

Reinforced Concrete Design

Notation:

a	= depth of the effective compression block in a concrete beam	f'_c	= concrete design compressive stress
A_g	= gross area, equal to the total area ignoring any reinforcement	f'_s	= compressive stress in the compression reinforcement for concrete beam design
A_s	= area of steel reinforcement in concrete beam design	f_y	= yield stress or strength
A'_s	= area of steel compression reinforcement in concrete beam design	F	= shorthand for fluid load
A_{st}	= area of steel reinforcement in concrete column design	F_y	= yield strength
A_v	= area of concrete shear stirrup reinforcement	h	= cross-section depth
ACI	= American Concrete Institute	H	= shorthand for lateral pressure load
b	= width, often cross-sectional	h_f	= depth of a flange in a T section
b_E	= effective width of the flange of a concrete T beam cross section	$I_{transformed}$	= moment of inertia of a multi-material section transformed to one material
b_f	= width of the flange	l_d	= development length for reinforcing steel
b_w	= width of the stem (web) of a concrete T beam cross section	l_{dh}	= development length for hooks
cc	= shorthand for clear cover	l_n	= clear span from face of support to face of support in concrete design
C	= name for centroid	L	= name for length or span length, as is l
	= name for a compression force		= shorthand for live load
C_c	= compressive force in the compression steel in a doubly reinforced concrete beam	L_r	= shorthand for live roof load
C_s	= compressive force in the concrete of a doubly reinforced concrete beam	LL	= shorthand for live load
d	= effective depth from the top of a reinforced concrete beam to the centroid of the tensile steel	M	= internal bending moment
d'	= effective depth from the top of a reinforced concrete beam to the centroid of the compression steel	M_n	= nominal flexure strength with the steel reinforcement at the yield stress and concrete at the concrete design strength for reinforced concrete beam design
d_b	= bar diameter of a reinforcing bar	M_u	= maximum moment from factored loads for LRFD beam design
D	= shorthand for dead load	n	= modulus of elasticity transformation coefficient for steel to concrete
DL	= shorthand for dead load	$n.a.$	= shorthand for neutral axis (N.A.)
e	= eccentricity	P_o	= maximum axial force with no concurrent bending moment in a reinforced concrete column
E	= modulus of elasticity or Young's modulus	P_n	= nominal column load capacity in concrete design
	= shorthand for earthquake load	P_u	= factored column load calculated from load factors in concrete design
E_c	= modulus of elasticity of concrete	R	= shorthand for rain or ice load
E_s	= modulus of elasticity of steel	R_n	= concrete beam design ratio = M_u/bd^2
f	= symbol for stress		

s	= spacing of stirrups in reinforced concrete beams	W	= shorthand for wind load = distance from the top to the neutral axis of a concrete beam
S	= shorthand for snow load	y	= vertical distance
t	= name for thickness (as is h)	β_1	= coefficient for determining stress block height, a , based on concrete strength, f'_c
T	= name for a tension force = shorthand for thermal load	ε	= strain
U	= factored design value	ϕ	= resistance factor
V_c	= shear force capacity in concrete	ϕ_c	= resistance factor for compression
V_n	= nominal shear force	γ	= density or unit weight
V_s	= shear force capacity in steel shear stirrups	ρ	= radius of curvature in beam deflection relationships = reinforcement ratio in concrete beam design = A_s/bd
V_u	= shear at a distance of d away from the face of support for reinforced concrete beam design	$\rho_{balanced}$	= balanced reinforcement ratio in concrete beam design
w_c	= unit weight of concrete	σ	= engineering symbol for normal stress
w_{DL}	= load per unit length on a beam from dead load	v_c	= shear strength in concrete design
w_{LL}	= load per unit length on a beam from live load		
$w_{self\ wt}$	= name for distributed load from self weight of member		
w_u	= load per unit length on a beam from load factors		

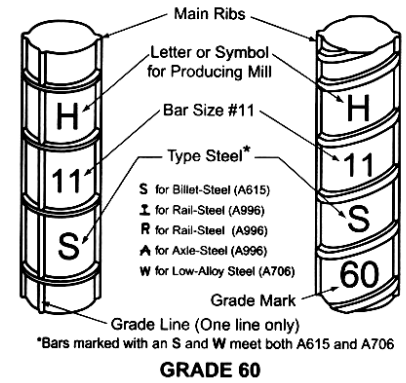
Reinforced Concrete Design

Structural design standards for reinforced concrete are established by the *Building Code and Commentary (ACI 318-11)* published by the American Concrete Institute International, and uses **ultimate** strength design (also known as *limit state* design).

Materials

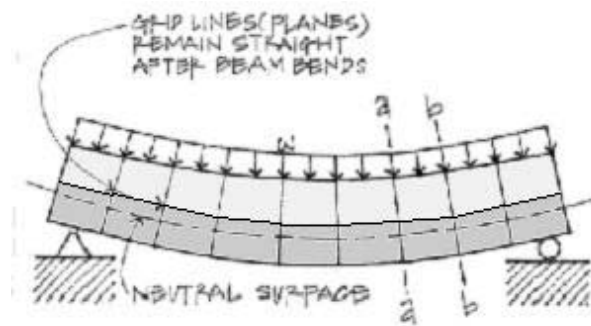
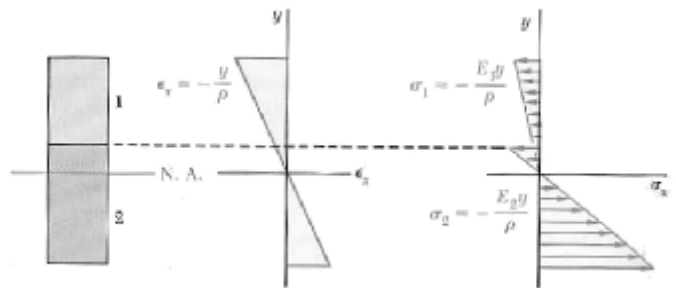
f'_c = concrete compressive design strength at 28 days (units of psi when used in equations)

Deformed reinforcing bars come in grades 40, 60 & 75 (for 40 ksi, 60 ksi and 75 ksi yield strengths). Sizes are given as # of 1/8" up to #8 bars. For #9 and larger, the number is a nominal size (while the actual size is larger).



Reinforced concrete is a composite material, and the average density is considered to be 150 lb/ft^3 . It has the properties that it will creep (deformation with long term load) and shrink (a result of hydration) that must be considered.

Plane sections of composite materials can still be assumed to be plane (strain is linear), *but* the stress distribution is *not* the same in both materials because the *modulus of elasticity* is different. ($f=E \cdot \epsilon$)



$$f_1 = E_1 \epsilon = -\frac{E_1 y}{\rho} \quad f_2 = E_2 \epsilon = -\frac{E_2 y}{\rho}$$

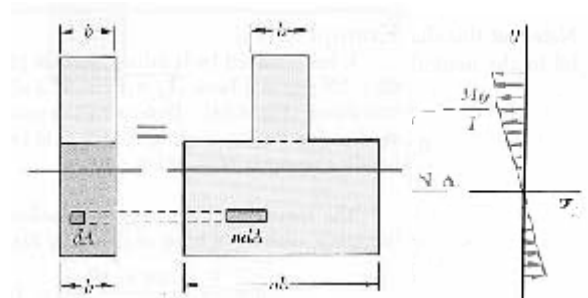
In order to determine the stress, we can define n as the ratio of the elastic moduli:

$$n = \frac{E_2}{E_1}$$

n is used to transform the width of the second material such that it sees the equivalent element stress.

Transformed Section y and I

In order to determine stresses in all types of material in the beam, we transform the materials into a single material, and calculate the location of the neutral axis and modulus of inertia for that material.



ex: When material 1 above is concrete and material 2 is steel

to transform steel into concrete
$$n = \frac{E_2}{E_1} = \frac{E_{steel}}{E_{concrete}}$$

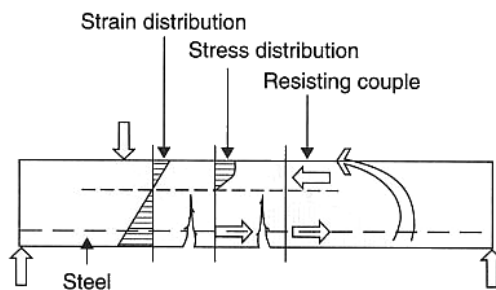
to find the neutral axis of the equivalent concrete member we transform the width of the steel by multiplying by n

to find the moment of inertia of the equivalent concrete member, $I_{transformed}$, use the new geometry resulting from transforming the width of the steel

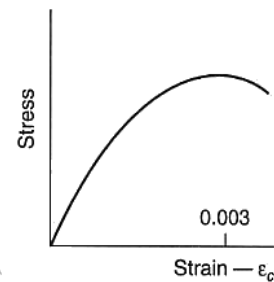
concrete stress:
$$f_{concrete} = -\frac{My}{I_{transformed}}$$

steel stress:
$$f_{steel} = -\frac{Myn}{I_{transformed}}$$

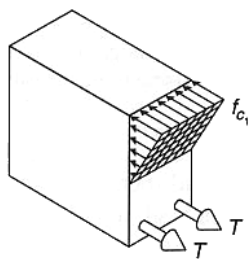
Reinforced Concrete Beam Members



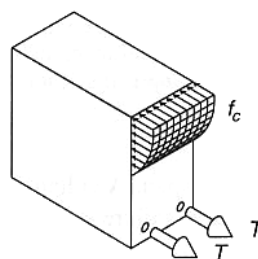
Stresses in the concrete above the neutral axis are compressive and nonlinearly distributed. In the tension zone below the neutral axis, the concrete is assumed to be cracked and the tensile force present to be taken up by reinforcing steel.



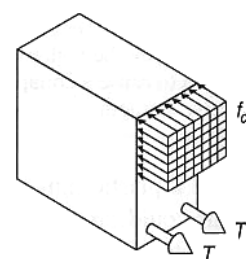
Typical stress-strain curve for concrete.



Working stress analysis. (Concrete stress distribution is assumed to be linear. Service loads are used in calculations.)



Actual stress distribution near ultimate strength (nonlinear).



Ultimate strength analysis. (A rectangular stress block is used to idealize the actual stress distribution. Calculations are based on ultimate loads and failure stresses.)

Ultimate Strength Design for Beams

The ultimate strength design method is similar to LRFD. There is a *nominal* strength that is reduced by a factor ϕ which must exceed the factored design stress. For beams, the concrete only works in compression over a rectangular “stress” block above the n.a. from elastic calculation, and the steel is exposed and reaches the yield stress, F_y

For stress analysis in reinforced concrete beams

- the steel is transformed to concrete
- any concrete in tension is assumed to be cracked and to have no strength
- the steel can be in tension, and is placed in the bottom of a beam that has positive bending moment

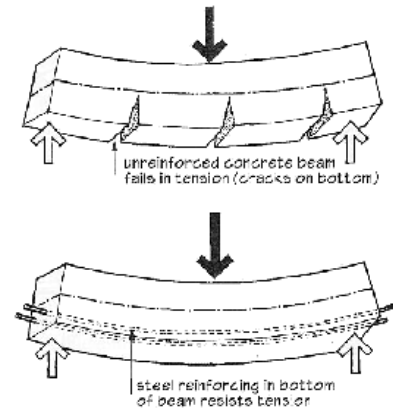
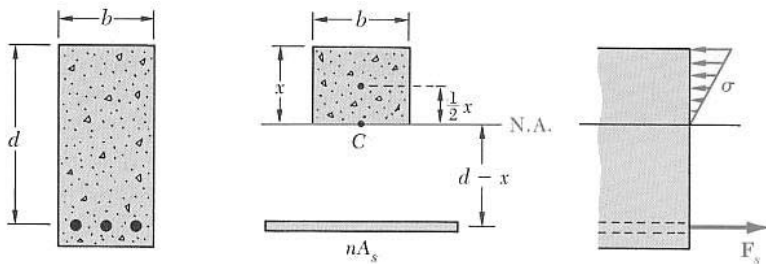


Figure 8.5: Bending in a concrete beam without and with steel reinforcing.

The neutral axis is where there is no stress and no strain. The concrete above the n.a. is in compression. The concrete below the n.a. is considered ineffective. The steel below the n.a. is in tension.

Because the n.a. is defined by the moment areas, we can solve for x knowing that d is the distance from the top of the concrete section to the centroid of the steel:

$$bx \cdot \frac{x}{2} - nA_s(d - x) = 0$$

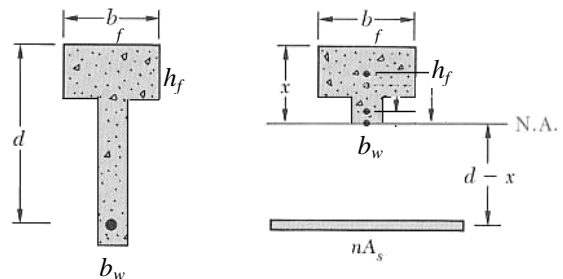
x can be solved for when the equation is rearranged into the generic format with a, b & c in the binomial equation:

$$ax^2 + bx + c = 0 \quad \text{by} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

T-sections

If the n.a. is *above* the bottom of a flange in a T section, x is found as for a rectangular section.

If the n.a. is *below* the bottom of a flange in a T section, x is found by including the flange and the stem of the web (b_w) in the moment area calculation:



$$b_f h_f \left(x - \frac{h_f}{2} \right) + (x - h_f) b_w \frac{(x - h_f)}{2} - nA_s(d - x) = 0$$

Load Combinations (Alternative values are allowed)

$$1.4D$$

$$1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$$

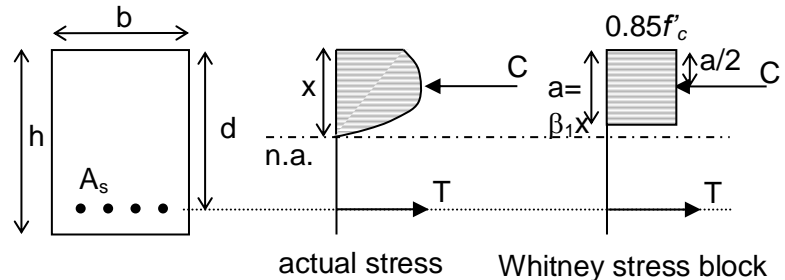
$$1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$$

$$1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$$

$$1.2D + 1.0E + 1.0L + 0.2S$$

$$0.9D + 1.0W$$

$$0.9D + 1.0E$$

*Internal Equilibrium*

C = compression in concrete = stress x area = $0.85 f'_c b a$

T = tension in steel = stress x area = $A_s f_y$

$C = T$ and $M_n = T(d-a/2)$

where f'_c = concrete compression strength

a = height of stress block

b = width of stress block

f_y = steel yield strength

A_s = area of steel reinforcement

d = effective depth of section

= depth to n.a. of reinforcement

ASTM STANDARD REINFORCING BARS

Bar size, no.	Nominal diameter, in.	Nominal area, in. ²	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.650
18	2.257	4.00	13.600

With $C=T$, $A_s f_y = 0.85 f'_c b a$ so a can be determined with $a = \frac{A_s f_y}{0.85 f'_c b}$

Criteria for Beam Design

For flexure design:

$$M_u \leq \phi M_n \quad \phi = 0.9 \text{ for flexure (when the section is } \underline{\text{tension}} \text{ controlled)}$$

so for design, M_u can be set to $\phi M_n = \phi T(d-a/2) = \phi A_s f_y (d-a/2)$

Reinforcement Ratio

The amount of steel reinforcement is *limited*. Too much reinforcement, or **over-reinforcing** will not allow the steel to yield before the concrete crushes and there is a sudden failure. A beam with the proper amount of steel to allow it to yield at failure is said to be **under reinforced**.

The reinforcement ratio is just a fraction: $\rho = \frac{A_s}{bd}$ (or p) and must be less than a value

determined with a concrete strain of 0.003 and tensile strain of 0.004 (minimum). When the strain in the reinforcement is 0.005 or greater, the section is **tension controlled**. (For smaller strains the resistance factor reduces to 0.65 – see tied columns - because the stress is less than the yield stress in the steel.) Previous codes limited the amount to $0.75 \rho_{balanced}$ where $\rho_{balanced}$ was determined from the amount of steel that would make the concrete start to crush at the exact same time that the steel would yield based on strain.

Flexure Design of Reinforcement

One method is to “wisely” estimate a height of the stress block, a , and solve for A_s , and calculate a new value for a using M_u .

1. guess a (less than $n.a.$)

$$2. A_s = \frac{0.85 f'_c b a}{f_y}$$

3. solve for a from

$$\text{setting } M_u = \phi A_s f_y (d - a/2) :$$

$$a = 2 \left(d - \frac{M_u}{\phi A_s f_y} \right)$$

4. repeat from 2. until a found from step 3 matches a used in step 2.

Maximum Reinforcement Ratio ρ for Singly Reinforced Rectangular Beams (tensile strain = 0.005) for which ϕ is permitted to be 0.9

f_y	$f'_c = 3000$ psi	$f'_c = 3500$ psi	$f'_c = 4000$ psi	$f'_c = 5000$ psi	$f'_c = 6000$ psi
	$\beta_1 = 0.85$	$\beta_1 = 0.85$	$\beta_1 = 0.85$	$\beta_1 = 0.80$	$\beta_1 = 0.75$
40,000 psi	0.0203	0.0237	0.0271	0.0319	0.0359
50,000 psi	0.0163	0.0190	0.0217	0.0255	0.0287
60,000 psi	0.0135	0.0158	0.0181	0.0213	0.0239

f_y	$f'_c = 20$ MPa	$f'_c = 25$ MPa	$f'_c = 30$ MPa	$f'_c = 35$ MPa	$f'_c = 40$ MPa
	$\beta_1 = 0.85$	$\beta_1 = 0.85$	$\beta_1 = 0.85$	$\beta_1 = 0.81$	$\beta_1 = 0.77$
300 MPa	0.0181	0.0226	0.0271	0.0301	0.0327
350 MPa	0.0155	0.0194	0.0232	0.0258	0.0281
400 MPa	0.0135	0.0169	0.0203	0.0226	0.0245
500 MPa	0.0108	0.0135	0.0163	0.0181	0.0196

Design Chart Method:

$$1. \text{ calculate } R_n = \frac{M_n}{bd^2}$$

2. find curve for f'_c and f_y to get ρ

3. calculate A_s and a , where:

$$A_s = \rho b d \text{ and } a = \frac{A_s f_y}{0.85 f'_c b}$$

Any method can simplify the size of d using $h = 1.1d$

Maximum Reinforcement

Based on the limiting strain of 0.005 in the steel, $x(\text{or } c) = 0.375d$ so

$$a = \beta_1 (0.375d) \text{ to find } A_{s-\text{max}}$$

(β_1 is shown in the table above)

Minimum Reinforcement

Minimum reinforcement is provided even if the concrete can resist the tension. This is a means to control cracking.

$$\text{Minimum required: } A_s = \frac{3\sqrt{f'_c}}{f_y} (b_w d)$$

$$\text{but not less than: } A_s = \frac{200}{f_y} (b_w d)$$

where f'_c is in psi.

$$\text{This can be translated to } \rho_{\text{min}} = \frac{3\sqrt{f'_c}}{f_y} \text{ but not less than } \frac{200}{f_y}$$

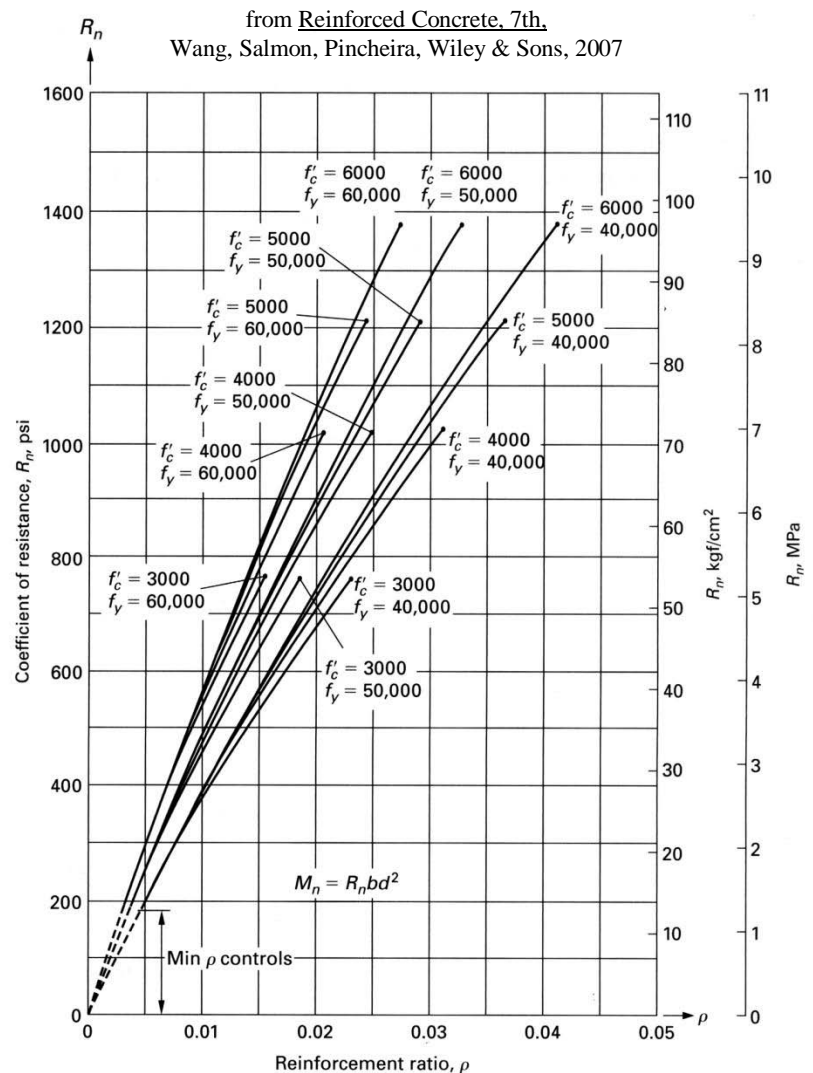


Figure 3.8.1 Strength curves (R_n vs ρ) for singly reinforced rectangular sections. Upper limit of curves is at ρ_{max} . (tensile strain of 0.004)

Cover for Reinforcement

Cover of concrete over/under the reinforcement must be provided to protect the steel from corrosion. For indoor exposure, 1.5 inch is typical for beams and columns, 0.75 inch is typical for slabs, and for concrete cast against soil, 3 inch minimum is required.

Bar Spacing

Minimum bar spacings are specified to allow proper consolidation of concrete around the reinforcement.

T-sections (pan joists)

T beams have an effective width, b_E , that sees compression stress in a wide flange beam or joist in a slab system.

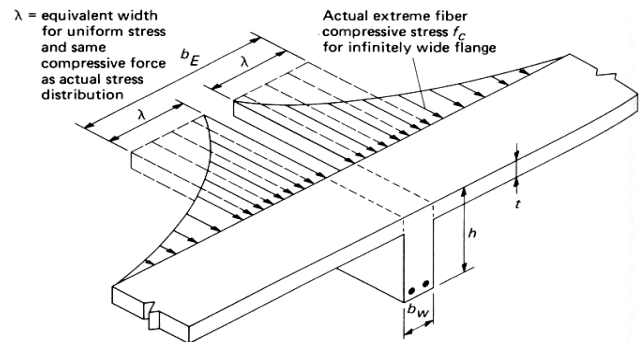


Figure 9.3.1 Actual and equivalent stress distribution over flange width.

For *interior* T-sections, b_E is the smallest of $L/4$, $b_w + 16t$, or center to center of beams

For *exterior* T-sections, b_E is the smallest of $b_w + L/12$, $b_w + 6t$, or $b_w + 1/2(\text{clear distance to next beam})$

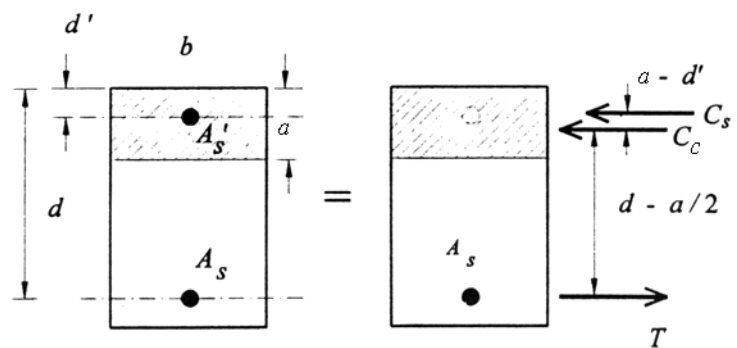
When the **web** is in tension the minimum reinforcement required is the same as for rectangular sections with the web width (b_w) in place of b .

When the **flange** is in tension (negative bending), the minimum reinforcement required is the greater value of $A_s = \frac{6\sqrt{f'_c}}{f_y}(b_w d)$ or $A_s = \frac{3\sqrt{f'_c}}{f_y}(b_f d)$

where f'_c is in psi, b_w is the beam width, and b_f is the effective flange width

Compression Reinforcement

If a section is *doubly reinforced*, it means there is steel in the beam seeing compression. The force in the compression steel at yield is equal to stress x area, $C_s = A_s' F_y$. The total compression that balances the tension is now: $T = C_c + C_s$. And the moment taken about the centroid of the compression stress is $M_n = T(d-a/2) + C_s(a-d')$



where A_s' is the area of compression reinforcement, and d' is the effective depth to the centroid of the compression reinforcement

Slabs

One way slabs can be designed as “one unit”-wide beams. Because they are thin, control of deflections is important, and minimum depths are specified, as is minimum reinforcement for shrinkage and crack control when not in flexure. Reinforcement is commonly small diameter bars and welded wire fabric. Maximum spacing between bars is also specified for shrinkage and crack control as five times the slab thickness not exceeding 18”. For required flexure reinforcement the spacing limit is three times the slab thickness not exceeding 18”.

TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE COMPUTED

Member	Minimum thickness, <i>h</i>			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Solid one-way slabs	$l/20$	$l/24$	$l/28$	$l/10$
Beams or ribbed one-way slabs	$l/16$	$l/18.5$	$l/21$	$l/8$

Notes:
 Values given shall be used directly for members with normalweight concrete and Grade 60 reinforcement. For other conditions, the values shall be modified as follows:
 a) For lightweight concrete having equilibrium density, w_c , in the range of 90 to 115 lb/ft³, the values shall be multiplied by $(1.65 - 0.005w_c)$ but not less than 1.09.
 b) For f_y other than 60,000 psi, the values shall be multiplied by $(0.4 + f_y/100,000)$.

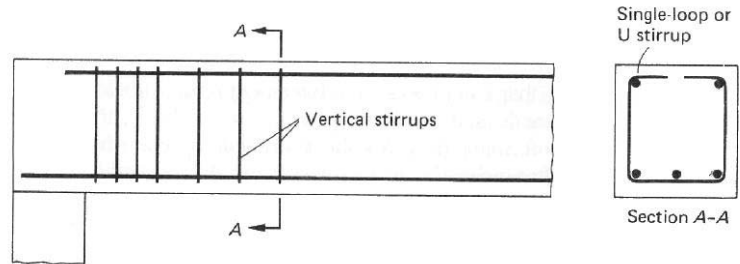
Shrinkage and temperature reinforcement (and minimum for flexure reinforcement):

Minimum for slabs with grade 40 or 50 bars: $\rho = \frac{A_s}{bt} = 0.002 \text{ or } A_{s-min} = 0.002bt$

Minimum for slabs with grade 60 bars: $\rho = \frac{A_s}{bt} = 0.0018 \text{ or } A_{s-min} = 0.0018bt$

Shear Behavior

Horizontal shear stresses occur along with bending stresses to cause tensile stresses where the concrete cracks. Vertical reinforcement is required to bridge the cracks which are called **shear stirrups** (or **stirrups**).



The maximum shear for design, V_u is the value at a distance of d from the face of the support.

Nominal Shear Strength

The shear force that can be resisted is the shear stress \times cross section area: $V_c = v_c \times b_w d$

The shear stress for beams (one way) $v_c = 2\sqrt{f'_c}$ so $\phi V_c = \phi 2\sqrt{f'_c} b_w d$

where b_w = the beam width or the minimum width of the stem.
 $\phi = 0.75$ for shear

One-way joists are allowed an increase of 10% V_c if the joists are closely spaced.

Stirrups are necessary for strength (as well as crack control): $V_s = \frac{A_v f_y d}{s} \leq 8\sqrt{f'_c} b_w d (max)$

where A_v = area of all vertical legs of stirrup
 s = spacing of stirrups
 d = effective depth

For shear design:

$$V_u \leq \phi V_c + \phi V_s \quad \phi = 0.75 \text{ for shear}$$

Spacing Requirements

Stirrups are required when V_u is greater than $\frac{\phi V_c}{2}$

Table 3-8 ACI Provisions for Shear Design*

		$V_u \leq \frac{\phi V_c}{2}$	$\phi V_c \geq V_u > \frac{\phi V_c}{2}$	$V_u > \phi V_c$
Required area of stirrups, A_v **		none	$\frac{50b_w s}{f_y}$	$\frac{(V_u - \phi V_c)s}{\phi f_y d}$
Stirrup spacing, s	Required	—	$\frac{A_v f_y}{50b_w}$	$\frac{\phi A_v f_y d}{V_u - \phi V_c}$
	Recommended Minimum†	—	—	4 in.
	Maximum†† (ACI 11.5.4)	—	$\frac{d}{2}$ or 24 in.	$\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \leq \phi 4\sqrt{f'_c} b_w d$ $\frac{d}{4}$ or 12 in. for $(V_u - \phi V_c) > \phi 4\sqrt{f'_c} b_w d$

*Members subjected to shear and flexure only; $\phi V_c = \phi 2\sqrt{f'_c} b_w d$, $\phi = 0.75$ (ACI 11.3.1.1)

** $A_v = 2 \times A_b$ for U stirrups; $f_y \leq 60$ ksi (ACI 11.5.2)

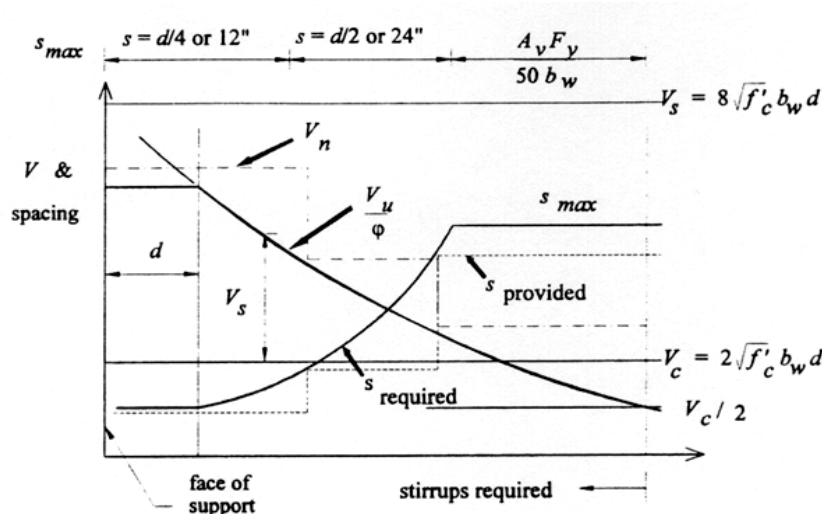
†A practical limit for minimum spacing is $d/4$

††Maximum spacing based on minimum shear reinforcement ($= A_v f_y / 50b_w$) must also be considered (ACI 11.5.5.3).

Economical spacing of stirrups is considered to be greater than $d/4$. Common spacings of $d/4$, $d/3$ and $d/2$ are used to determine the values of ϕV_s at which the spacings can be increased.

$$\phi V_s = \frac{\phi A_v f_y d}{s}$$

This figure shows the size of V_n provided by $V_c + V_s$ (long dashes) exceeds V_u/ϕ in a step-wise function, while the spacing provided (short dashes) is at or less than the required s (limited by the maximum allowed). (Note that the maximum shear permitted from the stirrups is $8\sqrt{f'_c} b_w d$)



The minimum recommended spacing for the first stirrup is 2 inches from the face of the support.

Torsional Shear Reinforcement

On occasion beam members will see twist along the axis caused by an eccentric shape supporting a load, like on an L-shaped spandrel (edge) beam. The torsion results in shearing stresses, and closed stirrups may be needed to resist the stress that the concrete cannot resist.

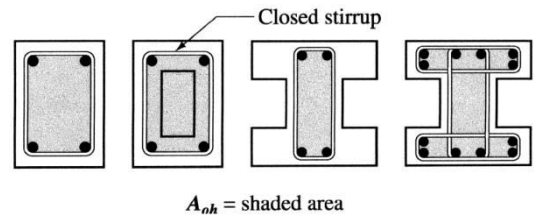


Fig. R11.6.3.6(b)—Definition of A_{oh}

Development Length for Reinforcement

Because the design is based on the reinforcement attaining the yield stress, the reinforcement needs to be properly bonded to the concrete for a finite length so it won't slip. This is referred to as the development length. Providing sufficient length to anchor bars that need to reach the yield stress near the end of connections are also specified with hook lengths. *Detailing reinforcement is a tedious job.* . Splices are also necessary to extend the length of reinforcement that come in standard lengths. The equations are not provided here.

Development Length in Tension

With the proper bar to bar spacing and cover, the common development length equations are:

#6 bars and smaller: $l_d = \frac{d_b F_y}{25 \sqrt{f'_c}}$ or 12 in. minimum

#7 bars and larger: $l_d = \frac{d_b F_y}{20 \sqrt{f'_c}}$ or 12 in. minimum

Development Length in Compression

$$l_d = \frac{0.02 d_b F_y}{\sqrt{f'_c}} \leq 0.0003 d_b F_y$$

Hook Bends and Extensions

The minimum hook length is $l_{dh} = \frac{1200 d_b}{\sqrt{f'_c}}$

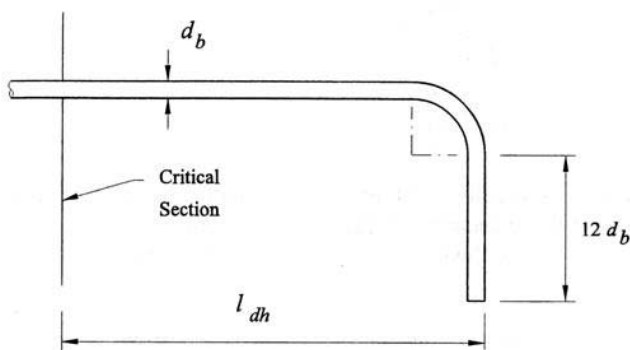


Figure 9-17: Minimum requirements for 90° bar hooks.

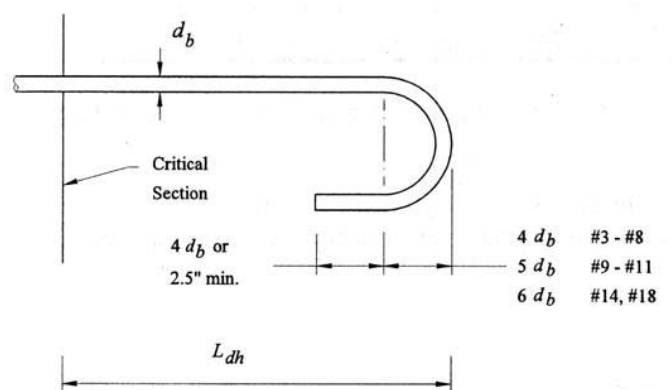


Figure 9-18: Minimum requirements for 180° bar hooks.

Modulus of Elasticity & Deflection

E_c for deflection calculations can be used with the transformed section modulus in the elastic range. After that, the cracked section modulus is calculated and E_c is adjusted.

Code values:

$$E_c = 57,000\sqrt{f'_c} \text{ (normal weight)} \quad E_c = w_c^{1.5} 33\sqrt{f'_c}, \quad w_c = 90 \text{ lb/ft}^3 - 160 \text{ lb/ft}^3$$

Deflections of beams and one-way slabs need not be computed if the overall member thickness meets the minimum specified by the code, and are shown in Table 9.5(a) (see *Slabs*).

Criteria for Flat Slab & Plate System Design

Systems with slabs and supporting beams, joists or columns typically have multiple bays. The horizontal elements can act as one-way or two-way systems. Most often the flexure resisting elements are continuous, having positive and negative bending moments. These moment and shear values can be found using beam tables, or from code specified approximate design factors. Flat slab two-way systems have drop panels (for shear), while flat plates do not.

Criteria for Column Design

(American Concrete Institute) ACI 318-02 Code and Commentary:

$$P_u \leq \phi_c P_n \quad \text{where}$$

P_u is a factored load

ϕ is a resistance factor

P_n is the nominal load capacity (strength)

Load combinations, ex:

1.4D (D is dead load)

1.2D + 1.6L (L is live load)

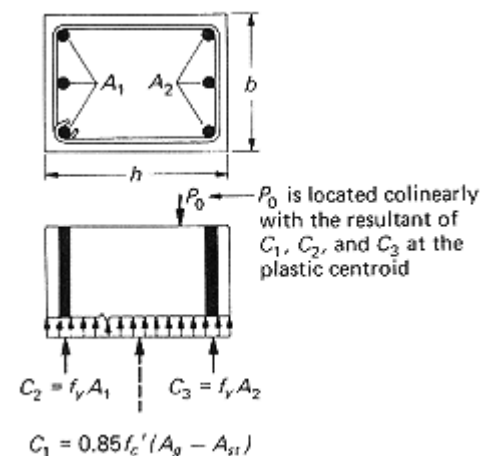
For compression, $\phi_c = 0.75$ and $P_n = 0.85P_o$ for spirally reinforced, $\phi_c = 0.65$ and $P_n = 0.8P_o$ for tied columns where $P_o = 0.85f'_c(A_g - A_{st}) + f_y A_{st}$ and P_o is the name of the maximum axial force with no concurrent bending moment.

Columns which have reinforcement ratios, $\rho_g = \frac{A_{st}}{A_g}$, in the

range of 1% to 2% will usually be the most economical, with 1% as a minimum and 8% as a maximum by code.

Bars are symmetrically placed, typically.

Spiral ties are harder to construct.



Columns with Bending (Beam-Columns)

Concrete columns rarely see only axial force and must be designed for the combined effects of axial load and bending moment. The **interaction** diagram shows the reduction in axial load a column can carry with a bending moment.

Design aids commonly present the interaction diagrams in the form of load vs. equivalent eccentricity for standard column sizes and bars used.

Rigid Frames

Monolithically cast frames with beams and column elements will have members with shear, bending and axial loads. Because the joints can rotate, the effective length must be determined from methods like that presented in the handout on Rigid Frames. The charts for evaluating k for non-sway and sway frames can be found in the ACI code.

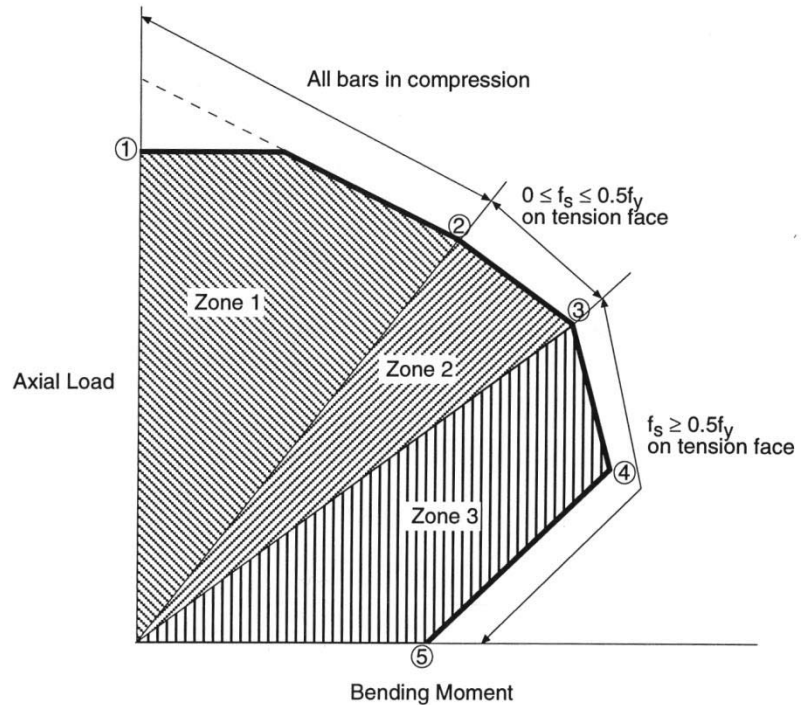


Figure 5-3 Transition Stages on Interaction Diagram

Example 1

(a) Determine the ultimate moment capacity of a beam with dimensions $b = 10$ in. and $d_{\text{effective}} = 15$ in. and that has three No. 9 bars (3.0 in.^2) of tension-reinforcing steel. Assume that $h = 18$ in., $F_y = 40$ ksi, and $f'_c = 5 \text{ ksi}$. (b) Assume also that the section is used as a cantilever beam 10 ft long, where the service loads are dead load = 400 lb/ft and live load = 300 lb/ft. Is the beam adequate in bending? Calculate the ultimate moment capacity of the beam first.

Solution:

$$(a) \quad a = A_s F_y / 0.85 f'_c b = (3)(40,000) / (0.85)(5000)(10) = 2.82 \text{ in.}$$

$$\phi M_n = \phi A_s F_y [d - a/2] = 0.9(3)(40,000)[15 - (2.82)/(2)] = 1,466,640 \text{ in.-lb}$$

Check for overreinforcement, $c = 0.375 \cdot 15 = 5.625$. Depth of stress block $a = 0.80 \cdot 5.625 \text{ in.} = 4.5 \text{ in.}$ $A_{s,\text{max}} = (0.85)(5 \text{ ksi})(4.5 \text{ in.})(10 \text{ in.}) / (40 \text{ ksi}) = 4.78 \text{ in.}^2$ The beam is not over reinforced. Check for minimum steel: $A_{s,\text{min}} = \frac{3\sqrt{f'_c}}{F_y} bd = 0.80 \text{ in.}^2$, so beam is sufficiently reinforced.

$$(b) \quad U = 1.2D + 1.6L = 1.2(400) + 1.6(300) = 960 \text{ lb/ft}$$

$$M_u = w_u L^2 / 2 = (960)(10^2) / 2 = 48,000 \text{ ft-lb} = 576,000 \text{ in.-lb}$$

Since $M_u = 576,000 < \phi M_n = 1,466,640$, the beam is adequate in bending.

EXAMPLE

Determine the ultimate moment capacity of a beam of dimensions $b = 250$ mm and $d = 350$ mm and that has 300 mm^2 of reinforcing steel. Assume that $F_y = 400$ MPa and $f'_c = 25$ MPa.

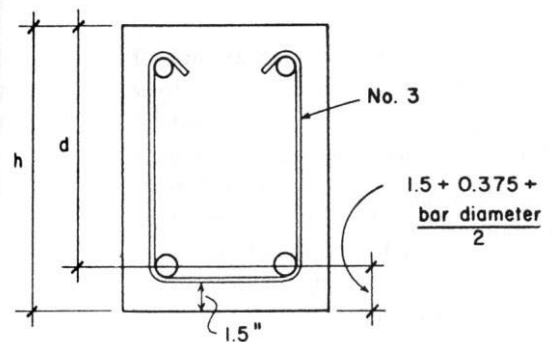
Solution:

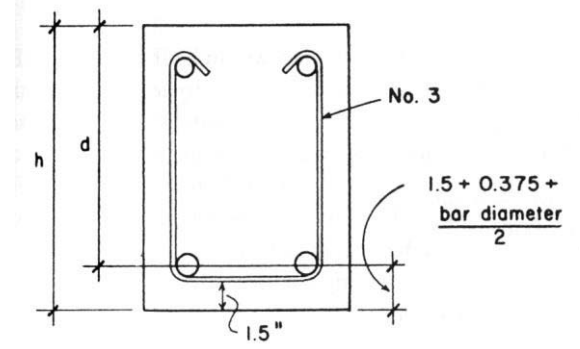
$$a = \frac{A_s F_y}{0.85 f'_c b} = \frac{(300)(400)}{(0.85)(25)(250)} = 22.6$$

$$\phi M_n = \phi A_s F_y \left(d - \frac{a}{2} \right) = 0.9(300)(400) \left(350 - \frac{22.6}{2} \right) = 36.5 \text{ kN} \cdot \text{m}$$

Example 2 (pg 423)

Example 1. The service load bending moments on a beam are 58 kip-ft [78.6 kN-m] for dead load and 38 kip-ft [51.5 kN-m] for live load. The beam is 10 in. [254 mm] wide, f'_c is 3000 psi [27.6 MPa], and f_y is 60 ksi [414 MPa]. Determine the depth of the beam and the tensile reinforcing required.



Example 2 (continued)

Example 3

A simply supported beam 20 ft long carries a service dead load of 300 lb/ft and a live load of 500 lb/ft. Design an appropriate beam (for flexure only). Use grade 40 steel and concrete strength of 5000 psi.

SOLUTION:

Find the design moment, M_u , from the factored load combination of $1.2D + 1.6L$. It is good practice to guess a beam size to include self weight in the dead load, because "service" means dead load of everything except the beam itself.

Guess a size of 10 in x 12 in. Self weight for normal weight concrete is the density of 150 lb/ft³ multiplied by the cross section area: self weight = $150 \frac{\text{lb}}{\text{ft}^3} (10 \text{ in})(12 \text{ in}) \cdot \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)^2 = 125 \text{ lb/ft}$

$$w_u = 1.2(300 \text{ lb/ft} + 125 \text{ lb/ft}) + 1.6(500 \text{ lb/ft}) = 1310 \text{ lb/ft}$$

$$\text{The maximum moment for a simply supported beam is } \frac{wl^2}{8} : \quad M_u = \frac{w_u l^2}{8} = \frac{1310 \frac{\text{lb}}{\text{ft}} (20 \text{ ft})^2}{8} = 65,500 \text{ lb-ft}$$

$$M_n \text{ required} = M_u / \phi = \frac{65,500 \text{ lb-ft}}{0.9} = 72,778 \text{ lb-ft}$$

To use the design chart aid, find $R_n = \frac{M_n}{bd^2}$, estimating that d is about 1.75 inches less than h :

$$d = 12 \text{ in} - 1.75 \text{ in} - (0.375) = 10.25 \text{ in} \quad (\text{NOTE: If there are stirrups, you must also subtract the diameter of the stirrup bar.})$$

$$R_n = \frac{72,778 \text{ lb-ft}}{(10 \text{ in})(10.25 \text{ in})^2} \cdot (12 \frac{\text{in}}{\text{ft}}) = 831 \text{ psi}$$

ρ corresponds to approximately 0.023 (which is less than that for 0.005 strain of 0.0319), so the estimated area required, A_s , can be found:

$$A_s = \rho b d = (0.023)(10 \text{ in})(10.25 \text{ in}) = 2.36 \text{ in}^2$$

The number of bars for this area can be found from handy charts.

(Whether the number of bars actually fit for the width with cover and space between bars must also be considered. If you are at ρ_{\max} do not choose an area bigger than the maximum!)

$$\text{Try } A_s = 2.37 \text{ in}^2 \text{ from } 3\#8 \text{ bars}$$

$$d = 12 \text{ in} - 1.5 \text{ in (cover)} - \frac{1}{2} (8/8 \text{ in diameter bar}) = 10 \text{ in}$$

$$\text{Check } \rho = 2.37 \text{ in}^2 / (10 \text{ in})(10 \text{ in}) = 0.0237 \text{ which is less than } \rho_{\max-0.005} = 0.0319 \text{ OK (We cannot have an over reinforced beam!!)}$$

Find the moment capacity of the beam as designed, ϕM_n

$$a = A_s f_y / 0.85 f_c b = 2.37 \text{ in}^2 (40 \text{ ksi}) / [0.85 (5 \text{ ksi}) 10 \text{ in}] = 2.23 \text{ in}$$

$$\phi M_n = \phi A_s f_y (d - a/2) = 0.9 (2.37 \text{ in}^2) (40 \text{ ksi}) \left(10 \text{ in} - \frac{2.23 \text{ in}}{2}\right) \cdot \left(\frac{1}{12 \frac{\text{in}}{\text{ft}}}\right) = 63.2 \text{ k-ft} < 65.5 \text{ k-ft needed (not OK)}$$

So, we can increase d to 13 in, and $\phi M_n = 70.3 \text{ k-ft}$ (OK). Or increase A_s to 2 # 10's (2.54 in²), for $a = 2.39 \text{ in}$ and ϕM_n of 67.1 k-ft (OK). Don't exceed ρ_{\max} Or $\rho_{\max-0.005}$ if you want to use $\phi=0.9$

Example 4

A simply supported beam 20 ft long carries a service dead load of 425 lb/ft (including self weight) and a live load of 500 lb/ft. Design an appropriate beam (for flexure only). Use grade 40 steel and concrete strength of 5000 psi.

SOLUTION:

Find the design moment, M_u , from the factored load combination of $1.2D + 1.6L$. *If self weight is not included in the service loads, you need to guess a beam size to include self weight in the dead load, because "service" means dead load of everything except the beam itself.*

$$w_u = 1.2(425 \text{ lb/ft}) + 1.6(500 \text{ lb/ft}) = 1310 \text{ lb/ft}$$

$$\text{The maximum moment for a simply supported beam is } \frac{wl^2}{8} : \quad M_u = \frac{w_u l^2}{8} = \frac{1310 \text{ lb/ft} (20 \text{ ft})^2}{8} = 65,500 \text{ lb-ft}$$

$$M_n \text{ required} = M_u / \phi = \frac{65,500 \text{ lb-ft}}{0.9} = 72,778 \text{ lb-ft}$$

To use the design chart aid, we can find $R_n = \frac{M_n}{bd^2}$, and estimate that h is roughly 1.5-2 times the size of b , and $h = 1.1d$ (rule of thumb): $d = h/1.1 = (2b)/1.1$, so $d \approx 1.8b$ or $b \approx 0.55d$.

We can find R_n at the maximum reinforcement ratio for our materials, keeping in mind ρ_{\max} at a strain = 0.005 is 0.0319 off of the chart at about 1070 psi, with $\rho_{\max} = 0.037$. Let's substitute b for a function of d :

$$R_n = 1070 \text{ psi} = \frac{72,778 \text{ lb-ft}}{(0.55d)(d)^2} \cdot (12 \text{ in/ft}) \quad \text{Rearranging and solving for } d = 11.4 \text{ inches}$$

That would make b a little over 6 inches, which is impractical. 10 in is commonly the smallest width.

So if h is commonly 1.5 to 2 times the width, b , h ranges from 14 to 20 inches. ($10 \times 1.5 = 15$ and $10 \times 2 = 20$)

Choosing a depth of 14 inches, $d \cong 14 - 1.5$ (clear cover) - $\frac{1}{2}$ (1" diameter bar guess) - $\frac{3}{8}$ in (stirrup diameter) = 11.625 in.

$$\text{Now calculating an updated } R_n = \frac{72,778 \text{ lb-ft}}{(10 \text{ in})(11.625 \text{ in})^2} \cdot (12 \text{ in/ft}) = 646.2 \text{ psi}$$

ρ now is 0.020 (under the limit at 0.005 strain of 0.0319), so the estimated area required, A_s , can be found:

$$A_s = \rho b d = (0.020)(10 \text{ in})(11.625 \text{ in}) = 2.33 \text{ in}^2$$

The number of bars for this area can be found from handy charts.

(Whether the number of bars actually fit for the width with cover and space between bars must also be considered. If you are at $\rho_{\max-0.005}$ do not choose an area bigger than the maximum!)

Try $A_s = 2.37 \text{ in}^2$ from 3#8 bars. (or 2.0 in^2 from 2 #9 bars. 4#7 bars don't fit...)

$d(\text{actually}) = 14 \text{ in.} - 1.5 \text{ in (cover)} - \frac{1}{2} (8/8 \text{ in bar diameter}) - \frac{3}{8} \text{ in. (stirrup diameter)} = 11.625 \text{ in.}$

Check $\rho = 2.37 \text{ in}^2 / (10 \text{ in})(11.625 \text{ in}) = 0.0203$ which is less than $\rho_{\max-0.005} = 0.0319$ OK (We cannot have an over reinforced beam!!)

Find the moment capacity of the beam as designed, ϕM_n

$$a = A_s f_y / 0.85 f_c b = 2.37 \text{ in}^2 (40 \text{ ksi}) / [0.85(5 \text{ ksi})10 \text{ in}] = 2.23 \text{ in}$$

$$\phi M_n = \phi A_s f_y (d - a/2) = 0.9(2.37 \text{ in}^2)(40 \text{ ksi})(11.625 \text{ in} - \frac{2.23 \text{ in}}{2}) \cdot (\frac{1}{12 \text{ in/ft}}) = 74.7 \text{ k-ft} > 65.5 \text{ k-ft needed}$$

OK! Note: If the section doesn't work, you need to increase d or A_s as long as you don't exceed $\rho_{\max-0.005}$

Example 5

A simply supported beam 25 ft long carries a service dead load of 2 k/ft, an estimated self weight of 500 lb/ft and a live load of 3 k/ft. Design an appropriate beam (for flexure only). Use grade 60 steel and concrete strength of 3000 psi.

SOLUTION:

Find the design moment, M_u , from the factored load combination of $1.2D + 1.6L$. If self weight is estimated, and the selected size has a larger self weight, the design moment must be adjusted for the extra load.

$$w_u = 1.2(2 \text{ k/ft} + 0.5 \text{ k/ft}) + 1.6(3 \text{ k/ft}) = 7.8 \text{ k/ft} \quad \text{So, } M_u = \frac{w_u L^2}{8} = \frac{7.8 \text{ k/ft} (25 \text{ ft})^2}{8} = 609.4 \text{ k-ft}$$

$$M_n \text{ required} = M_u / \phi = \frac{609.4 \text{ k-ft}}{0.9} = 677.1 \text{ k-ft}$$

To use the design chart aid, we can find $R_n = \frac{M_n}{bd^2}$, and estimate that h is roughly 1.5-2 times the size of b , and $h = 1.1d$ (rule of thumb): $d = h/1.1 = (2b)/1.1$, so $d \approx 1.8b$ or $b \approx 0.55d$.

We can find R_n at the maximum reinforcement ratio for our materials off of the chart at about 700 psi with $\rho_{\max-0.005} = 0.0135$. Let's substitute b for a function of d :

$$R_n = 700 \text{ psi} = \frac{677.1 \text{ k-ft} (1000 \text{ lb/k})}{(0.55d)^2} \cdot (12 \text{ in/ft}) \quad \text{Rearranging and solving for } d = 27.6 \text{ inches}$$

That would make b 15.2 in. (from 0.55d). Let's try 15. So,

$$h \cong d + 1.5 \text{ (clear cover)} + \frac{1}{2}(1" \text{ diameter bar guess}) + \frac{3}{8} \text{ in (stirrup diameter)} = 27.6 + 2.375 = 29.975 \text{ in.}$$

Choosing a depth of 30 inches, $d \cong 30 - 1.5 \text{ (clear cover)} - \frac{1}{2}(1" \text{ diameter bar guess}) - \frac{3}{8} \text{ in (stirrup diameter)} = 27.625 \text{ in.}$

$$\text{Now calculating an updated } R_n = \frac{677,100 \text{ lb-ft}}{(15 \text{ in})(27.625 \text{ in})^2} \cdot (12 \text{ in/ft}) = 710 \text{ psi} \quad \text{This is larger than } R_n \text{ for the } 0.005 \text{ strain limit!}$$

We can't just use $\rho_{\max-0.005}$. The way to reduce R_n is to increase b or d or both. Let's try increasing h to 31 in., then $R_n = 661 \text{ psi}$ with $d = 28.625 \text{ in.}$ That puts us under $\rho_{\max-0.005}$. We'd have to remember to keep UNDER the area of steel calculated, which is hard to do.

From the chart, $\rho \approx 0.013$, less than the $\rho_{\max-0.005}$ of 0.0135, so the estimated area required, A_s , can be found:

$$A_s = \rho b d = (0.013)(15 \text{ in})(29.625 \text{ in}) = 5.8 \text{ in}^2$$

The number of bars for this area can be found from handy charts. Our charts say there can be 3 – 6 bars that fit when $\frac{3}{4}"$ aggregate is used. We'll assume 1 inch spacing between bars. The actual limit is the maximum of 1 in, the bar diameter or 1.33 times the maximum aggregate size.

$$\text{Try } A_s = 6.0 \text{ in}^2 \text{ from 6\#9 bars. Check the width: } 15 - 3 \text{ (1.5 in cover each side)} - 0.75 \text{ (two \#3 stirrup legs)} - 6 * 1.128 - 5 * 1.128 \text{ in.} = -1.16 \text{ in NOT OK.}$$

$$\text{Try } A_s = 5.08 \text{ in}^2 \text{ from 4\#10 bars. Check the width: } 15 - 3 \text{ (1.5 in cover each side)} - 0.75 \text{ (two \#3 stirrup legs)} - 4 * 1.27 - 3 * 1.27 \text{ in.} = 2.36 \text{ OK.}$$

$$d(\text{actually}) = 31 \text{ in.} - 1.5 \text{ in (cover)} - \frac{1}{2}(1.27 \text{ in bar diameter}) - \frac{3}{8} \text{ in. (stirrup diameter)} = 28.49 \text{ in.}$$

Find the moment capacity of the beam as designed, ϕM_n

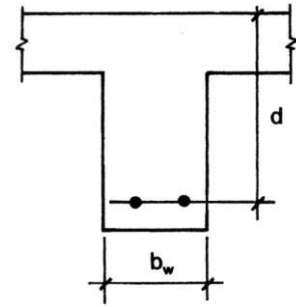
$$a = A_s f_y / 0.85 f'_c b = 5.08 \text{ in}^2 (60 \text{ ksi}) / [0.85(3 \text{ ksi})15 \text{ in}] = 8.0 \text{ in}$$

$$\phi M_n = \phi A_s f_y (d - a/2) = 0.9(5.08 \text{ in}^2)(60 \text{ ksi})(28.49 \text{ in} - \frac{8.0 \text{ in}}{2}) \cdot (\frac{1}{12 \text{ in/ft}}) = 559.8 \text{ k-ft} < 609 \text{ k-ft needed!! (NO GOOD)}$$

More steel isn't likely to increase the capacity much unless we are close. It looks like we need more steel **and** lever arm. Try $h = 32 \text{ in.}$ AND $b = 16 \text{ in.}$, then M_u^* (with the added self weight of 33.3 lb/ft) = 680.2 k-ft, $\rho \approx 0.012$, $A_s = 0.012(16 \text{ in})(29.42 \text{ in}) = 5.66 \text{ in}^2$. 6#9's won't fit, but 4#11's will: $\rho = 0.0132$ ✓, $a = 9.18 \text{ in}$, and $\phi M_n = 697.2 \text{ k-ft}$ which is finally larger than 680.2 k-ft **OK**

Example 6 (pg 437)

Example 4. A T-section is to be used for a beam to resist positive moment. The following data are given: beam span is 18 ft [5.49 m], beams are 9 ft [2.74 m] center to center, slab thickness is 4 in. [0.102 m], beam stem dimensions are $b_w = 15$ in. [0.381 m] and $d = 22$ in. [0.559 m], $f'_c = 4$ ksi [27.6 MPa], $f_y = 60$ ksi [414 MPa]. Find the required area of steel and select the reinforcing bars for a dead load moment of 125 kip-ft [170 kN-m] plus a live load moment of 100 kip-ft [136 kN-m].



Example 7

Design a T-beam for a floor with a 4 in slab supported by 22-ft-span-length beams cast monolithically with the slab. The beams are 8 ft on center and have a web width of 12 in. and a total depth of 22 in.; $f'_c = 3000$ psi and $f_y = 60$ ksi. Service loads are 125 psf and 200 psf dead load which does not include the weight of the floor system

SOLUTION:

1. Establish the design moment:

$$\text{slab weight} = \frac{96(4)}{144}(0.150) = 0.400 \text{ kip/ft}$$

$$\text{stem weight} = \frac{12(18)}{144}(0.150) = 0.225$$

$$\text{total} = 0.625 \text{ kip/ft}$$

$$\text{service DL} = 8(0.200) = 1.60 \text{ kips/ft}$$

$$\text{service LL} = 8(0.125) = 1.00 \text{ kip/ft}$$

Calculate the factored load and moment:

$$w_u = 1.2(0.625 + 1.60) + 1.6(1.00) = 4.27 \text{ kip/ft}$$

$$M_u = \frac{w_u \ell^2}{8} = \frac{4.27(22)^2}{8} = 258 \text{ ft-kips}$$

2. Assume an effective depth $d = h - 3$ in.:

$$d = 22 - 3 = 19 \text{ in.}$$

3. Determine the effective flange width:

$$\frac{1}{4} \text{ span length} = 0.25(22)(12) = 66 \text{ in.}$$

$$b_w + 16h_f = 12 + 16(4) = 76 \text{ in.}$$

$$\text{beam spacing} = 96 \text{ in.}$$

Use an effective flange width $b = 66$ in.

4. Determine whether the beam behaves as a true T-beam or as a rectangular beam by computing the practical moment strength ϕM_{nf} with the full effective flange assumed to be in compression. This assumes that the bottom of the compressive stress block coincides with the bottom of the flange, as shown in Figure 3-10. Thus

$$\begin{aligned} \phi M_{nf} &= \phi(0.85f'_c)bh_f \left(d - \frac{h_f}{2} \right) \\ &= 0.9(0.85)(3)(66) \frac{4(19 - 4/2)}{12} = 858 \text{ ft-kips} \end{aligned}$$

5. Since $858 \text{ ft-kips} > 258 \text{ ft-kips}$, the total effective flange need not be completely utilized in compression (i.e., $a < h_f$), and the T-beam behaves as a wide rectangular beam with a width b of 66 in.
6. Design as a rectangular beam with b and d as known values (see Section 2-15):

$$\text{required } R_n = \frac{M_u}{\phi b d^2} = \frac{258(12)}{0.9(66)(19)^2} = 0.1444 \text{ ksi}$$

7. From Table A-8, select the required steel ratio to provide a R_n of 0.1444 ksi:

$$\text{required } \rho = 0.0024$$

8. Calculate the required steel area:

$$\begin{aligned} \text{required } A_s &= \rho b d \\ &= 0.0024(66)(19) = 3.01 \text{ in.}^2 \end{aligned}$$

9. Select the steel bars. Use 3#9 ($A_s = 3.00 \text{ in.}^2$)

$$\text{minimum } b_w = 7.125 \text{ in.} \quad (\text{O.K.})$$

Check the effective depth d :

$$d = 22 - 1.5 - 0.38 \frac{1.129}{2} = 19.56 \text{ in.}$$

$$19.49 \text{ in.} > 19 \text{ in.}$$

(O.K.)

10. Check $A_{s,\min}$. From Table A-5:

$$\begin{aligned} A_{s,\min} &= 0.0033b_w d \\ &= 0.0033(12)(19) = 0.75 \text{ in.}^2 \\ 0.75 \text{ in.}^2 &< 3.00 \text{ in.}^2 \end{aligned}$$

11. Check $A_{s,\max}$:

$$\begin{aligned} A_{s,\max} &= 0.0135(66)(19) \\ &= 16.93 \text{ in.}^2 > 3.00 \text{ in.}^2 \quad (\text{O.K.}) \end{aligned}$$

12. Verify the moment capacity:

$$(\text{Is } M_u \leq \phi M_n)$$

$$a = (3.00)(60)/[0.85(3)(66)] = 1.07 \text{ in.}$$

$$\begin{aligned} \phi M_n &= 0.9(3.00)(60)(19.56 - \frac{1.07}{2}) \frac{1}{12} \\ &= 256.9.1 \text{ ft-kips} \quad (\text{Not O.K.}) \end{aligned}$$

Choose more steel, $A_s = 3.16 \text{ in.}^2$ from 4-#8's

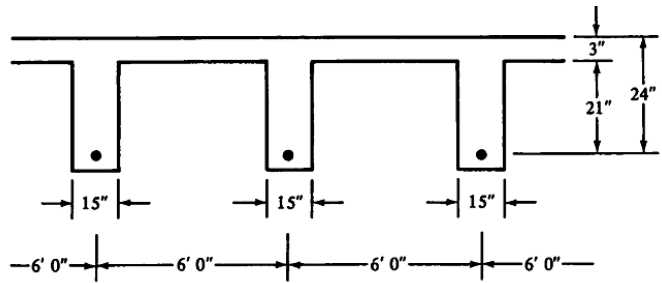
$$d = 19.62 \text{ in, } a = 1.13 \text{ in}$$

$$\phi M_n = 271.0 \text{ ft-kips, which is OK}$$

13. Sketch the design

Example 8

Design a T-beam for the floor system shown for which b_w and d are given. $M_D = 200$ ft-k, $M_L = 425$ ft-k, $f'_c = 3000$ psi and $f_y = 60$ ksi, and simple span = 18 ft.



SOLUTION

Effective Flange Width

- (a) $\frac{1}{4} \times 18' = 4'6'' = \underline{54''}$
- (b) $15'' + (2)(8)(3) = 63''$
- (c) $6'0'' = 72''$

Moments Assuming $\phi = 0.90$

$$M_u = (1.2)(200) + (1.6)(425) = 920 \text{ ft-k}$$

$$M_n = \frac{M_u}{0.90} = \frac{920}{0.90} = 1022 \text{ ft-k}$$

First assume $a \leq h_f$ (which is very often the case). Then the design would proceed like that of a rectangular beam with a width equal to the effective width of the T beam flange.

$$\frac{M_u}{\phi b d^2} = \frac{920(12,000)}{(0.9)(54)(24)^2} = 394.4 \text{ psi}$$

from Table A.12, $\rho = 0.0072$

$$a = \frac{\rho f_y d}{0.85 f'_c} = \frac{0.0072(60)(24)}{(0.85)(3)} = 4.06 \text{ in.} > h_f = 3 \text{ in.}$$

The beams acts like a T beam, not a rectangular beam, and the values for ρ and a above are not correct. If the value of a had been $\leq h_f$, the value of A_s would have been simply $\rho b d = 0.0072(54)(24) = 9.33 \text{ in}^2$. Now break the beam up into two parts (Figure 5.7) and design it as a T beam.

Assuming $\phi = 0.90$

$$A_{sf} = \frac{(0.85)(3)(54 - 15)(3)}{60} = 4.97 \text{ in.}^2$$

$$M_{uf} = (0.9)(4.97)(60)(24 - \frac{3}{2}) = 6039 \text{ in.-k} = 503 \text{ ft-k}$$

$$M_{uw} = 920 - 503 = 417 \text{ ft-k}$$

Designing a rectangular beam with $b_w = 15$ in. and $d = 24$ in. to resist 417 ft-k

$$\frac{M_{uw}}{\phi b_w d^2} = \frac{(12)(417)(1000)}{(0.9)(15)(24)^2} = 643.5$$

$\rho_w = 0.0126$ from Appendix Table A.12

$$A_{sw} = (0.0126)(15)(24) = 4.54 \text{ in.}^2$$

$$A_s = 4.97 + 4.54 = \underline{9.51 \text{ in.}^2}$$

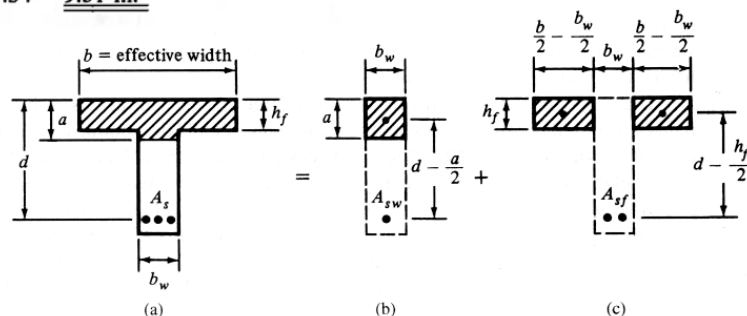


Figure 5.7 Separation of T beam into rectangular parts.

Example 9 (pg 448)

Example 7. A one-way solid concrete slab is to be used for a simple span of 14 ft [4.27 m]. In addition to its own weight, the slab carries a superimposed dead load of 30 psf [1.44 kPa] plus a live load of 100 psf [4.79 kPa]. Using $f'_c = 3$ ksi [20.7 MPa] and $f_y = 40$ ksi [276 MPa], design the slab for minimum overall thickness.

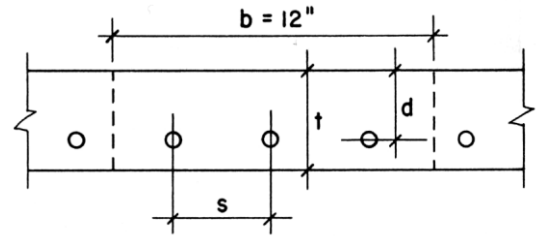


TABLE 13.7 Areas Provided By Spaced Reinforcement

Bar Spacing (in.)	Area Provided (in. ² /ft width)									
	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11	
3	0.44	0.80	1.24	1.76	2.40	3.16	4.00			
3.5	0.38	0.69	1.06	1.51	2.06	2.71	3.43	4.35		
4	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68	
4.5	0.29	0.53	0.83	1.17	1.60	2.11	2.67	3.39	4.16	
5	0.26	0.48	0.74	1.06	1.44	1.89	2.40	3.05	3.74	
5.5	0.24	0.44	0.68	0.96	1.31	1.72	2.18	2.77	3.40	
6	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12	
7	0.19	0.34	0.53	0.75	1.03	1.35	1.71	2.18	2.67	
8	0.16	0.30	0.46	0.66	0.90	1.18	1.50	1.90	2.34	
9	0.15	0.27	0.41	0.59	0.80	1.05	1.33	1.69	2.08	
10	0.13	0.24	0.37	0.53	0.72	0.95	1.20	1.52	1.87	
11	0.12	0.22	0.34	0.48	0.65	0.86	1.09	1.38	1.70	
12	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56	
13	0.10	0.18	0.29	0.40	0.55	0.73	0.92	1.17	1.44	
14	0.09	0.17	0.27	0.38	0.51	0.68	0.86	1.09	1.34	
15	0.09	0.16	0.25	0.35	0.48	0.63	0.80	1.01	1.25	
16	0.08	0.15	0.23	0.33	0.45	0.59	0.75	0.95	1.17	
18	0.07	0.13	0.21	0.29	0.40	0.53	0.67	0.85	1.04	
24	0.05	0.10	0.15	0.22	0.30	0.39	0.50	0.63	0.78	

Example 10

Example 2-9

Design a simple-span one-way slab to carry a uniformly distributed live load of 400 psf. The span is 10 ft (center to center of supports). Use $f'_c = 4000$ psi and $f_y = 60,000$ psi. Select the thickness to be not less than the ACI minimum thickness requirement.

Solution:

Determine the required minimum h and use this to estimate the slab dead weight.

1. From ACI Table 9.5(a), for a simply supported, solid, one-way slab,

$$\text{minimum } h = \frac{\ell}{20} = \frac{10(12)}{20} = 6.0 \text{ in.}$$

Try $h = 6$ in. and design a 12-in.-wide segment.

2. Determine the slab weight dead load:

$$\frac{6(12)}{144}(0.150) = 0.075 \text{ kip/ft}$$

The total design load is

$$\begin{aligned} w_u &= 1.2w_{DL} + 1.6w_{LL} \\ &= 1.2(0.075) + 1.6(0.400) \\ &= 0.730 \text{ kip/ft} \end{aligned}$$

3. Determine the design moment:

$$M_u = \frac{w_u \ell^2}{8} = \frac{0.73(10)^2}{8} = 9.125 \text{ ft-kips}$$

4. Establish the approximate d . Assuming No. 6 bars and minimum concrete cover on the bars of $\frac{3}{8}$ in.,

$$\text{assumed } d = 6.0 - 0.75 - 0.375 = 4.88 \text{ in.}$$

5. Determine the required R_n :

$$\begin{aligned} \text{required } R_n &= \frac{M_u}{\phi b d^2} \\ &= \frac{9.125(12)}{0.9(12)(4.88)^2} = 0.4257 \text{ ksi} \end{aligned}$$

6. From Table A-10, for a required $R_n = 0.4257$, the required $\rho = 0.0077$. (Note that the required ρ selected is the next higher value from Table A-10.) Thus

$$\rho_{\max} = 0.0181 > 0.0077 \quad (\text{O.K.})$$

Use $\rho = 0.0077$.

7. required $A_s = \rho b d = 0.0077(12)(4.88) = 0.45 \text{ in.}^2/\text{ft}$

8. Select the main steel (from Table A-4). Select No. 5 bars at $7\frac{1}{2}$ in. o.c. ($A_s = 0.50 \text{ in.}^2$). The assumption on bar size was satisfactory. The code requirements for maximum spacing have been discussed in Section 2-13. Minimum spacing of bars in slabs, practically, should not be less than 4 in., although the ACI Code allows bars to be placed closer together, as discussed in Example 2-7. Check the maximum spacing (ACI Code, Section 7.6.5):

$$\text{maximum spacing} = 3h \text{ or } 18 \text{ in.}$$

$$3h = 3(6) = 18 \text{ in.}$$

$$7\frac{1}{2} \text{ in.} < 18 \text{ in.} \quad (\text{O.K.})$$

Therefore use No. 5 bars at $7\frac{1}{2}$ in. o.c.

9. Select shrinkage and temperature reinforcement (ACI Code, Section 7.12):

$$\begin{aligned} \text{required } A_s &= 0.0018bh \\ &= 0.0018(12)(6) = 0.13 \text{ in.}^2/\text{ft} \end{aligned}$$

Select No. 3 bars at 10 in. o.c. ($A_s = 0.13 \text{ in.}^2$) or No. 4 bars at 18 in. o.c. ($A_s = 0.13 \text{ in.}^2$):

$$\text{maximum spacing} = 5h \text{ or } 18 \text{ in.}$$

Use No. 3 bars at 10 in. o.c.

10. The main steel area must exceed the area required for shrinkage and temperature steel (ACI Code, Section 10.5.4):

$$0.50 \text{ in.}^2 > 0.13 \text{ in.}^2 \quad (\text{O.K.})$$

11. Verify the moment capacity:

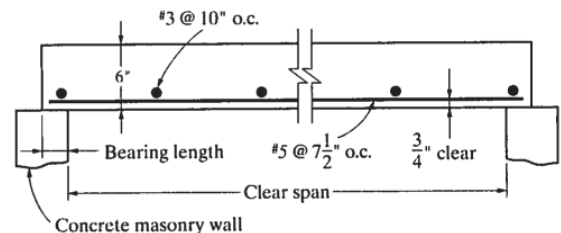
$$(\text{Is } M_u \leq \phi M_n)$$

$$a = \frac{(0.50)(60)}{0.85(4)(12)} = 0.74 \text{ in}$$

$$\phi M_n = 0.9(0.50)(60)(5.0625 - 0.74/2) \frac{1}{12}$$

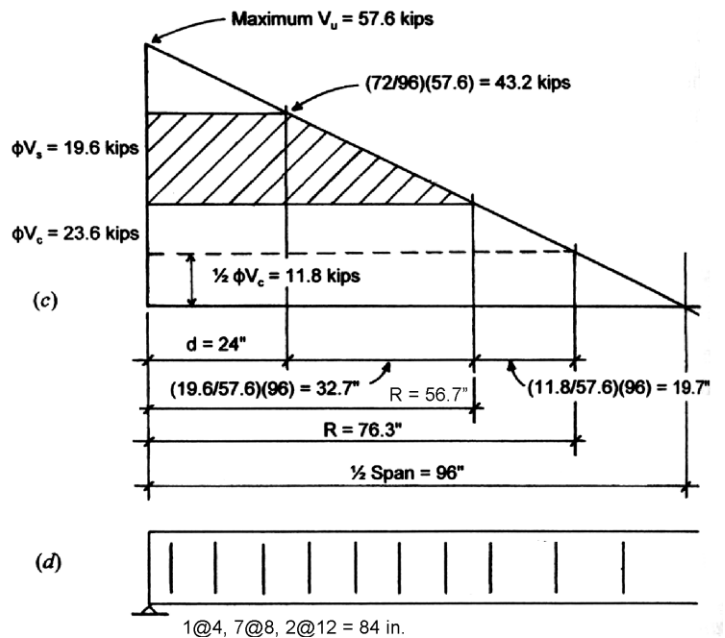
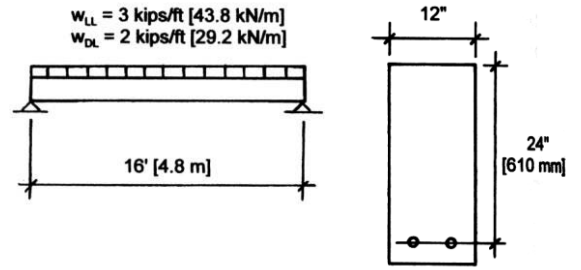
$$= 10.6 \text{ ft-kips} \quad (\text{OK})$$

12. A design sketch is drawn:



Example 11 (pg 461)

Example 8. Design the required shear reinforcement for the simple beam shown in Figure 13.18. Use $f'_c = 3$ ksi [20.7 MPa] and $f_y = 40$ ksi [276 MPa] and single U-shaped stirrups.



Example 12

For the simply supported concrete beam shown in Figure 5-61, determine the stirrup spacing (if required) using No. 3 U stirrups of Grade 60 ($f_y = 60$ ksi). Assume $f'_c = 3000$ psi.

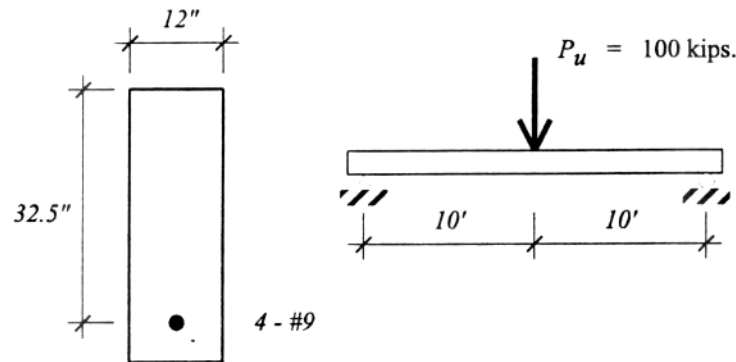


Figure 5-61: Simply supported concrete beam for Example 5-15.

$$f'_c = 3000 \text{ psi.}$$

$$F_y = 60 \text{ ksi.}$$

$$\text{For \#3 bars, } A_s = 0.11 \text{ in.}^2,$$

$$\text{with 2 legs, then } A_v = 0.22 \text{ in.}^2$$

Solution:

$$V_u = 50 \text{ kips (neglecting weight of the beam)}$$

$$\phi V_c = \phi 2\sqrt{f'_c} b_w d$$

$$= \frac{(0.75)2\sqrt{3000}(12)(32.5)}{1000} = 32.0 \text{ kips} < V_u \therefore \text{Need Stirrups}$$

Note: If $V_u = \frac{1}{2}\phi V_c$, minimum stirrups would still be required.

$$V_u = \phi V_c + \phi V_s$$

$$\therefore \phi V_s = V_u - \phi V_c = 50 - 32.0 = 18.0 \text{ kips} \quad (< \phi 4\sqrt{f'_c} b_w d = 64.1 \text{ kips})$$

$$s_{req'd} \leq \frac{\phi A_s F_y d}{\phi V_c} = \frac{(0.75)(0.22 \text{ in}^2)(60 \text{ ksi})(32.5 \text{ in})}{18.0 \text{ k}}$$

$$= 17.875 \text{ in.}$$

$$s_{max} = \frac{d}{2} = \frac{32.5}{2} = 16.2 \text{ in.} \quad \text{controls}$$

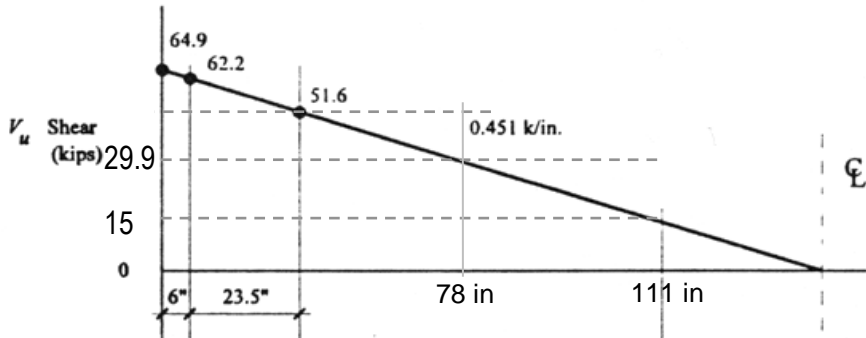
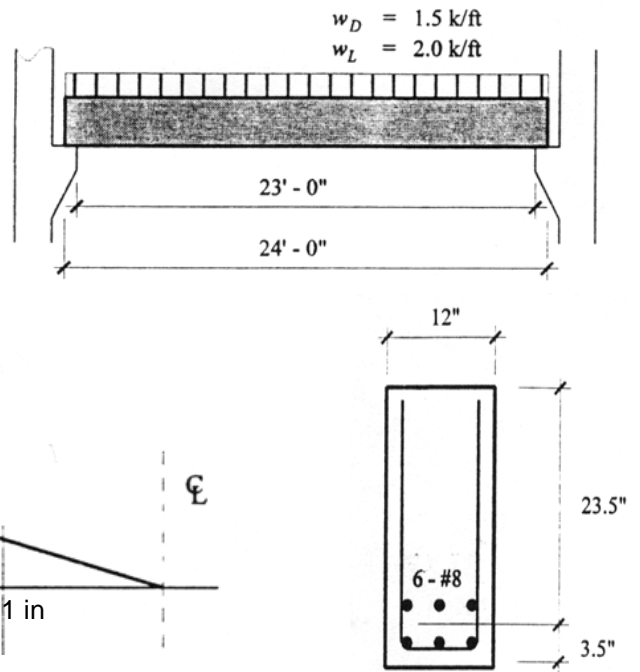
$$= 24 \text{ in.}$$

$$\left[\begin{array}{l} s_{req'd} \\ \text{if } \phi V_c > V_u > \frac{\phi V_c}{2} \end{array} = \frac{A_v F_y}{50 b_w} = \frac{(0.22)(60,000)}{50(12)} = 22.0 \text{ in., but } 16'' (d/2) \text{ would be the maximum as well.} \right]$$

\therefore Use #3 U @ 16" max spacing

Example 13

Design the shear reinforcement for the simply supported reinforced concrete beam shown with a dead load of 1.5 k/ft and a live load of 2.0 k/ft. Use 5000 psi concrete and Grade 60 steel. Assume that the point of reaction is at the end of the beam.



SOLUTION:

Shear diagram:

Find self weight = 1 ft x (27/12 ft) x 150 lb/ft³ = 338 lb/ft = 0.338 k/ft

$w_u = 1.2 (1.5 \text{ k/ft} + 0.338 \text{ k/ft}) + 1.6 (2 \text{ k/ft}) = 5.41 \text{ k/ft} (= 0.451 \text{ k/in})$

$V_{u(max)}$ is at the ends = $w_u L / 2 = 5.41 \text{ k/ft} (24 \text{ ft}) / 2 = 64.9 \text{ k}$

$V_{u(support)} = V_{u(max)} - w_u(\text{distance}) = 64.9 \text{ k} - 5.4 \text{ k/ft} (6/12 \text{ ft}) = 62.2 \text{ k}$

V_u for design is d away from the support = $V_{u(support)} - w_u(d) = 62.2 \text{ k} - 5.41 \text{ k/ft} (23.5/12 \text{ ft}) = 51.6 \text{ k}$

Concrete capacity:

We need to see if the concrete needs stirrups for strength or by requirement because $V_u \leq \phi V_c + \phi V_s$ (design requirement)

$\phi V_c = \phi 2 \sqrt{f'_c} b_w d = 0.75 (2) \sqrt{5000} \text{ psi} (12 \text{ in}) (23.5 \text{ in}) = 299106 \text{ lb} = 29.9 \text{ kips} (< 51.6 \text{ k!})$

Stirrups design and spacing

We need stirrups: $A_v = V_s / f_y d$

$\phi V_s \geq V_u - \phi V_c = 51.6 \text{ k} - 29.9 \text{ k} = 21.7 \text{ k}$

Spacing requirements are in Table 3-8 and depend on $\phi V_c / 2 = 15.0 \text{ k}$ and $2\phi V_c = 59.8 \text{ k}$

2 legs for a #3 is 0.22 in², so $s_{req'd} \leq \phi A_v f_y d / \phi V_s = 0.75 (0.22 \text{ in}^2) (60 \text{ ksi}) (23.5 \text{ in}) / 21.7 \text{ k} = 10.72 \text{ in}$ Use $s = 10''$

our maximum falls into the $d/2$ or $24''$, so $d/2$ governs with 11.75 in Our 10'' is ok.

This spacing is valid until $V_u = \phi V_c$ and that happens at $(64.9 \text{ k} - 29.9 \text{ k}) / 0.451 \text{ k/in} = 78 \text{ in}$

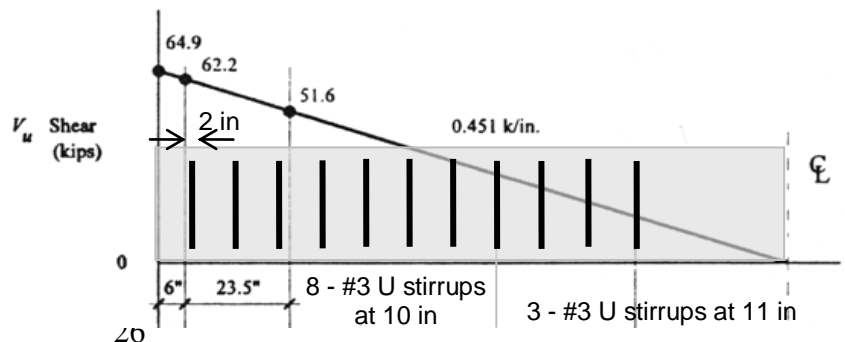
We can put the first stirrup at a minimum of 2 in fr support face, so we need 10'' spaces for $(78 - 2) / 10 = 7.6$ even (8 stirrups altogether ending at 78 in)

After 78'' we can change the spacing to the require more than the maximum of $d/2 = 11.75 \text{ in} \leq 24 \text{ in}$;

$s = A_v f_y / 50 b_w = 0.22 \text{ in}^2 (60,000 \text{ psi}) / 50 (1$

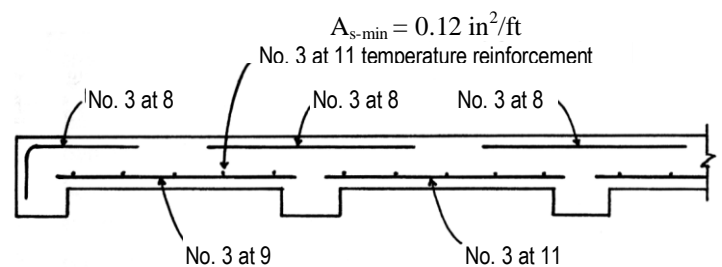
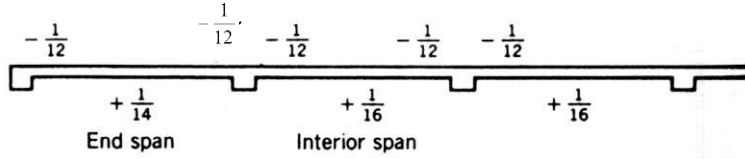
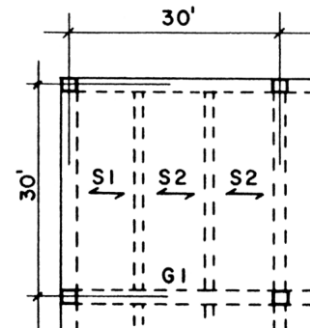
We need to continue to 111 in, so $(111 - 78 \text{ in}) / 10 = 3.3$ even

Locating end points:
 $29.9 \text{ k} = 64.9 \text{ k} - 0.451 \text{ k/in} \times (a)$
 $a = 78 \text{ in}$
 $15 \text{ k} = 64.9 \text{ k} - 0.451 \text{ k/in} \times (b)$
 $b = 111 \text{ in}.$



Example 14 (pg 483)

Example 1. A solid one-way slab is to be used for a framing system similar to that shown in Figure 14.1. Column spacing is 30 ft. with evenly spaced beams occurring at 10 ft. center to center. Superimposed loads on the structure (floor live load plus other construction dead load) are a dead load of 38 psf [1.82 kPa] and a live load of 100 psf [4.79 kPa]. Use $f'_c = 3$ ksi [20.7 MPa] and $f_y = 40$ ksi [275 MPa]. Determine the thickness for the slab and select its reinforcement.



Example 15

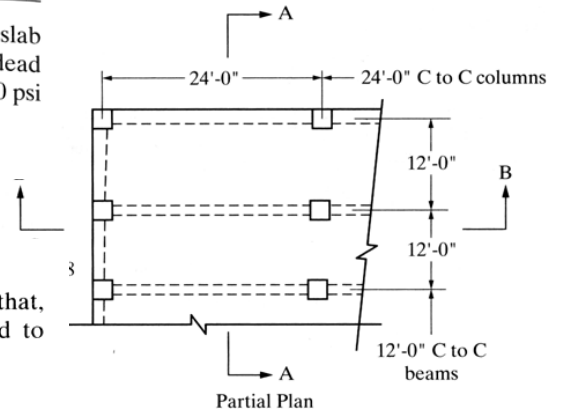
Example 6-1

The floor system shown in Figure 6-4 consists of a continuous one-way slab supported by continuous beams. The service loads on the floor are 25 psf dead load (does not include weight of slab) and 250 psf live load. Use $f'_c = 3000$ psi (normal-weight concrete) and $f_y = 60,000$ psi. The bars are uncoated.

Design the continuous one-way floor slab.

Solution:

The primary difference in this design from previous flexural designs is that, because of continuity, the ACI coefficients and equations will be used to determine design shears and moments.



A. Continuous one-way floor slab

1. Determine the slab thickness. The slab will be designed to satisfy the ACI minimum thickness requirements from Table 9.5(a) of the code and this thickness will be used to estimate slab weight.

With both ends continuous,

$$\text{minimum } h = \frac{1}{28} \ell_n = \frac{1}{28} (11)(12) = 4.71 \text{ in.}$$

With one end continuous,

$$\text{minimum } h = \frac{1}{24} \ell_n = \frac{1}{24} (11)(12) = 5.5 \text{ in.}$$

Try a 5½-in.-thick slab. Design a 12-in.-wide segment ($b = 12$ in.).

2. Determine the load:

$$\text{slab dead load} = \frac{5.5}{12} (150) = 68.8 \text{ psf}$$

$$\text{total dead load} = 25.0 + 68.8 = 93.8 \text{ psf}$$

$$\begin{aligned} w_u &= 1.2 w_{DL} + 1.6 w_{LL} \\ &= 1.2(93.8) + 1.6(250) \\ &= 112.6 + 400.0 \\ &= 512.6 \text{ psf} \quad (\text{design load}) \end{aligned}$$

Because we are designing a slab segment that is 12 in. wide, the foregoing loading is the same as 512.6 lb/ft or 0.513 kip/ft.

3. Determine the moments and shears. Moments are determined using the ACI moment equations. Refer to Figures 6-1 and 6-4. Thus

$$+M_u = \frac{1}{14} w_u \ell_n^2 = \frac{1}{14} (0.513)(11)^2 = 4.43 \text{ ft-kips}$$

$$+M_u = \frac{1}{16} w_u \ell_n^2 = \frac{1}{16} (0.513)(11)^2 = 3.88 \text{ ft-kips}$$

$$-M_u = \frac{1}{10} w_u \ell_n^2 = \frac{1}{10} (0.513)(11)^2 = 6.20 \text{ ft-kips}$$

$$-M_u = \frac{1}{11} w_u \ell_n^2 = \frac{1}{11} (0.513)(11)^2 = 5.64 \text{ ft-kips}$$

$$-M_u = \frac{1}{24} w_u \ell_n^2 = \frac{1}{24} (0.513)(11)^2 = 2.58 \text{ ft-kips}$$

4. The bars are selected in the same manner as for beams with minimum areas based on shrinkage and crack reinforcement. Moment and shear capacities should be satisfied.

5. Development length for the flexure reinforcement is required.

For example, #6 bars:

$$l_d = \frac{d_b F_y}{25 \sqrt{f'_c}} \quad \text{or } 12 \text{ in. minimum}$$

With grade 40 steel and 3000 psi concrete:

$$l_d = \frac{\%8 \text{ in}(40,000 \text{ psi})}{25 \sqrt{3000 \text{ psi}}} = 21.9 \text{ in}$$

(which is larger than 12 in.)

Similarly, the shears are determined using the ACI shear equations. In the end span at the face of the first interior support,

$$V_u = 1.15 \frac{w_u \ell_n}{2} = 1.15(0.513) \left(\frac{11}{2} \right) = 3.24 \text{ kips}$$

whereas at all other supports,

$$V_u = \frac{w_u \ell_n}{2} = (0.513) \left(\frac{11}{2} \right) = 2.82 \text{ kips}$$

Example 16

A building is supported on a grid of columns that is spaced at 30 ft on center in both the north-south and east-west directions. Hollow core planks with a 2 in. topping span 30 ft in the east-west direction and are supported on precast L and inverted T beams. Size the hollow core planks assuming a live load of 100 lb/ft². Choose the shallowest plank with the least reinforcement that will span the 30 ft while supporting the live load.

SOLUTION:

The shallowest that works is an 8 in. deep hollow core plank.

The one with the least reinforcing has a strand pattern of 68-S, which contains 6 strands of diameter 8/16 in. = 1/2 in. The S indicates that the strands are straight. The plank supports a superimposed service load of 124 lb/ft² at a span of 30 ft with an estimated camber at erection of 0.8 in. and an estimated long-time camber of 0.2 in.

The weight of the plank is 81 lb/ft².

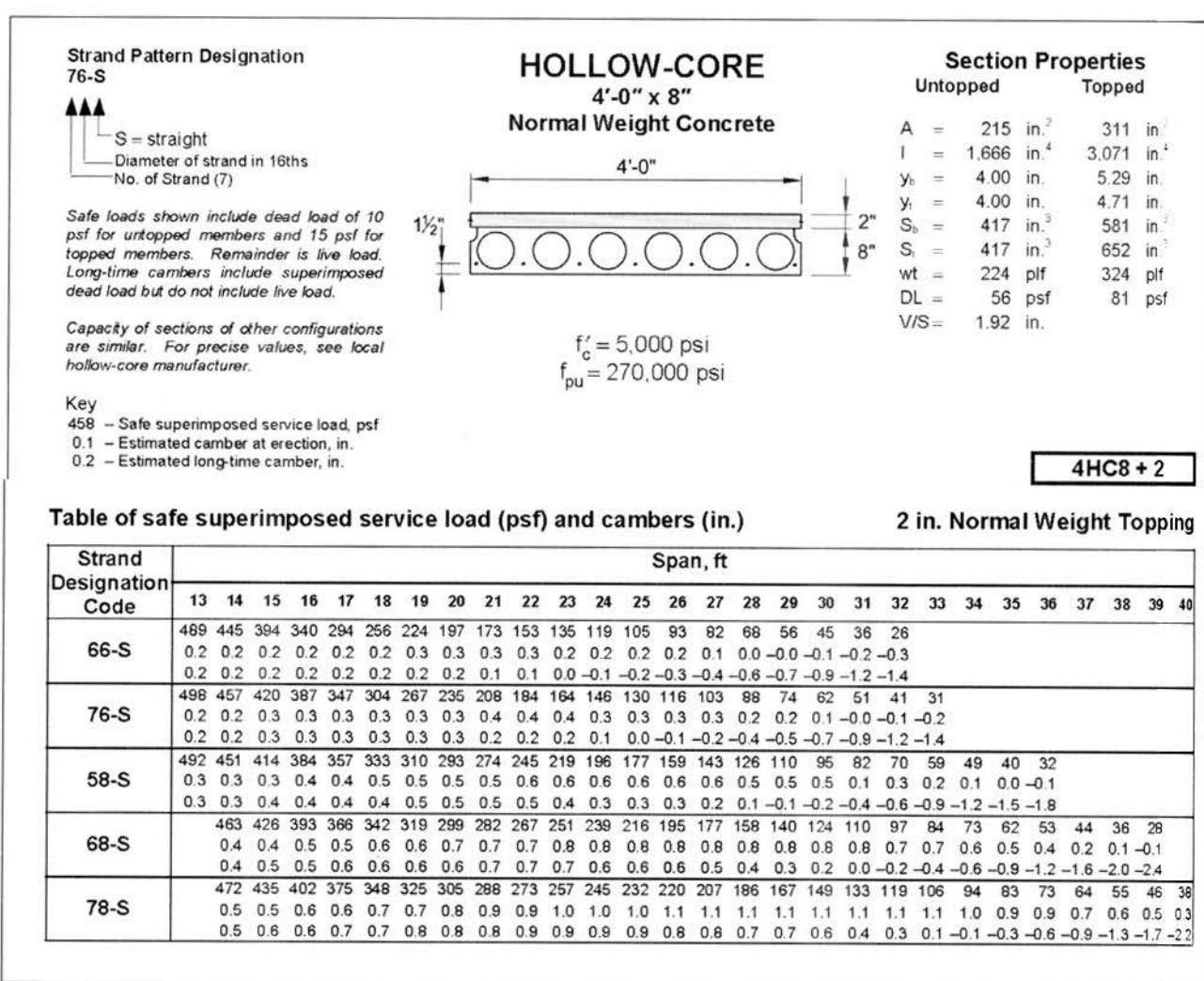
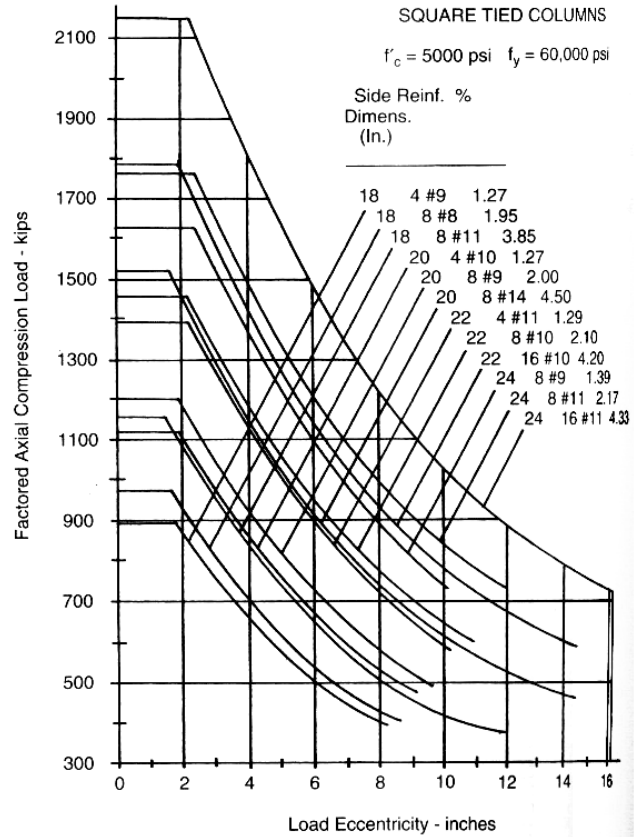
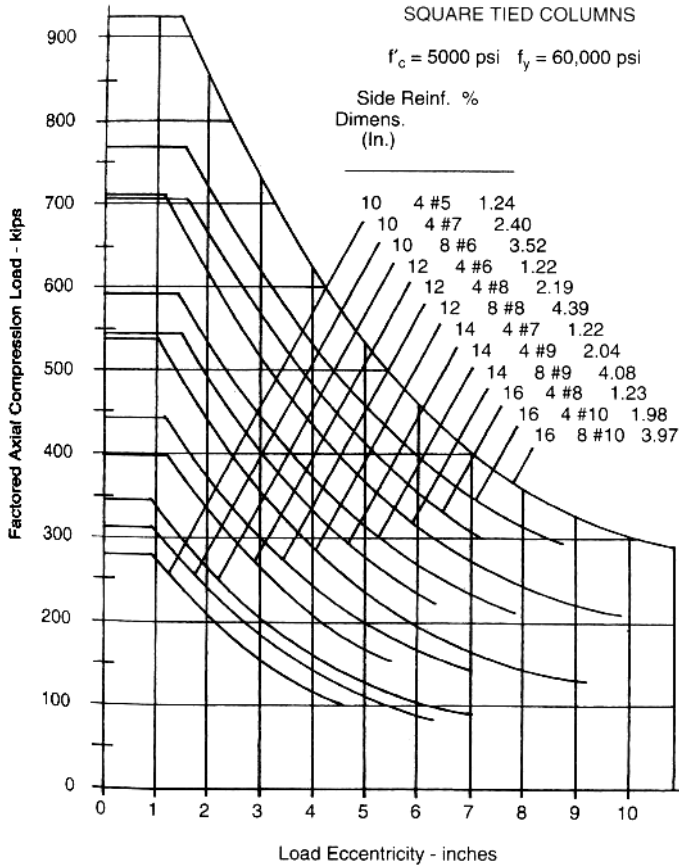
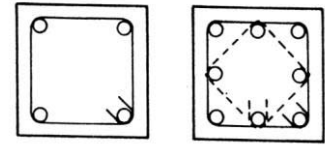


Figure 6.88 Allowed load on 4 ft-wide, 8 in.-deep hollow-core planks (HCPs). (Copyright Prestressed/Precast Concrete Institute (PCI). Reprinted with permission. All rights reserved.)

Example 17 (pg 510)

Example 1. A square tied column with $f'_c = 5$ ksi and steel with $f_y = 60$ ksi sustains an axial compression load of 150 kips dead load and 250 kips live load with no computed bending moment. Find the minimum practical column size if reinforcing is a maximum of 4% and the maximum size if reinforcing is a minimum of 1%. Also, design for $e = 6$ in.



Example 18

Determine the capacity of a 16" x 16" column with 8- #10 bars, tied. Grade 40 steel and 4000 psi concrete.

SOLUTION:

Find ϕP_n , with $\phi=0.65$ and $P_n = 0.80P_o$ for tied columns and

$$P_o = 0.85f'_c(A_g - A_{st}) + f_y A_{st}$$

Steel area (found from reinforcing bar table for the bar size):

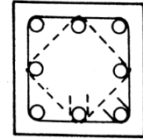
$$A_{st} = 8 \text{ bars} \times (1.27 \text{ in}^2) = 10.16 \text{ in}^2$$

Concrete area (gross):

$$A_g = 16 \text{ in} \times 16 \text{ in} = 256 \text{ in}^2$$

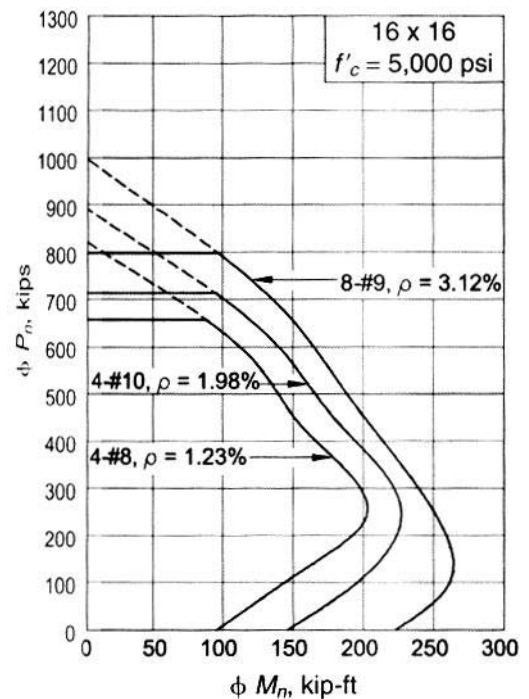
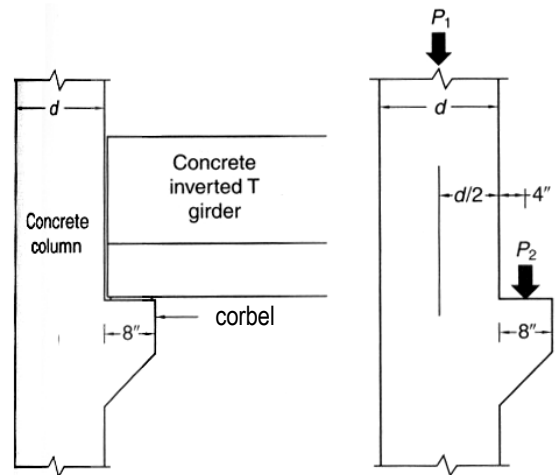
Grade 40 reinforcement has $f_y = 40,000 \text{ psi}$ and $f'_c = 4000 \text{ psi}$

$$\phi P_n = (0.65)(0.80)[0.85(4000 \text{ psi})(256 \text{ in}^2 - 10.16 \text{ in}^2) + (40,000 \text{ psi})(10.16 \text{ in}^2)] = 646,026 \text{ lb} = 646 \text{ kips}$$



Example 19

16" x 16" precast reinforced columns support inverted T girders on corbels as shown. The unfactored loads on the corbel are 81 k dead, and 72 k live. The unfactored loads on the column are 170 k dead and 150 k live. Determine the reinforcement required using the interaction diagram provided. Assume that half the moment is resisted by the column above the corbel and the other half is resisted by the column below. Use grade 50 steel and 5000 psi concrete.



Example 20

EXAMPLE 5-4

Design a short square tied column to carry an axial dead load of 300 kip and a live load of 200 kip. Assume that the applied moments on the column are negligible. Use $f'_c = 4,000$ psi and $f_y = 60,000$ psi.

Solution

Step 1 The factored load, P_u , is:

$$\begin{aligned} P_u &= 1.2P_D + 1.6P_L \\ P_u &= 1.2(300) + 1.6(200) \\ P_u &= 680 \text{ kip} \end{aligned}$$

Assume $\rho_g = 0.03$.

Step 2 The required area of the column, A_g , is:

$$\begin{aligned} A_g &= \frac{P_u}{0.8\phi[0.85f'_c(1 - \rho_g) + f_y\rho_g]} \\ A_g &= \frac{680}{0.80(0.65)[0.85(4)(1 - 0.03) + 60(0.03)]} \\ A_g &= 257 \text{ in}^2 \end{aligned}$$

Step 3 For a square column, the size, h , is:

$$\begin{aligned} h &= \sqrt{A_g} = \sqrt{257} \\ \therefore h &= 16.0 \text{ in.} \end{aligned}$$

Try a 16 in. \times 16 in. column:

$$A_g = (16)(16) = 256 \text{ in}^2$$

Step 4 The required amount of steel, A_{st} , is:

$$\begin{aligned} A_{st} &= \frac{P_u - 0.8\phi(0.85f'_c A_g)}{0.8\phi(f_y - 0.85f'_c)} \\ A_{st} &= \frac{680 - 0.8 \times 0.65(0.85 \times 4 \times 256)}{0.8 \times 0.65(60 - 0.85 \times 4)} = 7.73 \text{ in}^2 \end{aligned}$$

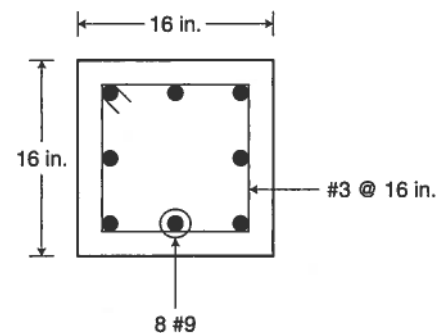
Step 5 Select the size and number of bars. For a square column with bars uniformly distributed along the edges, we keep the number of bars as multiples of four. Using Table A2-9, 8 #9 bars ($A_s = 8 \text{ in}^2$) are selected.

From Table A5-1 \rightarrow Maximum of 12 #9 bars \therefore ok

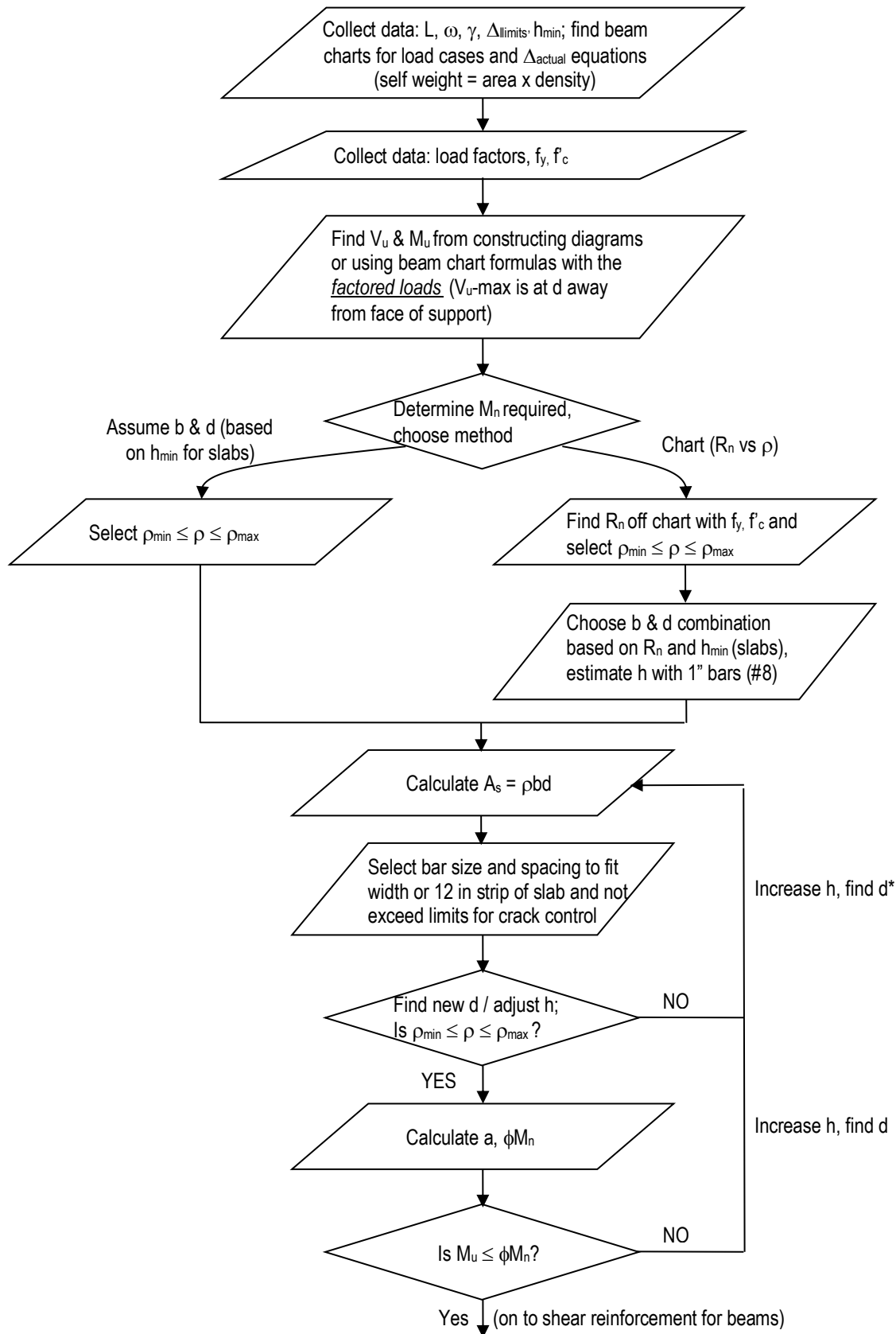
Step 6 Because the longitudinal bars are #9, select #3 bars for the ties. The maximum spacing of the ties (s_{\max}) is:

$$\begin{aligned} s_{\max} &= \min\{16d_b, 48d_t, b_{\min}\} \\ s_{\max} &= \min\{16(1.128), 48(\frac{3}{8}), 16\} \\ s_{\max} &= \min\{18.0, 18.0, 16.0\} \\ \therefore s_{\max} &= 16 \text{ in.} \end{aligned}$$

The selected ties are #3 @ 16 in.



Beam / One-Way Slab Design Flow Chart



Beam / One-Way Slab Design Flow Chart - continued

