ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN

ARCH 331

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lecture
twenty one

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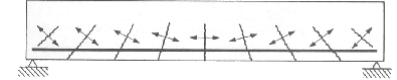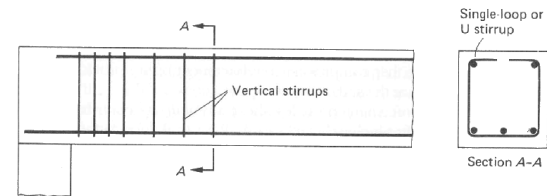
concrete construction:
shear & deflection

Shear in Concrete Beams

- flexure combines with shear to form diagonal cracks

- horizontal reinforcement doesn't help
- stirrups = vertical reinforcement

ACI Shear Values

- V_u is at distance d from face of support
- shear capacity:

$$V_c = \upsilon_c \times b_w d$$

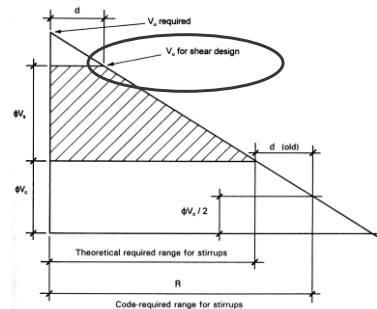 – where b_w means thickness of web at n.a.

Figure 13.16   Layout for shear stress analysis: ACI Code requirements.

ACI Shear Values

- shear stress (beams)
  – $\upsilon_c = 2\sqrt{f'_c}$ $\phi = 0.75$ for shear

  $\phi V_c = \phi 2\sqrt{f'_c}\, b_w d$ f'_c is in psi

- shear strength:

  $$V_u \leq \phi V_c + \phi V_s$$
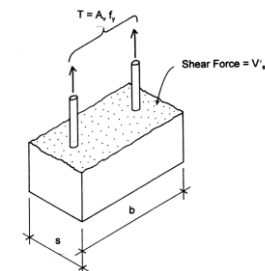
  – V_s is strength from stirrup reinforcement

Figure 13.17   Consideration for spacing of a single stirrup.
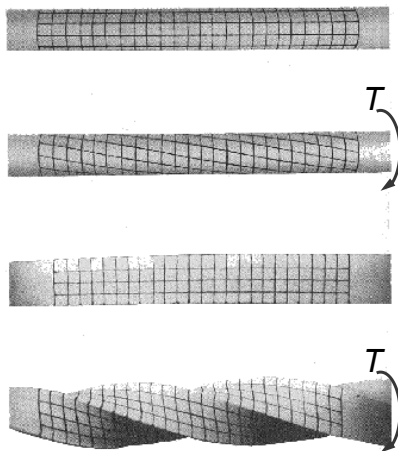
Stirrup Reinforcement

- shear capacity:

$$V_s = \frac{A_v f_y d}{s}$$

  - A_v = area in all legs of stirrups
  - s = spacing of stirrup

- may need stirrups when concrete has enough strength!

Required Stirrup Reinforcement

- spacing limits

Table 3-8 ACI Provisions for Shear Design*

		$V_u \leq \frac{\phi V_c}{2}$	$\phi V_c \geq V_u > \frac{\phi V_c}{2}$	$V_u > \phi V_c$
Required area of stirrups, A_v**		none	$\frac{50 b_w s}{f_y}$	$\frac{(V_u - \phi V_c)s}{\phi f_y d}$
Stirrup spacing, s	Required	—	$\frac{A_v f_y}{50 b_w}$	$\frac{\phi A_v f_y d}{V_u - \phi V_c}$
	Recommended Minimum†	—	—	4 in.
	Maximum†† (ACI 11.5.4)	—	$\frac{d}{2}$ or 24 in.	$\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \leq \phi 4\sqrt{f_c'}\, b_w d$
				$\frac{d}{4}$ or 12 in. for $(V_u - \phi V_c) > \phi 4\sqrt{f_c'}\, b_w d$

*Members subjected to shear and flexure only; $\phi V_c = \phi 2\sqrt{f_c'}\, b_w d$, $\phi = 0.85$ (ACI 11.3.1.1)
      0.75
**$A_v = 2 \times A_b$ for U stirrups; $f_y \leq 60$ ksi (ACI 11.5.2)
†A practical limit for minimum spacing is d/4
††Maximum spacing based on minimum shear reinforcement (= $A_v f_y / 50 b_w$) must also be considered (ACI 11.5.5.3).

Torsional Stress & Strain

- can see torsional stresses & twisting of axi-symmetrical cross sections
  - torque
  - remain plane
  - undistorted
  - rotates
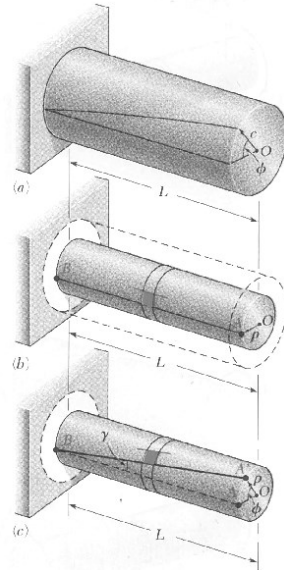- not true for square sections....

Shear Stress Distribution

- depend on the deformation
- ϕ = angle of twist
  - measure
- can prove planar section doesn't distort
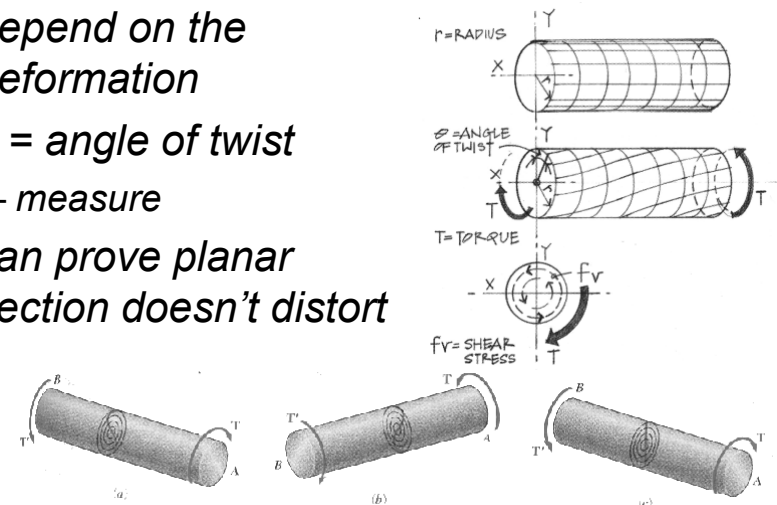
Shearing Strain

- related to ϕ

$$\gamma = \frac{\rho\phi}{L}$$

- ρ is the radial distance from the centroid to the point under strain

- shear strain varies linearly along the radius: γ_{max} is at outer diameter

Torsional Stress - Strain

- know $f_v = \tau = G \cdot \gamma$ and $\gamma = \frac{\rho\phi}{L}$

- so $\tau = G \cdot \frac{\rho\phi}{L}$
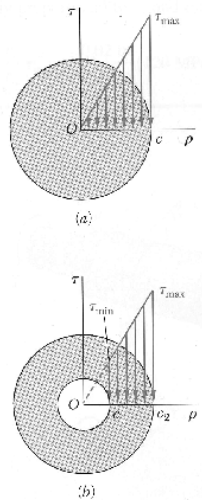
- where G is the <u>Shear Modulus</u>

Torsional Stress - Strain

- from $T = \Sigma\tau(\rho)\Delta A$

- can derive $T = \frac{\tau J}{\rho}$

  – where J is the polar moment of inertia
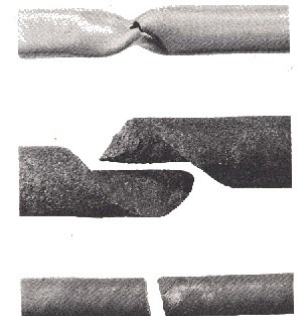 – elastic range $\tau = \frac{T\rho}{J}$

Shear Stress

- τ_{max} happens at <u>outer diameter</u>

- combined shear and axial stresses

  – maximum shear stress at 45° "twisted" plane

Shear Strain

- knowing $\tau = G \cdot \dfrac{\rho\phi}{L}$ and $\tau = \dfrac{T\rho}{J}$

- solve: $\phi = \dfrac{TL}{JG}$

- composite shafts: $\phi = \sum_i \dfrac{T_i L_i}{J_i G_i}$

Noncircular Shapes

- torsion depends on J
- plane sections don't remain plane
- τ_{max} is still at outer diameter

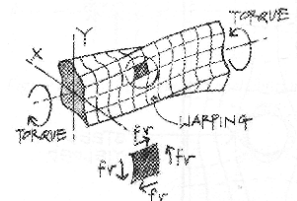$$\tau_{max} = \dfrac{T}{c_1 ab^2} \quad \phi = \dfrac{TL}{c_2 ab^3 G}$$

  – where a is longer side (> b)

TABLE 3.1. Coefficients for Rectangular Bars in Torsion

a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

Open Thin-Walled Sections

- with very large a/b ratios:
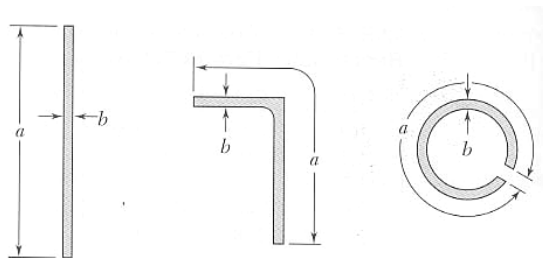
$$\tau_{max} = \dfrac{T}{\frac{1}{3} ab^2} \qquad \phi = \dfrac{TL}{\frac{1}{3} ab^3 G}$$
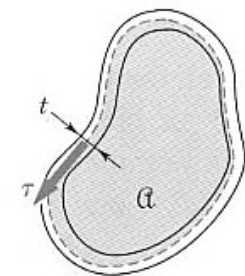
Shear Flow in Closed Sections

- q is the internal shear force/unit length

$$\tau = \dfrac{T}{2t\,a}$$

$$\phi = \dfrac{TL}{4t\,a^2} \sum_i \dfrac{s_i}{t_i}$$

- a is the area bounded by the centerline
- s_i is the length segment, t_i is the thickness
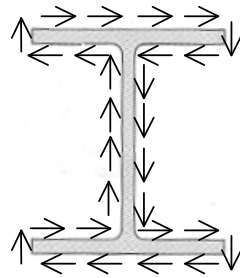
Shear Flow in Open Sections

- each segment has proportion of T with respect to torsional rigidity,

$$\tau_{max} = \frac{Tt_{max}}{\frac{1}{3}\Sigma b_i t_i^3}$$

- total angle of twist:

$$\phi = \frac{TL}{\frac{1}{3}G\Sigma b_i t_i^3}$$

- I beams - web is thicker, so τ_{max} is in _web_

Torsional Shear Stress

- twisting moment
- _and_ beam shear
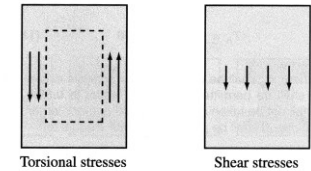
Fig. R11.6.3.1—Addition of torsional and shear stresses

Torsional Shear Reinforcement

- closed stirrups
- more longitudinal reinforcement

Fig. R11.6.3.6(a)—Space truss analogy

- area enclosed by shear flow

A_{oh} = shaded area

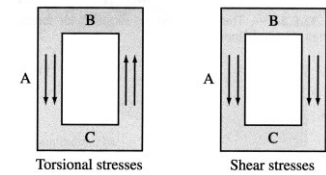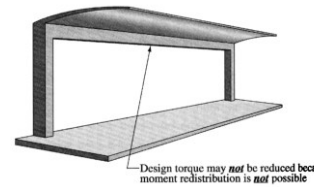Fig. R11.6.3.6(b)—Definition of A_{oh}

Development Lengths

- required to allow steel to yield (f_y)
- standard hooks
  - moment at beam end
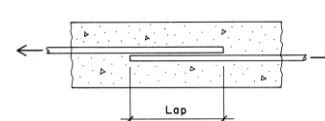
Figure 13.24 The lapped splice for steel reinforcing bars.

- splices
  - lapped
  - mechanical connectors
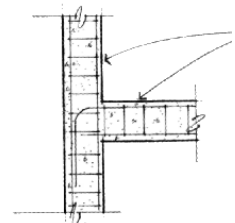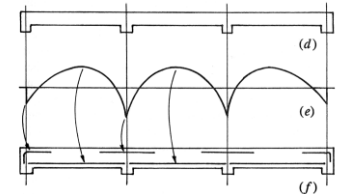
Development Lengths

- ℓ_d , embedment required <u>both</u> sides

- proper cover, spacing:
  - No. 6 or smaller

$$l_d = \frac{d_b F_y}{25\sqrt{f_c'}} \quad \text{or 12 in. minimum}$$

  - No. 7 or larger

$$l_d = \frac{d_b F_y}{20\sqrt{f_c'}} \quad \text{or 12 in. minimum}$$

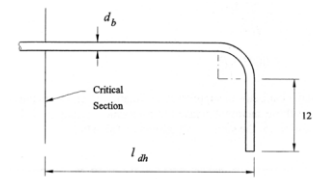## Development Lengths

- hooks
  - bend and extension

Figure 9-17: Minimum requirements for 90°bar hooks.   Figure 9-18: Minimum requirements for 180° bar hooks.
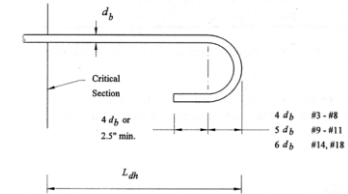
- minimum

$$l_{dh} = \frac{1200\, d_b}{\sqrt{f_c'}}$$

Development Lengths

- bars in compression

$$l_d = \frac{0.02\, d_b F_y}{\sqrt{f_c'}} \le 0.0003\, d_b F_y$$

- splices
  - tension minimum is function of l_d and splice classification
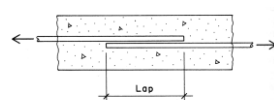 - compression minimum
  - is function of d_b and F_y

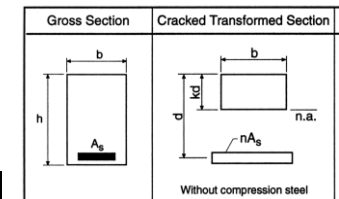Figure 13.24 The lapped splice for steel reinforcing bars.

Concrete Deflections

- elastic range
  - I transformed
  - E_c (with f'_c in <u>psi</u>)
    - normal weight concrete (~ 145 lb/ft3)

    - concrete between 90 and 160 lb/ft3

- cracked
  - I cracked
  - E adjusted

Deflection Limits

- relate to whether or not beam supports or is attached to a damageable non-structural element

- need to check <u>service</u> live load and long term deflection against these

L/180	roof systems (typical) – live
L/240	floor systems (typical) – live + long term
L/360	supporting plaster – live
L/480	supporting masonry – live + long term