

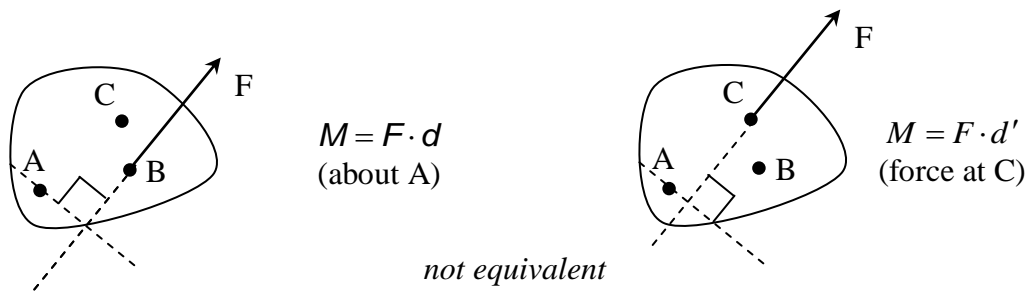
## Moments & Supports

**Notation:**

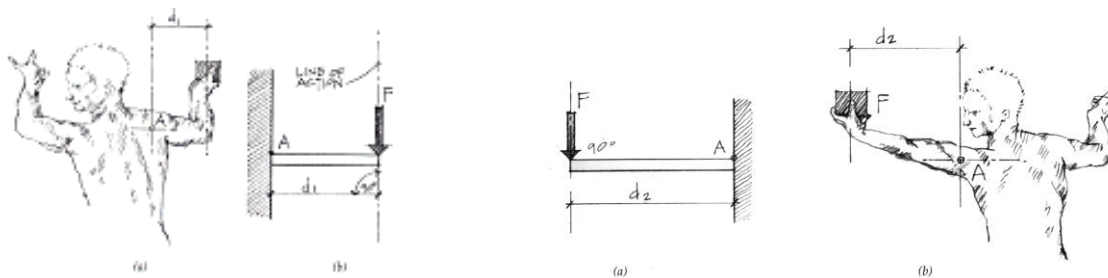
<p><math>d</math> = perpendicular distance to a force from a point</p> <p><math>F</math> = name for force vectors or magnitude of a force, as is <math>P, Q, R</math></p> <p><math>F_x</math> = force component in the x direction</p> <p><math>F_y</math> = force component in the y direction</p> <p><math>FBD</math> = free body diagram</p> <p><math>L</math> = beam span length</p> <p><math>M</math> = moment due to a force</p> <p><math>R_x</math> = resultant component in the x direction</p>	<p><math>R_y</math> = resultant component in the y direction</p> <p><math>w</math> = name for distributed load</p> <p><math>W</math> = name for total force due to distributed load</p> <p><math>x</math> = horizontal distance</p> <p><math>x</math> = x axis direction</p> <p><math>y</math> = y axis direction</p> <p><math>\theta</math> = angle, in a trig equation, ex. <math>\sin \theta</math>, that is measured between the x axis and <i>tail</i> of a vector</p>
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**Moment of a Force About an Axis**

- Two forces of the same size and direction acting at different points *are not equivalent*. They may cause the same **translation**, but they cause different **rotation**.
- DEFINITION: *Moment* – A moment is the tendency of a force to make a body rotate about an axis. It is measured by  $F \times d$ , where  $d$  is the distance **perpendicular** to the line of action of the force and through the axis of rotation.

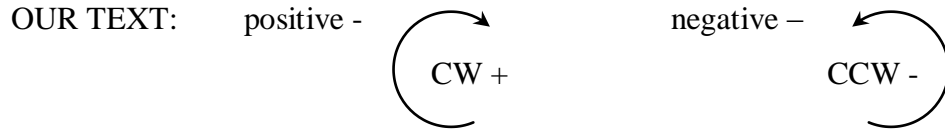


- For the same force, the bigger the **lever arm (or moment arm)**, the bigger the moment magnitude, i.e.  $M_A = F \cdot d_1 < M_A = F \cdot d_2$



- Units: SI: N·m, KN·m  
Engr. English: lb-ft, kip-ft

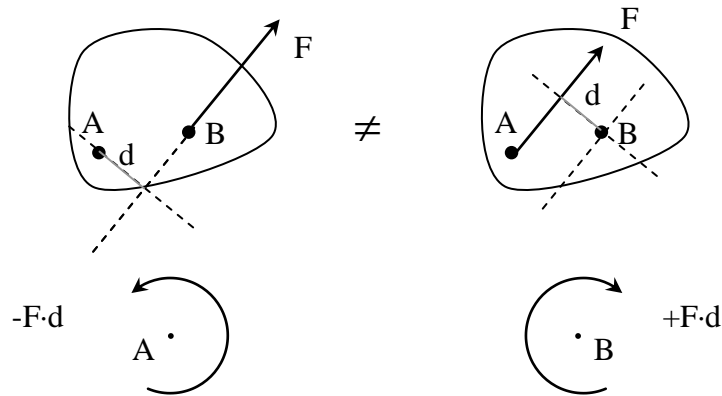
- Sign conventions: Moments have magnitude *and* rotational direction:



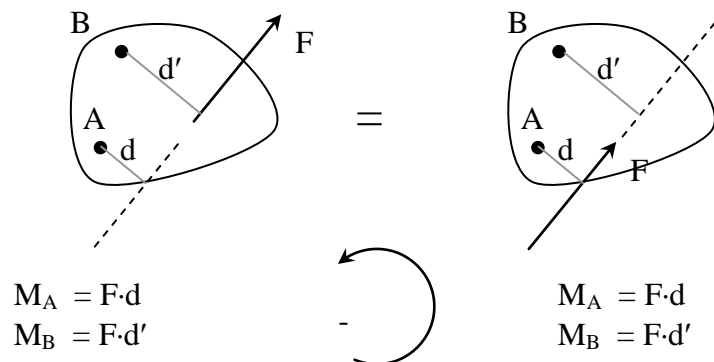
*MOST OTHER TEXTS, INCLUDING PHYSICS TEXTS:*



- Moments can be added as scalar quantities when there is a sign convention.

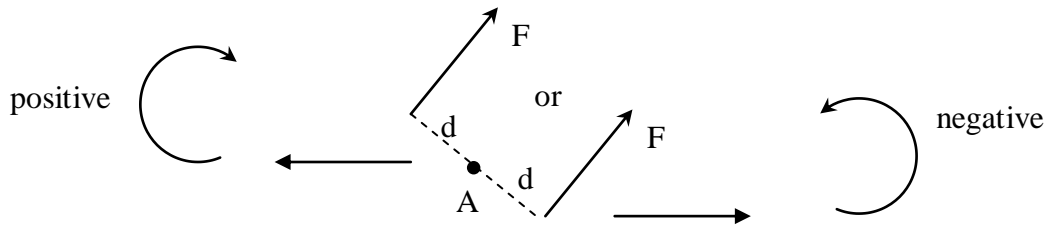


- Repositioning a force along its line of action results in the same moment about any axis.

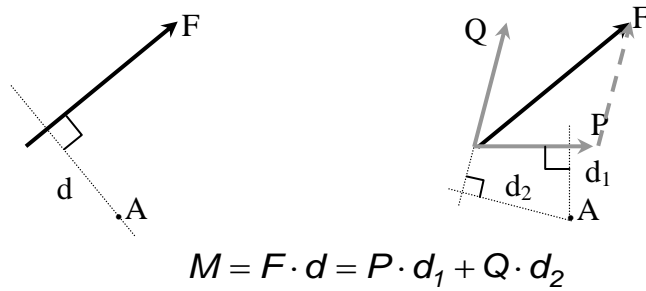


- A force is completely defined (except for its exact position on the line of action) by  $F_x$ ,  $F_y$ , and  $M_A$  about A (size and direction).

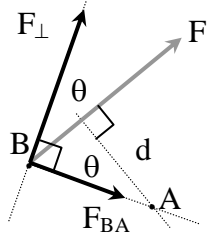
- The *sign* of the moment is determined by which side of the axis the force is on.



- Varignon's Theorem:* The moment of a force about any axis is equal to the sum of moments of the components about that axis.



- Proof 1: Resolve F into components along line BA and perpendicular to it (90°).



$d$  from A to line AB = 0

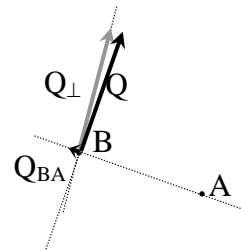
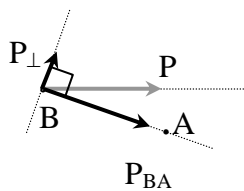
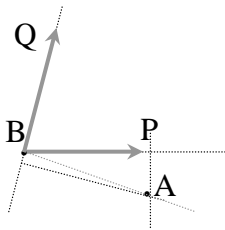
$$d \text{ from A to } F_{\perp} = d_{BA} = \frac{d}{\cos \theta}$$

$$F_{BA} = F \sin \theta$$

$$F_{\perp} = F \cos \theta$$

$$\sum M = F \cdot d = F_{BA} \cdot 0 + F_{\perp} \cdot d_{BA} = F \cos \theta \cdot \frac{d}{\cos \theta} = F \cdot d$$

- Proof 2: Resolve P and Q into  $P_{BA}$  &  $P_{\perp}$ , and  $Q_{BA}$  &  $Q_{\perp}$ .



$d$  from A to line AB = 0

$$M_{A \text{ by } P} = P_{\perp} \cdot d_{BA}$$

$$M_{A \text{ by } Q} = Q_{\perp} \cdot d_{BA}$$

$$\sum M = P_{\perp} \cdot d_{BA} + Q_{\perp} \cdot d_{BA}$$

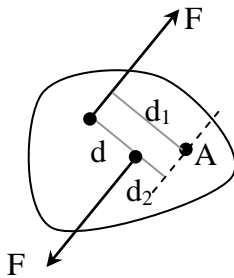
and we know  $d_{BA}$  from Proof 1, and by definition:  $P_{\perp} + Q_{\perp} = F_{\perp}$ . We know  $d_{BA}$  and  $F_{\perp}$  from above, so again  $M = F_{\perp} \cdot d_{BA} = F \cdot d$

- By choosing component directions such that  $d = 0$  to one of the components, we can simplify many problems.
- EQUILIBRIUM is the state where the resultant of the forces on a particle or a rigid is *zero*. There will be no rotation or translation. The forces are referred to as balanced.
- NEWTON'S FIRST LAW: If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

$$R_x = \sum F_x = \sum H = 0 \quad R_y = \sum F_y = \sum V = 0 \quad \text{AND} \quad \sum M = 0$$

### Moment Couples

- *Moment Couple*: Two forces with equal magnitude, parallel lines of action and opposite sense tend to make our body rotate even though the sum of forces is 0. The sum of the moment of the forces about any axis *is not zero*.

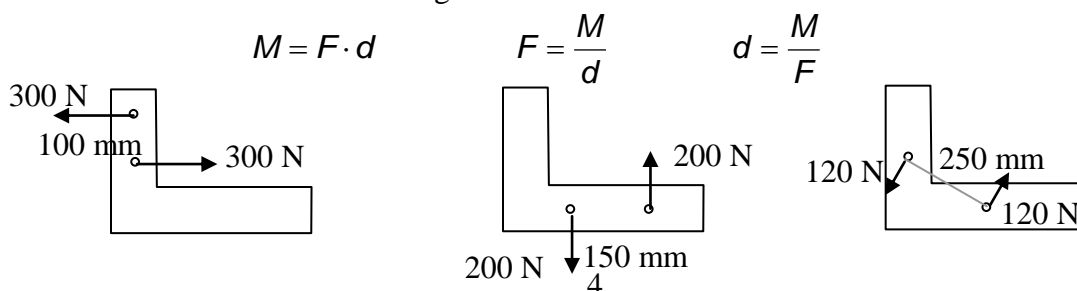


$$\sum M = -F \cdot d_2 + F \cdot d_1 = M$$

$$M = F(d_1 - d_2)$$

$$M = F \cdot d : \text{moment of the couple (CW)}$$

- $M$  does *not depend on where A is*.  $M$  depends on the perpendicular distance between the line of action of the parallel forces.
- $M$  for a couple (defined by  $F$  and  $d$ ) is a constant. And the sense (+/-) is obtained by observation.
- Just as there are equivalent moments (other values of  $F$  and  $d$  that result in  $M$ ) there are equivalent couples. The magnitude is the same for different values of  $F$  and resulting  $d$  or different values of  $d$  and resulting  $F$ .



## Equivalent Force Systems

- Two systems of forces are equivalent if we can transform one of them into the other with:

- 1.) replacing *two forces on a point* by their **resultant**
- 2.) resolving a *force* into two components
- 3.) canceling two equal and opposite forces on a point
- 4.) attaching two equal and opposite forces to a point
- 5.) moving a force along its line of action'
- 6.) replacing a force and moment on a point with a force on another (specific) point
- 7.) replacing a force on point with a force and moment on another (specific) point

\* based on the parallelogram rule and the principle of transmissibility

- The size and direction are important for a moment. The location on a body doesn't matter because couples with the same moment will have the same effect on the rigid body.

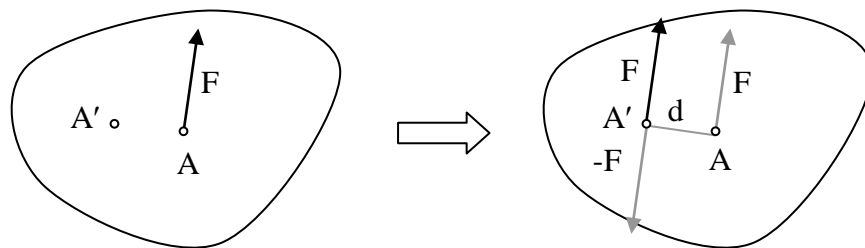
## Addition of Couples

- Couples can be added as *scalars*.
- Two couples can be *replaced* by a single couple with the magnitude of the algebraic sum of the two couples.

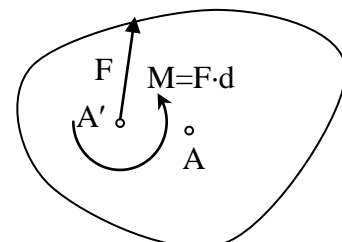
## Resolution of a Force into a Force and a Couple

- The equivalent action of a force on a body can be reproduced by that force and a force couple:

If we'd rather have  $F$  acting at  $A'$  which isn't in the line of action, we can instead add  $F$  and  $-F$  at  $A'$  with no change of action by  $F$ . Now it becomes a couple of  $F$  separated by  $d$  and the force  $F$  moved to  $A'$ . The size is  $F \cdot d = M$



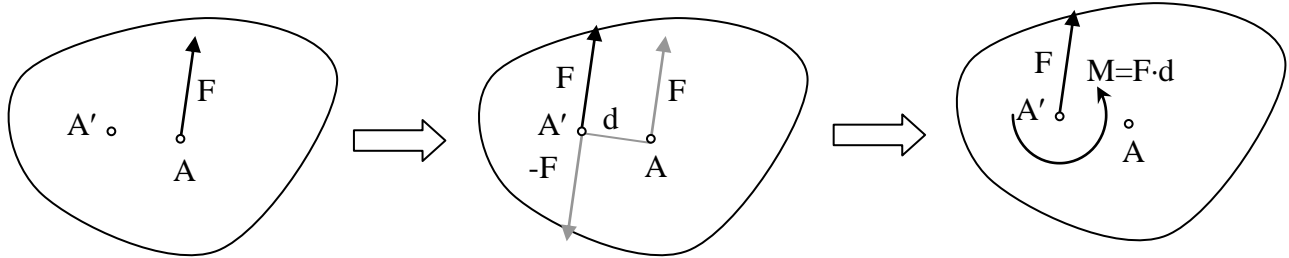
The couple can be represented by a moment symbol:



- Any force can be replaced by itself at another point and the moment equal to the force multiplied by the distance between the original line of action and *new* line of action.

## Resolution of a Force into a Force and a Moment

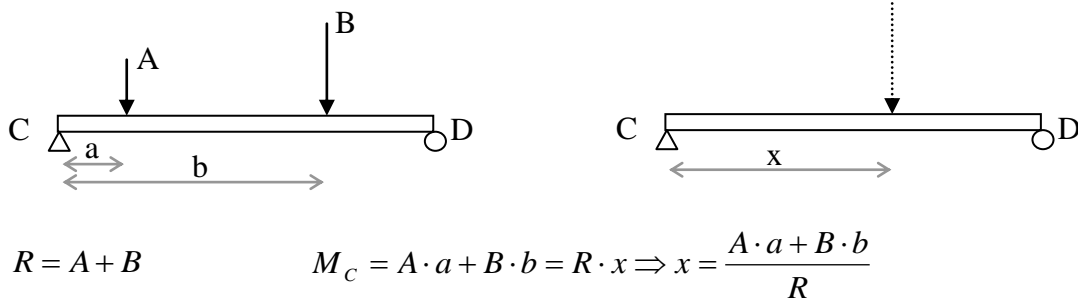
- *Principle:* Any force  $\mathbf{F}$  acting on a rigid body (say the one at A) may be moved to any given point  $A'$ , provided that a couple  $\mathbf{M}$  is added: the moment  $\mathbf{M}$  of the couple must equal the moment of  $\mathbf{F}$  (in its original position at A) about  $A'$ .



IN REVERSE: A force  $\mathbf{F}$  acting at  $A'$  and a couple  $\mathbf{M}$  may be combined into a single resultant force  $\mathbf{F}$  acting at A (a distance  $d$  away) where the moment of  $\mathbf{F}$  about  $A'$  is equal to  $\mathbf{M}$ .

## Resultant of Two Parallel Forces

- Gravity loads act in one direction, so we may have parallel forces on our structural elements. We know how to find the resultant **force**, but the *location* of the resultant must provide the equivalent total moment from each individual force.



## Equilibrium for a Rigid Body

### FREE BODY DIAGRAM STEPS:

1. Determine the free body of interest. (What body is in equilibrium?)
2. Detach the body from the ground and all other bodies (“free” it).
3. Indicate all external forces and moments which include:
  - action on the free body by the **supports & connections**
  - action on the free body by other bodies
  - the weigh effect (=force) of the free body itself (force due to gravity)

4. All forces and moments should be clearly marked with magnitudes and direction. The sense of forces and moments should be those acting *on the body* not by the body.
5. Dimensions/angles should be included for moment computations and force component computations.
6. Indicate the unknown angles, distances, forces or moments, such as those reactions or constraining forces where the body is supported or connected.
  - *Reactions* can be categorized by the type of connections or supports. A reaction is a force with known line of action, or a force of unknown direction, or a moment. The line of action of the force or direction of the moment is directly related to the motion that is prevented.
  - The line of action should be indicated on the FBD. The sense of direction is determined by the type of support. (Cables are in tension, etc...) *If the sense isn't obvious, assume a sense.* When the reaction value comes out positive, the assumption was correct. When the reaction value comes out negative, the assumption was *opposite* the actual sense. ***DON'T CHANGE THE ARROWS ON YOUR FBD OR SIGNS IN YOUR EQUATIONS.***
  - With the 3 equations of equilibrium, there can be no more than 3 unknowns. ***COUNT THE NUMBER OF UNKNOWN REACTIONS.***

**Reactions and Support Connections**

Structural Analysis, 4<sup>th</sup> ed., R.C. Hibbeler

**Table 2-1 Supports for Coplanar Structures**

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1) light cable weightless link			One unknown. The reaction is a force that acts in the direction of the cable or link.
(2) rollers rocker			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(3) smooth contacting surface			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(4) smooth pin-connected collar			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(5) smooth pin or hinge			Two unknowns. The reactions are two force components.
(6) slider fixed-connected collar			Two unknowns. The reactions are a force and a moment.
(7) fixed support			Three unknowns. The reactions are the moment and the two force components.

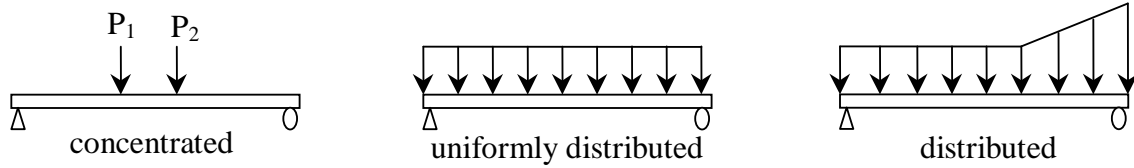


### Loads, Support Conditions & Reactions for Beams

#### Types of Forces

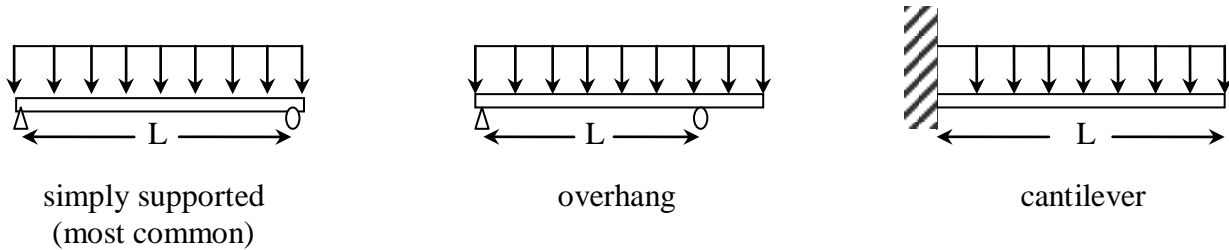
*Concentrated* – single load at one point

*Distributed* – loading spread over a distance or area

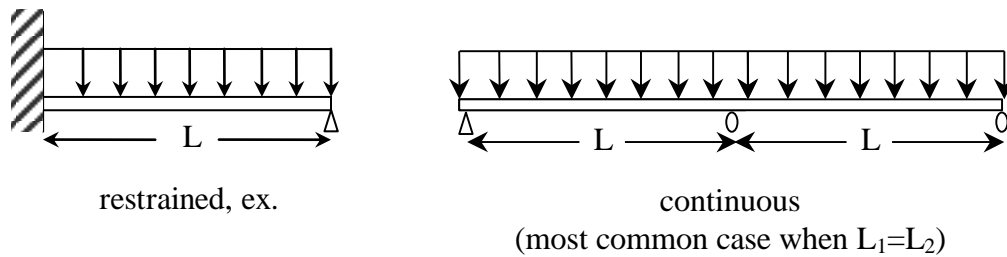


#### Types of supports:

- Statically determinate  
(number of unknowns ≤ number of equilibrium equations)



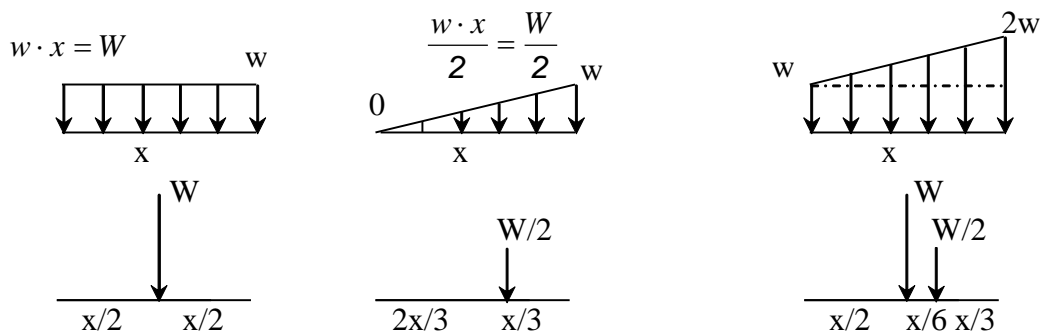
- Statically indeterminate: (need more equations from somewhere)



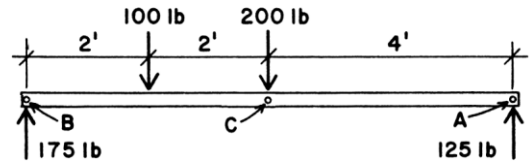
#### Distributed Loads

Distributed loads may be replaced by concentrated loads acting through the balance/center of the distribution or *load area*: **THIS IS AN EQUIVALENT FORCE SYSTEM.**

- $w$  is the symbol used to describe the *load* per unit **length**.
- $W$  is the symbol used to describe the *total load*.

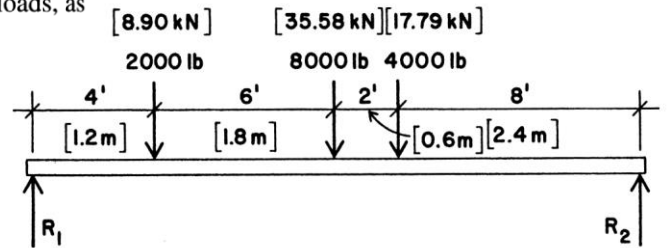


Example 1 (pg 31) Verify that the beam reactions satisfy rotational equilibrium for the rigid body. Check the summation of moments at points A, B & C.



Example 2 (pg 32)

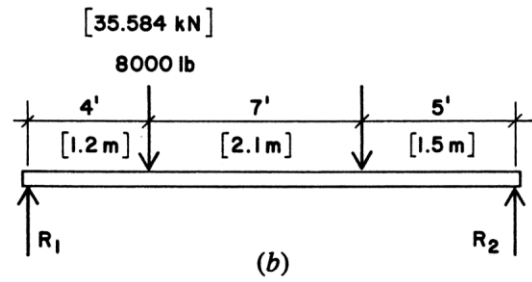
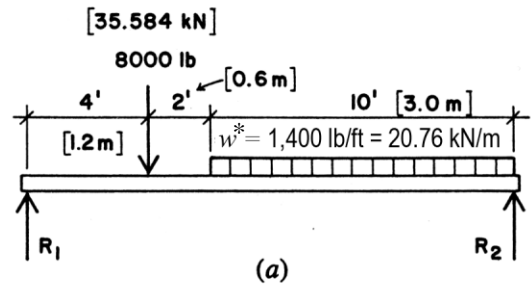
**Example 1.** A simple beam 20 ft long has three concentrated loads, as indicated in Figure 3.6. Find the magnitudes of the reactions.



Example 3 (pg 34)

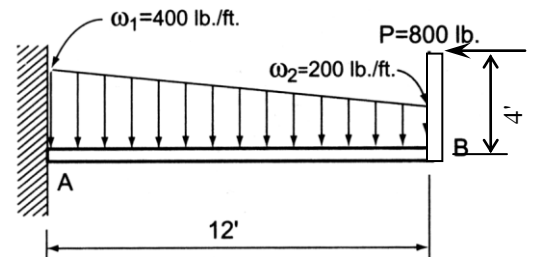
**Example 8.** A simple beam 16 ft long carries the loading shown in Figure 3.7a. Find the reactions.

\*Note: The figure has been changed to show  $w$  rather than  $W$ .



Example 4

Determine the support reactions developed at A for a cantilever beam supporting a trapezoidal load and a point load (horizontal) on the bar at the free end.



Example 5

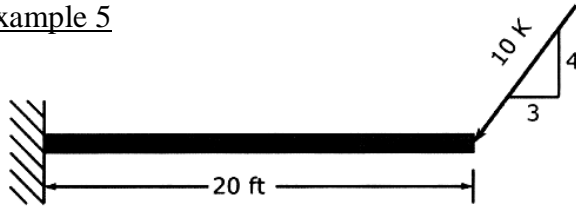


Figure 3.45 Cantilevered beam with point force at the end.

Example Problem 3

Find the reactions for the beam in Figure 3.45.

As a first step:

- Resolve the 10 K applied force into its components.
- Replace the support by the forces and moment that it exerts on the beam (see Figure 3.46).

All of the reactions (i.e., the two reactive forces and the reactive moment) have been drawn in the directions conventionally designated as positive. This is an assumption that can be made without thinking. The machinery of the mathematics will tell whether or not these assumptions are correct. For example, simple inspection will reveal that  $M$  must be in the opposite direction shown to maintain equilibrium. However, there is no need to be concerned about that at this point in working the problem. The fact that one can make an arbitrary assumption and rely on the mathematics to tell whether it was correct is fortunate, inasmuch as there will be many complex structural situations in which determining the actual direction of a force a moment by simple inspection will be impossible. The way one is going about the problem to this point is exactly the way that any computer program would solve it.

Next, begin applying the equations of equilibrium:

$$\sum P_x = 0 \rightarrow$$

$$\sum P_y = 0 \uparrow +$$

$$\sum M = 0 \curvearrowright$$

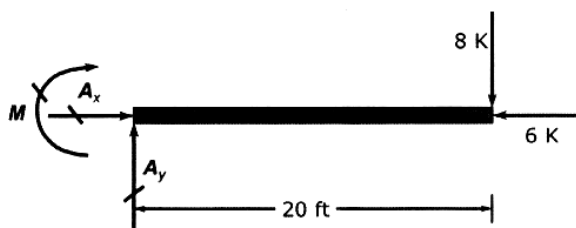


Figure 3.46 Resolving applied force into its components.

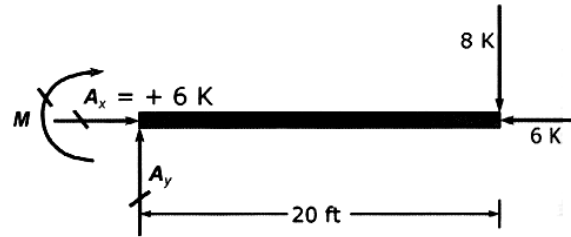


Figure 3.47 Resolved horizontal reaction.

Applying the equation for the horizontal forces,

$$\sum P_x = 0 \rightarrow \text{yields } A_x - 6 K = 0,$$

which solves to give  $A_x = +6 K$ .

The plus sign means, Yes, the original direction assumed for  $A_x$  was correct; that is, the actual force is to the right.

With this new piece of information, the diagram now looks like that in Figure 3.47.

Applying the equation for the vertical forces,

$$\sum P_y = 0 \uparrow +, \text{ yields } A_y - 8 K = 0,$$

which solves to give  $A_y = +8 K$

The plus sign means, Yes, the original direction assumed for  $A_y$  was correct; that is, the actual force is upward.

With this new piece of information, the diagram now looks like that in Figure 3.48.

Summing the moments about the point of support,

$$\sum M = 0 \curvearrowright \text{ yields } M + 8 K(20\text{ft}) = 0,$$

which solves to give  $M = -160 K \text{ ft}$ .

The minus sign means, No, the original direction assumed for  $M$  was wrong; that is, the actual moment is counterclockwise, rather than the clockwise direction assumed in the original drawing.

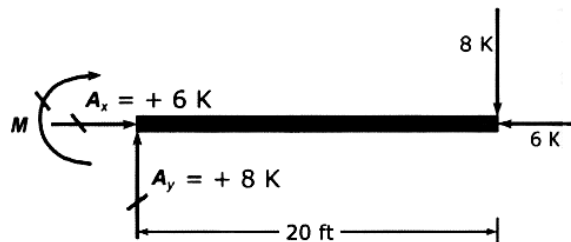


Figure 3.48 Resolved vertical reaction.