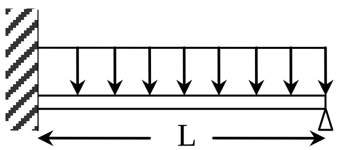
Advanced Beam Analysis

Notation:

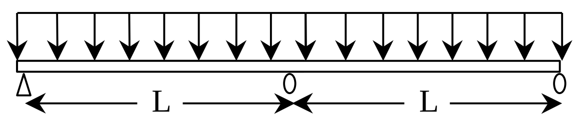
I	=	moment of inertia with respect to neutral axis bending	*P*	=	name for a force vector
L	=	beam span length	*R*	=	name for reaction force vector
M	=	internal bending moment	*w*	=	name for distributed load
n	=	relative location of the load on a span	*Σ*	=	summation symbol

Statically indeterminate beams have more unknowns than equations provided by statics. But by adding more restraints, the deflections are significantly impacted.

This means that the maximum moment, if the beam was statically determinate with simple supports, can be said to be **redistributed** between positive and negative moments (which means the absolute value of the moments sum).



restrained, ex.

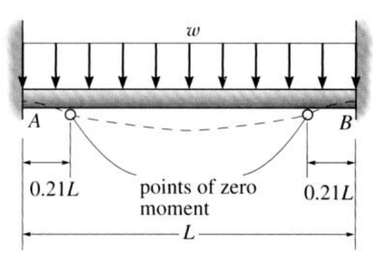


continuous
(most common case when L_1=L_2)

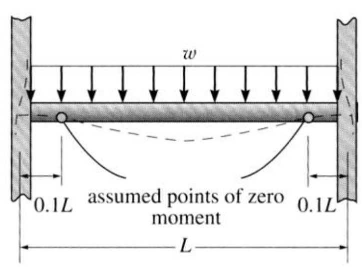
Approximate Analysis Methods

There are analysis methods based on the way the structure deforms which assume where the inflection points (having zero moment) may be. These inflection points are treated as *hinges.*

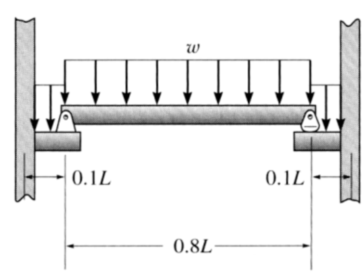
For example, the following beam has supports that aren't entirely rigid, so the inflection points can be assumed to be closer to the supports than a rigidly supported beam (see *Beam Diagrams and Formulas*). Statics is used to isolate the center span, find the support forces reactions that end up as loads on the remaining bodies.



fixed supported



approximate case



model

Analysis Methods

There are two general methods for analysis of statically indeterminate structures; the *force* or *flexibility method,* and the *stiffness* or *displacement method.*

- Force Method – The method obtains additional equations from writing equations that satisfy compatibility (consistent displacements) and force-displacement requirements. The *Theorem of Three Moments* is a force method.

- Displacement Method – The method is based on writing force-displacement relations for the members and then satisfying the equilibrium requirements for the structures. The unknowns are the displacements. Matrix methods use this format, as do most computer programs (like Multiframe3D).

Theorem of Three Moments

The general three-moment equation applies to continuous beams of constant section, the supports either being unyielding or settling* by known amounts. It gives a relationship between the moments at three adjacent supports, in terms of the loading on the two associated spans.

General relation for fixed supports: (*The settling equation has more terms.)

$$M_1\frac{L_1}{I_1}+2M_2\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right)+M_3\frac{L_2}{I_2}=-\sum\frac{P_1L_1^2}{I_1}\left(n_1-n_1^3\right)-\sum\frac{P_2L_2^2}{I_2}\left(n_2-n_2^3\right)-\frac{w_1L_1^3}{4I_1}-\frac{w_2L_2^3}{4I_2}$$

where:
M_1 is the moment at the left support of the two spans
M_2 is the moment at the center support of the two spans
M_3 is the moment at the right support of the two spans
L_1 is the length of the left span
L_2 is the length of the right span
I_1 is the moment of inertia of the left span
I_2 is the moment of inertia of the right span
P_1 is the concentrated load on the left span
P_2 is the concentrated of the right span
n_1 is the relative location of the concentrated load on the left span with respect to the span length
n_1 is the relative location of the concentrated load on the left span with respect to the span length

For spans with only distributed loading AND constant moment of inertia ($I_1 = I_2$), the general equation becomes:

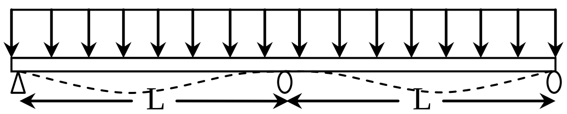
$$M_1L_1+2M_2\left(L_1+L_2\right)+M_3L_2=-\frac{w_1L_1^3}{4}-\frac{w_2L_2^3}{4}$$

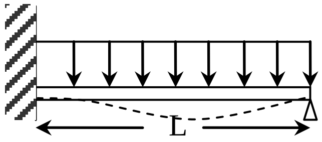
For spans with only concentrated loading AND constant moment of inertia ($I_1 = I_2$), the general equation becomes:

$$M_1L_1 + 2M_2(L_1 + L_2) + M_3L_2 = -\sum P_1L_1^2(n_1 - n_1^3) - \sum P_2L_2^2(n_2 - n_2^3)$$

Continuous Beams with Two Spans and Symmetrical Loading

With symmetrical loading, the center support of a two equal-span continuous beam acts like a fixed support preventing any rotation and displacement. We can treat one span like a beam fixed at one end, supported at the other and use beam formulas and diagrams.





Example 1 (pg 128)

Example 18. Construct the shear and moment diagrams for the beam in Figure 3.28.