Reference Formulas

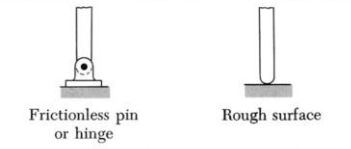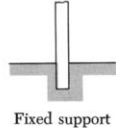|		
$\sum F_x = 0$	$C^2 = A^2 + B^2 - 2AB\cos\gamma$	$\pi(\,radians\,) = 180°$
$\sum F_y = 0$	$\dfrac{A}{\sin\alpha} = \dfrac{B}{\sin\beta} = \dfrac{C}{\sin\gamma}$	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$\sum M = 0$	$\sin\theta = \dfrac{O}{H}$	$y = mx + b$
$F_x = F\cos\theta$	$\cos\theta = \dfrac{A}{H}$	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$
$F_y = F\sin\theta$	$\tan\theta = \dfrac{O}{A}$	$\hat{x} = \dfrac{\Sigma(\bar{x}A)}{\Sigma A}$
$F = \sqrt{F_x^2 + F_y^2}$	$p = 2\pi r = \pi d$	$Q_y = \bar{x}A = \sum_{i=1}^{n}\bar{x}_i A_i$
$\tan\theta = \dfrac{F_y}{F_x}$	$A = \pi r^2 = \dfrac{\pi d^2}{4}$	$\hat{y} = \dfrac{\Sigma(\bar{y}A)}{\Sigma A}$
$M = Fd$	$A = W \cdot l = t \cdot d$	$Q_x = \bar{y}A = \sum_{i=1}^{n}\bar{y}_i A_i$
$g = 9.81\,\dfrac{m}{s^2}$	$V = W \cdot h \cdot l = A \cdot l$	$I = \bar{I} + Ad^2$
$s(\,t\,) = v(\,0\,)t + \tfrac{1}{2}at^2$	$F = mg$	$I = \sum I_c + \sum Ad^2$
$\dfrac{dV}{dx} = -w$	$r = \sqrt{\dfrac{I}{A}}$	$d_x = \hat{x} - \bar{x}$
$\dfrac{dM}{dx} = V$	$x = \dfrac{V_A}{w}$	$d_y = \hat{y} - \bar{y}$
$Pa = \dfrac{N}{m^2}$	$N = \dfrac{kg \cdot m}{s^2}$	$F = \mu N$
$1\,kPa = 1{,}000\,Pa$	$psi = \dfrac{lb}{in^2}$	$ksi = \dfrac{kip}{in^2}$
$1\,kPa = 1\,\dfrac{kN}{m^2}$	$1\,kip = 1000\,lb$	$12\,in = 1\,ft$
$1\,MPa = 10^6\,Pa$	$1\,GPa = 10^9\,Pa$	$1\,m = 1000\,mm$
$f_c = \dfrac{P}{A}$	$F.S = \dfrac{ultimate}{allowable}$	$\varepsilon = \dfrac{\delta}{L}$
$f_t = \dfrac{P}{A}$ or $\dfrac{P}{A_e}$	$f_v = \dfrac{P}{A} = \dfrac{P}{td}$	$f = E\varepsilon$
$f_p = \dfrac{P}{A} = \dfrac{P}{td}$	$f_v = \tau = \dfrac{T\rho}{J}$	$f = \dfrac{\delta}{L}E$
$f_y = \dfrac{My}{I}$	$f_{v-ave} = \dfrac{VQ}{Ib}$	$f_v = \dfrac{P}{2A}$
$S = \dfrac{I}{c}$	$f_{v-\max} = \dfrac{3V}{2A}$ for a rectangle	$\delta = \dfrac{PL}{AE}$

$$f_{b-max} = \frac{Mc}{I} = \frac{M}{S}$$

$$f_{v-max} \cong \frac{V}{A_{web}} = \frac{V}{t_w d} \text{ for an I beam}$$

$$\delta_T = \alpha(\Delta T)L$$

$$S_{req} \geq \frac{M}{F_b}$$

$$\varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E}$$

$$\varepsilon_T = \alpha(\Delta T)$$

$$w = \gamma A$$

$$W = \gamma V$$

$$W = \gamma t A$$

$$nF_{connector} \geq \frac{VQ_{connected\,area}}{I} \cdot p$$

$$M_1 L_1 + 2M_2(L_1 + L_2) + M_3 L_2 = -\frac{w_1 L_1^3}{4} - \frac{w_2 L_2^3}{4}$$

$$M_1 L_1 + 2M_2(L_1 + L_2) + M_3 L_2 = -\sum P_1 L_1^2(n_1 - n_1^3) - \sum P_2 L_2^2(n_2 - n_2^3)$$

$$V_{longitudinal} = \frac{V_T Q}{I} \Delta x$$

$$\gamma = \frac{\rho\phi}{L}$$

$$f_v = \tau = G \cdot \frac{\rho\phi}{L}$$

$$\tau_{max} = \frac{T}{c_1 ab^2}$$

$$\phi = \frac{TL}{c_2 ab^3 G}$$

$$\phi = \frac{TL}{JG}$$

$$\tau_{max} = \frac{T}{\frac{1}{3}ab^2}$$

$$\phi = \frac{TL}{\frac{1}{3}ab^3 G}$$

$$\tau_{max} = \frac{Tt_{max}}{\frac{1}{3}\Sigma b_i t_i^3}$$

$$\tau_{max} = \frac{T}{2t\mathcal{A}}$$

$$\phi = \frac{TL}{4t\mathcal{A}^2} \sum_i \frac{s_i}{t_i}$$

$$\phi = \frac{TL}{\frac{1}{3}G\Sigma b_i t_i^3}$$

$$\frac{1}{R} = \frac{M}{EI}$$

$$\Delta = \iint \frac{M(x)}{EI} dx$$

$$2n = b + 3$$

$$P_U = P_L \gamma_L + P_D \gamma_D \leq \phi P_n$$

$$1.4D$$

$$1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.5W)$$

$$L_e = Kl$$

$$1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$$

$$1.2D + 1.0W + L + 0.5(L_r \text{ or } S \text{ or } R)$$

$$P_{cr} = \frac{\pi^2 EI}{(L_e)^2} = \frac{\pi^2 EA}{\left(\frac{L_e}{r}\right)^2}$$

AISC – ASD:
$$R_a \leq \frac{R_n}{\Omega}$$

$$\frac{l_e}{r} \geq C_c \quad F_a = \frac{F_{cr}}{F.S.} = \frac{12\pi^2 E}{23\left(\frac{Kl}{r}\right)^2}$$

$$f_{cr} = \frac{\pi^2 E}{\left(\frac{L_e}{r}\right)^2}$$

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$

$$\frac{l_e}{r} < C_c \quad F_a = \left[1 - \frac{\left(\frac{Kl}{r}\right)^2}{2C_c^2}\right]\frac{F_y}{F.S.}$$

$$f_{max} = \frac{P}{A} + \frac{Mc}{I}$$

$$P_n = F_{cr}A_g$$

$$\Omega = 1.67 \text{ (bending)}$$

$$F.S. = \frac{5}{3} + \frac{3}{8} \cdot \frac{\frac{L_e}{r}}{C_c} - \frac{1}{8} \cdot \left(\frac{\frac{L_e}{r}}{C_c}\right)^3$$

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0$$

$$\Omega = 1.67 \text{ (beam shear)}$$

$$\Omega = 2.00 \text{ (bolt shear)}$$

$$\frac{P_u}{\phi_c P_n} \geq 0.2 : \frac{P}{\frac{P_n}{\Omega}} + \frac{8}{9}\left(\frac{M_x}{\frac{M_{nx}}{\Omega}} + \frac{M_y}{\frac{M_{ny}}{\Omega}}\right) \leq 1.0$$

$$f_{max} = \frac{P}{A} + \frac{M_1 y}{I} + \frac{M_2 z}{I}$$

$$\Omega = 2.00 \text{ (weld shear)}$$

$$\Omega = 1.50 \text{ (bearing)}$$

$$\frac{P_u}{\phi_c P_n} < 0.2 : \frac{P}{\frac{2P_n}{\Omega}} + \left(\frac{M_x}{\frac{M_{nx}}{\Omega}} + \frac{M_y}{\frac{M_{ny}}{\Omega}}\right) \leq 1.0$$

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0$$

$$\Omega = 1.67 \text{ (compression)}$$

ACI-318: $A_s f_y = 0.85 f'_c ba$	$a = \dfrac{A_s f_y}{0.85 f'_c b}$	$\rho = \dfrac{A_s}{bd}$
$M_u \le \phi M_n = \phi A_s f_y \left(d - \dfrac{a}{2}\right)$ $\phi = 0.9$	min: $A_s = \dfrac{3\sqrt{f'_c}}{f_y}(bd)$, not less than $A_s = \dfrac{200}{f_y}(bd)$	
$R_n = \dfrac{M_n}{bd^2}$	T, min: $A_s = \dfrac{6\sqrt{f'_c}}{f_y}(b_w d)$, not less than $A_s = \dfrac{3\sqrt{f'_c}}{f_y}(b_f d)$	
slab (<60): $A_s = 0.002b(t \text{ or } h)$	slab (60): $A_s = 0.0018b(t \text{ or } h)$	$V_u \le \phi V_c + \phi V_s \quad \phi = 0.75$
one-way: $V_c = 2\sqrt{f'_c} b_w d$	$V_s = \dfrac{A_v f_y d}{s}$	$E_c = w_c^{1.5} 33\sqrt{f'_c}$
two-way: $V_c = 4\sqrt{f'_c} b_w d$	$E_c = 57{,}000\sqrt{f'_c}$	$\le \#6 : l_d = \dfrac{d_b F_y}{25\sqrt{f'_c}}$
$l_{dh} = \dfrac{1200 d_b}{\sqrt{f'_c}}$	$(c): l_d = \dfrac{0.02 d_b F_y}{\sqrt{f'_c}} \le 0.0003 d_b F_y$	$> \#6 : l_d = \dfrac{d_b F_y}{20\sqrt{f'_c}}$
$\dfrac{P}{A} \le q_{net}$	tied: $\phi_c P_n = \phi_c(0.8 P_o) \quad \phi_c = 0.65$	$P_o = 0.85 f'_c(A_g - A_{st}) + f_y A_{st}$
$q_u = \dfrac{P_u}{A}$	spiral: $\phi_c P_n = \phi_c(0.85 P_o) \quad \phi_c = 0.75$	$q_{net} = q_{allowable} - h_f(\gamma_c - \gamma_s)$
$G = \Psi = \dfrac{\Sigma \, EI/l_c}{\Sigma \, EI/l_b}$	$P_u \le \phi_b P_n = \phi_b(0.85 f'_c A_1)\sqrt{\dfrac{A_2}{A_1}}$ $\phi_p = 0.65$	$V_{u2} = P_u - q_u(c+d)(b+d)$
$volume = \dfrac{wpx}{2} = N$	$b_o = 2(c+d) + 2(b+d)$	$V_{u1} = BL' q_u$
$p_{max} = \dfrac{2N}{wx}$	$V_{u2} \le \phi\left(2 + \dfrac{4}{\beta_c}\right)\sqrt{f'_c} b_o d \le \phi 4\sqrt{f'_c} b_o d$	$V_{u1} \le \phi 2\sqrt{f'_c} Bd$
$SF = \dfrac{M_{resist}}{M_{overturning}} \ge 1.5$	$SF = \dfrac{F_{horizontal\text{-}resist}}{F_{sliding}} \ge 1.5$	$M_u = q_u \dfrac{BL_m^2}{2}$

Wood: $F' = C_D C_M C_F \ldots \times F_{tabulated}$	$K_{cE} = 0.3$ sawn, 0.418 glulam	$P_a = F'_c A$
$F'_c = F_c^* C_p = (F_c C_D) C_p$	$F_{cE} = \dfrac{K_{cE} E}{\left(l_e/d\right)^2}$	$\left[\dfrac{f_c}{F'_c}\right]^2 + \dfrac{f_{bx}}{F'_{bx}\left[1 - \dfrac{f_c}{F_{cEx}}\right]} \le 1.0$

AISC-LRFD: $\phi_b = 0.9$ $M_u \le \phi_b M_n = 0.9 F_y Z$	$k = Z/S$	$Z = \dfrac{M_p}{f_y}$
$M_{ult} = M_p = f_y \Sigma A_i y_i = f_y Z$	$L_p = 1.76 r_y \sqrt{\dfrac{E}{F_y}}$	$M_{max} = \dfrac{w_{equivalent} L^2}{8}$
$V_u \le \phi_v (0.6 F_{yw} A_w)\ \ \phi_v = 1.0$	$I_{req'd} \ge \dfrac{\Delta_{toobig}}{\Delta_{limit}} I_{trial}$	$F_e = \dfrac{\pi^2 E}{\left(KL/r\right)^2}$
$P_u \le \phi_c F_{cr} A_g\ \ \phi_c = 0.90$	$F_e \ge 0.44 F_y \qquad F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y$	
$P_{n(max-end)} = (N + 2.5k) F_{yw} t_w \quad \phi = 1.0$ $P_{n(max-interior)} = (N + 5k) F_{yw} t_w \quad \phi = 1.0$	$F_e < 0.44 F_y \qquad F_{cr} = 0.877 F_e$	
$R_u \le \phi_t F_y A_g\ \ \phi_t = 0.9$ $R_u \le \phi_t F_u A_e\ \ \phi_t = 0.75$	$\dfrac{P_u}{\phi_c P_n} \ge 0.2 : \dfrac{P_u}{\phi_c P_n} + \dfrac{8}{9}\left(\dfrac{M_{ux}}{\phi_b M_{nx}} + \dfrac{M_{uy}}{\phi_b M_{ny}}\right) \le 1.0$	
$R_u \le \phi 0.6 F_{EXX} T l \quad \phi = 0.75$ $C_m = 0.6 - 0.4 \left(M_1/M_2\right)$	$\dfrac{P_u}{\phi_c P_n} < 0.2 : \dfrac{P_u}{2\phi_c P_n} + \left(\dfrac{M_{ux}}{\phi_b M_{nx}} + \dfrac{M_{uy}}{\phi_b M_{ny}}\right) \le 1.0$	
$A_n = A_g - A_{of\,all\,holes} + t\Sigma \dfrac{s}{4g}$	$B_1 = \dfrac{C_m}{1 - (P_u/P_{e1})} \le 1.0$	$P_{e1} = \dfrac{\pi^2 EA}{\left(Kl/r\right)^2}$
$A_e = A_n U$	$R_u \le \phi(0.6 F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.6 F_y A_{gv} + U_{bs} F_u A_{nt})\quad \phi = 0.75$	

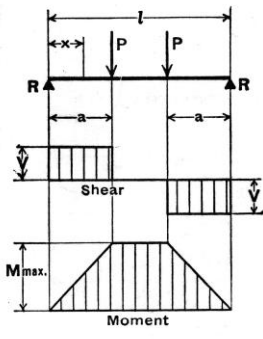|		
Masonry: plain: $F_b = \frac{1}{3} f'_m$	$A_s f_s = \dfrac{f_m b(kd)}{2}$	$M_m = \dfrac{f_m b d^2 jk}{2}$
plain: $F_v = 1.5 \sqrt{f'_m} \le 120 psi$	$F_v = 3.0 \sqrt{f'_m}$ when $M/(Vd) \ge 0.25$ $F_v = 2.0 \sqrt{f'_m}$ when $M(Vd) \ge 1.0$	$f_v \le V/A_{nv}$
$F_v = F_{vm} + F_{vs}$	$F_{vm} = \dfrac{1}{2}\left[\left(4.0 - 1.75\left(\dfrac{M}{Vd}\right)\right)\sqrt{f'_m}\right] + 0.25\dfrac{P}{A_n}$ $F_{vs} = 0.5\left(\dfrac{A_v F_s d}{A_{nv} s}\right)$	
$f_b - f_a \le F_t$	$h'/r \le 99\quad P_a = \left[0.25 f'_m A_n + 0.65 A_{st} F_s\right]\left[1 - \left(\dfrac{h'}{140r}\right)^2\right]$	
$f_a + f_b \le F_b$	$h'/r > 99\quad P_a = \left[0.25 f'_m A_n + 0.65 A_{st} F_s\right]\left(\dfrac{70r}{h'}\right)^2$	
$e_1 = \dfrac{M}{P}$	$h'/r \le 99\ \ F_a = 0.25 f'_m\left[1 - \left(\dfrac{h'}{140r}\right)^2\right]$	$h'/r > 99\ \ F_a = 0.25 f'_m\left(\dfrac{70r}{h'}\right)^2$

Reference Diagrams

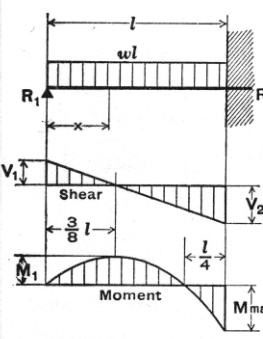| | Support or Connection | Reaction |
Rollers	Rocker	Frictionless surface
Short cable	Short link
Frictionless pin or hinge	Rough surface
Fixed support

75%

25%

75%

25%
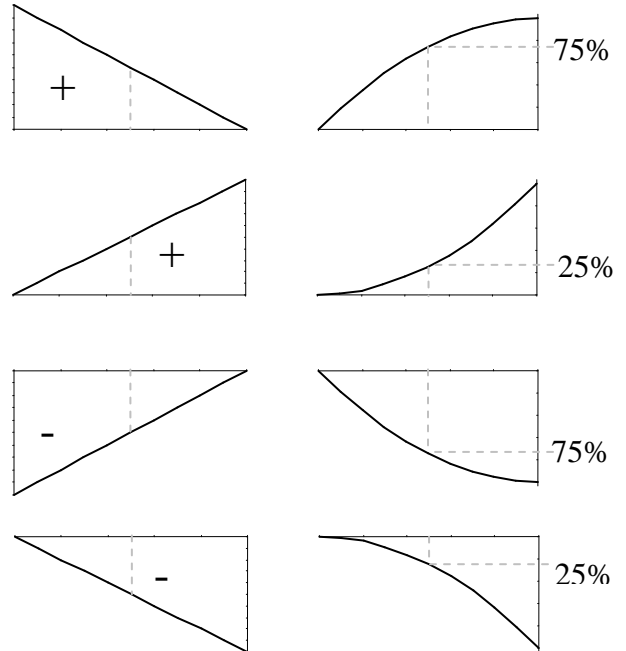
Reference Beam Diagrams

9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED

Total Equiv. Uniform Load $= \dfrac{8Pa}{l}$

$R = V$ $= P$

M max. (between loads) $= Pa$

M_x (when $x < a$) $= Px$

Δmax. (at center) $= \dfrac{Pa}{24EI}(3l^2 - 4a^2)$

Δ_x (when $x < a$) $= \dfrac{Px}{6EI}(3la - 3a^2 - x^2)$

Δ_x (when $x > a$ and $< (l-a)$) $= \dfrac{Pa}{6EI}(3lx - 3x^2 - a^2)$

12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER— UNIFORMLY DISTRIBUTED LOAD

Total Equiv. Uniform Load $= wl$

$R_1 = V_1$ $= \dfrac{3wl}{8}$

$R_2 = V_2$ max. $= \dfrac{5wl}{8}$

V_x $= R_1 - wx$

M max. $= \dfrac{wl^2}{8}$

M_1 (at $x = \dfrac{3}{8}l$) $= \dfrac{9}{128}wl^2$

M_x $= R_1 x - \dfrac{wx^2}{2}$

Δmax. (at $x = \dfrac{l}{16}(1+\sqrt{33}) = .4215l$) $= \dfrac{wl^4}{185EI}$

Δ_x $= \dfrac{wx}{48EI}(l^3 - 3lx^2 + 2x^3)$
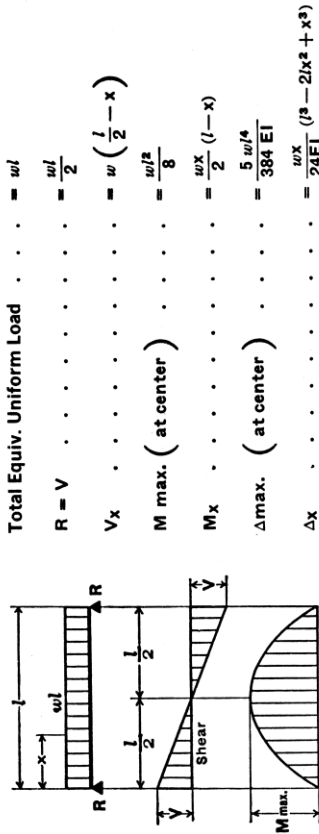
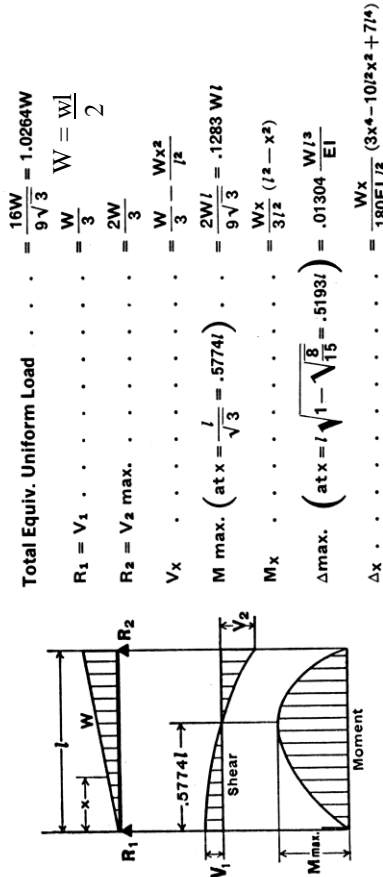## 5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END
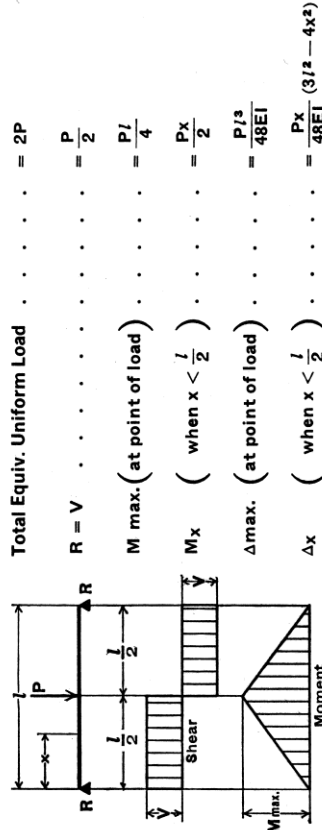
$$R_1 = V_1 \text{ max.} = \frac{wa}{2l}(2l-a)$$
$$R_2 = V_2 = \frac{wa^2}{2l}$$
$$V_x \quad (\text{when } x < a) = R_1 - wx$$
$$M \text{ max.} \left(\text{at } x = \frac{R_1}{w}\right) = \frac{R_1^2}{2w}$$
$$M_x \quad (\text{when } x < a) = R_1 x - \frac{wx^2}{2}$$
$$M_x \quad (\text{when } x > a) = R_2(l-x)$$
$$\Delta_x \quad (\text{when } x < a) = \frac{wx}{24EIl}\left(a^2(2l-a)^2 - 2ax^2(2l-a) + lx^3\right)$$
$$\Delta_x \quad (\text{when } x > a) = \frac{wa^2(l-x)}{24EIl}(4xl - 2x^2 - a^2)$$

7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER

$$\text{Total Equiv. Uniform Load} = 2P$$
$$R = V = \frac{P}{2}$$
$$M \text{ max.} (\text{at point of load}) = \frac{Pl}{4}$$
$$M_x \left(\text{when } x < \frac{l}{2}\right) = \frac{Px}{2}$$
$$\Delta \text{ max.} (\text{at point of load}) = \frac{Pl^3}{48EI}$$
$$\Delta_x \left(\text{when } x < \frac{l}{2}\right) = \frac{Px}{48EI}(3l^2 - 4x^2)$$

8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT

$$\text{Total Equiv. Uniform Load} = \frac{8Pab}{l^2}$$
$$R_1 = V_1 \left(\text{max. when } a < b\right) = \frac{Pb}{l}$$
$$R_2 = V_2 \left(\text{max. when } a > b\right) = \frac{Pa}{l}$$
$$M \text{ max.} (\text{at point of load}) = \frac{Pab}{l}$$
$$M_x \quad (\text{when } x < a) = \frac{Pbx}{l}$$
$$\Delta \text{ max.} \left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b\right) = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EIl}$$
$$\Delta a \quad (\text{at point of load}) = \frac{Pa^2b^2}{3EIl}$$
$$\Delta_x \quad (\text{when } x < a) = \frac{Pbx}{6EIl}(l^2 - b^2 - x^2)$$

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD

$$\text{Total Equiv. Uniform Load} = wl$$
$$R = V = \frac{wl}{2}$$
$$V_x = w\left(\frac{l}{2} - x\right)$$
$$M \text{ max.} (\text{at center}) = \frac{wl^2}{8}$$
$$M_x = \frac{wx}{2}(l-x)$$
$$\Delta \text{ max.} (\text{at center}) = \frac{5wl^4}{384EI}$$
$$\Delta_x = \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$$

2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END

$$\text{Total Equiv. Uniform Load} = \frac{16W}{9\sqrt{3}} = 1.0264W$$
$$R_1 = V_1 = \frac{W}{3} \qquad W = \frac{wl}{2}$$
$$R_2 = V_2 \text{ max.} = \frac{2W}{3}$$
$$V_x = \frac{W}{3} - \frac{Wx^2}{l^2}$$
$$M \text{ max.} \left(\text{at } x = \frac{l}{\sqrt{3}} = .5774l\right) = \frac{2Wl}{9\sqrt{3}} = .1283Wl$$
$$M_x = \frac{Wx}{3l^2}(l^2 - x^2)$$
$$\Delta \text{ max.} \left(\text{at } x = l\sqrt{1 - \sqrt{\tfrac{8}{15}}} = .5193l\right) = .01304\frac{Wl^3}{EI}$$
$$\Delta_x = \frac{Wx}{180EIl^2}(3x^4 - 10l^2x^2 + 7l^4)$$

3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER

$$\text{Total Equiv. Uniform Load} = \frac{4W}{3} \qquad W = \frac{wl}{2}$$
$$R = V = \frac{W}{2}$$
$$V_x \left(\text{when } x < \frac{l}{2}\right) = \frac{W}{2l^2}(l^2 - 4x^2)$$
$$M \text{ max.} (\text{at center}) = \frac{Wl}{6}$$
$$M_x \left(\text{when } x < \frac{l}{2}\right) = Wx\left(\frac{1}{2} - \frac{2x^2}{3l^2}\right)$$
$$\Delta \text{ max.} (\text{at center}) = \frac{Wl^3}{60EI}$$
$$\Delta_x \left(\text{when } x < \frac{l}{2}\right) = \frac{Wx}{480EIl^2}(5l^2 - 4x^2)^2$$

### 15. BEAM FIXED AT BOTH ENDS—UNIFORMLY DISTRIBUTED LOADS

Total Equiv. Uniform Load $= \dfrac{2wl}{3}$

$R = V = \dfrac{wl}{2}$

$V_x = w\left(\dfrac{l}{2} - x\right)$

M max. (at ends) $= \dfrac{wl^2}{12}$

M_1 (at center) $= \dfrac{wl^2}{24}$

$M_x = \dfrac{w}{12}(6lx - l^2 - 6x^2)$

Δmax. (at center) $= \dfrac{wl^4}{384EI}$

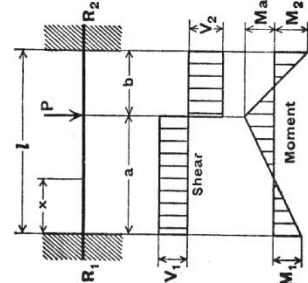$\Delta_x = \dfrac{wx^2}{24EI}(l - x)^2$
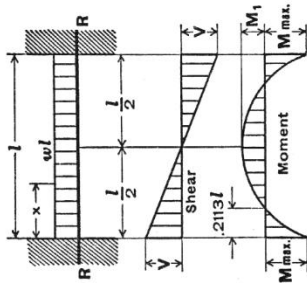
16. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT CENTER

Total Equiv. Uniform Load $= P$

$R = V = \dfrac{P}{2}$

M max. (at center and ends) $= \dfrac{Pl}{8}$

$M_x \left(\text{when } x < \dfrac{l}{2}\right) = \dfrac{P}{8}(4x - l)$

Δmax. (at center) $= \dfrac{Pl^3}{192EI}$

$\Delta_x \left(\text{when } x < \dfrac{l}{2}\right) = \dfrac{Px^2}{48EI}(3l - 4x)$
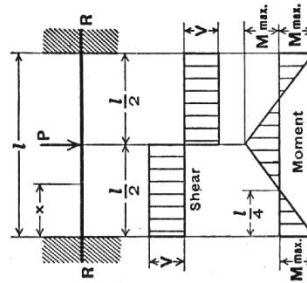
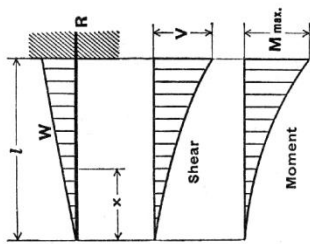### 17. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT ANY POINT

$R_1 = V_1$ (max. when a < b) $= \dfrac{Pb^2}{l^3}(3a + b)$

$R_2 = V_2$ (max. when a > b) $= \dfrac{Pa^2}{l^3}(a + 3b)$

M_1 (max. when a < b) $= \dfrac{Pab^2}{l^2}$

M_2 (max. when a > b) $= \dfrac{Pa^2b}{l^2}$

M_a (at point of load) $= \dfrac{2Pa^2b^2}{l^3}$

M_x (when x < a) $= R_1 x - \dfrac{Pab^2}{l^2}$

Δmax. $\left(\text{when } a > b \text{ at } x = \dfrac{2al}{3a + b}\right) = \dfrac{2Pa^3b^2}{3EI(3a + b)^2}$

Δ_a (at point of load) $= \dfrac{Pa^3b^3}{3EI\,l^3}$

Δ_x (when x < a) $= \dfrac{Pb^2x^2}{6EI\,l^3}(3al - 3ax - bx)$

13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT CENTER

Total Equiv. Uniform Load $= \dfrac{3P}{2}$

$R_1 = V_1 = \dfrac{5P}{16}$

$R_2 = V_2$ max. $= \dfrac{11P}{16}$

M max. (at fixed end) $= \dfrac{3Pl}{16}$

M_1 (at point of load) $= \dfrac{5Pl}{32}$

$M_x \left(\text{when } x < \dfrac{l}{2}\right) = \dfrac{5Px}{16}$

$M_x \left(\text{when } x > \dfrac{l}{2}\right) = P\left(\dfrac{l}{2} - \dfrac{11x}{16}\right)$

Δmax. $\left(\text{at } x = l\sqrt{\dfrac{1}{5}} = .4472l\right) = \dfrac{Pl^3}{48EI\sqrt{5}} = .009317\,\dfrac{Pl^3}{EI}$

Δ_x (at point of load) $= \dfrac{7Pl^3}{768EI}$

$\Delta_x \left(\text{when } x < \dfrac{l}{2}\right) = \dfrac{Px}{96EI}(3l^2 - 5x^2)$

$\Delta_x \left(\text{when } x > \dfrac{l}{2}\right) = \dfrac{P}{96EI}(x - l)^2(11x - 2l)$

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT ANY POINT

$R_1 = V_1 = \dfrac{Pb^2}{2l^3}(a + 2l)$

$R_2 = V_2 = \dfrac{Pa}{2l^3}(3l^2 - a^2)$

M_1 (at point of load) $= R_1 a$

M_2 (at fixed end) $= \dfrac{Pab}{2l^2}(a + l)$

M_x (when x < a) $= R_1 x$

M_x (when x > a) $= R_1 x - P(x - a)$

Δmax. $\left(\text{when } a < .414l \text{ at } x = l\dfrac{l^2 + a^2}{3l^2 - a^2}\right) = \dfrac{Pa}{3EI}\dfrac{(l^2 - a^2)^3}{(3l^2 - a^2)^2}$

Δmax. $\left(\text{when } a > .414l \text{ at } x = l\sqrt{\dfrac{a}{2l + a}}\right) = \dfrac{Pab^2}{6EI}\sqrt{\dfrac{a}{2l + a}}$

Δ_a (at point of load) $= \dfrac{Pa^2b^3}{12EI\,l^3}(3l + a)$

Δ_x (when x < a) $= \dfrac{Pb^2x}{12EI\,l^3}(3al^2 - 2l x^2 - ax^2)$

Δ_x (when x > a) $= \dfrac{Pa}{12EI\,l^3}(l - x)^2(3l^2x - a^2x - 2a^2l)$

21. CANTILEVER BEAM—CONCENTRATED LOAD AT ANY POINT

Total Equiv. Uniform Load $= \dfrac{8Pb}{l}$

$R = V \quad \dots \dots \dots = P$

$M \text{ max.} \left(\text{at fixed end} \right) = Pb$

$M_x \quad \left(\text{when } x > a \right) = P(x - a)$

$\Delta \text{max.} \left(\text{at free end} \right) = \dfrac{Pb^2}{6EI}(3l - b)$

$\Delta a \quad \left(\text{at point of load} \right) = \dfrac{Pb^3}{3EI}$

$\Delta_x \quad \left(\text{when } x < a \right) = \dfrac{Pb^2}{6EI}(3l - 3x - b)$

$\Delta_x \quad \left(\text{when } x > a \right) = \dfrac{P(l - x)^2}{6EI}(3b - l + x)$

22. CANTILEVER BEAM—CONCENTRATED LOAD AT FREE END

Total Equiv. Uniform Load $= 8P$

$R = V \quad \dots \dots \dots = P$

$M \text{ max.} \left(\text{at fixed end} \right) = Pl$

$M_x \quad \dots \dots \dots = Px$

$\Delta \text{max.} \left(\text{at free end} \right) = \dfrac{Pl^3}{3EI}$

$\Delta_x \quad \dots \dots = \dfrac{P}{6EI}(2l^3 - 3l^2x + x^3)$

23. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—CONCENTRATED LOAD AT DEFLECTED END

Total Equiv. Uniform Load $= 4P$

$R = V \quad \dots \dots \dots = P$

$M \text{ max.} \left(\text{at both ends} \right) = \dfrac{Pl}{2}$

$M_x \quad \dots \dots = P\left(\dfrac{l}{2} - x \right)$

$\Delta \text{max.} \left(\text{at deflected end} \right) = \dfrac{Pl^3}{12EI}$

$\Delta_x \quad \dots = \dfrac{P(l - x)^2}{12EI}(l + 2x)$

18. CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END

Total Equiv. Uniform Load $= \dfrac{8}{3}W \qquad W = \dfrac{wl}{2}$

$R = V \quad \dots \dots \dots = W$

$V_x \quad \dots \dots \dots = W\dfrac{x^2}{l^2}$

$M \text{ max.} \left(\text{at fixed end} \right) = \dfrac{Wl}{3}$

$M_x \quad \dots \dots \dots = \dfrac{Wx^3}{3l^2}$

$\Delta \text{max.} \left(\text{at free end} \right) = \dfrac{Wl^3}{15EI}$

$\Delta_x \quad \dots = \dfrac{W}{60EIl^2}(x^5 - 5l^4x + 4l^5)$

19. CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD
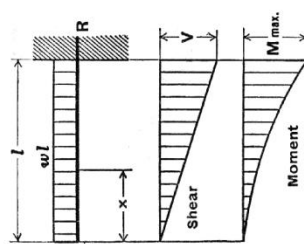
Total Equiv. Uniform Load $= 4wl$

$R = V \quad \dots \dots \dots = wl$

$V_x \quad \dots \dots \dots = wx$

$M \text{ max.} \left(\text{at fixed end} \right) = \dfrac{wl^2}{2}$

$M_x \quad \dots \dots \dots = \dfrac{wx^2}{2}$

$\Delta \text{max.} \left(\text{at free end} \right) = \dfrac{wl^4}{8EI}$

$\Delta_x \quad \dots = \dfrac{w}{24EI}(x^4 - 4l^3x + 3l^4)$

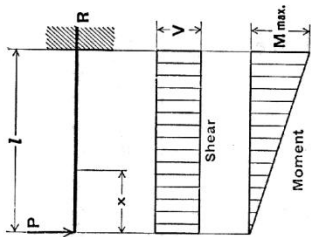## 20. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—UNIFORMLY DISTRIBUTED LOAD
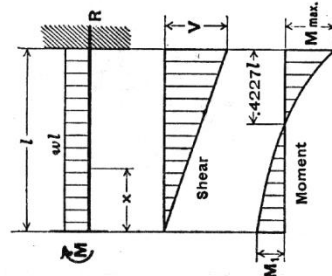
Total Equiv. Uniform Load $= \dfrac{8}{3}wl$

$R = V \quad \dots \dots \dots = wl$

$V_x \quad \dots \dots \dots = wx$

$M \text{ max.} \left(\text{at fixed end} \right) = \dfrac{wl^2}{3}$

$M_1 \quad \left(\text{at deflected end} \right) = \dfrac{wl^2}{6}$

$M_x \quad \dots \dots = \dfrac{w}{6}(l^2 - 3x^2)$

$\Delta \text{max.} \left(\text{at deflected end} \right) = \dfrac{wl^4}{24EI}$

$\Delta_x \quad \dots = \dfrac{w(l^2 - x^2)^2}{24EI}$

26. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT END OF OVERHANG

$R_1 = V_1 = \dfrac{Pa}{l}$

$R_2 = V_1 + V_2 = \dfrac{P}{l}(l+a)$

$V_2 = P$

M_{max} (at R_2) $= Pa$

M_x (between supports) $= \dfrac{Pax}{l}$

M_{x_1} (for overhang) $= P(a-x_1)$

Δ_{max} (between supports at $x = \dfrac{l}{\sqrt{3}}$) $= \dfrac{Pal^2}{9\sqrt{3}\,EI} = .06415\,\dfrac{Pal^2}{EI}$

Δ_{max} (for overhang at $x_1 = a$) $= \dfrac{Pa^2}{3EI}(l+a)$

Δ_x (between supports) $= \dfrac{Pax}{6EI\,l}(l^2-x^2)$

Δ_{x_1} (for overhang) $= \dfrac{Px_1}{6EI}(2al+3ax_1-x_1^2)$

27. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD BETWEEN SUPPORTS

Total Equiv. Uniform Load $= wl$

$R = V = \dfrac{wl}{2}$

$V_x = w\left(\dfrac{l}{2}-x\right)$

M_{max} (at center) $= \dfrac{wl^2}{8}$

$M_x = \dfrac{wx}{2}(l-x)$

Δ_{max} (at center) $= \dfrac{5wl^4}{384EI}$

$\Delta_x = \dfrac{wx}{24EI}(l^3-2lx^2+x^3)$

$\Delta_{x_1} = \dfrac{wl^3x_1}{24EI}$

28. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT ANY POINT BETWEEN SUPPORTS

Total Equiv. Uniform Load $= \dfrac{8Pab}{l^2}$

$R_1 = V_1$ (max. when a < b) $= \dfrac{Pb}{l}$

$R_2 = V_2$ (max. when a > b) $= \dfrac{Pa}{l}$

M_{max} (at point of load) $= \dfrac{Pab}{l}$

M_x (when x < a) $= \dfrac{Pbx}{l}$

Δ_{max} $\left(\text{at } x=\sqrt{\dfrac{a(a+2b)}{3}} \text{ when a > b}\right) = \dfrac{Pab\,(a+2b)\sqrt{3a\,(a+2b)}}{27EI\,l}$

Δ_a (at point of load) $= \dfrac{Pa^2b^2}{3EI\,l}$

Δ_x (when x < a) $= \dfrac{Pbx}{6EI\,l}(l^2-b^2-x^2)$

Δ_x (when x > a) $= \dfrac{Pa(l-x)}{6EI\,l}(2lx-x^2-a^2)$

$\Delta_{x_1} = \dfrac{Pabx_1}{6EI\,l}(l+a)$

24. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD

$R_1 = V_1 = \dfrac{w}{2l}(l^2-a^2)$

$R_2 = V_2 + V_3 = \dfrac{w}{2l}(l+a)^2$

$V_2 = wa$

$V_3 = \dfrac{w}{2l}(l^2+a^2)$

V_x (between supports) $= R_1 - wx$

V_{x_1} (for overhang) $= w(a-x_1)$

M_1 $\left(\text{at } x = \dfrac{l}{2}\left[1-\dfrac{a^2}{l^2}\right]\right) = \dfrac{w}{8l^2}(l+a)^2(l-a)^2$

M_2 (at R_2) $= \dfrac{wa^2}{2}$

M_x (between supports) $= \dfrac{wx}{2l}(l^2-a^2-xl)$

M_{x_1} (for overhang) $= \dfrac{w}{2}(a-x_1)^2$

Δ_x (between supports) $= \dfrac{wx}{24EI\,l}(l^4-2l^2x^2+lx^3-2a^2l^2+2a^2x^2)$

Δ_{x_1} (for overhang) $= \dfrac{wx_1}{24EI}(4a^2l-l^3+6a^2x_1-4ax_1^2+x_1^3)$

25. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD ON OVERHANG

$R_1 = V_1 = \dfrac{wa^2}{2l}$

$R_2 = V_1 + V_2 = \dfrac{wa}{2l}(2l+a)$

$V_2 = wa$

V_{x_1} (for overhang) $= w(a-x_1)$

M_{max} (at R_2) $= \dfrac{wa^2}{2}$

M_x (between supports) $= \dfrac{wa^2x}{2l}$

M_{x_1} (for overhang) $= \dfrac{w}{2}(a-x_1)^2$

Δ_{max} (between supports at $x = \dfrac{l}{\sqrt{3}}$) $= \dfrac{wa^2l^2}{18\sqrt{3}\,EI} = .03208\,\dfrac{wa^2l^2}{EI}$

Δ_{max} (for overhang at $x_1 = a$) $= \dfrac{wa^3}{24EI}(4l+3a)$

Δ_x (between supports) $= \dfrac{wa^2x}{12EI\,l}(l^2-x^2)$

Δ_{x_1} (for overhang) $= \dfrac{wx_1}{24EI}(4a^2l+6a^2x_1-4ax_1^2+x_1^3)$

Reference Diagrams

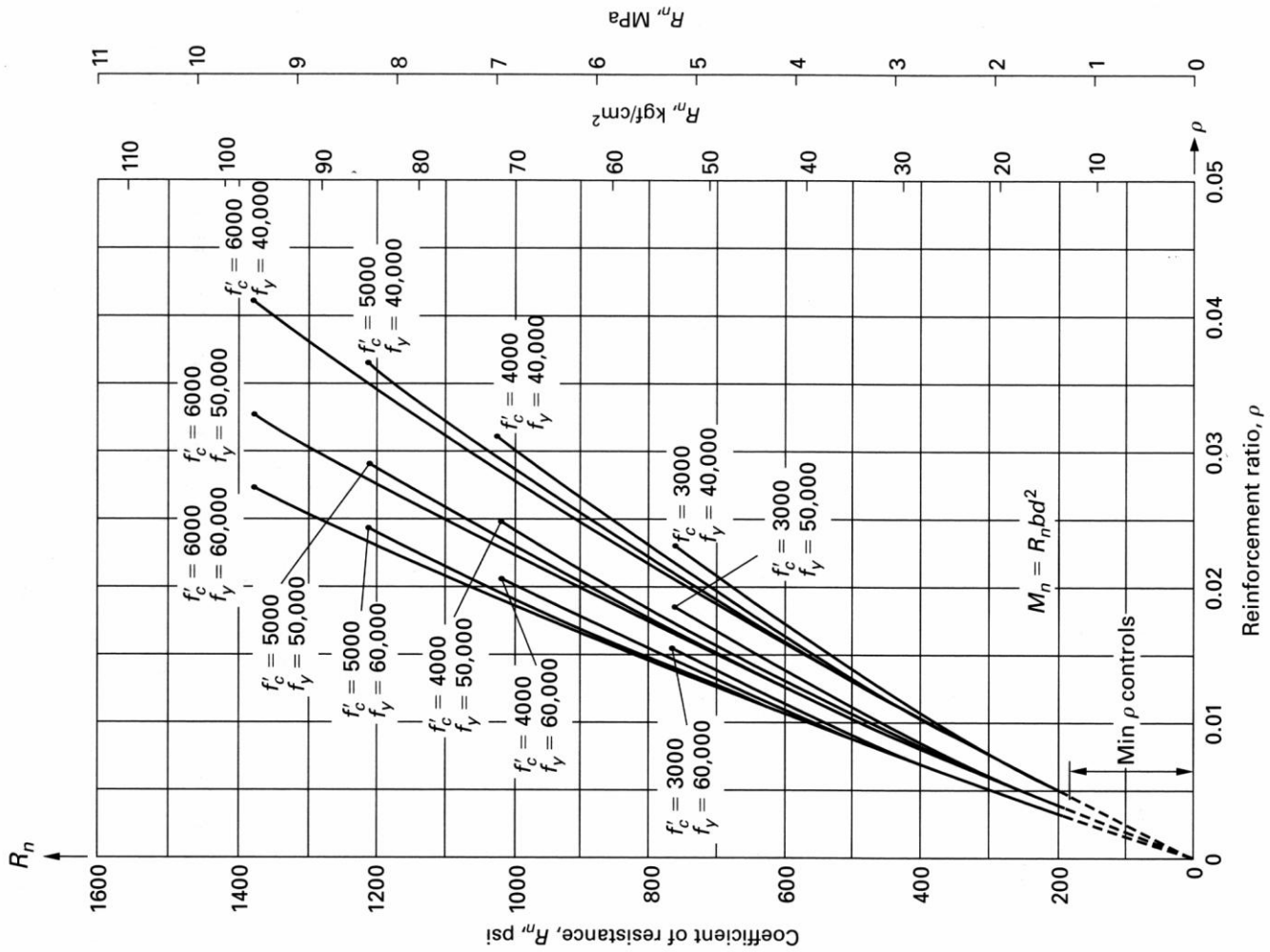**Figure 3.8.1** Strength curves (R_n vs ρ) for singly reinforced rectangular sections. Upper limit of curves is at (tensile strain of 0.004)

Table 3-8 ACI Provisions for Shear Design*

		$V_u \leq \dfrac{\phi V_c}{2}$	$\phi V_c \geq V_u > \dfrac{\phi V_c}{2}$	$V_u > \phi V_c$
Required area of stirrups, A_v**		none	$\dfrac{50 b_w s}{f_y}$	$\dfrac{(V_u - \phi V_c)s}{\phi f_y d}$
Stirrup spacing, s	Required	—	$\dfrac{A_v f_y}{50 b_w}$	$\dfrac{\phi A_v f_y d}{V_u - \phi V_c}$
	Recommended Minimum†	—	—	4 in.
	Maximum†† (ACI 11.5.4)	—	$\dfrac{d}{2}$ or 24 in.	$\dfrac{d}{2}$ or 24 in. for $\left(V_u - \phi V_c\right) \leq \phi 4\sqrt{f_c'}\, b_w d$
				$\dfrac{d}{4}$ or 12 in. for $\left(V_u - \phi V_c\right) > \phi 4\sqrt{f_c'}\, b_w d$

*Members subjected to shear and flexure only; $\phi V_c = \phi 2\sqrt{f_c'}\, b_w d$, $\phi = 0.75$ (ACI 11.3.1.1)

**$A_v = 2 \times A_b$ for U stirrups; $f_y \leq 60$ ksi (ACI 11.5.2)

†A practical limit for minimum spacing is d/4

††Maximum spacing based on minimum shear reinforcement (= $A_v f_y / 50 b_w$) must also be considered (ACI 11.5.5.3).

Reference Diagrams

TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE COMPUTED

	Minimum thickness, *h*			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

Notes:
Values given shall be used directly for members with normalweight concrete and Grade 60 reinforcement. For other conditions, the values shall be modified as follows:
a) For lightweight concrete having equilibrium density, w_c, in the range of 90 to 115 lb/ft³, the values shall be multiplied by $(1.65 - 0.005w_c)$ but not less than 1.09.
b) For f_y other than 60,000 psi, the values shall be multiplied by $(0.4 + f_y/100,000)$.

LOAD DURATION FACTOR - C_D

$C_D = 1.6$

$C_D = 1.25$
$C_D = 1.15$
$C_D = 1.0$

DURATION OF LOAD (TIME)
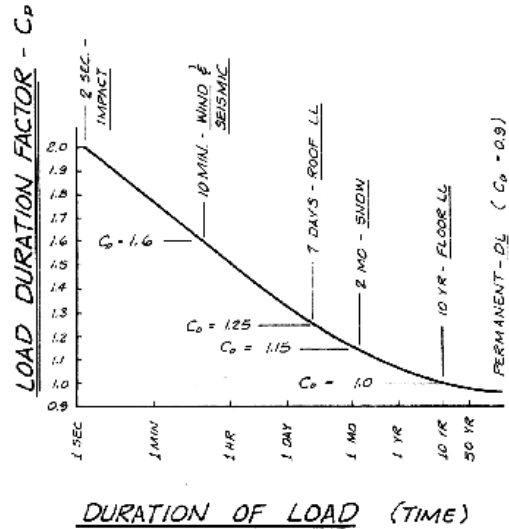
TABLE 13.6   Areas Provided By Spaced Reinforcement

Bar Spacing (in.)	Area Provided (in.²/ft width)									
	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11
3	0.20	0.44	0.80	1.24	1.76	2.40	3.16	4.00		
3.5	0.17	0.38	0.69	1.06	1.51	2.06	2.71	3.43	4.35	
4	0.15	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4.5	0.13	0.29	0.53	0.83	1.17	1.60	2.11	2.67	3.39	4.16
5	0.12	0.26	0.48	0.74	1.06	1.44	1.89	2.40	3.05	3.74
5.5	0.11	0.24	0.44	0.68	0.96	1.31	1.72	2.18	2.77	3.40
6	0.10	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
7	0.08	0.19	0.34	0.53	0.75	1.03	1.35	1.71	2.18	2.67
8	0.07	0.16	0.30	0.46	0.66	0.90	1.18	1.50	1.90	2.34
9	0.07	0.15	0.27	0.41	0.59	0.80	1.05	1.33	1.69	2.08
10	0.06	0.13	0.24	0.37	0.53	0.72	0.95	1.20	1.52	1.87
11	0.05	0.12	0.22	0.34	0.48	0.65	0.86	1.09	1.38	1.70
12	0.05	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
13	0.05	0.10	0.18	0.29	0.40	0.55	0.73	0.92	1.17	1.44
14	0.04	0.09	0.17	0.27	0.38	0.51	0.68	0.86	1.09	1.34
15	0.04	0.09	0.16	0.25	0.35	0.48	0.63	0.80	1.01	1.25
16	0.04	0.08	0.15	0.23	0.33	0.45	0.59	0.75	0.95	1.17
18	0.03	0.07	0.13	0.21	0.29	0.40	0.53	0.67	0.85	1.04
24	0.02	0.05	0.10	0.15	0.22	0.30	0.39	0.50	0.63	0.78

Table 3.7.1
Total Areas for Various Numbers of Reinforcing Bars

Bar Size	Nominal Diameter (in.)	Weight (lb/ft)	Number of Bars									
			1	2	3	4	5	6	7	8	9	10
#3	0.375	0.376	0.11	0.22	0.33	0.44	0.55	0.66	0.77	0.88	0.99	1.10
#4	0.500	0.668	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
#5	0.625	1.043	0.31	0.62	0.93	1.24	1.55	1.86	2.17	2.48	2.79	3.10
#6	0.750	1.502	0.44	0.88	1.32	1.76	2.20	2.64	3.08	3.52	3.96	4.40
#7	0.875	2.044	0.60	1.20	1.80	2.40	3.00	3.60	4.20	4.80	5.40	6.00
#8	1.000	2.670	0.79	1.58	2.37	3.16	3.95	4.74	5.53	6.32	7.11	7.90
#9	1.128	3.400	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
#10	1.270	4.303	1.27	2.54	3.81	5.08	6.35	7.62	8.89	10.16	11.43	12.70
#11	1.410	5.313	1.56	3.12	4.68	6.24	7.80	9.36	10.92	12.48	14.04	15.60
#14ᵃ	1.693	7.65	2.25	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50
#18ᵃ	2.257	13.60	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00

ᵃ #14 and #18 bars are used primarily as column reinforcement and are rarely used in beams.
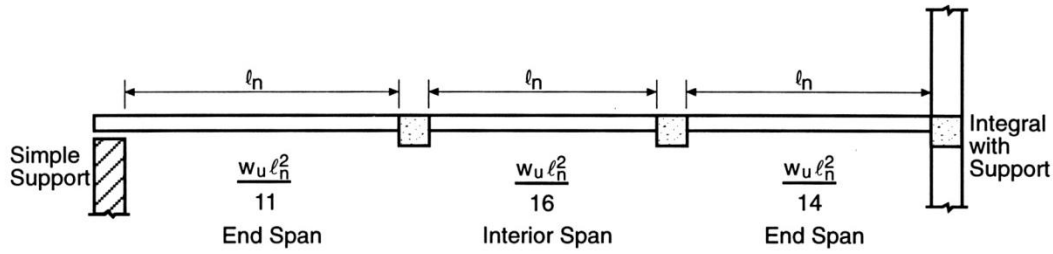
Reference Diagrams

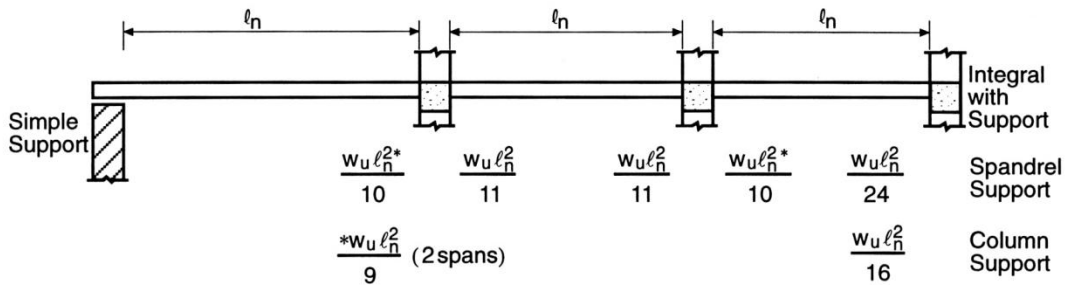*Figure 2-3 Positive Moments—All Cases*
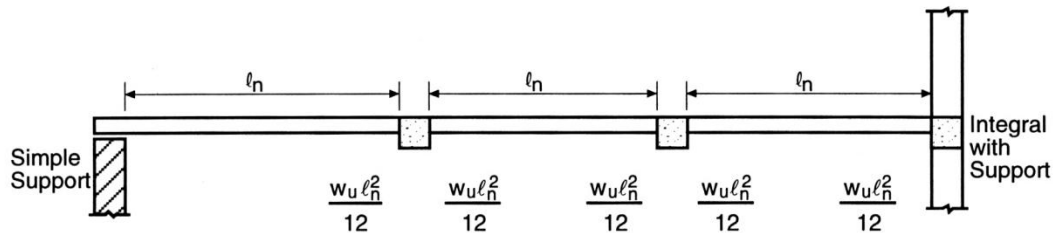
Figure 2-4 Negative Moments—Beams and Slabs

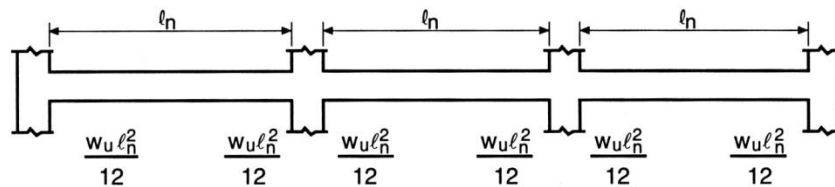*Figure 2-5 Negative Moments—Slabs with spans ≤ 10 ft*

Figure 2-6 Negative Moments—Beams with Stiff Columns ($\Sigma K_c/\Sigma K_b > 8$)
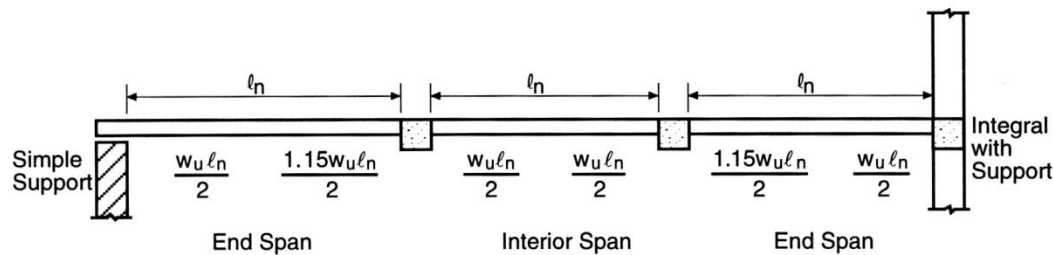
Figure 2-7 End Shears—All Cases

Reference Diagrams

Table 14   Column Stability Factor C_p

$$\text{"}C_p\text{"} \qquad F_c' = C_p \cdot F_c^* \qquad F_{CE} = \frac{30\,E}{(l/d)^2} \text{ for sawed posts} \qquad F_{CE} = \frac{.418\,E}{(l/d)^2} \text{ for glu-lam posts}$$

$\dfrac{F_{CE}}{F_c^*}$	Sawed C_p	Glu-Lam C_p
0.00	0.000	0.000
0.01	0.010	0.010
0.02	0.020	0.020
0.03	0.030	0.030
0.04	0.040	0.040
0.05	0.049	0.050
0.06	0.059	0.060
0.07	0.069	0.069
0.08	0.079	0.079
0.09	0.088	0.089
0.10	0.098	0.099
0.11	0.107	0.109
0.12	0.117	0.118
0.13	0.126	0.128
0.14	0.136	0.138
0.15	0.145	0.147
0.16	0.154	0.157
0.17	0.164	0.167
0.18	0.173	0.176
0.19	0.182	0.186
0.20	0.191	0.195
0.21	0.200	0.205
0.22	0.209	0.214
0.23	0.218	0.224
0.24	0.227	0.233
0.25	0.235	0.242
0.26	0.244	0.252
0.27	0.253	0.261
0.28	0.261	0.270
0.29	0.270	0.279
0.30	0.278	0.288
0.31	0.287	0.297
0.32	0.295	0.306
0.33	0.304	0.315
0.34	0.312	0.324
0.35	0.320	0.333
0.36	0.328	0.342
0.37	0.336	0.351
0.38	0.344	0.360
0.39	0.352	0.368

$\dfrac{F_{CE}}{F_c^*}$	Sawed C_p	Glu-Lam C_p
0.40	0.360	0.377
0.41	0.367	0.386
0.42	0.375	0.394
0.43	0.383	0.403
0.44	0.390	0.411
0.45	0.398	0.420
0.46	0.405	0.428
0.47	0.412	0.436
0.48	0.419	0.444
0.49	0.427	0.453
0.50	0.434	0.461
0.51	0.441	0.469
0.52	0.448	0.477
0.53	0.454	0.484
0.54	0.461	0.492
0.55	0.468	0.500
0.56	0.474	0.508
0.57	0.481	0.515
0.58	0.487	0.523
0.59	0.494	0.530
0.60	0.500	0.538
0.61	0.506	0.545
0.62	0.512	0.552
0.63	0.518	0.559
0.64	0.524	0.566
0.65	0.530	0.573
0.66	0.536	0.580
0.67	0.542	0.587
0.68	0.548	0.593
0.69	0.553	0.600
0.70	0.559	0.607
0.71	0.564	0.613
0.72	0.569	0.619
0.73	0.575	0.626
0.74	0.580	0.632
0.75	0.585	0.638
0.76	0.590	0.644
0.77	0.595	0.650
0.78	0.600	0.655
0.79	0.605	0.661

$\dfrac{F_{CE}}{F_c^*}$	Sawed C_p	Glu-Lam C_p
0.80	0.610	0.667
0.81	0.614	0.672
0.82	0.619	0.678
0.83	0.623	0.683
0.84	0.628	0.688
0.85	0.632	0.693
0.86	0.637	0.698
0.87	0.641	0.703
0.88	0.645	0.708
0.89	0.649	0.713
0.90	0.653	0.718
0.91	0.658	0.722
0.92	0.661	0.727
0.93	0.665	0.731
0.94	0.669	0.735
0.95	0.673	0.740
0.96	0.677	0.744
0.97	0.680	0.748
0.98	0.684	0.752
0.99	0.688	0.756
1.00	0.691	0.760
1.01	0.694	0.764
1.02	0.698	0.767
1.03	0.701	0.771
1.04	0.704	0.774
1.05	0.708	0.778
1.06	0.711	0.781
1.07	0.714	0.784
1.08	0.717	0.788
1.09	0.720	0.791
1.10	0.723	0.794
1.11	0.726	0.797
1.12	0.729	0.800
1.13	0.731	0.803
1.14	0.734	0.806
1.15	0.737	0.809
1.16	0.740	0.811
1.17	0.742	0.814
1.18	0.745	0.817
1.19	0.747	0.819

$\dfrac{F_{CE}}{F_c^*}$	Sawed C_p	Glu-Lam C_p
1.20	0.750	0.822
1.22	0.755	0.826
1.24	0.760	0.831
1.26	0.764	0.836
1.28	0.769	0.840
1.30	0.773	0.844
1.32	0.777	0.848
1.34	0.781	0.852
1.36	0.785	0.855
1.38	0.789	0.859
1.40	0.793	0.862
1.42	0.796	0.865
1.44	0.800	0.868
1.46	0.803	0.871
1.48	0.807	0.874
1.50	0.810	0.877
1.52	0.813	0.879
1.54	0.816	0.882
1.56	0.819	0.884
1.58	0.822	0.887
1.60	0.825	0.889
1.62	0.827	0.891
1.64	0.830	0.893
1.66	0.832	0.895
1.68	0.835	0.897
1.70	0.837	0.899
1.72	0.840	0.901
1.74	0.842	0.903
1.76	0.844	0.904
1.78	0.846	0.906
1.80	0.849	0.908
1.82	0.851	0.909
1.84	0.853	0.911
1.86	0.855	0.912
1.88	0.857	0.914
1.90	0.858	0.915
1.92	0.860	0.916
1.94	0.862	0.918
1.96	0.864	0.919
1.98	0.868	0.920

$\dfrac{F_{CE}}{F_c^*}$	Sawed C_p	Glu-Lam C_p
2.00	0.867	0.921
2.02	0.869	0.922
2.04	0.870	0.924
2.06	0.872	0.925
2.08	0.874	0.926
2.10	0.875	0.927
2.12	0.876	0.928
2.14	0.878	0.929
2.16	0.879	0.930
2.18	0.881	0.931
2.20	0.882	0.932
2.22	0.883	0.932
2.24	0.885	0.933
2.26	0.886	0.934
2.28	0.887	0.935
2.30	0.888	0.936
2.32	0.889	0.937
2.34	0.891	0.937
2.36	0.892	0.938
2.38	0.893	0.939
2.40	0.894	0.940
2.45	0.897	0.941
2.50	0.899	0.943
2.55	0.901	0.944
2.60	0.904	0.946
2.65	0.906	0.947
2.70	0.908	0.949
2.75	0.910	0.950
2.80	0.912	0.951
2.85	0.914	0.952
2.90	0.916	0.953
2.95	0.917	0.954
3.00	0.919	0.955
3.05	0.920	0.956
3.10	0.922	0.957
3.15	0.923	0.958
3.20	0.925	0.959
3.25	0.926	0.960
3.30	0.927	0.961
3.35	0.929	0.961

$\dfrac{F_{CE}}{F_c^*}$	Sawed C_p	Glu-Lam C_p
3.40	0.930	0.962
3.45	0.931	0.963
3.50	0.932	0.963
3.55	0.933	0.964
3.60	0.934	0.965
3.65	0.936	0.965
3.70	0.937	0.966
3.75	0.938	0.966
3.80	0.938	0.967
3.85	0.939	0.968
3.90	0.940	0.968
3.95	0.941	0.969
4.00	0.942	0.969
4.05	0.943	0.969
4.10	0.944	0.970
4.15	0.944	0.970
4.20	0.945	0.970
4.25	0.946	0.971
4.30	0.947	0.971
4.35	0.947	0.972
4.40	0.948	0.972
4.45	0.949	0.973
4.50	0.949	0.973
4.55	0.950	0.974
4.60	0.950	0.974
4.65	0.951	0.974
4.70	0.952	0.975
4.75	0.952	0.975
4.80	0.953	0.975
4.85	0.953	0.975
4.90	0.954	0.976
5.00	0.955	0.976
6.00	0.963	0.981
8.00	0.973	0.986
10.0	0.979	0.989
20.0	0.990	0.995
40.0	0.995	0.997
60.0	0.997	0.998
100.0	0.998	0.999
200.0	0.999	0.999

Table developed and permission for use granted by Professor Ed Lebert, Dept. of Architecture, University of Washington.

Reference Diagrams

Maximum Reinforcement Ratio ρ for Singly Reinforced Rectangular Beams
(tensile strain = 0.005) for which ϕ is permitted to be 0.9

f_y	$f'_c = 3000$ psi $\beta_1 = 0.85$	$f'_c = 3500$ psi $\beta_1 = 0.85$	$f'_c = 4000$ psi $\beta_1 = 0.85$	$f'_c = 5000$ psi $\beta_1 = 0.80$	$f'_c = 6000$ psi $\beta_1 = 0.75$
40,000 psi	0.0203	0.0237	0.0271	0.0319	0.0359
50,000 psi	0.0163	0.0190	0.0217	0.0255	0.0287
60,000 psi	0.0135	0.0158	0.0181	0.0213	0.0239
f_y	$f'_c = 20$ MPa $\beta_1 = 0.85$	$f'_c = 25$ MPa $\beta_1 = 0.85$	$f'_c = 30$ MPa $\beta_1 = 0.85$	$f'_c = 35$ MPa $\beta_1 = 0.81$	$f'_c = 40$ MPa $\beta_1 = 0.77$
300 MPa	0.0181	0.0226	0.0271	0.0301	0.0327
350 MPa	0.0155	0.0194	0.0232	0.0258	0.0281
400 MPa	0.0135	0.0169	0.0203	0.0226	0.0245
500 MPa	0.0108	0.0135	0.0163	0.0181	0.0196

Table 7-1
Available Shear Strength of Bolts, kips

Nominal Bolt Diameter, d, in.				5/8		3/4		7/8		1	
Nominal Bolt Area, in.²				0.307		0.442		0.601		0.785	
ASTM Desig.	Thread Cond.	F_{nv}/Ω (ksi) ASD	ϕF_{nv} (ksi) LRFD	Load-ing	r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD
Group A	N	27.0	40.5	S	8.29	12.4	11.9	17.9	16.2	24.3	21.2	31.8
				D	16.6	24.9	23.9	35.8	32.5	48.7	42.4	63.6
	X	34.0	51.0	S	10.4	15.7	15.0	22.5	20.4	30.7	26.7	40.0
				D	20.9	31.3	30.1	45.1	40.9	61.3	53.4	80.1
Group B	N	34.0	51.0	S	10.4	15.7	15.0	22.5	20.4	30.7	26.7	40.0
				D	20.9	31.3	30.1	45.1	40.9	61.3	53.4	80.1
	X	42.0	63.0	S	12.9	19.3	18.6	27.8	25.2	37.9	33.0	49.5
				D	25.8	38.7	37.1	55.7	50.5	75.7	65.9	98.9
A307	–	13.5	20.3	S	4.14	6.23	5.97	8.97	8.11	12.2	10.6	15.9
				D	8.29	12.5	11.9	17.9	16.2	24.4	21.2	31.9

Nominal Bolt Diameter, d, in.				1 1/8		1 1/4		1 3/8		1 1/2	
Nominal Bolt Area, in.²				0.994		1.23		1.48		1.77	
ASTM Desig.	Thread Cond.	F_{nv}/Ω (ksi) ASD	ϕF_{nv} (ksi) LRFD	Load-ing	r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD
Group A	N	27.0	40.5	S	26.8	40.3	33.2	49.8	40.0	59.9	47.8	71.7
				D	53.7	80.5	66.4	99.6	79.9	120	95.6	143
	X	34.0	51.0	S	33.8	50.7	41.8	62.7	50.3	75.5	60.2	90.3
				D	67.6	101	83.6	125	101	151	120	181
Group B	N	34.0	51.0	S	33.8	50.7	41.8	62.7	50.3	75.5	60.2	90.3
				D	67.6	101	83.6	125	101	151	120	181
	X	42.0	63.0	S	41.7	62.6	51.7	77.5	62.2	93.2	74.3	112
				D	83.5	125	103	155	124	186	149	223
A307	–	13.5	20.3	S	13.4	20.2	16.6	25.0	20.0	30.0	23.9	35.9
				D	26.8	40.4	33.2	49.9	40.0	60.1	47.8	71.9

ASD	LRFD	For end loaded connections greater than 38 in., see AISC *Specification* Table J3.2 footnote b.
$\Omega = 2.00$	$\phi = 0.75$	

Available Strength of Fillet Welds per inch of weld (ϕS)		
Weld Size (in.)	E60XX (k/in.)	E70XX (k/in.)
3/16	3.58	4.18
1/4	4.77	5.57
5/16	5.97	6.96
3/8	7.16	8.35
7/16	8.35	9.74
1/2	9.55	11.14
5/8	11.93	13.92
3/4	14.32	16.70

(not considering increase in throat with submerged arc weld process)

Reference Diagrams

Table 7-3 (continued) — Slip-Critical Connections
Available Shear Strength, kips (Class A Faying Surface, μ = 0.30)

Group B Bolts — A490, A490M, F2280, A354 Grade BD

Hole Type	Loading	5/8 (24) ASD r_n/Ω	5/8 (24) LRFD ϕr_n	3/4 (35) ASD	3/4 (35) LRFD	7/8 (49) ASD	7/8 (49) LRFD	1 (64) ASD	1 (64) LRFD
STD/SSLT	S	5.42	8.14	7.91	11.9	11.1	16.6	14.5	21.7
STD/SSLT	D	10.8	16.3	15.8	23.7	22.1	33.2	28.9	43.4
OVS/SSLP	S	4.62	6.92	6.74	10.1	9.44	14.1	12.3	18.4
OVS/SSLP	D	9.25	13.8	13.5	20.2	18.9	28.2	24.7	36.9
LSL	S	3.80	5.70	5.54	8.31	7.76	11.6	10.1	15.2
LSL	D	7.60	11.4	11.1	16.6	15.5	23.3	20.3	30.4

Hole Type	Loading	1 1/8 (80) ASD r_n/Ω	1 1/8 (80) LRFD ϕr_n	1 1/4 (102) ASD	1 1/4 (102) LRFD	1 3/8 (121) ASD	1 3/8 (121) LRFD	1 1/2 (148) ASD	1 1/2 (148) LRFD
STD/SSLT	S	18.1	27.1	23.1	34.6	27.3	41.0	33.4	50.2
STD/SSLT	D	36.2	54.2	46.1	69.2	54.7	82.0	66.9	100
OVS/SSLP	S	15.4	23.1	19.6	29.4	23.3	34.9	28.5	42.6
OVS/SSLP	D	30.8	46.1	39.3	58.8	46.6	69.7	57.0	85.3
LSL	S	12.7	19.0	16.2	24.2	19.2	28.7	23.4	35.1
LSL	D	25.3	38.0	32.3	48.4	38.3	57.4	46.9	70.2

STD = standard hole   S = single shear
OVS = oversized hole   D = double shear
SSLT = short-slotted hole transverse to the line of force
SSLP = short-slotted hole parallel to the line of force
LSL = long-slotted hole transverse or parallel to the line of force

Hole Type	ASD	LRFD
STD and SSLT	Ω = 1.50	φ = 1.00
OVS and SSLP	Ω = 1.76	φ = 0.85
LSL	Ω = 2.14	φ = 0.70

Note: Slip-critical bolt values assume no more than one filler has been provided or bolts have been added to distribute loads in the fillers.
See AISC *Specification* Sections J3.8 and J5 for provisions when fillers are present.
For Class B faying surfaces, multiply the tabulated available strength by 1.67.

Table 7-3 — Slip-Critical Connections
Available Shear Strength, kips (Class A Faying Surface, μ = 0.30)

Group A Bolts — A325, A325M, F1852, A354 Grade BC, A449

Hole Type	Loading	5/8 (19) ASD r_n/Ω	5/8 (19) LRFD ϕr_n	3/4 (28) ASD	3/4 (28) LRFD	7/8 (39) ASD	7/8 (39) LRFD	1 (51) ASD	1 (51) LRFD
STD/SSLT	S	4.29	6.44	6.33	9.49	8.81	13.2	11.5	17.3
STD/SSLT	D	8.59	12.9	12.7	19.0	17.6	26.4	23.1	34.6
OVS/SSLP	S	3.66	5.47	5.39	8.07	7.51	11.2	9.82	14.7
OVS/SSLP	D	7.32	10.9	10.8	16.1	15.0	22.5	19.6	29.4
LSL	S	3.01	4.51	4.44	6.64	6.18	9.25	8.08	12.1
LSL	D	6.02	9.02	8.87	13.3	12.4	18.5	16.2	24.2

Hole Type	Loading	1 1/8 (56) ASD r_n/Ω	1 1/8 (56) LRFD ϕr_n	1 1/4 (71) ASD	1 1/4 (71) LRFD	1 3/8 (85) ASD	1 3/8 (85) LRFD	1 1/2 (103) ASD	1 1/2 (103) LRFD
STD/SSLT	S	12.7	19.0	16.0	24.1	19.2	28.8	23.3	34.9
STD/SSLT	D	25.3	38.0	32.1	48.1	38.4	57.6	46.6	69.8
OVS/SSLP	S	10.8	16.1	13.7	20.5	16.4	24.5	19.8	29.7
OVS/SSLP	D	21.6	32.3	27.4	40.9	32.7	49.0	39.7	59.4
LSL	S	8.87	13.3	11.2	16.8	13.5	20.2	16.3	24.4
LSL	D	17.7	26.6	22.5	33.7	26.9	40.3	32.6	48.9

STD = standard hole   S = single shear
OVS = oversized hole   D = double shear
SSLT = short-slotted hole transverse to the line of force
SSLP = short-slotted hole parallel to the line of force
LSL = long-slotted hole transverse or parallel to the line of force

Hole Type	ASD	LRFD
STD and SSLT	Ω = 1.50	φ = 1.00
OVS and SSLP	Ω = 1.76	φ = 0.85
LSL	Ω = 2.14	φ = 0.70

Note: Slip-critical bolt values assume no more than one filler has been provided or bolts have been added to distribute loads in the fillers.
See AISC *Specification* Sections J3.8 and J5 for provisions when fillers are present.
For Class B faying surfaces, multiply the tabulated available strength by 1.67.

Reference Diagrams

Table 7-4 (continued)
Available Bearing Strength at Bolt Holes Based on Bolt Spacing
kips/in. thickness

Hole Type	Bolt Spacing, s, in.	F_u ksi	Nominal Bolt Diameter, d, in.							
			1 1/8		1 1/4		1 3/8		1 1/2	
			r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD
STD SSLT	2²/₃ d_b	58	63.1	94.6	70.3	105	77.6	116	84.8	127
		65	70.7	106	78.8	118	86.9	130	95.1	143
	3 in.	58	63.1	94.6	—	—	—	—	—	—
		65	70.7	106	—	—	—	—	—	—
SSLP	2²/₃ d_b	58	52.2	78.3	59.5	89.2	66.7	100	74.0	111
		65	58.5	87.8	66.6	99.9	74.8	112	82.9	124
	3 in.	58	52.2	78.3	—	—	—	—	—	—
		65	58.5	87.8	—	—	—	—	—	—
OVS	2²/₃ d_b	58	54.4	81.6	61.6	92.4	68.9	103	76.1	114
		65	60.9	91.4	69.1	104	77.2	116	85.3	128
	3 in.	58	54.4	81.6	—	—	—	—	—	—
		65	60.9	91.4	—	—	—	—	—	—
LSLP	2²/₃ d_b	58	6.53	9.79	7.25	10.9	7.98	12.0	8.70	13.1
		65	7.31	11.0	8.13	12.2	8.94	13.4	9.75	14.6
	3 in.	58	6.53	9.79	—	—	—	—	—	—
		65	7.31	11.0	—	—	—	—	—	—
LSLT	2²/₃ d_b	58	52.6	78.8	58.6	87.9	64.6	97.0	70.7	106
		65	58.9	88.4	65.7	98.5	72.4	109	79.2	119
	3 in.	58	52.6	78.8	—	—	—	—	—	—
		65	58.9	88.4	—	—	—	—	—	—
STD, SSLT, SSLP, OVS, LSLP	$s \geq s_{full}$	58	78.3	117	87.0	131	95.7	144	104	157
		65	87.8	132	97.5	146	107	161	117	176
LSLT	$s \geq s_{full}$	58	65.3	97.9	72.5	109	79.8	120	87.0	131
		65	73.1	110	81.3	122	89.4	134	97.5	146
Spacing for full bearing strength s_{full}[a], in.	STD, SSLT, LSLT		3⁷/₁₆		3¹³/₁₆		4³/₁₆		4⁹/₁₆	
	OVS		3¹¹/₁₆		4¹/₁₆		4⁷/₁₆		4¹³/₁₆	
	SSLP		3³/₄		4¹/₈		4¹/₂		4⁷/₈	
	LSLP		5¹/₁₆		5⁵/₈		6³/₁₆		6³/₄	
Minimum Spacing[a] = 2²/₃d, in.			3		3⁵/₁₆		3¹¹/₁₆		4	

STD = standard hole
SSLT = short-slotted hole oriented transverse to the line of force
SSLP = short-slotted hole oriented parallel to the line of force
OVS = oversized hole
LSLP = long-slotted hole oriented parallel to the line of force
LSLT = long-slotted hole oriented transverse to the line of force

ASD	$\Omega = 2.00$	LRFD	$\phi = 0.75$

— indicates spacing less than minimum spacing required per AISC Specification Section J3.3.
Note: Spacing indicated is from the center of the hole or slot to the center of the adjacent hole or slot in the line of force. Hole deformation is not considered, see AISC Specification Section J3.10.
[a] A decimal value has been rounded to the nearest sixteenth of an inch.

Table 7-4
Available Bearing Strength at Bolt Holes Based on Bolt Spacing
kips/in. thickness

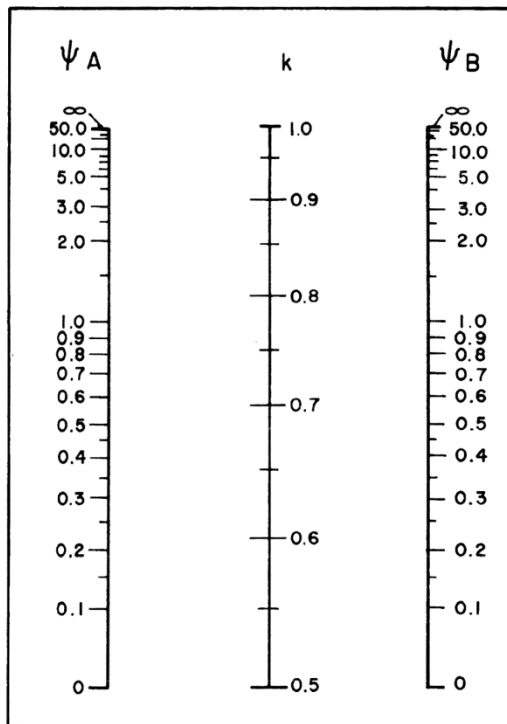| Hole Type	Bolt Spacing, s, in.	F_u ksi	Nominal Bolt Diameter, d, in.							
			5/8		3/4		7/8		1	
			r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD
STD SSLT	2²/₃ d_b	58	34.1	51.1	41.3	62.0	48.6	72.9	55.8	83.7
		65	38.2	57.3	46.3	69.5	54.4	81.7	62.6	93.8
	3 in.	58	43.5	65.3	52.2	78.3	60.9	91.4	67.4	101
		65	48.8	73.1	58.5	87.8	68.3	102	75.6	113
SSLP	2²/₃ d_b	58	27.6	41.3	34.8	52.2	42.1	63.1	47.1	70.7
		65	30.9	46.3	39.0	58.5	47.1	70.7	52.8	79.2
	3 in.	58	43.5	65.3	52.2	78.3	60.9	91.4	58.7	88.1
		65	48.8	73.1	58.5	87.8	68.3	102	65.8	98.7
OVS	2²/₃ d_b	58	29.7	44.6	37.0	55.5	44.2	66.3	49.3	74.0
		65	33.3	50.0	41.4	62.2	49.6	74.3	55.3	82.9
	3 in.	58	43.5	65.3	52.2	78.3	60.9	91.4	60.9	91.4
		65	48.8	73.1	58.5	87.8	68.3	102	68.3	102
LSLP	2²/₃ d_b	58	3.62	5.44	4.35	6.53	5.08	7.61	5.80	8.70
		65	4.06	6.09	4.88	7.31	5.69	8.53	6.50	9.75
	3 in.	58	43.5	65.3	39.2	58.7	28.3	42.4	17.4	26.1
		65	48.8	73.1	43.9	65.8	31.7	47.5	19.5	29.3
LSLT	2²/₃ d_b	58	28.4	42.6	34.4	51.7	40.5	60.7	46.5	69.8
		65	31.8	47.7	38.6	57.9	45.4	68.0	52.1	78.2
	3 in.	58	36.3	54.4	43.5	65.3	50.8	76.1	56.2	84.3
		65	40.6	60.9	48.8	73.1	56.9	85.3	63.0	94.5
STD, SSLT, SSLP, OVS, LSLP	$s \geq s_{full}$	58	43.5	65.3	52.2	78.3	60.9	91.4	69.6	104
		65	48.8	73.1	58.5	87.8	68.3	102	78.0	117
LSLT	$s \geq s_{full}$	58	36.3	54.4	43.5	65.3	50.8	76.1	58.0	87.0
		65	40.6	60.9	48.8	73.1	56.9	85.3	65.0	97.5
Spacing for full bearing strength s_{full}[a], in.	STD, SSLT, LSLT		1¹⁵/₁₆		2⁵/₁₆		2¹¹/₁₆		3¹/₁₆	
	OVS		2¹/₁₆		2⁷/₁₆		2¹³/₁₆		3³/₄	
	SSLP		2¹/₈		2¹/₂		2⁷/₈		3⁵/₁₆	
	LSLP		2¹³/₁₆		3³/₈		3¹⁵/₁₆		4¹/₂	
Minimum Spacing[a] = 2²/₃d, in.			1¹¹/₁₆		2		2⁵/₁₆		2¹¹/₁₆	

STD = standard hole
SSLT = short-slotted hole oriented transverse to the line of force
SSLP = short-slotted hole oriented parallel to the line of force
OVS = oversized hole
LSLP = long-slotted hole oriented parallel to the line of force
LSLT = long-slotted hole oriented transverse to the line of force

ASD	$\Omega = 2.00$	LRFD	$\phi = 0.75$

Note: Spacing indicated is from the center of the hole or slot to the center of the adjacent hole or slot in the line of force. Hole deformation is considered. When hole deformation is not considered, see AISC Specification Section J3.10.
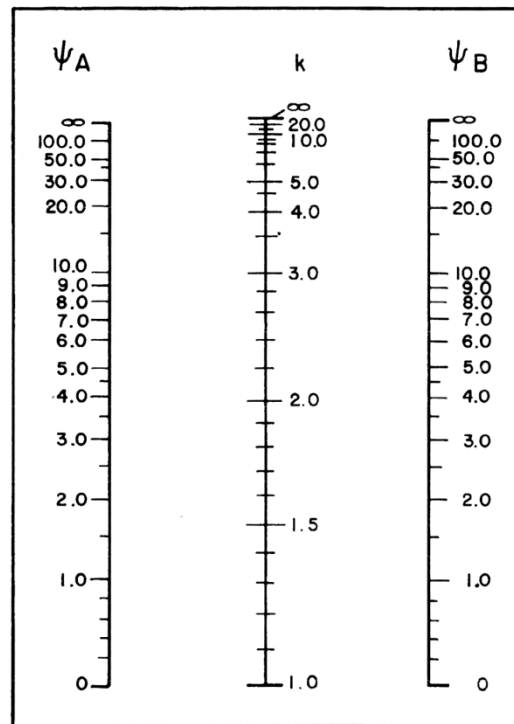[a] A decimal value has been rounded to the nearest sixteenth of an inch.

Reference Diagrams

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column shown by dashed line						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design values when ideal conditions are approximated	0.65	0.80	1.0	1.2	2.10	2.0

End conditions code

Rotation fixed, Translation fixed

Rotation free, Translation fixed
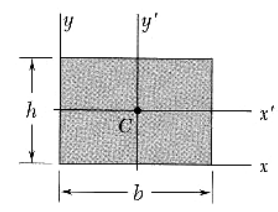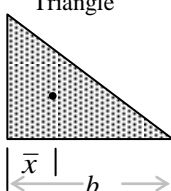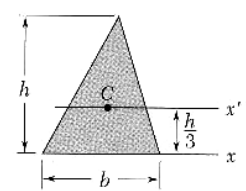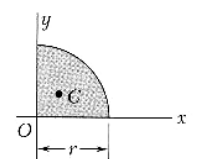
Rotation fixed, Translation free

Rotation free, Translation free
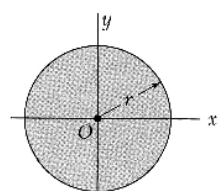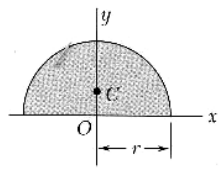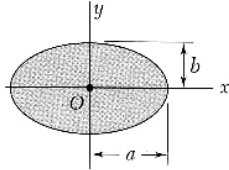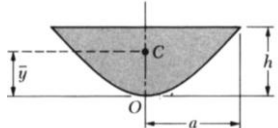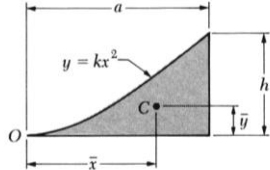
(a)

Nonsway Frames

(b)

Sway Frames

Reference Geometry

Shape	Figure	Moments of Inertia	Properties
Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ *about* $I_y = \frac{1}{3}b^3h$ *bottom* *left* $J_C = \frac{1}{12}bh(b^2+h^2)$	Area = bh $\bar{x} = b/2$ $\bar{y} = h/2$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{36}b^3h$	Area = $\frac{bh}{2}$ $\bar{x} = \frac{b}{3}$ $\bar{y} = \frac{h}{3}$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$	Area $= \pi r^2 = \frac{\pi d^2}{4}$ $\bar{x} = 0$ $\bar{y} = 0$
Semicircle		$\bar{I}_x = 0.1098r^4$ $\bar{I}_y = \pi r^4 / 8$	Area $= \frac{\pi r^2}{2} = \frac{\pi d^2}{8}$ $\bar{x} = 0$ $\bar{y} = \frac{4r}{3\pi}$
Quarter circle		$\bar{I}_x = 0.0549r^4$ $\bar{I}_y = 0.0549r^4$	Area $= \frac{\pi r^2}{4} = \frac{\pi d^2}{16}$ $\bar{x} = \frac{4r}{3\pi}$ $\bar{y} = \frac{4r}{3\pi}$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2+b^2)$	Area $= \pi ab$ $\bar{x} = 0$ $\bar{y} = 0$
Semiparabolic area Parabolic area		$\bar{I}_x = 16ah^3/175$ $\bar{I}_y = 4a^3h/15$	Area $= \frac{4ah}{3}$ $\bar{x} = 0$ $\bar{y} = \frac{3h}{5}$
Parabolic span-drel		$\bar{I}_x = 37ah^3/2100$ $\bar{I}_y = a^3h/80$	Area $= \frac{ah}{3}$ $\bar{x} = \frac{3a}{4}$ $\bar{y} = \frac{3h}{10}$