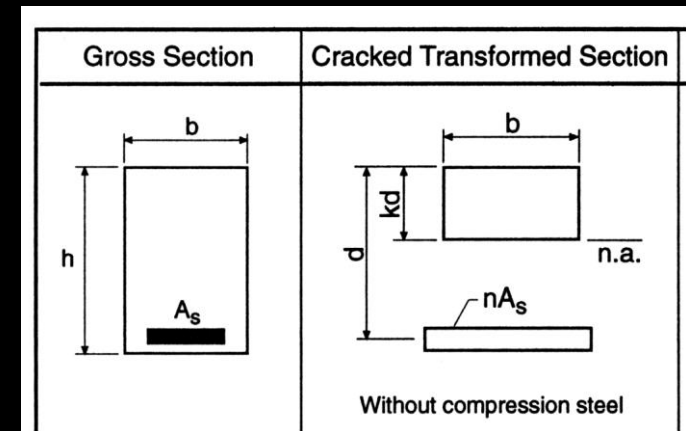
$$E_c = 57,000 \sqrt{f'_c}$$

- *concrete between 90 and 160 lb/ft³*

- *cracked*

- *I cracked*

- *E adjusted*



Deflection Limits

- *relate to whether or not beam supports or is attached to a damageable non-structural element*
- *need to check service live load and long term deflection against these*

L/180

roof systems (typical) – live

L/240

floor systems (typical) – live + long term

L/360

supporting plaster – live

L/480

supporting masonry – live + long term