

**ELEMENTS OF ARCHITECTURAL STRUCTURES:  
FORM, BEHAVIOR, AND DESIGN**

**ARCH 614**

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**SPRING 2013**

**lecture  
ten**

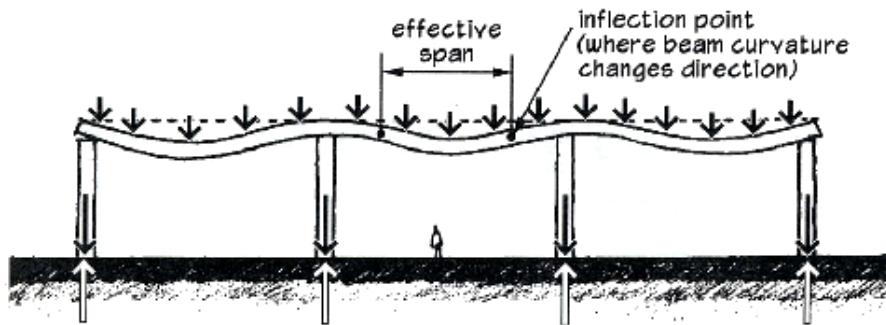
**other beams &  
pinned frames**



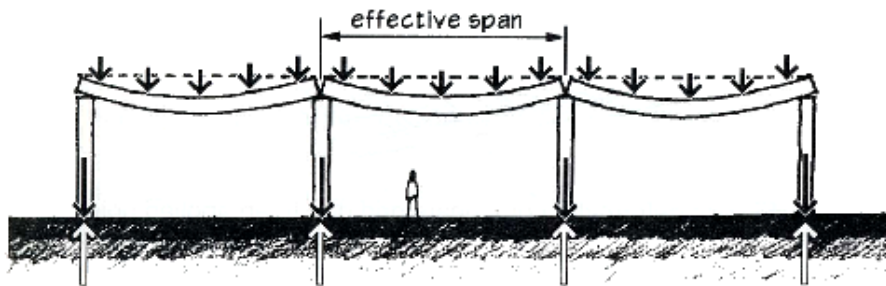
*Continental train platform, Grimshaw 1993*

# Continuous Beams

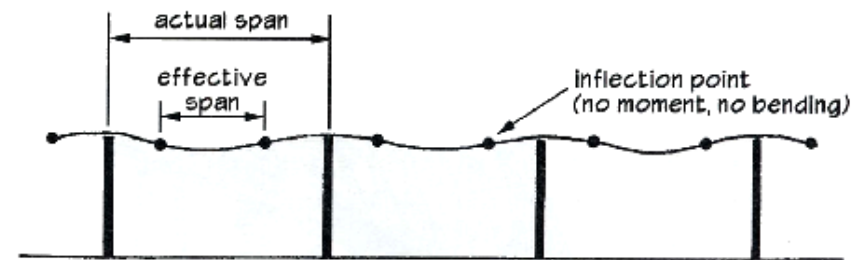
- *statically indeterminate*
- *reduced moments than simple beam*



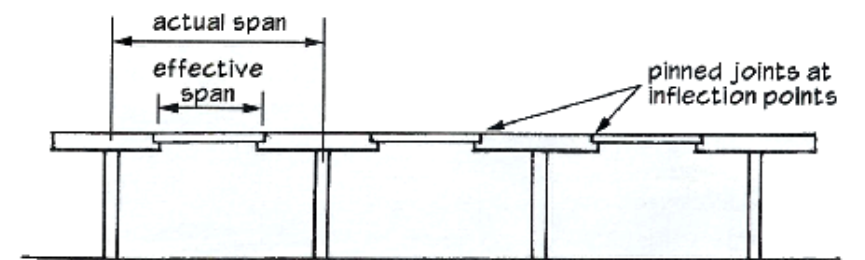
(a) CONTINUOUS BEAM



(b) SIMPLE BEAM



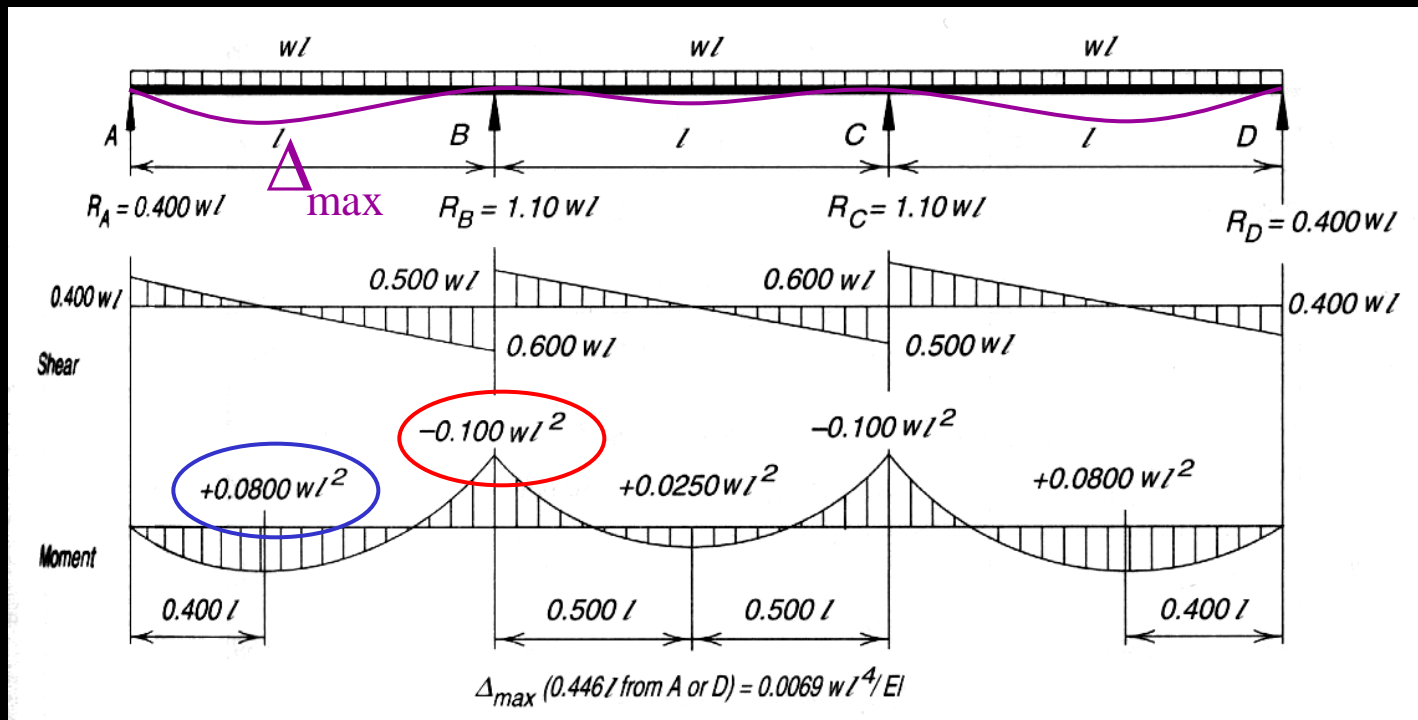
CONTINUOUS BEAMS (deflection diagram)



GERBER BEAMS

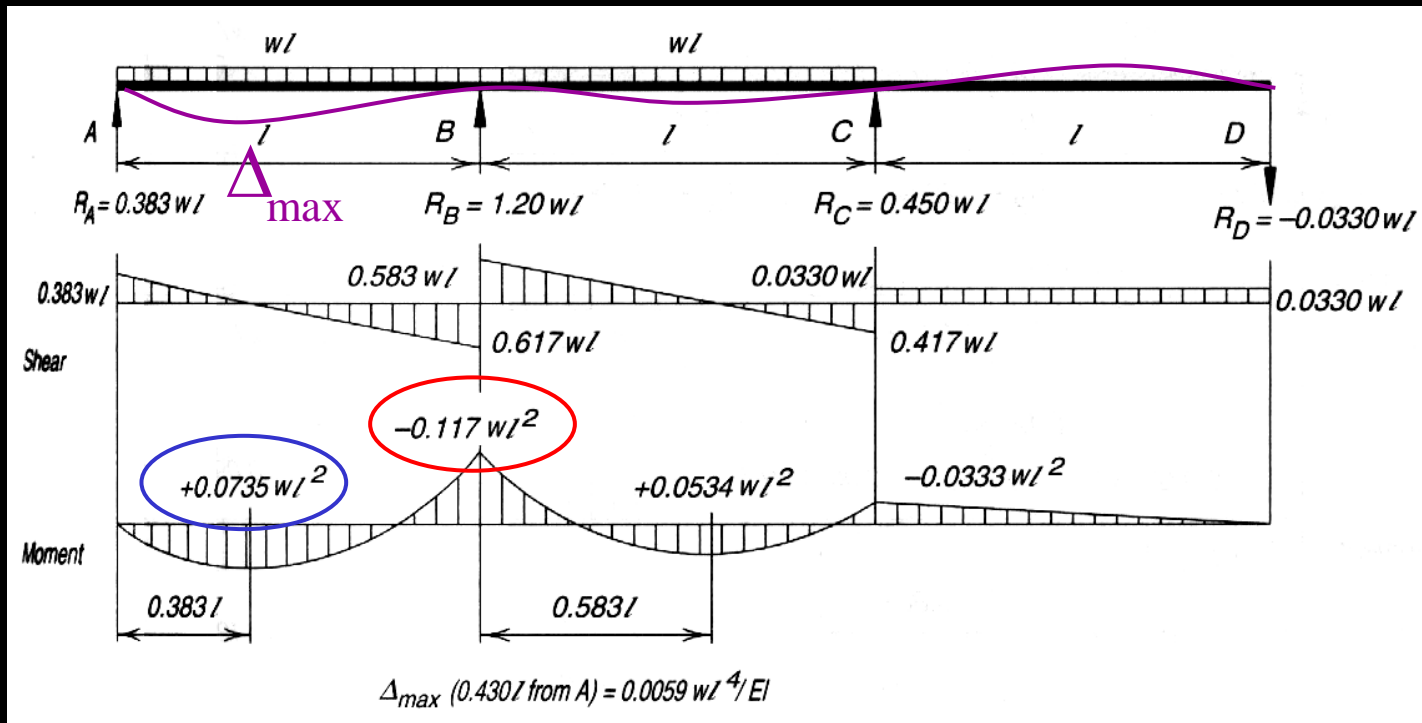
# Continuous Beams

- loading pattern affects
  - moments & deflection



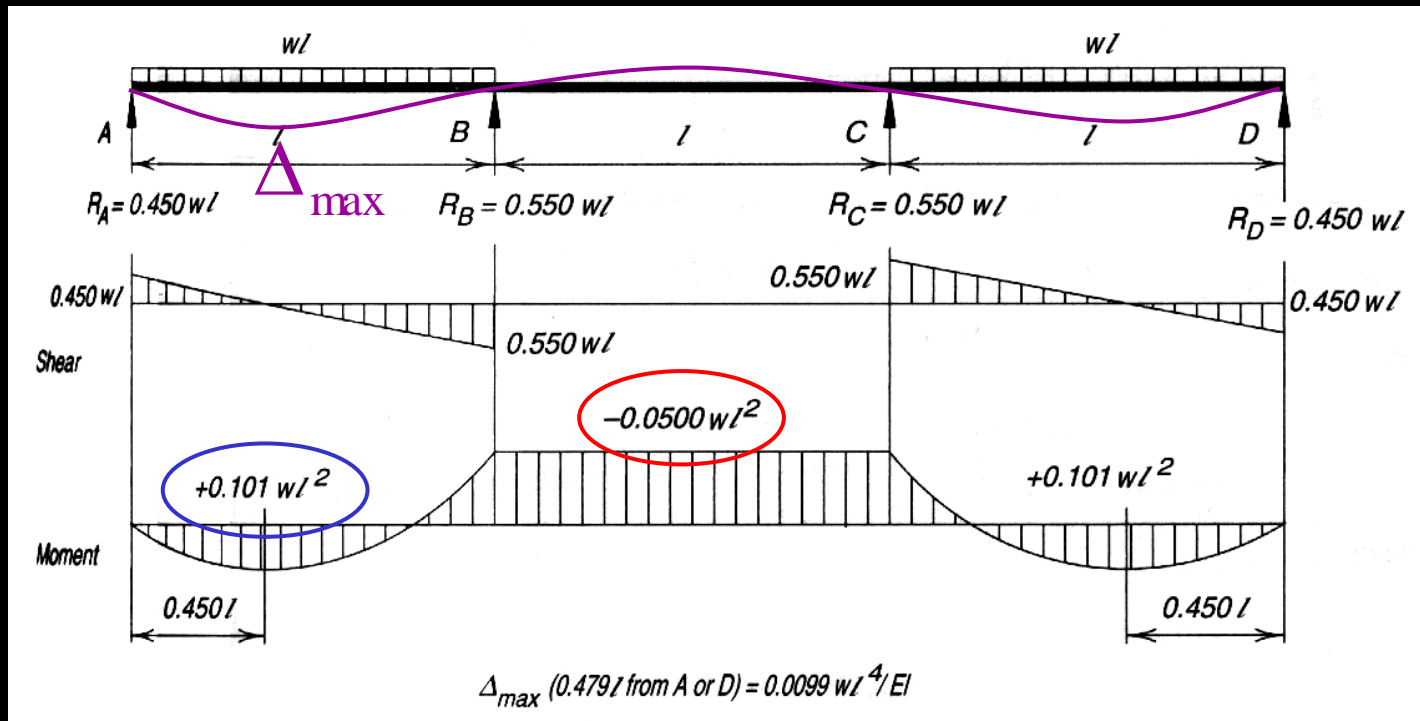
# Continuous Beams

- *unload end span*



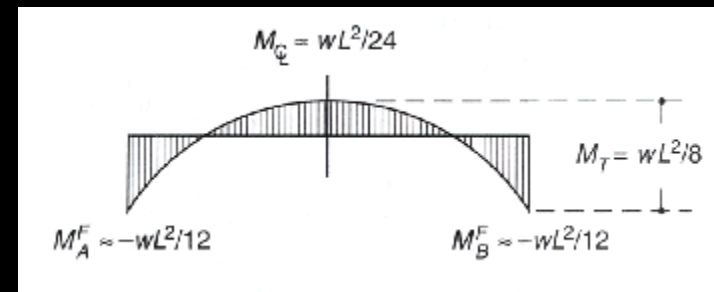
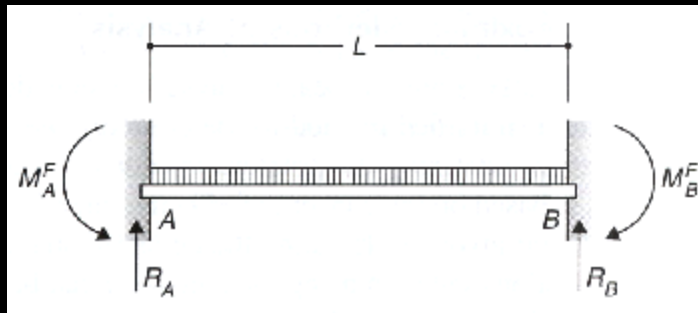
# Continuous Beams

- *unload middle span*



# Moment Redistribution

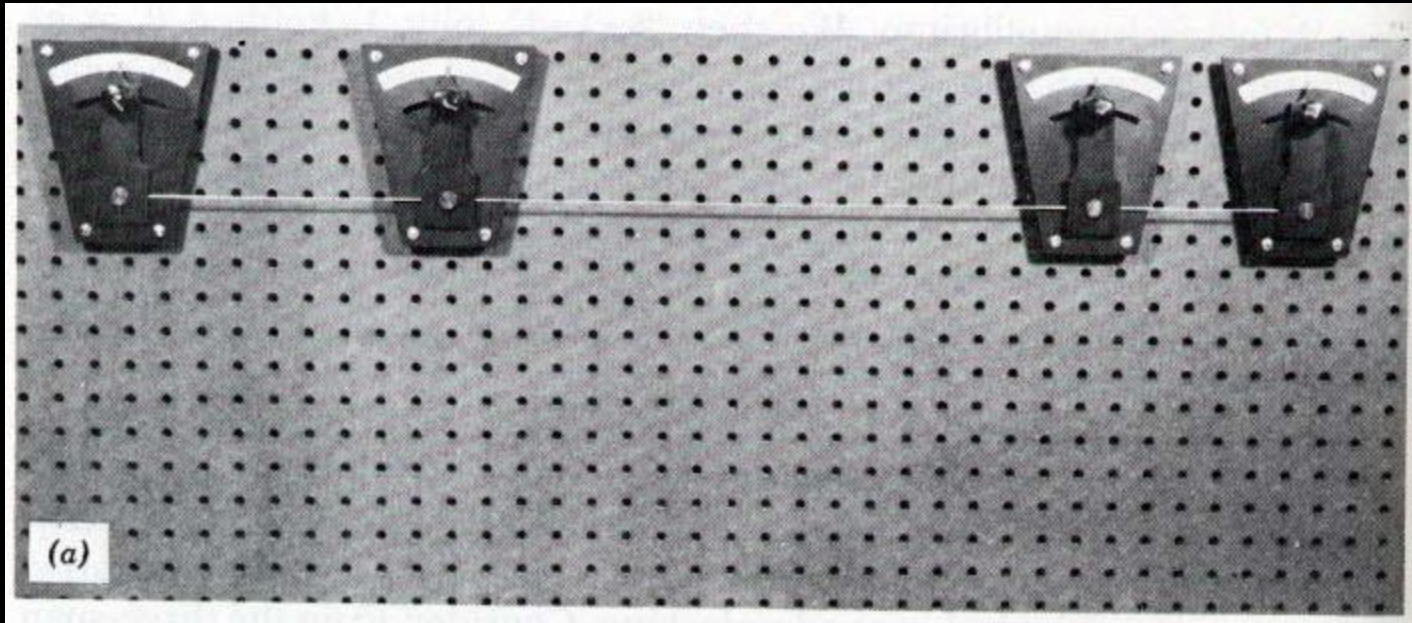
- continuous slabs & beams with uniform loading
  - joints similar to fixed ends, but can rotate
- change in moment to center =  $\frac{wL^2}{8}$ 
  - $M_{max}$  for simply supported beam



# Moment Distribution (a)

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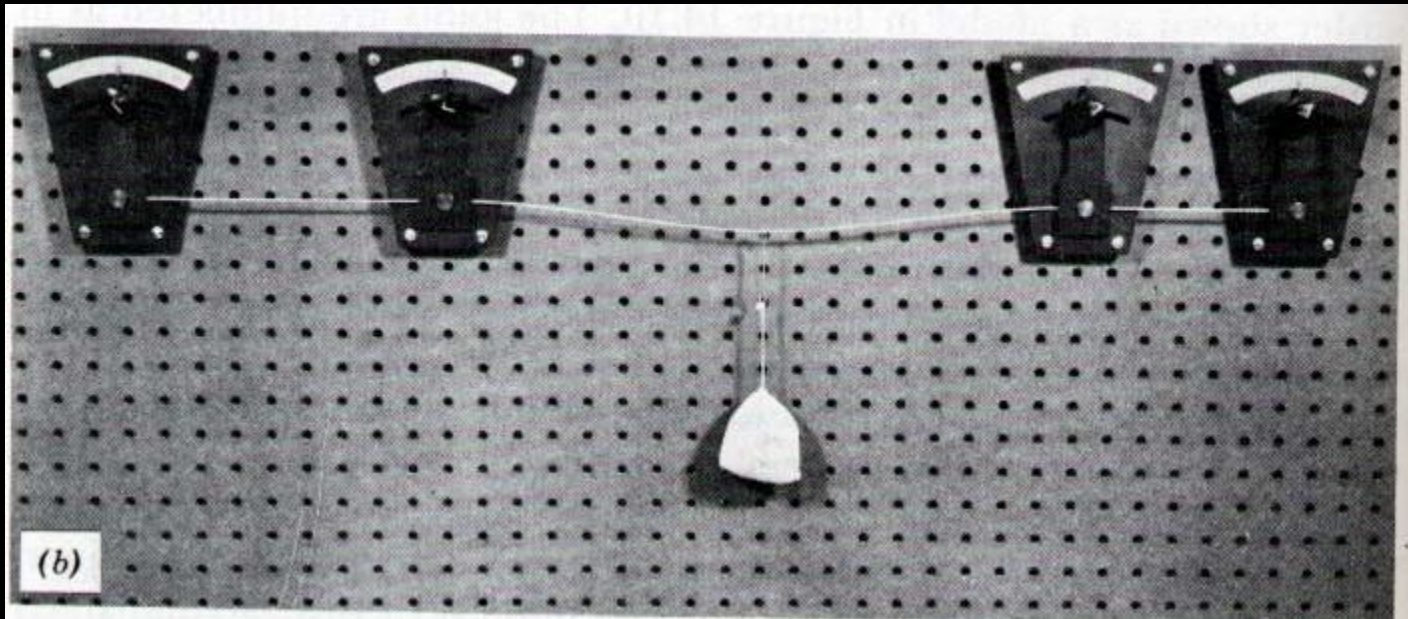
- *no load*



<http://nisee.berkeley.edu/godden>

# Moment Distribution (b)

- *add load*

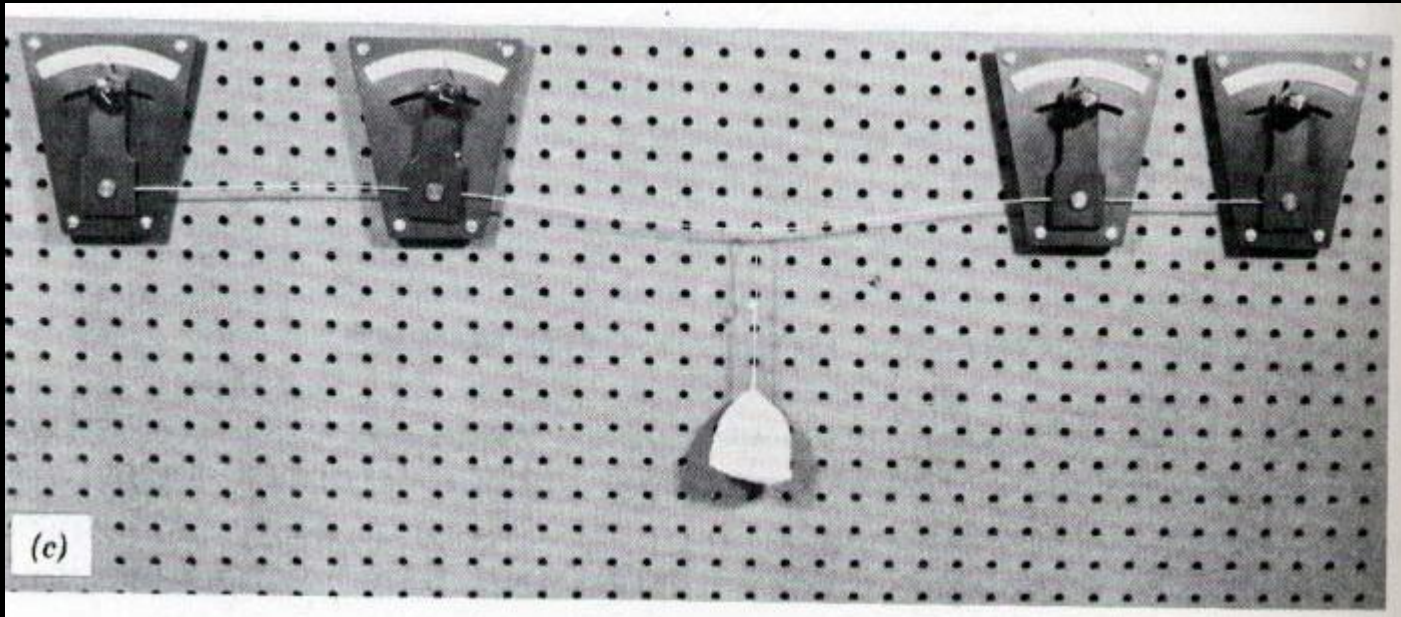


<http://nisee.berkeley.edu/godden>



# Moment Distribution Method (c)

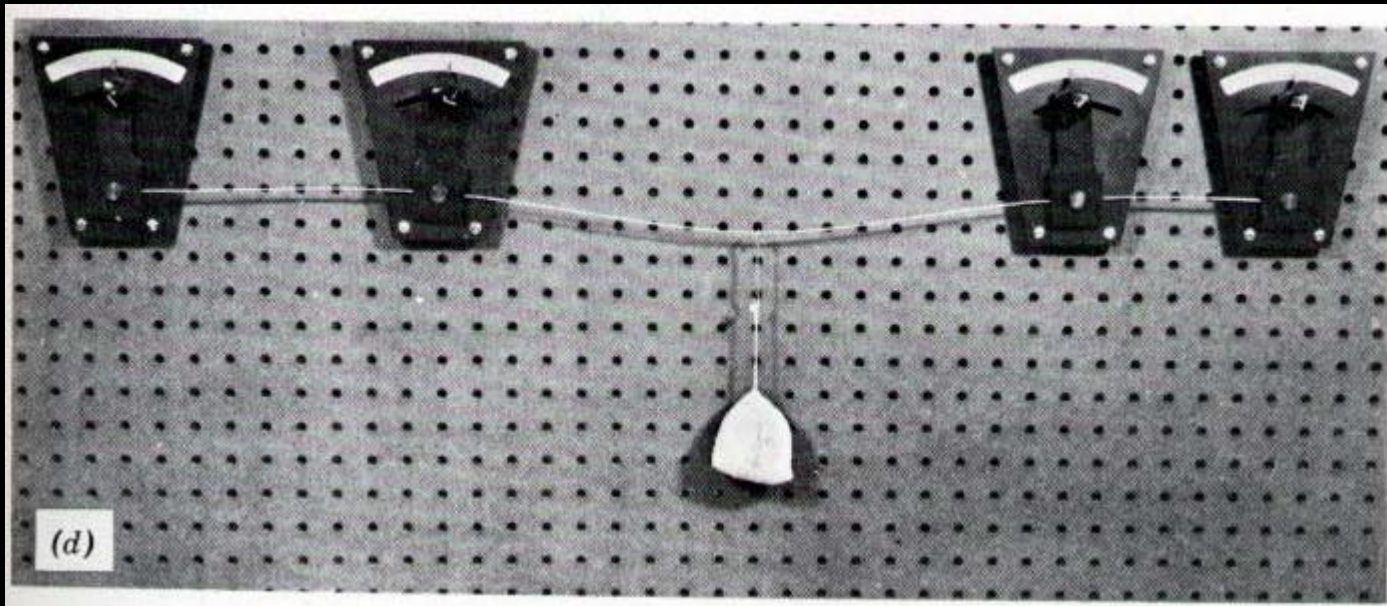
- *release joint 2*



<http://nisee.berkeley.edu/godden>

# Moment Distribution Method (d)

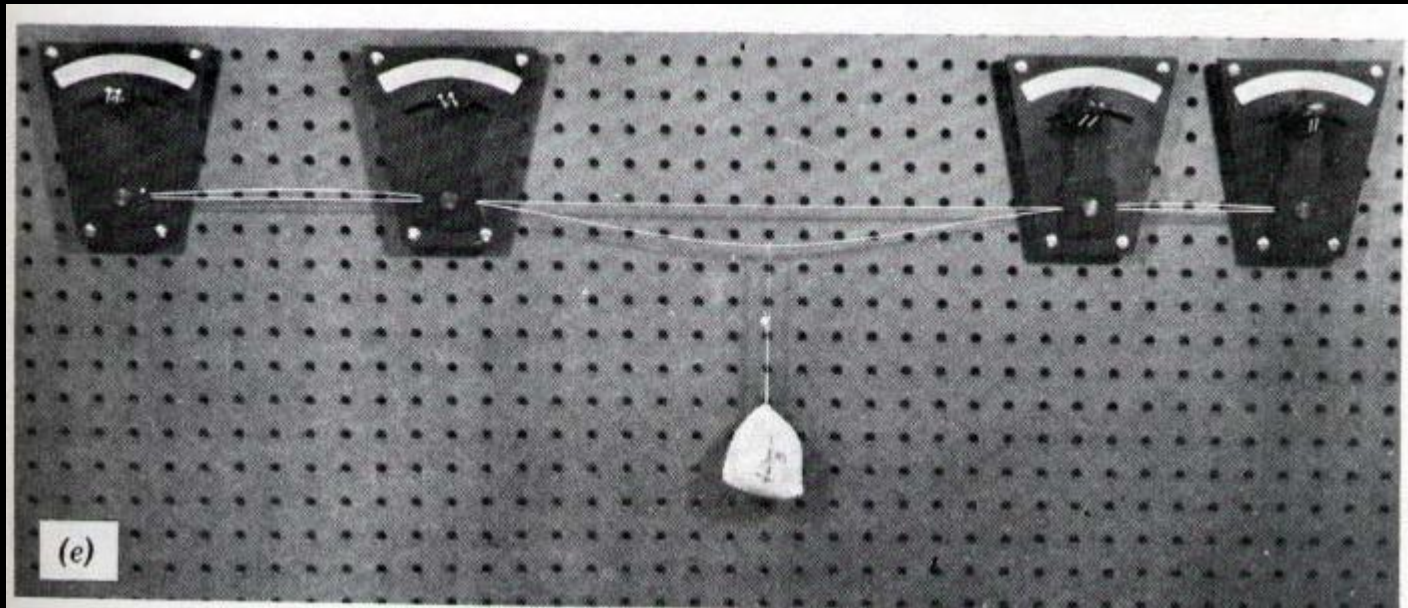
- release joint 3



<http://nisee.berkeley.edu/godden>

# Moment Distribution Method (e)

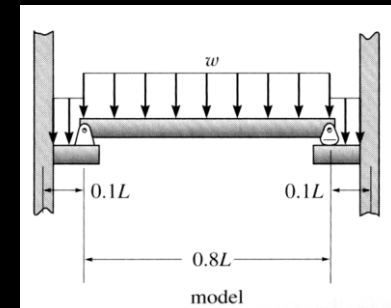
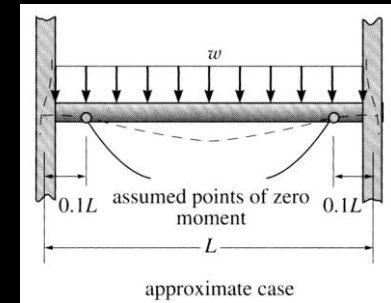
- exposure of final shape after cycles over initial shape



<http://nisee.berkeley.edu/godden>

# Analysis Methods

- *Approximate Methods*
  - *location of inflection points*
- *Force Method*
  - *forces are unknowns*
- *Displacement Method*
  - *displacements are unknowns*



# Theorem of Three Moments

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- *moments at three adjacent supports  
(2 spans)*
- *distributed load and same I:*

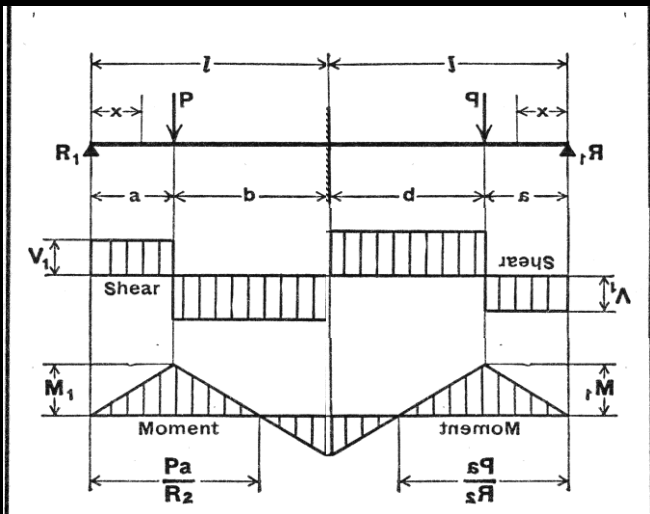
$$M_1 L_1 + 2M_2 (L_1 + L_2) + M_3 L_2 = -\frac{w_1 L_1^3}{4} - \frac{w_2 L_2^3}{4}$$

- *concentrated loads and same I:*

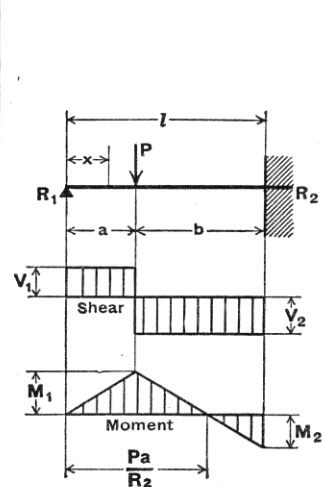
$$M_1 L_1 + 2M_2 (L_1 + L_2) + M_3 L_2 = -\sum P_1 L_1^2 (n_1 - n_1^3) - \sum P_2 L_2^2 (n_2 - n_2^3)$$

# Two Span Beams & Charts

- equal spans & symmetrical loading
- middle support as flat slope



## 14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER— CONCENTRATED LOAD AT ANY POINT



$$R_1 = V_1 \dots \dots \dots = \frac{Pb^2}{2l^3} (a + 2l)$$

$$R_2 = V_2 \dots \dots \dots = \frac{Pa}{2l^3} (3l^2 - a^2)$$

$$M_1 \text{ (at point of load)} \dots \dots \dots = R_1 a$$

$$M_2 \text{ (at fixed end)} \dots \dots \dots = \frac{Pab}{2l^2} (a + l)$$

$$M_x \text{ (when } x < a) \dots \dots \dots = R_1 x$$

$$M_x \text{ (when } x > a) \dots \dots \dots = R_1 x - P(x - a)$$

$$\Delta_{\max.} \text{ (when } a < .414l \text{ at } x = l \sqrt{\frac{l^2 + a^2}{3l^2 - a^2}}) = \frac{Pa}{3EI} \frac{(l^2 - a^2)^3}{(3l^2 - a^2)^2}$$

$$\Delta_{\max.} \text{ (when } a > .414l \text{ at } x = l \sqrt{\frac{a}{2l + a}}) = \frac{Pab^2}{6EI} \sqrt{\frac{a}{2l + a}}$$

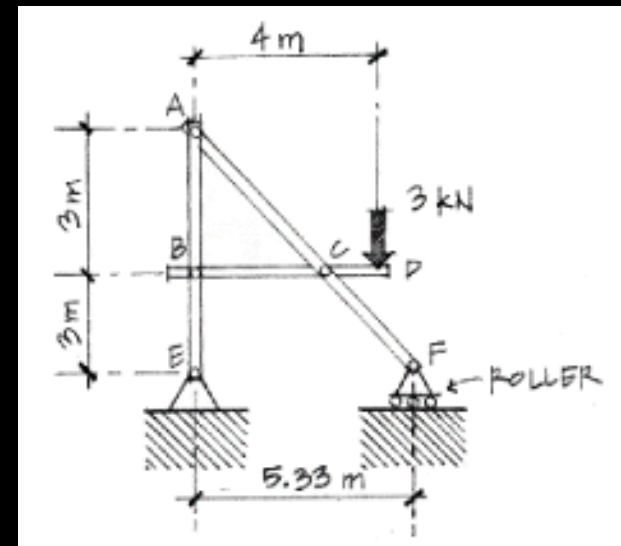
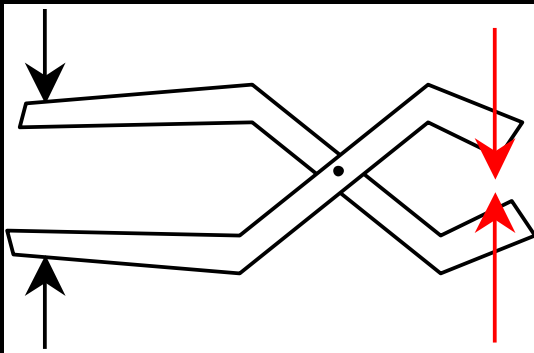
$$\Delta a \text{ (at point of load)} \dots \dots \dots = \frac{Pa^2 b^3}{12EI^3} (3l + a)$$

$$\Delta x \text{ (when } x < a) \dots \dots \dots = \frac{Pb^2 x}{12EI^3} (3a l^2 - 2l x^2 - a x^2)$$

$$\Delta x \text{ (when } x > a) \dots \dots \dots = \frac{Pa}{12EI^3} (l - x)^2 (3l^2 x - a^2 x - 2a^2 l)$$

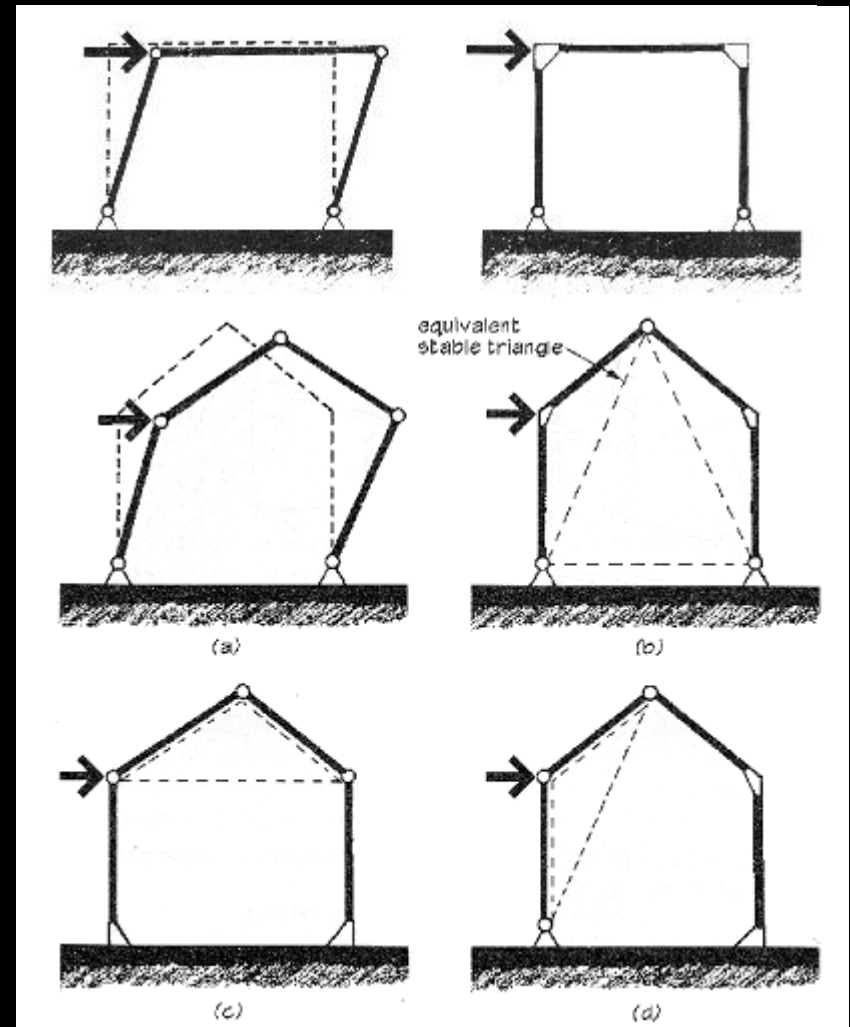
# Pinned Frames

- structures with at least one 3 force body
- connected with pins
- reactions are equal and opposite
  - non-rigid
  - rigid



# Rigid Frames

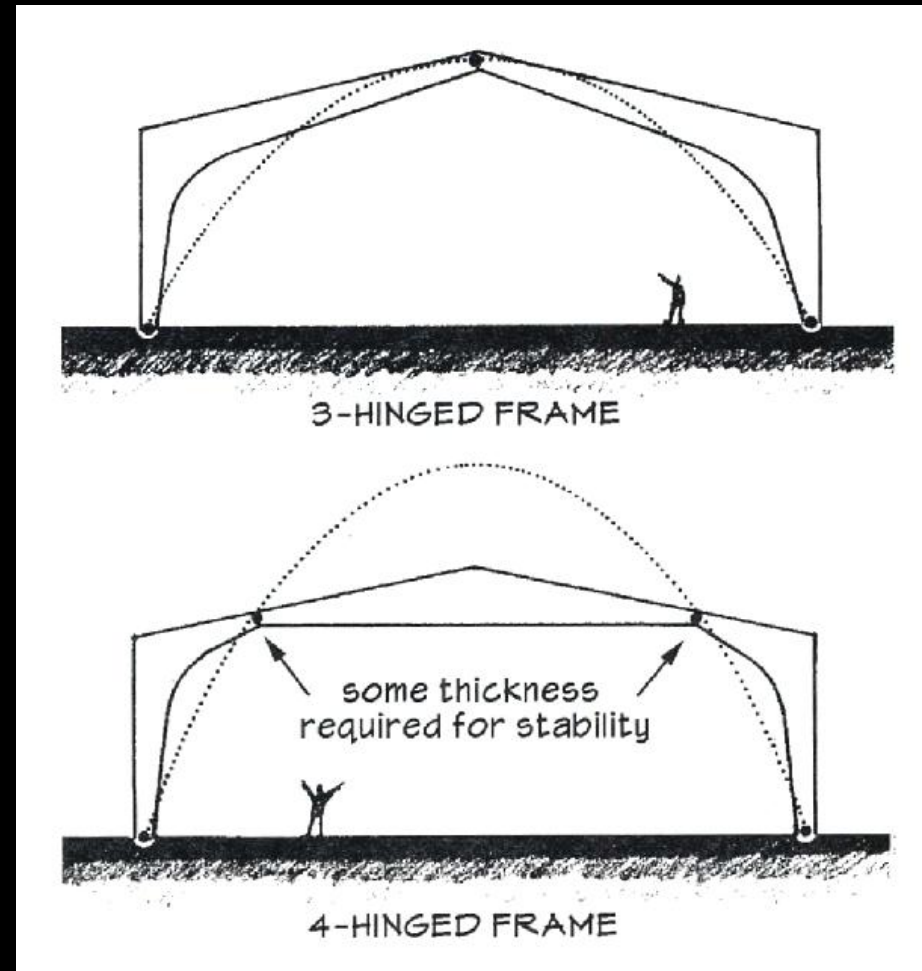
- rigid frames have no pins
- frame is all one body
- typically statically indeterminate
- types
  - portal
  - gable





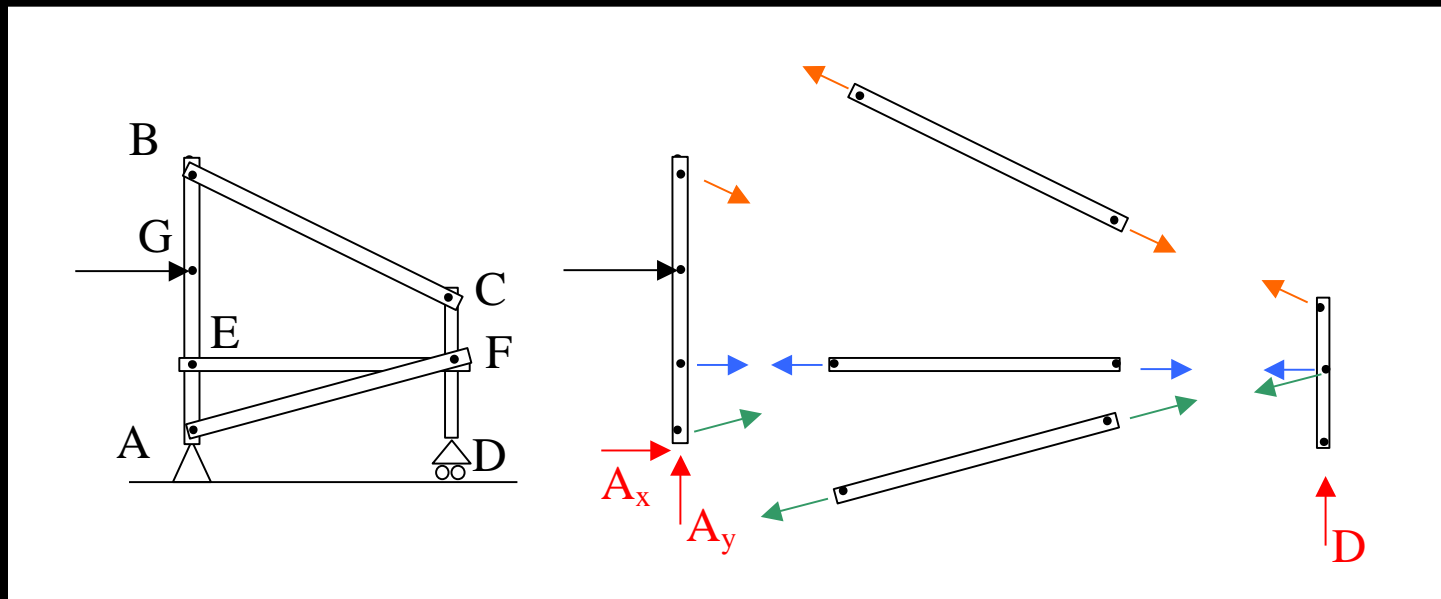
# Rigid Frames with PINS

- *frame pieces with connecting pins*
- *not necessarily symmetrical*



# Internal Pin Connections

- *statically determinant*
  - 3 equations per body
  - 2 reactions per pin + support forces



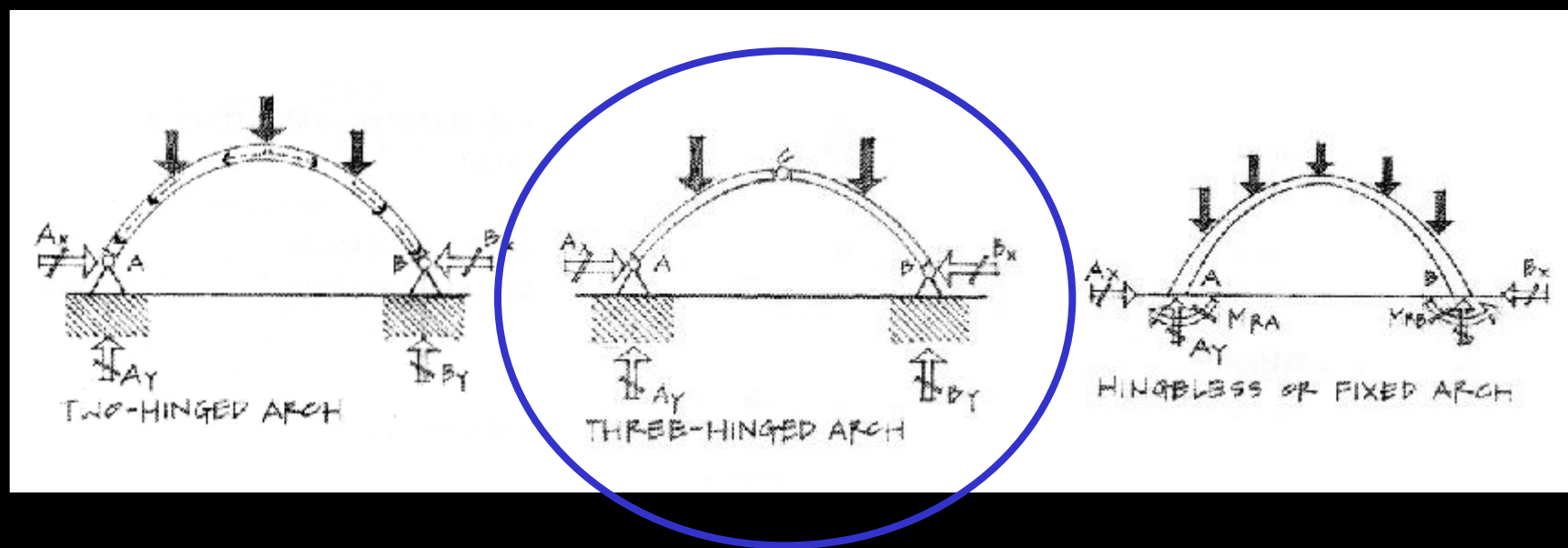
# Arches

- *ancient*
- *traditional shape to span long distances*



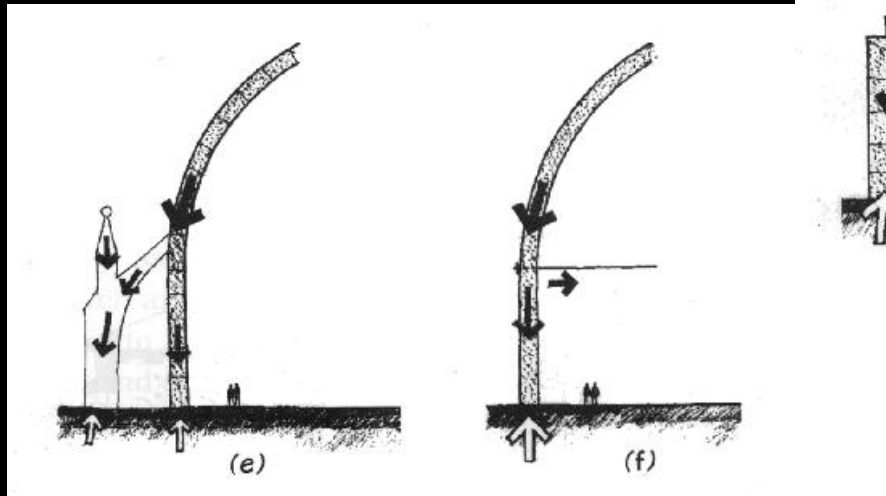
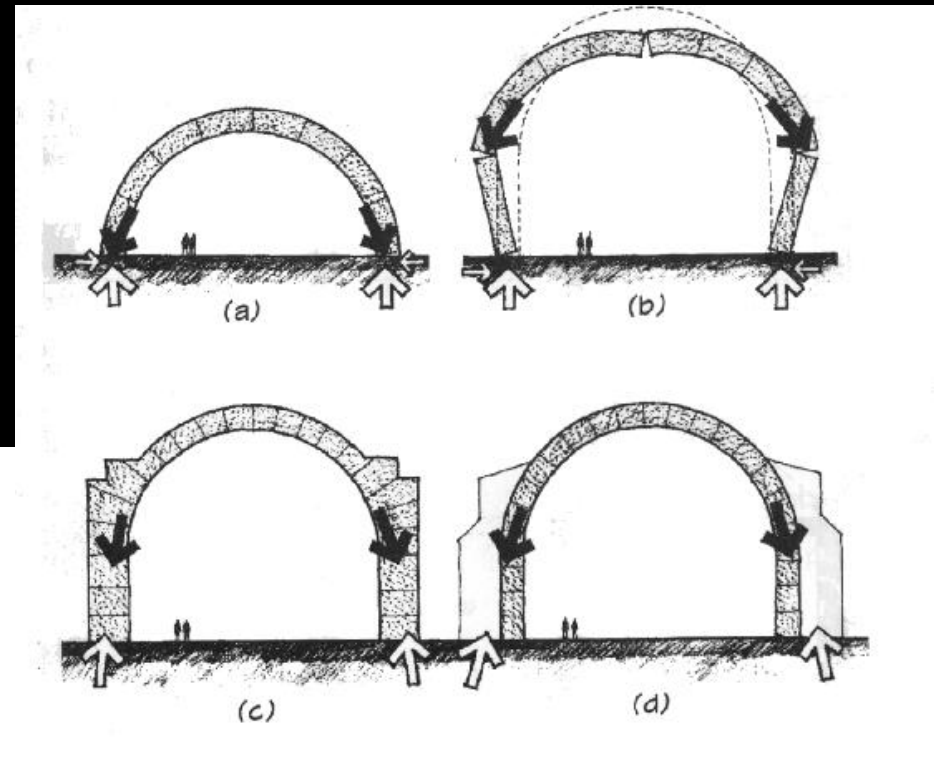
# Arches

- primarily sees compression
- a brick “likes an arch”



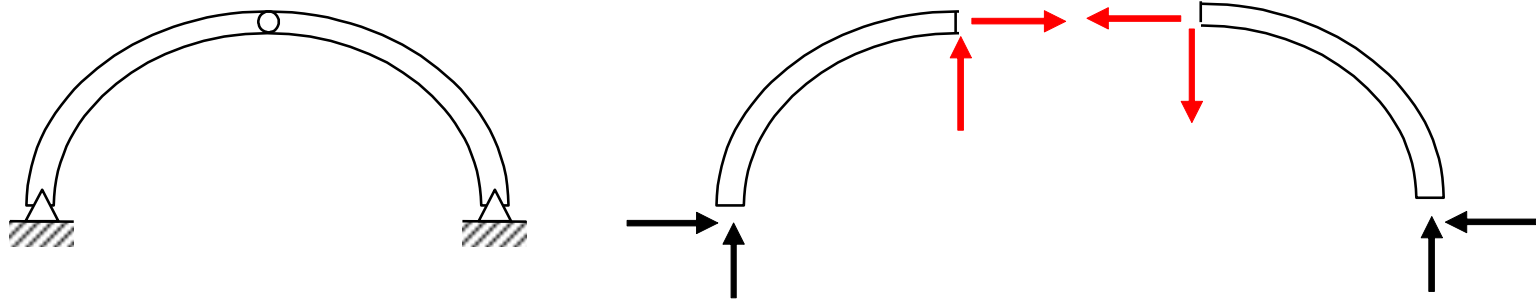
# Arches

- *behavior*
  - *thrust related to height to width*



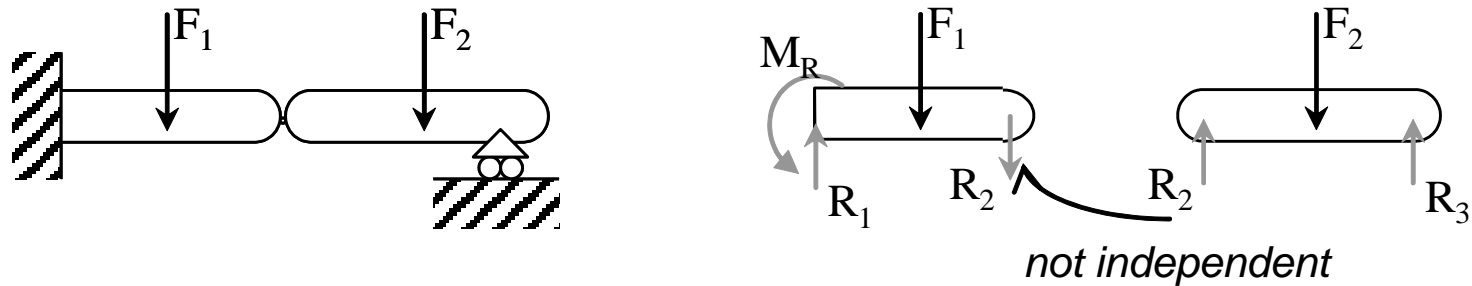
# Three-Hinged Arch

- *statically determinant*
  - 2 bodies, 6 equilibrium equations
  - 4 support, 2 pin reactions (= 6)



# Beams with Internal Pins

- *statically determinant when*
  - 3 equilibrium equations per link =>
  - total of support & pin reactions (properly constrained)
- *zero moment at pins*



# Procedure

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- *solve for all support forces you can*
- *draw a FBD of each member*
  - *pins are integral with member*
  - *pins with loads should belong to 3+ force bodies*
  - *pin forces are equal and opposite on connecting bodies*
  - *identify 2 force bodies vs. 3+ force bodies*
  - *use all equilibrium equations*