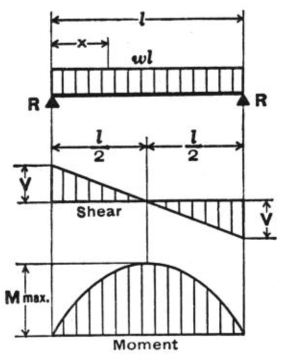
BEAM DIAGRAMS AND FORMULAS For Various Static Loading Conditions, AISC ASD 8th ed.

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . $= wl$

$R = V$ . . . . $= \frac{wl}{2}$

V_x . . . . $= w\left(\frac{l}{2} - x\right)$

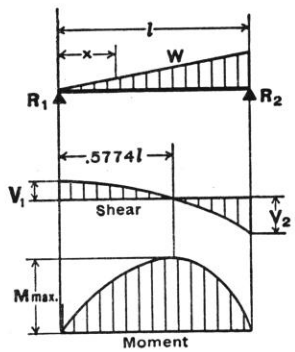
M max. (at center) . . . . $= \frac{wl^2}{8}$

M_x . . . . $= \frac{wx}{2}(l - x)$

Δmax. (at center) . . . . $= \frac{5wl^4}{384EI}$

Δ_x . . . . $= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END



Total Equiv. Uniform Load . . . . $= \frac{16W}{9\sqrt{3}} = 1.0264W$

$R_1 = V_1$ . . . . $= \frac{W}{3}$

$R_2 = V_2$ max. . . . . $= \frac{2W}{3}$

V_x . . . . $= \frac{W}{3} - \frac{Wx^2}{l^2}$

M max. $\left(\text{at } x = \frac{l}{\sqrt{3}} = .5774l\right)$ . . $= \frac{2Wl}{9\sqrt{3}} = .1283\,Wl$

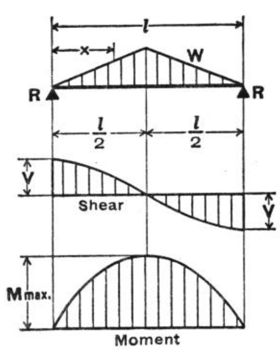
M_x . . . . $= \frac{Wx}{3l^2}(l^2 - x^2)$

Δmax. $\left(\text{at } x = l\sqrt{1 - \sqrt{\frac{8}{15}}} = .5193l\right) = .01304\frac{Wl^3}{EI}$

Δ_x . . . . $= \frac{Wx}{180EIl^2}(3x^4 - 10l^2x^2 + 7l^4)$

$W = \frac{wl}{2}$

3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER



Total Equiv. Uniform Load . . . . $= \frac{4W}{3}$

$R = V$ . . . . $= \frac{W}{2}$

V_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . $= \frac{W}{2l^2}(l^2 - 4x^2)$

M max. (at center) . . . . $= \frac{Wl}{6}$

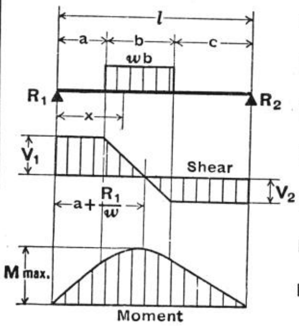
M_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . $= Wx\left(\frac{1}{2} - \frac{2x^2}{3l^2}\right)$

Δmax. (at center) . . . . $= \frac{Wl^3}{60EI}$

Δ_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . $= \frac{Wx}{480EIl^2}(5l^2 - 4x^2)^2$

$W = \frac{wl}{2}$

4. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED



$R_1 = V_1$ (max. when $a < c$) . . $= \frac{wb}{2l}(2c + b)$

$R_2 = V_2$ (max. when $a > c$) . . $= \frac{wb}{2l}(2a + b)$

V_x (when $x > a$ and $< (a + b)$) . $= R_1 - w(x - a)$

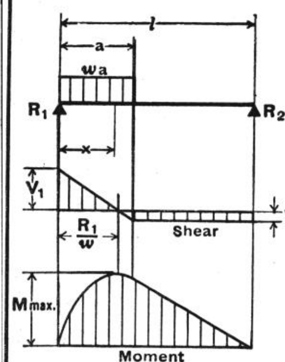
M max. $\left(\text{at } x = a + \frac{R_1}{w}\right)$ . . . . $= R_1\left(a + \frac{R_1}{2w}\right)$

M_x (when $x < a$) . . . . . . $= R_1x$

M_x (when $x > a$ and $< (a + b)$) . $= R_1x - \frac{w}{2}(x - a)^2$

M_x (when $x > (a + b)$) . . . . $= R_2(l - x)$

5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END



$R_1 = V_1$ max. . . . . $= \frac{wa}{2l}(2l - a)$

$R_2 = V_2$ . . . . $= \frac{wa^2}{2l}$

V_x (when $x < a$) . . . . . $= R_1 - wx$

M max. $\left(\text{at } x = \frac{R_1}{w}\right)$ . . . . . $= \frac{R_1^2}{2w}$

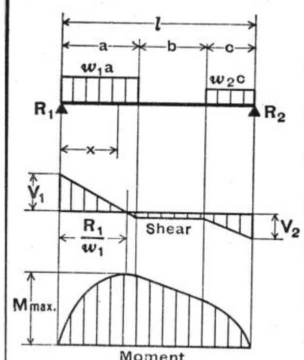
M_x (when $x < a$) . . . . . $= R_1x - \frac{wx^2}{2}$

M_x (when $x > a$) . . . . . $= R_2(l - x)$

Δ_x (when $x < a$) . . . . . $= \frac{wx}{24EIl}\left(a^2(2l - a)^2 - 2ax^2(2l - a) + lx^3\right)$

Δ_x (when $x > a$) . . . . . $= \frac{wa^2(l - x)}{24EIl}(4xl - 2x^2 - a^2)$

6. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END



$R_1 = V_1$ . . . . $= \frac{w_1a(2l - a) + w_2c^2}{2l}$

$R_2 = V_2$ . . . . $= \frac{w_2c(2l - c) + w_1a^2}{2l}$

V_x (when $x < a$) . . . . . . $= R_1 - w_1x$

V_x (when $x > a$ and $< (a + b)$) . $= R_1 - w_1a$

V_x (when $x > (a + b)$) . . . . $= R_2 - w_2(l - x)$

M max. $\left(\text{at } x = \frac{R_1}{w_1} \text{ when } R_1 < w_1a\right)$. $= \frac{R_1^2}{2w_1}$

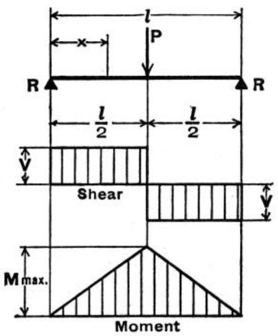
M max. $\left(\text{at } x = l - \frac{R_2}{w_2} \text{ when } R_2 < w_2c\right) = \frac{R_2^2}{2w_2}$

M_x (when $x < a$) . . . . . . $= R_1x - \frac{w_1x^2}{2}$

M_x (when $x > a$ and $< (a + b)$) . $= R_1x - \frac{w_1a}{2}(2x - a)$

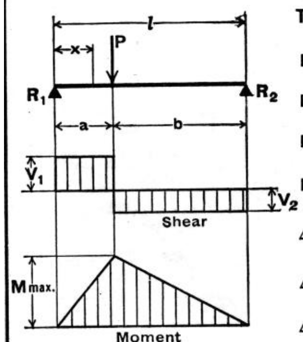
M_x (when $x > (a + b)$) . . . . $= R_2(l - x) - \frac{w_2(l - x)^2}{2}$

7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER



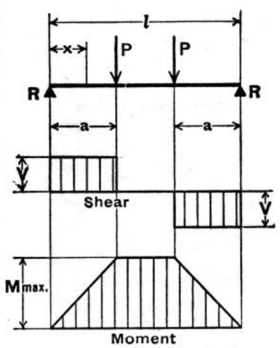
Total Equiv. Uniform Load		$= 2P$
$R = V$		$= \frac{P}{2}$
M max.	(at point of load)	$= \frac{Pl}{4}$
M_x	(when $x < \frac{l}{2}$)	$= \frac{Px}{2}$
Δmax.	(at point of load)	$= \frac{Pl^3}{48EI}$
Δ_x	(when $x < \frac{l}{2}$)	$= \frac{Px}{48EI}(3l^2 - 4x^2)$

8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT



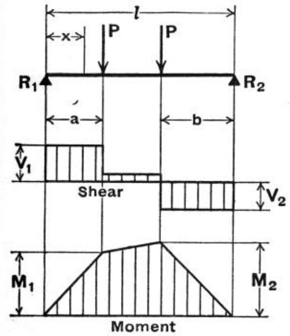
Total Equiv. Uniform Load		$= \frac{8\,Pab}{l^2}$
$R_1 = V_1$	(max. when $a < b$)	$= \frac{Pb}{l}$
$R_2 = V_2$	(max. when $a > b$)	$= \frac{Pa}{l}$
M max.	(at point of load)	$= \frac{Pab}{l}$
M_x	(when $x < a$)	$= \frac{Pbx}{l}$
Δmax.	(at $x = \sqrt{\frac{a(a+2b)}{3}}$ when $a > b$)	$= \frac{Pab\,(a+2b)\,\sqrt{3a\,(a+2b)}}{27\,EI\,l}$
Δa	(at point of load)	$= \frac{Pa^2b^2}{3EI\,l}$
Δ_x	(when $x < a$)	$= \frac{Pbx}{6EI\,l}(l^2 - b^2 - x^2)$

9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED



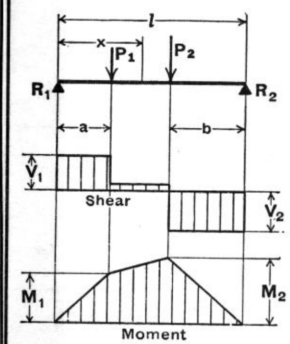
Total Equiv. Uniform Load		$= \frac{8\,Pa}{l}$
$R = V$		$= P$
M max.	(between loads)	$= Pa$
M_x	(when $x < a$)	$= Px$
Δmax.	(at center)	$= \frac{Pa}{24EI}(3l^2 - 4a^2)$
Δ_x	(when $x < a$)	$= \frac{Px}{6EI}(3la - 3a^2 - x^2)$
Δ_x	(when $x > a$ and $< (l - a)$)	$= \frac{Pa}{6EI}(3lx - 3x^2 - a^2)$

10. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



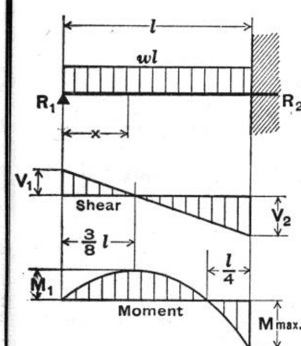
$R_1 = V_1$	(max. when $a < b$)	$= \frac{P}{l}(l - a + b)$
$R_2 = V_2$	(max. when $a > b$)	$= \frac{P}{l}(l - b + a)$
V_x	(when $x > a$ and $< (l - b)$)	$= \frac{P}{l}(b - a)$
M_1	(max. when $a > b$)	$= R_1 a$
M_2	(max. when $a < b$)	$= R_2 b$
M_x	(when $x < a$)	$= R_1 x$
M_x	(when $x > a$ and $< (l - b)$)	$= R_1 x - P(x - a)$

11. SIMPLE BEAM—TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



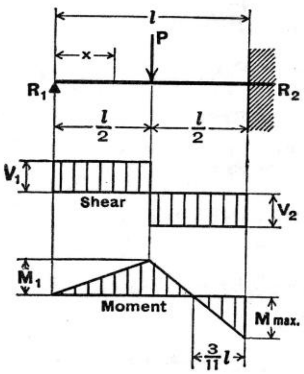
$R_1 = V_1$		$= \frac{P_1(l - a) + P_2 b}{l}$
$R_2 = V_2$		$= \frac{P_1 a + P_2(l - b)}{l}$
V_x	(when $x > a$ and $< (l - b)$)	$= R_1 - P_1$
M_1	(max. when $R_1 < P_1$)	$= R_1 a$
M_2	(max. when $R_2 < P_2$)	$= R_2 b$
M_x	(when $x < a$)	$= R_1 x$
M_x	(when $x > a$ and $< (l - b)$)	$= R_1 x - P_1(x - a)$

12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—UNIFORMLY DISTRIBUTED LOAD



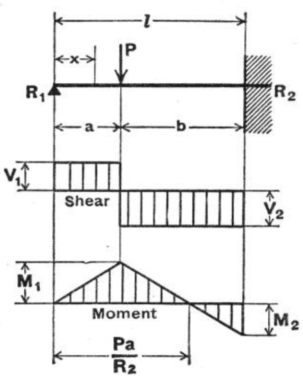
Total Equiv. Uniform Load		$= wl$
$R_1 = V_1$		$= \frac{3wl}{8}$
$R_2 = V_2$ max.		$= \frac{5wl}{8}$
V_x		$= R_1 - wx$
M max.		$= \frac{wl^2}{8}$
M_1	(at $x = \frac{3}{8}l$)	$= \frac{9}{128}wl^2$
M_x		$= R_1 x - \frac{wx^2}{2}$
Δmax.	(at $x = \frac{l}{16}(1 + \sqrt{33}) = .4215l$)	$= \frac{wl^4}{185EI}$
Δ_x		$= \frac{wx}{48EI}(l^3 - 3lx^2 + 2x^3)$

13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT CENTER



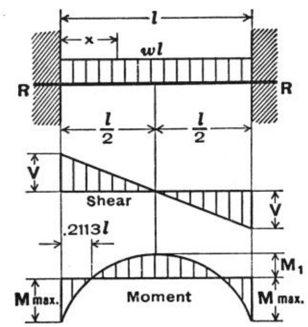
Total Equiv. Uniform Load	$= \frac{3P}{2}$
$R_1 = V_1$	$= \frac{5P}{16}$
$R_2 = V_2$ max.	$= \frac{11P}{16}$
M max. (at fixed end)	$= \frac{3Pl}{16}$
M_1 (at point of load)	$= \frac{5Pl}{32}$
M_x $\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{5Px}{16}$
M_x $\left(\text{when } x > \frac{l}{2}\right)$	$= P\left(\frac{l}{2} - \frac{11x}{16}\right)$
Δmax. $\left(\text{at } x = l\sqrt{\frac{1}{5}} = .4472l\right)$	$= \frac{Pl^3}{48EI\sqrt{5}} = .009317\frac{Pl^3}{EI}$
Δx (at point of load)	$= \frac{7Pl^3}{768EI}$
Δx $\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{Px}{96EI}(3l^2 - 5x^2)$
Δx $\left(\text{when } x > \frac{l}{2}\right)$	$= \frac{P}{96EI}(x-l)^2(11x - 2l)$

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT ANY POINT



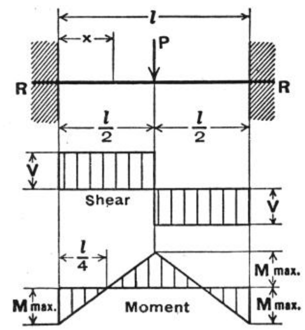
$R_1 = V_1$	$= \frac{Pb^2}{2l^3}(a + 2l)$
$R_2 = V_2$	$= \frac{Pa}{2l^3}(3l^2 - a^2)$
M_1 (at point of load)	$= R_1 a$
M_2 (at fixed end)	$= \frac{Pab}{2l^2}(a + l)$
M_x (when $x < a$)	$= R_1 x$
M_x (when $x > a$)	$= R_1 x - P(x - a)$
Δmax. $\left(\text{when } a < .414l \text{ at } x = l\frac{l^2+a^2}{3l^2-a^2}\right)$	$= \frac{Pa}{3EI}\frac{(l^2-a^2)^3}{(3l^2-a^2)^2}$
Δmax. $\left(\text{when } a > .414l \text{ at } x = l\sqrt{\frac{a}{2l+a}}\right)$	$= \frac{Pab^2}{6EI}\sqrt{\frac{a}{2l+a}}$
Δa (at point of load)	$= \frac{Pa^2b^3}{12EIl^3}(3l + a)$
Δx (when $x < a$)	$= \frac{Pb^2x}{12EIl^3}(3al^2 - 2lx^2 - ax^2)$
Δx (when $x > a$)	$= \frac{Pa}{12EIl^3}(l-x)^2(3l^2x - a^2x - 2a^2l)$

15. BEAM FIXED AT BOTH ENDS—UNIFORMLY DISTRIBUTED LOADS



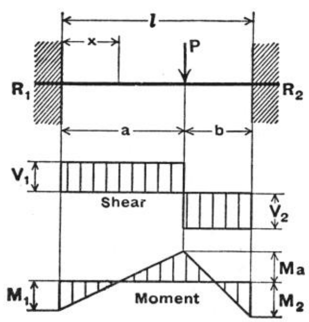
Total Equiv. Uniform Load	$= \frac{2wl}{3}$
$R = V$	$= \frac{wl}{2}$
V_x	$= w\left(\frac{l}{2} - x\right)$
M max. (at ends)	$= \frac{wl^2}{12}$
M_1 (at center)	$= \frac{wl^2}{24}$
M_x	$= \frac{w}{12}(6lx - l^2 - 6x^2)$
Δmax. (at center)	$= \frac{wl^4}{384EI}$
Δx	$= \frac{wx^2}{24EI}(l - x)^2$

16. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT CENTER



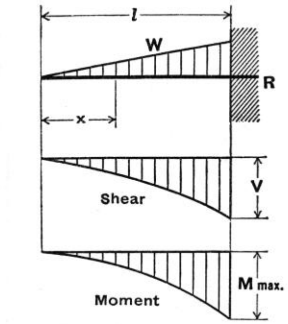
Total Equiv. Uniform Load	$= P$
$R = V$	$= \frac{P}{2}$
M max. (at center and ends)	$= \frac{Pl}{8}$
M_x $\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{P}{8}(4x - l)$
Δmax. (at center)	$= \frac{Pl^3}{192EI}$
Δx $\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{Px^2}{48EI}(3l - 4x)$

17. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$ (max. when $a < b$)	$= \frac{Pb^2}{l^3}(3a + b)$
$R_2 = V_2$ (max. when $a > b$)	$= \frac{Pa^2}{l^3}(a + 3b)$
M_1 (max. when $a < b$)	$= \frac{Pab^2}{l^2}$
M_2 (max. when $a > b$)	$= \frac{Pa^2b}{l^2}$
M_a (at point of load)	$= \frac{2Pa^2b^2}{l^3}$
M_x (when $x < a$)	$= R_1 x - \frac{Pab^2}{l^2}$
Δmax. $\left(\text{when } a > b \text{ at } x = \frac{2al}{3a+b}\right)$	$= \frac{2Pa^3b^2}{3EI\,(3a+b)^2}$
Δa (at point of load)	$= \frac{Pa^3b^3}{3EIl^3}$
Δx (when $x < a$)	$= \frac{Pb^2x^2}{6EIl^3}(3al - 3ax - bx)$

18. CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END



Total Equiv. Uniform Load . . . . . $= \frac{8}{3}W$

R = V . . . . $= W$ $\qquad W = \frac{wl}{2}$

V_x . . . . $= W\frac{x^2}{l^2}$

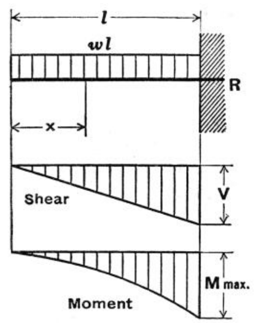
M max. (at fixed end) . . . . . . $= \frac{Wl}{3}$

M_x . . . . $= \frac{Wx^3}{3l^2}$

Δmax. (at free end) . . . . . . $= \frac{Wl^3}{15EI}$

Δx . . . . $= \frac{W}{60EIl^2}(x^5 - 5l^4x + 4l^5)$

19. CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . . $= 4wl$

R = V . . . . $= wl$

V_x . . . . $= wx$

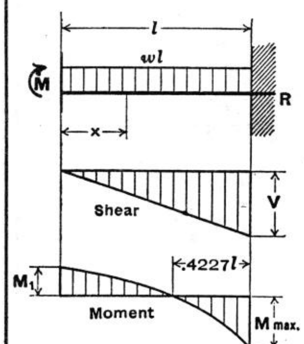
M max. (at fixed end) . . . . . . $= \frac{wl^2}{2}$

M_x . . . . $= \frac{wx^2}{2}$

Δmax. (at free end) . . . . . . $= \frac{wl^4}{8EI}$

Δx . . . . $= \frac{w}{24EI}(x^4 - 4l^3x + 3l^4)$

20. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . . $= \frac{8}{3}wl$

R = V . . . . $= wl$

V_x . . . . $= wx$

M max. (at fixed end) . . . . . . $= \frac{wl^2}{3}$

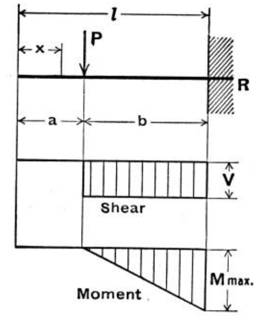
M_1 (at deflected end) . . . . . $= \frac{wl^2}{6}$

M_x . . . . $= \frac{w}{6}(l^2 - 3x^2)$

Δmax. (at deflected end) . . . . $= \frac{wl^4}{24EI}$

Δx . . . . $= \frac{w(l^2 - x^2)^2}{24EI}$

21. CANTILEVER BEAM—CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load . . . . . $= \frac{8Pb}{l}$

R = V . . . . $= P$

M max. (at fixed end) . . . . . . $= Pb$

M_x (when $x > a$) . . . . . . $= P(x - a)$

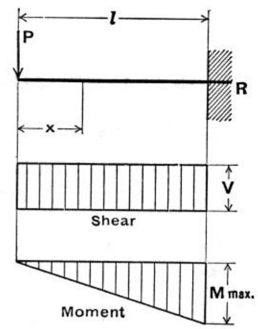
Δmax. (at free end) . . . . . $= \frac{Pb^2}{6EI}(3l - b)$

Δa (at point of load) . . . . . $= \frac{Pb^3}{3EI}$

Δx (when $x < a$) . . . . . . $= \frac{Pb^2}{6EI}(3l - 3x - b)$

Δx (when $x > a$) . . . . . . $= \frac{P(l - x)^2}{6EI}(3b - l + x)$

22. CANTILEVER BEAM—CONCENTRATED LOAD AT FREE END



Total Equiv. Uniform Load . . . . . $= 8P$

R = V . . . . $= P$

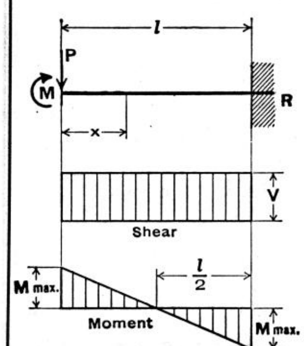
M max. (at fixed end) . . . . . . $= Pl$

M_x . . . . $= Px$

Δmax. (at free end) . . . . . $= \frac{Pl^3}{3EI}$

Δx . . . . $= \frac{P}{6EI}(2l^3 - 3l^2x + x^3)$

23. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—CONCENTRATED LOAD AT DEFLECTED END



Total Equiv. Uniform Load . . . . . $= 4P$

R = V . . . . $= P$

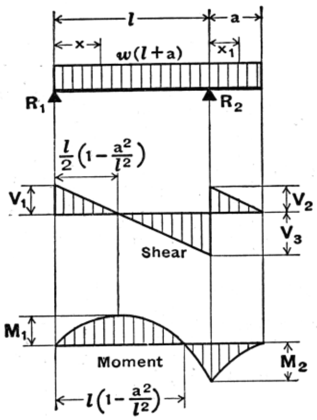
M max. (at both ends) . . . . . $= \frac{Pl}{2}$

M_x . . . . $= P\left(\frac{l}{2} - x\right)$

Δmax. (at deflected end) . . . . $= \frac{Pl^3}{12EI}$

Δx . . . . $= \frac{P(l - x)^2}{12EI}(l + 2x)$

24. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD



$R_1 = V_1$ . . . . $= \frac{w}{2l}(l^2 - a^2)$

$R_2 = V_2 + V_3$ . . . . $= \frac{w}{2l}(l + a)^2$

V_2 . . . . $= wa$

V_3 . . . . $= \frac{w}{2l}(l^2 + a^2)$

V_x (between supports) . . . $= R_1 - wx$

V_{x_1} (for overhang) . . . . $= w(a - x_1)$

M_1 $\left(\text{at } x = \frac{l}{2}\left[1 - \frac{a^2}{l^2}\right]\right)$ . . . $= \frac{w}{8l^2}(l + a)^2(l - a)^2$

M_2 (at R_2) . . . . . . $= \frac{wa^2}{2}$

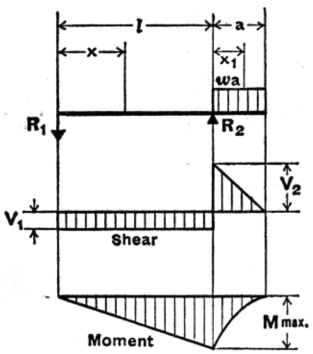
M_x (between supports) . . . $= \frac{wx}{2l}(l^2 - a^2 - xl)$

M_{x_1} (for overhang) . . . . $= \frac{w}{2}(a - x_1)^2$

Δx (between supports) . . . $= \frac{wx}{24EIl}(l^4 - 2l^2x^2 + lx^3 - 2a^2l^2 + 2a^2x^2)$

Δx_1 (for overhang) . . . . $= \frac{wx_1}{24EI}(4a^2l - l^3 + 6a^2x_1 - 4ax_1^2 + x_1^3)$

25. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD ON OVERHANG



$R_1 = V_1$ . . . . $= \frac{wa^2}{2l}$

$R_2 = V_1 + V_2$ . . . . $= \frac{wa}{2l}(2l + a)$

V_2 . . . . $= wa$

V_{x_1} (for overhang) . . . . $= w(a - x_1)$

M max. (at R_2) . . . . . . $= \frac{wa^2}{2}$

M_x (between supports) . . . $= \frac{wa^2x}{2l}$

M_{x_1} (for overhang) . . . . $= \frac{w}{2}(a - x_1)^2$

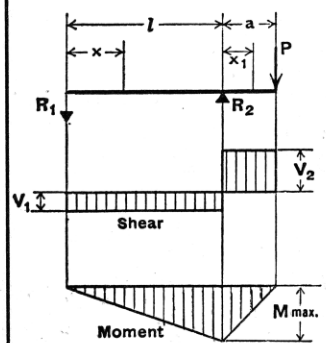
Δmax. $\left(\text{between supports at } x = \frac{l}{\sqrt{3}}\right) = \frac{wa^2l^2}{18\sqrt{3}EI} = .03208\frac{wa^2l^2}{EI}$

Δmax. (for overhang at $x_1 = a$) . $= \frac{wa^3}{24EI}(4l + 3a)$

Δx (between supports) . . $= \frac{wa^2x}{12EIl}(l^2 - x^2)$

Δx_1 (for overhang) . . . . $= \frac{wx_1}{24EI}(4a^2l + 6a^2x_1 - 4ax_1^2 + x_1^3)$

26. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT END OF OVERHANG



$R_1 = V_1$ . . . . $= \frac{Pa}{l}$

$R_2 = V_1 + V_2$ . . . . $= \frac{P}{l}(l + a)$

V_2 . . . . $= P$

M max. (at R_2) . . . . $= Pa$

M_x (between supports) . . . $= \frac{Pax}{l}$

M_{x_1} (for overhang) . . . . $= P(a - x_1)$

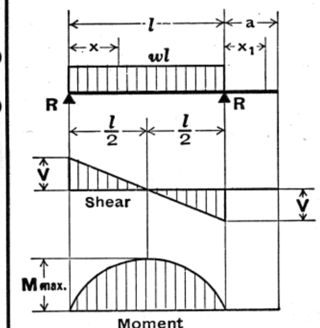
Δmax. $\left(\text{between supports at } x = \frac{l}{\sqrt{3}}\right) = \frac{Pal^2}{9\sqrt{3}EI} = .06415\frac{Pal^2}{EI}$

Δmax. (for overhang at $x_1 = a$) . $= \frac{Pa^2}{3EI}(l + a)$

Δx (between supports) . . $= \frac{Pax}{6EIl}(l^2 - x^2)$

Δx_1 (for overhang) . . . . $= \frac{Px_1}{6EI}(2al + 3ax_1 - x_1^2)$

27. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD BETWEEN SUPPORTS



Total Equiv. Uniform Load . . . $= wl$

$R = V$ . . . . $= \frac{wl}{2}$

V_x . . . . $= w\left(\frac{l}{2} - x\right)$

M max. (at center) . . . . . $= \frac{wl^2}{8}$

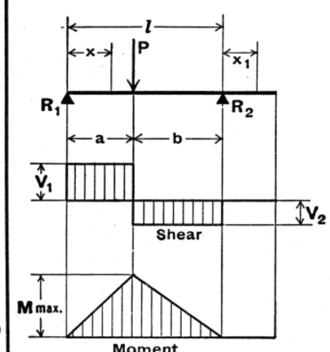
M_x . . . . $= \frac{wx}{2}(l - x)$

Δmax. (at center) . . . . . $= \frac{5wl^4}{384EI}$

Δx . . . . $= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

Δx_1 . . . . $= \frac{wl^3x_1}{24EI}$

28. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT ANY POINT BETWEEN SUPPORTS



Total Equiv. Uniform Load . . . $= \frac{8Pab}{l^2}$

$R_1 = V_1$ (max. when $a < b$) . . . $= \frac{Pb}{l}$

$R_2 = V_2$ (max. when $a > b$) . . . $= \frac{Pa}{l}$

M max. (at point of load) . . . $= \frac{Pab}{l}$

M_x (when $x < a$) . . . . $= \frac{Pbx}{l}$

Δmax. $\left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b\right) = \frac{Pab(a + 2b)\sqrt{3a(a + 2b)}}{27EIl}$

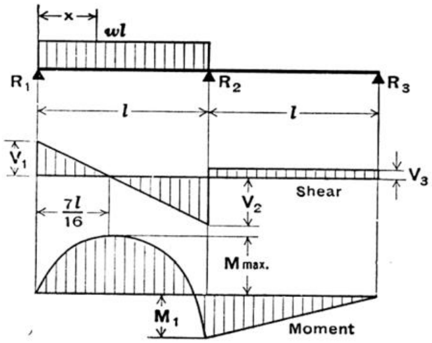
Δa (at point of load) . . . $= \frac{Pa^2b^2}{3EIl}$

Δx (when $x < a$) . . . . $= \frac{Pbx}{6EIl}(l^2 - b^2 - x^2)$

Δx (when $x > a$) . . . . $= \frac{Pa(l - x)}{6EIl}(2lx - x^2 - a^2)$

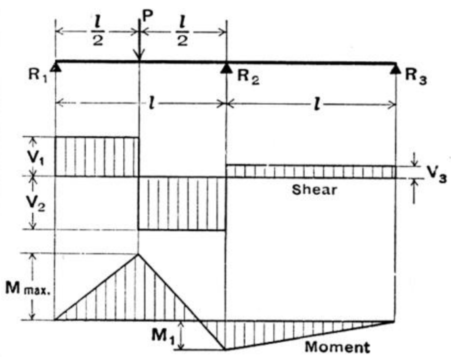
Δx_1 . . . . $= \frac{Pabx_1}{6EIl}(l + a)$

29. CONTINUOUS BEAM—TWO EQUAL SPANS—UNIFORM LOAD ON ONE SPAN



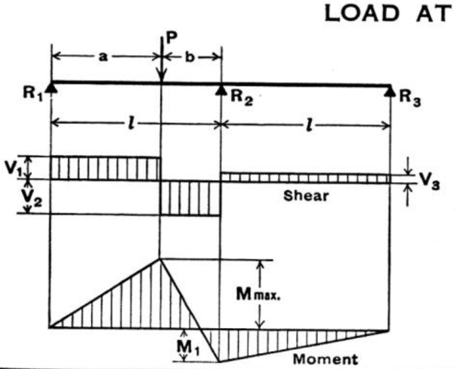
Total Equiv. Uniform Load	=	$\frac{49}{64} wl$
$R_1 = V_1$	=	$\frac{7}{16} wl$
$R_2 = V_2 + V_3$	=	$\frac{5}{8} wl$
$R_3 = V_3$	=	$-\frac{1}{16} wl$
V_2	=	$\frac{9}{16} wl$
M max. $\left(\text{at } x = \frac{7}{16} l\right)$ . . .	=	$\frac{49}{512} wl^2$
M_1 (at support R_2) . .	=	$\frac{1}{16} wl^2$
M_x (when $x < l$) . .	=	$\frac{wx}{16}(7l - 8x)$
Δ Max. (0.472 l from R_1)	=	$0.0092\ wl^4/EI$

30. CONTINUOUS BEAM—TWO EQUAL SPANS—CONCENTRATED LOAD AT CENTER OF ONE SPAN



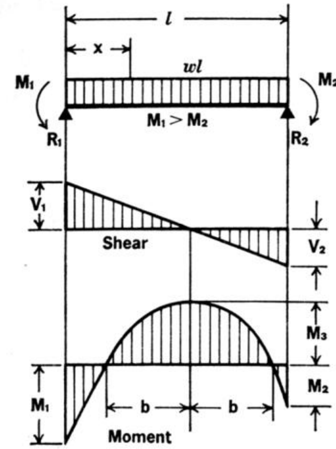
Total Equiv. Uniform Load .	=	$\frac{13}{8} P$
$R_1 = V_1$	=	$\frac{13}{32} P$
$R_2 = V_2 + V_3$	=	$\frac{11}{16} P$
$R_3 = V_3$	=	$-\frac{3}{32} P$
V_2	=	$\frac{19}{32} P$
M max. (at point of load) .	=	$\frac{13}{64} Pl$
M_1 (at support R_2) .	=	$\frac{3}{32} Pl$
Δ Max. (0.480 l from R_1)	=	$0.015\ Pl^3/EI$

31. CONTINUOUS BEAM—TWO EQUAL SPANS—CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$	=	$\frac{Pb}{4l^3}\left(4l^2 - a(l+a)\right)$
$R_2 = V_2 + V_3$	=	$\frac{Pa}{2l^3}\left(2l^2 + b(l+a)\right)$
$R_3 = V_3$	=	$-\frac{Pab}{4l^3}(l+a)$
V_2	=	$\frac{Pa}{4l^3}\left(4l^2 + b(l+a)\right)$
M max. (at point of load) .	=	$\frac{Pab}{4l^3}\left(4l^2 - a(l+a)\right)$
M_1 (at support R_2) .	=	$\frac{Pab}{4l^2}(l+a)$

32. BEAM—UNIFORMLY DISTRIBUTED LOAD AND VARIABLE END MOMENTS



$$R_1 = V_1 = \frac{wl}{2} + \frac{M_1 - M_2}{l}$$

$$R_2 = V_2 = \frac{wl}{2} - \frac{M_1 - M_2}{l}$$

$$V_x = w\left(\frac{l}{2} - x\right) + \frac{M_1 - M_2}{l}$$

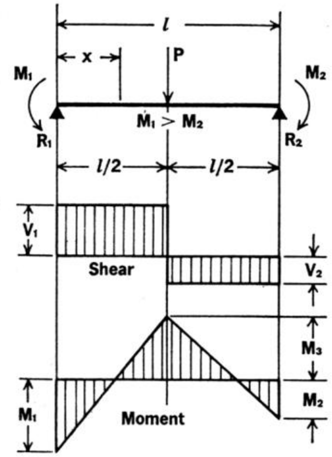
$$M_3\left(\text{at } x = \frac{l}{2} + \frac{M_1 - M_2}{wl}\right) = \frac{wl^2}{8} - \frac{M_1 + M_2}{2} + \frac{(M_1 - M_2)^2}{2wl^2}$$

$$M_x = \frac{wx}{2}(l - x) + \left(\frac{M_1 - M_2}{l}\right)x - M_1$$

$$b\left(\begin{array}{l}\text{To locate}\\ \text{inflection points}\end{array}\right) = \sqrt{\frac{l^2}{4} - \left(\frac{M_1 + M_2}{w}\right) + \left(\frac{M_1 - M_2}{wl}\right)^2}$$

$$\Delta_x = \frac{wx}{24EI}\left[x^3 - \left(2l + \frac{4M_1}{wl} - \frac{4M_2}{wl}\right)x^2 + \frac{12M_1}{w}x + l^3 - \frac{8M_1 l}{w} - \frac{4M_2 l}{w}\right]$$

33. BEAM—CONCENTRATED LOAD AT CENTER AND VARIABLE END MOMENTS



$$R_1 = V_1 = \frac{P}{2} + \frac{M_1 - M_2}{l}$$

$$R_2 = V_2 = \frac{P}{2} - \frac{M_1 - M_2}{l}$$

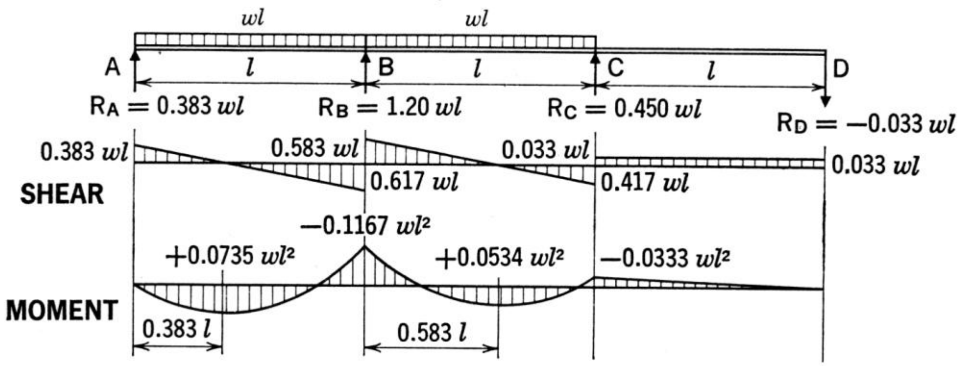
$$M_3 \text{ (At center)} = \frac{Pl}{4} - \frac{M_1 + M_2}{2}$$

$$M_x\left(\text{When } x < \frac{l}{2}\right) = \left(\frac{P}{2} + \frac{M_1 - M_2}{l}\right)x - M_1$$

$$M_x\left(\text{When } x > \frac{l}{2}\right) = \frac{P}{2}(l - x) + \frac{(M_1 - M_2)x}{l} - M_1$$

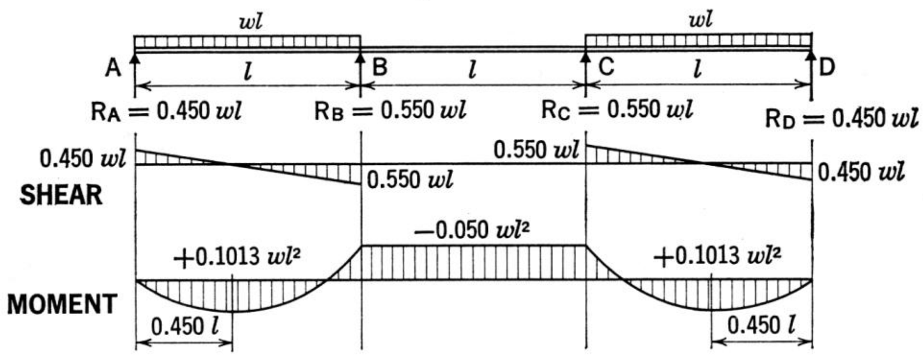
$$\Delta_x\left(\text{When } x < \frac{l}{2}\right) = \frac{Px}{48EI}\left(3l^2 - 4x^2 - \frac{8(l - x)}{Pl}\left[M_1(2l - x) + M_2(l + x)\right]\right)$$

34. CONTINUOUS BEAM—THREE EQUAL SPANS—ONE END SPAN UNLOADED



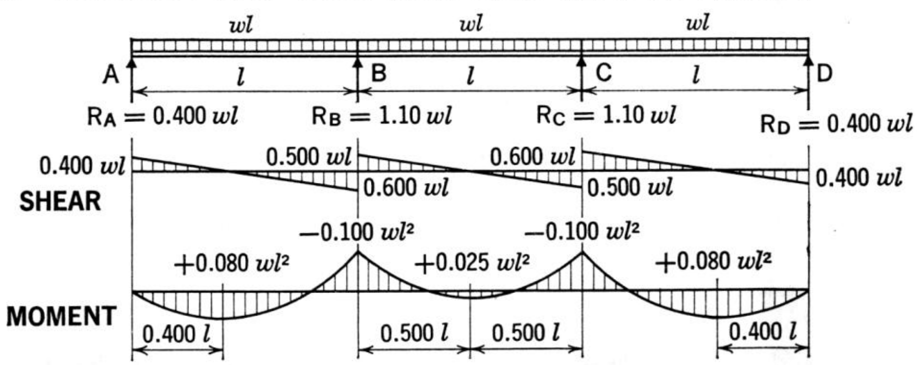
△ Max. (0.430 l from A) = 0.0059 wl^4/EI

35. CONTINUOUS BEAM—THREE EQUAL SPANS—END SPANS LOADED



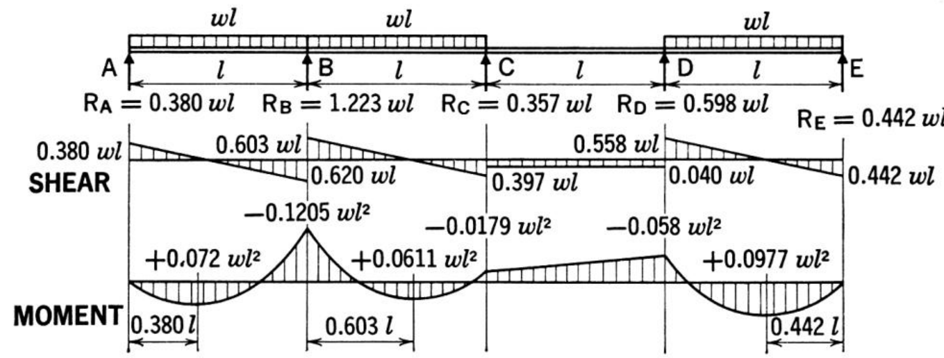
△ Max. (0.479 l from A or D) = 0.0099 wl^4/EI

36. CONTINUOUS BEAM—THREE EQUAL SPANS—ALL SPANS LOADED



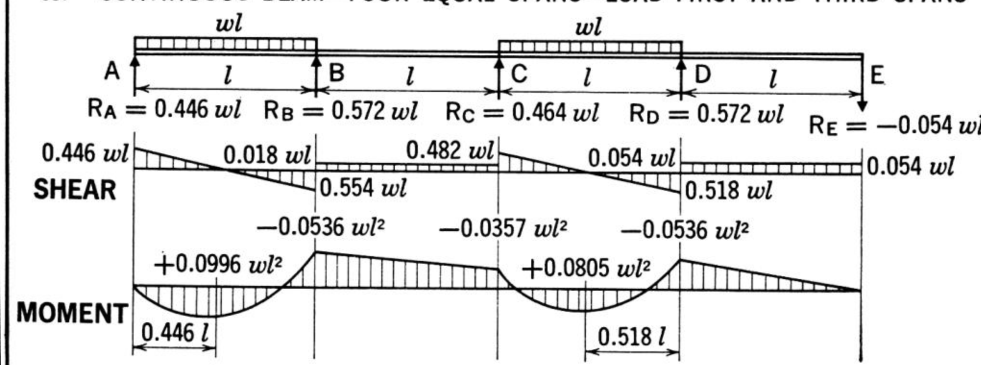
△ Max. (0.446 l from A or D) = 0.0069 wl^4/EI

37. CONTINUOUS BEAM—FOUR EQUAL SPANS—THIRD SPAN UNLOADED



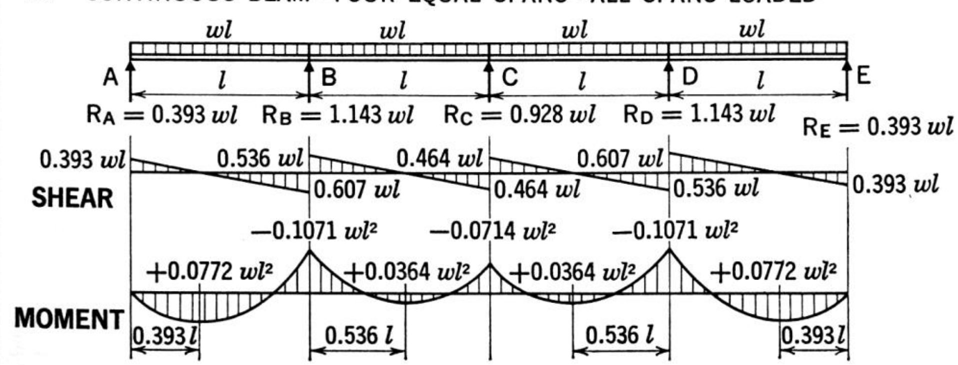
△ Max. (0.475 l from E) = 0.0094 wl^4/EI

38. CONTINUOUS BEAM—FOUR EQUAL SPANS—LOAD FIRST AND THIRD SPANS



△ Max. (0.477 l from A) = 0.0097 wl^4/EI

39. CONTINUOUS BEAM—FOUR EQUAL SPANS—ALL SPANS LOADED



△ Max. (0.440 l from A and E) = 0.0065 wl^4/EI