ARCH 614: Practice Quiz 8

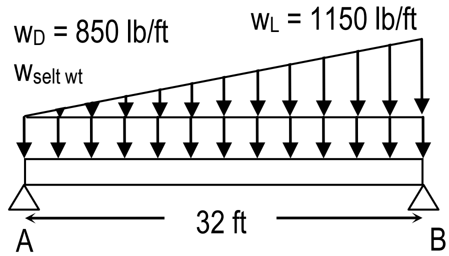
Note: No aids are allowed for part 1. One side of a letter sized paper with notes is allowed during part 2, along with a silent, **non-programmable** *calculator. There are reference charts on pages 2-6 for part 2.*

Clearly show your work and answer.

Part 1) Worth 5 points (conceptual questions)

Part 2) Worth 45 points

(NOTE: The loading type [ex, live, dead, wind...] and sizes <u>*can and will*</u> *be changed for the quiz with respect to the beam diagrams and formula provided.)*



A wide flange beam of A992 steel (F_y = 50 ksi, E = 30 x 10^3 ksi) is needed to span 32 ft and support uniformly distributed loads of 850 lb/ft of dead load (from materials), the self weight, and 1150 lb/ft of linearly distributed live load. The beam is simply supported with a maximum unbraced length of 11 ft.

a) Select the most economical beam based on flexural strength using the provided chart (including self weight). *Assume that the dead load will determine the location of the maximum bending moment and superimpose the live load moment at that location.*

b) If a W21 x 44 (A = 13.0 in.^{2}, d = 20.66 in., t_w = 0.35 in., b_f = 6.50 in., t_f = 0.45 in., I_x = 843 in.^{4}) is chosen, is it adequate for shear?

c) Determine the moment of inertia required such that the total [*or live load or dead load...*] deflection, ignoring self weight, does not exceed 1.25 inches. *Assume that the distributed load determines the location of the maximum deflection.*

Answers – <u>*Not provided on actual quiz!*</u>

Disclaimer: Answers have NOT been painstakingly researched.

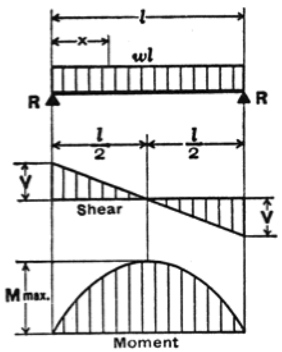
a) M_u = 248.3 k-ft, use W14x48 (M_u* > 250.5 k-ft)

b) V_u* = 36.8 k, ϕV_n = 216.9 k, ∴ OK

c) $I_{req'd}$ = 897 in^4 [$I_{req'd-dead}$ = 535 in^4, $I_{req'd-live}$ = 362.3 in^4]

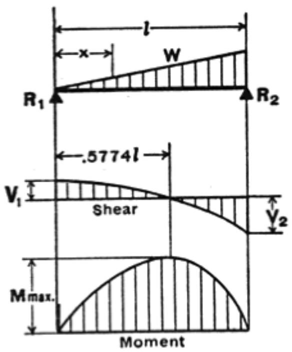
REFERENCE CHARTS FOR QUIZ 8

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load		$= wl$
$R = V$		$= \frac{wl}{2}$
V_x		$= w\left(\frac{l}{2} - x\right)$
M max.	(at center)	$= \frac{wl^2}{8}$
M_x		$= \frac{wx}{2}(l - x)$
Δmax.	(at center)	$= \frac{5wl^4}{384EI}$
Δx		$= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

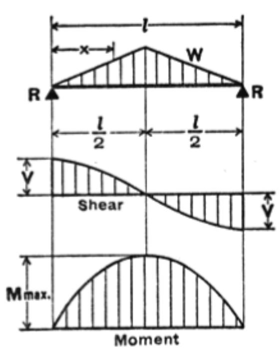
2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END



Total Equiv. Uniform Load		$= \frac{16W}{9\sqrt{3}} = 1.0264W$
$R_1 = V_1$		$= \frac{W}{3}$
$R_2 = V_2$ max.		$= \frac{2W}{3}$
V_x		$= \frac{W}{3} - \frac{Wx^2}{l^2}$
M max.	$\left(\text{at } x = \frac{l}{\sqrt{3}} = .5774l\right)$	$= \frac{2Wl}{9\sqrt{3}} = .1283\,Wl$
M_x		$= \frac{Wx}{3l^2}(l^2 - x^2)$
Δmax.	$\left(\text{at } x = l\sqrt{1 - \sqrt{\frac{8}{15}}} = .5193l\right)$	$= .01304\frac{Wl^3}{EI}$
Δx		$= \frac{Wx}{180EIl^2}(3x^4 - 10l^2x^2 + 7l^4)$

$W = \frac{wl}{2}$

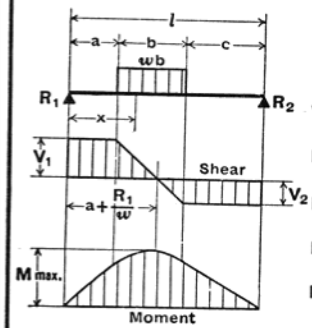
3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER



Total Equiv. Uniform Load		$= \frac{4W}{3}$
$R = V$		$= \frac{W}{2}$
V_x	$\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{W}{2l^2}(l^2 - 4x^2)$
M max.	(at center)	$= \frac{Wl}{6}$
M_x	$\left(\text{when } x < \frac{l}{2}\right)$	$= Wx\left(\frac{1}{2} - \frac{2x^2}{3l^2}\right)$
Δmax.	(at center)	$= \frac{Wl^3}{60EI}$
Δx	$\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{Wx}{480EIl^2}(5l^2 - 4x^2)^2$

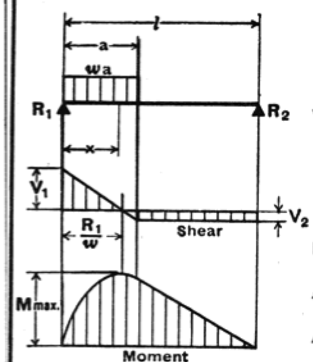
$W = \frac{wl}{2}$

4. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED



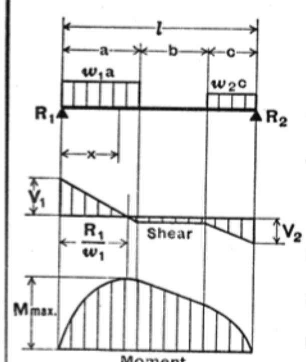
$R_1 = V_1$	(max. when $a < c$)	$= \frac{wb}{2l}(2c + b)$
$R_2 = V_2$	(max. when $a > c$)	$= \frac{wb}{2l}(2a + b)$
V_x	(when $x > a$ and $< (a + b)$)	$= R_1 - w(x - a)$
M max.	$\left(\text{at } x = a + \frac{R_1}{w}\right)$	$= R_1\left(a + \frac{R_1}{2w}\right)$
M_x	(when $x < a$)	$= R_1 x$
M_x	(when $x > a$ and $< (a + b)$)	$= R_1 x - \frac{w}{2}(x - a)^2$
M_x	(when $x > (a + b)$)	$= R_2(l - x)$

5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END



$R_1 = V_1$ max.		$= \frac{wa}{2l}(2l - a)$
$R_2 = V_2$		$= \frac{wa^2}{2l}$
V_x	(when $x < a$)	$= R_1 - wx$
M max.	$\left(\text{at } x = \frac{R_1}{w}\right)$	$= \frac{R_1^2}{2w}$
M_x	(when $x < a$)	$= R_1 x - \frac{wx^2}{2}$
M_x	(when $x > a$)	$= R_2(l - x)$
Δx	(when $x < a$)	$= \frac{wx}{24EIl}\left(a^2(2l - a)^2 - 2ax^2(2l - a) + lx^3\right)$
Δx	(when $x > a$)	$= \frac{wa^2(l - x)}{24EIl}(4xl - 2x^2 - a^2)$

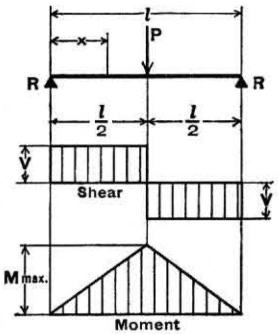
6. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END



$R_1 = V_1$		$= \frac{w_1 a(2l - a) + w_2 c^2}{2l}$
$R_2 = V_2$		$= \frac{w_2 c(2l - c) + w_1 a^2}{2l}$
V_x	(when $x < a$)	$= R_1 - w_1 x$
V_x	(when $x > a$ and $< (a + b)$)	$= R_1 - w_1 a$
V_x	(when $x > (a + b)$)	$= R_2 - w_2(l - x)$
M max.	$\left(\text{at } x = \frac{R_1}{w_1} \text{ when } R_1 < w_1 a\right)$	$= \frac{R_1^2}{2w_1}$
M max.	$\left(\text{at } x = l - \frac{R_2}{w_2} \text{ when } R_2 < w_2 c\right)$	$= \frac{R_2^2}{2w_2}$
M_x	(when $x < a$)	$= R_1 x - \frac{w_1 x^2}{2}$
M_x	(when $x > a$ and $< (a + b)$)	$= R_1 x - \frac{w_1 a}{2}(2x - a)$
M_x	(when $x > (a + b)$)	$= R_2(l - x) - \frac{w_2(l - x)^2}{2}$

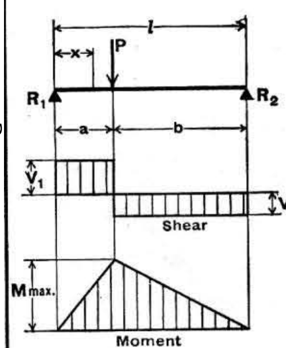
REFERENCE CHARTS FOR QUIZ 8

7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER



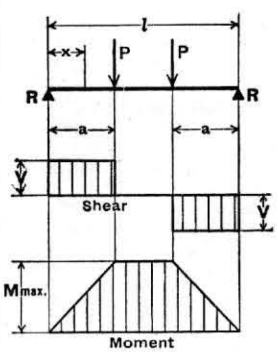
Total Equiv. Uniform Load		$= 2P$
$R = V$		$= \frac{P}{2}$
M max.	(at point of load)	$= \frac{Pl}{4}$
M_x	(when $x < \frac{l}{2}$)	$= \frac{Px}{2}$
Δmax.	(at point of load)	$= \frac{Pl^3}{48EI}$
Δ_x	(when $x < \frac{l}{2}$)	$= \frac{Px}{48EI}(3l^2 - 4x^2)$

8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT



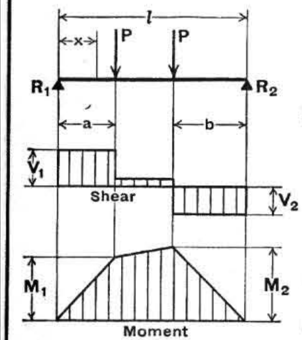
Total Equiv. Uniform Load		$= \frac{8Pab}{l^2}$
$R_1 = V_1$	(max. when $a < b$)	$= \frac{Pb}{l}$
$R_2 = V_2$	(max. when $a > b$)	$= \frac{Pa}{l}$
M max.	(at point of load)	$= \frac{Pab}{l}$
M_x	(when $x < a$)	$= \frac{Pbx}{l}$
Δmax.	(at $x = \sqrt{\frac{a(a+2b)}{3}}$ when $a > b$)	$= \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EIl}$
Δa	(at point of load)	$= \frac{Pa^2b^2}{3EIl}$
Δ_x	(when $x < a$)	$= \frac{Pbx}{6EIl}(l^2 - b^2 - x^2)$

9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED



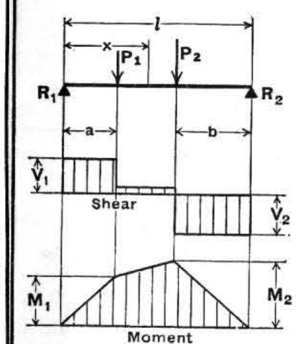
Total Equiv. Uniform Load		$= \frac{8Pa}{l}$
$R = V$		$= P$
M max.	(between loads)	$= Pa$
M_x	(when $x < a$)	$= Px$
Δmax.	(at center)	$= \frac{Pa}{24EI}(3l^2 - 4a^2)$
Δ_x	(when $x < a$)	$= \frac{Px}{6EI}(3la - 3a^2 - x^2)$
Δ_x	(when $x > a$ and $< (l-a)$)	$= \frac{Pa}{6EI}(3lx - 3x^2 - a^2)$

10. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



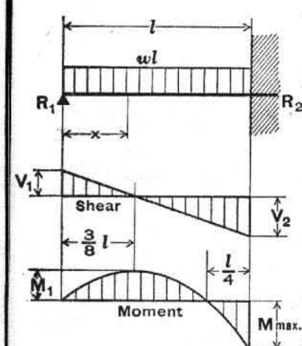
$R_1 = V_1$	(max. when $a < b$)	$= \frac{P}{l}(l - a + b)$
$R_2 = V_2$	(max. when $a > b$)	$= \frac{P}{l}(l - b + a)$
V_x	(when $x > a$ and $< (l-b)$)	$= \frac{P}{l}(b - a)$
M_1	(max. when $a > b$)	$= R_1 a$
M_2	(max. when $a < b$)	$= R_2 b$
M_x	(when $x < a$)	$= R_1 x$
M_x	(when $x > a$ and $< (l-b)$)	$= R_1 x - P(x - a)$

11. SIMPLE BEAM—TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



$R_1 = V_1$		$= \frac{P_1(l-a) + P_2 b}{l}$
$R_2 = V_2$		$= \frac{P_1 a + P_2(l-b)}{l}$
V_x	(when $x > a$ and $< (l-b)$)	$= R_1 - P_1$
M_1	(max. when $R_1 < P_1$)	$= R_1 a$
M_2	(max. when $R_2 < P_2$)	$= R_2 b$
M_x	(when $x < a$)	$= R_1 x$
M_x	(when $x > a$ and $< (l-b)$)	$= R_1 x - P_1(x - a)$

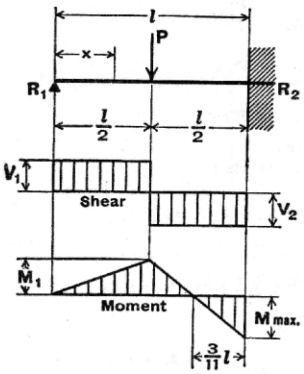
12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load		$= wl$
$R_1 = V_1$		$= \frac{3wl}{8}$
$R_2 = V_2$ max.		$= \frac{5wl}{8}$
V_x		$= R_1 - wx$
M max.		$= \frac{wl^2}{8}$
M_1	(at $x = \frac{3}{8}l$)	$= \frac{9}{128}wl^2$
M_x		$= R_1 x - \frac{wx^2}{2}$
Δmax.	(at $x = \frac{l}{16}(1 + \sqrt{33}) = .4215l$)	$= \frac{wl^4}{185EI}$
Δ_x		$= \frac{wx}{48EI}(l^3 - 3lx^2 + 2x^3)$

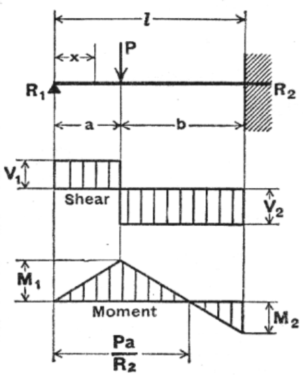
REFERENCE CHARTS FOR QUIZ 8

13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT CENTER



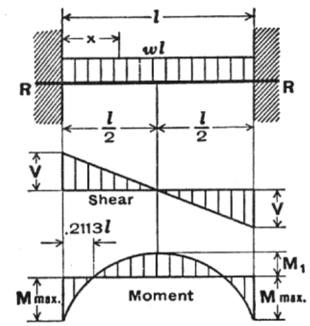
Total Equiv. Uniform Load		$= \frac{3P}{2}$
$R_1 = V_1$		$= \frac{5P}{16}$
$R_2 = V_2$ max.		$= \frac{11P}{16}$
M max.	(at fixed end)	$= \frac{3Pl}{16}$
M_1	(at point of load)	$= \frac{5Pl}{32}$
M_x	(when $x < \frac{l}{2}$)	$= \frac{5Px}{16}$
M_x	(when $x > \frac{l}{2}$)	$= P\left(\frac{l}{2} - \frac{11x}{16}\right)$
Δmax.	(at $x = l\sqrt{\frac{1}{5}} = .4472l$)	$= \frac{Pl^3}{48EI\sqrt{5}} = .009317\frac{Pl^3}{EI}$
Δx	(at point of load)	$= \frac{7Pl^3}{768EI}$
Δx	(when $x < \frac{l}{2}$)	$= \frac{Px}{96EI}(3l^2 - 5x^2)$
Δx	(when $x > \frac{l}{2}$)	$= \frac{P}{96EI}(x-l)^2(11x - 2l)$

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT ANY POINT



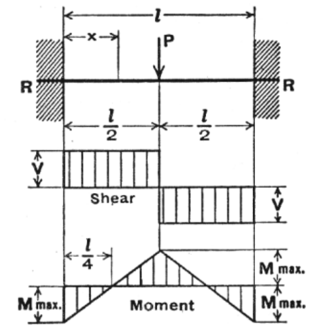
$R_1 = V_1$		$= \frac{Pb^2}{2l^3}(a + 2l)$
$R_2 = V_2$		$= \frac{Pa}{2l^3}(3l^2 - a^2)$
M_1	(at point of load)	$= R_1 a$
M_2	(at fixed end)	$= \frac{Pab}{2l^2}(a + l)$
M_x	(when $x < a$)	$= R_1 x$
M_x	(when $x > a$)	$= R_1 x - P(x - a)$
Δmax.	(when $a < .414l$ at $x = l\frac{l^2+a^2}{3l^2-a^2}$)	$= \frac{Pa}{3EI}\frac{(l^2-a^2)^3}{(3l^2-a^2)^2}$
Δmax.	(when $a > .414l$ at $x = l\sqrt{\frac{a}{2l+a}}$)	$= \frac{Pab^2}{6EI}\sqrt{\frac{a}{2l+a}}$
Δa	(at point of load)	$= \frac{Pa^2b^3}{12EIl^3}(3l + a)$
Δx	(when $x < a$)	$= \frac{Pb^2x}{12EIl^3}(3al^2 - 2lx^2 - ax^2)$
Δx	(when $x > a$)	$= \frac{Pa}{12EIl^3}(l-x)^2(3l^2x - a^2x - 2a^2l)$

15. BEAM FIXED AT BOTH ENDS—UNIFORMLY DISTRIBUTED LOADS



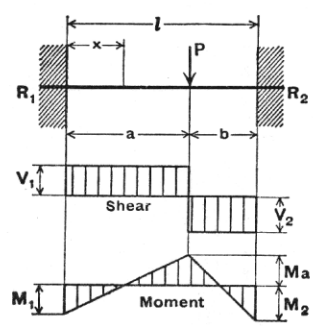
Total Equiv. Uniform Load		$= \frac{2wl}{3}$
$R = V$		$= \frac{wl}{2}$
V_x		$= w\left(\frac{l}{2} - x\right)$
M max.	(at ends)	$= \frac{wl^2}{12}$
M_1	(at center)	$= \frac{wl^2}{24}$
M_x		$= \frac{w}{12}(6lx - l^2 - 6x^2)$
Δmax.	(at center)	$= \frac{wl^4}{384EI}$
Δx		$= \frac{wx^2}{24EI}(l - x)^2$

16. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT CENTER



Total Equiv. Uniform Load		$= P$
$R = V$		$= \frac{P}{2}$
M max.	(at center and ends)	$= \frac{Pl}{8}$
M_x	(when $x < \frac{l}{2}$)	$= \frac{P}{8}(4x - l)$
Δmax.	(at center)	$= \frac{Pl^3}{192EI}$
Δx	(when $x < \frac{l}{2}$)	$= \frac{Px^2}{48EI}(3l - 4x)$

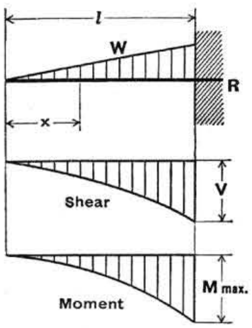
17. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$	(max. when $a < b$)	$= \frac{Pb^2}{l^3}(3a + b)$
$R_2 = V_2$	(max. when $a > b$)	$= \frac{Pa^2}{l^3}(a + 3b)$
M_1	(max. when $a < b$)	$= \frac{Pab^2}{l^2}$
M_2	(max. when $a > b$)	$= \frac{Pa^2b}{l^2}$
M_a	(at point of load)	$= \frac{2Pa^2b^2}{l^3}$
M_x	(when $x < a$)	$= R_1x - \frac{Pab^2}{l^2}$
Δmax.	(when $a > b$ at $x = \frac{2al}{3a+b}$)	$= \frac{2Pa^3b^2}{3EI(3a+b)^2}$
Δa	(at point of load)	$= \frac{Pa^3b^3}{3EIl^3}$
Δx	(when $x < a$)	$= \frac{Pb^2x^2}{6EIl^3}(3al - 3ax - bx)$

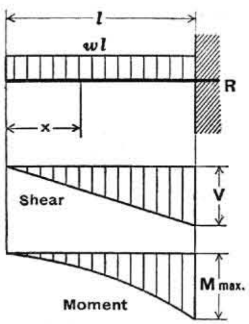
REFERENCE CHARTS FOR QUIZ 8

18. CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END



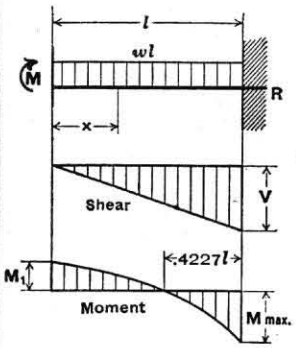
Total Equiv. Uniform Load	$= \frac{8}{3} W$
R = V	$= W$
V_x	$= W \frac{x^2}{l^2}$
M max. (at fixed end)	$= \frac{Wl}{3}$
M_x	$= \frac{Wx^3}{3l^2}$
Δmax. (at free end)	$= \frac{Wl^3}{15EI}$
Δx	$= \frac{W}{60EIl^2}(x^5 - 5l^4x + 4l^5)$

19. CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD



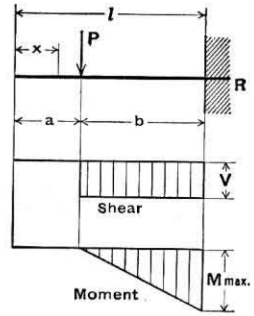
Total Equiv. Uniform Load	$= 4wl$
R = V	$= wl$
V_x	$= wx$
M max. (at fixed end)	$= \frac{wl^2}{2}$
M_x	$= \frac{wx^2}{2}$
Δmax. (at free end)	$= \frac{wl^4}{8EI}$
Δx	$= \frac{w}{24EI}(x^4 - 4l^3x + 3l^4)$

20. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—UNIFORMLY DISTRIBUTED LOAD



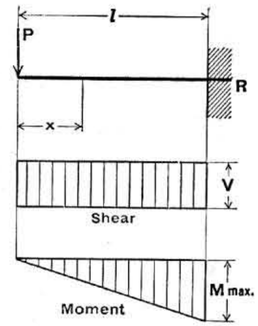
Total Equiv. Uniform Load	$= \frac{8}{3} wl$
R = V	$= wl$
V_x	$= wx$
M max. (at fixed end)	$= \frac{wl^2}{3}$
M_1 (at deflected end)	$= \frac{wl^2}{6}$
M_x	$= \frac{w}{6}(l^2 - 3x^2)$
Δmax. (at deflected end)	$= \frac{wl^4}{24EI}$
Δx	$= \frac{w(l^2 - x^2)^2}{24EI}$

21. CANTILEVER BEAM—CONCENTRATED LOAD AT ANY POINT



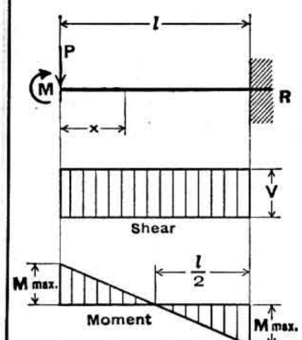
Total Equiv. Uniform Load	$= \frac{8Pb}{l}$
R = V	$= P$
M max. (at fixed end)	$= Pb$
M_x (when x > a)	$= P(x - a)$
Δmax. (at free end)	$= \frac{Pb^2}{6EI}(3l - b)$
Δa (at point of load)	$= \frac{Pb^3}{3EI}$
Δx (when x < a)	$= \frac{Pb^2}{6EI}(3l - 3x - b)$
Δx (when x > a)	$= \frac{P(l - x)^2}{6EI}(3b - l + x)$

22. CANTILEVER BEAM—CONCENTRATED LOAD AT FREE END



Total Equiv. Uniform Load	$= 8P$
R = V	$= P$
M max. (at fixed end)	$= Pl$
M_x	$= Px$
Δmax. (at free end)	$= \frac{Pl^3}{3EI}$
Δx	$= \frac{P}{6EI}(2l^3 - 3l^2x + x^3)$

23. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—CONCENTRATED LOAD AT DEFLECTED END



Total Equiv. Uniform Load	$= 4P$
R = V	$= P$
M max. (at both ends)	$= \frac{Pl}{2}$
M_x	$= P\left(\frac{l}{2} - x\right)$
Δmax. (at deflected end)	$= \frac{Pl^3}{12EI}$
Δx	$= \frac{P(l - x)^2}{12EI}(l + 2x)$

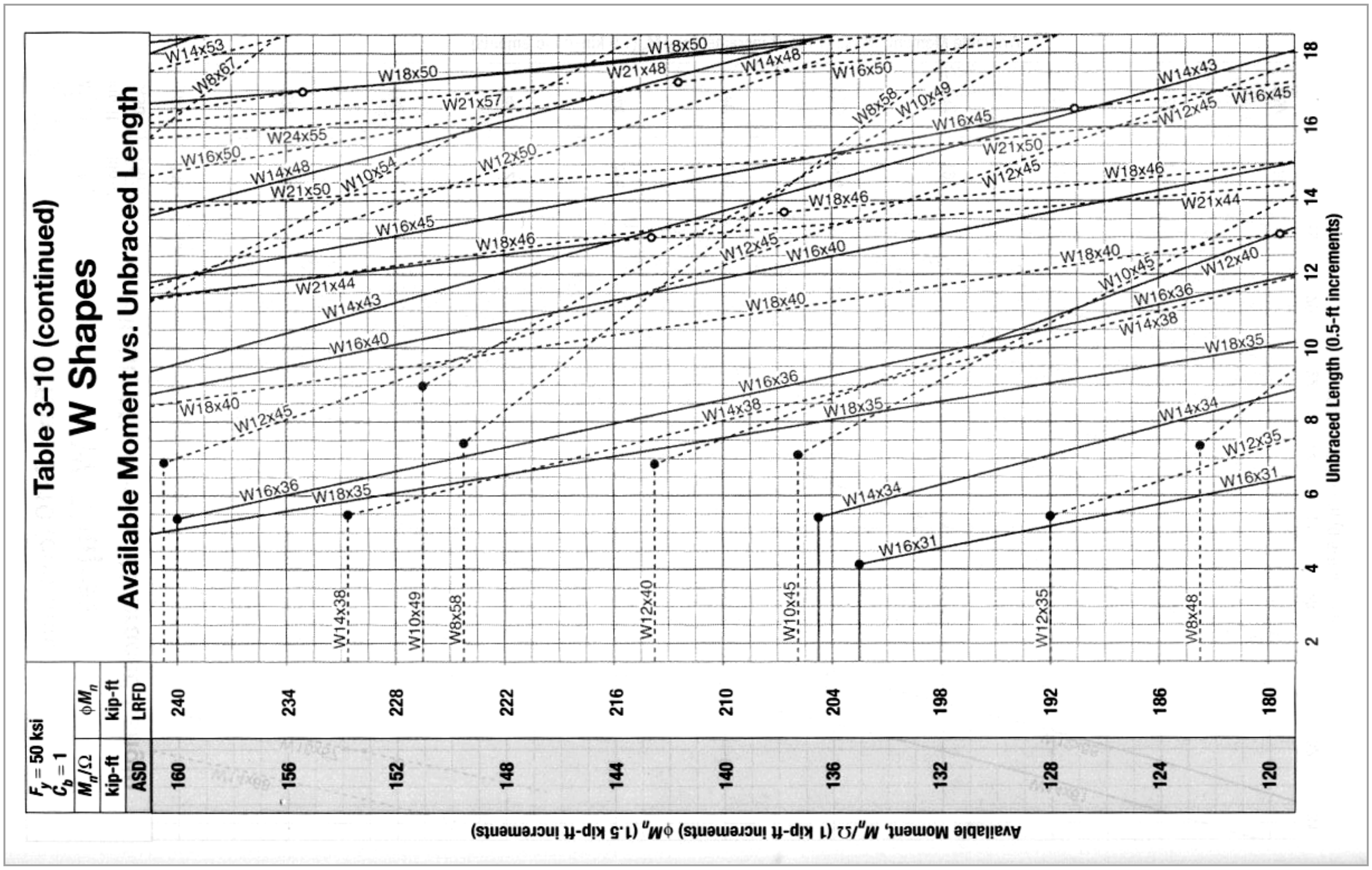


Table 3–10 (continued)
W Shapes
Available Moment vs. Unbraced Length
F_y = 50 ksi
C_b = 1
Available Moment, M_n/Ω (1 kip-ft increments) ϕM_n (1.5 kip-ft increments)
Unbraced Length (0.5-ft increments)

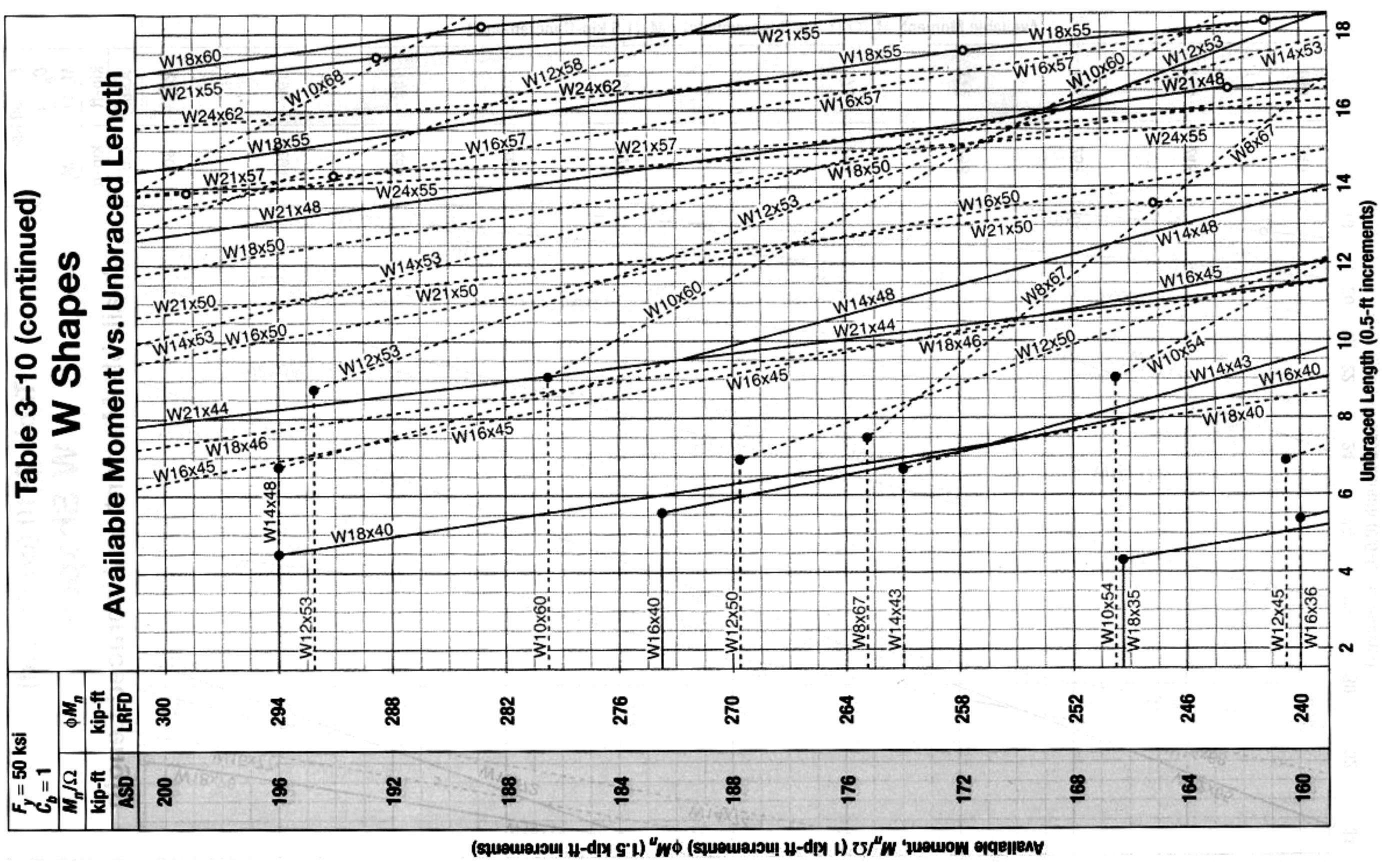


Table 3–10 (continued)
W Shapes
Available Moment vs. Unbraced Length
F_y = 50 ksi
C_b = 1
Available Moment, M_n/Ω (1 kip-ft increments) ϕM_n (1.5 kip-ft increments)
Unbraced Length (0.5-ft increments)