Frame & Pinned Systems

Notation:

|			
F	= name for force vectors	R	= name for reaction force vector
F_x	= force component in the x direction	w	= name for distributed load
F_y	= force component in the y direction	W	= name for total force due to
FBD	= free body diagram		distributed load
M	= name for reaction moment, as is M_R	Σ	= summation symbol

Frame Systems

- A FRAME is made up of members where at least <u>one</u> member has more than 3 forces on it
 - Usually stationary and fully constrained
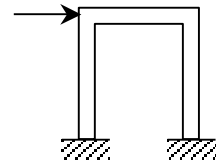
- A PINNED FRAME has member connected by pins
  - Considered *non-rigid* if it would collapse when the supports are removed
  - Considered *rigid* if it retains it's original shape when the supports are removed
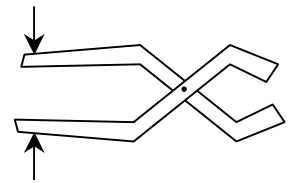
- A RIGID FRAME is all one member with no internal pins
  - Typically *statically indeterminate*
  - **Portal** frames look like door frames
  - Gable frames have a peak.
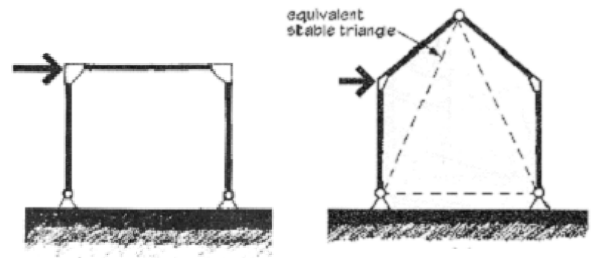
- INTERNAL PIN CONNECTIONS:
  - Pin connection forces are **equal** and **opposite** between the bodies they connect.
  - There are 2 unknown forces at a pin, but if we know a body is a **two-force** body the direction of the *resultant* force is known.
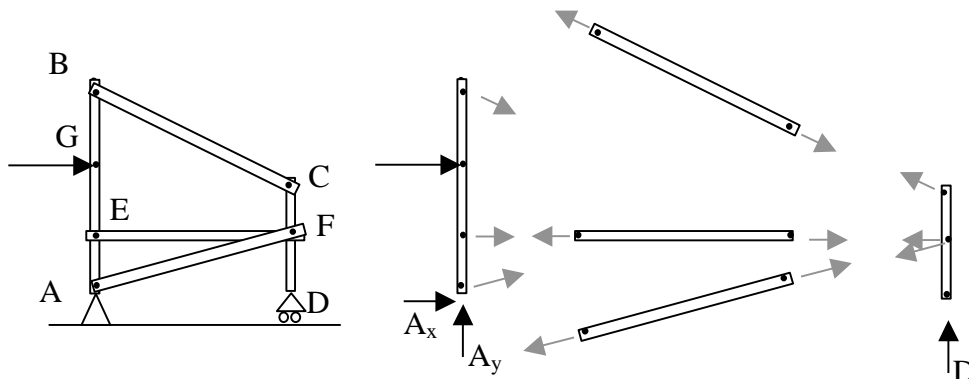
- AN ARCH is a structural shape that can span large distances and sees compression along its slope. It may have no hinges (or pins), two hinges at the supports, or two hinges at the supports with a hinge at the apex. The three-hinged arch types are statically determinate with **2** bodies and **6** unknown forces.

- CONTINUOUS BEAMS WITH PINS:
  - If pins within the span of a beam over multiple supports result in static determinacy (the right number of unknowns for the number of equilibrium equations), the internal forces at the pins are applied as reactions to the adjacent span.
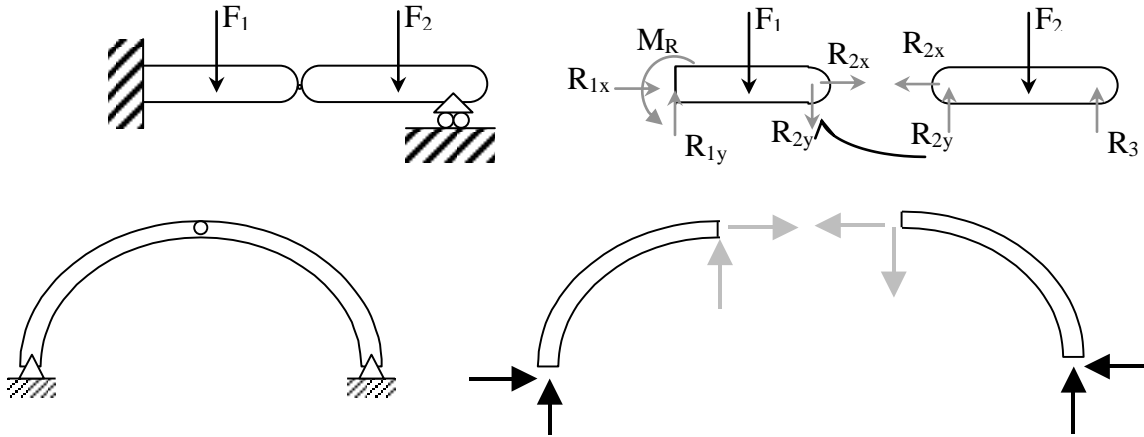

  - The location of the internal pins can be chosen to increase or decrease the moments in order to make the section economical for both positive bending and negative bending (similar values for the moments).

Solution Procedure

1. Solve for the support forces on the entire frame (FBD) if possible.
2. Draw a FBD of each member:
   - Consider all two-force bodies first.
   - Pins are integral with members
   - Pins with applied forces should belong to members with greater than two forces [Same if pins connect 3 or more members]
   - Draw forces on either side of a pin <u>equal</u> and <u>opposite</u> with arbitrary direction chosen for the first side
   - Consider all multi-force bodies
   - Represent connection forces <u>not known</u> by x & y components
   - There are still three equilibrium equations available, but the moment equations may be more helpful when the number of unknowns is greater than two.

Example 1

Example 23.   Find the components of the reactions for the structure shown in Figure 3.44*a*.
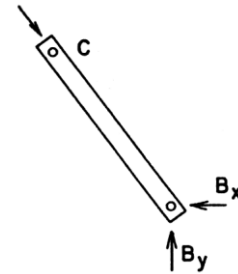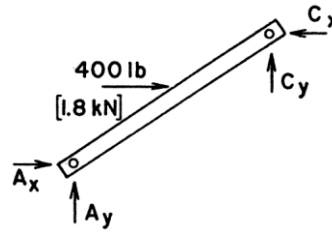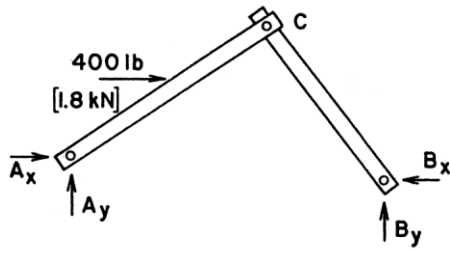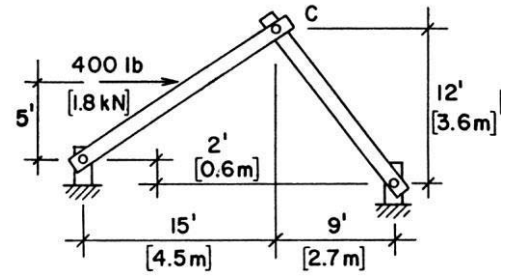
Example 2

Example 4.13 (Three-Hinged Arch)

An industrial building is framed using tapered steel sections (haunches) and connected with three hinges (Figure 4.70). Assuming that the loads shown are from gravity loads and wind, determine the support reactions at A and D and the pin reactions at B.
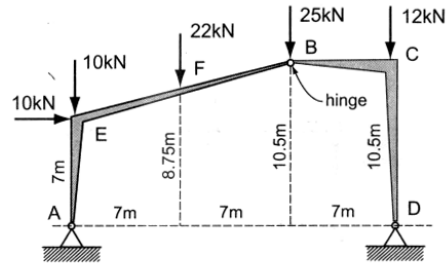

Figure 4.70   Frame as a three-hinged arch.

Solution:

Construct a FBD (Figure 4.71) of the entire three-hinged arch and determine as many of the support reactions as possible using the three available equations of equilibrium.

Since there are four support reactions and only three equations of equilibrium, only A_y and D_y can be solved at this time.

$$\sum M_A = + 10 \text{ kN}(7 \text{ m}) + 22 \text{ kN}(7 \text{ m}) + 25 \text{ kN}(14 \text{ m})$$
$$+ 12 \text{ kN}(21 \text{ m}) - D_y(21 \text{ m}) = 0$$
$$\therefore D_y = 39.3 \text{ kN} (\uparrow)$$

$$\sum F_y = + A_y - 10 \text{ kN} - 22 \text{ kN} - 25 \text{ kN}$$
$$- 12 \text{ kN} + 39.3 \text{ kN} = 0$$
$$\therefore A_y = + 29.7 \text{ kN} (\uparrow)$$

Figure 4.71   FBD of the entire three-hinged arch.

To determine A_x and D_x and the pin reactions at B, additional free-body diagrams are needed.

Separate the three-hinged arch into its two main components and draw the FBD of each.

Using Figure 4.72a, note that the 25-kN load at B was assigned to member AEB (an assumption).

$$\sum M_B = - 22 \text{ kN}(7 \text{ m}) - 10 \text{ kN}(14 \text{ m})$$
$$- 10 \text{ kN}(3.5 \text{ m}) + 29.7 \text{ kN}(14 \text{ m})$$
$$- A_x(10.5 \text{ m}) = 0$$
$$\therefore A_x = 8.3 \text{ kN} (\rightarrow)$$

$$\sum F_x = + \underset{(A_x)}{8.3 \text{ kN}} + 10 \text{ kN} - B_x = 0$$
$$\therefore B_x = + 18.3 \text{ kN}$$

$$\sum F_y = + \underset{(A_y)}{29.7 \text{ kN}} - 10 \text{ kN} - 22 \text{ kN}$$
$$- 25 \text{ kN} + B_y = 0$$
$$\therefore B_y = + 27.3 \text{ kN}$$

(a)

The remaining unknown D_x can be solved using free-body diagrams Figure 4.71 or Figure 4.72b.

Using Figure 4.72b:

$$\sum F_x = + \underset{(B_x)}{18.3 \text{ kN}} - D_x = 0$$
$$\therefore D_x = + 18.3 \text{ kN} (\leftarrow)$$

(b)

Figure 4.72   FBD of each haunch section.

As a check, substitute the answer for D_x into an equation for the horizontal condition of equilibrium using Figure 4.71.
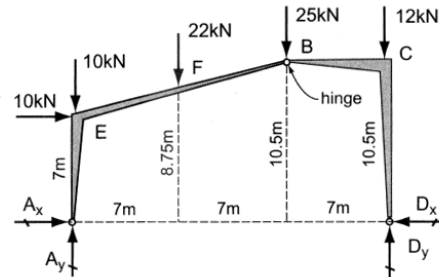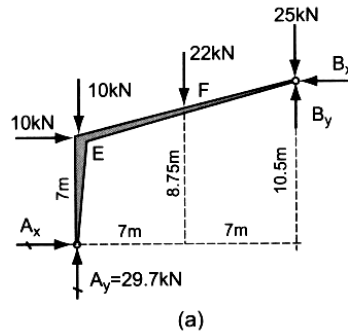
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Example 3

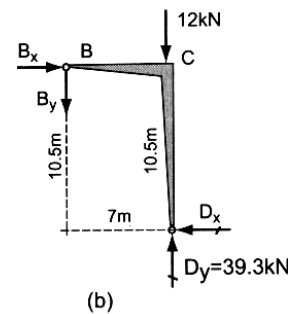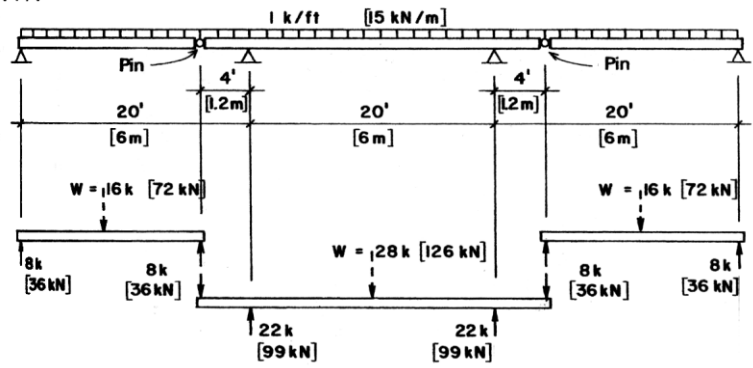*Example 25.*   Investigate the beam shown in Figure 3.47.

Example 4

Example 24.   Investigate the beam shown in Figure 3.46*a*. Find the reactions, draw the shear and moment diagrams, and sketch the deflected shape.

Solution: Because of the internal pin, the first 12 ft of the left-hand span acts as a simple beam. Its two reactions are therefore equal, being one half the total load, and its shear, moment, and deflected shape diagrams are those for a simple beam with a uniformly distributed load. (See Case 2, Figure 3.25). As shown in *b* and *c* in Figure 3.46, the simple beam reaction at the right end of the 12-ft portion of the left span becomes a 6-kip concentrated load at the left end of the remainder of the beam. This beam (Figure 3.46*c*) is then investigated as a beam with one overhanging end, carrying a single concentrated load at the cantilevered end and the total distributed load of 20 kips. (Note that on the diagram the total uniformly distributed load is indicated in the form of a single force, representing its resultant.) The second portion of the beam is statically determinate, and its reactions can now be determined by statics equations.

For left beam, the reaction forces are equal because it is symmetrical:

$R = W/2 = 12^k/2 = 6^k$

With the equal and opposite for to the reaction of the 6 k at the pin, the right hand beam reactions can be found:

$\Sigma F_x: R_{1x} = 0$

$\Sigma F_y: -6^k + R_{1y} - 20^k + R_2 = 0$

$\Sigma M_1: -6^k(4^{ft}) + 20^k(6^{ft}) - R_2(16^{ft}) = 0$

So, $R_2 = 6^k$ and $R_{1y} = 20^k$

The loaded area from the first support to the second support is $-(16^{ft})(1^{k/ft}) = -16^k$, while the area from second to third supports is -16^k.

The shear starts at 0, adds 6^k, subtracts 16^k at the middle support (--10), adds 20^k (10), subtracts 16^k (-6), and adds 6^k.

The triangle base is:

$x = 6^k/1^{k/ft} = 6_{ft}$

The first area is $6^k(6^{ft})/2 = 18^{k-ft}$

The second area is $-10^k(10^{ft})/2 = -50$ k-ft.

(Mirrored negatively on the right half.)

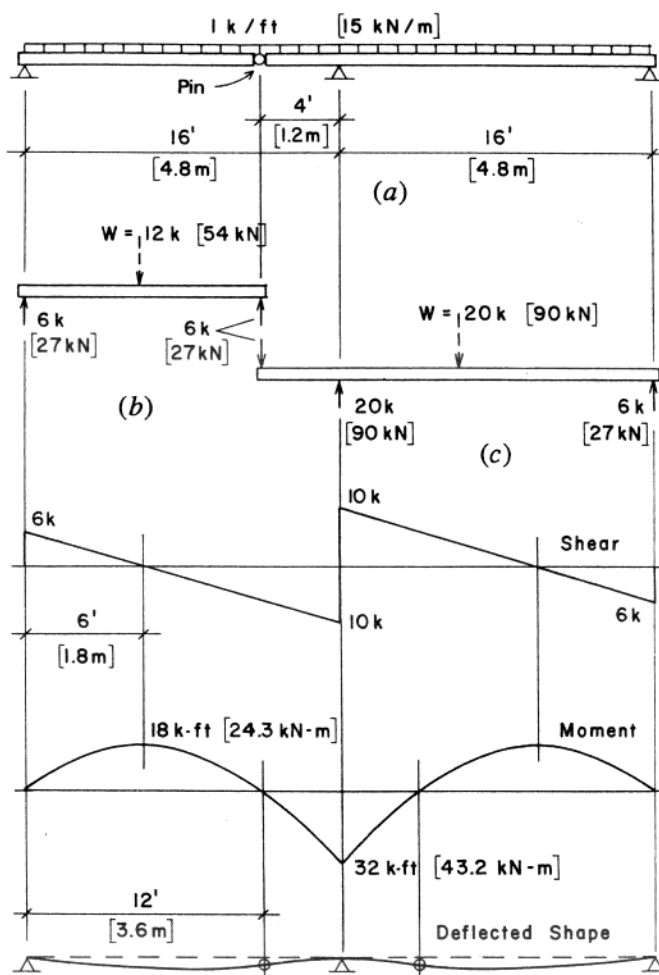The moment starts at 0, adds 18^{k-ft}, subtracts 50^{k-ft} (-32^{k-ft}), adds 50^{k-ft} (18^{k-ft}) and subtracts 18^{k-ft} to close.

Figure 3.46   Reference for Example 24.