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beams – internal forces & diagrams

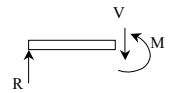
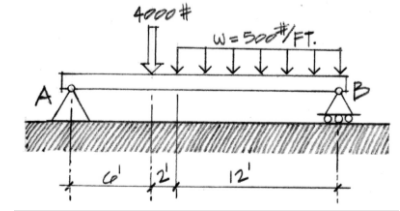
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Beams

- span horizontally
 - floors
 - bridges
 - roofs
- loaded transversely by gravity loads
- may have internal axial force
- will have internal shear force
- will have internal moment (bending)



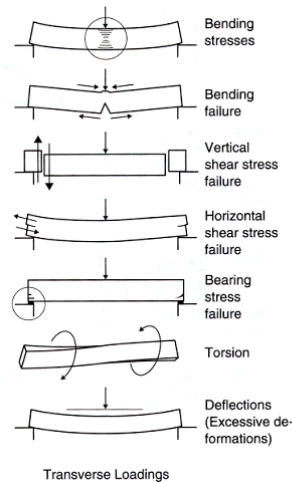
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Beams

- transverse loading
- sees:
 - bending
 - shear
 - deflection
 - torsion
 - bearing
- behavior depends on cross section shape



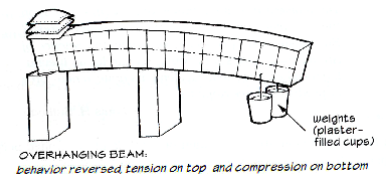
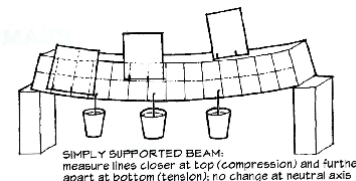
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Beams

- bending
 - bowing of beam with loads
 - one edge surface stretches
 - other edge surface squishes



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Beam Stresses

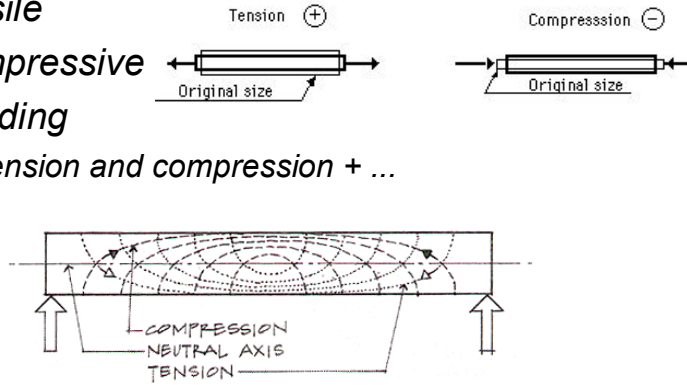
- *stress = relative force over an area*

- *tensile*

- *compressive*

- *bending*

- *tension and compression + ...*

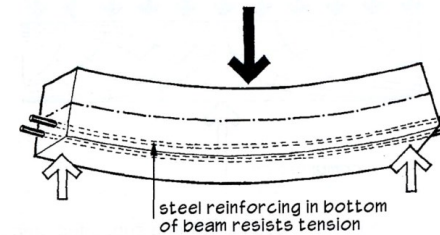
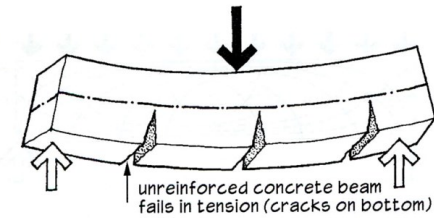


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Beam Stresses



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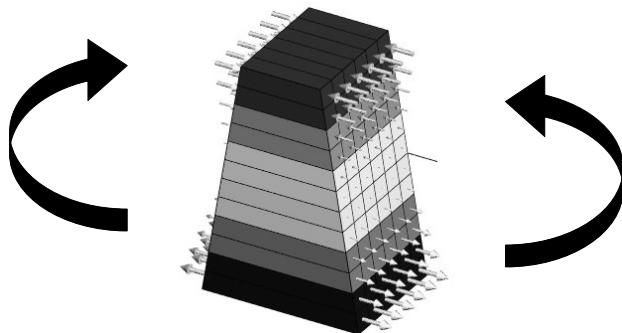
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Beam Stresses

- *tension and compression*

- *causes moments*



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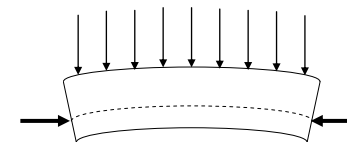
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Beam Stresses

- *prestress or post-tensioning*

- *put stresses in tension area to “pre-compress”*



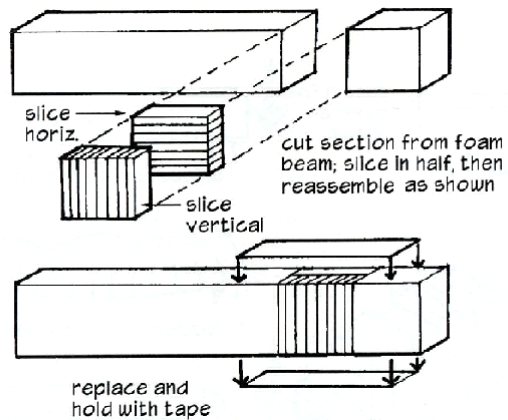
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Beam Stresses

- shear – horizontal & vertical



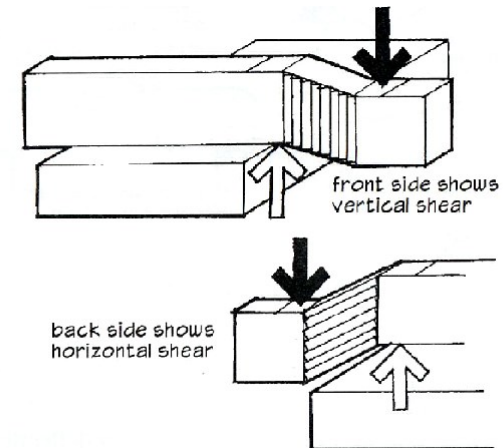
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Beam Stresses

- shear – horizontal & vertical



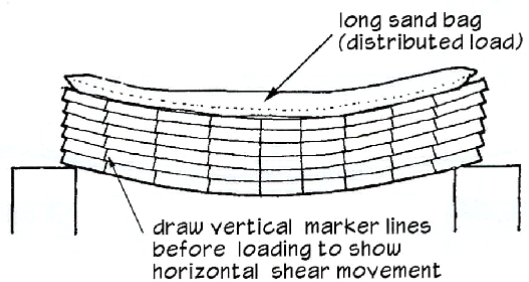
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Beam Stresses

- shear – horizontal



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Beam Deflections

- depends on
 - load
 - section
 - material

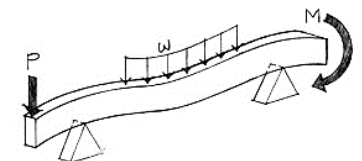
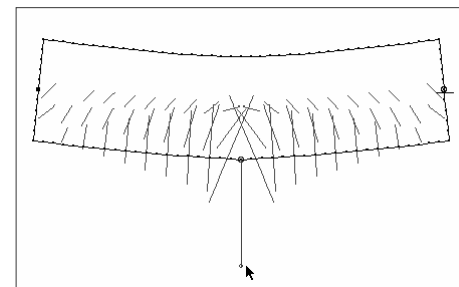


Figure 5.4 Bending (flexural) loads on a beam.



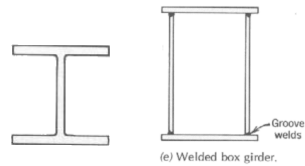
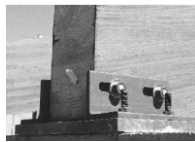
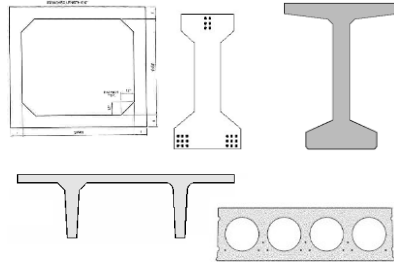
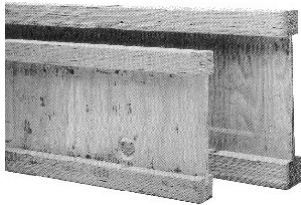
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Beam Deflections

- “moment of inertia”



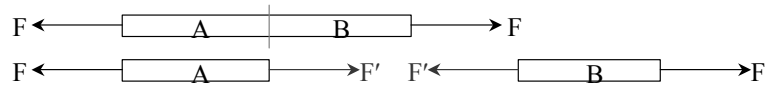
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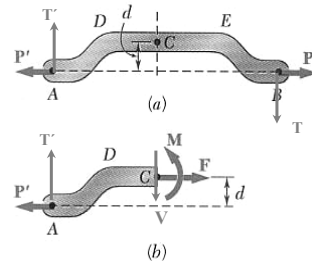
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Internal Forces

- *trusses*
 - axial only, (compression & tension)



- *in general*
 - axial force
 - shear force, V
 - bending moment, M



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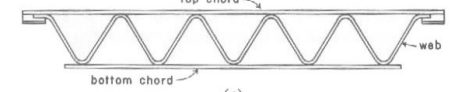
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Beam Styles

- *vierendeel*
- *open web joists*
- *manufactured*



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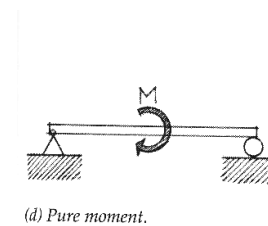
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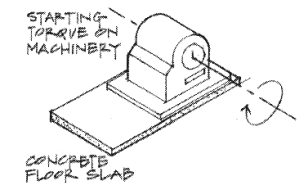
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Beam Loading

- *concentrated force*
- *concentrated moment*
 - *spandrel beams*



(d) Pure moment.



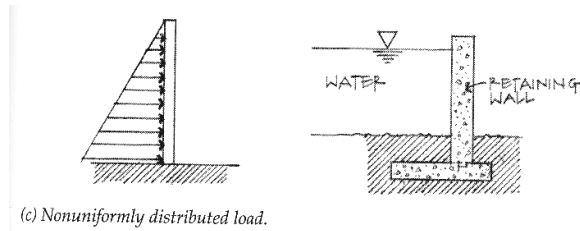
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Beam Loading

- *uniformly distributed load (line load)*
- *non-uniformly distributed load*
 - hydrostatic pressure = γh
 - wind loads



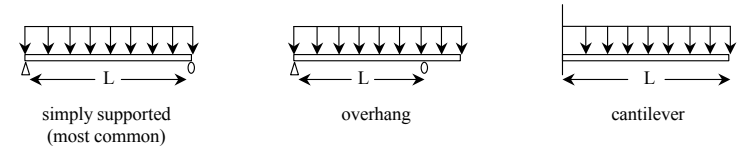
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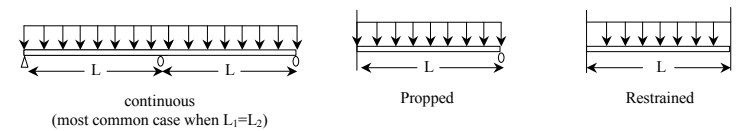
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Beam Supports

- *statically determinate*



- *statically indeterminate*



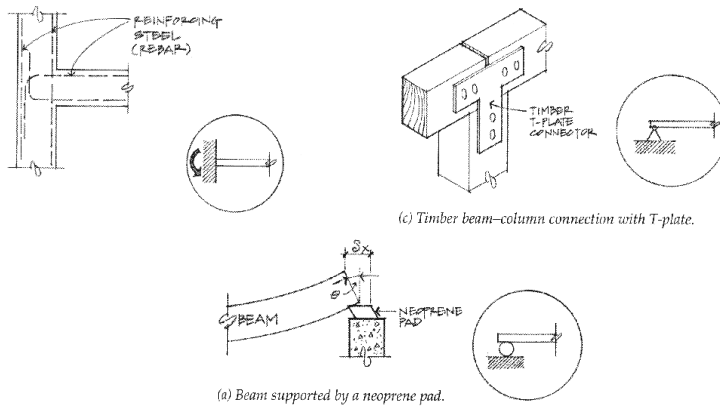
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Beam Supports

- *in the real world, modeled type*



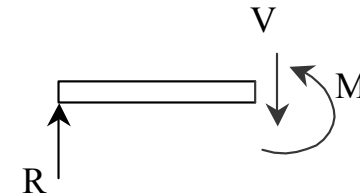
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Internal Forces in Beams

- *like method of sections / joints*
 - no axial forces
- *section must be in equilibrium*
- *want to know where biggest internal forces and moments are for designing*



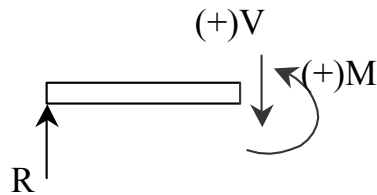
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V & M Diagrams

- tool to locate V_{max} and M_{max} (at $V = 0$)
- necessary for designing
- have a different sign convention than external forces, moments, and reactions



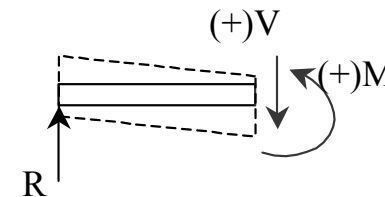
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Sign Convention

- shear force, V :
 - cut section to LEFT
 - if $\sum F_y$ is positive by statics, V acts down and is POSITIVE
 - beam has to resist shearing apart by V

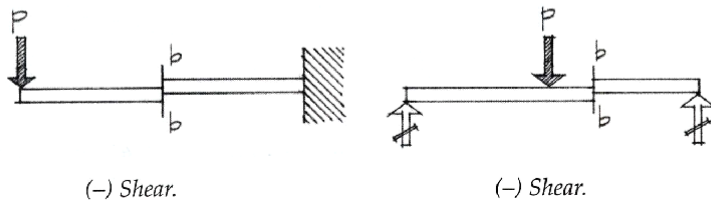
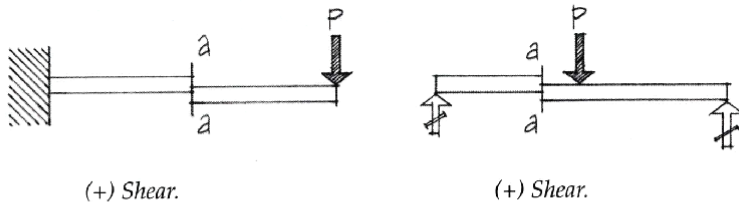


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Shear Sign Convention



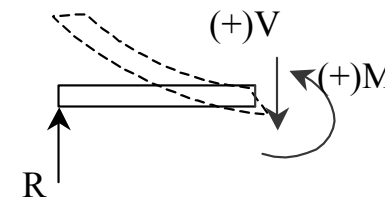
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Sign Convention

- bending moment, M :
 - cut section to LEFT
 - if $\sum M_{cut}$ is clockwise, M acts ccw and is POSITIVE – flexes into a “smiley” beam has to resist bending apart by M

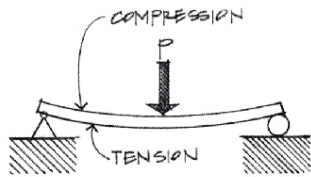


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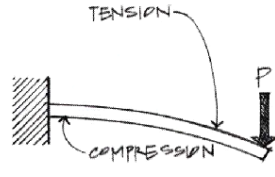
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Bending Moment Sign Convention



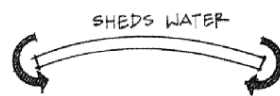
(+) Moment.



(-) Moment.



(+) Moment.



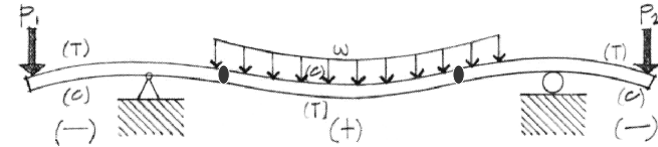
(-) Moment.

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Deflected Shape



- **positive bending moment**
 - tension in bottom, compression in top
- **negative bending moment**
 - tension in top, compression in bottom
- **zero bending moment**
 - inflection point

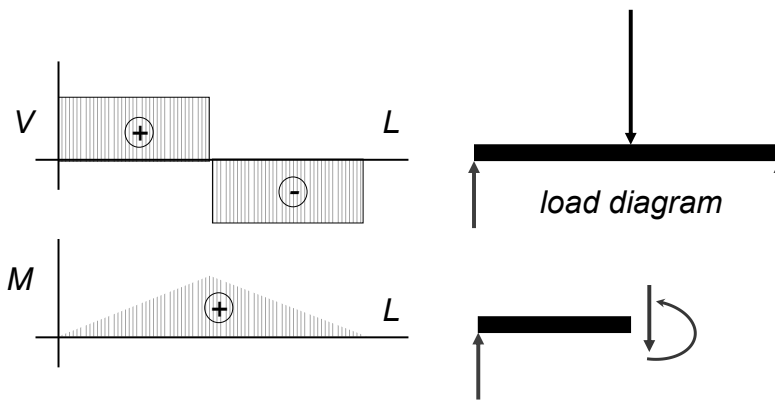
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Constructing V & M Diagrams

- along the beam length, plot V, plot M



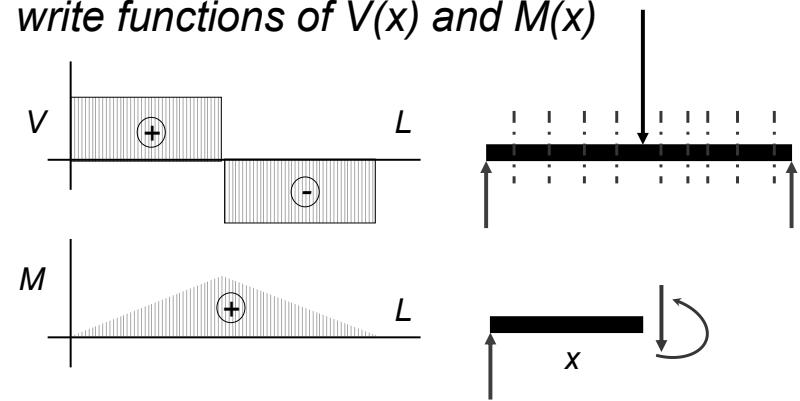
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Mathematical Method

- cut sections with x as width
- write functions of $V(x)$ and $M(x)$



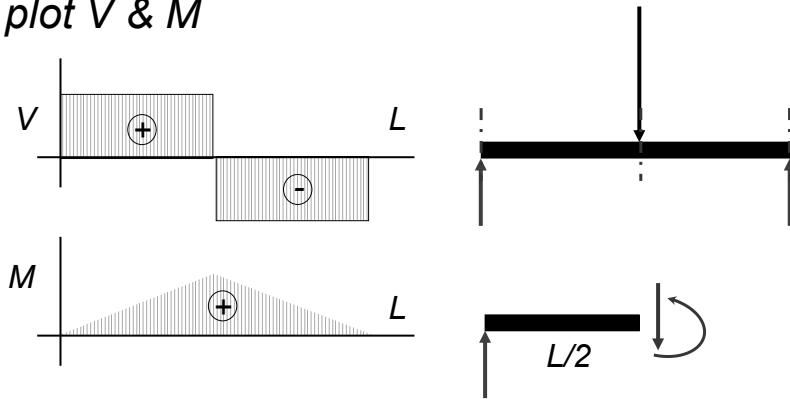
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Method 1: Equilibrium

- cut sections at important places
- plot V & M



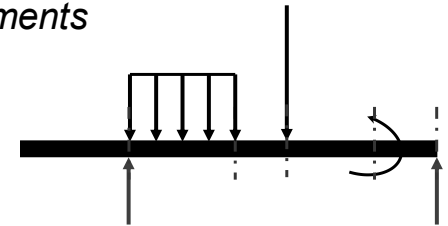
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Method 1: Equilibrium

- important places
 - supports
 - concentrated loads
 - start and end of distributed loads
 - concentrated moments
- free ends
 - zero forces



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Method 2: Semigraphical

- by knowing
 - area under loading curve = change in V
 - area under shear curve = change in M
 - concentrated forces cause “jump” in V
 - concentrated moments cause “jump” in M

$$V_D - V_C = - \int_{x_C}^{x_D} w dx \quad M_D - M_C = \int_{x_C}^{x_D} V dx$$

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Method 2

- relationships

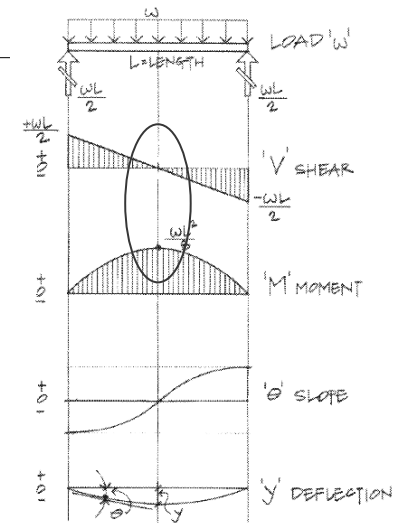


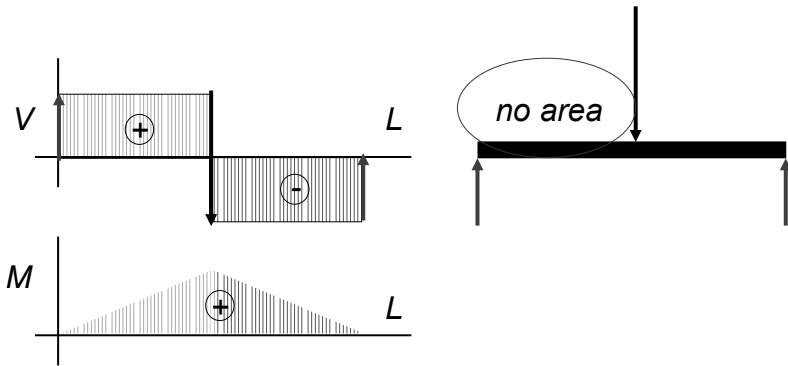
Figure 7.11 Relationship of load, shear, moment, slope, and deflection diagrams.

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Method 2: Semigraphical

- M_{max} occurs where $V = 0$ (calculus)



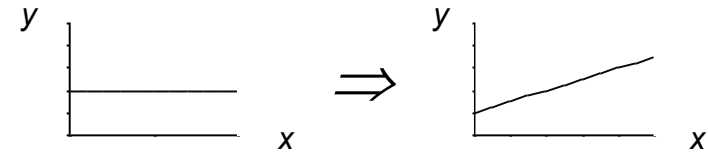
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Curve Relationships

- integration of functions
- line with 0 slope, integrates to sloped



- ex: load to shear, shear to moment

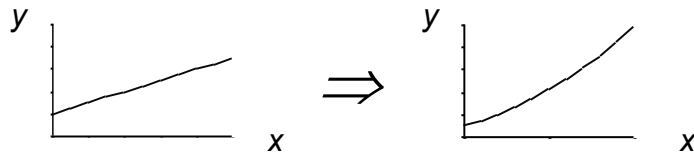
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Curve Relationships

- line with slope, integrates to parabola



- ex: load to shear, shear to moment

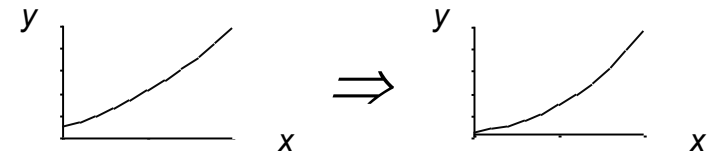
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Curve Relationships

- parabola, integrates to 3rd order curve



- ex: load to shear, shear to moment

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Basic Procedure with Sections

1. Find reaction forces & moments
Plot axes, underneath beam load diagram

V:

2. Starting at left
3. Shear is 0 at free ends
4. Shear has 2 values at point loads
5. Sum vertical forces at each section

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Basic Procedure with Sections

M:

6. Starting at left
7. Moment is 0 at free ends
8. Moment has 2 values at moments
9. Sum moments at each section
10. Maximum moment is where shear = 0!
(locate where $V = 0$)

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Basic Procedure by Curves

1. Find reaction forces & moments
Plot axes, underneath beam load diagram

V:

2. Starting at left
3. Shear is 0 at free ends
4. Shear jumps with concentrated load
5. Shear changes with area under load

V & M Diagrams 9
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Basic Procedure by Curves

M:

6. Starting at left
7. Moment is 0 at free ends
8. Moment jumps with moment
9. Moment changes with area under V
10. Maximum moment is where shear = 0!
(locate where $V = 0$)

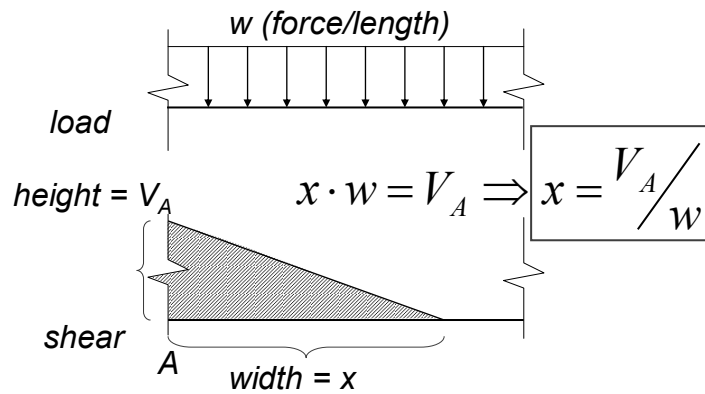
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Shear Through Zero

- slope of V is w ($-w:1$)



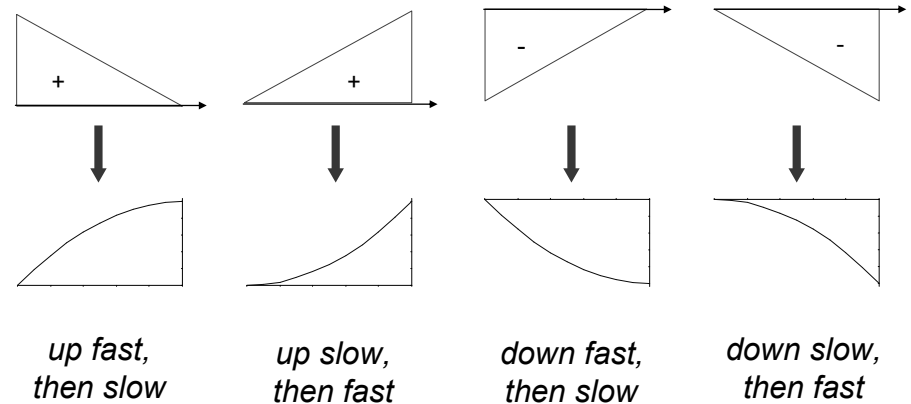
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Parabolic Shapes

- cases



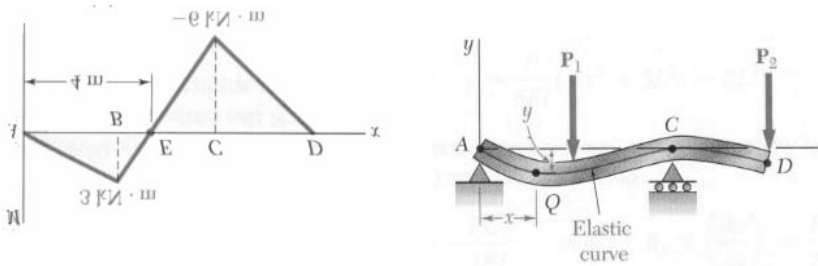
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Deflected Shape & $M(x)$

- $-M(x)$ gives shape indication
- boundary conditions must be met



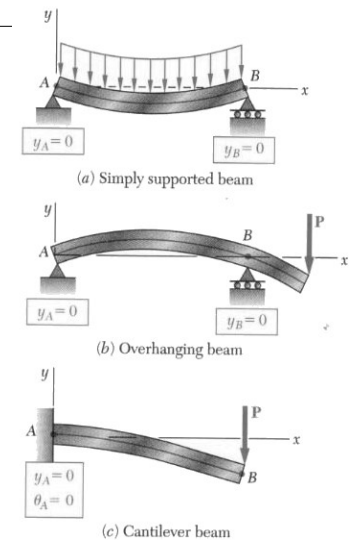
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Boundary Conditions

- at pins, rollers, fixed supports: $y = 0$
- at fixed supports: $\theta = 0$
- at inflection points from symmetry: $\theta = 0$
- y_{max} at $\frac{dy}{dx} = 0$



V & M Diagrams 14
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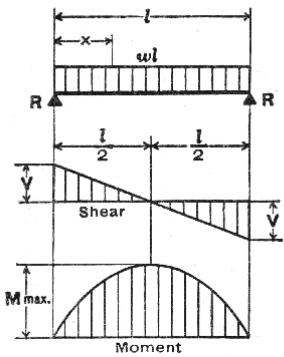
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Tabulated Beam Formulas

- how to read charts

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . = wl

$R = V$ = $\frac{wl}{2}$

V_x = $w\left(\frac{l}{2} - x\right)$

$M_{max.} \text{ (at center)}$ = $\frac{wl^2}{8}$

M_x = $\frac{wx}{2}(l-x)$

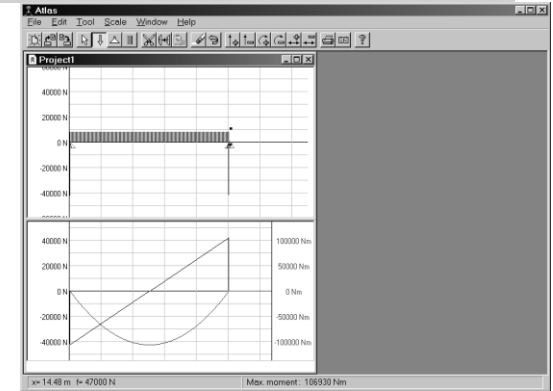
$\Delta_{max.} \text{ (at center)}$ = $\frac{5wl^4}{384EI}$

Δ_x = $\frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

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Tools

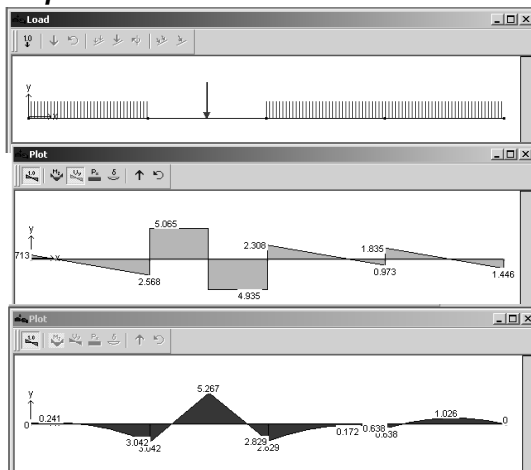
- software & spreadsheets help
- <http://www.rekenwonder.com/atlas.htm>



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Tools – Multiframe

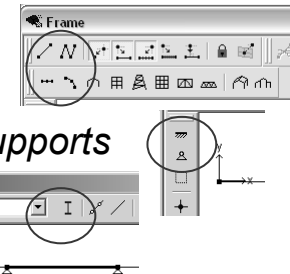
- in computer lab



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Tools – Multiframe

- frame window
 - define beam members
 - select points, assign supports
 - select members, assign section
- load window
 - select point or member, add point or distributed loads



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Tools – Multiframe

- to run analysis choose
 - Analyze menu
 - Linear
- plot
 - choose options
 - double click (all)
- results
 - choose options

