Reference Formulas

$\sum F_x = 0$	$C^2 = A^2 + B^2 - 2AB\cos\gamma$	$\pi(radians) = 180°$
$\sum F_y = 0$	$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$\sum M = 0$	$sin\,\theta = \frac{O}{H}$	$y = mx + b$
$F_x = F\cos\theta$	$cos\,\theta = \frac{A}{H}$	$m = \frac{y_2 - y_1}{x_2 - x_1}$
$F_y = F\sin\theta$	$tan\,\theta = \frac{O}{A}$	$\hat{x} = \frac{\Sigma(\bar{x}A)}{\Sigma A}$
$F = \sqrt{F_x^2 + F_y^2}$	$p = 2\pi r = \pi d$	$Q_y = \bar{x}A = \sum_{i=1}^{n} \bar{x}_i A_i$
$\tan\theta = \frac{F_y}{F_x}$	$A = \pi r^2 = \frac{\pi d^2}{4}$	$\hat{y} = \frac{\Sigma(\bar{y}A)}{\Sigma A}$
$M = Fd$	$A = W \cdot l = t \cdot d$	$Q_x = \bar{y}A = \sum_{i=1}^{n} \bar{y}_i A_i$
$g = 9.81\, m/s^2$	$V = W \cdot h \cdot l = A \cdot l$	$I = \bar{I} + Ad^2$
$s(t) = v(0)t + \frac{1}{2}at^2$	$F = mg$	$I = \sum I_c + \sum Ad^2$
$\frac{dV}{dx} = -w$	$r = \sqrt{\frac{I}{A}}$	$d_x = \hat{x} - \bar{x}$
$\frac{dM}{dx} = V$	$x = V_A / w$	$d_y = \hat{y} - \bar{y}$
$Pa = N/m^2$	$N = kg \cdot m / s^2$	$F = \mu N$
$1\, kPa = 1{,}000\, Pa$	$psi = lb/in^2$	$ksi = kip/in^2$
$1\, kPa = 1\, kN/m^2$	$1\, kip = 1000\, lb$	$12\, in = 1\, ft$
$1\, MPa = 10^6\, Pa$	$1\, GPa = 10^9\, Pa$	$1\, m = 1000\, mm$
$f_c = \frac{P}{A}$	$F.S = \frac{ultimate}{allowable}$	$\varepsilon = \frac{\delta}{L}$
$f_t = \frac{P}{A}\ or\ \frac{P}{A_e}$	$f_v = \frac{P}{A} = \frac{P}{td}$	$f = E\varepsilon$
$f_p = \frac{P}{A} = \frac{P}{td}$	$f_v = \tau = \frac{T\rho}{J}$	$f = \frac{\delta}{L}E$
$f_y = \frac{My}{I}$	$f_{v-ave} = \frac{VQ}{Ib}$	$f_v = \frac{P}{2A}$
$S = \frac{I}{c}$	$f_{v-\max} = \frac{3V}{2A}$ for a rectangle	$\delta = \frac{PL}{AE}$

$f_{b-max}=\frac{Mc}{I}=\frac{M}{S}$	$f_{v-max}\cong\frac{V}{A_{web}}=\frac{V}{t_w d}$ for an I beam	$\delta_T=\alpha(\Delta T)L$
$S_{req}\geq\frac{M}{F_b}$	$\varepsilon_y=\varepsilon_z=-\frac{\mu f_x}{E}$	$\varepsilon_T=\alpha(\Delta T)$
$w=\gamma A$	$W=\gamma V$	$W=\gamma t A$
$nF_{connector}\geq\frac{VQ_{connected\ area}}{I}\cdot p$	$M_1L_1+2M_2(L_1+L_2)+M_3L_2=-\frac{w_1L_1^3}{4}-\frac{w_2L_2^3}{4}$ $M_1L_1+2M_2(L_1+L_2)+M_3L_2=-\sum P_1L_1^2(n_1-n_1^3)-\sum P_2L_2^2(n_2-n_2^3)$	
$V_{longitudinal}=\frac{V_TQ}{I}\Delta x$	$\gamma=\frac{\rho\phi}{L}$	$f_v=\tau=G\cdot\frac{\rho\phi}{L}$
$\tau_{max}=\frac{T}{c_1ab^2}$	$\phi=\frac{TL}{c_2ab^3G}$	$\phi=\frac{TL}{JG}$
$\tau_{max}=\frac{T}{\frac{1}{3}ab^2}$	$\phi=\frac{TL}{\frac{1}{3}ab^3G}$	$\tau_{max}=\frac{Tt_{max}}{\frac{1}{3}\Sigma b_it_i^3}$
$\tau_{max}=\frac{T}{2t\mathcal{a}}$	$\phi=\frac{TL}{4t\mathcal{a}^2}\sum_i\frac{s_i}{t_i}$	$\phi=\frac{TL}{\frac{1}{3}G\Sigma b_it_i^3}$
$\frac{1}{R}=\frac{M}{EI}$	$\Delta=\iint\frac{M(x)}{EI}dx$	$2n=b+3$
$P_U=P_L\gamma_L+P_D\gamma_D\leq\phi P_n$	$1.4D$	$1.2D+1.6(L_r\ or\ S\ or\ R)+(L\ or\ 0.5W)$
$L_e=Kl$	$1.2D+1.6L+0.5(L_r\ or\ S\ or\ R)$	$1.2D+1.0W+L+0.5(L_r\ or\ S\ or\ R)$

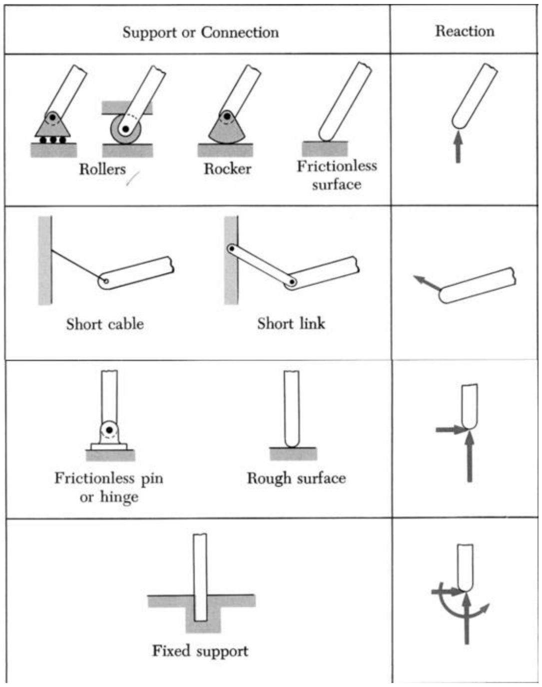
$P_{cr}=\frac{\pi^2EI}{(L_e)^2}=\frac{\pi^2EA}{\left(\frac{L_e}{r}\right)^2}$	AISC – ASD: $R_a\leq\frac{R_n}{\Omega}$	$\frac{l_e}{r}\geq C_c\quad F_a=\frac{F_{cr}}{F.S.}=\frac{12\pi^2E}{23\left(\frac{Kl}{r}\right)^2}$
$f_{cr}=\frac{\pi^2E}{\left(\frac{L_e}{r}\right)^2}$	$C_c=\sqrt{\frac{2\pi^2E}{F_y}}$	$\frac{l_e}{r}<C_c\quad F_a=\left[1-\frac{\left(\frac{Kl}{r}\right)^2}{2C_c^2}\right]\frac{F_y}{F.S.}$
$f_{max}=\frac{P}{A}+\frac{Mc}{I}$	$P_n=F_{cr}A_g$ Ω =1.67 (bending)	$F.S.=\frac{5}{3}+\frac{3}{8}\cdot\frac{\frac{L_e}{r}}{C_c}-\frac{1}{8}\cdot\left(\frac{\frac{L_e}{r}}{C_c}\right)^3$
$\frac{f_a}{F_a}+\frac{f_b}{F_b}\leq1.0$	Ω =1.67 (beam shear) Ω =2.00 (bolt shear)	$\frac{P_u}{\phi_cP_n}\geq0.2:\ \frac{P}{\frac{P_n}{\Omega}}+\frac{8}{9}\left(\frac{M_x}{\frac{M_{nx}}{\Omega}}+\frac{M_y}{\frac{M_{ny}}{\Omega}}\right)\leq1.0$
$f_{max}=\frac{P}{A}+\frac{M_1y}{I}+\frac{M_2z}{I}$	Ω =2.00 (weld shear) Ω =1.50 (bearing)	$\frac{P_u}{\phi_cP_n}<0.2:\ \frac{P}{\frac{2P_n}{\Omega}}+\left(\frac{M_x}{\frac{M_{nx}}{\Omega}}+\frac{M_y}{\frac{M_{ny}}{\Omega}}\right)\leq1.0$
$\frac{f_a}{F_a}+\frac{f_{bx}}{F_{bx}}+\frac{f_{by}}{F_{by}}\leq1.0$	Ω =1.67 (compression)	

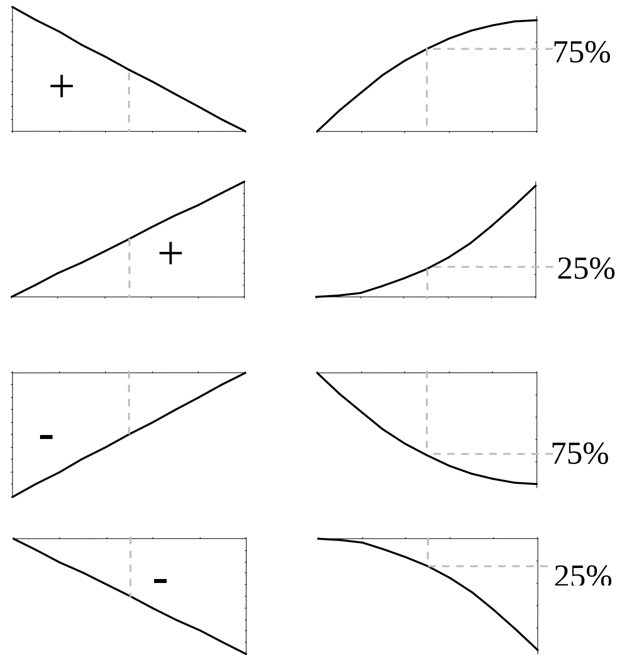
ACI-318: $A_s f_y = 0.85 f'_c ba$	$a = \frac{A_s f_y}{0.85 f'_c b}$	$\rho = \frac{A_s}{bd}$
$M_u \le \phi M_n = \phi A_s f_y \left(d - a/2\right)$ $\phi = 0.9$	$min: A_s = \frac{3\sqrt{f'_c}}{f_y}(bd)$, not less than $A_s = \frac{200}{f_y}(bd)$	
$R_n = \frac{M_n}{bd^2}$	$T, min: A_s = \frac{6\sqrt{f'_c}}{f_y}(b_w d)$, not less than $A_s = \frac{3\sqrt{f'_c}}{f_y}(b_f d)$	
slab (<60): $A_s = 0.002b(t\ or\ h)$	slab (60): $A_s = 0.0018b(t\ or\ h)$	$V_u \le \phi V_c + \phi V_s \quad \phi = 0.75$
one-way: $V_c = 2\sqrt{f'_c} b_w d$	$V_s = \frac{A_v f_y d}{s}$	$E_c = w_c^{1.5} 33\sqrt{f'_c}$
two-way: $V_c = 4\sqrt{f'_c} b_w d$	$E_c = 57{,}000\sqrt{f'_c}$	$\le\#6: l_d = \frac{d_b F_y}{25\sqrt{f'_c}}$
$l_{dh} = \frac{1200 d_b}{\sqrt{f'_c}}$	(c): $l_d = \frac{0.02 d_b F_y}{\sqrt{f'_c}} \le 0.0003 d_b F_y$	$>\#6: l_d = \frac{d_b F_y}{20\sqrt{f'_c}}$
$\frac{P}{A} \le q_{net}$	tied: $\phi_c P_n = \phi_c (0.8 P_o) \quad \phi_c = 0.65$	$P_o = 0.85 f'_c (A_g - A_{st}) + f_y A_{st}$
$q_u = \frac{P_u}{A}$	spiral: $\phi_c P_n = \phi_c (0.85 P_o) \quad \phi_c = 0.75$	$q_{net} = q_{allowable} - h_f(\gamma_c - \gamma_s)$
$G = \Psi = \frac{\Sigma EI/l_c}{\Sigma EI/l_b}$	$P_u \le \phi_b P_n = \phi_b (0.85 f'_c A_1)\sqrt{A_2/A_1}$ $\phi_p = 0.65$	$V_{u2} = P_u - q_u (c+d)(b+d)$
$volume = \frac{wpx}{2} = N$	$b_o = 2(c+d) + 2(b+d)$	$V_{u1} = BL' q_u$
$p_{max} = \frac{2N}{wx}$	$V_{u2} \le \phi\left(2 + \frac{4}{\beta_c}\right)\sqrt{f'_c} b_o d \le \phi 4\sqrt{f'_c} b_o d$	$V_{u1} \le \phi 2\sqrt{f'_c} Bd$
$SF = \frac{M_{resist}}{M_{overturning}} \ge 1.5$	$SF = \frac{F_{horizontal-resist}}{F_{sliding}} \ge 1.5$	$M_u = q_u \frac{BL_m^2}{2}$

Wood: $F' = C_D C_M C_F \ldots \times F_{tabulated}$	K_{cE} = 0.3 sawn, 0.418 glulam	$P_a = F'_c A$
$F'_c = F_c^* C_p = (F_c C_D) C_p$	$F_{cE} = \frac{K_{cE} E}{\left(l_e/d\right)^2}$	$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_{bx}}{F'_{bx}\left[1 - f_c/F_{cEx}\right]} \le 1.0$

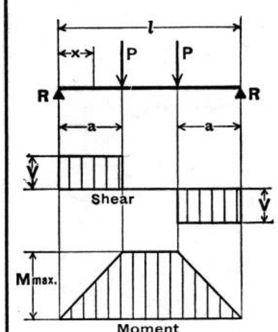
AISC-LRFD: $\phi_b = 0.9$ $M_u \le \phi_b M_n = 0.9F_y Z$	$k = Z/S$	$Z = \frac{M_p}{f_y}$
$M_{ult} = M_p = f_y \Sigma A_i y_i = f_y Z$	$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}}$	$M_{max} = \frac{w_{equivalent} L^2}{8}$
$V_u \le \phi_v (0.6F_{yw} A_w)\ \ \phi_v = 1.0$	$I_{req'd} \ge \frac{\Delta_{too\,big}}{\Delta_{limit}} I_{trial}$	$F_e = \frac{\pi^2 E}{(KL/r)^2}$
$P_u \le \phi_c F_{cr} A_g\ \ \phi_c = 0.90$	$F_e \ge 0.44F_y \quad F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y$	
$P_{n(max-end)} = (N + 2.5k)F_{yw} t_w \quad \phi = 1.0$ $P_{n(max-interior)} = (N + 5k)F_{yw} t_w \quad \phi = 1.0$	$F_e < 0.44F_y \quad F_{cr} = 0.877F_e$	
$R_u \le \phi_t F_y A_g\ \ \phi_t = 0.9$ $R_u \le \phi_t F_u A_e\ \ \phi_t = 0.75$	$\frac{P_u}{\phi_c P_n} \ge 0.2:\ \frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \le 1.0$	
$R_u \le \phi 0.6F_{EXX} Tl \quad \phi = 0.75$ $C_m = 0.6 - 0.4\left(M_1/M_2\right)$	$\frac{P_u}{\phi_c P_n} < 0.2:\ \frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \le 1.0$	
$A_n = A_g - A_{of\ all\ holes} + t\Sigma \frac{s}{4g}$	$B_1 = \frac{C_m}{1-(P_u/P_{e1})} \le 1.0$	$P_{e1} = \frac{\pi^2 EA}{(Kl/r)^2}$
$A_e = A_n U$	$R_u \le \phi(0.6F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.6F_y A_{gv} + U_{bs} F_u A_{nt})\quad \phi = 0.75$	
Masonry: *plain:* $F_b = \frac{1}{3} f'_m$	$A_s f_s = \frac{f_m b(kd)}{2}$	$M_m = \frac{f_m bd^2 jk}{2}$
plain: $F_v = 1.5\sqrt{f'_m} \le 120psi$	$F_v = 3.0\sqrt{f'_m}$ when M/(Vd) ≥ 0.25 $F_v = 2.0\sqrt{f'_m}$ when M(Vd) ≥ 1.0	$f_v \le V/A_{nv}$
$F_v = F_{vm} + F_{vs}$	$F_{vm} = \frac{1}{2}\left[\left(4.0 - 1.75\left(\frac{M}{Vd}\right)\right)\sqrt{f'_m}\right] + 0.25\frac{P}{A_n}$	$F_{vs} = 0.5\left(\frac{A_v F_s d}{A_{nv} s}\right)$
$f_b - f_a \le F_t$	$h'/r \le 99\quad P_a = [0.25 f'_m A_n + 0.65 A_{st} F_s]\left[1 - \left(\frac{h'}{140r}\right)^2\right]$	
$f_a + f_b \le F_b$	$h'/r > 99\quad P_a = [0.25 f'_m A_n + 0.65 A_{st} F_s]\left(\frac{70r}{h'}\right)^2$	
$e_1 = \frac{M}{P}$	$h'/r \le 99\quad F_a = 0.25 f'_m \left[1 - \left(\frac{h'}{140r}\right)^2\right]$	$h'/r > 99\quad F_a = 0.25 f'_m \left(\frac{70r}{h'}\right)^2$

Reference Diagrams





Reference Beam Diagrams



9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED

Total Equiv. Uniform Load . . . . . . $= \frac{8\,Pa}{l}$

R = V . . . . $= P$

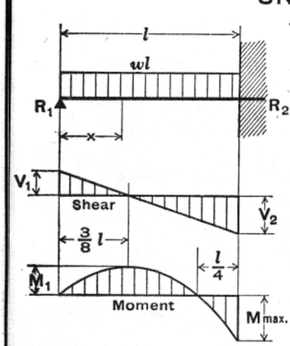
M max. (between loads) . . . . . . $= Pa$

M_x (when x < a) . . . . . . $= Px$

Δmax. (at center) . . . . $= \frac{Pa}{24EI}(3l^2 - 4a^2)$

Δx (when x < a) . . . . $= \frac{Px}{6EI}(3la - 3a^2 - x^2)$

Δx (when x > a and < (l − a)) . . $= \frac{Pa}{6EI}(3lx - 3x^2 - a^2)$



12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—UNIFORMLY DISTRIBUTED LOAD

Total Equiv. Uniform Load . . . . . . $= wl$

$R_1 = V_1$ . . . . $= \frac{3wl}{8}$

$R_2 = V_2$ max. . . . . $= \frac{5wl}{8}$

V_x . . . . $= R_1 - wx$

M max. . . . . $= \frac{wl^2}{8}$

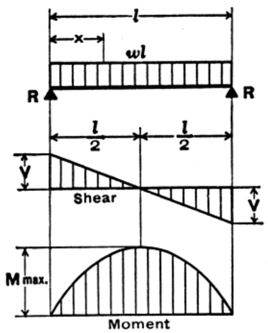
M_1 (at $x = \frac{3}{8}l$) . . . . $= \frac{9}{128}wl^2$

M_x . . . . $= R_1 x - \frac{wx^2}{2}$

Δmax. (at $x = \frac{l}{16}(1 + \sqrt{33}) = .4215l$) . $= \frac{wl^4}{185EI}$

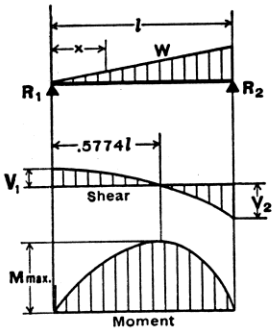
Δx . . . . $= \frac{wx}{48EI}(l^3 - 3lx^2 + 2x^3)$

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



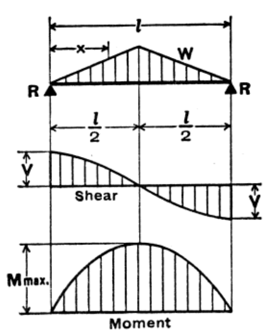
- Total Equiv. Uniform Load . . . . $= wl$
- R = V . . . . $= \frac{wl}{2}$
- V_x . . . . $= w\left(\frac{l}{2} - x\right)$
- M max. (at center) . . . . . $= \frac{wl^2}{8}$
- M_x . . . . $= \frac{wx}{2}(l - x)$
- Δmax. (at center) . . . . . $= \frac{5\,wl^4}{384\,EI}$
- Δx . . . . $= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END



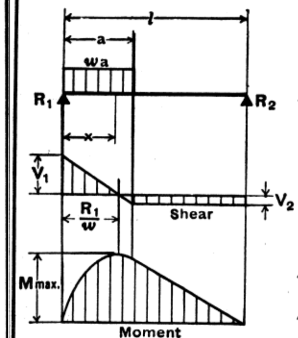
- Total Equiv. Uniform Load . . . . $= \frac{16W}{9\sqrt{3}} = 1.0264W$
- $R_1 = V_1$ . . . . $= \frac{W}{3}$ W = wl/2
- $R_2 = V_2$ max. . . . . $= \frac{2W}{3}$
- V_x . . . . $= \frac{W}{3} - \frac{Wx^2}{l^2}$
- M max. $\left(\text{at } x = \frac{l}{\sqrt{3}} = .5774l\right)$ . . $= \frac{2Wl}{9\sqrt{3}} = .1283\,Wl$
- M_x . . . . $= \frac{Wx}{3l^2}(l^2 - x^2)$
- Δmax. $\left(\text{at } x = l\sqrt{1 - \sqrt{\frac{8}{15}}} = .5193l\right) = .01304\,\frac{Wl^3}{EI}$
- Δx . . . . $= \frac{Wx}{180EI\,l^2}(3x^4 - 10l^2x^2 + 7l^4)$

3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER



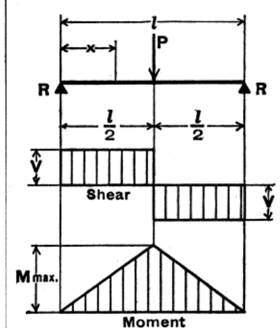
- Total Equiv. Uniform Load . . . . $= \frac{4W}{3}$ W = wl/2
- R = V . . . . $= \frac{W}{2}$
- V_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . $= \frac{W}{2l^2}(l^2 - 4x^2)$
- M max. (at center) . . . . $= \frac{Wl}{6}$
- M_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . $= Wx\left(\frac{1}{2} - \frac{2x^2}{3l^2}\right)$
- Δmax. (at center) . . . . $= \frac{Wl^3}{60EI}$
- Δx $\left(\text{when } x < \frac{l}{2}\right)$ . . . . $= \frac{Wx}{480\,EI\,l^2}(5l^2 - 4x^2)^2$

5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END



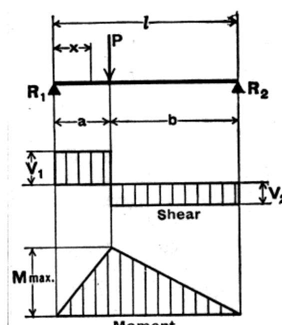
- $R_1 = V_1$ max. . . . . $= \frac{wa}{2l}(2l - a)$
- $R_2 = V_2$ . . . . $= \frac{wa^2}{2l}$
- V_x (when x < a) . . . . . . $= R_1 - wx$
- M max. $\left(\text{at } x = \frac{R_1}{w}\right)$ . . . . . . $= \frac{R_1^2}{2w}$
- M_x (when x < a) . . . . . . $= R_1x - \frac{wx^2}{2}$
- M_x (when x > a) . . . . . . $= R_2(l - x)$
- Δx (when x < a) . . . . . . $= \frac{wx}{24EIl}\left(a^2(2l-a)^2 - 2ax^2(2l-a) + lx^3\right)$
- Δx (when x > a) . . . . . . $= \frac{wa^2(l-x)}{24EIl}(4xl - 2x^2 - a^2)$

7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER



- Total Equiv. Uniform Load . . . . . . $= 2P$
- R = V . . . . $= \frac{P}{2}$
- M max. (at point of load) . . . . . . $= \frac{Pl}{4}$
- M_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . . . $= \frac{Px}{2}$
- Δmax. (at point of load) . . . . . . $= \frac{Pl^3}{48EI}$
- Δx $\left(\text{when } x < \frac{l}{2}\right)$ . . . . . . $= \frac{Px}{48EI}(3l^2 - 4x^2)$

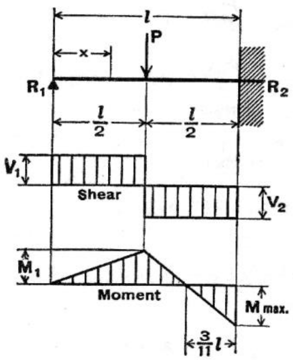
8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT



- Total Equiv. Uniform Load . . . . . . $= \frac{8\,Pab}{l^2}$
- $R_1 = V_1$ (max. when a < b) . . . . . $= \frac{Pb}{l}$
- $R_2 = V_2$ (max. when a > b) . . . . . $= \frac{Pa}{l}$
- M max. (at point of load) . . . . . $= \frac{Pab}{l}$
- M_x (when x < a) . . . . . $= \frac{Pbx}{l}$
- Δmax. $\left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b\right) = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27\,EI\,l}$
- Δa (at point of load) . . . . . $= \frac{Pa^2b^2}{3EI\,l}$
- Δx (when x < a) . . . . . $= \frac{Pbx}{6EI\,l}(l^2 - b^2 - x^2)$

Δx

13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT CENTER



Total Equiv. Uniform Load . . . . . $= \frac{3P}{2}$

$R_1 = V_1$ . . . . $= \frac{5P}{16}$

$R_2 = V_2$ max. . . . . $= \frac{11P}{16}$

M max. (at fixed end) . . . . . . $= \frac{3Pl}{16}$

M_1 (at point of load) . . . . . $= \frac{5Pl}{32}$

M_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . . . $= \frac{5Px}{16}$

M_x $\left(\text{when } x > \frac{l}{2}\right)$ . . . . . . $= P\left(\frac{l}{2} - \frac{11x}{16}\right)$

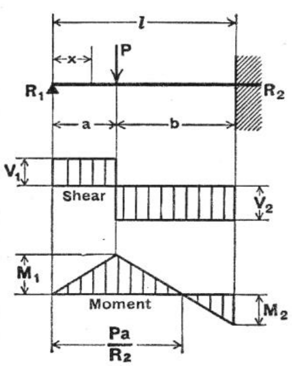
Δmax. $\left(\text{at } x = l\sqrt{\frac{1}{5}} = .4472l\right)$ . . . $= \frac{Pl^3}{48EI\sqrt{5}} = .009317\,\frac{Pl^3}{EI}$

Δx (at point of load) . . . . . $= \frac{7Pl^3}{768EI}$

Δx $\left(\text{when } x < \frac{l}{2}\right)$ . . . . . . $= \frac{Px}{96EI}(3l^2 - 5x^2)$

Δx $\left(\text{when } x > \frac{l}{2}\right)$ . . . . . . $= \frac{P}{96EI}(x - l)^2(11x - 2l)$

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$ . . . . $= \frac{Pb^2}{2l^3}(a + 2l)$

$R_2 = V_2$ . . . . $= \frac{Pa}{2l^3}(3l^2 - a^2)$

M_1 (at point of load) . . . . . $= R_1 a$

M_2 (at fixed end) . . . . . . $= \frac{Pab}{2l^2}(a + l)$

M_x (when $x < a$) . . . . . . $= R_1 x$

M_x (when $x > a$) . . . . . . $= R_1 x - P(x - a)$

Δmax. $\left(\text{when } a < .414l \text{ at } x = l\,\frac{l^2 + a^2}{3l^2 - a^2}\right) = \frac{Pa}{3EI}\,\frac{(l^2 - a^2)^3}{(3l^2 - a^2)^2}$

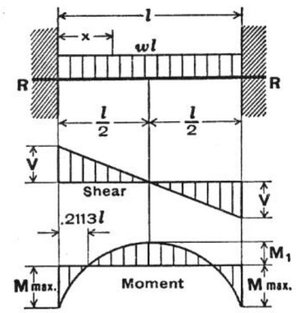
Δmax. $\left(\text{when } a > .414l \text{ at } x = l\sqrt{\frac{a}{2l + a}}\right) = \frac{Pab^2}{6EI}\sqrt{\frac{a}{2l + a}}$

Δa (at point of load) . . . . $= \frac{Pa^2b^3}{12EIl^3}(3l + a)$

Δx (when $x < a$) . . . . . . $= \frac{Pb^2x}{12EIl^3}(3al^2 - 2lx^2 - ax^2)$

Δx (when $x > a$) . . . . . . $= \frac{Pa}{12EIl^3}(l - x)^2(3l^2x - a^2x - 2a^2l)$

15. BEAM FIXED AT BOTH ENDS—UNIFORMLY DISTRIBUTED LOADS



Total Equiv. Uniform Load . . . . . $= \frac{2wl}{3}$

$R = V$ . . . . $= \frac{wl}{2}$

V_x . . . . $= w\left(\frac{l}{2} - x\right)$

M max. (at ends) . . . . . . $= \frac{wl^2}{12}$

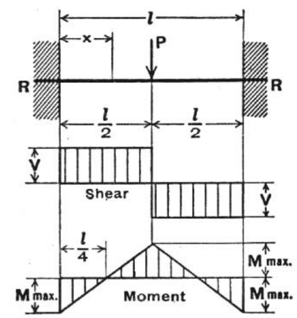
M_1 (at center) . . . . . . $= \frac{wl^2}{24}$

M_x . . . . $= \frac{w}{12}(6lx - l^2 - 6x^2)$

Δmax. (at center) . . . . . . $= \frac{wl^4}{384EI}$

Δx . . . . $= \frac{wx^2}{24EI}(l - x)^2$

16. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT CENTER



Total Equiv. Uniform Load . . . . . $= P$

$R = V$ . . . . $= \frac{P}{2}$

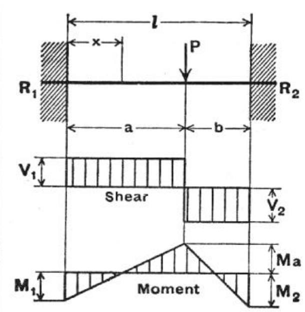
M max. (at center and ends) . . . . $= \frac{Pl}{8}$

M_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . . . $= \frac{P}{8}(4x - l)$

Δmax. (at center) . . . . . . $= \frac{Pl^3}{192EI}$

Δx $\left(\text{when } x < \frac{l}{2}\right)$ . . . . . . $= \frac{Px^2}{48EI}(3l - 4x)$

17. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$ (max. when $a < b$) . . . $= \frac{Pb^2}{l^3}(3a + b)$

$R_2 = V_2$ (max. when $a > b$) . . . $= \frac{Pa^2}{l^3}(a + 3b)$

M_1 (max. when $a < b$) . . . $= \frac{Pab^2}{l^2}$

M_2 (max. when $a > b$) . . . $= \frac{Pa^2b}{l^2}$

M_a (at point of load) . . . $= \frac{2Pa^2b^2}{l^3}$

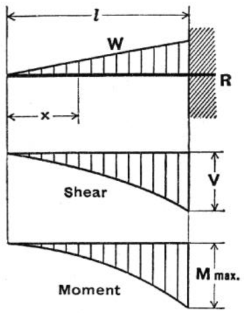
M_x (when $x < a$) . . . . . . $= R_1 x - \frac{Pab^2}{l^2}$

Δmax. $\left(\text{when } a > b \text{ at } x = \frac{2al}{3a + b}\right)$. $= \frac{2Pa^3b^2}{3EI(3a + b)^2}$

Δa (at point of load) . . . $= \frac{Pa^3b^3}{3EIl^3}$

Δx (when $x < a$) . . . . . . $= \frac{Pb^2x^2}{6EIl^3}(3al - 3ax - bx)$

18. CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END



Total Equiv. Uniform Load . . . . . $= \frac{8}{3}W$

R = V . . . . $= W$ $W = \frac{wl}{2}$

V_x . . . . $= W\frac{x^2}{l^2}$

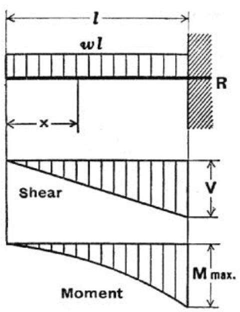
M max. (at fixed end) . . . . . $= \frac{Wl}{3}$

M_x . . . . $= \frac{Wx^3}{3l^2}$

Δmax. (at free end) . . . . . $= \frac{Wl^3}{15EI}$

Δx . . . . $= \frac{W}{60EIl^2}(x^5 - 5l^4x + 4l^5)$

19. CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . . $= 4wl$

R = V . . . . $= wl$

V_x . . . . $= wx$

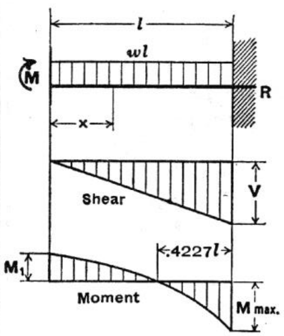
M max. (at fixed end) . . . . . $= \frac{wl^2}{2}$

M_x . . . . $= \frac{wx^2}{2}$

Δmax. (at free end) . . . . . $= \frac{wl^4}{8EI}$

Δx . . . . $= \frac{w}{24EI}(x^4 - 4l^3x + 3l^4)$

20. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . . $= \frac{8}{3}wl$

R = V . . . . $= wl$

V_x . . . . $= wx$

M max. (at fixed end) . . . . . $= \frac{wl^2}{3}$

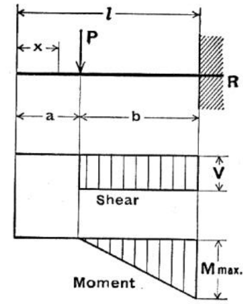
M_1 (at deflected end) . . . . . $= \frac{wl^2}{6}$

M_x . . . . $= \frac{w}{6}(l^2 - 3x^2)$

Δmax. (at deflected end) . . . . . $= \frac{wl^4}{24EI}$

Δx . . . . $= \frac{w(l^2 - x^2)^2}{24EI}$

21. CANTILEVER BEAM—CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load . . . . . $= \frac{8Pb}{l}$

R = V . . . . $= P$

M max. (at fixed end) . . . . . $= Pb$

M_x (when x > a) . . . . . $= P(x - a)$

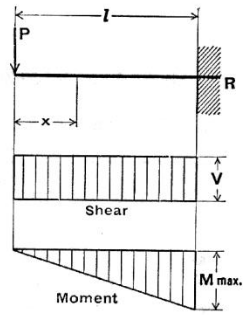
Δmax. (at free end) . . . . . $= \frac{Pb^2}{6EI}(3l - b)$

Δa (at point of load) . . . . . $= \frac{Pb^3}{3EI}$

Δx (when x < a) . . . . . $= \frac{Pb^2}{6EI}(3l - 3x - b)$

Δx (when x > a) . . . . . $= \frac{P(l - x)^2}{6EI}(3b - l + x)$

22. CANTILEVER BEAM—CONCENTRATED LOAD AT FREE END



Total Equiv. Uniform Load . . . . . $= 8P$

R = V . . . . $= P$

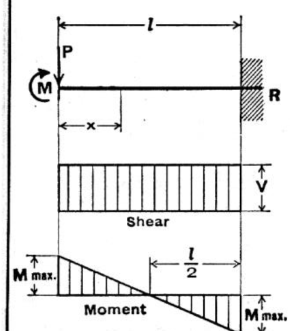
M max. (at fixed end) . . . . . $= Pl$

M_x . . . . $= Px$

Δmax. (at free end) . . . . . $= \frac{Pl^3}{3EI}$

Δx . . . . $= \frac{P}{6EI}(2l^3 - 3l^2x + x^3)$

23. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—CONCENTRATED LOAD AT DEFLECTED END



Total Equiv. Uniform Load . . . . . $= 4P$

R = V . . . . $= P$

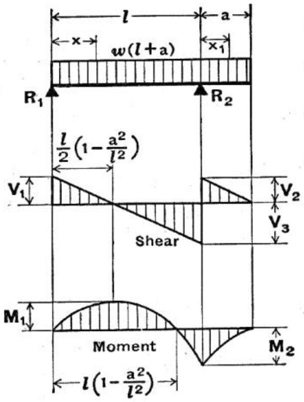
M max. (at both ends) . . . . . $= \frac{Pl}{2}$

M_x . . . . $= P\left(\frac{l}{2} - x\right)$

Δmax. (at deflected end) . . . . . $= \frac{Pl^3}{12EI}$

Δx . . . . $= \frac{P(l - x)^2}{12EI}(l + 2x)$

24. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD



$R_1 = V_1$ . . . . $= \frac{w}{2l}(l^2 - a^2)$

$R_2 = V_2 + V_3$ . . . . $= \frac{w}{2l}(l + a)^2$

V_2 . . . . $= wa$

V_3 . . . . $= \frac{w}{2l}(l^2 + a^2)$

V_x (between supports) . . . $= R_1 - wx$

V_{x_1} (for overhang) . . . . . $= w(a - x_1)$

M_1 $\left(\text{at } x = \frac{l}{2}\left[1 - \frac{a^2}{l^2}\right]\right)$ . . . $= \frac{w}{8l^2}(l+a)^2(l-a)^2$

M_2 (at R_2) . . . . $= \frac{wa^2}{2}$

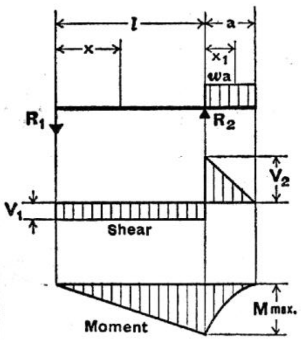
M_x (between supports) . . . $= \frac{wx}{2l}(l^2 - a^2 - xl)$

M_{x_1} (for overhang) . . . . . $= \frac{w}{2}(a - x_1)^2$

Δx (between supports) . . . $= \frac{wx}{24EIl}(l^4 - 2l^2x^2 + lx^3 - 2a^2l^2 + 2a^2x^2)$

Δx_1 (for overhang) . . . . . $= \frac{wx_1}{24EI}(4a^2l - l^3 + 6a^2x_1 - 4ax_1^2 + x_1^3)$

25. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD ON OVERHANG



$R_1 = V_1$ . . . . $= \frac{wa^2}{2l}$

$R_2 = V_1 + V_2$ . . . . $= \frac{wa}{2l}(2l + a)$

V_2 . . . . $= wa$

V_{x_1} (for overhang) . . . . . $= w(a - x_1)$

M max. (at R_2) . . . . $= \frac{wa^2}{2}$

M_x (between supports) . . . $= \frac{wa^2x}{2l}$

M_{x_1} (for overhang) . . . . . $= \frac{w}{2}(a - x_1)^2$

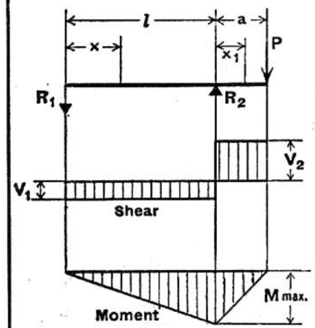
Δmax. $\left(\text{between supports at } x = \frac{l}{\sqrt{3}}\right) = \frac{wa^2l^2}{18\sqrt{3}EI} = .03208\,\frac{wa^2l^2}{EI}$

Δmax. (for overhang at $x_1 = a$) . $= \frac{wa^3}{24EI}(4l + 3a)$

Δx (between supports) . . . $= \frac{wa^2x}{12EIl}(l^2 - x^2)$

Δx_1 (for overhang) . . . . . $= \frac{wx_1}{24EI}(4a^2l + 6a^2x_1 - 4ax_1^2 + x_1^3)$

26. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT END OF OVERHANG



$R_1 = V_1$ . . . . $= \frac{Pa}{l}$

$R_2 = V_1 + V_2$ . . . . $= \frac{P}{l}(l + a)$

V_2 . . . . $= P$

M max. (at R_2) . . . . $= Pa$

M_x (between supports) . . . $= \frac{Pax}{l}$

M_{x_1} (for overhang) . . . . . $= P(a - x_1)$

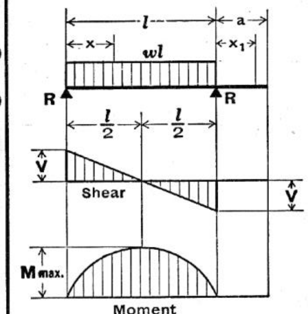
Δmax. $\left(\text{between supports at } x = \frac{l}{\sqrt{3}}\right) = \frac{Pal^2}{9\sqrt{3}EI} = .06415\,\frac{Pal^2}{EI}$

Δmax. (for overhang at $x_1 = a$) . $= \frac{Pa^2}{3EI}(l + a)$

Δx (between supports) . . . $= \frac{Pax}{6EIl}(l^2 - x^2)$

Δx_1 (for overhang) . . . . . $= \frac{Px_1}{6EI}(2al + 3ax_1 - x_1^2)$

27. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD BETWEEN SUPPORTS



Total Equiv. Uniform Load . . . . $= wl$

$R = V$ . . . . $= \frac{wl}{2}$

V_x . . . . $= w\left(\frac{l}{2} - x\right)$

M max. (at center) . . . . . . $= \frac{wl^2}{8}$

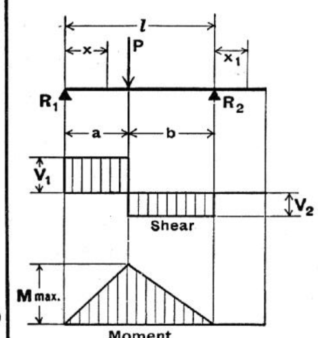
M_x . . . . $= \frac{wx}{2}(l - x)$

Δmax. (at center) . . . . . . $= \frac{5wl^4}{384EI}$

Δx . . . . $= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

Δx_1 . . . . $= \frac{wl^3x_1}{24EI}$

28. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT ANY POINT BETWEEN SUPPORTS



Total Equiv. Uniform Load . . . . $= \frac{8Pab}{l^2}$

$R_1 = V_1$ (max. when $a < b$) . . . $= \frac{Pb}{l}$

$R_2 = V_2$ (max. when $a > b$) . . . $= \frac{Pa}{l}$

M max. (at point of load) . . . $= \frac{Pab}{l}$

M_x (when $x < a$) . . . . . $= \frac{Pbx}{l}$

Δmax. $\left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b\right) = \frac{Pab\,(a+2b)\,\sqrt{3a\,(a+2b)}}{27EIl}$

Δa (at point of load) . . . $= \frac{Pa^2b^2}{3EIl}$

Δx (when $x < a$) . . . . . $= \frac{Pbx}{6EIl}(l^2 - b^2 - x^2)$

Δx (when $x > a$) . . . . . $= \frac{Pa\,(l - x)}{6EIl}(2lx - x^2 - a^2)$

Δx_1 . . . . $= \frac{Pabx_1}{6EIl}(l + a)$

Reference Diagrams

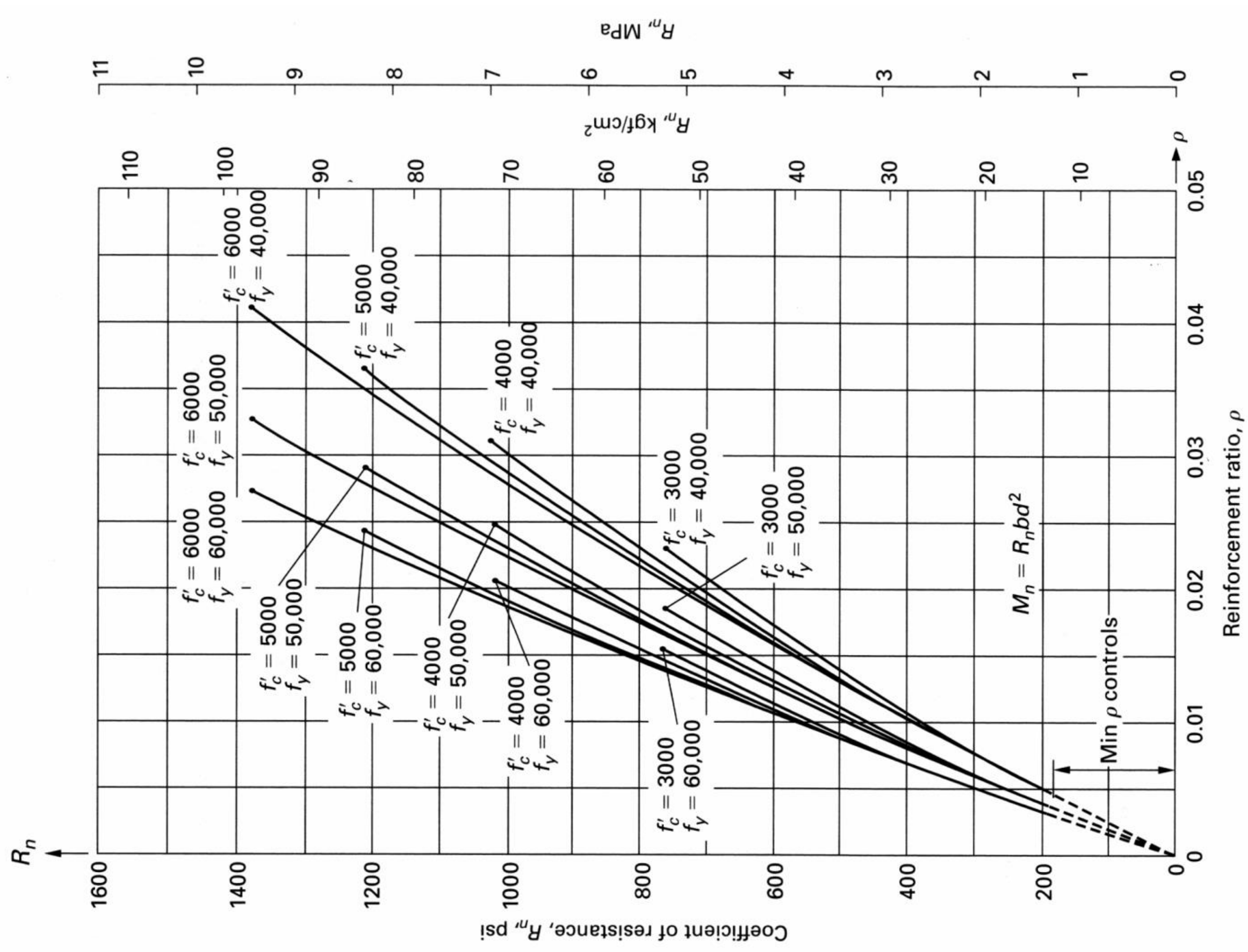


Figure 3.8.1 Strength curves (R_n vs ρ) for singly reinforced rectangular sections. Upper limit of curves is at (tensile strain of 0.004)

Table 3-8 ACI Provisions for Shear Design*

		$V_u \le \frac{\phi V_c}{2}$	$\phi V_c \ge V_u > \frac{\phi V_c}{2}$	$V_u > \phi V_c$
Required area of stirrups, A_v**		none	$\frac{50 b_w s}{f_y}$	$\frac{(V_u - \phi V_c)s}{\phi f_y d}$
Stirrup spacing, s	Required	—	$\frac{A_v f_y}{50 b_w}$	$\frac{\phi A_v f_y d}{V_u - \phi V_c}$
	Recommended Minimum†	—	—	4 in.
	Maximum†† (ACI 11.5.4)	—	$\frac{d}{2}$ or 24 in.	$\frac{d}{2}$ or 24 in. for $\left(V_u - \phi V_c\right) \le \phi 4\sqrt{f'_c}\, b_w d$
				$\frac{d}{4}$ or 12 in. for $\left(V_u - \phi V_c\right) > \phi 4\sqrt{f'_c}\, b_w d$

*Members subjected to shear and flexure only; $\phi V_c = \phi 2\sqrt{f'_c}\, b_w d$, $\phi = 0.75$ (ACI 11.3.1.1)
**$A_v = 2 \times A_b$ for U stirrups; $f_y \le 60$ ksi (ACI 11.5.2)
†A practical limit for minimum spacing is d/4
††Maximum spacing based on minimum shear reinforcement ($= A_v f_y/50 b_w$) must also be considered (ACI 11.5.5.3).

Reference Diagrams

TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE COMPUTED

	Minimum thickness, *h*			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

Notes:
Values given shall be used directly for members with normalweight concrete and Grade 60 reinforcement. For other conditions, the values shall be modified as follows:
a) For lightweight concrete having equilibrium density, w_c, in the range of 90 to 115 lb/ft^3, the values shall be multiplied by $(1.65 - 0.005w_c)$ but not less than 1.09.
b) For f_y other than 60,000 psi, the values shall be multiplied by $(0.4 + f_y/100,000)$.

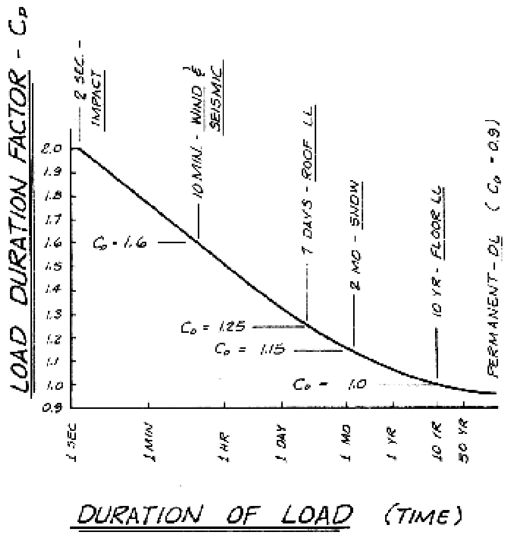


TABLE 13.6 Areas Provided By Spaced Reinforcement

Bar Spacing (in.)	Area Provided (in.^{2}/ft width) No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11
3	0.20	0.44	0.80	1.24	1.76	2.40	3.16	4.00		
3.5	0.17	0.38	0.69	1.06	1.51	2.06	2.71	3.43	4.35	
4	0.15	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4.5	0.13	0.29	0.53	0.83	1.17	1.60	2.11	2.67	3.39	4.16
5	0.12	0.26	0.48	0.74	1.06	1.44	1.89	2.40	3.05	3.74
5.5	0.11	0.24	0.44	0.68	0.96	1.31	1.72	2.18	2.77	3.40
6	0.10	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
7	0.08	0.19	0.34	0.53	0.75	1.03	1.35	1.71	2.18	2.67
8	0.07	0.16	0.30	0.46	0.66	0.90	1.18	1.50	1.90	2.34
9	0.07	0.15	0.27	0.41	0.59	0.80	1.05	1.33	1.69	2.08
10	0.06	0.13	0.24	0.37	0.53	0.72	0.95	1.20	1.52	1.87
11	0.05	0.12	0.22	0.34	0.48	0.65	0.86	1.09	1.38	1.70
12	0.05	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
13	0.05	0.10	0.18	0.29	0.40	0.55	0.73	0.92	1.17	1.44
14	0.04	0.09	0.17	0.27	0.38	0.51	0.68	0.86	1.09	1.34
15	0.04	0.09	0.16	0.25	0.35	0.48	0.63	0.80	1.01	1.25
16	0.04	0.08	0.15	0.23	0.33	0.45	0.59	0.75	0.95	1.17
18	0.03	0.07	0.13	0.21	0.29	0.40	0.53	0.67	0.85	1.04
24	0.02	0.05	0.10	0.15	0.22	0.30	0.39	0.50	0.63	0.78

Table 3.7.1
Total Areas for Various Numbers of Reinforcing Bars

Bar Size	*Nominal Diameter (in.)*	*Weight (lb/ft)*	*Number of Bars* 1	2	3	4	5	6	7	8	9	10
#3	0.375	0.376	0.11	0.22	0.33	0.44	0.55	0.66	0.77	0.88	0.99	1.10
#4	0.500	0.668	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
#5	0.625	1.043	0.31	0.62	0.93	1.24	1.55	1.86	2.17	2.48	2.79	3.10
#6	0.750	1.502	0.44	0.88	1.32	1.76	2.20	2.64	3.08	3.52	3.96	4.40
#7	0.875	2.044	0.60	1.20	1.80	2.40	3.00	3.60	4.20	4.80	5.40	6.00
#8	1.000	2.670	0.79	1.58	2.37	3.16	3.95	4.74	5.53	6.32	7.11	7.90
#9	1.128	3.400	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
#10	1.270	4.303	1.27	2.54	3.81	5.08	6.35	7.62	8.89	10.16	11.43	12.70
#11	1.410	5.313	1.56	3.12	4.68	6.24	7.80	9.36	10.92	12.48	14.04	15.60
#14[a]	1.693	7.65	2.25	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50
#18[a]	2.257	13.60	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00

[a] #14 and #18 bars are used primarily as column reinforcement and are rarely used in beams.

Reference Diagrams

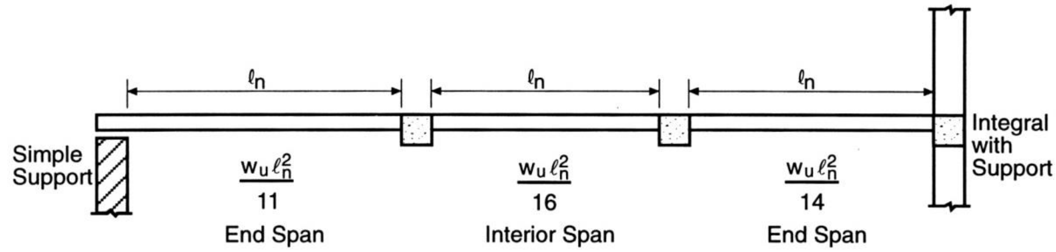


Figure 2-3 Positive Moments—All Cases

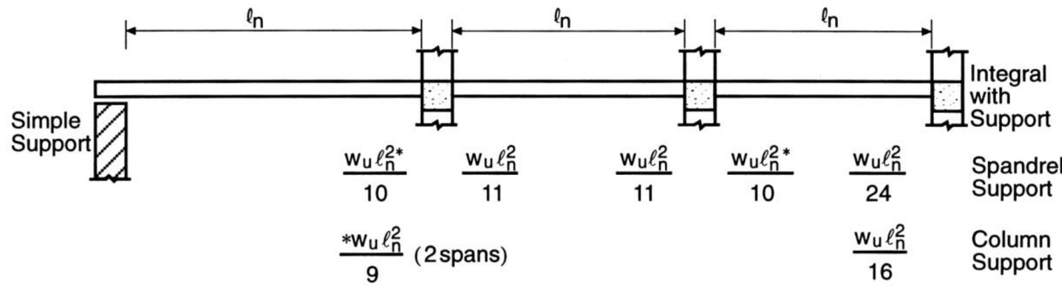


Figure 2-4 Negative Moments—Beams and Slabs

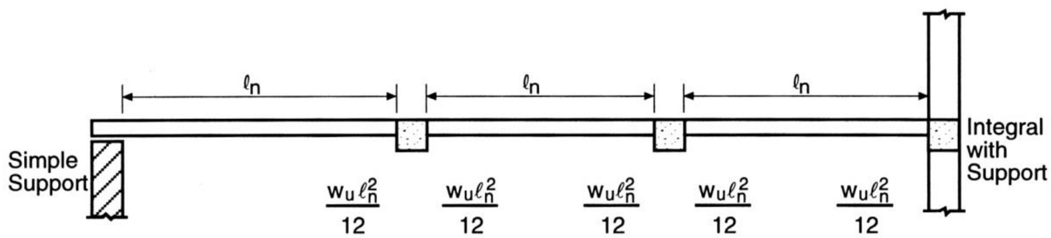


Figure 2-5 Negative Moments—Slabs with spans ≤ 10 ft

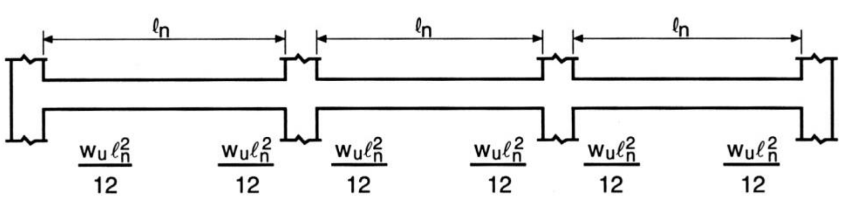


Figure 2-6 Negative Moments—Beams with Stiff Columns ($\Sigma K_c / \Sigma K_b > 8$)

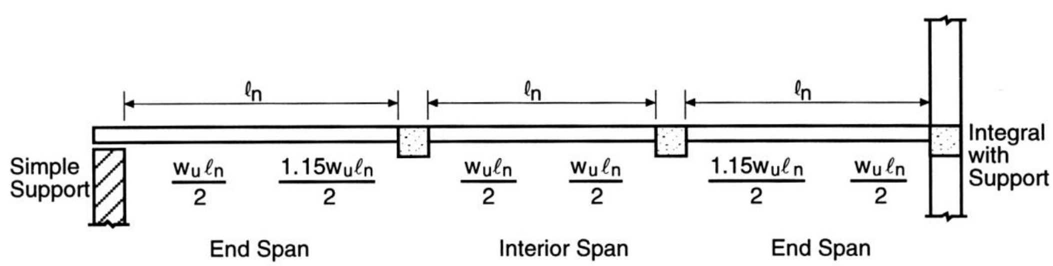


Figure 2-7 End Shears—All Cases

Reference Diagrams

Table 14 Column Stability Factor C_p.

"C_p" $F_c' = C_p \cdot F_c^*$ $F_{CE} = \frac{.30\,E}{(l/d)^2}$ for sawed posts $F_{CE} = \frac{.418\,E}{(l/d)^2}$ for glu-lam posts

F_{CE}/F_c^*	Sawed C_p	Glu-Lam C_p	F_{CE}/F_c^*	Sawed C_p	Glu-Lam C_p	F_{CE}/F_c^*	Sawed C_p	Glu-Lam C_p	F_{CE}/F_c^*	Sawed C_p	Glu-Lam C_p	F_{CE}/F_c^*	Sawed C_p	Glu-Lam C_p	F_{CE}/F_c^*	Sawed C_p	Glu-Lam C_p
0.00	0.000	0.000	0.40	0.360	0.377	0.80	0.610	0.667	1.20	0.750	0.822	2.00	0.867	0.921	3.40	0.930	0.962
0.01	0.010	0.010	0.41	0.367	0.386	0.81	0.614	0.672	1.22	0.755	0.826	2.02	0.869	0.922	3.45	0.931	0.963
0.02	0.020	0.020	0.42	0.375	0.394	0.82	0.619	0.678	1.24	0.760	0.831	2.04	0.870	0.924	3.50	0.932	0.963
0.03	0.030	0.030	0.43	0.383	0.403	0.83	0.623	0.683	1.26	0.764	0.836	2.06	0.872	0.925	3.55	0.933	0.964
0.04	0.040	0.040	0.44	0.390	0.411	0.84	0.628	0.688	1.28	0.769	0.840	2.08	0.874	0.926	3.60	0.934	0.965
0.05	0.049	0.050	0.45	0.398	0.420	0.85	0.632	0.693	1.30	0.773	0.844	2.10	0.875	0.927	3.65	0.936	0.965
0.06	0.059	0.060	0.46	0.405	0.428	0.86	0.637	0.698	1.32	0.777	0.848	2.12	0.876	0.928	3.70	0.937	0.966
0.07	0.069	0.069	0.47	0.412	0.436	0.87	0.641	0.703	1.34	0.781	0.852	2.14	0.878	0.929	3.75	0.938	0.966
0.08	0.079	0.079	0.48	0.419	0.444	0.88	0.645	0.708	1.36	0.785	0.855	2.16	0.879	0.930	3.80	0.938	0.967
0.09	0.088	0.089	0.49	0.427	0.453	0.89	0.649	0.713	1.38	0.789	0.859	2.18	0.881	0.931	3.85	0.939	0.968
0.10	0.098	0.099	0.50	0.434	0.461	0.90	0.653	0.718	1.40	0.793	0.862	2.20	0.882	0.932	3.90	0.940	0.968
0.11	0.107	0.109	0.51	0.441	0.469	0.91	0.658	0.722	1.42	0.796	0.865	2.22	0.883	0.932	3.95	0.941	0.969
0.12	0.117	0.118	0.52	0.448	0.477	0.92	0.661	0.727	1.44	0.800	0.868	2.24	0.885	0.933	4.00	0.942	0.969
0.13	0.126	0.128	0.53	0.454	0.484	0.93	0.665	0.731	1.46	0.803	0.871	2.26	0.886	0.934	4.05	0.943	0.969
0.14	0.136	0.138	0.54	0.461	0.492	0.94	0.669	0.735	1.48	0.807	0.874	2.28	0.887	0.935	4.10	0.944	0.970
0.15	0.145	0.147	0.55	0.468	0.500	0.95	0.673	0.740	1.50	0.810	0.877	2.30	0.888	0.936	4.15	0.944	0.970
0.16	0.154	0.157	0.56	0.474	0.508	0.96	0.677	0.744	1.52	0.813	0.879	2.32	0.889	0.937	4.20	0.945	0.971
0.17	0.164	0.167	0.57	0.481	0.515	0.97	0.680	0.748	1.54	0.816	0.882	2.34	0.891	0.937	4.25	0.946	0.971
0.18	0.173	0.176	0.58	0.487	0.523	0.98	0.684	0.752	1.56	0.819	0.884	2.36	0.892	0.938	4.30	0.947	0.972
0.19	0.182	0.186	0.59	0.494	0.530	0.99	0.688	0.756	1.58	0.822	0.887	2.38	0.893	0.939	4.35	0.947	0.972
0.20	0.191	0.195	0.60	0.500	0.538	1.00	0.691	0.760	1.60	0.825	0.889	2.40	0.894	0.940	4.40	0.948	0.972
0.21	0.200	0.205	0.61	0.506	0.545	1.01	0.694	0.764	1.62	0.827	0.891	2.45	0.897	0.941	4.45	0.949	0.973
0.22	0.209	0.214	0.62	0.512	0.552	1.02	0.698	0.767	1.64	0.830	0.893	2.50	0.899	0.943	4.50	0.949	0.973
0.23	0.218	0.224	0.63	0.518	0.559	1.03	0.701	0.771	1.66	0.832	0.895	2.55	0.901	0.944	4.55	0.950	0.974
0.24	0.227	0.233	0.64	0.524	0.566	1.04	0.704	0.774	1.68	0.835	0.897	2.60	0.904	0.946	4.60	0.950	0.974
0.25	0.235	0.242	0.65	0.530	0.573	1.05	0.708	0.778	1.70	0.837	0.899	2.65	0.906	0.947	4.65	0.951	0.974
0.26	0.244	0.252	0.66	0.536	0.580	1.06	0.711	0.781	1.72	0.840	0.901	2.70	0.908	0.949	4.70	0.952	0.975
0.27	0.253	0.261	0.67	0.542	0.587	1.07	0.714	0.784	1.74	0.842	0.903	2.75	0.910	0.950	4.75	0.952	0.975
0.28	0.261	0.270	0.68	0.548	0.593	1.08	0.717	0.788	1.76	0.844	0.904	2.80	0.912	0.951	4.80	0.953	0.975
0.29	0.270	0.279	0.69	0.553	0.600	1.09	0.720	0.791	1.78	0.846	0.906	2.85	0.914	0.952	4.85	0.953	0.975
0.30	0.278	0.288	0.70	0.559	0.607	1.10	0.723	0.794	1.80	0.849	0.908	2.90	0.916	0.953	4.90	0.954	0.976
0.31	0.287	0.297	0.71	0.564	0.613	1.11	0.726	0.797	1.82	0.851	0.909	2.95	0.917	0.954	5.00	0.955	0.976
0.32	0.295	0.306	0.72	0.569	0.619	1.12	0.729	0.800	1.84	0.853	0.911	3.00	0.919	0.955	6.00	0.963	0.981
0.33	0.304	0.315	0.73	0.575	0.626	1.13	0.731	0.803	1.86	0.855	0.912	3.05	0.920	0.956	8.00	0.973	0.986
0.34	0.312	0.324	0.74	0.580	0.632	1.14	0.734	0.806	1.88	0.857	0.914	3.10	0.922	0.957	10.0	0.979	0.989
0.35	0.320	0.333	0.75	0.585	0.638	1.15	0.737	0.809	1.90	0.858	0.915	3.15	0.923	0.958	20.0	0.990	0.995
0.36	0.328	0.342	0.76	0.590	0.644	1.16	0.740	0.811	1.92	0.860	0.916	3.20	0.925	0.959	40.0	0.995	0.997
0.37	0.336	0.351	0.77	0.595	0.650	1.17	0.742	0.814	1.94	0.862	0.918	3.25	0.926	0.960	60.0	0.997	0.998
0.38	0.344	0.360	0.78	0.600	0.655	1.18	0.745	0.817	1.96	0.864	0.919	3.30	0.927	0.961	100.0	0.998	0.999
0.39	0.352	0.368	0.79	0.605	0.661	1.19	0.747	0.819	1.98	0.868	0.920	3.35	0.929	0.961	200.0	0.999	0.999

Table developed and permission for use granted by Professor Ed Lebert, Dept. of Architecture, University of Washington.

Reference Diagrams

Maximum Reinforcement Ratio ρ for Singly Reinforced Rectangular Beams
(tensile strain = 0.005) for which ϕ is permitted to be 0.9

f_y	f'_c = 3000 psi β_1 = 0.85	f'_c = 3500 psi β_1 = 0.85	f'_c = 4000 psi β_1 = 0.85	f'_c = 5000 psi β_1 = 0.80	f'_c = 6000 psi β_1 = 0.75
40,000 psi	0.0203	0.0237	0.0271	0.0319	0.0359
50,000 psi	0.0163	0.0190	0.0217	0.0255	0.0287
60,000 psi	0.0135	0.0158	0.0181	0.0213	0.0239
f_y	f'_c = 20 MPa β_1 = 0.85	f'_c = 25 MPa β_1 = 0.85	f'_c = 30 MPa β_1 = 0.85	f'_c = 35 MPa β_1 = 0.81	f'_c = 40 MPa β_1 = 0.77
300 MPa	0.0181	0.0226	0.0271	0.0301	0.0327
350 MPa	0.0155	0.0194	0.0232	0.0258	0.0281
400 MPa	0.0135	0.0169	0.0203	0.0226	0.0245
500 MPa	0.0108	0.0135	0.0163	0.0181	0.0196

Table 7-1
Available Shear Strength of Bolts, kips

Nominal Bolt Diameter, *d*, in.					5/8		3/4		7/8		1	
Nominal Bolt Area, in.²					0.307		0.442		0.601		0.785	
ASTM Desig.	Thread Cond.	F_{nv}/Ω (ksi) ASD	ϕF_{nv} (ksi) LRFD	Loading	r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD
Group A	N	27.0	40.5	S	8.29	12.4	11.9	17.9	16.2	24.3	21.2	31.8
				D	16.6	24.9	23.9	35.8	32.5	48.7	42.4	63.6
	X	34.0	51.0	S	10.4	15.7	15.0	22.5	20.4	30.7	26.7	40.0
				D	20.9	31.3	30.1	45.1	40.9	61.3	53.4	80.1
Group B	N	34.0	51.0	S	10.4	15.7	15.0	22.5	20.4	30.7	26.7	40.0
				D	20.9	31.3	30.1	45.1	40.9	61.3	53.4	80.1
	X	42.0	63.0	S	12.9	19.3	18.6	27.8	25.2	37.9	33.0	49.5
				D	25.8	38.7	37.1	55.7	50.5	75.7	65.9	98.9
A307	–	13.5	20.3	S	4.14	6.23	5.97	8.97	8.11	12.2	10.6	15.9
				D	8.29	12.5	11.9	17.9	16.2	24.4	21.2	31.9
Nominal Bolt Diameter, *d*, in.					1 1/8		1 1/4		1 3/8		1 1/2	
Nominal Bolt Area, in.²					0.994		1.23		1.48		1.77	
ASTM Desig.	Thread Cond.	F_{nv}/Ω (ksi) ASD	ϕF_{nv} (ksi) LRFD	Loading	r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD	r_n/Ω ASD	ϕr_n LRFD
Group A	N	27.0	40.5	S	26.8	40.3	33.2	49.8	40.0	59.9	47.8	71.7
				D	53.7	80.5	66.4	99.6	79.9	120	95.6	143
	X	34.0	51.0	S	33.8	50.7	41.8	62.7	50.3	75.5	60.2	90.3
				D	67.6	101	83.6	125	101	151	120	181
Group B	N	34.0	51.0	S	33.8	50.7	41.8	62.7	50.3	75.5	60.2	90.3
				D	67.6	101	83.6	125	101	151	120	181
	X	42.0	63.0	S	41.7	62.6	51.7	77.5	62.2	93.2	74.3	112
				D	83.5	125	103	155	124	186	149	223
A307	–	13.5	20.3	S	13.4	20.2	16.6	25.0	20.0	30.0	23.9	35.9
				D	26.8	40.4	33.2	49.9	40.0	60.1	47.8	71.9
ASD	LRFD	For end loaded connections greater than 38 in., see AISC *Specification* Table J3.2 footnote b.										
Ω = 2.00	ϕ = 0.75											

Available Strength of Fillet Welds
per inch of weld (ϕS)

Weld Size (in.)	E60XX (k/in.)	E70XX (k/in.)
3/16	3.58	4.18
¼	4.77	5.57
5/16	5.97	6.96
3/8	7.16	8.35
7/16	8.35	9.74
½	9.55	11.14
5/8	11.93	13.92
¾	14.32	16.70

(not considering increase in throat with submerged arc weld process)

Reference Diagrams

Group A Bolts

A325, A325M
F1858
A354 Grade BC
A449

Table 7-3
Slip-Critical Connections
Available Shear Strength, kips
(Class A Faying Surface, $\mu = 0.30$)

Group A Bolts									
Hole Type	Loading	Nominal Bolt Diameter, d, in.							
		5/8		3/4		7/8		1	
		Minimum Group A Bolt Pretension, kips							
		19		28		39		51	
		r_n/Ω	ϕr_n	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
STD/SSLT	S	4.29	6.44	6.33	9.49	8.81	13.2	11.5	17.3
	D	8.59	12.9	12.7	19.0	17.6	26.4	23.1	34.6
OVS/SSLP	S	3.66	5.47	5.39	8.07	7.51	11.2	9.82	14.7
	D	7.32	10.9	10.8	16.1	15.0	22.5	19.6	29.4
LSL	S	3.01	4.51	4.44	6.64	6.18	9.25	8.08	12.1
	D	6.02	9.02	8.87	13.3	12.4	18.5	16.2	24.2
Hole Type	Loading	Nominal Bolt Diameter, d, in.							
		1 1/8		1 1/4		1 3/8		1 1/2	
		Minimum Group A Bolt Pretension, kips							
		56		71		85		103	
		r_n/Ω	ϕr_n	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
STD/SSLT	S	12.7	19.0	16.0	24.1	19.2	28.8	23.3	34.9
	D	25.3	38.0	32.1	48.1	38.4	57.6	46.6	69.8
OVS/SSLP	S	10.8	16.1	13.7	20.5	16.4	24.5	19.8	29.7
	D	21.6	32.3	27.4	40.9	32.7	49.0	39.7	59.4
LSL	S	8.87	13.3	11.2	16.8	13.5	20.2	16.3	24.4
	D	17.7	26.6	22.5	33.7	26.9	40.3	32.6	48.9

STD = standard hole
OVS = oversized hole
SSLT = short-slotted hole transverse to the line of force
SSLP = short-slotted hole parallel to the line of force
LSL = long-slotted hole transverse or parallel to the line of force

S = single shear
D = double shear

Hole Type	ASD	LRFD	
STD and SSLT	$\Omega = 1.50$	$\phi = 1.00$	Note: Slip-critical bolt values assume no more than one filler has been provided or bolts have been added to distribute loads in the fillers.
OVS and SSLP	$\Omega = 1.76$	$\phi = 0.85$	See AISC *Specification* Sections J3.8 and J5 for provisions when fillers are present.
LSL	$\Omega = 2.14$	$\phi = 0.70$	For Class B faying surfaces, multiply the tabulated available strength by 1.67.

Table 7-3 (continued)
Slip-Critical Connections
Available Shear Strength, kips
(Class A Faying Surface, $\mu = 0.30$)

Group B Bolts

A490, A490M
F2280
A354 Grade BD

Group B Bolts									
Hole Type	Loading	Nominal Bolt Diameter, d, in.							
		5/8		3/4		7/8		1	
		Minimum Group B Bolt Pretension, kips							
		24		35		49		64	
		r_n/Ω	ϕr_n	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
STD/SSLT	S	5.42	8.14	7.91	11.9	11.1	16.6	14.5	21.7
	D	10.8	16.3	15.8	23.7	22.1	33.2	28.9	43.4
OVS/SSLP	S	4.62	6.92	6.74	10.1	9.44	14.1	12.3	18.4
	D	9.25	13.8	13.5	20.2	18.9	28.2	24.7	36.9
LSL	S	3.80	5.70	5.54	8.31	7.76	11.6	10.1	15.2
	D	7.60	11.4	11.1	16.6	15.5	23.3	20.3	30.4
Hole Type	Loading	Nominal Bolt Diameter, d, in.							
		1 1/8		1 1/4		1 3/8		1 1/2	
		Minimum Group B Bolt Pretension, kips							
		80		102		121		148	
		r_n/Ω	ϕr_n	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
STD/SSLT	S	18.1	27.1	23.1	34.6	27.3	41.0	33.4	50.2
	D	36.2	54.2	46.1	69.2	54.7	82.0	66.9	100
OVS/SSLP	S	15.4	23.1	19.6	29.4	23.3	34.9	28.5	42.6
	D	30.8	46.1	39.3	58.8	46.6	69.7	57.0	85.3
LSL	S	12.7	19.0	16.2	24.2	19.2	28.7	23.4	35.1
	D	25.3	38.0	32.3	48.4	38.3	57.4	46.9	70.2

STD = standard hole
OVS = oversized hole
SSLT = short-slotted hole transverse to the line of force
SSLP = short-slotted hole parallel to the line of force
LSL = long-slotted hole transverse or parallel to the line of force

S = single shear
D = double shear

Hole Type	ASD	LRFD	
STD and SSLT	$\Omega = 1.50$	$\phi = 1.00$	Note: Slip-critical bolt values assume no more than one filler has been provided or bolts have been added to distribute loads in the fillers.
OVS and SSLP	$\Omega = 1.76$	$\phi = 0.85$	See AISC *Specification* Sections J3.8 and J5 for provisions when fillers are present.
LSL	$\Omega = 2.14$	$\phi = 0.70$	For Class B faying surfaces, multiply the tabulated available strength by 1.67.

Reference Diagrams

Table 7-4
Available Bearing Strength at Bolt Holes Based on Bolt Spacing
kips/in. thickness

Hole Type	Bolt Spacing, *s*, in.	F_u ksi	5/8 r_n/Ω ASD	5/8 ϕr_n LRFD	3/4 r_n/Ω ASD	3/4 ϕr_n LRFD	7/8 r_n/Ω ASD	7/8 ϕr_n LRFD	1 r_n/Ω ASD	1 ϕr_n LRFD
STD SSLT	$2\frac{2}{3}d_b$	58	34.1	51.1	41.3	62.0	48.6	72.9	55.8	83.7
		65	38.2	57.3	46.3	69.5	54.4	81.7	62.6	93.8
	3 in.	58	43.5	65.3	52.2	78.3	60.9	91.4	67.4	101
		65	48.8	73.1	58.5	87.8	68.3	102	75.6	113
SSLP	$2\frac{2}{3}d_b$	58	27.6	41.3	34.8	52.2	42.1	63.1	47.1	70.7
		65	30.9	46.3	39.0	58.5	47.1	70.7	52.8	79.2
	3 in.	58	43.5	65.3	52.2	78.3	60.9	91.4	58.7	88.1
		65	48.8	73.1	58.5	87.8	68.3	102	65.8	98.7
OVS	$2\frac{2}{3}d_b$	58	29.7	44.6	37.0	55.5	44.2	66.3	49.3	74.0
		65	33.3	50.0	41.4	62.2	49.6	74.3	55.3	82.9
	3 in.	58	43.5	65.3	52.2	78.3	60.9	91.4	60.9	91.4
		65	48.8	73.1	58.5	87.8	68.3	102	68.3	102
LSLP	$2\frac{2}{3}d_b$	58	3.62	5.44	4.35	6.53	5.08	7.61	5.80	8.70
		65	4.06	6.09	4.88	7.31	5.69	8.53	6.50	9.75
	3 in.	58	43.5	65.3	39.2	58.7	28.3	42.4	17.4	26.1
		65	48.8	73.1	43.9	65.8	31.7	47.5	19.5	29.3
LSLT	$2\frac{2}{3}d_b$	58	28.4	42.6	34.4	51.7	40.5	60.7	46.5	69.8
		65	31.8	47.7	38.6	57.9	45.4	68.0	52.1	78.2
	3 in.	58	36.3	54.4	43.5	65.3	50.8	76.1	56.2	84.3
		65	40.6	60.9	48.8	73.1	56.9	85.3	63.0	94.5
STD, SSLT, SSLP, OVS, LSLP	$s \geq s_{full}$	58	43.5	65.3	52.2	78.3	60.9	91.4	69.6	104
		65	48.8	73.1	58.5	87.8	68.3	102	78.0	117
LSLT	$s \geq s_{full}$	58	36.3	54.4	43.5	65.3	50.8	76.1	58.0	87.0
		65	40.6	60.9	48.8	73.1	56.9	85.3	65.0	97.5
Spacing for full bearing strength s_{full}[a], in.		STD, SSLT, LSLT	$1\frac{15}{16}$		$2\frac{5}{16}$		$2\frac{11}{16}$		$3\frac{1}{16}$	
		OVS	$2\frac{1}{16}$		$2\frac{7}{16}$		$2\frac{13}{16}$		$3\frac{1}{4}$	
		SSLP	$2\frac{1}{8}$		$2\frac{1}{2}$		$2\frac{7}{8}$		$3\frac{5}{16}$	
		LSLP	$2\frac{13}{16}$		$3\frac{3}{8}$		$3\frac{15}{16}$		$4\frac{1}{2}$	
Minimum Spacing[a] = $2\frac{2}{3}d$, in.			$1\frac{11}{16}$		2		$2\frac{5}{16}$		$2\frac{11}{16}$	

STD = standard hole
SSLT = short-slotted hole oriented transverse to the line of force
SSLP = short-slotted hole oriented parallel to the line of force
OVS = oversized hole
LSLP = long-slotted hole oriented parallel to the line of force
LSLT = long-slotted hole oriented transverse to the line of force

ASD	LRFD	
$\Omega = 2.00$	$\phi = 0.75$	Note: Spacing indicated is from the center of the hole or slot to the center of the adjacent hole or slot in the line of force. Hole deformation is considered. When hole deformation is not considered, see AISC *Specification* Section J3.10. [a] Decimal value has been rounded to the nearest sixteenth of an inch.

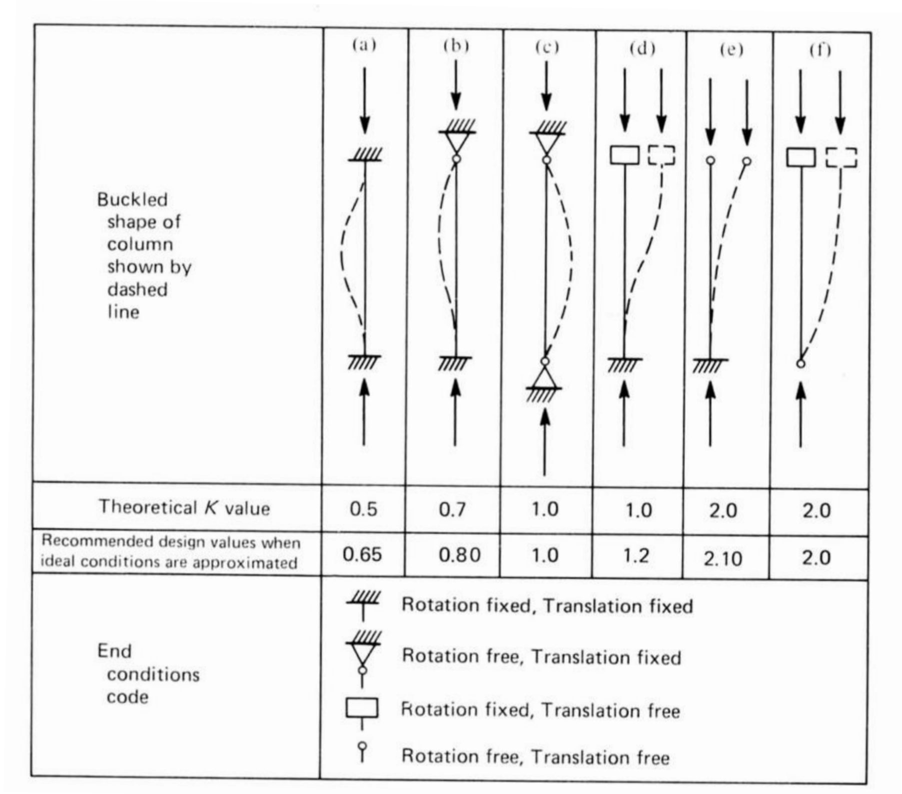
Table 7-4 (continued)
Available Bearing Strength at Bolt Holes Based on Bolt Spacing
kips/in. thickness

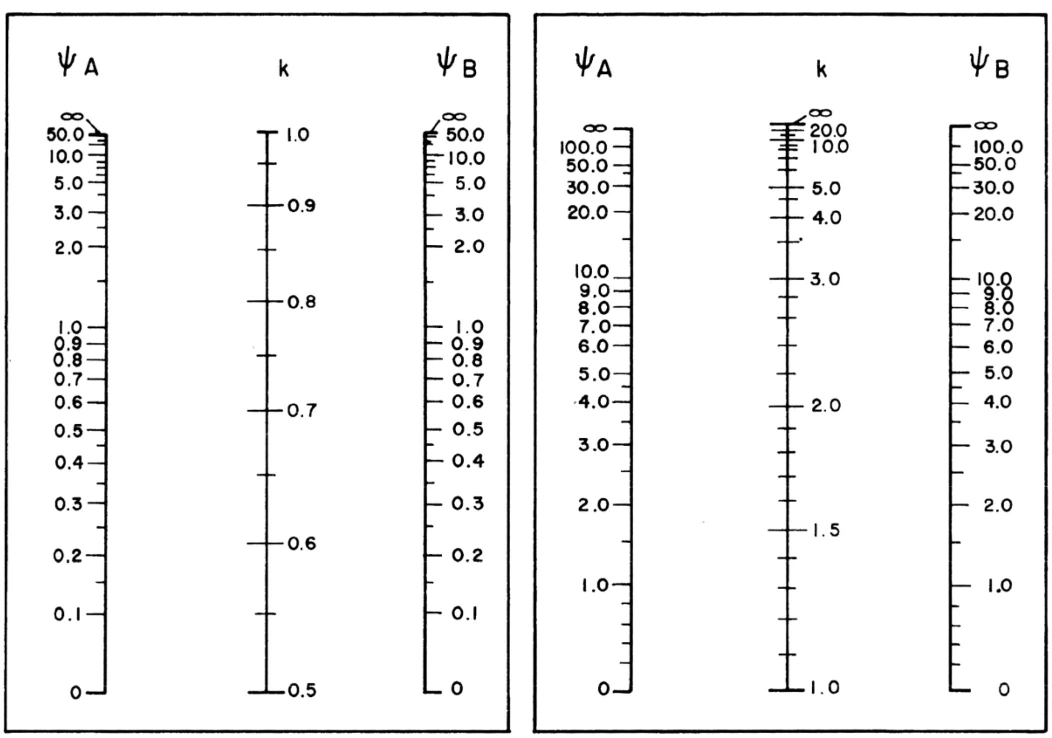
Hole Type	Bolt Spacing, *s*, in.	F_u ksi	1 1/8 r_n/Ω ASD	1 1/8 ϕr_n LRFD	1 1/4 r_n/Ω ASD	1 1/4 ϕr_n LRFD	1 3/8 r_n/Ω ASD	1 3/8 ϕr_n LRFD	1 1/2 r_n/Ω ASD	1 1/2 ϕr_n LRFD
STD SSLT	$2\frac{2}{3}d_b$	58	63.1	94.6	70.3	105	77.6	116	84.8	127
		65	70.7	106	78.8	118	86.9	130	95.1	143
	3 in.	58	63.1	94.6	—	—	—	—	—	—
		65	70.7	106	—	—	—	—	—	—
SSLP	$2\frac{2}{3}d_b$	58	52.2	78.3	59.5	89.2	66.7	100	74.0	111
		65	58.5	87.8	66.6	99.9	74.8	112	82.9	124
	3 in.	58	52.2	78.3	—	—	—	—	—	—
		65	58.5	87.8	—	—	—	—	—	—
OVS	$2\frac{2}{3}d_b$	58	54.4	81.6	61.6	92.4	68.9	103	76.1	114
		65	60.9	91.4	69.1	104	77.2	116	85.3	128
	3 in.	58	54.4	81.6	—	—	—	—	—	—
		65	60.9	91.4	—	—	—	—	—	—
LSLP	$2\frac{2}{3}d_b$	58	6.53	9.79	7.25	10.9	7.98	12.0	8.70	13.1
		65	7.31	11.0	8.13	12.2	8.94	13.4	9.75	14.6
	3 in.	58	6.53	9.79	—	—	—	—	—	—
		65	7.31	11.0	—	—	—	—	—	—
LSLT	$2\frac{2}{3}d_b$	58	52.6	78.8	58.6	87.9	64.6	97.0	70.7	106
		65	58.9	88.4	65.7	98.5	72.4	109	79.2	119
	3 in.	58	52.6	78.8	—	—	—	—	—	—
		65	58.9	88.4	—	—	—	—	—	—
STD, SSLT, SSLP, OVS, LSLP	$s \geq s_{full}$	58	78.3	117	87.0	131	95.7	144	104	157
		65	87.8	132	97.5	146	107	161	117	176
LSLT	$s \geq s_{full}$	58	65.3	97.9	72.5	109	79.8	120	87.0	131
		65	73.1	110	81.3	122	89.4	134	97.5	146
Spacing for full bearing strength s_{full}[a], in.		STD, SSLT, LSLT	$3\frac{7}{16}$		$3\frac{13}{16}$		$4\frac{3}{16}$		$4\frac{9}{16}$	
		OVS	$3\frac{11}{16}$		$4\frac{1}{16}$		$4\frac{7}{16}$		$4\frac{13}{16}$	
		SSLP	$3\frac{3}{4}$		$4\frac{1}{8}$		$4\frac{1}{2}$		$4\frac{7}{8}$	
		LSLP	$5\frac{1}{16}$		$5\frac{5}{8}$		$6\frac{3}{16}$		$6\frac{3}{4}$	
Minimum Spacing[a] = $2\frac{2}{3}d$, in.			3		$3\frac{5}{16}$		$3\frac{11}{16}$		4	

STD = standard hole
SSLT = short-slotted hole oriented transverse to the line of force
SSLP = short-slotted hole oriented parallel to the line of force
OVS = oversized hole
LSLP = long-slotted hole oriented parallel to the line of force
LSLT = long-slotted hole oriented transverse to the line of force

ASD	LRFD	
$\Omega = 2.00$	$\phi = 0.75$	— indicates spacing less than minimum spacing required per AISC *Specification* Section J3.3. Note: Spacing indicated is from the center of the hole or slot to the center of the adjacent hole or slot in the line of force. Hole deformation is considered. When hole deformation is not considered, see AISC *Specification* Section J3.10. [a] Decimal value has been rounded to the nearest sixteenth of an inch.

Reference Diagrams





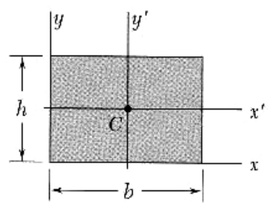
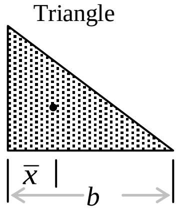
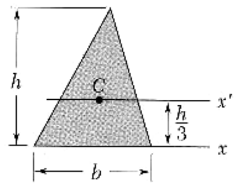
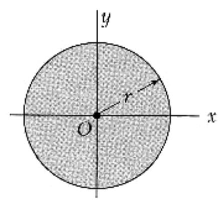
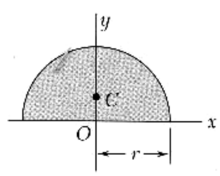
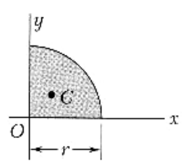
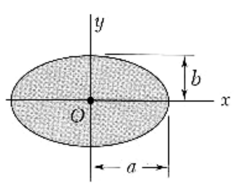
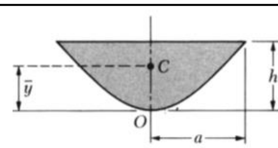
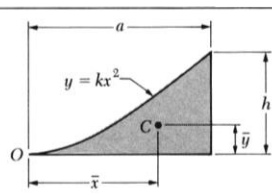
(a)

Nonsway Frames

(b)

Sway Frames

Reference Geometry

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ *about bottom left* $J_C = \frac{1}{12}bh(b^2 + h^2)$	Area = bh $\bar{x}$ = b/2 $\bar{y}$ = h/2
Triangle 		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{36}b^3h$	Area = $bh/2$ $\bar{x} = b/3$ $\bar{y} = h/3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$	Area = $\pi r^2 = \pi d^2/4$ $\bar{x}$ = 0 $\bar{y}$ = 0
Semicircle		$\bar{I}_x = 0.1098r^4$ $\bar{I}_y = \pi r^4/8$	Area = $\pi r^2/2 = \pi d^2/8$ $\bar{x}$ = 0 $\bar{y} = 4r/3\pi$
Quarter circle		$\bar{I}_x = 0.0549r^4$ $\bar{I}_y = 0.0549r^4$	Area = $\pi r^2/4 = \pi d^2/16$ $\bar{x} = 4r/3\pi$ $\bar{y} = 4r/3\pi$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$	Area = πab $\bar{x}$ = 0 $\bar{y}$ = 0
Parabolic area		$\bar{I}_x = 16ah^3/175$ $\bar{I}_y = 4a^3h/15$	Area = $4ah/3$ $\bar{x}$ = 0 $\bar{y} = 3h/5$
Parabolic span-drel		$\bar{I}_x = 37ah^3/2100$ $\bar{I}_y = a^3h/80$	Area = $ah/3$ $\bar{x} = 3a/4$ $\bar{y} = 3h/10$