Wood Design

Notation:

- $=$ constant in C_p expression
- c_1 = coefficient for shear stress for a rectangular bar in torsion
- C_D = load duration factor
- C_{fu} = flat use factor for other than decks
- C_F = size factor
- C_H = shear stress factor
- C_i = incising factor
- C_L = beam stability factor
- C_M = wet service factor
- C_p = column stability factor for wood design
- C_r = repetitive member factor for wood design
- C_V = volume factor for glue laminated timber design
- C_t = temperature factor for wood design
- $d =$ name for depth
	- = calculus symbol for differentiation
- *dmin* = dimension of timber critical for buckling
- $D =$ shorthand for dead load $=$ name for diameter
- *DL* = shorthand for dead load
- E = modulus of elasticity
- $f =$ stress (strength is a stress limit)
- f_b = bending stress
- f_c = compressive stress
- $f_{from table}$ = tabular strength (from table)
- f_p = bearing stress
- f_v = shear stress
- $f_{v\text{-}max}$ = maximum shear stress
- $F_{\text{allow}} = \text{allowable stress}$
- F_b = tabular bending strength
	-
- $F'_{\mathbf{b}}$ = allowable bending stress (adjusted) F_c = tabular compression strength parallel to the grain
- *Fc* = allowable compressive stress (adjusted)
- F^* _c intermediate compressive stress for dependant on load duration
- F_{cE} = theoretical allowed buckling stress
- F_{c} = tabular compression strength perpendicular to the grain
- $F_{\text{connector}}$ = shear force capacity per connector
- F_p = tabular bearing strength parallel to the grain
	- = allowable bearing stress
- F_t = tabular tensile strength
- F_u = ultimate strength
- F_v = tabular bending strength
	- = allowable shear stress
- h = height of a rectangle
- $I =$ moment of inertia with respect to neutral axis bending
- I_{trial} = moment of inertia of trial section
- $I_{\text{real'}d}$ = moment of inertia required at limiting deflection
- I_y = moment of inertia about the y axis
- $J =$ polar moment of inertia
- $K =$ effective length factor for columns
- K_{cE} = material factor for wood column design
- $L =$ name for length or span length
- L_e = effective length that can buckle for column design, as is ℓ_{ϵ}
- *LL* = shorthand for live load

LRFD = load and resistance factor design

- $M =$ internal bending moment
- M_{max} = maximum internal bending moment
- $M_{max-adj}$ = maximum bending moment adjusted to include self weight
- $n =$ number of connectors across a joint, as is *N*
- $p =$ pitch of connector spacing
	- = safe connector load parallel to the grain

Wood or Timber Design

Structural design standards for wood are established by the *National Design Specification (NDS)* published by the National Forest Products Association. There is a combined specification (from 2005) for **Allowable** Stress Design and limit state design (LRFD).

Tabulated wood strength values are used as the base allowable strength and modified by appropriate **adjustment** factors: $f = C_D C_M C_F ... \times f_{frontable}$

Size and Use Categories

Adjustment Factors *(partial list)*

 C_D load duration factor

- C_M wet service factor
	- $(1.0 \text{ dry} < 16\% \text{ moisture content})$
- C_F size factor for visually graded sawn lumber and round timber > 12 " depth

$$
C_F = (12/d)^{\frac{1}{9}} \le 1.0
$$

- C_{fn} flat use factor (excluding decking)
- C_i incising factor (from increasing the depth of pressure treatment)
- C_t temperature factor (at high temperatures **strength** decreases)
 C_r repetitive member factor
- repetitive member factor
- C_H shear stress factor (amount of splitting)
 C_V volume factor for glued laminated timbe
- volume factor for glued laminated timber (similar to C_F)
- C^L beam stability factor (for beams without full lateral support)

Tabular Design Values

- F_b :
 F_t : bending stress
- tensile stress
- Fv: horizontal shear stress
- $F_{c\perp}$: compression stress (perpendicular to grain)
 F_c : compression stress (parallel to grain)
- compression stress (parallel to grain)
- E: modulus of elasticity
- F_p : bearing stress (parallel to grain)

Wood is significantly weakest in **shear** and strongest along the direction of the grain (tension and compression).

Load Combinations and Deflection

The critical load combination (ASD) is determined by the largest of either:

$$
\frac{dead\ load}{0.9}
$$
 or
$$
\frac{(dead\ load + any\ combination\ of\ live\ load}{C_D}
$$

The deflection limits may be increased for less stiffness with total load: $LL + 0.5(DL)$

Criteria for Design of Beams

Allowable normal stress or normal stress from LRFD should not be exceeded:

$$
F'_{b} \text{ or } \Phi F_{u} \ge f_{b} = \frac{Mc}{I}
$$

M

Knowing M and F_b , the minimum section modulus fitting the limit is:

$$
S_{req'd} \geq \frac{M}{F_b'}
$$

Besides strength, we also need to be concerned about *serviceability.* This involves things like limiting deflections & cracking, controlling noise and vibrations, preventing excessive settlements of foundations and durability. When we know about a beam section and its material, we can determine beam deformations.

Determining Maximum Bending Moment

Drawing V and M diagrams will show us the maximum values for design. Remember:

$$
V = \Sigma(-w)dx
$$

\n
$$
dV = \Sigma(V)dx
$$

\n
$$
\frac{dV}{dx} = -w
$$

\n
$$
\frac{dM}{dx} = V
$$

Determining Maximum Bending Stress

For a prismatic member (constant cross section), the maximum normal stress will occur at the maximum moment.

For a *non-prismatic* member, the stress varies with the cross section AND the moment.

Deflections

If the bending moment changes, $M(x)$ across a beam of constant material and cross section then the curvature will change:

The slope of the n.a. of a beam, θ , will be tangent to the radius of curvature, R: *EI M ^x R* $\frac{1}{1} = \frac{M(x)}{1}$

$$
\theta = slope = \frac{1}{EI} \int M(x) dx
$$

The equation for deflection, y, along a beam is:

$$
y = \frac{1}{EI} \int \theta dx = \frac{1}{EI} \iint M(x) dx
$$

Elastic curve equations can be found in handbooks, textbooks, design manuals, etc...Computer programs can be used as well (like *Multiframe*).

Elastic curve equations can be **superpositioned** ONLY if the stresses are in the elastic range. *The deflected shape is roughly the same shape flipped as the bending moment diagram but is constrained by supports and geometry.*

Boundary Conditions

The boundary conditions are geometrical values that we know – slope or deflection – which may be restrained by supports or symmetry.

At Pins, Rollers, Fixed Supports: $y = 0$

At Fixed Supports: $\theta = 0$

At Inflection Points From Symmetry: $\theta = 0$

The Slope Is Zero At The Maximum Deflection y_{max} :

$$
\theta = \frac{dy}{dx} = slope = 0
$$

Allowable Deflection Limits

All building codes and design codes limit deflection for beam types and damage that could happen based on service condition and severity.

With compression stresses in the top of a beam, a sudden "popping" or **buckling** can happen even at low stresses. In order to prevent it, we need to brace it along the top, or laterally brace it, or provide a bigger Iy.

Beam Loads & Load Tracing

In order to determine the loads on a beam (or girder, joist, column, frame, foundation...) we can start at the top of a structure and determine the *tributary area* that a load acts over and the beam needs to support. Loads come from material weights, people, and the environment. This area is assumed to be from half the distance to the next beam over to halfway to the next beam.

The reactions must be supported by the next lower structural element *ad infinitum*, to the ground.

Design Procedure

The intent is to find the most light weight member satisfying the section modulus size.

- 1. Know F_{all} (F'_b) for the material or F_U for LRFD.
- 2. Draw V & M, finding M_{max} .
- 3. Calculate S_{req'd}. This step is equivalent to determining $f_b = \frac{f_b f_{\text{max}}}{S} \leq F_b$ $f_b = \frac{M_{\text{max}}}{g} \leq F'_b$
- 4. For rectangular beams 6 $S = \frac{bh^2}{2}$
	- For timber: use the section charts to find S that will work *and remember that the beam self weight will increase Sreq'd.* $W_{self_wt} = \gamma A$

*****Determine the "updated" Vmax and Mmax including the beam self weight, and verify that the updated Sreq'd has been met.*******

- 5. Consider lateral stability.
- 6. Evaluate horizontal shear stresses using V_{max} to determine if $f_v \leq F'_v$
	- *A V* $f_{v-\text{max}} = \frac{24}{24} = 1.5$ 2 3 $\frac{1}{\sqrt{2}}$ = $\frac{1}{\sqrt{24}}$ = For rectangular beams
- 7. Provide adequate bearing area at supports:
- 8. Evaluate shear due to torsion $\frac{d}{dt}$ = $\frac{d}{dt}$ *or* $\frac{d}{dt}$ $\leq F_v$ $\frac{T\rho}{J}$ or $\frac{T}{c,ai}$ $f_v = \frac{T\rho}{I}$ or $\frac{T}{I_1^2} \leq F'_v$ 1

(circular section or rectangular)

9. Evaluate the deflection to determine if $\Delta_{maxLL} \leq \Delta_{LL-allowed}$ and/or $\Delta_{maxTotal} \leq \Delta_{Total-allowed}$ **** *note:* when Δ _{calculated} > Δ limit, I_{required} can be found with: *and Sreq'd will be satisfied for similar self weight ***** trial limit too big* $I_{req'd} \geq \frac{1000n_0}{4}I$ \varDelta \varDelta ≥

FOR ANY EVALUATION:

Redesign (with a new section) at any point that a stress or serviceability criteria is NOT satisfied and re-evaluate each condition until it is satisfactory.

Load Tables for Uniformly Loaded Joists & Rafters

Tables exists for the common loading situation for joists and rafters – that of uniformly distributed load. The tables either provide the safe distributed load based on bending and deflection limits, they give the allowable span for specific live and dead loads. If the load is *not uniform,* an *equivalent distributed load* can be calculated from the maximum moment equation. w L^2 $M_{max} = \frac{W_{equivalent}}{2}$

If the deflection limit is less, the design live load to check against allowable must be increased, ex.

$$
W_{adjusted} = W_{ll-have} \left(\frac{L/400}{L/360} \right) \text{wanted}
$$

8

 $\frac{F}{A} \leq F'_p$ (from F_c or $F_{c\perp}$) $f_p = \frac{P}{A} \leq F'_p$ (from F_c or $F_{c\perp}$)

A

V

Decking

Flat panels or planks that span several joists or evenly spaced support behave as continuous beams. Design tables consider a "1 unit" wide strip across the supports and determine maximum bending moment and deflections in order to provide allowable loads depending on the depth of the material.

The other structural use of decking is to construct what is called a *diaphragm,* which is a horizontal or vertical (if the panels are used in a shear wall) unit tying the sheathing to the joists or studs that resists forces parallel to the surface of the diaphragm.

Criteria for Design of Columns

If we know the loads, we can select a section that is adequate for strength $\&$ buckling.

If we know the length, we can find the limiting load satisfying strength $\&$ buckling.

Any slenderness ratio, $L_e/d \le 50$:

$$
f_c = \frac{P}{A} \leq F'_c
$$

$$
F'_c = F_c(C_D)(C_M)(C_r)(C_F)(C_p)
$$

The allowable stress equation uses factors to replicate the combination crushing-buckling curve:

where:

 F_c ^{$\dot{ }$} = allowable compressive stress parallel to the grain F_c = compressive strength parallel to the grain $c = 0.8$ for sawn lumber, 0.85 for poles, 0.9 for glulam timber C_D = load duration factor C_M = wet service factor (1.0 for dry) C_t = temperature factor C_F = size factor C_p = column stability factor off chart or equation: $F_{\rm \scriptscriptstyle SF}$ / F *C* $\,{}^+$ ═

$$
C_p = \frac{1 + (F_{cE} / F_c^*)}{2c} - \sqrt{\left[\frac{1 + F_{cE} / F_c^*}{2c}\right]^2 - \frac{F_{cE} / F_c^*}{c}}
$$

For preliminary column design:

$$
F_c' = F_c^* C_p = (F_c C_p) C_p
$$

Procedure for Analysis

- 1. Calculate L_e/d_{min} (KL/d for each axis and chose largest)
- 2. Obtain F_c

compute
$$
F_{cE} = \frac{K_{cE}E}{(\frac{l_e}{d})^2}
$$
 or $\frac{0.822E'_{min}}{(\frac{l_e}{d})^2}$ with $K_{cE} = 0.3$ for sawn, = 0.418 for glu-lam

- 3. Compute $F_c^* \cong F_c C_D$ $F_c^* \cong F_c C_D$ with $C_D = 1$, normal, $C_D = 1.25$ for 7 day roof...
- 4. Calculate F_{cE}/F_c^* and get C_p from table or calculation
- 5. Calculate $F_c' = F_c^* C_p$
- 6. Compute $P_{\text{allowable}} = F'_{c}$ A *or alternatively* compute $f_{\text{actual}} = P/A$
- 7. Is the design satisfactory?

Is $P \leq P_{\text{allowable}}$? \Rightarrow yes, it is; no, it is no good

or Is $f_{actual} \leq F_c$? \Rightarrow yes, it is; no, it is no good

Procedure for Design

- 1. Guess a size by picking a section
- 2. Calculate $L_{\rm e}/d_{\rm min}$ (KL/d for each axis and chose largest)
- 3. Obtain F_c

compute
$$
F_{cE} = \frac{K_{cE}E}{(\frac{l_e}{d})^2}
$$
 or $\frac{0.822E'_{min}}{(\frac{l_e}{d})^2}$ with $K_{cE} = 0.3$ for sawn, = 0.418 for glu-lam

- 4. Compute $F_c^* \cong F_c C_D$ $F_c^* \cong F_c C_D$ with $C_D = 1$, normal, $C_D = 1.25$ for 7 day roof...
- 5. Calculate F_{cE}/F_c^* and get C_p from table or calculation
- 6. Calculate $F_c' = F_c^* C_p$
- 7. Compute $P_{\text{allowable}} = F'_{c}$ A *or alternatively* compute $f_{\text{actual}} = P/A$
- 8. Is the design satisfactory?

Is $P \leq P_{\text{allowable}}$? \Rightarrow yes, it is; *no, pick a bigger section and go back to step 2.*

or Is $f_{\text{actual}} \leq F_c$? \Rightarrow yes, it is; *no, pick a bigger section and go back to step 2.*

Columns with Bending (Beam-Columns)

The modification factors are included in the form: where:

$$
\left[\frac{f_c}{F_c'}\right]^2 + \frac{f_{bx}}{F_{bx}'\left[1 - \frac{f_c}{F_{cEx}}\right]} \le 1.0
$$

$$
1 - \frac{f_c}{F_{cEx}} =
$$
magnification factor accounting for P-A
 F'_{bx} = allowable bending stress

$$
f_{bx}
$$
 = working stress from bending about x-x axis

In order to *design* an adequate section for allowable stress, we have to start somewhere:

- 1. Make assumptions about the limiting stress from:
	- buckling
	- axial stress
	- combined stress
- 2. See if we can find values for <u>r</u> or \overline{A} or \overline{S} (=I/c_{max})
- 3. Pick a trial section based on if we think r or A is going to govern the section size.
- 4. Analyze the stresses and compare to allowable using the allowable stress method or interaction formula for eccentric columns.
- 5. Did the section pass the stress test?
	- If not, do you *increase* r or A or S?
	- If so, is the difference really big so that you could *decrease* r or A or S to make it more efficient (economical)?
- 6. Change the section choice and go back to step 4. Repeat until the section meets the stress criteria.

Criteria for Design of Connections

Connections for wood are typically mechanical fasteners. Shear plates and split ring connectors are common in trusses. Bolts of metal bear on holes in wood, and nails rely on shear resistance transverse and parallel to the nail shaft. Timber rivets with steel side plates are allowed with glue laminated timber.

Connections must be able to transfer any axial force, shear, or moment from member to member or from beam to column.

Bolted Joints

Stress must be evaluated in the member being connected using the load being transferred and the reduced cross section area called *net area.* Bolt capacities are usually provided in tables and take into account the allowable shearing stress across the diameter for *single* and *double shear*, and the allowable bearing stress of the connected material based on the direction of the load with

respect to the grain. Problems, such as ripping of the bolt hole at the end of the member, are avoided by following code guidelines on minimum edge distances and spacing. Nailed Joints

Because nails rely on shear resistance, a common problem when nailing is splitting of the wood at the end of the member, which is a shear failure. Tables list the shear force capacity per unit length of embedment per nail. Jointed members used for beams will have shear stress across the connector, and the pitch spacing, *p*, can be determined from the shear stress equation when the capacity, *F*, is known:

$$
nF_{connector} \geq \frac{VQ_{connected\,area}}{I} \cdot p
$$

Example 1 (pg 204)

Example A simple beam has a span of 16 ft [4.88 m] and supports a total uniformly distributed load, including its own weight, of 6500 lb [28.9 kN]. Using Douglas fir-larch, select structural grade, determine the size of the beam with the least cross-sectional area on the basis of limiting bending stress. Density of douglas fir-larch is 32 lb/ft^3

Example 2 (pg 207)

Example 3. A 6×10 beam of Douglas fir-larch, No. 2 grade, has a total horizontally distributed load of 6000 lb [26.7 kN]. Investigate for shear stress.

Example 3 (pg 209)

Example 6. A two-span 3×12 beam of Douglas fir-larch, No. 1 grade, bears on a 3×14 beam at its center support. If the reaction force is 4200 lb $[18.7 \text{ kN}]$, is this safe for bearing?

Example 4 (pg 212)

Example 7. An 8×12 wood beam with $E = 1,600,000$ psi is used to carry a total uniformly distributed load of 10 kips on a simple span of 16 ft. Find the maximum deflection of the beam.

Example 5 (pg 223)

Example 13. Using Table 5.10 select joists to carry a live load of 40 psf on a span of 15 ft 6 in. if the spacing is 16 in. on center.

TABLE 5.10 Maximum Spans for Floor Joists (ft-in.)^a Joist Size 2×10 Spacing (in.) 2×6 2×8 2×12 Live load = 40 psf, Dead load = 10 psf, Maximum live-load deflection = $L/360$ 12 $10-9$ $14-2$ $17-9$ $20 - 7$ $9-9$ $12 - 7$ $15-5$ $17 - 10$ 16 19.2 $9 - 1$ $11-6$ $14-1$ $16 - 3$ $10-3$ $12-7$ $14-7$ 24 $8 - 1$ Live load = 40 psf, Dead load = 20 psf, Maximum live-load deflection = $L/360$ $13-3$ $16 - 3$ 12 $10-6$ $18 - 10$ $11-6$ $14-1$ $16-3$ 16 $9 - 1$ $8 - 3$ $10-6$ $12 - 10$ $14 - 10$ 19.2 $9 - 50$ $11-6$ 24 $7 - 5$ $13-4$

Source: Compiled from data in the International Building Code (Ref. 4), with permission of the publisher, International Code Council.

 a Joists are Douglas fir-larch, No. 2 grade. Assumed maximum available length of single piece is 26ft .

Example 6 (pg 239)

Example 1. A wood column consists of a 6×6 of Douglas fir-larch, No. 1 grade. Find the safe axial compression load for unbraced lengths of: (1) 2 ft, (2) 8 ft, (3) 16 ft. using the ASD method.

Example 7

Example Problem 10.18 (Figures 10.60 and 10.61)

An 18' tall 6×8 Southern pine column supports a roof load
(dead load plus a 7-day live load) equal to 16 kips. The
weak axis of buckling is braced at a point 9'6" from the bottom support. Determine the adequacy of the column.

$$
F_c = 975
$$
psi, $E = 1.6 \times 10^6$ psi

Figure 10.61 (a) Strong axis. (b) Weak axis.

Example 7 (fully worked)

Example Problem 10.18 (Figures 10.60 and 10.61)

An 18' tall 6×8 Southern pine column supports a roof load (dead load plus a 7-day live load) equal to 16 kips. The weak axis of buckling is braced at a point 9'6" from the bottom support. Determine the adequacy of the column.

Solution:

6×8 S4S Southern pine post: $(A = 41.25 \text{ in.}^2, F_c = 975 \text{ psi.}$ $E = 1.6 \times 10^6 \text{ psi}$

Check the slenderness ratio about the weak axis:

$$
\frac{L_e}{d} = \frac{(9.5' \times 12 \text{ in.}/\text{ft.})}{5.5''} = 20.7
$$

The slenderness ratio about the strong axis is:

$$
\frac{L_e}{d} = \frac{(18' \times 12 \text{ in.}/\text{ft.})}{7.5''} = 28.8 \leftarrow \text{govers}
$$

$$
F_{cE} = \frac{0.3E}{(L_e/d)^2} = \frac{0.3 (1.6 \times 10^6 \text{ lb.}/\text{in.}^2)}{(28.8)^2} = 579 \text{ psi}
$$

$$
F_c^* \approx F_c C_D = (975 \text{ lb.}/\text{in.}^2)(1.25) = 1220 \text{ psi}
$$

where: C_D = 1.25 for 7-day-duration load

From Appendix Table 14: $C_p = 0.412$

$$
\therefore F_c' = F_c^* C_p = 1220 \text{ lb.}/\text{ in.}^2 \times 0.412 = 503 \text{ psi}
$$

\n
$$
P_a = F_c' \times A = (503 \text{ lb.}/\text{ in.}^2) \times (41.25 \text{ in.}^2)
$$

\n= 20,700 lb.
\n
$$
P_a = 20.7 \text{ k} > P_{\text{actual}} = 16 \text{ k}
$$

The column is adequate.

Figure 10.61 (a) Strong axis. (b) Weak axis.

Example 8 (pg 251)
Example 4. An exterior wall stud of Douglas fir-larch, stud grade, is *Example 4.* An exterior wall stud of Douglas fir-larch, stud grade, is loaded as shown in Figure 6.5*a*. Investigate the stud for the combined loading. (Note: This is the wall stud from the building example in Chap-(*Wind load duration does apply*.)

Example 9 (pg 264)

Example 2. The truss heel joint shown in Figure 7.5 is made with 2-in. nominal thickness lumber and gusset plates of ¹/₂-in.-thick plywood. Nails are 6d common wire with the nail layout shown occurring in both sides of the joint. Find the tension load capacity for the bottom chord member (load 3 in the figure).

TABLE 7.1 Reference Lateral Load Values for Common Wire Nails (Ib/in.)

Example 10

A nominal 4 x 6 in. redwood beam is to be supported by two 2 x 6 in. members acting as a spaced column. The minimum spacing and edge distances for the ½ inch bolts are shown. How many $\frac{1}{2}$ in. bolts will be required to safely carry a load of 1500 lb? Use the chart provided.

 $1,500$ pounds

TABLE 23-I-F-HOLDING POWER OF BOLTS^{1,2,3} FOR DOUGLAS FIR-LARCH,
CALIFORNIA REDWOOD (CLOSE GRAIN) AND SOUTHERN PINE
(See U.B.C. Standard 23-17 where members are not) of equal size and for values in other species.)

¹Tabulated values are on a normal load-duration basis and apply to joints made of seasoned lumber used in dry locations. See Division III for other service conditions.

2Double shear values are for joints consisting of three wood members in which the side members are one half the thickness of the main member. Single shear values are for joints consisting of two wood members having a minimum thickness not less than that specified.

³See Division III for wood-to-metal bolted joints.

The length specified is the length of the bolt in the main member of double shear joints or the length of the bolt in the thinner member of single shear joints.

ASD Beam Design Flow Chart

Column Stability Factors

from Statics and Strength of Materials: Foundations for Structural Design, Onouye

Table 14 Column Stability Factor C_p.

ផ្ល

Table developed and permission for use granted by Professor Ed Lebert, Dept. of Architecture, University of Washington.