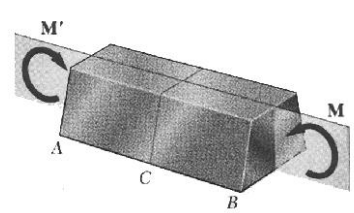
Beam Stresses – Bending and Shear

Notation:

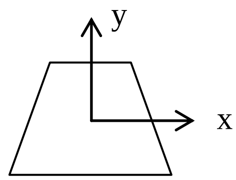
A = name for area
A_{web} = area of the web of a wide flange section
b = width of a rectangle
= total width of material at a horizontal section
c = largest distance from the neutral axis to the top or bottom edge of a beam
d = calculus symbol for differentiation
= depth of a wide flange section
d_y = difference in the y direction between an area centroid ($\bar{y}$) and the centroid of the composite shape ($\hat{y}$)
E = modulus of elasticity or Young's modulus
f_b = bending stress
f_c = compressive stress
f_{max} = maximum stress
f_t = tensile stress
f_v = shear stress
F_b = allowable bending stress
$F_{connector}$ = shear force capacity per connector
h = height of a rectangle
I = moment of inertia with respect to neutral axis bending
I_x = moment of inertia with respect to an x-axis
L = name for length
M = internal bending moment
= name for a moment vector
n = number of connectors across a joint
$n.a.$ = shorthand for neutral axis (N.A.)
O = name for reference origin
p = pitch of connector spacing
P = name for a force vector
q = shear per length (shear flow)
Q = first moment area about a neutral axis
$Q_{connected}$ = first moment area about a neutral axis for the connected part
R = radius of curvature of a deformed beam
S = section modulus
$S_{req'd}$ = section modulus required at allowable stress
t_w = thickness of web of wide flange
V = internal shear force
$V_{longitudinal}$ = longitudinal shear force
V_T = transverse shear force
w = name for distributed load
x = horizontal distance
y = vertical distance
$\bar{y}$ = the distance in the y direction from a reference axis (*n.a*) to the centroid of a shape
$\hat{y}$ = the distance in the y direction from a reference axis to the centroid of a composite shape
Δ = calculus symbol for small quantity
δ = elongation or length change
ε = strain
θ = arc angle
Σ = summation symbol

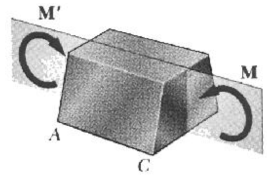
Pure Bending in Beams

With bending moments along the axis of the member only, a beam is said to be in pure bending.



Normal stresses due to bending can be found for homogeneous materials having a plane of symmetry in the y axis that follow Hooke's law.





Maximum Moment and Stress Distribution

In a member of constant cross section, the maximum bending moment will govern the design of the section size when we know what kind of normal stress is caused by it.

For internal equilibrium to be maintained, the bending moment will be equal to the ΣM from the normal stresses $\times$ the areas $\times$ the moment arms. Geometric fit helps solve this statically indeterminate problem:

1. The normal planes remain normal for pure bending.
2. There is no net internal axial force.
3. Stress varies linearly over cross section.
4. Zero stress exists at the centroid and the line of centroids is the *neutral axis* (n. a)

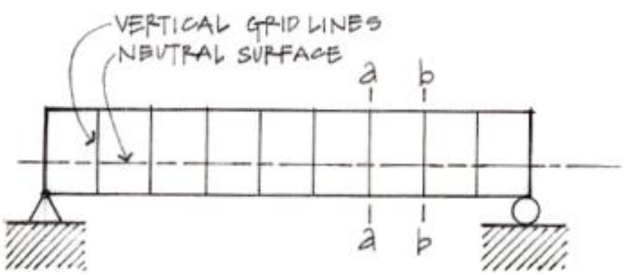
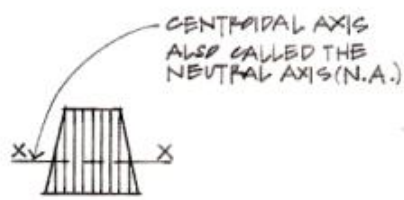


Figure 8.5(a) Beam elevation before loading.



Beam cross section.

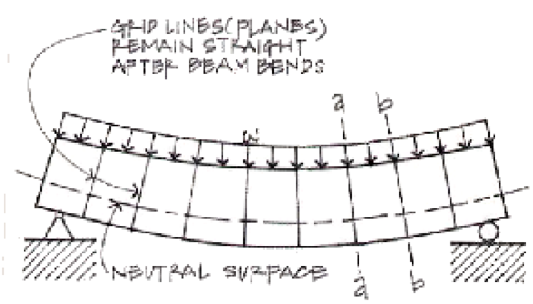


Figure 8.5(b) Beam bending under load.

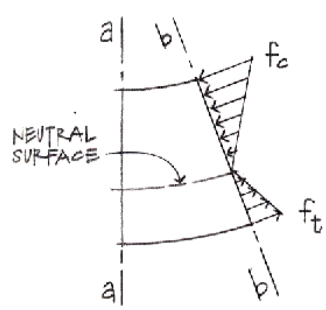
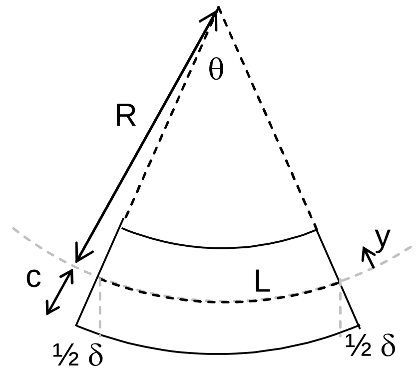


Figure 8.8 Bending stresses on section b-b.



Relations for Beam Geometry and Stress

Pure bending results in a circular arc deflection. R is the distance to the center of the arc; θ is the angle of the arc (radians); c is the distance from the n.a. to the *extreme fiber*; f_{max} is the maximum normal stress at the *extreme fiber*; y is a distance in y from the n.a.; M is the bending moment; I is the moment of in δ zertia; S is the *section modulus.*

$$L = R\theta \qquad \varepsilon = \frac{\delta}{L} = R \qquad f = E\varepsilon = \frac{y}{c} f_{\max}$$

$$M = \Sigma f_i A_i \qquad M = \frac{f_{\max}}{c}\Sigma y_i^2 A_i \qquad I = \Sigma y^2 A \qquad S = \frac{I}{c} \qquad f_{\max} = \frac{Mc}{I} = \frac{M}{S}$$

Now: $f_b = \dfrac{My}{I}$ *for a rectangle of height h and width b:* $S = \dfrac{bh^3}{12\,{}^{h}\!/_{2}} = \dfrac{bh^2}{6}$

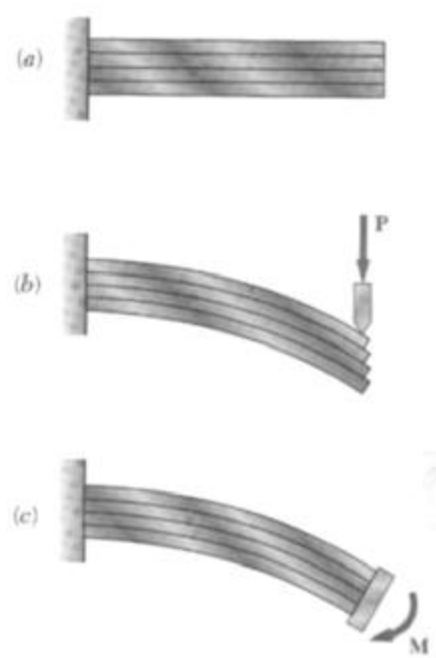
<u>RELATIONS</u>:

$$\boxed{\frac{1}{R} = \frac{M}{EI}} \qquad \boxed{f_b = \frac{My}{I}}^{*} \qquad \boxed{S = \frac{I}{c}}$$

$$\boxed{f_{b-\max} = \frac{Mc}{I} = \frac{M}{S}} \qquad \boxed{S_{required} \geq \frac{M}{F_b}}$$

**Note: y positive goes DOWN. With a positive M and y to the bottom fiber as positive, it results in a TENSION stress (we've called positive).*

Transverse Loading in Beams



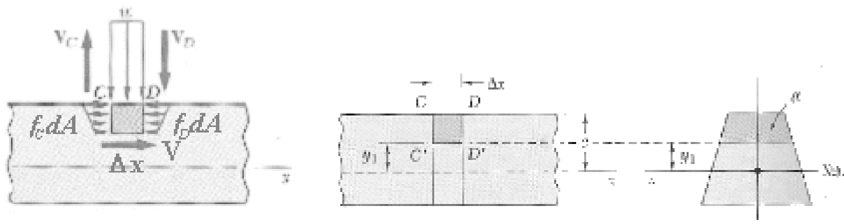
We are aware that transverse beam loadings result in internal shear and bending moments.

We designed sections based on bending stresses, since this stress dominates beam behavior.

There can be shear stresses *horizontally* within a beam member. It can be shown that $f_{horizontal} = f_{vertical}$

Equilibrium and Derivation

In order for equilibrium for any element CDD'C', there needs to be a horizontal force ΔH.



$$V = f_D dA - f_C dA$$

Q is a moment area with respect to the neutral axis of the area *above or below* the horizontal where the ΔH occurs.

Q is a maximum when y = 0 (at the **neutral axis**).

$$V_{longitudinal} = \frac{V_T Q}{I}\Delta x$$

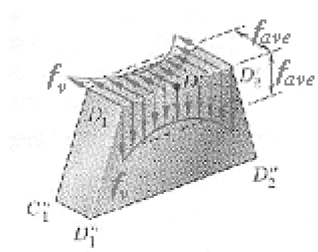
q is a horizontal shear per unit length → *shear flow*

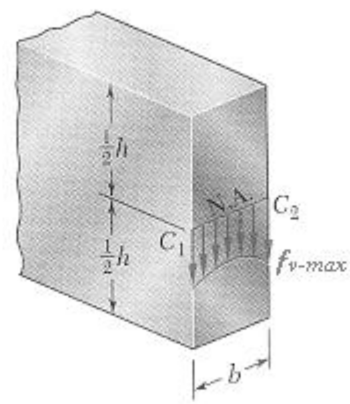
$$q = \frac{V_{longitudinal}}{\Delta x} = \frac{V_T Q}{I}$$

Shearing Stresses

f_{v-ave} = 0 on the beam's surface. Even if Q is a maximum at y = 0, we don't know that the thickness is a *minimum* there.

$$f_v = \frac{V}{\Delta A} = \frac{V}{b \cdot \Delta x} \qquad \boxed{f_{v-ave} = \frac{VQ}{Ib}}$$





Rectangular Sections

$f_{v-\max}$ occurs at the neutral axis:

$$I = \frac{bh^3}{12} \qquad Q = A\bar{y} = b\,{}^{h}\!/_{2} \cdot {}^{1}\!/_{2}\,{}^{h}\!/_{2} = {}^{bh^2}\!/_{8}$$

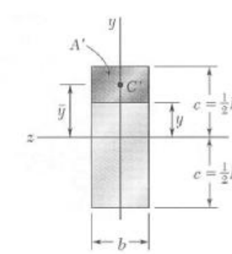
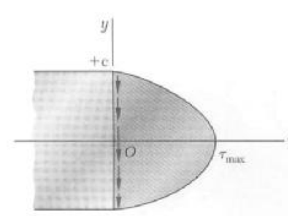


Fig. 6.15

then:

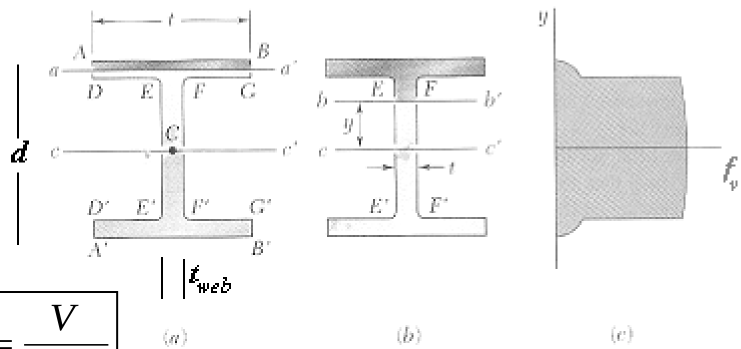
$$f_v = \frac{VQ}{Ib} = \frac{V\,{}^{1}\!/_{8}\,bh^2}{{}^{1}\!/_{12}\,bh^3 b} = \frac{3V}{2bh} \qquad \boxed{f_v = \frac{3V}{2A}}$$



Webs of Beams

In steel W or S sections the thickness varies from the flange to the web.

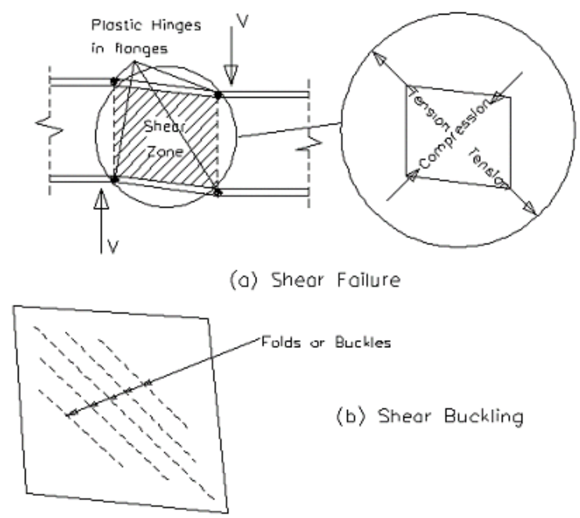
We neglect the shear stress in the flanges and consider the shear stress in the web to be constant:

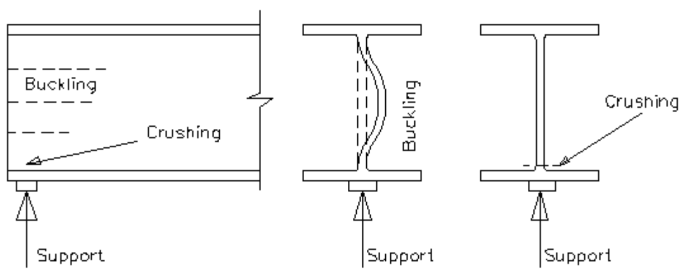


$$f_{v-\max} = \frac{3V}{2A} \approx \frac{V}{A_{web}}$$

$$\boxed{f_{v-\max} = \frac{V}{t_{web}d}}$$

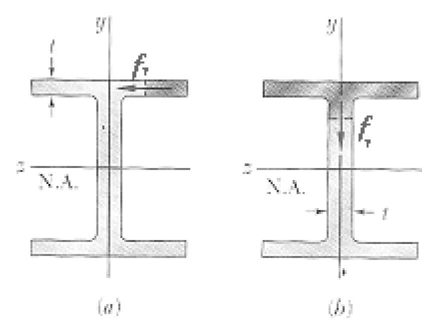
Webs of I beams can fail in tension shear across a panel with stiffeners or the web can buckle.



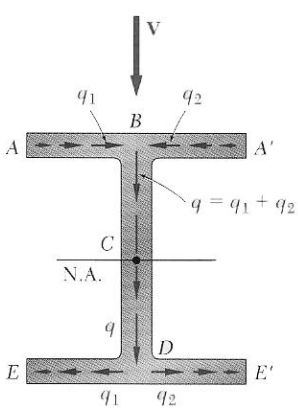


Shear Flow

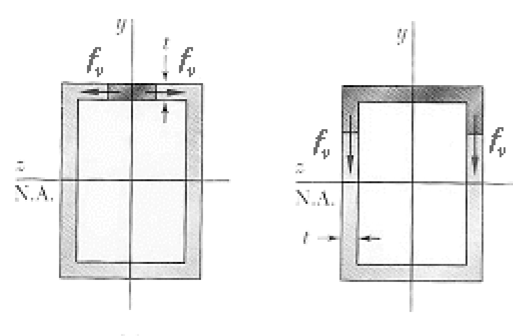
Even if the cut we make to find Q is not horizontal, but arbitrary, we can still find the shear flow, q, as long as the loads on thin-walled sections are applied in a plane of symmetry, and the cut is made *perpendicular* to the surface of the member.



$$q = \frac{VQ}{I}$$



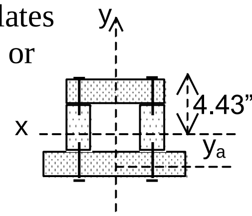
The shear flow magnitudes can be sketched by knowing Q.

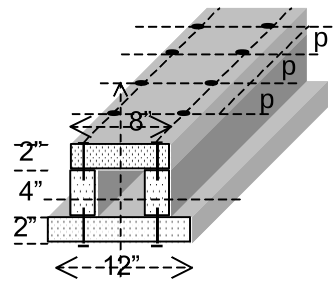


Connectors to Resist Horizontal Shear in Composite Members

Typical connections needing to resist shear are plates with nails or rivets or bolts in composite sections or splices.

The pitch (spacing) can be determined by the capacity in shear of the connector(s) to the shear flow over the spacing interval, p.





$$\frac{V_{longitudinal}}{p} = \frac{VQ}{I} \qquad V_{longitudinal} = \frac{VQ}{I} \cdot p$$

where

p = pitch length

$$nF_{connector} \geq \frac{VQ_{connected\,area}}{I} \cdot p$$

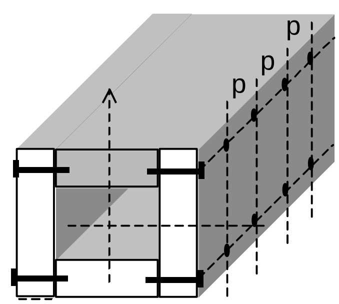
n = number of connectors connecting the connected area to the rest of the cross section

F = force capacity in one connector

$Q_{connected\ area} = A_{connected\ area} \times y_{connected\ area}$

$y_{connected\ area}$ = distance from the centroid of the connected area to the neutral axis

Connectors to Resist Horizontal Shear in Composite Members



Even vertical connectors have shear flow across them.

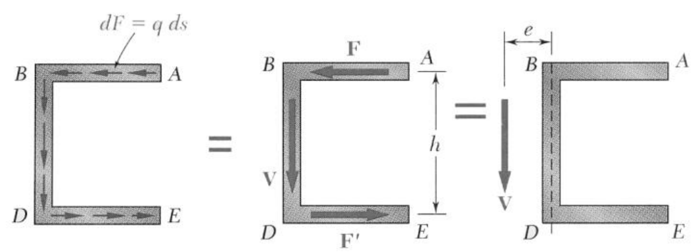
The spacing can be determined by the capacity in shear of the connector(s) to the shear flow over the spacing interval, p.

$$p \leq \frac{nF_{connector} I}{VQ_{connected\,area}}$$

Unsymmetrical Sections or Shear

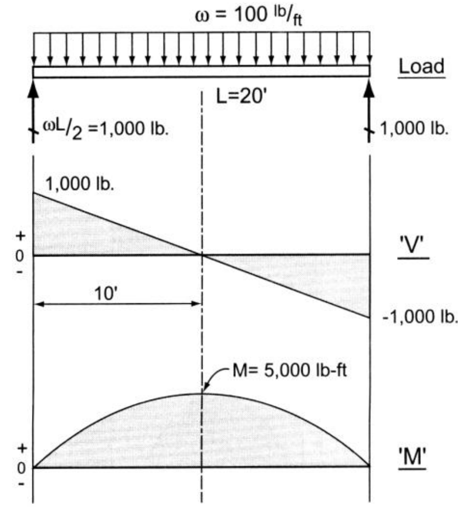
If the section is <u>not</u> symmetric, or has a shear <u>not</u> in that plane, the member can bend and twist.

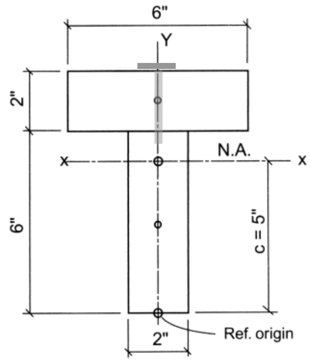
If the load is applied at the *shear center* there will not be twisting. This is the location where the moment caused by shear flow = the moment of the shear force about the shear center.



Example 1

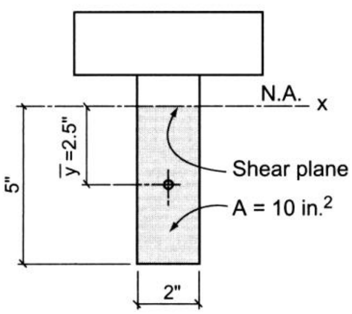
Calculate the maximum bending and shear stress for the beam shown.
ALSO: Determine the minimum nail spacing required (pitch) if the shear capacity of a nail (F) is 250 lb.

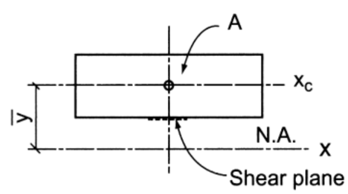




Component	A (in.^{2})	$\overline{y}$ (in.)	$\overline{y}\Delta A$ (in.^{3})
	12	7	84
	12	3	36

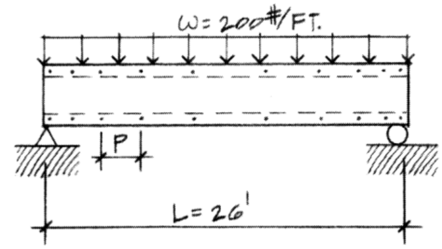
Component	I_{xc} (in.^{4})	A (in.^{2})	d_y (in.)	Ad_y^2 (in.^{4})
	4	12	2	48
	36	12	2	48





Example 2

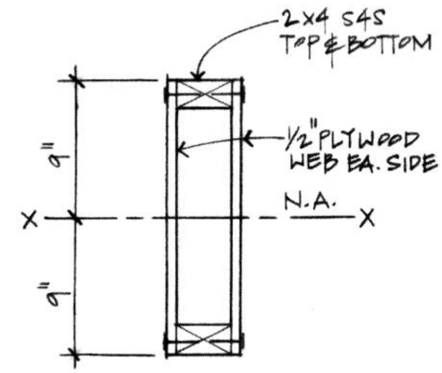
8.11 A built-up plywood box beam with 2×4 S4S top and bottom flanges is held together by nails. Determine the pitch (spacing) of the nails if the beam supports a uniform load of 200 #/ft. along the 26-foot span. Assume the nails have a shear capacity of 80# each.



Solution:

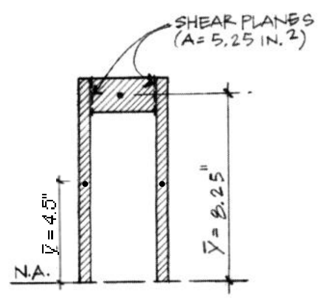
Construct the shear (V) diagram to obtain the critical shear condition and its location

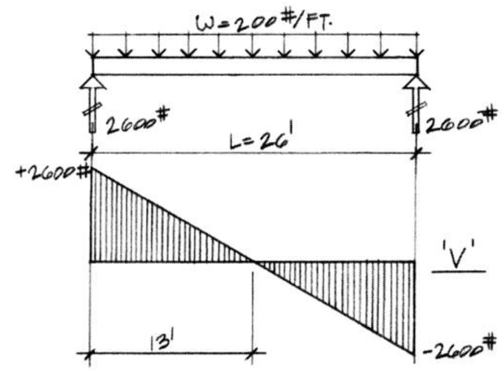
Note that the condition of shear is critical at the supports, and the shear intensity decreases as you approach the center line of the beam. This would indicate that the nail spacing P varies from the support to midspan. Nails are closely spaced at the support, but increasing spacing occurs toward midspan, following the shear diagram.



$$f_v = \frac{VQ}{Ib}$$

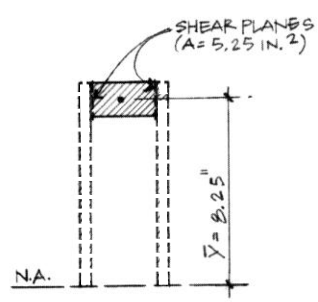
$$I_x = \frac{(4.5'')(18'')^3}{12} - \frac{(3.5'')(15'')^3}{12} = 1{,}202.6 \text{ in.}^4$$





$Q = \Sigma A\bar{y}$ = (9")(½")(4.5")+(9")(½")(4.5")+(1.5")(3.5")(8.25") = 83.8 in^3

$$f_{v-\max} = \frac{(2{,}600\#)(83.3 in.^3)}{(1{,}202.6 in.^4)(\frac{1}{2}"+\frac{1}{2}")} = 180.2\, psi$$



$$Q = A\bar{y} = (5.25 \text{ in.}^2)(8.25'') = 43.3 \text{ in.}^3$$

Shear force $= f_v \times A_v$

where:

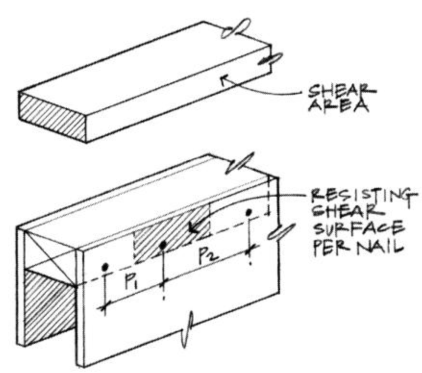
A_v = shear area

Assume:

F = Capacity of two nails (one each side) at the flange; representing two shear surfaces

$$(\text{n})F \geq f_v \times b \times p = \frac{VQ}{Ib} \times bp$$

$$\therefore\ (\text{n})F \geq p \times \frac{VQ}{I}; \quad p \leq \frac{(\text{n})FI}{VQ}$$



At the maximum shear location (support) where V = 2,600#

$$p \leq \frac{(2 \text{ nails} \times 80\ \#/\text{nail})(1{,}202.6 \text{ in.}^4)}{(2{,}600\#)(43.3 \text{ in.}^3)} = 1.71''$$