ARCHITECTURAL STRUCTURES: FORM, BEHAVIOR, AND DESIGN

ARCH 331

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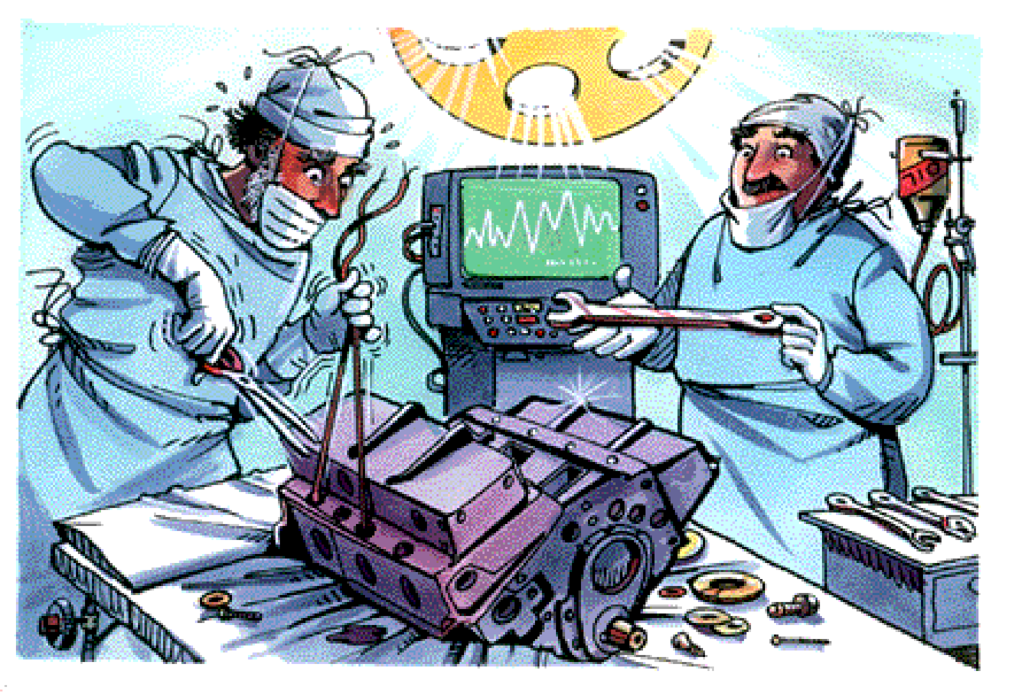
SUMMER 2014

lecture

five



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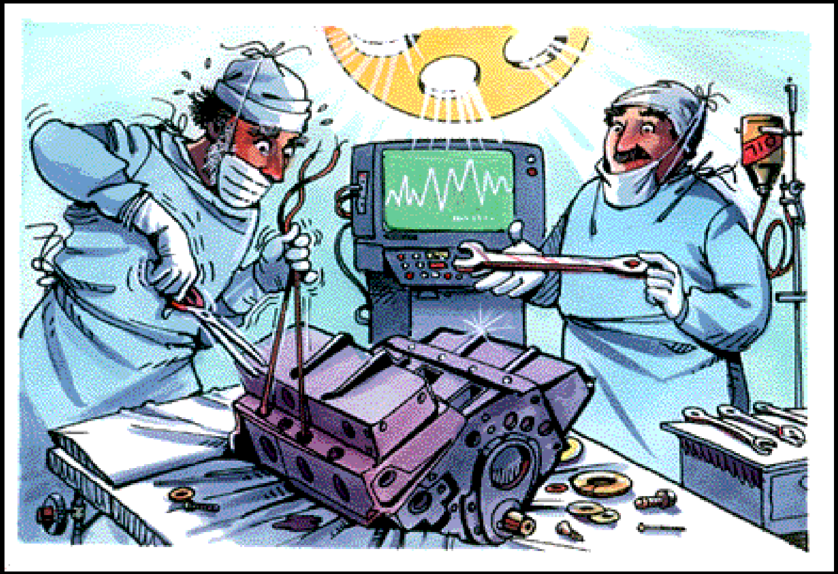
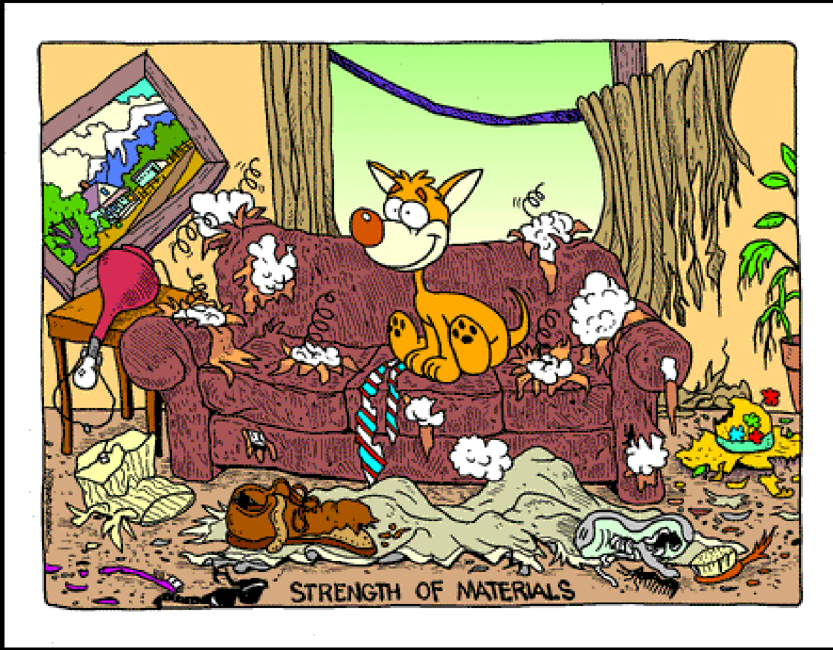
mechanics of materials

Mechanics of Materials

- MECHANICS



- MATERIALS



Mechanics of Materials

- *external loads and their effect on deformable bodies*
- *use it to answer question if structure meets requirements of*
  - *stability and equilibrium*
  - *strength and stiffness*
- *other principle building requirements*
    - *economy, functionality and aesthetics*

Knowledge Required

- *material properties*
- *member cross sections*
- *ability of a material to resist breaking*
- *structural elements that resist excessive*
  - *deflection*
  - *deformation*

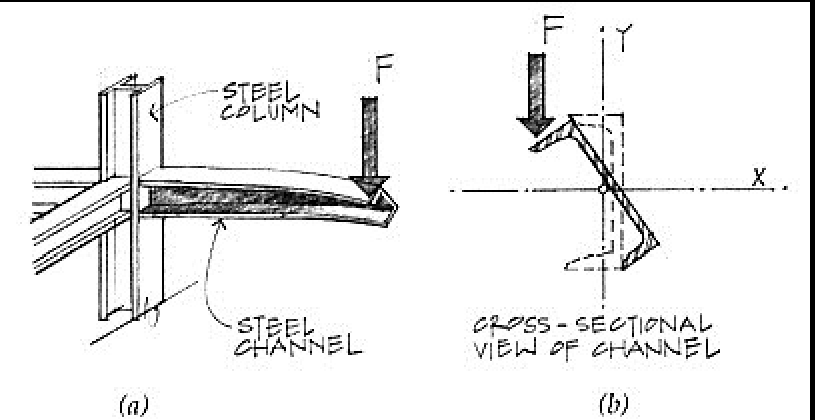


Figure 2.34 An example of torsion on a cantilever beam.

Problem Solving

1. *STATICS:*

   equilibrium of external forces, internal forces, <u>stresses</u>

2. *GEOMETRY:*

   cross section properties, deformations and conditions of geometric fit, <u>strains</u>

3. *MATERIAL PROPERTIES:*

   <u>stress-strain relationship</u> for each material obtained from testing

Stress

- *stress is a term for the intensity of a force, like a pressure*
- *internal or applied*
- *force per unit area*

$$stress = f = \frac{P}{A}$$

Design

- *materials have a critical stress value where they could break or yield*
  - *ultimate stress*
  - *yield stress*
  - *compressive stress*
  - *fatigue strength*
  - *(creep & temperature)*

acceptance vs. failure

Design (cont)

- *we'd like*

$$f_{actual} << F_{allowable}$$

- *stress distribution may vary: <u>average</u>*
- *uniform distribution exists IF the member is loaded axially (concentric)*

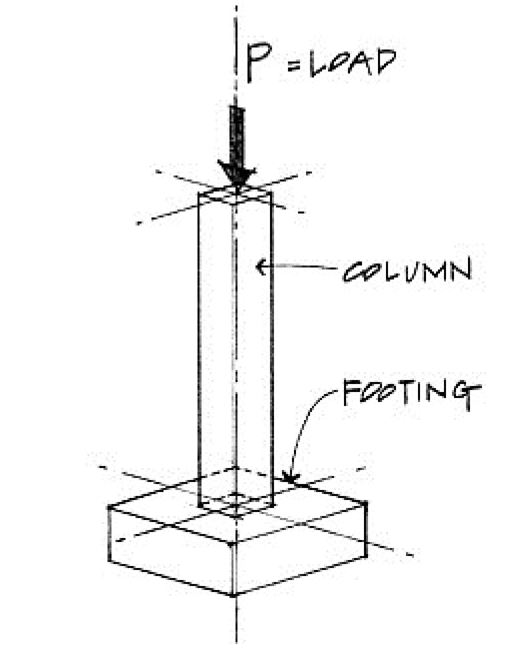
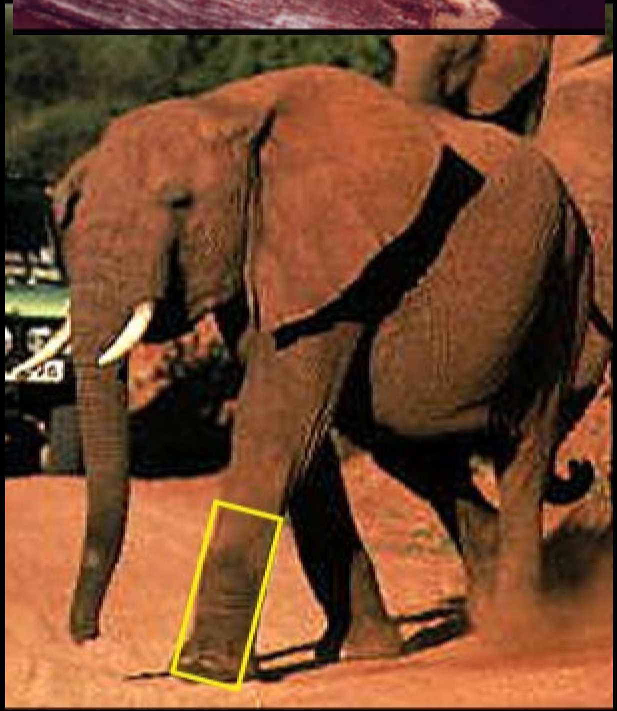


Figure 5.3 Centric loads.

Scale Effect

- *model scale*
  - *material weights by volume, small section areas*
- *structural scale*
  - *much more material weight, bigger section areas*
- *scale for strength is not proportional:*

$$\frac{\gamma L^3}{L^2} = \gamma L$$



Normal Stress (direct)

- <u>normal</u> stress is normal to the cross section
  - stressed area is perpendicular to the load

$$f_{t\ or\ c}\ (\sigma) = \frac{P}{A}$$

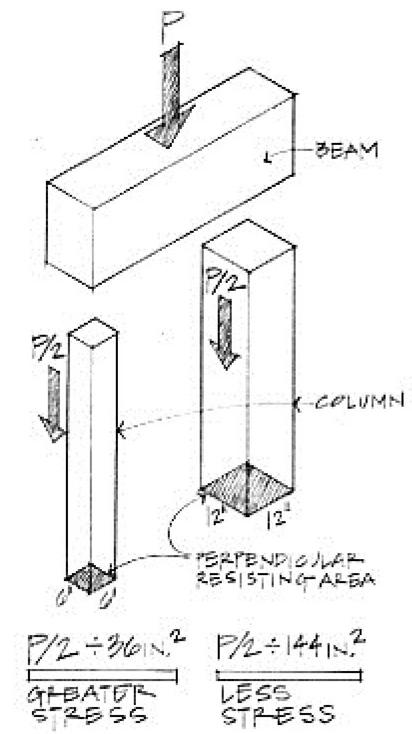


Figure 5.7 Two columns with the same load, different stress.

Shear Stress

- stress parallel to a surface

$$f_v = \frac{P}{A} = \frac{P}{td}$$
$$(\tau_{ave})$$

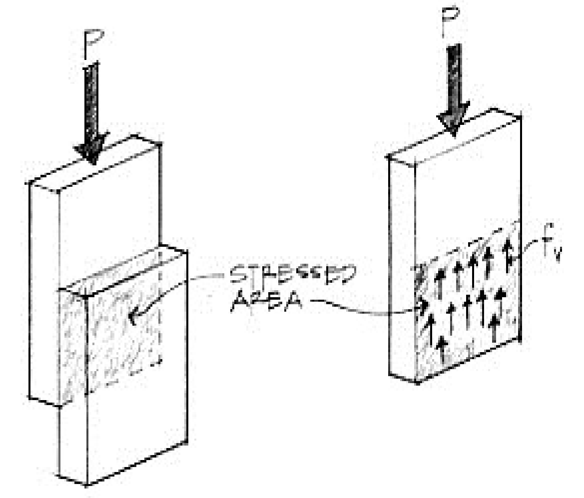


Figure 5.10 Shear stress between two glued blocks.

Bearing Stress

- stress on a surface by <u>contact</u> in compression

$$f_p = \frac{P}{A} = \frac{P}{td}$$

$$(\sigma)$$

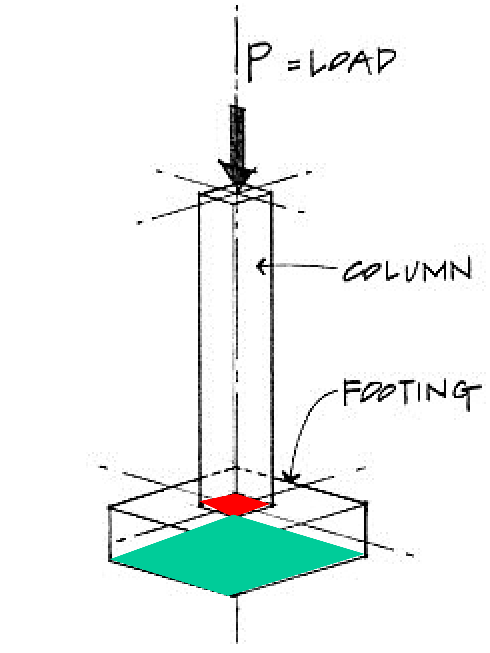


Figure 5.3 Centric loads.

Bending Stress

- *normal stress caused by bending*

$$f_b = \frac{Mc}{I} = \frac{M}{S}$$
$$(\sigma)$$

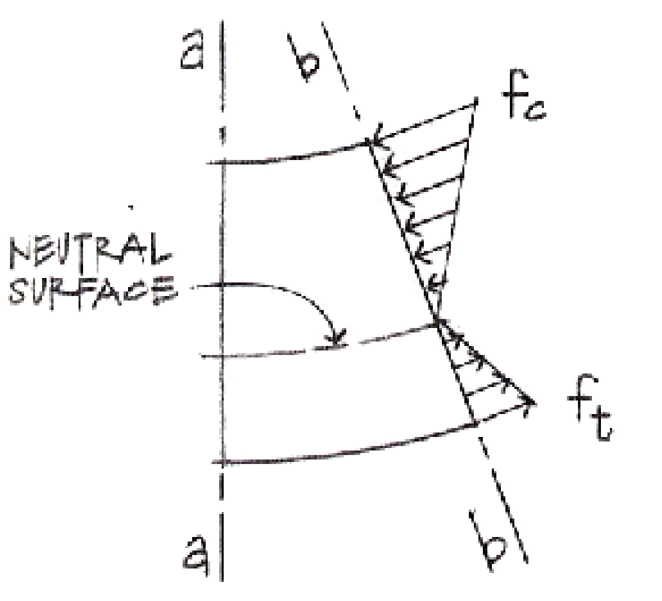
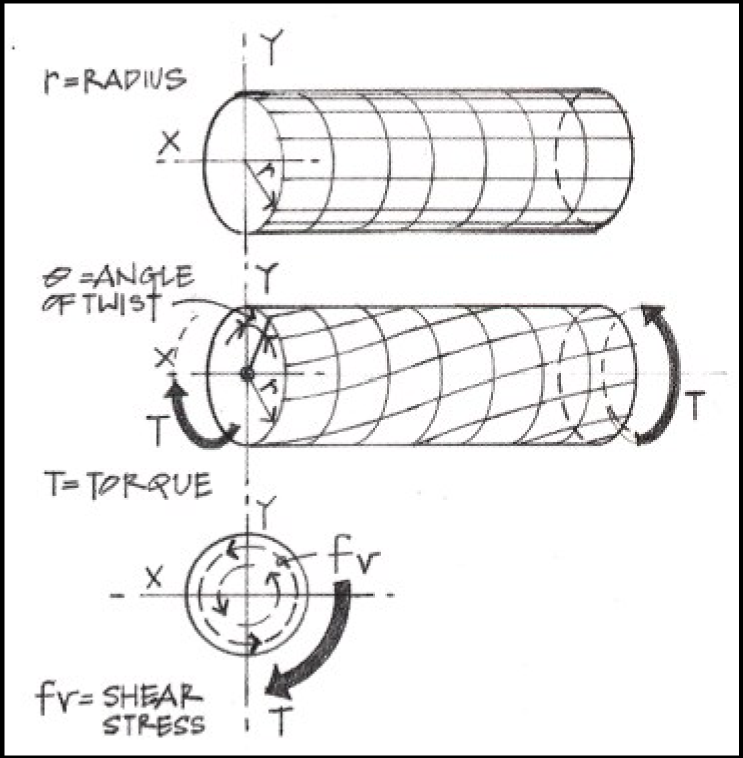


Figure 8.8 Bending stresses on section b-b.

Torsional Stress

- shear stress caused by twisting

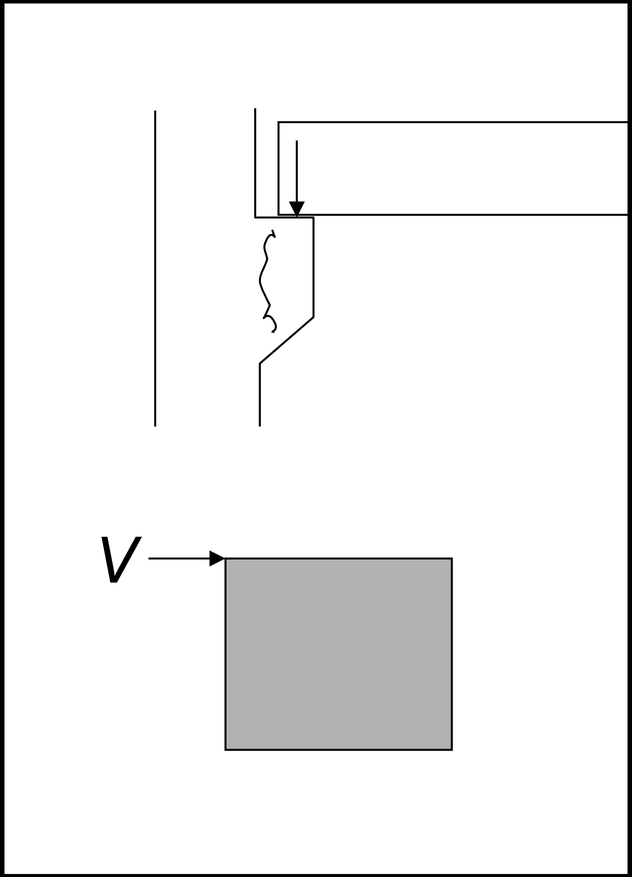
$$f_v = \frac{T\rho}{J}$$
$$(\tau)$$



Structures and Shear

- what structural elements see shear?
  - beams
  - bolts
  - splices
  - slabs
  - footings
  - walls
    - wind
    - seismic loads

connections (beams, bolts, splices)



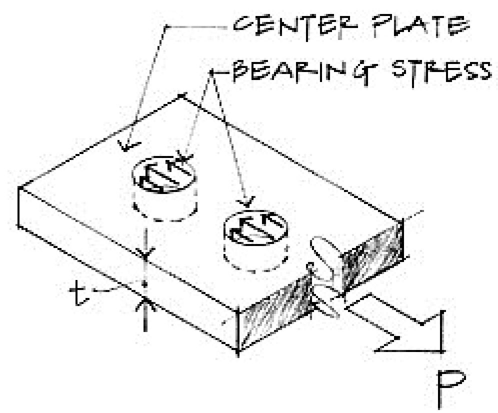
Bolts

- connected members in tension cause shear stress

(a) Two steel plates bolted using one bolt.

(b) Elevation showing the bolt in

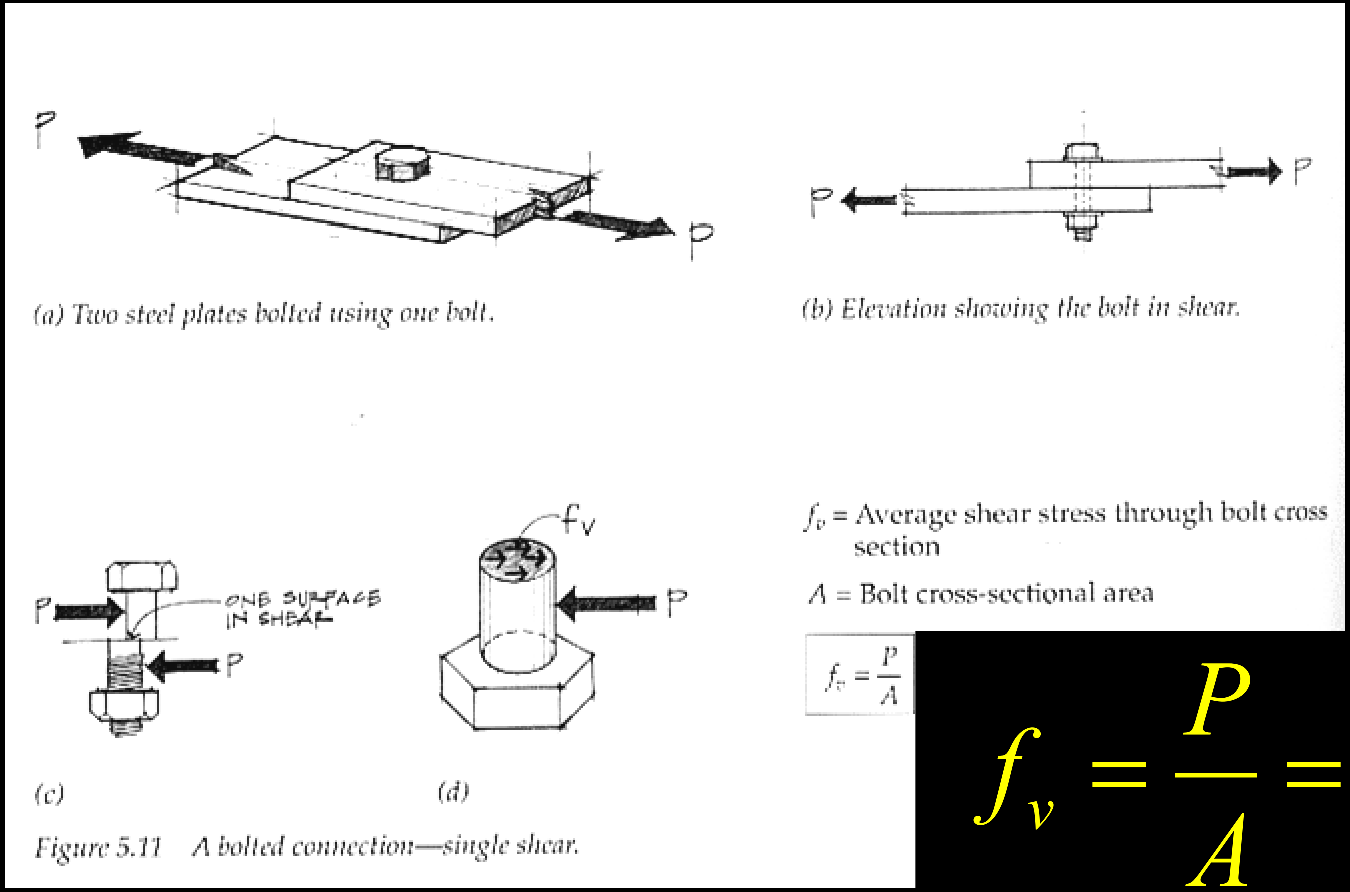
- connected members in compression cause bearing stress



Bearing stress on plate.

Single Shear

- *seen when 2 members are connected*



(a) Two steel plates bolted using one bolt.

(b) Elevation showing the bolt in shear.

(c)

(d)

Figure 5.11 A bolted connection—single shear.

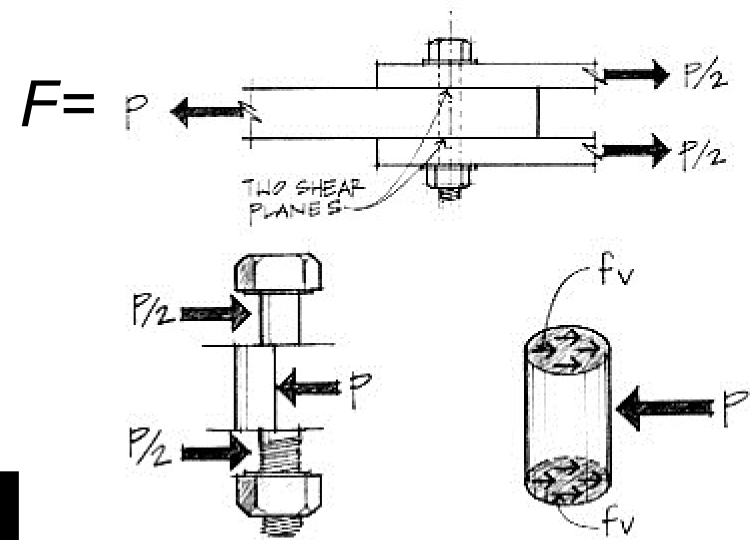
$$f_v = \frac{P}{A} = \frac{P}{\pi \, d^2/4}$$

Double Shear

- *seen when 3 members are connected*
- *<u>two</u> areas*

$$f_v = \frac{P}{2A}$$

(two shear planes)



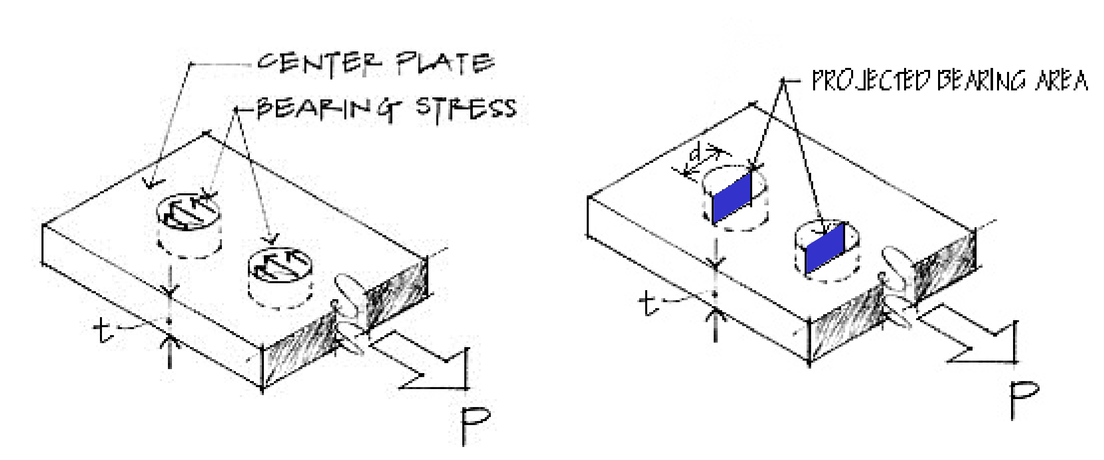
Free-body diagram of middle section of the bolt in shear.

Figure 5.12 A bolted connection in double shear.

$$f_v = \frac{P}{2A} = \frac{P/2}{A} = \frac{P/2}{\pi\, d^2/4}$$

Bolt Bearing Stress

- *compression & contact*
- *projected area*

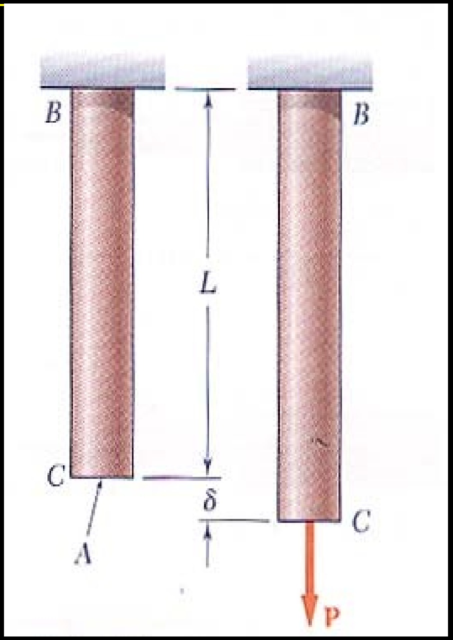


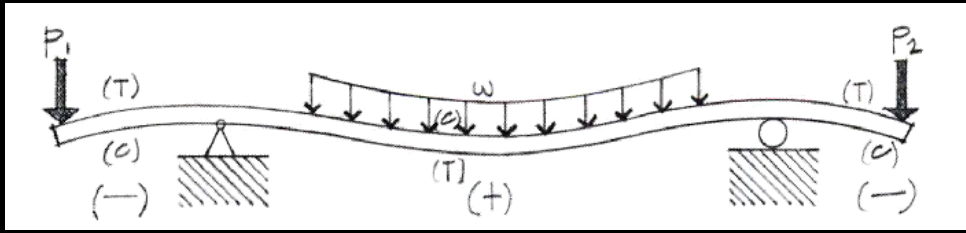
Bearing stress on plate.

$$f_p = \frac{P}{A_{projected}} = \frac{P}{td}$$

Strain

- *materials deform*
- *axially loaded materials change length*
- *bending materials deflect*



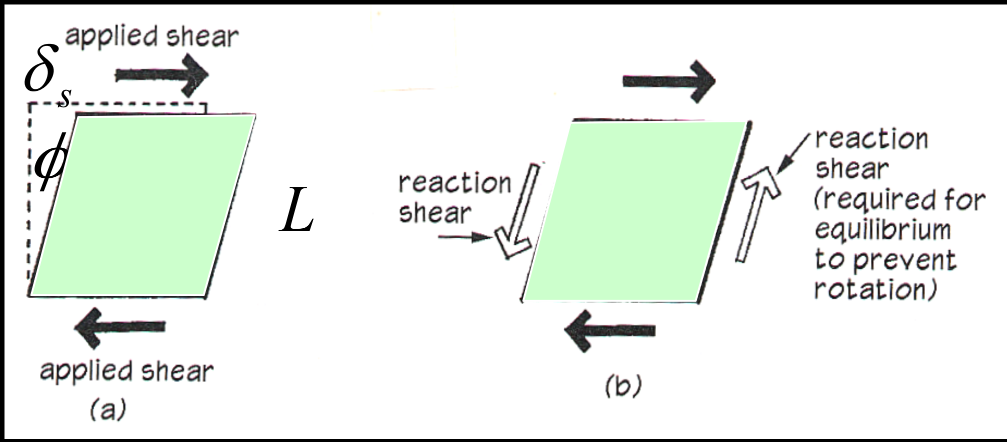


- *STRAIN:*
  - *change in length over length + UNITLESS*

$$strain = \varepsilon = \frac{\Delta L}{L}$$

Shearing Strain

- *deformations with shear*
- *parallelogram*
- *change in angles*
- *stress:* τ
- *strain:* γ
  - *unitless (radians)*

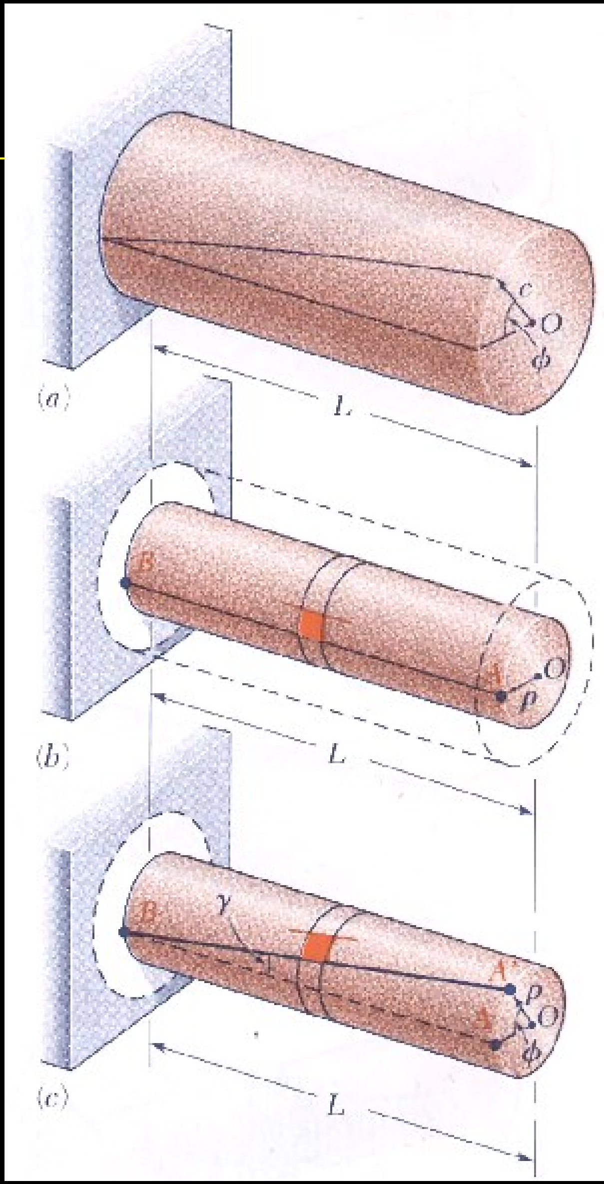


$$\gamma = \frac{\delta_s}{L} = \tan\phi \cong \phi$$

Shearing Strain

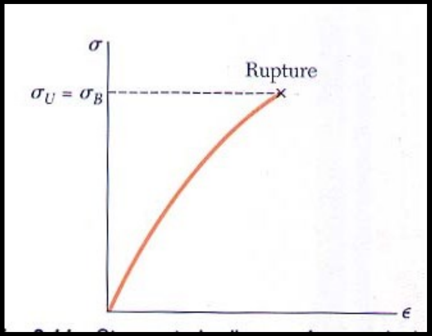
- *deformations with torsion*
- *twist*
- *change in angle of line*
- *stress:* τ
- *strain:* γ
  - *– unitless (radians)*

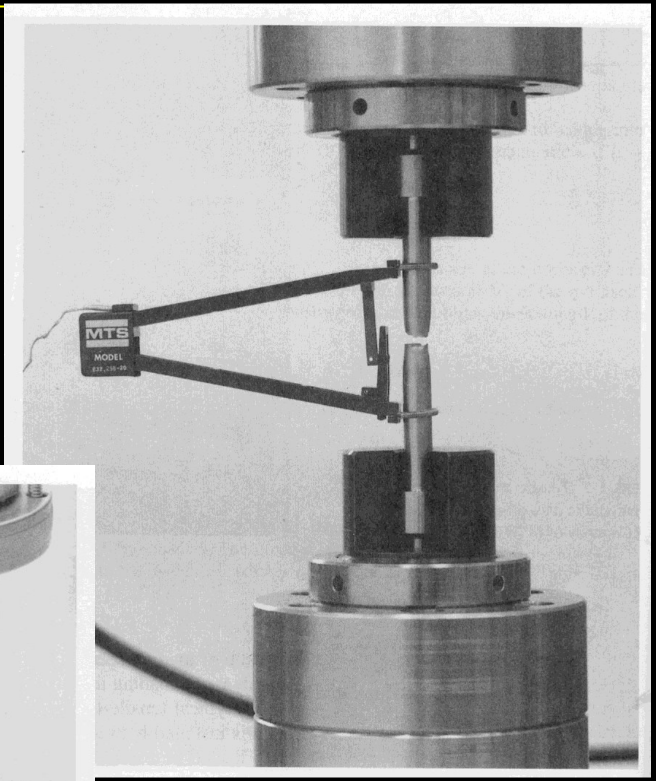
$$\gamma = \frac{\rho\phi}{L}$$

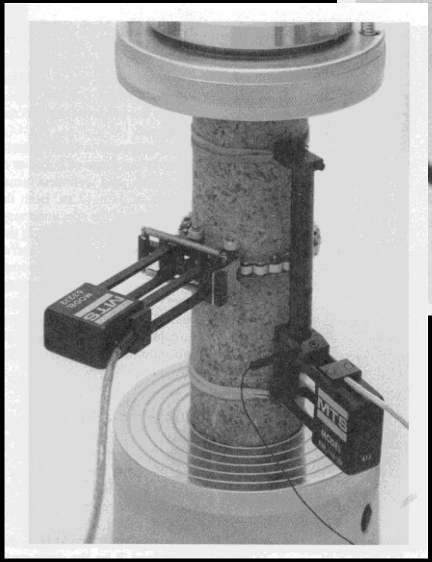


Load and Deformation

- for stress, need P & A
- for strain, need δ & L
  - how?
  - TEST with load and measure
  - plot P/A vs. ε







Material Behavior

- every material has its own response
  - 10,000 psi
  - L = 10 in
  - Douglas Fir vs. steel?

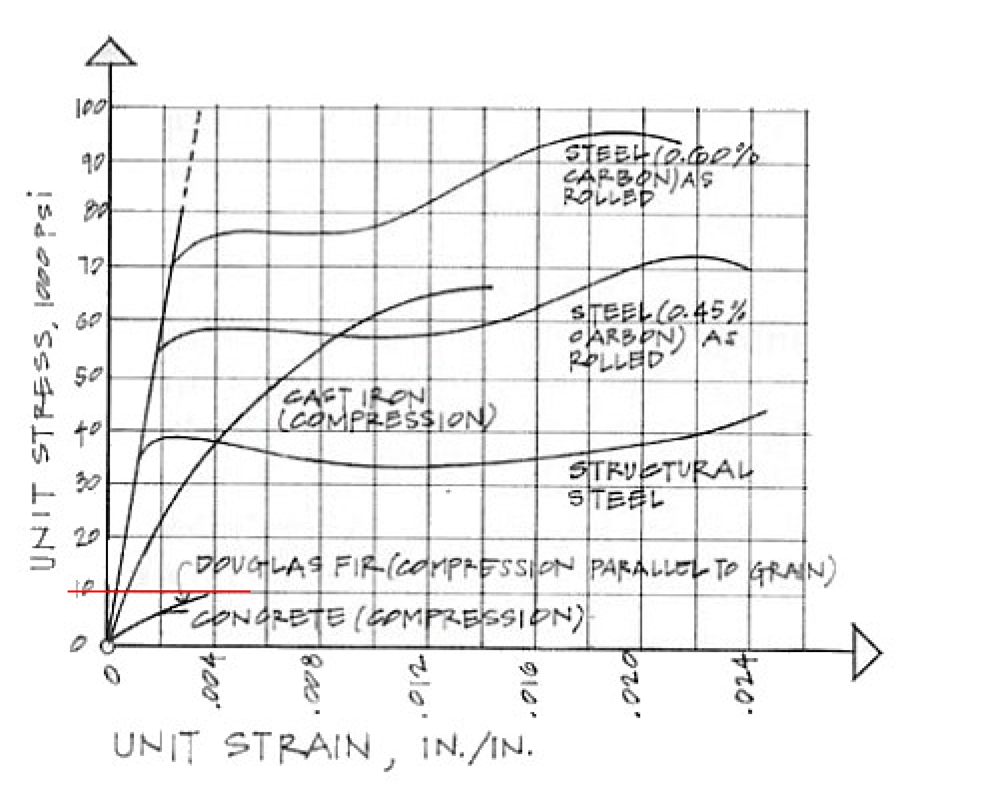


Figure 5.20 Stress-strain diagram for various materials.

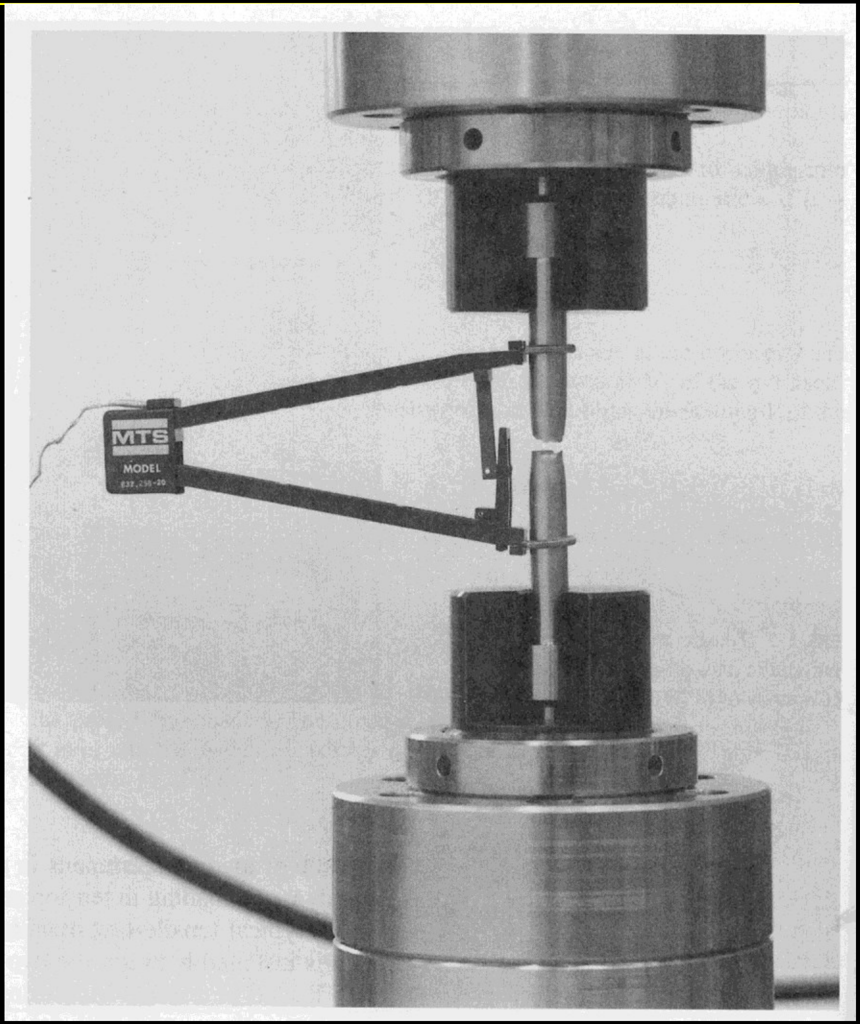
Behavior Types

- *ductile - "necking"*
- *true stress*

$$f = \frac{P}{A}$$

- *engineering stress*
  - *(simplified)*

$$f = \frac{P}{A_o}$$



Behavior Types

- *brittle*

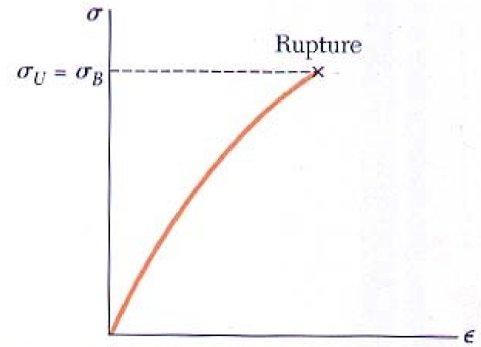


Fig. 2.11 Stress-strain diagram for a typical brittle material.

- *semi-brittle*

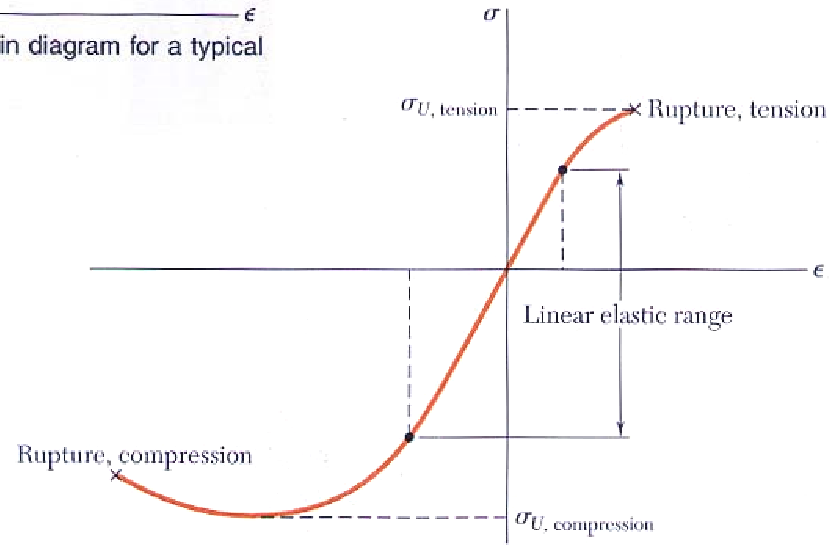


Fig. 2.14 Stress-strain diagram for concrete.

Stress to Strain

- *important to us in f-ε diagrams:*
  - *straight section*
  - *LINEAR-ELASTIC*
  - *recovers shape (no permanent deformation)*

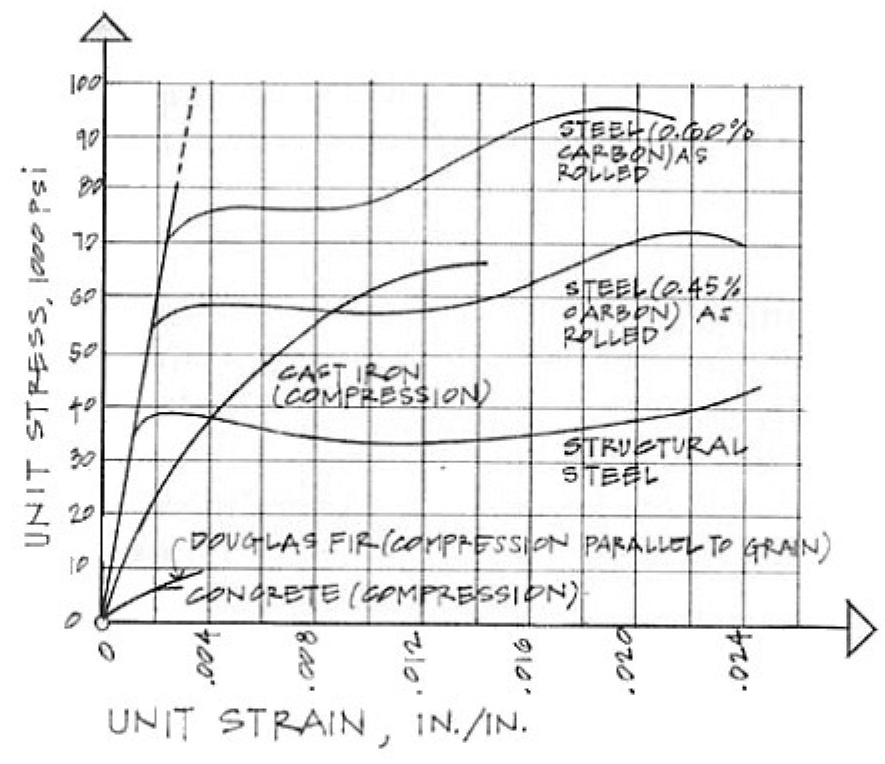
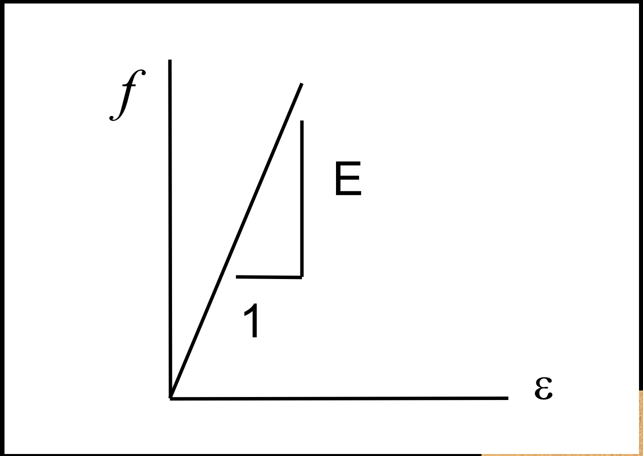


Figure 5.20 Stress-strain diagram for various materials.

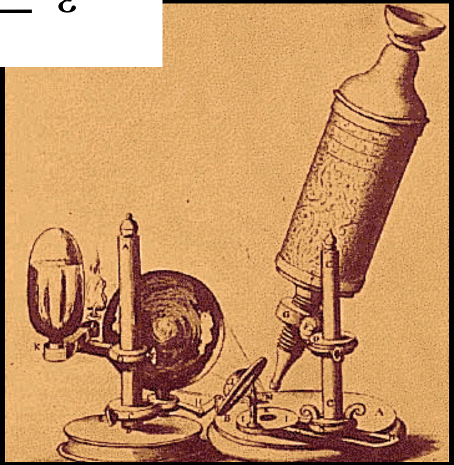
Hooke's Law

- *straight line has constant slope*
- *Hooke's Law*

$$f = E \cdot \varepsilon$$



- *E*
  - *Modulus of elasticity*
  - *Young's modulus*
  - *units just like stress*



Stiffness

- *ability to resist strain*
- *steels*
  - *same E*
  - *different <u>yield points</u>*
  - *different <u>ultimate strength</u>*

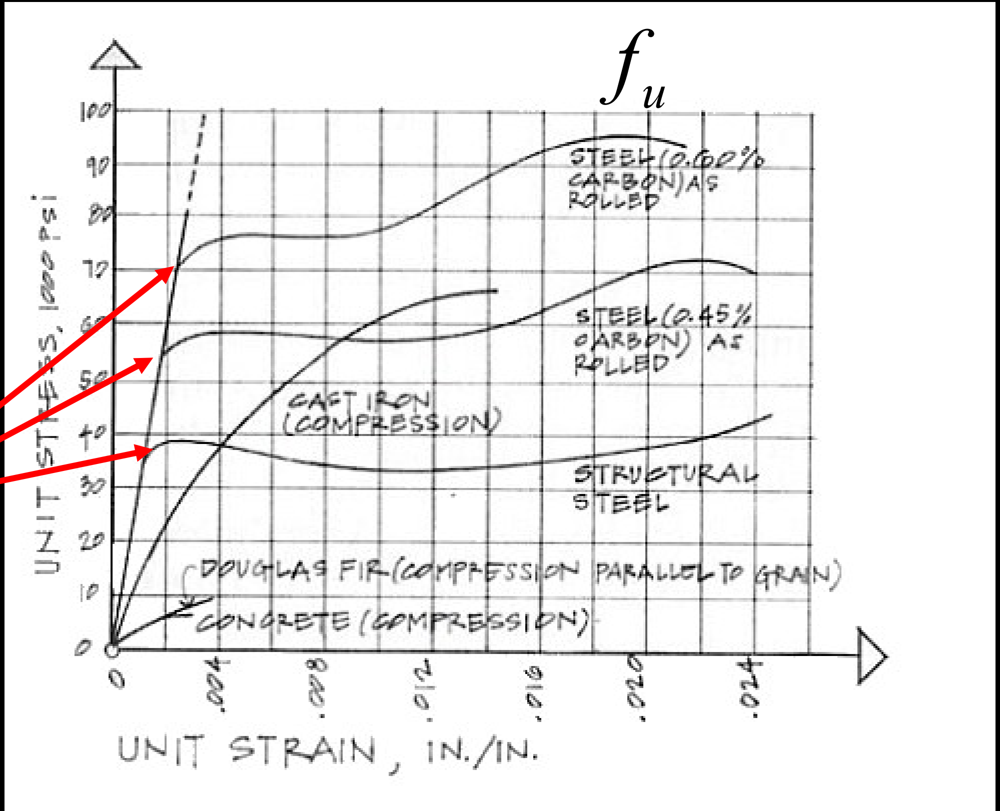
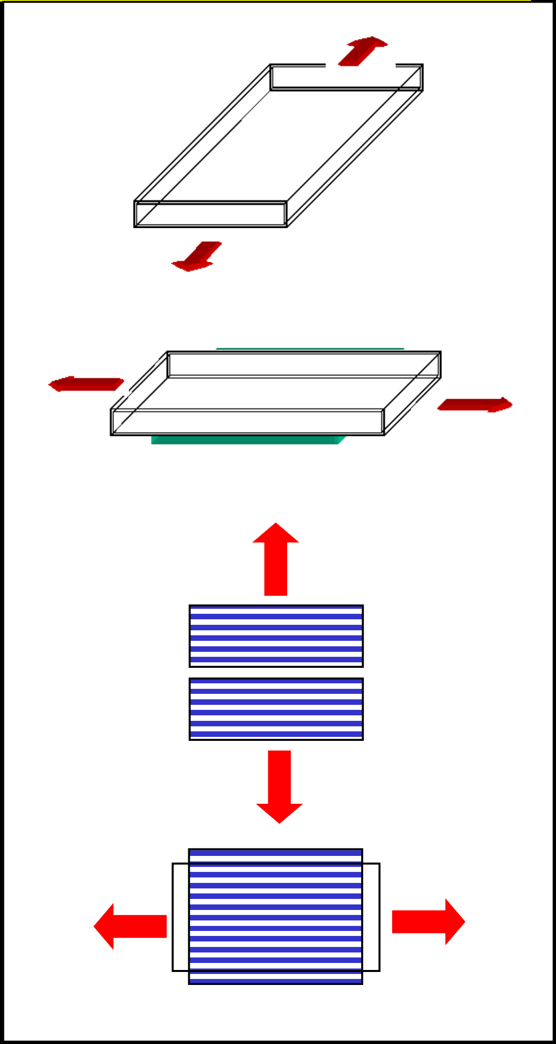


Figure 5.20 Stress-strain diagram for various materials.

Isotropy & Anisotropy

- ISOTROPIC
  - materials with E same at any direction of loading
  - ex. steel



- ANISOTROPIC
  - materials with different E at any direction of loading
  - ex. wood is <u>orthotropic</u>

Elastic, Plastic, Fatigue

- *elastic springs back*
- *plastic has permanent deformation*
- *fatigue caused by reversed loading cycles*

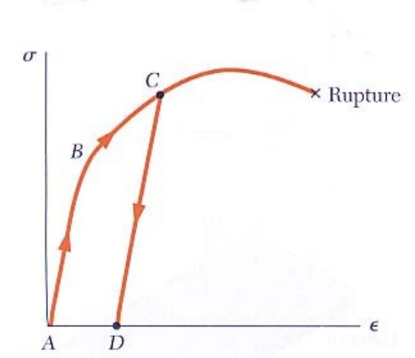


Fig. 2.18

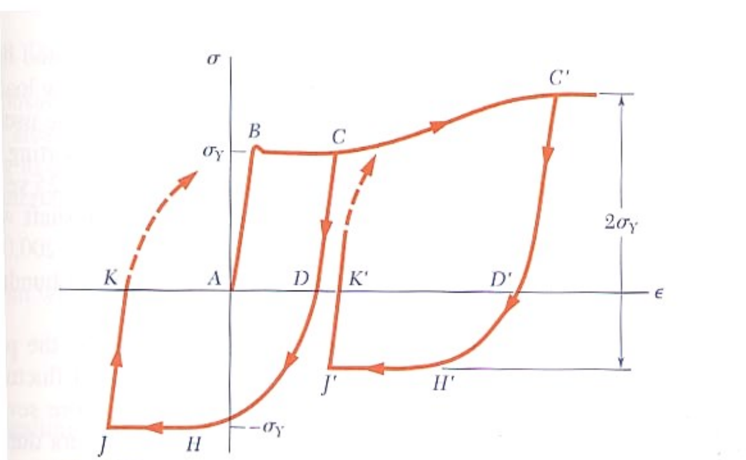


Fig. 2.20

Plastic Behavior

- *ductile*

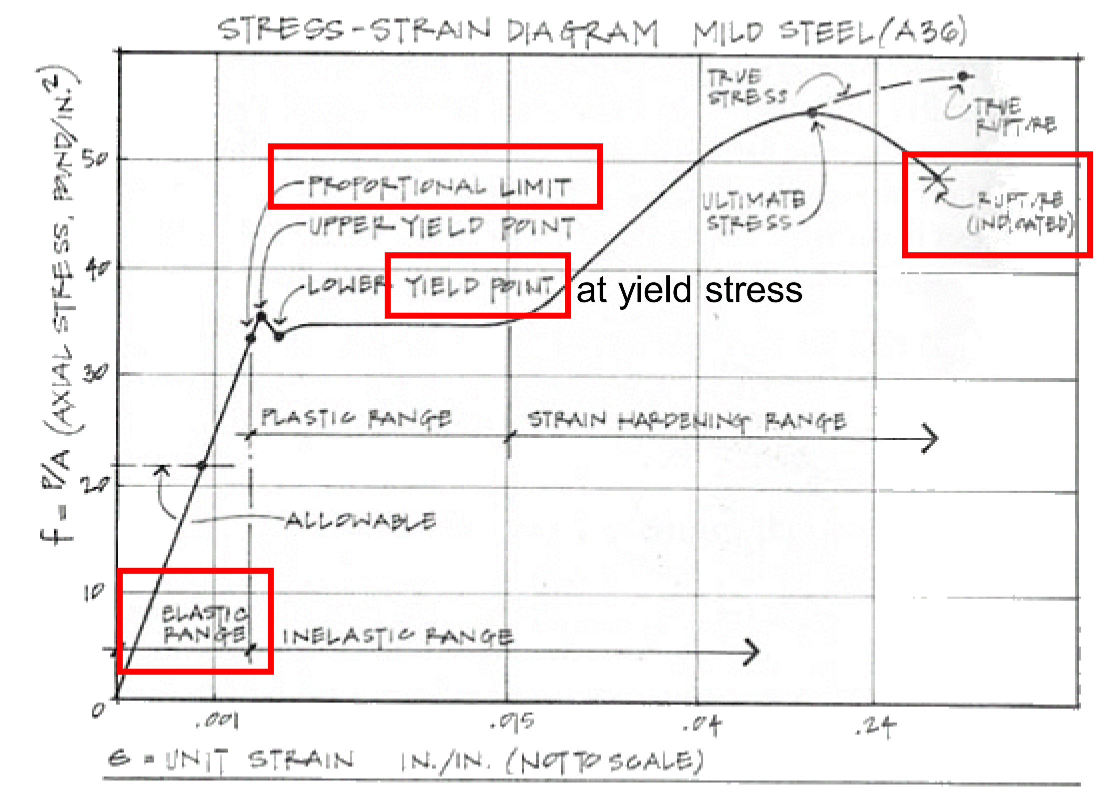
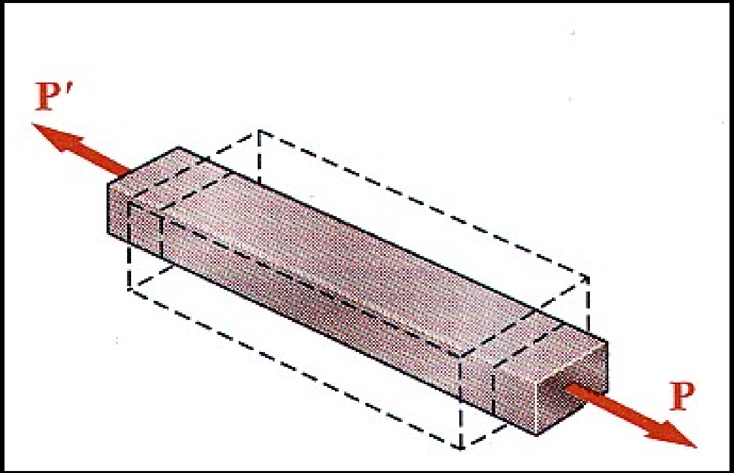


Figure 5.22 Stress-strain diagram for mild steel (A36) with key points highlighted.

Lateral Strain

- *or "what happens to the cross section with axial stress"*



$$\varepsilon_x = \frac{f_x}{E}$$

$$f_y = f_z = 0$$

- *strain in lateral direction*
  - *negative*
  - *equal for isometric materials*

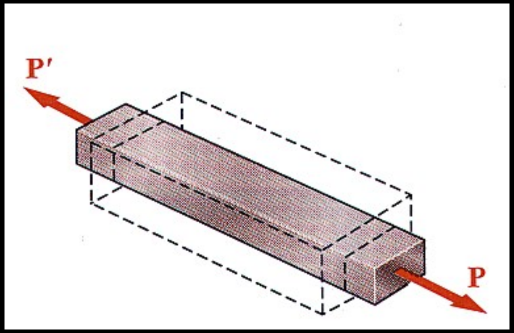
$$\varepsilon_y = \varepsilon_z$$

Poisson's Ratio

- *constant relationship between longitudinal strain and lateral strain*

$$\mu = -\frac{lateral\ strain}{axial\ strain} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

$$\varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E}$$



- *sign!* $$0 < \mu < 0.5$$

Calculating Strain

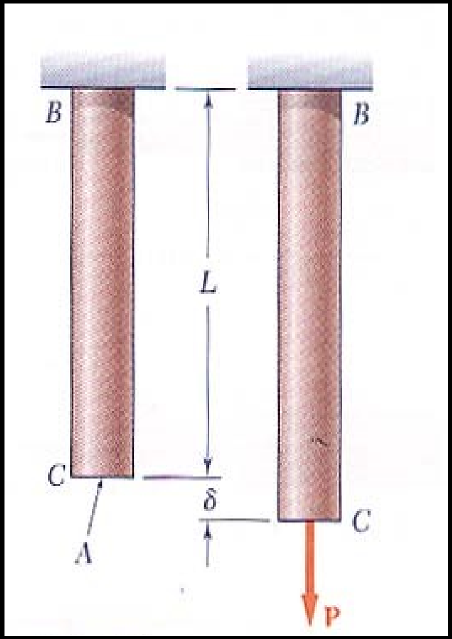
- *from Hooke's law*

$$f = E \cdot \varepsilon$$

- *substitute*

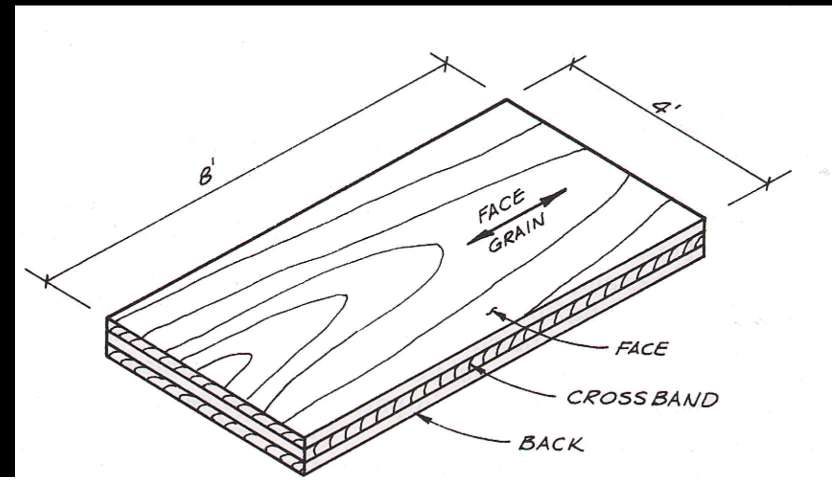
$$\frac{P}{A} = E \cdot \frac{\delta}{L}$$

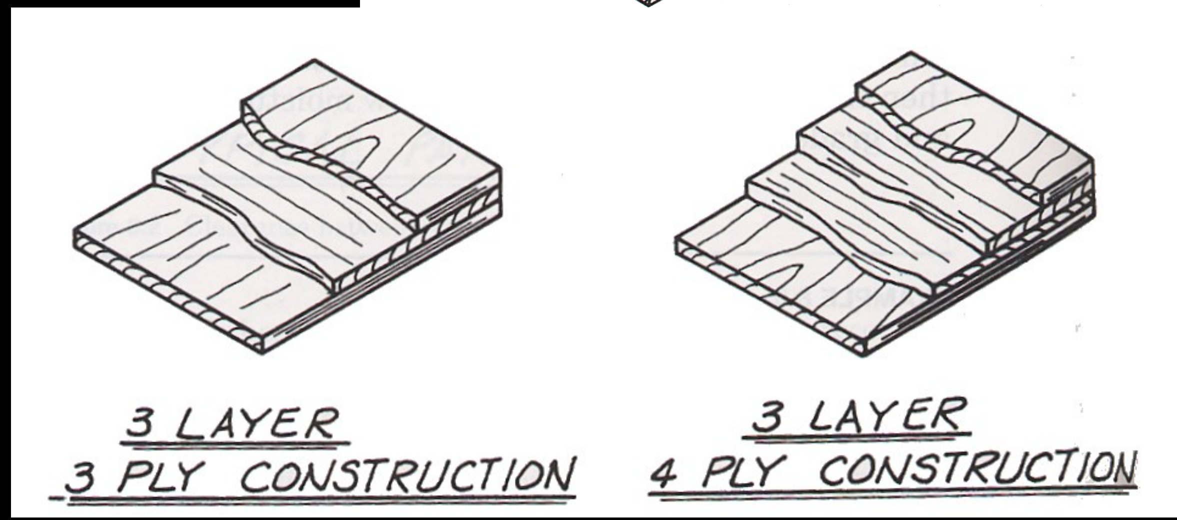
- *get* $\Rightarrow$ $$\delta = \frac{PL}{AE}$$



Orthotropic Materials

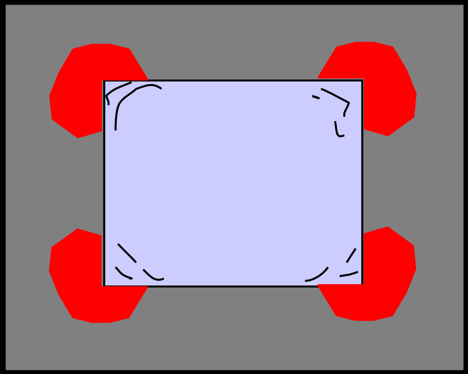
- *non-isometric*
- *directional values of E and μ*
- *ex:*
  - *plywood*
  - *laminates*
  - *polymer composites*

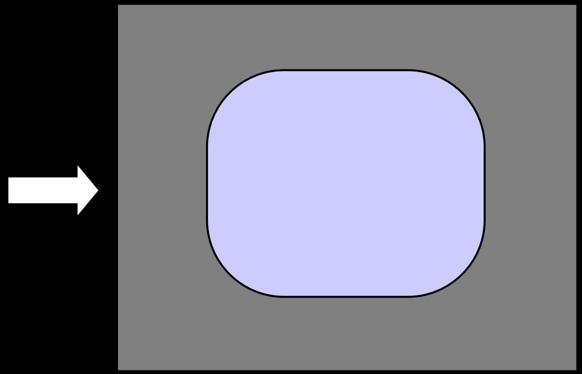




Stress Concentrations

- *why we use f_{ave}*
- *increase in stress at changes in geometry*
  - *sharp notches*
  - *holes*
  - *corners*





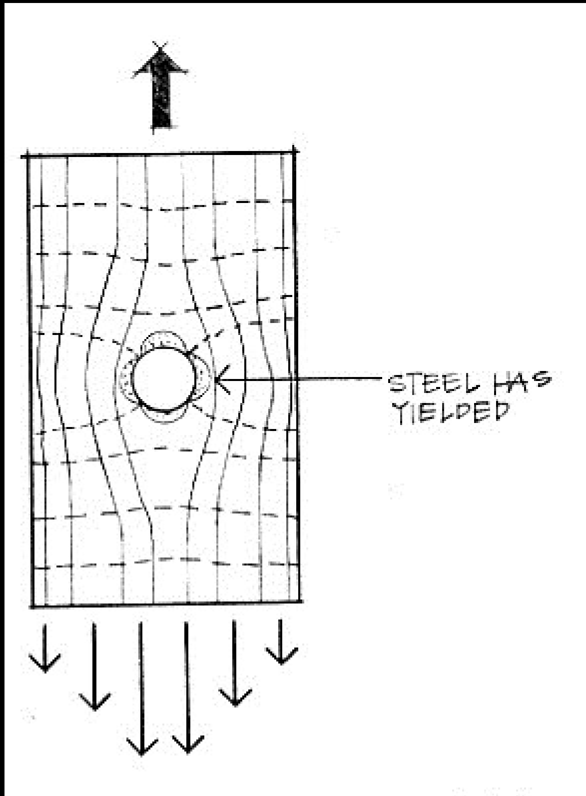
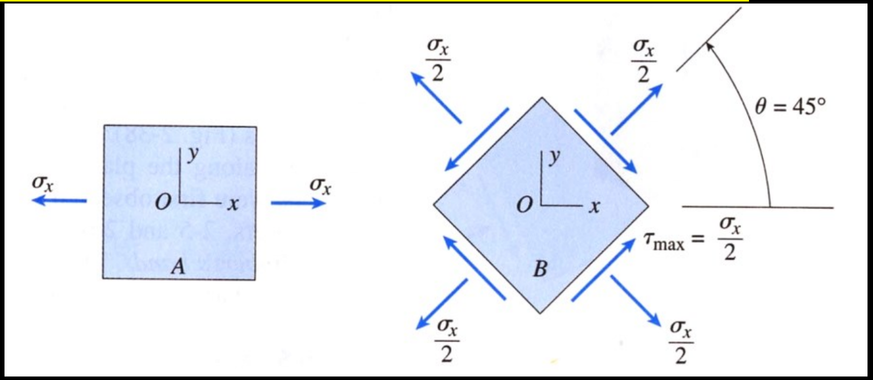


Figure 5.35 Stress trajectories around a hole.

Maximum Stresses

- *if we need to know where max f and f_v happen:*



$$\theta = 0° \rightarrow \cos\theta = 1 \qquad f_{max} = \frac{P}{A_o}$$

$$\theta = 45° \rightarrow \cos\theta = \sin\theta = \sqrt{0.5}$$

$$f_{v-max} = \frac{P}{2A_o} = \frac{f_{max}}{2}$$

Maximum Stresses

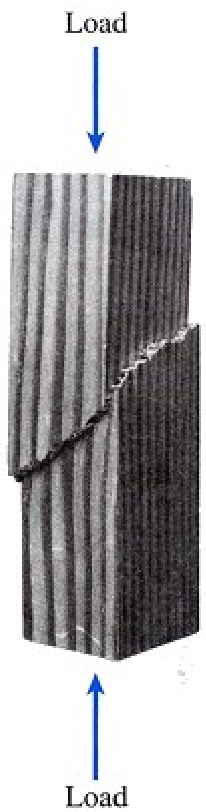
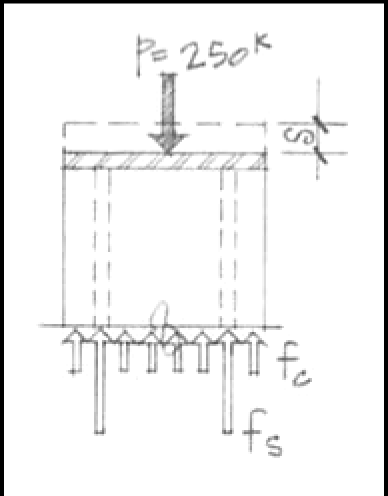


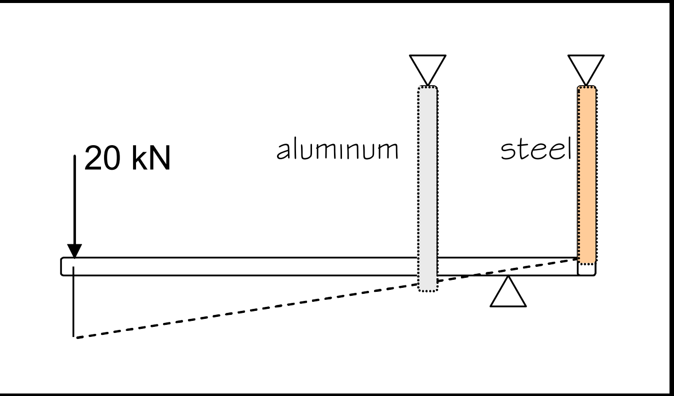
FIG. 2-37 Shear failure along a 45° plane of a wood block loaded in compression

FIG. 2-38 Slip bands (or Lüders' bands) in a polished steel specimen loaded in tension

Deformation Relationships

- *physical movement*
  - *axially (same or zero)*
  - *rotations from axial changes*

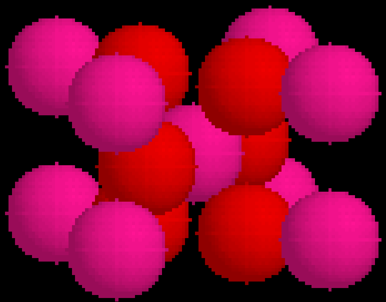


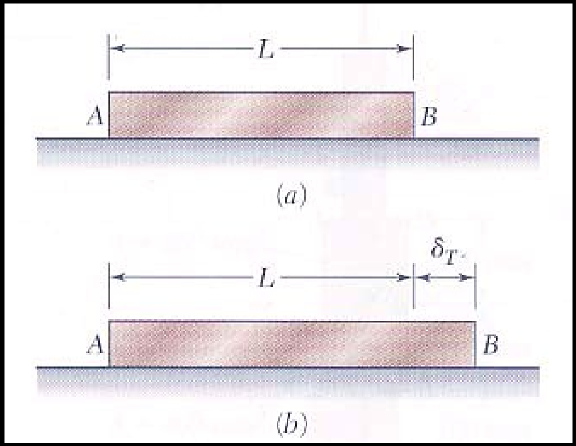


- $\delta = \frac{PL}{AE}$ *relates* δ *to P*

Deformations from Temperature

- atomic chemistry reacts to changes in energy
- solid materials
  - can contract with decrease in temperature
  - can expand with increase in temperature
- linear change can be measured per degree



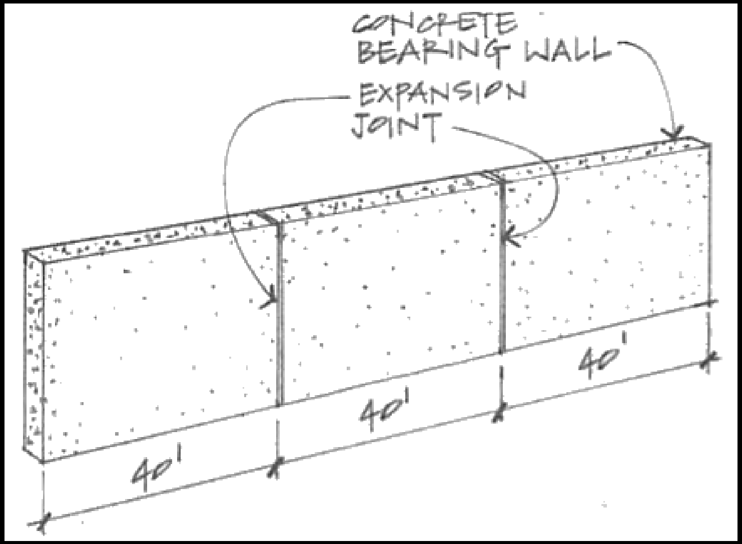


Thermal Deformation

- α - the rate of strain per degree
- UNITS : $/{}^\circ F$, $/{}^\circ C$
- length change: $$\delta_T = \alpha(\Delta T)L$$
- thermal strain: $$\varepsilon_T = \alpha(\Delta T)$$
  - <u>no stress</u> when movement allowed

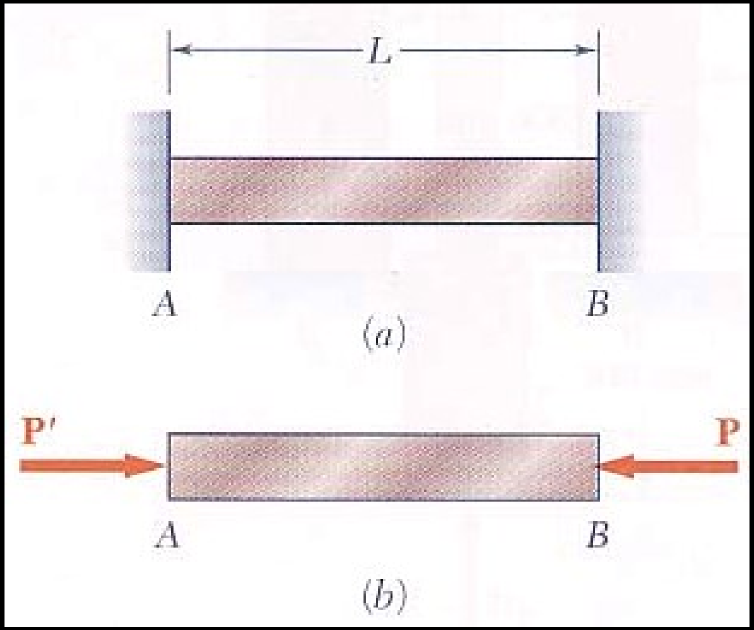
Coefficients of Thermal Expansion

Material	Coefficients (α) [in./in./°F]
Wood	3.0×10^{-6}
Glass	4.4×10^{-6}
Concrete	5.5×10^{-6}
Cast Iron	5.9×10^{-6}
Steel	6.5×10^{-6}
Wrought Iron	6.7×10^{-6}
Copper	9.3×10^{-6}
Bronze	10.1×10^{-6}
Brass	10.4×10^{-6}
Aluminum	12.8×10^{-6}



Stresses and Thermal Strains

- *if thermal movement is restrained* *stresses are induced*

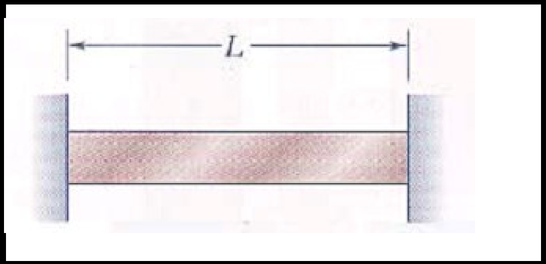


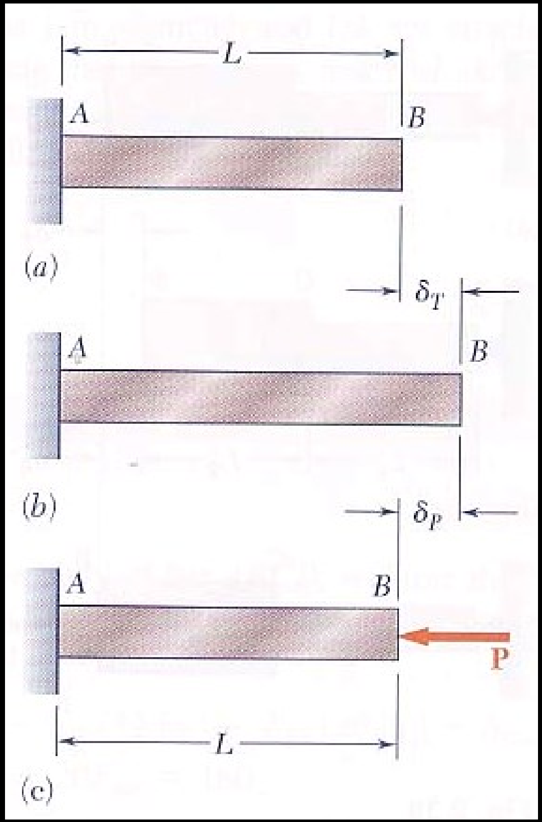
1. *bar pushes on supports*
2. *support pushes back*
3. *reaction causes internal stress*

$$f = \frac{P}{A} = \frac{\delta}{L}E$$

Superposition Method

- can remove a support to make it look determinant
- replace the support with a reaction
- enforce the geometry constraint





Superposition Method

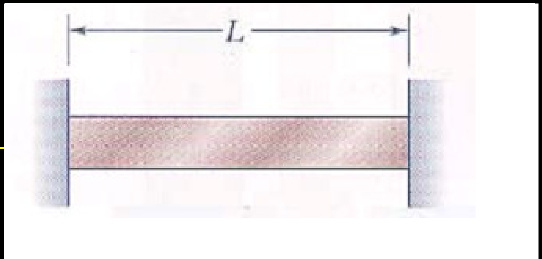
– total length change restrained to <u>zero</u>

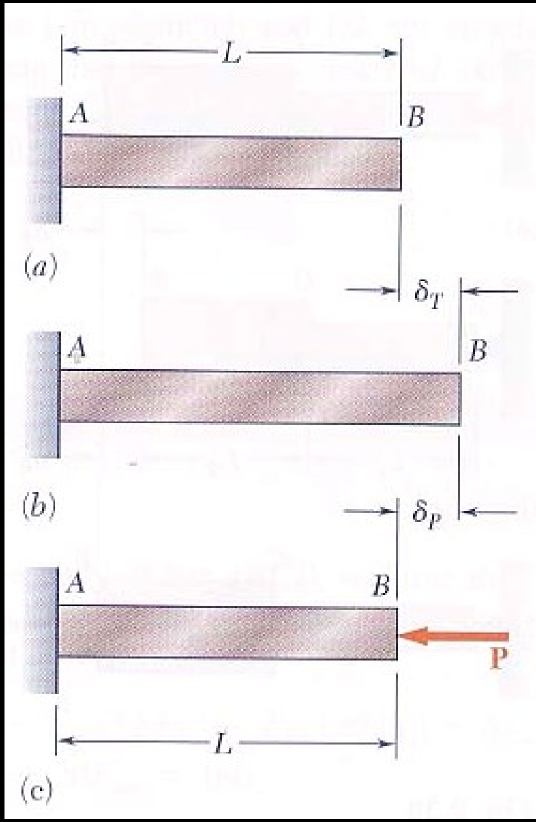
constraint: $\delta_P + \delta_T = 0$

$$\delta_p = -\frac{PL}{AE} \qquad \delta_T = \alpha(\Delta T)L$$

sub: $$-\frac{PL}{AE} + \alpha(\Delta T)L = 0$$

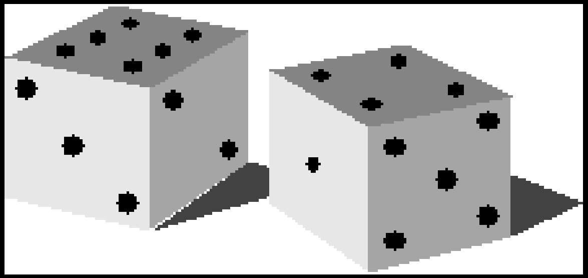
$$f = -\frac{P}{A} = -\alpha(\Delta T)E$$





Design of Members

- *beyond allowable stress...*
- *materials aren't uniform 100% of the time*
  - *ultimate strength or capacity to failure may be different and some strengths hard to test for*
- *RISK & UNCERTAINTY*



$$f_u = \frac{P_u}{A}$$

Factor of Safety

- *accommodate uncertainty with a safety factor:*

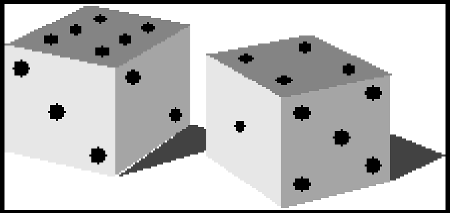
$$allowable\ load = \frac{ultimate\ load}{F.S}$$

- *with linear relation between load and stress:*

$$F.S = \frac{ultimate\ load}{allowable\ load} = \frac{ultimate\ stress}{allowable\ stress}$$

Load and Resistance Factor Design

- loads on structures are
  - not constant
  - can be more influential on failure
  - happen more or less often
  - UNCERTAINTY



$$R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n$$

ϕ - resistance factor

γ - load factor for (D)ead & (L)ive load