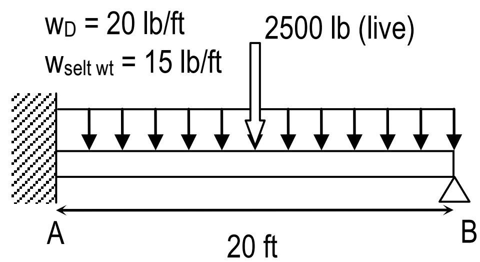
ARCH 614: Practice Quiz 6

Note: No aids are allowed for part 1. One side of a letter sized paper with notes is allowed during part 2, along with a silent, ***non-programm****able calculator.*

<u>Clearly show your work and answer.</u>

Part 1) Worth 5 points (conceptual questions)

Part 2) Worth 45 points



(NOTE: The superpositioned loads and configuration can and will be changed for the quiz! The material will not change. Load duration factor should be the only adjustment factor to consider.)

One wood beam is needed to span 20 ft from a fixed support to a wall, and support a roof having 20 lb/ft of dead load and a 2500 lb seven-day roof live load at midspan. The beam is fully braced. Idaho White Pine will be used and has the following tabular design values for bending for single member uses and modulus of elasticity:

F_b = 1150 psi F_v = 70 psi $E = 1.4 \times 10^6$ psi $\gamma = 26$ lb/ft^3

a) Including an estimated self weight of 15 lb/ft, determine the required section modulus, and choose the most economical section based on stress only.

b) If a section has been chosen having a moment of inertia of 231 in^4 and self weight of 15 lb/ft, is the design adequate for deflection at midspan when the limit is L/240 for live load only [*or* L/180 for total load], where L is the span between supports? *(Note: Be careful if the deflection is up as indicated by the bending moment diagrams! Up deflection has a negative value).*

Disclaimer: Answers have NOT been painstakingly researched.

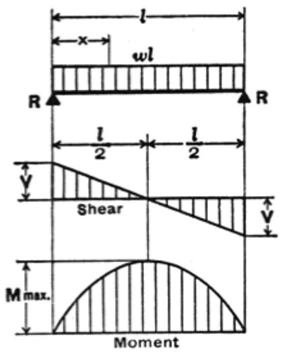
Answers – <u>Not provided on actual quiz!</u>

a) $S_{req'd} \geq 92.9$ in^3 $A_{req'd} \geq 37.0$ in^2 (*pick should consider the actual weight listed to the assumed self weight.)*

b) $\Delta_L = 0.97$ in. $\therefore$OK [or $\Delta_T = 1.13$ in. $\therefore$O.K.]

REFERENCE CHARTS FOR QUIZ 6

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . $= wl$

$R = V$ . . . . $= \frac{wl}{2}$

V_x . . . . $= w\left(\frac{l}{2} - x\right)$

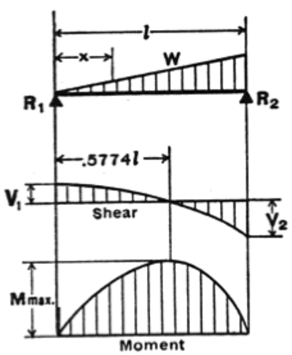
M max. (at center) . . . . . $= \frac{wl^2}{8}$

M_x . . . . $= \frac{wx}{2}(l - x)$

Δmax. (at center) . . . . . $= \frac{5wl^4}{384EI}$

Δx . . . . $= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END



Total Equiv. Uniform Load . . . . $= \frac{16W}{9\sqrt{3}} = 1.0264W$

$R_1 = V_1$ . . . . $= \frac{W}{3}$

$R_2 = V_2$ max. . . . . $= \frac{2W}{3}$

V_x . . . . $= \frac{W}{3} - \frac{Wx^2}{l^2}$

M max. $\left(\text{at } x = \frac{l}{\sqrt{3}} = .5774l\right)$ . . $= \frac{2Wl}{9\sqrt{3}} = .1283\,Wl$

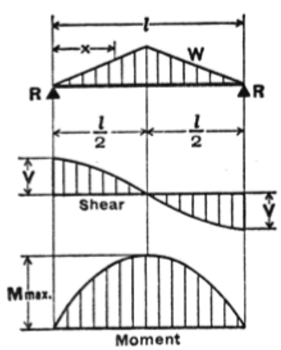
M_x . . . . $= \frac{Wx}{3l^2}(l^2 - x^2)$

Δmax. $\left(\text{at } x = l\sqrt{1-\sqrt{\frac{8}{15}}} = .5193l\right) = .01304\,\frac{Wl^3}{EI}$

Δx . . . . $= \frac{Wx}{180EIl^2}(3x^4 - 10l^2x^2 + 7l^4$

$W = \frac{wl}{2}$

3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER



Total Equiv. Uniform Load . . . . $= \frac{4W}{3}$

$R = V$ . . . . $= \frac{W}{2}$

V_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . $= \frac{W}{2l^2}(l^2 - 4x^2)$

M max. (at center) . . . . $= \frac{Wl}{6}$

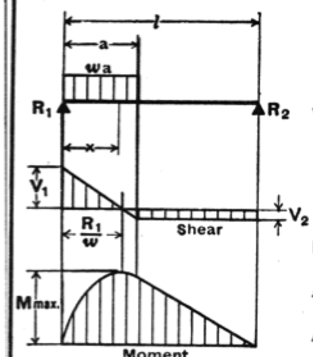
M_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . $= Wx\left(\frac{1}{2} - \frac{2x^2}{3l^2}\right)$

Δmax. (at center) . . . . $= \frac{Wl^3}{60EI}$

Δx $\left(\text{when } x < \frac{l}{2}\right)$ . . . . $= \frac{Wx}{480EIl^2}(5l^2 - 4x^2)^2$

$W = \frac{wl}{2}$

5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END



$R_1 = V_1$ max. . . . . $= \frac{wa}{2l}(2l - a)$

$R_2 = V_2$ . . . . $= \frac{wa^2}{2l}$

V_x (when $x < a$) . . . . . $= R_1 - wx$

M max. $\left(\text{at } x = \frac{R_1}{w}\right)$ . . . . . $= \frac{R_1^2}{2w}$

M_x (when $x < a$) . . . . . $= R_1x - \frac{wx^2}{2}$

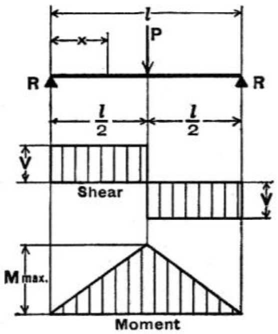
M_x (when $x > a$) . . . . . $= R_2(l - x)$

Δx (when $x < a$) . . . . . $= \frac{wx}{24EIl}\left(a^2(2l-a)^2 - 2ax^2(2l-a) + lx^3\right)$

Δx (when $x > a$) . . . . . $= \frac{wa^2(l-x)}{24EIl}(4xl - 2x^2 - a^2)$

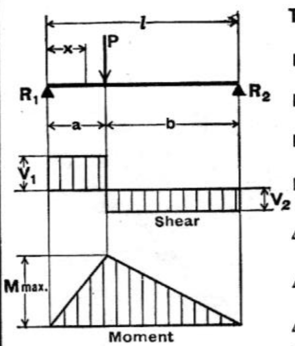
REFERENCE CHARTS FOR QUIZ 6

7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER



Total Equiv. Uniform Load	$= 2P$
$R = V$	$= \frac{P}{2}$
M max. (at point of load)	$= \frac{Pl}{4}$
M_x (when $x < \frac{l}{2}$)	$= \frac{Px}{2}$
Δmax. (at point of load)	$= \frac{Pl^3}{48EI}$
Δ_x (when $x < \frac{l}{2}$)	$= \frac{Px}{48EI}(3l^2 - 4x^2)$

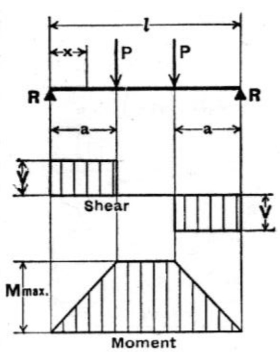
8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load	$= \frac{8\,Pab}{l^2}$
$R_1 = V_1$ (max. when $a < b$)	$= \frac{Pb}{l}$
$R_2 = V_2$ (max. when $a > b$)	$= \frac{Pa}{l}$
M max. (at point of load)	$= \frac{Pab}{l}$
M_x (when $x < a$)	$= \frac{Pbx}{l}$
Δmax. (at $x = \sqrt{\frac{a(a+2b)}{3}}$ when $a > b$)	$= \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27\,EI\,l}$
Δa (at point of load)	$= \frac{Pa^2b^2}{3EI\,l}$
Δ_x (when $x < a$)	$= \frac{Pbx}{6EI\,l}(l^2 - b^2 - x^2)$

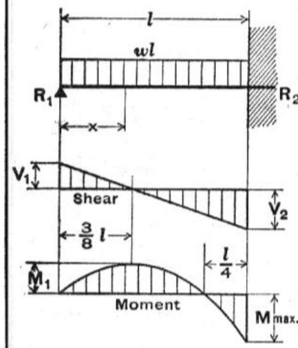
Δx

9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED



Total Equiv. Uniform Load	$= \frac{8\,Pa}{l}$
$R = V$	$= P$
M max. (between loads)	$= Pa$
M_x (when $x < a$)	$= Px$
Δmax. (at center)	$= \frac{Pa}{24EI}(3l^2 - 4a^2)$
Δ_x (when $x < a$)	$= \frac{Px}{6EI}(3la - 3a^2 - x^2)$
Δ_x (when $x > a$ and $< (l - a)$)	$= \frac{Pa}{6EI}(3lx - 3x^2 - a^2)$

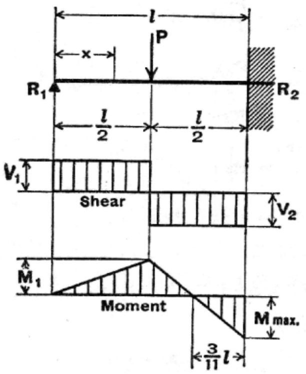
12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load	$= wl$
$R_1 = V_1$	$= \frac{3wl}{8}$
$R_2 = V_2$ max.	$= \frac{5wl}{8}$
V_x	$= R_1 - wx$
M max.	$= \frac{wl^2}{8}$
M_1 (at $x = \frac{3}{8}l$)	$= \frac{9}{128}wl^2$
M_x	$= R_1x - \frac{wx^2}{2}$
Δmax. (at $x = \frac{l}{16}(1 + \sqrt{33}) = .4215l$)	$= \frac{wl^4}{185EI}$
Δ_x	$= \frac{wx}{48EI}(l^3 - 3lx^2 + 2x^3)$

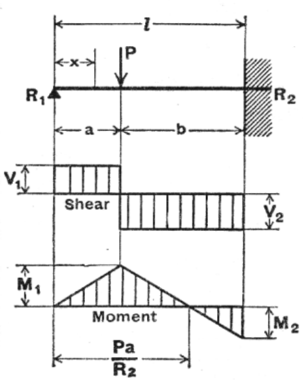
REFERENCE CHARTS FOR QUIZ 6

13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT CENTER



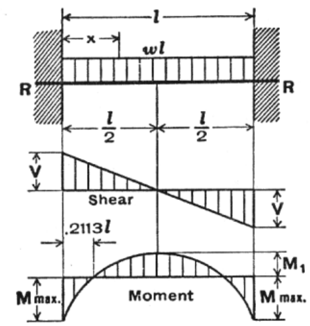
Total Equiv. Uniform Load		$= \frac{3P}{2}$
$R_1 = V_1$		$= \frac{5P}{16}$
$R_2 = V_2$ max.		$= \frac{11P}{16}$
M max.	(at fixed end)	$= \frac{3Pl}{16}$
M_1	(at point of load)	$= \frac{5Pl}{32}$
M_x	$\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{5Px}{16}$
M_x	$\left(\text{when } x > \frac{l}{2}\right)$	$= P\left(\frac{l}{2} - \frac{11x}{16}\right)$
Δmax.	$\left(\text{at } x = l\sqrt{\frac{1}{5}} = .4472l\right)$	$= \frac{Pl^3}{48EI\sqrt{5}} = .009317\frac{Pl^3}{EI}$
Δx	(at point of load)	$= \frac{7Pl^3}{768EI}$
Δx	$\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{Px}{96EI}(3l^2 - 5x^2)$
Δx	$\left(\text{when } x > \frac{l}{2}\right)$	$= \frac{P}{96EI}(x - l)^2(11x - 2l)$

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT ANY POINT



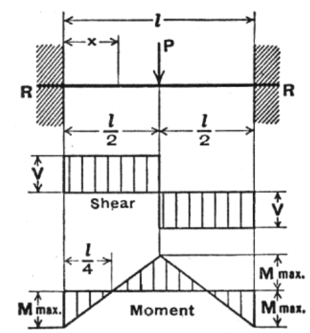
$R_1 = V_1$		$= \frac{Pb^2}{2l^3}(a + 2l)$
$R_2 = V_2$		$= \frac{Pa}{2l^3}(3l^2 - a^2)$
M_1	(at point of load)	$= R_1a$
M_2	(at fixed end)	$= \frac{Pab}{2l^2}(a + l)$
M_x	(when $x < a$)	$= R_1x$
M_x	(when $x > a$)	$= R_1x - P(x - a)$
Δmax.	$\left(\text{when } a < .414l \text{ at } x = l\frac{l^2+a^2}{3l^2-a^2}\right)$	$= \frac{Pa}{3EI}\frac{(l^2 - a^2)^3}{(3l^2 - a^2)^2}$
Δmax.	$\left(\text{when } a > .414l \text{ at } x = l\sqrt{\frac{a}{2l+a}}\right)$	$= \frac{Pab^2}{6EI}\sqrt{\frac{a}{2l+a}}$
Δa	(at point of load)	$= \frac{Pa^2b^3}{12EIl^3}(3l + a)$
Δx	(when $x < a$)	$= \frac{Pb^2x}{12EIl^3}(3al^2 - 2lx^2 - ax^2)$
Δx	(when $x > a$)	$= \frac{Pa}{12EIl^3}(l-x)^2(3l^2x - a^2x - 2a^2l)$

15. BEAM FIXED AT BOTH ENDS—UNIFORMLY DISTRIBUTED LOADS



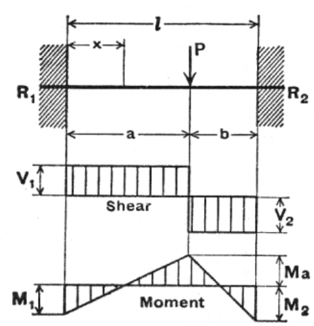
Total Equiv. Uniform Load		$= \frac{2wl}{3}$
$R = V$		$= \frac{wl}{2}$
V_x		$= w\left(\frac{l}{2} - x\right)$
M max.	(at ends)	$= \frac{wl^2}{12}$
M_1	(at center)	$= \frac{wl^2}{24}$
M_x		$= \frac{w}{12}(6lx - l^2 - 6x^2)$
Δmax.	(at center)	$= \frac{wl^4}{384EI}$
Δx		$= \frac{wx^2}{24EI}(l - x)^2$

16. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT CENTER



Total Equiv. Uniform Load		$= P$
$R = V$		$= \frac{P}{2}$
M max.	(at center and ends)	$= \frac{Pl}{8}$
M_x	$\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{P}{8}(4x - l)$
Δmax.	(at center)	$= \frac{Pl^3}{192EI}$
Δx	$\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{Px^2}{48EI}(3l - 4x)$

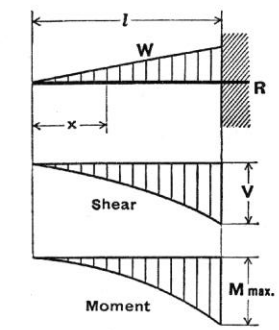
17. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$	(max. when $a < b$)	$= \frac{Pb^2}{l^3}(3a + b)$
$R_2 = V_2$	(max. when $a > b$)	$= \frac{Pa^2}{l^3}(a + 3b)$
M_1	(max. when $a < b$)	$= \frac{Pab^2}{l^2}$
M_2	(max. when $a > b$)	$= \frac{Pa^2b}{l^2}$
Ma	(at point of load)	$= \frac{2Pa^2b^2}{l^3}$
M_x	(when $x < a$)	$= R_1x - \frac{Pab^2}{l^2}$
Δmax.	$\left(\text{when } a > b \text{ at } x = \frac{2al}{3a + b}\right)$	$= \frac{2Pa^3b^2}{3EI(3a + b)^2}$
Δa	(at point of load)	$= \frac{Pa^3b^3}{3EIl^3}$
Δx	(when $x < a$)	$= \frac{Pb^2x^2}{6EIl^3}(3al - 3ax - bx)$

REFERENCE CHARTS FOR QUIZ 6

18. CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END



$W = \frac{wl}{2}$

Total Equiv. Uniform Load . . . . . $= \frac{8}{3}W$

R = V . . . . $= W$

V_x . . . . $= W\frac{x^2}{l^2}$

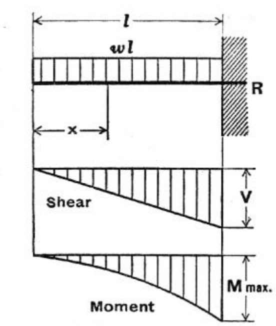
M max. (at fixed end) . . . . . . $= \frac{Wl}{3}$

M_x . . . . $= \frac{Wx^3}{3l^2}$

Δmax. (at free end) . . . . . . $= \frac{Wl^3}{15EI}$

Δx . . . . $= \frac{W}{60EIl^2}(x^5 - 5l^4x + 4l^5)$

19. CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . . $= 4wl$

R = V . . . . $= wl$

V_x . . . . $= wx$

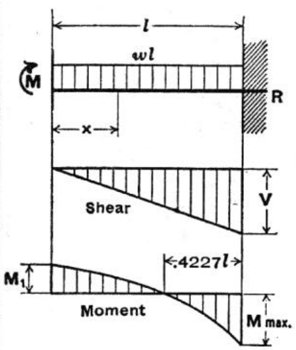
M max. (at fixed end) . . . . . . $= \frac{wl^2}{2}$

M_x . . . . $= \frac{wx^2}{2}$

Δmax. (at free end) . . . . . . $= \frac{wl^4}{8EI}$

Δx . . . . $= \frac{w}{24EI}(x^4 - 4l^3x + 3l^4)$

20. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . . $= \frac{8}{3}wl$

R = V . . . . $= wl$

V_x . . . . $= wx$

M max. (at fixed end) . . . . . . $= \frac{wl^2}{3}$

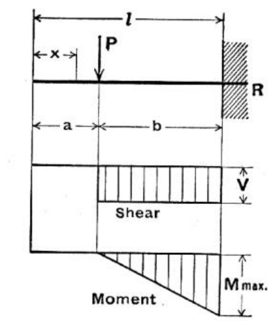
M_1 (at deflected end) . . . . . $= \frac{wl^2}{6}$

M_x . . . . $= \frac{w}{6}(l^2 - 3x^2)$

Δmax. (at deflected end) . . . . . $= \frac{wl^4}{24EI}$

Δx . . . . $= \frac{w(l^2 - x^2)^2}{24EI}$

21. CANTILEVER BEAM—CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load . . . . . $= \frac{8Pb}{l}$

R = V . . . . $= P$

M max. (at fixed end) . . . . . . $= Pb$

M_x (when x > a) . . . . . . $= P(x - a)$

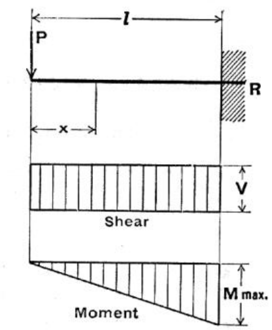
Δmax. (at free end) . . . . . . $= \frac{Pb^2}{6EI}(3l - b)$

Δa (at point of load) . . . . . $= \frac{Pb^3}{3EI}$

Δx (when x < a) . . . . . . $= \frac{Pb^2}{6EI}(3l - 3x - b)$

Δx (when x > a) . . . . . . $= \frac{P(l - x)^2}{6EI}(3b - l + x)$

22. CANTILEVER BEAM—CONCENTRATED LOAD AT FREE END



Total Equiv. Uniform Load . . . . . $= 8P$

R = V . . . . $= P$

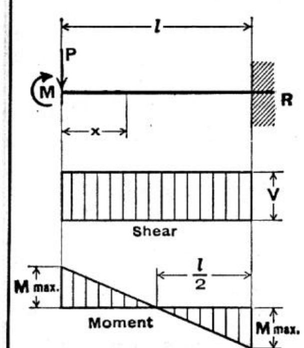
M max. (at fixed end) . . . . . . $= Pl$

M_x . . . . $= Px$

Δmax. (at free end) . . . . . $= \frac{Pl^3}{3EI}$

Δx . . . . $= \frac{P}{6EI}(2l^3 - 3l^2x + x^3)$

23. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—CONCENTRATED LOAD AT DEFLECTED END



Total Equiv. Uniform Load . . . . . $= 4P$

R = V . . . . $= P$

M max. (at both ends) . . . . . $= \frac{Pl}{2}$

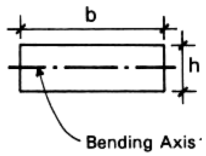
M_x . . . . $= P\left(\frac{l}{2} - x\right)$

Δmax. (at deflected end) . . . . . $= \frac{Pl^3}{12EI}$

Δx . . . . $= \frac{P(l - x)^2}{12EI}(l + 2x)$

REFERENCE CHARTS FOR QUIZ 6

TABLE A.8 Properties of Structural Lumber



Nominal b h (Dimensions (in.))	Actual b h (Dimensions (in.))	Area A (in.^{2})	Section Modulus S (in.^{3})	Moment of Inertia I (in.^{4})	Weight[a] (lb/ft)
2 × 3	1.5 × 2.5	3.75	1.563	1.953	0.9
2 × 4	1.5 × 3.5	5.25	3.063	5.359	1.3
2 × 6	1.5 × 5.5	8.25	7.563	20.797	2.0
2 × 8	1.5 × 7.25	10.875	13.141	47.635	2.6
2 × 10	1.5 × 9.25	13.875	21.391	98.932	3.4
2 × 12	1.5 × 11.25	16.875	31.641	177.979	4.1
2 × 14	1.5 × 13.25	19.875	43.891	290.775	4.8
3 × 2	2.5 × 1.5	3.75	0.938	0.703	0.9
3 × 4	2.5 × 3.5	8.75	5.104	8.932	2.1
3 × 6	2.5 × 5.5	13.75	12.604	34.661	3.3
3 × 8	2.5 × 7.25	18.125	21.901	79.391	4.4
3 × 10	2.5 × 9.25	23.125	35.651	164.886	5.6
3 × 12	2.5 × 11.25	28.125	52.734	296.631	6.8
3 × 14	2.5 × 13.25	33.125	73.151	484.625	8.1
3 × 16	2.5 × 15.25	38.125	96.901	738.870	9.3
4 × 2	3.5 × 1.5	5.25	1.313	0.984	1.3
4 × 3	3.5 × 2.5	8.75	3.646	4.557	2.1
4 × 4	3.5 × 3.5	12.25	7.146	12.505	3.0
4 × 6	3.5 × 5.5	19.25	17.646	48.526	4.7
4 × 8	3.5 × 7.25	25.375	30.661	111.148	6.2
4 × 10	3.5 × 9.25	32.375	49.911	230.840	7.9
4 × 12	3.5 × 11.25	39.375	73.828	415.283	9.6
4 × 14	3.5 × 13.25	46.375	102.411	678.475	11.3
4 × 16	3.5 × 15.25	53.375	135.661	1034.418	13.0
6 × 2	5.5 × 1.5	8.25	2.063	1.547	2.0
6 × 3	5.5 × 2.5	13.75	5.729	7.161	3.3
6 × 4	5.5 × 3.5	19.25	11.229	19.651	4.7
6 × 6	5.5 × 5.5	30.25	27.729	76.255	7.4
6 × 8	5.5 × 7.5	41.25	51.563	193.359	10.0
6 × 10	5.5 × 9.5	52.25	82.729	392.963	12.7
6 × 12	5.5 × 11.5	63.25	121.229	697.068	15.4
6 × 14	5.5 × 13.5	74.25	167.063	1127.672	18.0
6 × 16	5.5 × 15.5	85.25	220.229	1706.776	20.7
8 × 2	7.25 × 1.5	10.875	2.719	2.039	2.6
8 × 3	7.25 × 2.5	18.125	7.552	9.440	4.4
8 × 4	7.25 × 3.5	25.375	14.802	25.904	6.2
8 × 6	7.5 × 5.5	41.25	37.813	103.984	10.0
8 × 8	7.5 × 7.5	56.25	70.313	263.672	13.7

TABLE A.8 *(Continued)*

Nominal b h (Dimensions (in.))	Actual b h (Dimensions (in.))	Area A (in.^{2})	Section Modulus S (in.^{3})	Moment of Inertia I (in.^{4})	Weight[a] (lb/ft)
8 × 10	7.5 × 9.5	71.25	112.813	535.859	17.3
8 × 12	7.5 × 11.5	86.25	165.313	950.547	21.0
8 × 14	7.5 × 13.5	101.25	227.813	1537.734	24.6
8 × 16	7.5 × 15.5	116.25	300.313	2327.422	28.3
8 × 18	7.5 × 17.5	131.25	382.813	3349.609	31.9
8 × 20	7.5 × 19.5	146.25	475.313	4634.297	35.5
10 × 10	9.5 × 9.5	90.25	142.896	678.755	21.9
10 × 12	9.5 × 11.5	109.25	209.396	1204.026	26.6
10 × 14	9.5 × 13.5	128.25	288.563	1947.797	31.2
10 × 16	9.5 × 15.5	147.25	380.396	2948.068	35.8
10 × 18	9.5 × 17.5	166.25	484.896	4242.836	40.4
10 × 20	9.5 × 19.5	185.25	602.063	5870.109	45.0
12 × 12	11.5 × 11.5	132.25	253.479	1457.505	32.1
12 × 14	11.5 × 13.5	155.25	349.313	2357.859	37.7
12 × 16	11.5 × 15.5	178.25	460.479	3568.713	43.3
12 × 18	11.5 × 17.5	201.25	586.979	5136.066	48.9
12 × 20	11.5 × 19.5	224.25	728.813	7105.922	54.5
12 × 22	11.5 × 21.5	247.25	885.979	9524.273	60.1
12 × 24	11.5 × 23.5	270.25	1058.479	12437.129	65.7
14 × 14	13.5 × 13.5	182.25	410.063	2767.922	44.3
16 × 16	15.5 × 15.5	240.25	620.646	4810.004	58.4

[a] Based on an assumed average density of 35 pcf.

Source: Compiled from data in the *National Design Specification for Wood Construction* (Ref. 3), with permission of the publishers, National Forest Products Association.

TABLE 5.2 Modification Factors for Design Values for Structural Lumber for Load Duration[a]

Load Duration	Multiply Design Values by:	Typical Design Loads
Permanent	0.9	Dead load
Ten years	1.0	Occupancy live load
Two months	1.15	Snow load
Seven days	1.25	Construction load
Ten minutes	1.6	Wind or earthquake load
Impact[b]	2.00	Impact load

Source: Adapted from the *National Design Specification for Wood Construction*, 2001 edition (Ref. 3), with permission of the publishers, American Forest & Paper Association.

[a] Load duration factors shall not apply to modulus of elasticity, E, nor to compression perpendicular to grain design values, $F_{c\perp}$, based on a deformation limit.

[b] Load duration factors greater than 1.6 shall not apply to structural members pressure-treated with water-borne preservatives, or fire retardant chemicals. The impact load duration factor shall not apply to connections.