

mechanics of materials



www.cartalk.com

Mechanics of Materials

• MECHANICS

• MATERIALS



Mechanics of Materials 1
Lecture 5

Architectural Structures
ARCH 331

F2009abn

Mechanics of Materials 2
Lecture 5

Foundations Structures
ARCH 331

F2008abn

Mechanics of Materials

- external loads and their effect on deformable bodies
- use it to answer question if structure meets requirements of
 - stability and equilibrium
 - strength and stiffness
- other principle building requirements
 - economy, functionality and aesthetics

Knowledge Required

- material properties
- member cross sections
- ability of a material to resist breaking
- structural elements that resist excessive
 - deflection
 - deformation

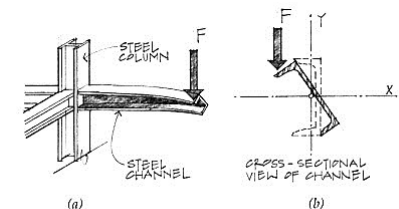


Figure 2.34 An example of torsion on a cantilever beam.

Mechanics of Materials 3
Lecture 5

Foundations Structures
ARCH 331

F2008abn

Mechanics of Materials 4
Lecture 5

Foundations Structures
ARCH 331

F2008abn

Problem Solving

1. STATICS:

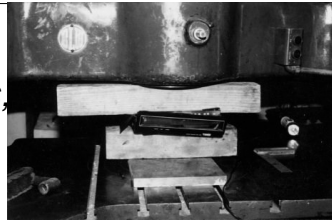
equilibrium of external forces,
internal forces, stresses

2. GEOMETRY:

cross section properties, deformations and
conditions of geometric fit, strains

3. MATERIAL PROPERTIES:

stress-strain relationship for each material
obtained from testing



Stress

- stress is a term for the intensity of a force, like a pressure
- internal or applied
- force per unit area

$$\text{stress} = f = \frac{P}{A}$$



Design

- materials have a critical stress value where they could break or yield
 - ultimate stress
 - yield stress
 - compressive stress
 - fatigue strength
 - (creep & temperature)

acceptance
vs. failure

Design (cont)

- we'd like

$$f_{\text{actual}} \ll F_{\text{allowable}}$$
- stress distribution may vary: average
- uniform distribution exists IF the member is loaded axially (concentric)

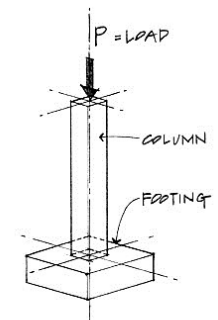


Figure 5.3 Centric loads.

Scale Effect

- **model scale**
 - material weights by volume, small section areas
- **structural scale**
 - much more material weight, bigger section areas
- **scale for strength is not proportional:** $\frac{\gamma L^3}{L^2} = \gamma L$



Normal Stress (direct)

- **normal stress is normal to the cross section**
 - stressed area is perpendicular to the load

$$f_{t \text{ or } c} = \frac{P}{A}$$

$$(\sigma)$$

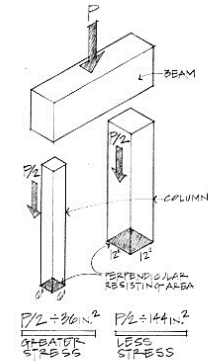


Figure 5.7 Two columns with the same load, different stress.

Shear Stress

- **stress parallel to a surface**

$$f_v = \frac{P}{A} = \frac{P}{td}$$

$$(\tau_{ave})$$

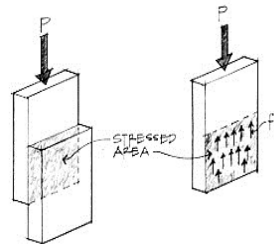


Figure 5.10 Shear stress between two glued blocks.

Bearing Stress

- **stress on a surface by contact in compression**

$$f_p = \frac{P}{A} = \frac{P}{td}$$

$$(\sigma)$$

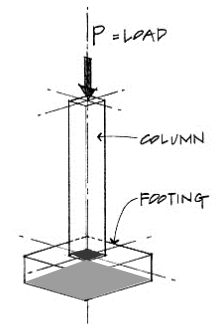


Figure 5.3 Centric loads.

Bending Stress

- normal stress caused by bending

$$f_b = \frac{Mc}{I} = \frac{M}{S}$$

(σ)

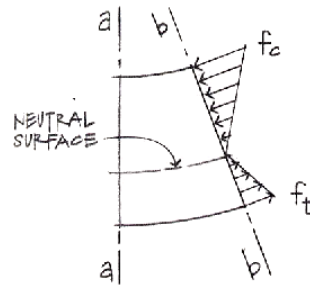


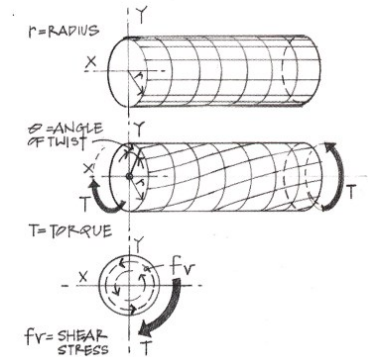
Figure 8.8 Bending stresses on section b-b.

Torsional Stress

- shear stress caused by twisting

$$f_v = \frac{T\rho}{J}$$

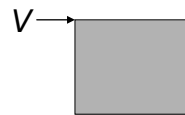
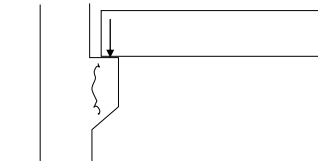
(τ)



Structures and Shear

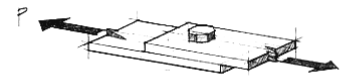
- what structural elements see shear?
 - beams
 - bolts
 - splices
 - slabs
 - footings
 - walls
 - wind
 - seismic loads

connections

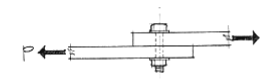


Bolts

- connected members in tension cause shear stress

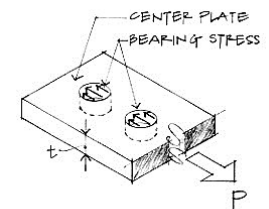


(a) Two steel plates bolted using one bolt.



(b) Elevation showing the bolt in

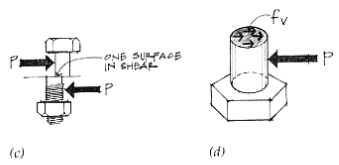
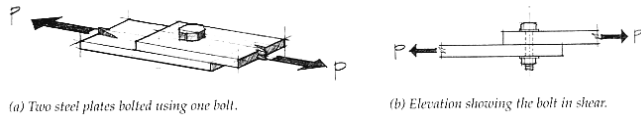
- connected members in compression cause bearing stress



Bearing stress on plate.

Single Shear

- seen when 2 members are connected



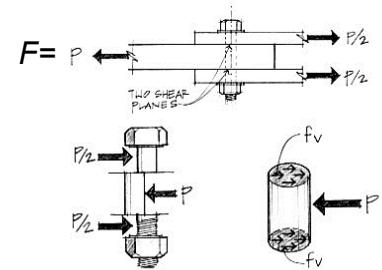
$f_v = \text{Average shear stress through bolt cross section}$
 $A = \text{Bolt cross-sectional area}$

$$f_v = \frac{P}{A} = \frac{P}{\pi d^2/4}$$

Figure 5.11 A bolted connection—single shear.

Double Shear

- seen when 3 members are connected
- two areas



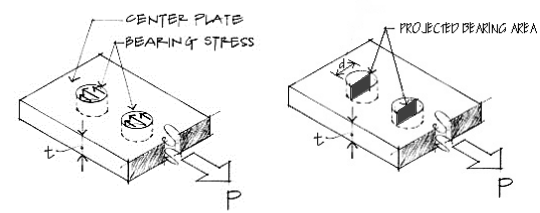
$f_v = \frac{P}{2A}$
 (two shear planes)

$$f_v = \frac{P}{2A} = \frac{P/2}{A} = \frac{P/2}{\pi d^2/4}$$

Free-body diagram of middle section of the bolt in shear. Figure 5.12 A bolted connection in double shear.

Bolt Bearing Stress

- compression & contact
- projected area

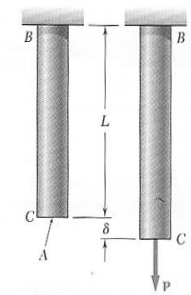
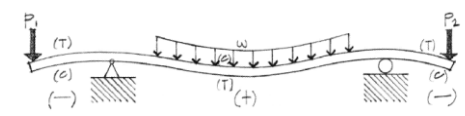


Bearing stress on plate.

$$f_p = \frac{P}{A_{projected}} = \frac{P}{td}$$

Strain

- materials deform
- axially loaded materials change length
- bending materials deflect

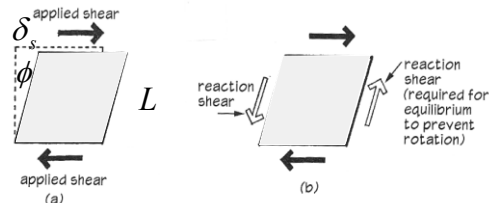


• STRAIN:
 – change in length over length + UNITLESS

$$\text{strain} = \epsilon = \frac{\Delta L}{L}$$

Shearing Strain

- deformations with shear
- parallelogram
- change in angles
- stress: τ
- strain: γ – unitless (radians)



$$\gamma = \frac{\delta_s}{L} = \tan \phi \cong \phi$$

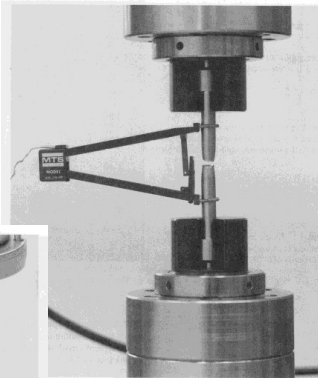
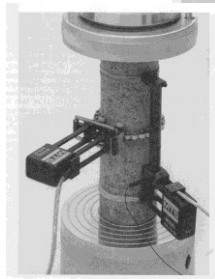
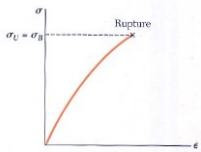
Mechanics of Materials 21
Lecture 5

Foundations Structures
ARCH 331

F2008abn

Load and Deformation

- for stress, need P & A
- for strain, need δ & L
 - how?
 - TEST with load and measure
 - plot P/A vs. ϵ



Mechanics of Materials 23
Lecture 5

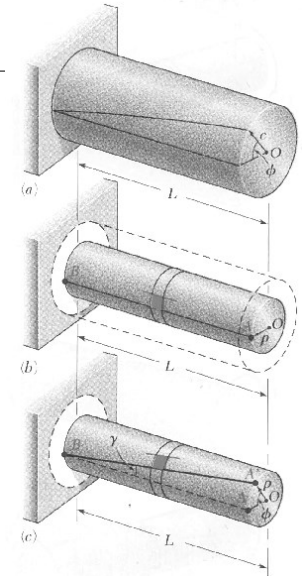
Foundations Structures
ARCH 331

F2008abn

Shearing Strain

- deformations with torsion
- twist
- change in angle of line
- stress: τ
- strain: γ – unitless (radians)

$$\gamma = \frac{\rho\phi}{L}$$



Mechanics of Materials 22
Lecture 5

Foundations Structures
ARCH 331

F2008abn

Material Behavior

- every material has its own response
 - 10,000 psi
 - $L = 10$ in
 - Douglas Fir vs. steel?

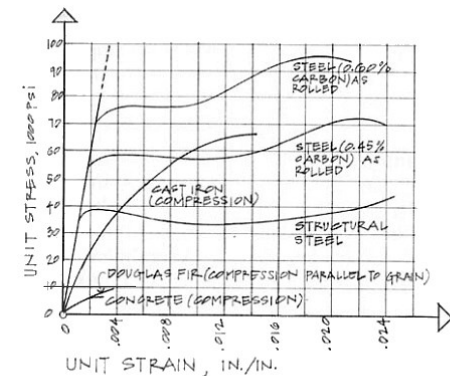


Figure 5.20 Stress-strain diagram for various materials.

Mechanics of Materials 24
Lecture 5

Foundations Structures
ARCH 331

F2008abn

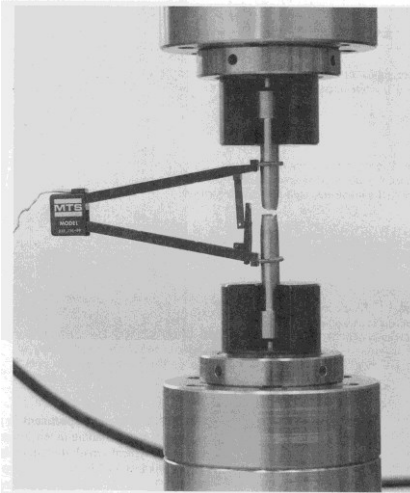
Behavior Types

- ductile - “necking”
- true stress

$$f = \frac{P}{A}$$

- engineering stress
– (simplified)

$$f = \frac{P}{A_0}$$



Behavior Types

- brittle

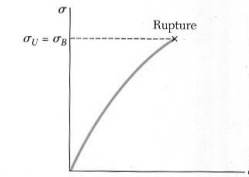


Fig. 2.11 Stress-strain diagram for a typical brittle material.

- semi-brittle

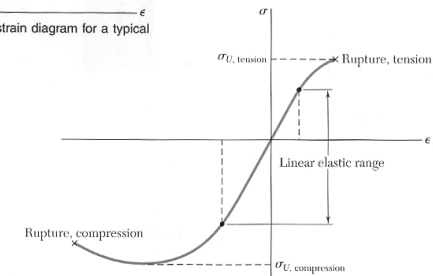


Fig. 2.14 Stress-strain diagram for concrete.

Stress to Strain

- important to us in f - ϵ diagrams:
 - straight section
 - LINEAR-ELASTIC
 - recovers shape (no permanent deformation)

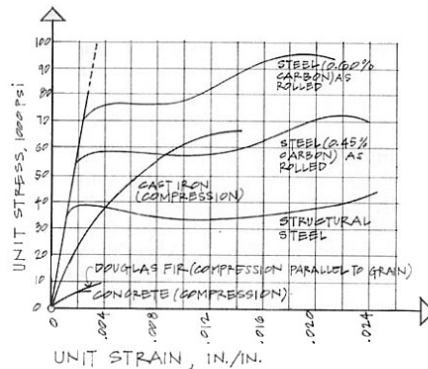
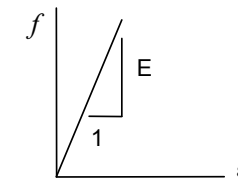


Figure 5.20 Stress-strain diagram for various materials.

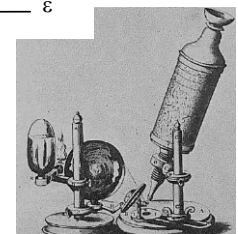
Hooke's Law

- straight line has constant slope
- Hooke's Law

$$f = E \cdot \epsilon$$



- E
 - Modulus of elasticity
 - Young's modulus
 - units just like stress



Stiffness

- ability to resist strain
- steels
 - same E
 - different yield points
 - different ultimate strength

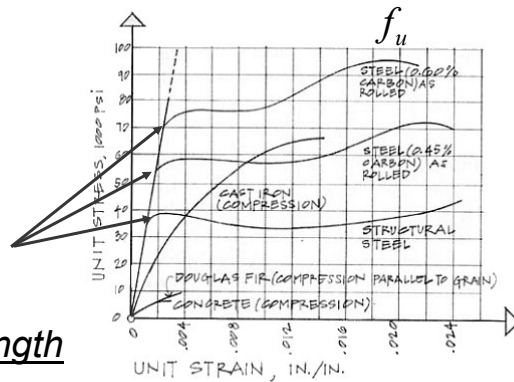
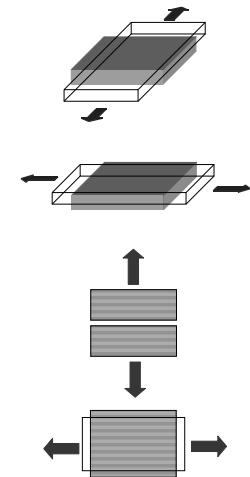


Figure 5.20 Stress-strain diagram for various materials.

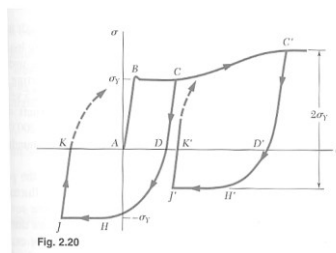
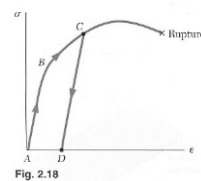
Isotropy & Anisotropy

- ISOTROPIC
 - materials with E same at any direction of loading
 - ex. steel
- ANISOTROPIC
 - materials with different E at any direction of loading
 - ex. wood is orthotropic



Elastic, Plastic, Fatigue

- elastic springs back
- plastic has permanent deformation
- fatigue caused by reversed loading cycles



Plastic Behavior

- ductile

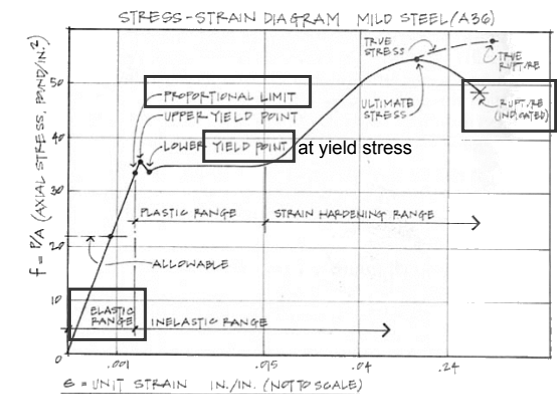


Figure 5.22 Stress-strain diagram for mild steel (A36) with key points highlighted.

Lateral Strain

- or “what happens to the cross section with axial stress”

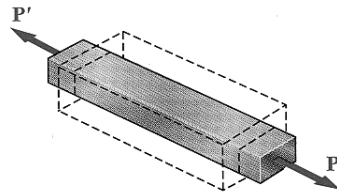
$$\varepsilon_x = \frac{f_x}{E}$$

$$f_y = f_z = 0$$

- strain in lateral direction

– negative

– equal for isometric materials



$$\varepsilon_y = \varepsilon_z$$

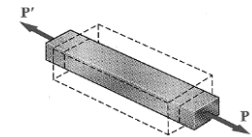
Poisson's Ratio

- constant relationship between longitudinal strain and lateral strain

$$\mu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

$$\varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E}$$

- sign! $0 < \mu < 0.5$



Calculating Strain

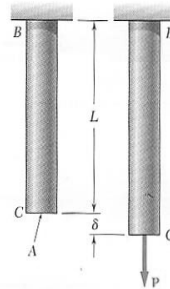
- from Hooke's law

$$f = E \cdot \varepsilon$$

- substitute

$$\frac{P}{A} = E \cdot \frac{\delta}{L}$$

- get \Rightarrow
$$\delta = \frac{PL}{AE}$$

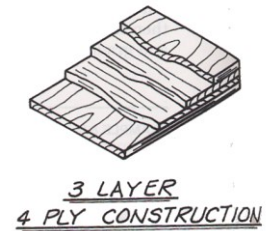
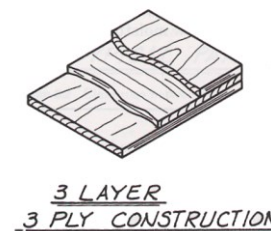
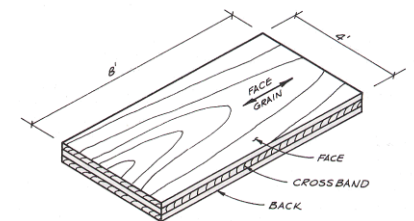


Orthotropic Materials

- non-isometric
- directional values of E and μ

- ex:

- plywood
- laminates
- polymer composites



Stress Concentrations

- why we use f_{ave}
- increase in stress at changes in geometry
 - sharp notches
 - holes
 - corners

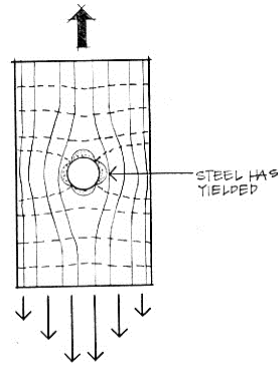
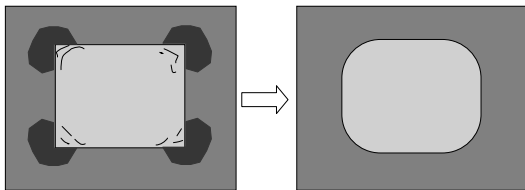


Figure 5.35 Stress trajectories around a hole.

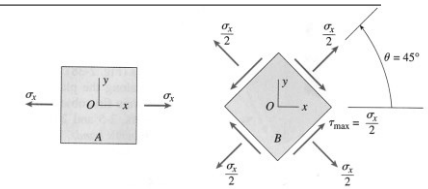
Mechanics of Materials 37
Lecture 5

Foundations Structures
ARCH 331

F2008abn

Maximum Stresses

- if we need to know where max f and f_v happen:



$$\theta = 0^\circ \rightarrow \cos \theta = 1 \quad f_{\max} = \frac{P}{A_0}$$

$$\theta = 45^\circ \rightarrow \cos \theta = \sin \theta = \sqrt{0.5}$$

$$f_{v-\max} = \frac{P}{2A_0} = \frac{f_{\max}}{2}$$

Mechanics of Materials 38
Lecture 5

Foundations Structures
ARCH 331

F2008abn

Maximum Stresses

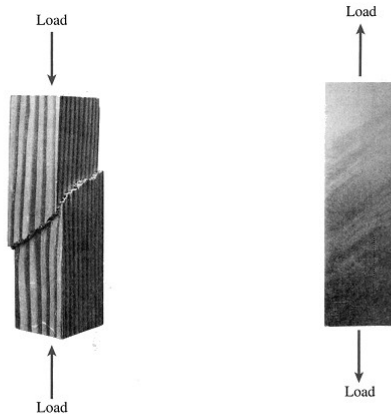


FIG. 2-37 Shear failure along a 45° plane of a wood block loaded in compression

FIG. 2-38 Slip bands (or Lüders' bands) in a polished steel specimen loaded in tension

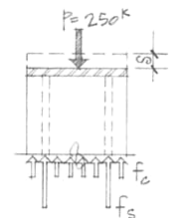
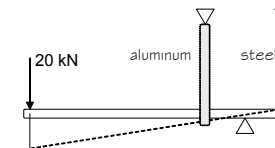
Mechanics of Materials 39
Lecture 5

Foundations Structures
ARCH 331

F2008abn

Deformation Relationships

- physical movement
 - axially (same or zero)
 - rotations from axial changes



$$\delta = \frac{PL}{AE} \quad \text{relates } \delta \text{ to } P$$

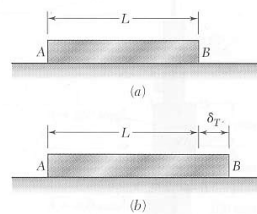
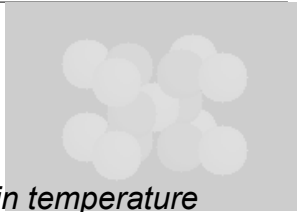
Mechanics of Materials 40
Lecture 5

Foundations Structures
ARCH 331

F2008abn

Deformations from Temperature

- atomic chemistry reacts to changes in energy
- solid materials
 - can contract with decrease in temperature
 - can expand with increase in temperature
- linear change can be measured per degree



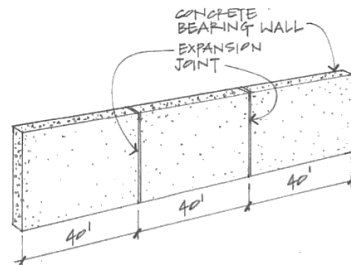
Mechanics of Materials 41
Lecture 5

Foundations Structures
ARCH 331

F2008abn

Coefficients of Thermal Expansion

Material	Coefficients (α) [in./in./°F]
Wood	3.0×10^{-6}
Glass	4.4×10^{-6}
Concrete	5.5×10^{-6}
Cast Iron	5.9×10^{-6}
Steel	6.5×10^{-6}
Wrought Iron	6.7×10^{-6}
Copper	9.3×10^{-6}
Bronze	10.1×10^{-6}
Brass	10.4×10^{-6}
Aluminum	12.8×10^{-6}



Mechanics of Materials 43
Lecture 5

Foundations Structures
ARCH 331

F2008abn

Thermal Deformation

- α - the rate of strain per degree
- UNITS : $/^{\circ}F$, $/^{\circ}C$
- length change: $\delta_T = \alpha(\Delta T)L$
- thermal strain: $\epsilon_T = \alpha(\Delta T)$
- no stress when movement allowed

Mechanics of Materials 42
Lecture 5

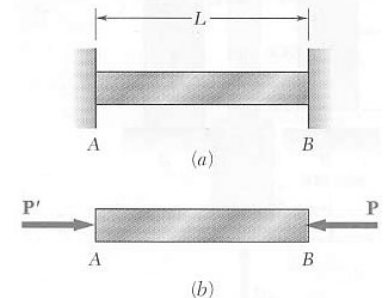
Foundations Structures
ARCH 331

F2008abn

Stresses and Thermal Strains

- if thermal movement is restrained stresses are induced

1. bar pushes on supports
2. support pushes back
3. reaction causes internal stress $f = \frac{P}{A} = \frac{\delta}{L} E$



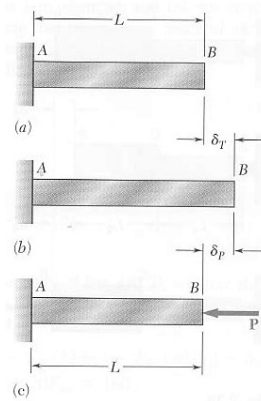
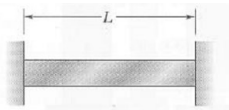
Mechanics of Materials 44
Lecture 5

Foundations Structures
ARCH 331

F2008abn

Superposition Method

- can remove a support to make it look determinant
- replace the support with a reaction
- enforce the geometry constraint



Mechanics of Materials 45
Lecture 5

Foundations Structures
ARCH 331

F2008abn

Superposition Method

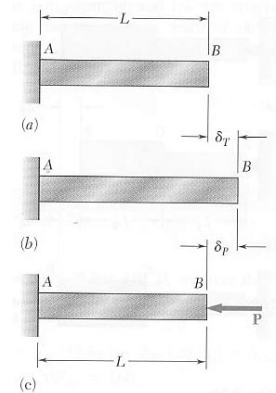
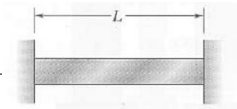
- total length change restrained to zero

$$\text{constraint: } \delta_P + \delta_T = 0$$

$$\delta_P = -\frac{PL}{AE} \quad \delta_T = \alpha(\Delta T)L$$

$$\text{sub: } -\frac{PL}{AE} + \alpha(\Delta T)L = 0$$

$$f = -\frac{P}{A} = -\alpha(\Delta T)E$$



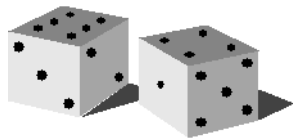
Mechanics of Materials 46
Lecture 5

Foundations Structures
ARCH 331

Design of Members

- beyond allowable stress...
- materials aren't uniform 100% of the time
 - ultimate strength or capacity to failure may be different and some strengths hard to test for

RISK & UNCERTAINTY



$$f_u = \frac{P_u}{A}$$

Mechanics of Materials 47
Lecture 5

Foundations Structures
ARCH 331

F2008abn

Factor of Safety

- accommodate uncertainty with a safety factor:

$$\text{allowable load} = \frac{\text{ultimate load}}{F.S}$$

- with linear relation between load and stress:

$$F.S = \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

Mechanics of Materials 48
Lecture 5

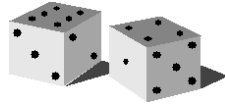
Foundations Structures
ARCH 331

F2008abn

Load and Resistance Factor Design

- loads on structures are

- not constant
- can be more influential on failure
- happen more or less often
- **UNCERTAINTY**



$$R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n$$

ϕ - resistance factor

γ - load factor for (D)ead & (L)ive load