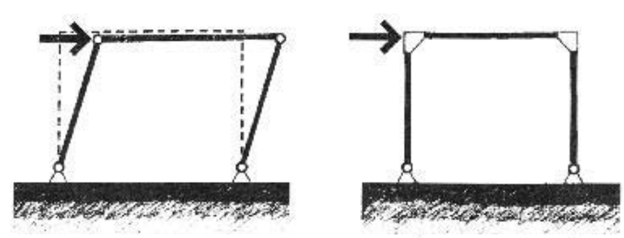
Rigid Frames - Compression & Buckling

Notation:

A = name for area
d = name for depth
E = modulus of elasticity or Young's modulus
f_a = axial stress
f_b = bending stress
f_z = stress in the x direction
F_a = allowable axial stress
F_b = allowable bending stress
F_x = force component in the x direction
F_y = force component in the y direction
FBD = free body diagram
G = relative stiffness of columns to beams in a rigid connection, as is Ψ
I = moment of inertia with respect to neutral axis bending
k = effective length factor for columns
ℓ_b = length of beam in rigid joint
ℓ_c = length of column in rigid joint
L = name for length
L_e = effective length that can buckle for column design, as is ℓ_e
M = internal bending moment
= name for a moment vector
P = name for axial force vector, as is P'
P_{crit} = critical buckling load in column calculations, as is $P_{critical}$, P_{cr}
r = radius of gyration
V = internal shear force
y = vertical distance
Δ = displacement due to bending
π = pi (180°)
Σ = summation symbol
Ψ = relative stiffness of columns to beams in a rigid connection, as is G

Rigid Frames

Rigid frames are identified by the lack of pinned joints within the frame. The joints are *rigid* and resist rotation. They may be supported by pins or fixed supports. They are typically statically indeterminate.

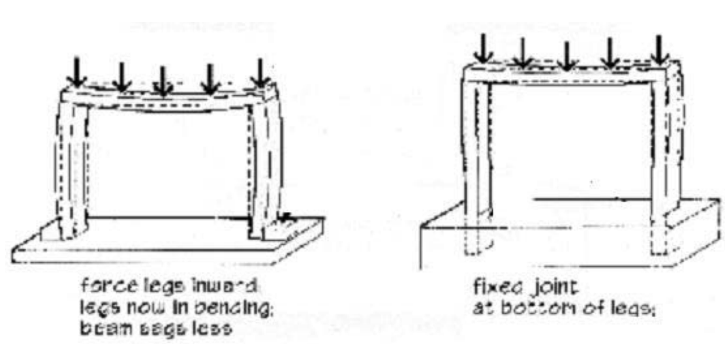


Frames are useful to resist **lateral** loads.

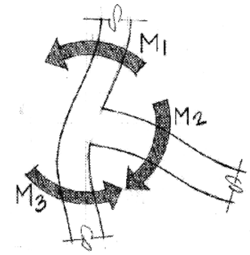
Frame members will see

- shear
- bending
- axial forces

and behave like *beam-columns.*



Behavior



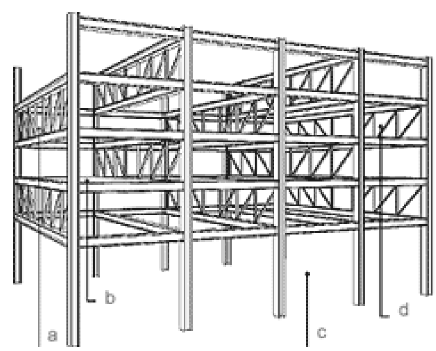
The relation between the joints has to be maintained, but the whole joint can *rotate*. The amount of rotation and distribution of moment depends on the *stiffness* (EI/L) of the members in the joint.

End restraints on columns reduce the effective length, allowing columns to be more slender. Because of the rigid joints, deflections and moments in beams are reduced as well.

Frames are sensitive to settlement because it induces strains and changes the stress distribution.

Types

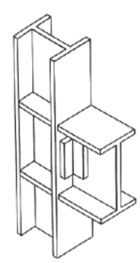
Gabled – has a peak

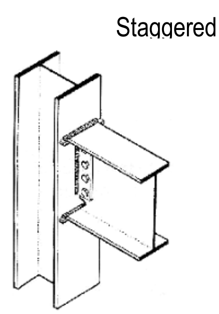


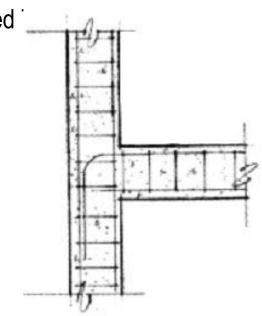
Portal – resembles a door. Multi-story, multiple bay portal frames are commonly used for commercial and industrial construction. The floor behavior is similar to that of continuous beams.

Staggered Truss – Full story trusses are staggered through the frame bays, allowing larger clear stories.

Connections







Steel – Flanges of members are fully attached to the flanges of the other member. This can be done with welding, or bolted plates.

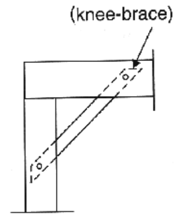
Reinforced Concrete – Joints are monolithic with continuous reinforcement for bending. Shear is resisted with stirrups and ties.

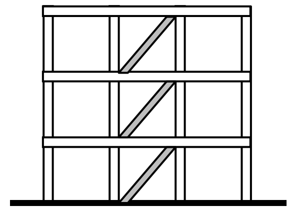
Braced Frames

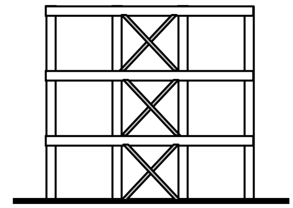
Braced frames have beams and columns that are “pin” connected with bracing to resist lateral loads.

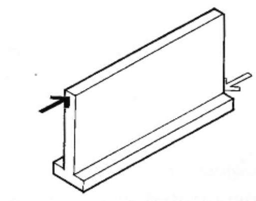
Types of Bracing

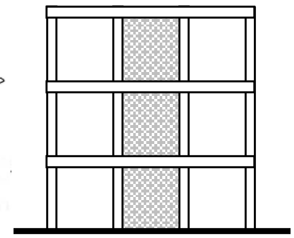
- knee-bracing
- diagonal (including eccentric)
- X
- K or chevron
- shear walls – which resist lateral forces in the plane of the wall

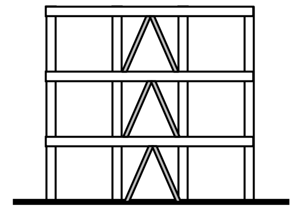



diagonal


X

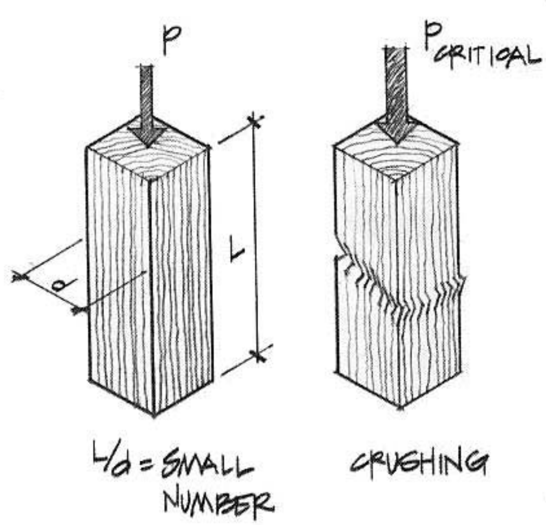



shear walls


K (chevron)

Compression Members - Columns

Including strength (stresses) and servicability (including deflections), another requirement is that the structure or structural member be *stable*.

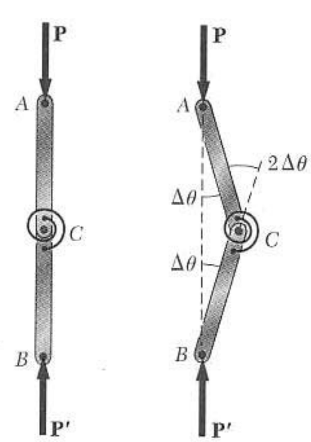


Stability is the ability of the structure to support a specified load without undergoing unacceptable (or sudden) deformations.

A column loaded centrically can experience unstable equilibrium, called *buckling*, because of how tall and slender they are. This instability is sudden and not good.

Buckling can occur in sheets (like my "memory metal" cookie sheet), pressure vessels or slender (narrow) beams not braced laterally.

Buckling can be thought of with the loads and motion of a column having a stiff spring at mid-height. There exists a load where the spring can't resist the moment in it any longer.



Short (stubby) columns will experience crushing before buckling.

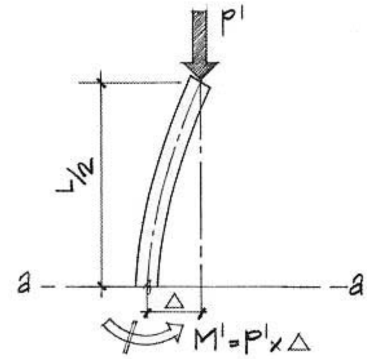
Critical Buckling Load

The critical axial load to cause buckling is related to the deflected shape we could get (or determine from bending moment of P·Δ).

The buckled shape will be in the form of a *sine wave*.

Euler Formula

Swiss mathematician Euler determined the relationship between the critical buckling load, the material, section and effective length (as long as the material stays in the elastic range):



$$P_{critical} = \frac{\pi^2 EI}{L^2} \quad \text{or} \quad P_{cr} = \frac{\pi^2 EI}{(L_e)^2} = \frac{\pi^2 EA}{\left(L_e/r\right)^2}$$

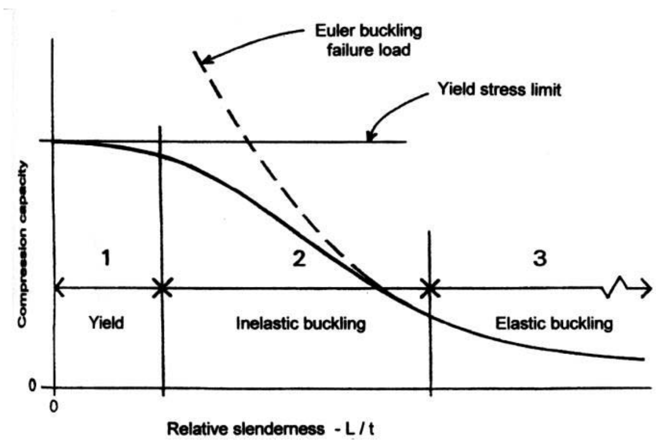
and the critical stress (*if less than the normal stress)* is:

$$f_{critical} = \frac{P_{critical}}{A} = \frac{\pi^2 EAr^2}{A(L_e)^2} = \frac{\pi^2 E}{\left(L_e/r\right)^2}$$

where $I = Ar^2$ and L_e/r is called the slenderness ratio. The smallest I of the section will govern, if the effective length is the same for box axes.

Yield Stress and Buckling Stress

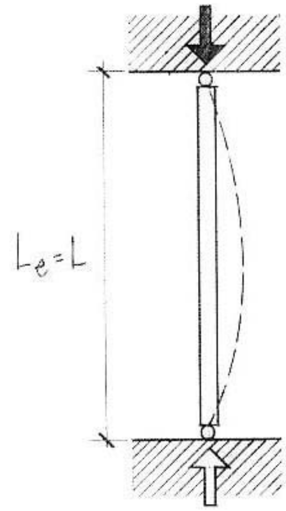
The two design criteria for columns are that they do not buckle and the strength is not exceeded. Depending on slenderness, one will control over the other.



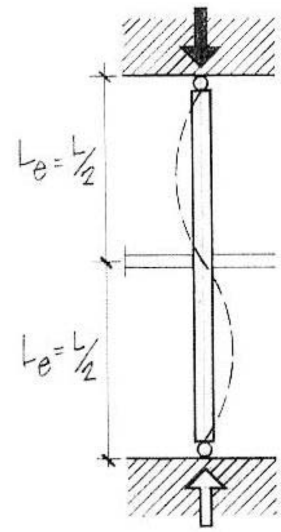
But, because in the real world, things are rarely perfect – and columns will not actually be loaded concentrically, but will see eccentricity – Euler's formula is used only if the critical stress is less than half of the yield point stress, in the elastic buckling region. A transition formula is used for inelastic buckling.

Effective Length and Bracing

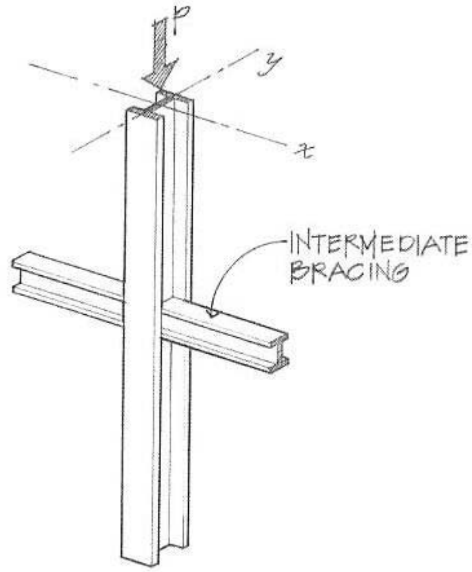
Depending on the end support conditions for a column, the effective length can be found from the deflected shape (elastic equations). If a very long column is braced intermittently along its length, the column length that will buckle can be determined. The effective length can be found by multiplying the column length by an effective length factor, K. $L_e = K \cdot L$

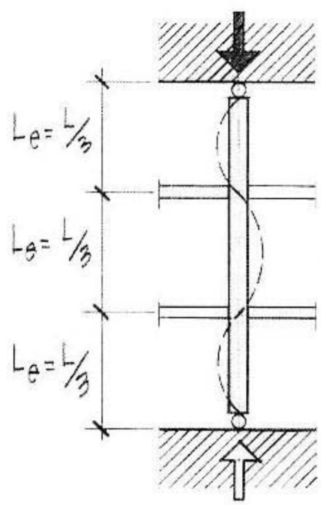


(a) No bracing.

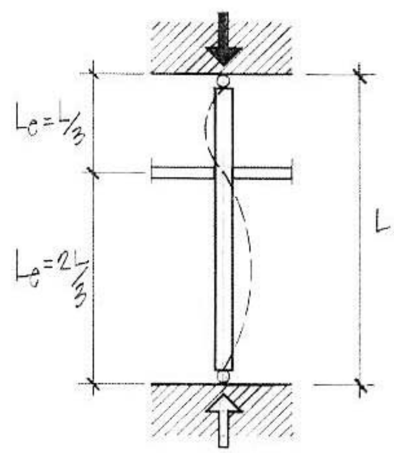


(b) Braced at midpoint.

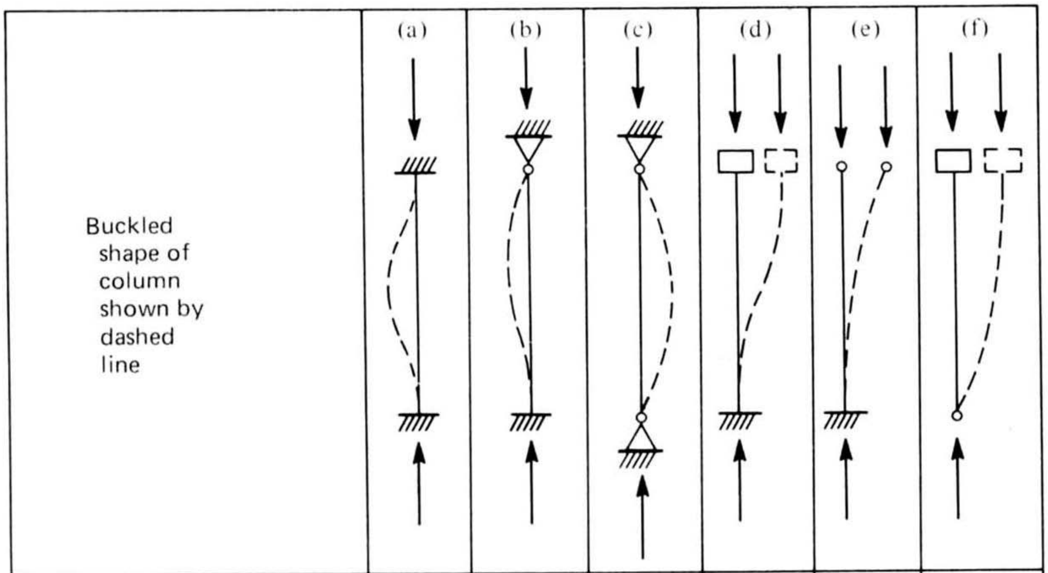




(c) Third-point bracing.



(d) Asymmetric bracing.



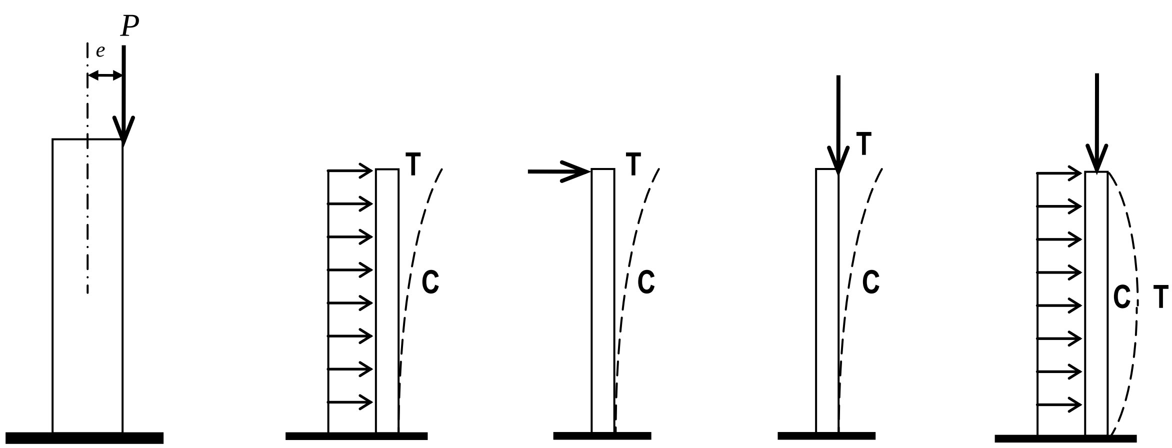
	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design values when ideal conditions are approximated	0.65	0.80	1.0	1.2	2.10	2.0

End conditions code:

- Rotation fixed, Translation fixed
- Rotation free, Translation fixed
- Rotation fixed, Translation free
- Rotation free, Translation free

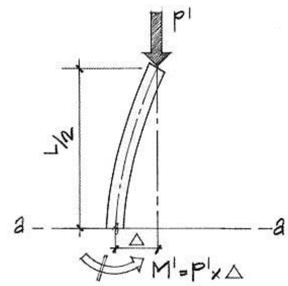
Bending in Columns

Bending can occur in column like members when there are transverse loads such as wind and seismic loads, when the column is in a frame, or when the column load <u>does not</u> go through the axes. This situation is referred to as *eccentric loading* and the moment is of size P x e.



P-Δ (delta) Effect

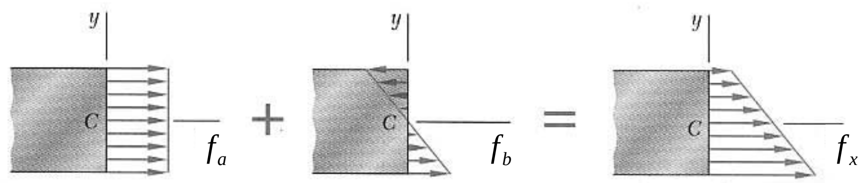
The bending moment on a column will produce a lateral deflection. Because there is an axial load P on the column, there will be an addition moment produced of the size P x Δ, which in turn will cause more deflection, increasing the moment, etc.. This non-linear increase in moment is called the *P-Δ effect*. Design methods usually take this into account with magnification factors.

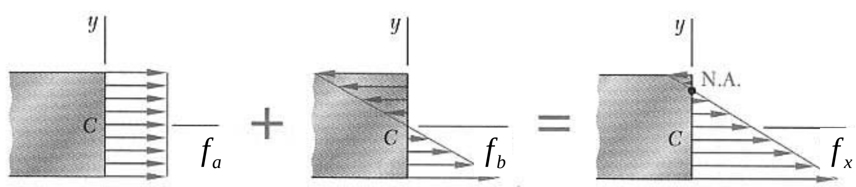


Combined Stresses

Within the elastic range (linear stresses) we can *superposition* or add up the normal and bending stresses (where M can be from Pe or calculated):

$$f_x = f_a + f_b = \frac{P}{A} + \frac{My}{I}$$



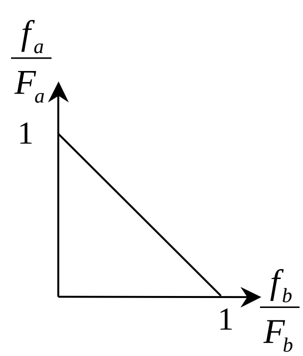


The resulting stress distribution is still *linear.* And the n.a. can *move* (if there is one)!

Interaction Design

Because there are combined stresses, we can't just compare the axial stress to a limit axial stress or a bending stress to a limit bending stress. We use a limit called the **interaction** diagram. The diagram can be simplified as a straight line from the ratio of axial stress to allowable stress= 1 (no bending) to the ratio of bending stress to allowable stress = 1 (no axial load).

The interaction diagram can be more sophisticated (represented by a curve instead of a straight line). These types of diagrams take the effect of the bending moment increasing because the beam deflects. This is called the **P-Δ (P-delta)** effect.



Limit Criteria Methods

1) $\dfrac{f_a}{F_a}+\dfrac{f_b}{F_b}\le 1.0$ interaction formula (bending in one direction)

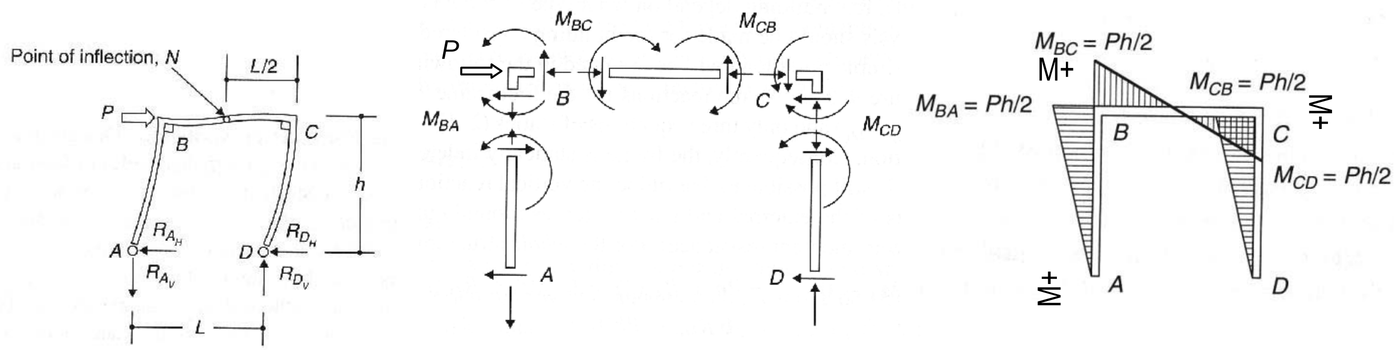
2) $\dfrac{f_a}{F_a}+\dfrac{f_{bx}}{F_{bx}}+\dfrac{f_{by}}{F_{by}}\le 1.0$ interaction formula (biaxial bending)

3) $\dfrac{f_a}{F_a}+\dfrac{f_b\times(Magnification\ factor)}{F_{bx}}\le 1.0$ interaction formula (P-Δ effect)

Rigid Frame Analysis

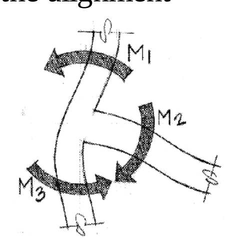
Structural analysis methods such as the *portal method* (approximate), the *method of virtual work, Castigliano's theorem*, the *force method*, the *slope-displacement method*, the *stiffness method*, and *matrix analysis,* can be used to solve for internal forces and moments and support reactions.

Shear and bending moment diagrams can be drawn for frame members by isolating the member from a joint and drawing a free body diagram. The internal forces at the end will be equal and opposite, just like for connections in *pinned frames*. Direction of the "beam-like" member is usually drawn by looking from the "inside" of the frame.



Frame Columns

Because joints can rotate in frames, the effective length of the column in a frame is harder to determine. The stiffness (EI/L) of each member in a joint determines how rigid or flexible it is. To find k, the relative stiffness, G or Ψ, must be found for both ends, plotted on the alignment charts, and connected by a line for braced and unbraced fames.

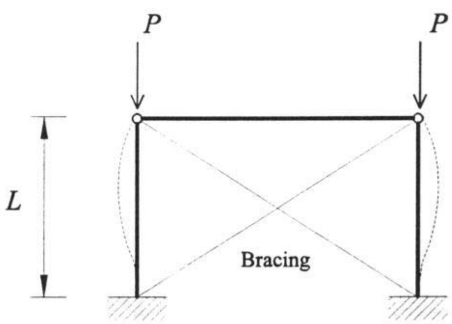


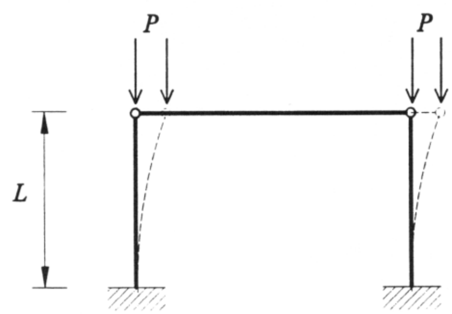
$$G=\Psi=\frac{\Sigma EI/l_c}{\Sigma EI/l_b}$$

where

E = modulus of elasticity for a member
I = moment of inertia of for a member
l_c = length of the column from center to center
l_b = length of the beam from center to center

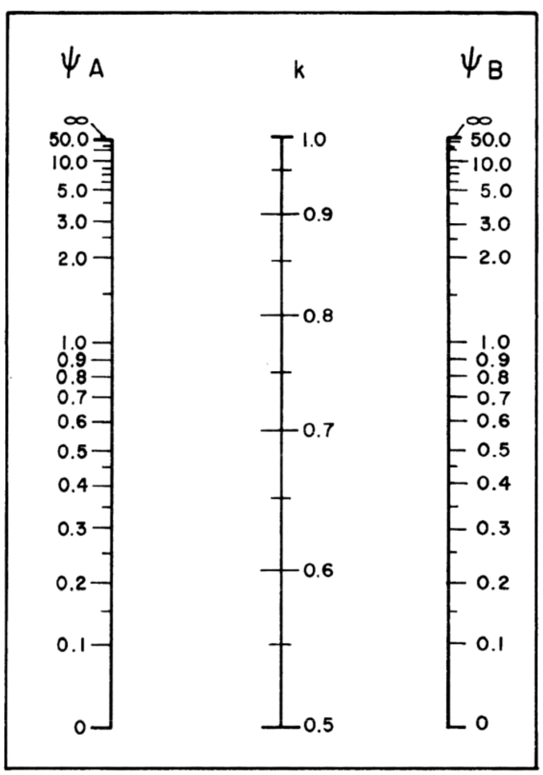
- For pinned connections we typically use a value of 10 for Ψ.
- For fixed connections we typically use a value of 1 for Ψ.





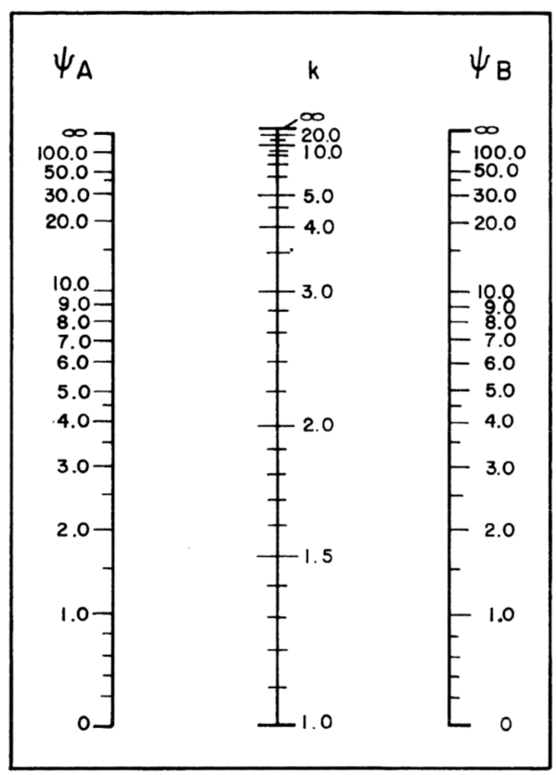
Braced – non-sway frame

Unbraced – sway frame



(a)

Nonsway Frames



(b)

Sway Frames

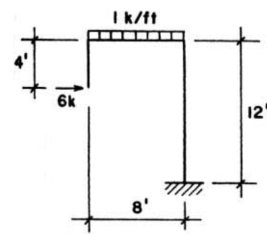
Lateral Buckling in Beams

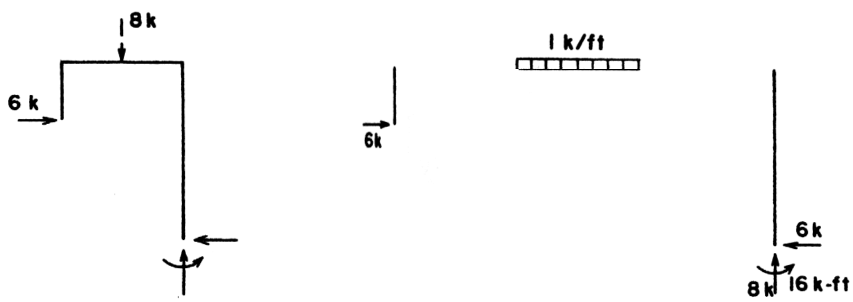
With compression stresses in the top of a beam, a sudden "popping" or **buckling** can happen even at low stresses. In order to prevent it, we need to brace it along the top, or laterally brace it, or provide a bigger I_y.

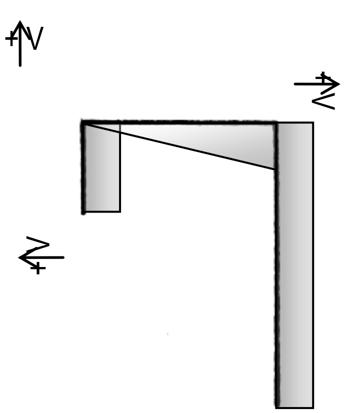
Torsional buckling can result with simultaneous twisting and bending, which can be a problem with thin walled, non-symmetric sections.

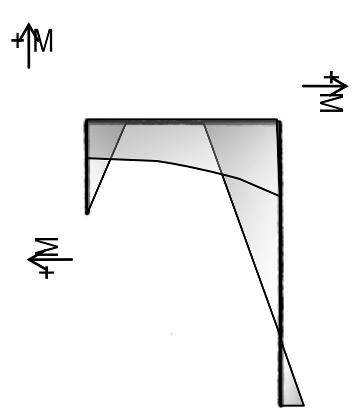
Example 1 (pg 152)

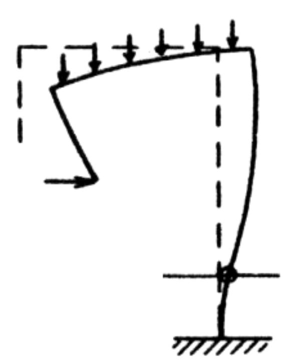
Example 23. Find the components of the reactions and draw the shear and moment diagrams and the deformed shape of the frame in Figure 3.58*a*.





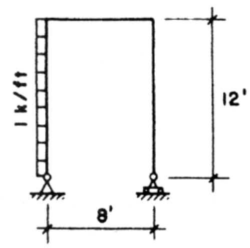


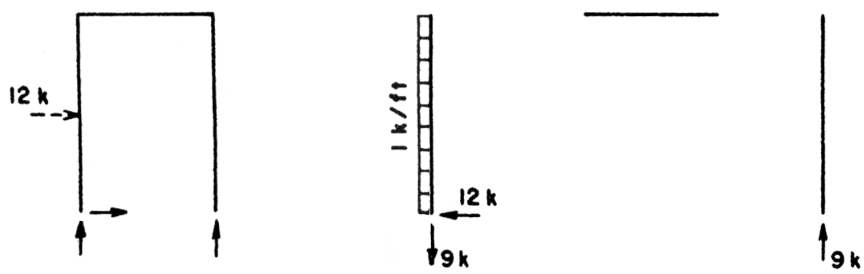


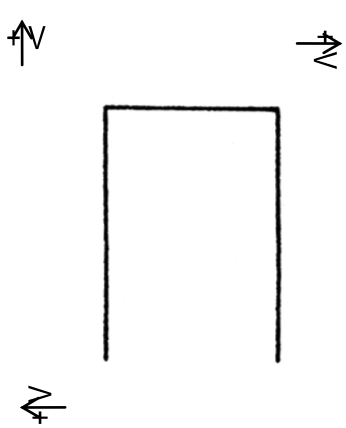


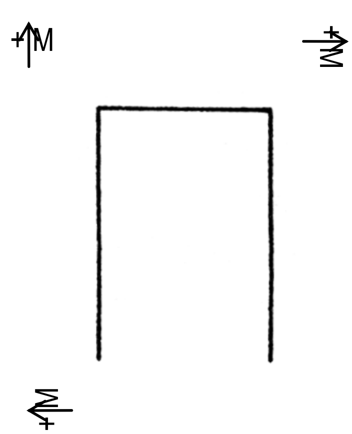
Example 2 (pg 154)

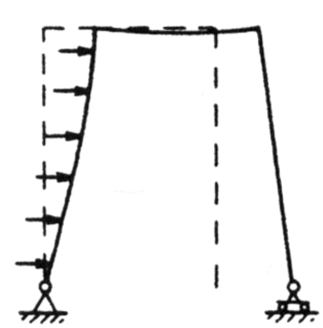
Example 24. Investigate the frame shown in Figure 3.60 for the reactions and internal conditions. Note that the right-hand support allows for an upward vertical reaction only, whereas the left-hand support allows for both vertical and horizontal components. Neither support provides moment resistance.





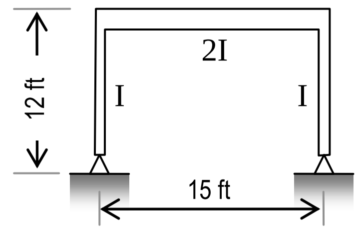


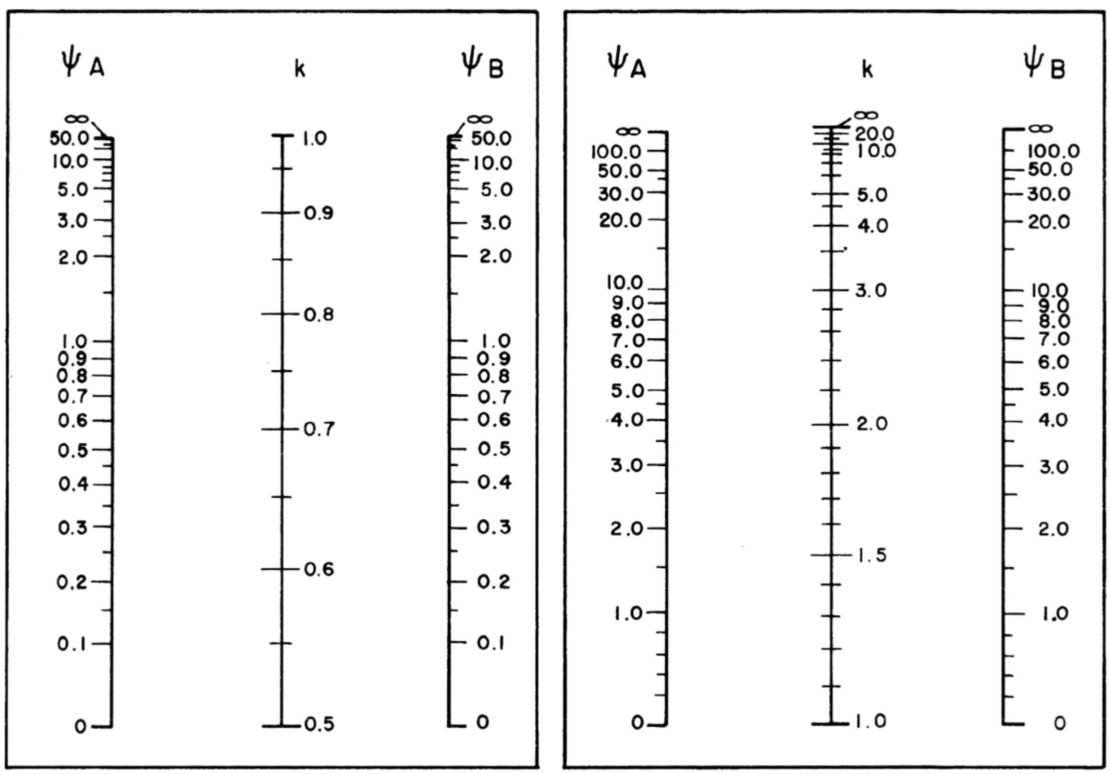




Example 3

Find the column effective lengths for a steel frame with 12 ft columns, a 15 ft beam when the support connections are pins for a) when it is braced and b) when it is allowed to sway. The relative stiffness of the beam is twice that of the columns (2I).





(a)
Nonsway Frames

(b)
Sway Frames