ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN

ARCH 331

DR. ANNE NICHOLS

SUMMER 2014

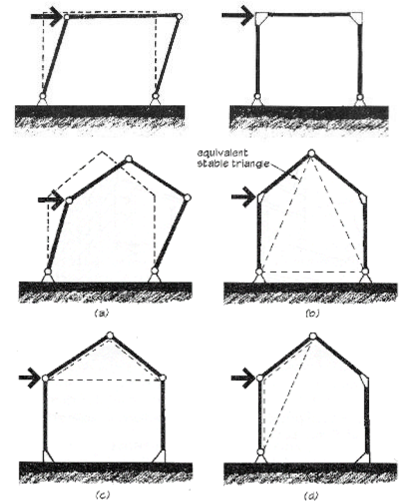
lecture
ten

rigid frames: compression & buckling

http:// nisee.berkeley.edu/godden

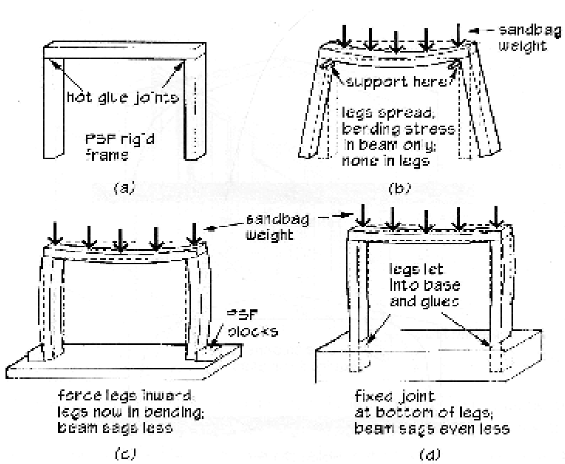
Rigid Frames

- *rigid frames have no pins*
- *frame is all one body*
- *joints transfer moments and shear*
- *typically statically indeterminate*
- *types*
  - *portal*
  - *gable*



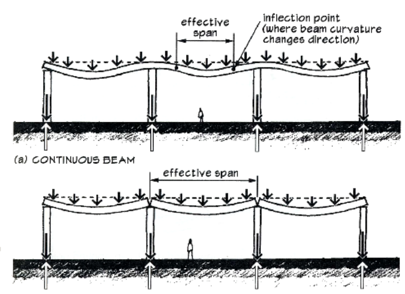
Rigid Frames

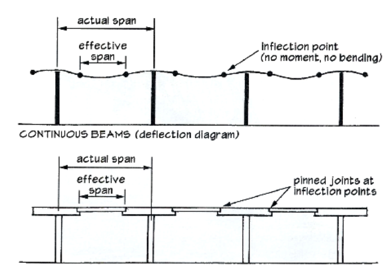
- *behavior*



Rigid Frames

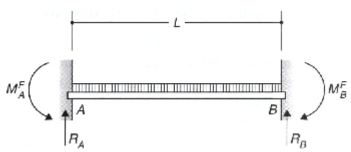
- *moments get redistributed*
- *deflections are smaller*
- *effective column lengths are shorter*
- *very sensitive to settling*

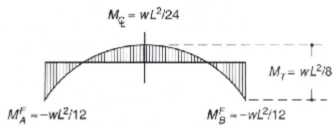




Moment Redistribution

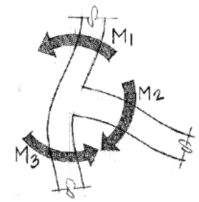
- continuous slabs & beams with uniform loading
  - joints similar to fixed ends, but can rotate
- change in moment to center = $\frac{wL^2}{8}$
  - M_{max} for simply supported beam





Rigid Frames

- resists lateral loadings
- shape depends on stiffness of beams and columns
- 90° maintained



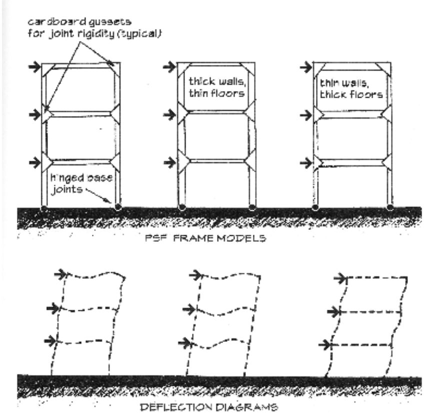
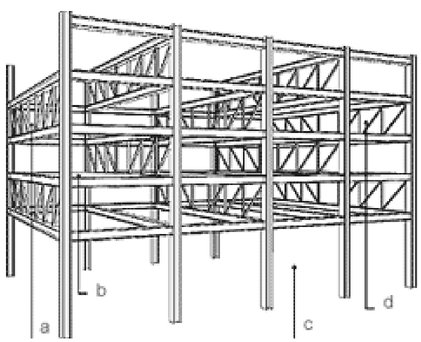


Figure 9.19: Model demonstration of the effects of varying the stiffness of beams and columns when a building frame is subjected to lateral loads.

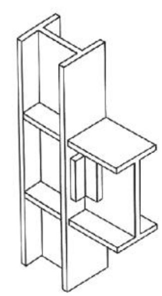
Rigid Frames

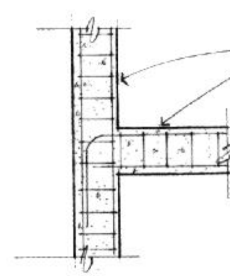
- staggered truss
  - rigidity
  - clear stories



Rigid Frames

- connections
  - steel
  - concrete





Fixed

http:// nisee.berkeley.edu/godden

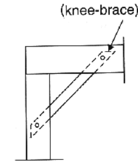
Braced Frames

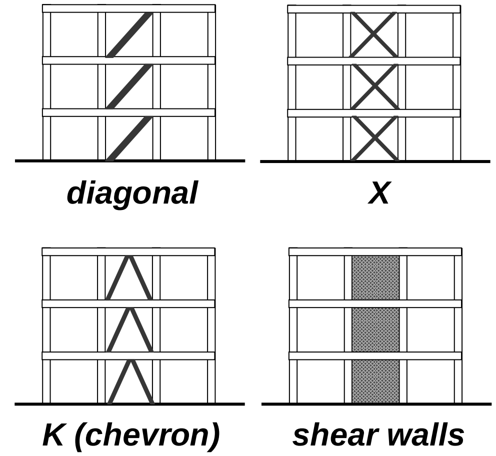
- *pin connections*
- *bracing to prevent lateral movements*

http:// nisee.berkeley.edu/godden

Braced Frames

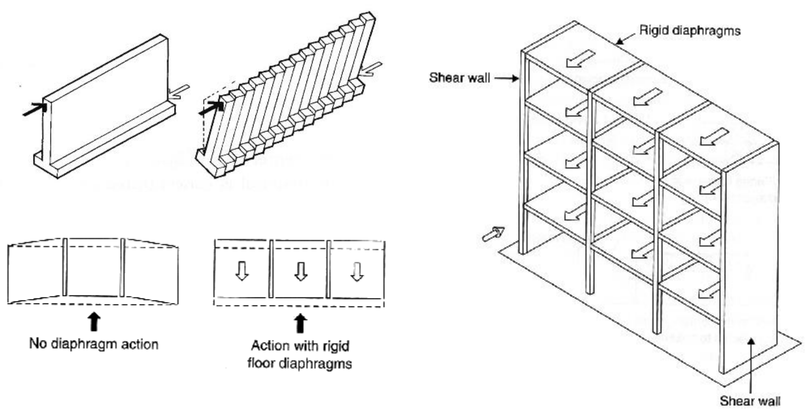
- *types of bracing*
  - *knee-bracing*
  - *diagonal*
  - *X*
  - *K or chevron*
  - *shear walls*





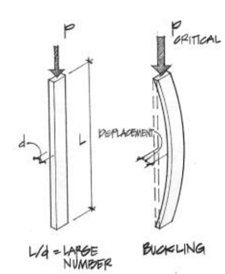
Shear Walls

- *resist lateral load in plane with wall*



Compression Members

- *designed for strength & stresses*
- *designed for serviceability & deflection*
- *need to design for <u>stability</u>*
  - *ability to support a specified load without sudden or unacceptable deformations*



Column Buckling

- *axially loaded columns*
- *long & slender*
  - *unstable equilibrium = buckling*
  - *sudden and not good*

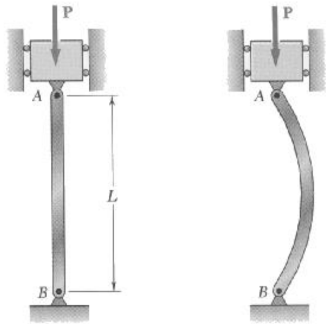
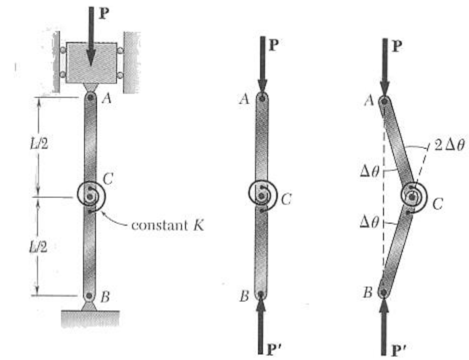


Fig. 10.1 Fig. 10.2

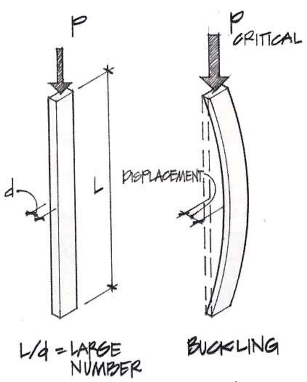
Modeling

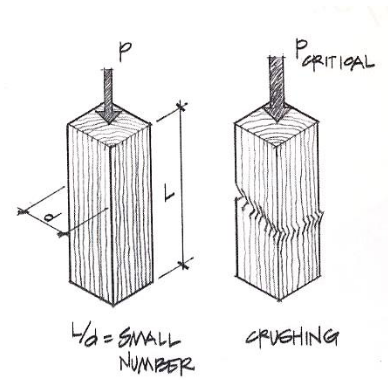
- *can be modeled with a spring at mid-height*
- *when moment from deflection exceeds the spring capacity ... "boing"*
- *critical load P*



Effect of Length

- *long & slender*
- *short & stubby*





Buckling Load

- *related to deflected shape ($P\Delta$)*
- *shape of sine wave*
- *Euler's Formula*
- *smallest I governs*

$$P_{critical} = \frac{\pi^2 EI}{(L)^2}$$

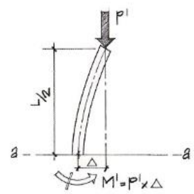


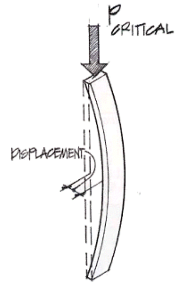
Figure 9.3 Leonhard Euler (1707–1783).

Critical Stress

- short columns

$$f_{critical} = \frac{P_{actual}}{A} < F_a$$

- slenderness ratio = L_e/r (L/d)
- radius of gyration = $r = \sqrt{\frac{I}{A}}$



weak axis

$$f_{critical} = \frac{P_{critical}}{A} = \frac{\pi^2 EAr^2}{A(L_e)^2} = \frac{\pi^2 E}{\left(\frac{L_e}{r}\right)^2} \qquad P_{critical} = \frac{\pi^2 EA}{\left(\frac{L_e}{r}\right)^2}$$

Critical Stresses

- when a column gets stubby, F_y will limit the load
- real world has loads with eccentricity
- C_c for steel and allowable stress

$$\frac{L_e}{r} > C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$

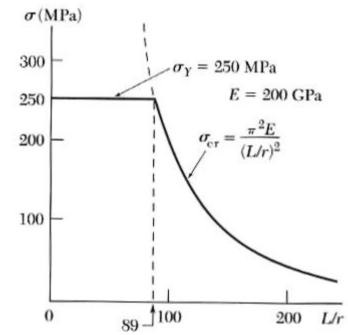
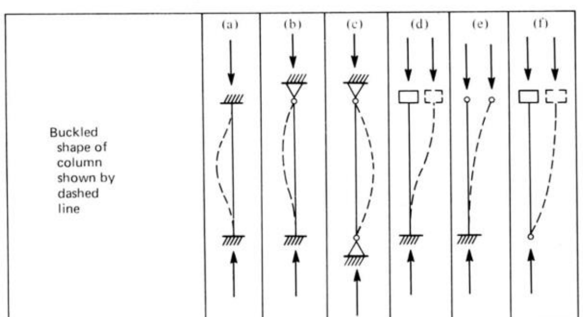


Fig. 10.9

Effective Length

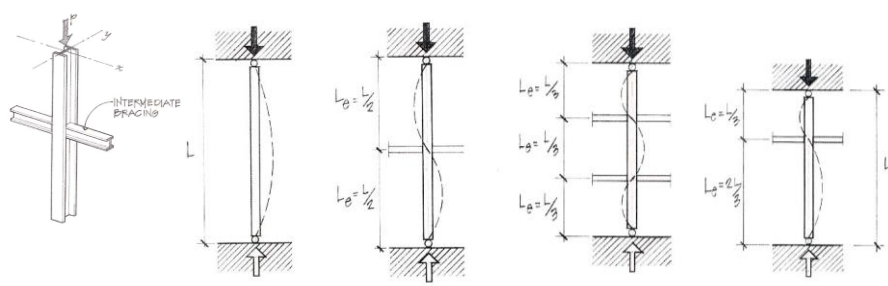
- end conditions affect shape
- effective length factor, K $\quad L_e = K \cdot L$



	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design values when ideal conditions are approximated	0.65	0.80	1.0	1.2	2.10	2.0

Bracing

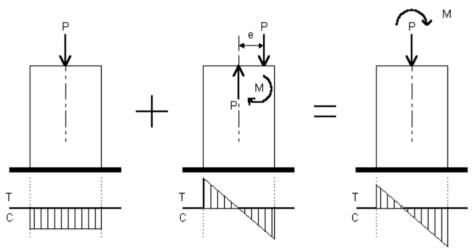
- bracing affects shape of buckle in one direction
- both should be checked!

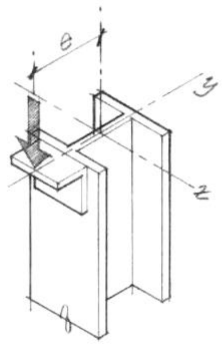


(a) No bracing. (b) Braced at midpoint. (c) Third-point bracing. (d) Asymmetric bracing.

Centric & Eccentric Loading

- *centric*
  - – *allowable stress from strength or buckling*
- *eccentric*
  - – *combined stresses*



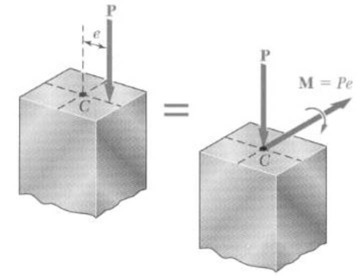


Combined Stresses

– *axial + bending*

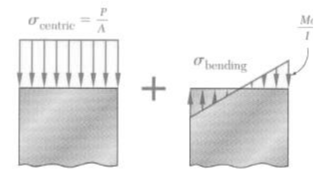
$$f_{\max} = \frac{P}{A} + \frac{Mc}{I}$$

$$M = P \cdot e$$



– *design*

$$f_{\max} \le F_{cr} = \frac{f_{cr}}{F.S.}$$



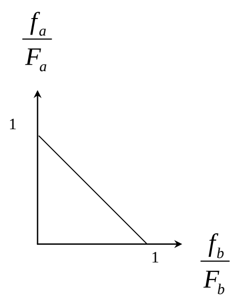
Stress Limit Conditions

– *ASD interaction formula*

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \le 1.0$$

– *with biaxial bending*

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \le 1.0$$



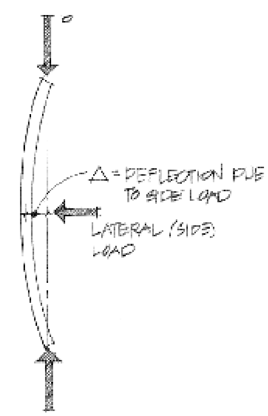
<u>*interaction diagram*</u>

Stress Limit Conditions

– *in reality, as the column flexes, the moment increases*

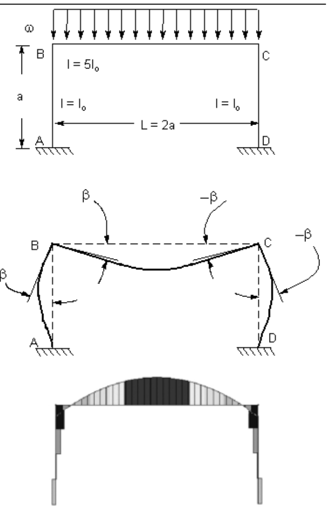
– <u>*P-Δ effect*</u>

$$\frac{f_a}{F_a} + \frac{f_b \times (Magnification\ \ factor)}{F_{bx}} \le 1.0$$



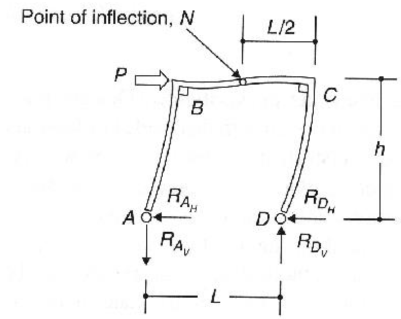
Rigid Frame Analysis

- members see
  - shear
  - axial force
  - bending
- V & M diagrams
  - plot on "outside"



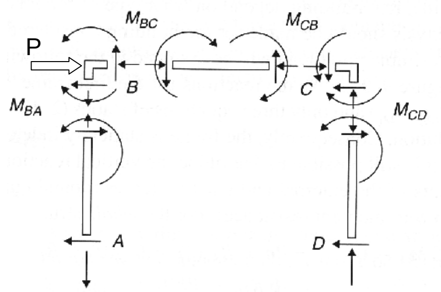
Rigid Frame Analysis

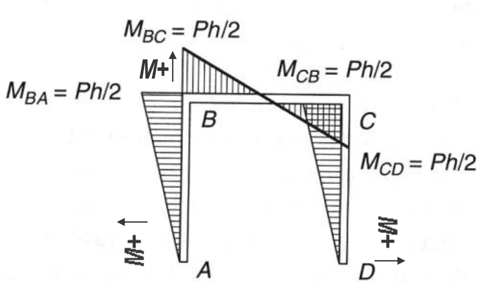
- need support reactions
- free body diagram each member
- end reactions are <u>equal and opposite</u> on next member
- "turn" member like beam
- draw V & M



Rigid Frame Analysis

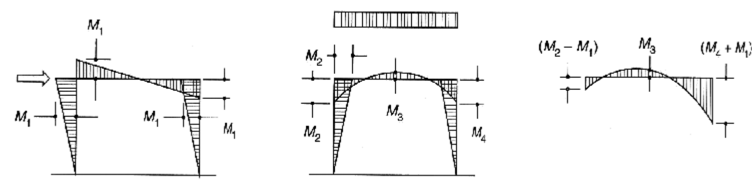
- FBD & M
  - opposite end reactions at joints





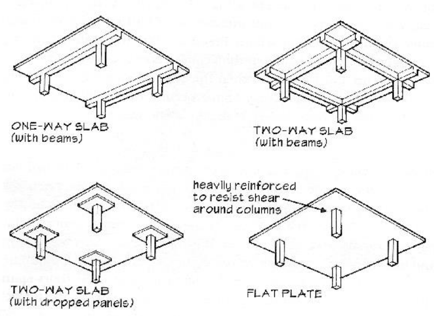
Rigid Frame Design

- loads and combinations
  - usually uniformly distributed gravity loads
  - worst case for largest moments...
  - wind direction can increase moments



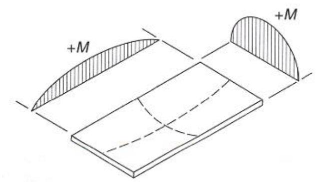
Rigid Frame Design

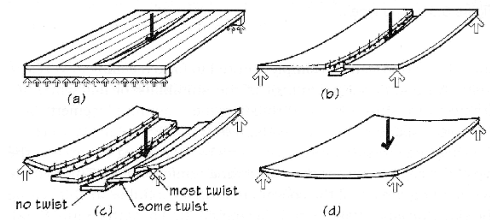
- frames & floors
  - rigid frame can have slab floors or slab with connecting beams
- other
  - slabs or plates on columns



Rigid Frame Design

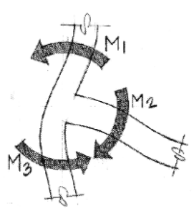
- floors – plates & slabs
  - one-way behavior
    - side ratio > 1.5
    - "strip" beam
  - two-way behavior
    - more complex





Rigid Frame Design

- columns in frames
  - ends can be "flexible"
  - stiffness affected by beams <u>and</u> column = EI/L

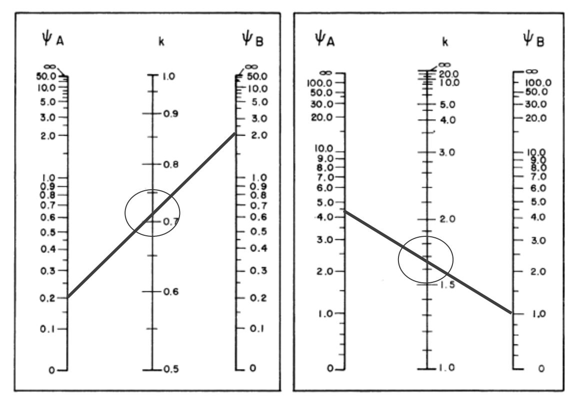


$$G = \Psi = \frac{\Sigma \, EI/l_c}{\Sigma \, EI/l_b}$$

  - for the joint
    - l_c is the column length of each column
    - l_b is the beam length of each beam
    - measured center to center

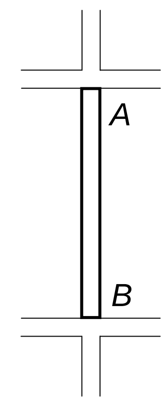
Rigid Frame Design

- column effective length, k



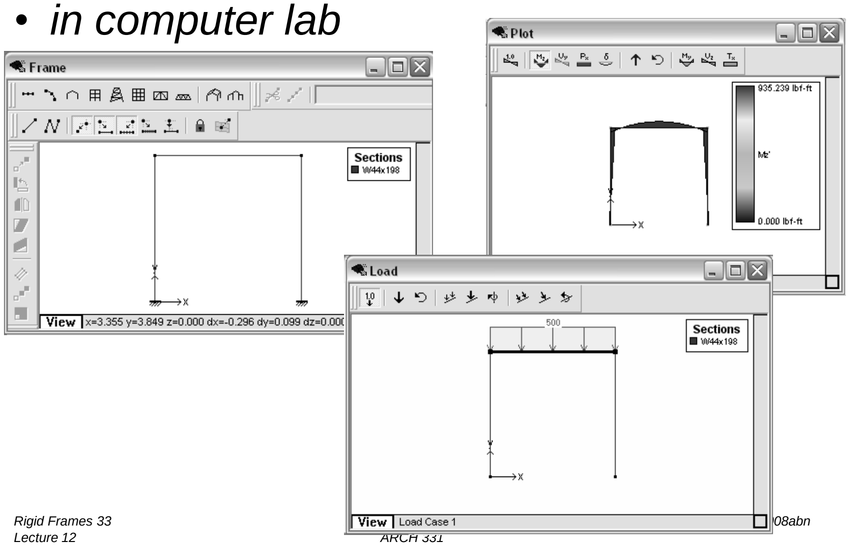
(a) Nonsway Frames

(b) Sway Frames



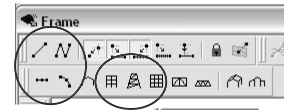
Tools – Multiframe

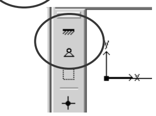
- *in computer lab*

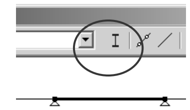


Tools – Multiframe

- *frame window*
  - *define frame members*
    - *or pre-defined frame*
  - *select points, assign supports*
  - *select members, assign <u>section</u>*
  - *load window*
  - *select point or member, add point or distributed loads*

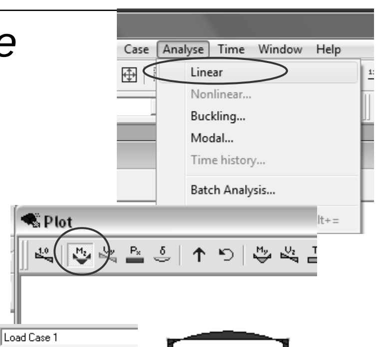


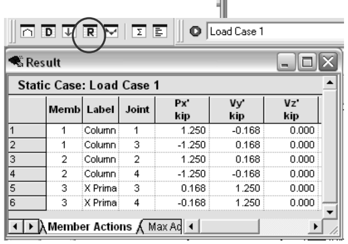




Tools – Multiframe

- *to run analysis choose*
  - *Analyze menu*
    - *Linear*
- *plot*
  - *choose options*
- *results*
  - *choose options*





Static Case: Load Case 1

	Memb	Label	Joint	Px' kip	Vy' kip	Vz' kip
1	1	Column	1	1.250	-0.168	0.000
2	1	Column	3	-1.250	0.168	0.000
3	2	Column	2	1.250	0.168	0.000
4	2	Column	4	-1.250	-0.168	0.000
5	3	X Prima	3	0.168	1.250	0.000
6	3	X Prima	4	-0.168	1.250	0.000