#### ARCHITECTURAL STRUCTURES:

FORM, BEHAVIOR, AND DESIGN

ARCH 331

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SUMMER 2014

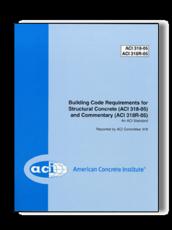
lecture NINETEEN



# concrete construction. http://nisee.berkeley.edu/godden materials & beams

# Concrete Beam Design

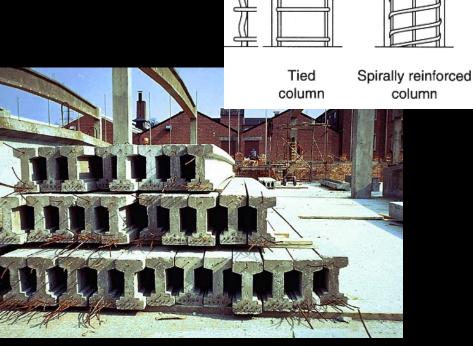
- composite of concrete and steel
- American Concrete Institute (ACI)
  - design for maximum stresses
  - limit state design
    - service loads x load factors
    - concrete holds no tension
    - failure criteria is yield of reinforcement
    - failure capacity x reduction factor
    - factored loads < reduced capacity</li>
  - concrete strength =  $f'_c$



## Concrete Construction

- cast-in-place
- tilt-up
- prestressing
- post-tensioning

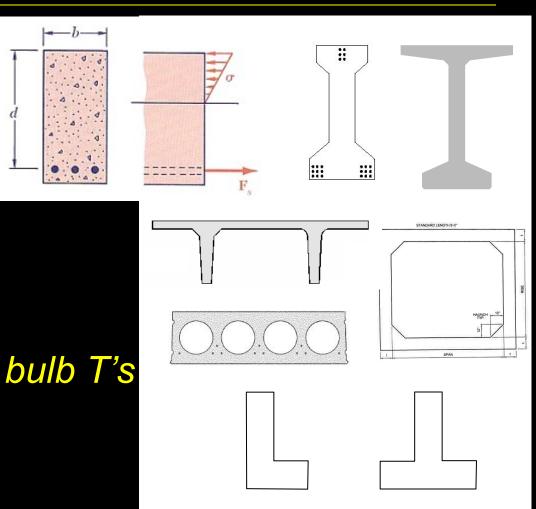




arch.mcgill.ca

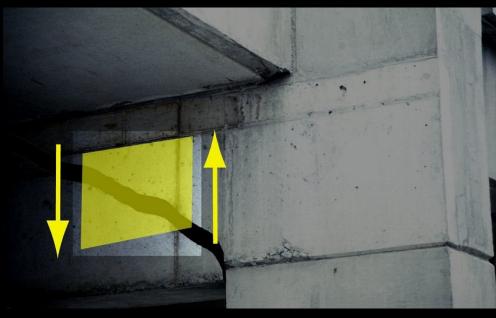
## Concrete Beams

- types
  - reinforced
  - precast
  - prestressed
- shapes
  - rectangular, I
  - T, double T's, bulb T's
  - -box
  - spandrel

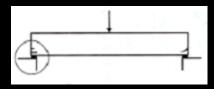


## Concrete Beams

- shear
  - vertical
  - horizontal
  - combination:
    - tensile stresses at 45°
- bearing
  - crushing

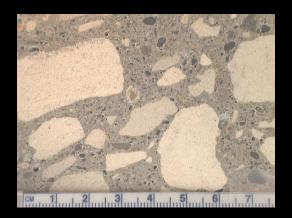


http://urban.arch.virginia.edu



## Concrete

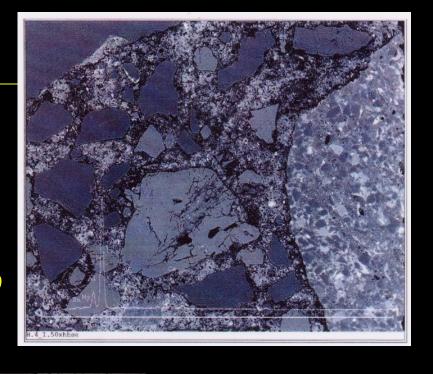
- low strength to weight ratio
- relatively inexpensive
  - Portland cement
    - types I V
  - aggregate
    - · course & fine
  - water
  - admixtures
    - air entraining
    - superplasticizers





## Concrete

- hydration
  - chemical reaction
  - workability
  - water to cement ratio
  - mix design
- fire resistant
- cover for steel
- creep & shrinkage









## Concrete

placement (not pouring!)

vibrating

screeding

floating

troweling

- curing
- finishing





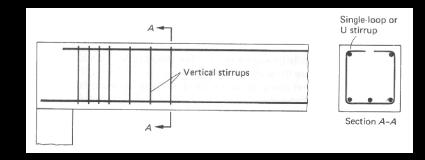


### Reinforcement

- deformed steel bars (rebar)
  - Grade 40,  $F_{v} = 40 \text{ ksi}$
  - Grade 60,  $F_v$  = 60 ksi most common
  - *Grade 75,*  $F_y = 75 \text{ ksi}$
  - US customary in # of 1/8" φ

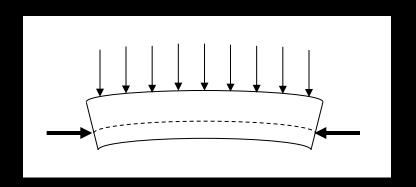


- longitudinally placed
  - bottom
  - top for compression reinforcement



## Reinforcement

- prestressing strand
- post-tensioning
- stirrups
- detailing
  - development length
  - anchorage
  - splices



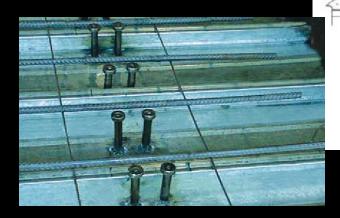


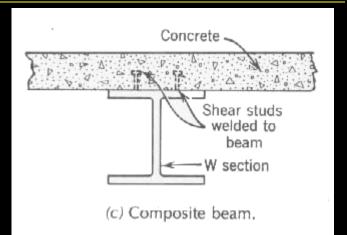
http://nisee.berkeley.edu/godden Su2014ab

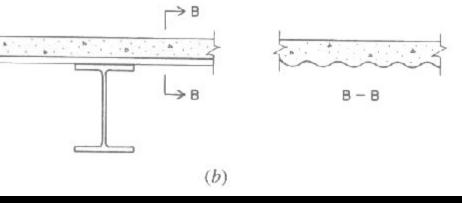
# Composite Beams

- concrete
  - in compression
- steel
  - in tension

shear studs

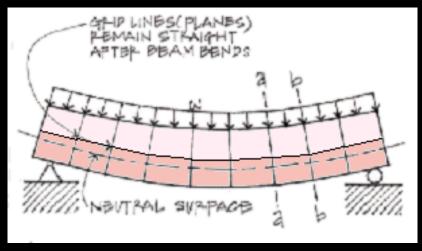


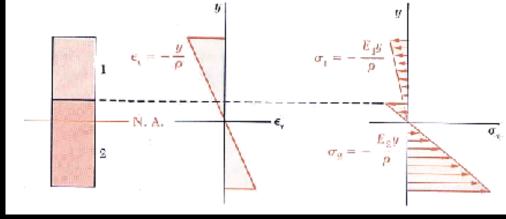




# Behavior of Composite Members

- plane sections remain plane
- stress distribution changes





$$f_1 = E_1 \varepsilon = -\frac{E_1 y}{\rho}$$

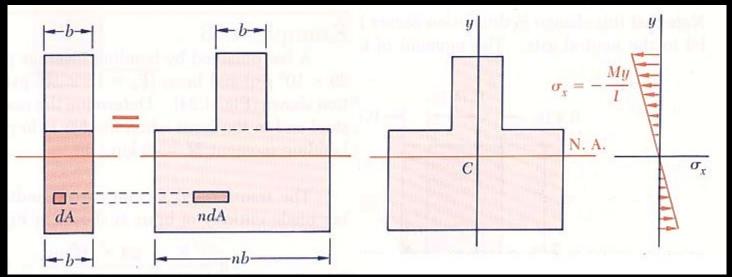
$$f_2 = E_2 \varepsilon = -\frac{E_2 y}{\rho}$$

## Transformation of Material

n is the ratio of E's

$$n=rac{E_2}{E_1}$$

 effectively widens a material to get same stress distribution



# Stresses in Composite Section

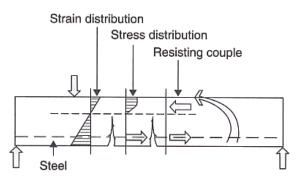
- with a section transformed to one material, new I
  - stresses in that material are determined as usual
  - stresses in the other material need to be adjusted by n

$$n = rac{E_2}{E_1} = rac{E_{steel}}{E_{concrete}}$$

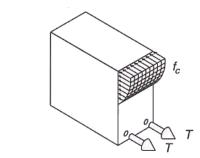
$$f_c = -rac{My}{I_{transformel}}$$

$$f_{s} = -\frac{Myn}{I_{transformel}}$$

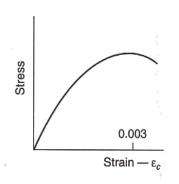
## Reinforced Concrete - stress/strain



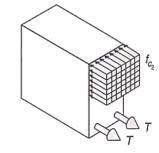
Stresses in the concrete above the neutral axis are compressive and nonlinearly distributed. In the tension zone below the neutral axis, the concrete is assumed to be cracked and the tensile force present to be taken up by reinforcing steel.



Actual stress distribution near ultimate strength (nonlinear).



Typical stress-strain curve for concrete.



Ultimate strength analysis. (A rectangular stress block is used to idealize the actual stress distribution. Calculations are based on ultimate loads and failure stresses.)

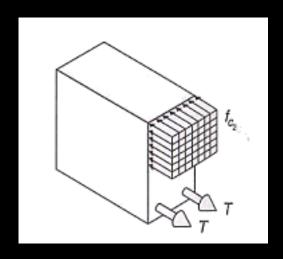


Working stress analysis. (Concrete stress distribution is assumed to be linear. Service loads are used in calculations.)

FIGURE 6-37 Reinforced concrete beams.

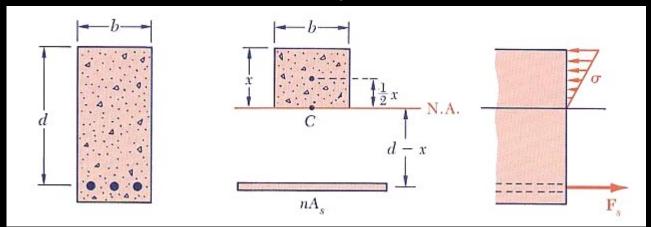
# Reinforced Concrete Analysis

- for stress calculations
  - steel is transformed to concrete
  - concrete is in compression above n.a. and represented by an equivalent <u>stress block</u>
  - concrete takes <u>no tension</u>
  - steel takes tension
  - force <u>ductile</u> failure



## Location of n.a.

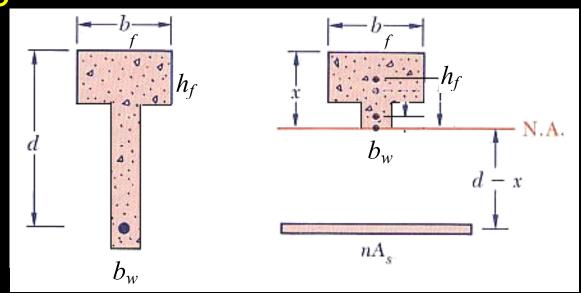
- ignore concrete below n.a.
- transform steel
- same area moments, solve for x



$$bx \cdot \frac{x}{2} - nA_s(d - x) = 0$$

## T sections

 n.a. equation is different if n.a. below flange



$$b_f h_f \left( x - \frac{h_f}{2} \right) + \left( x - h_f \right) b_w \frac{\left( x - h_f \right)}{2} - n A_s (d - x) = 0$$

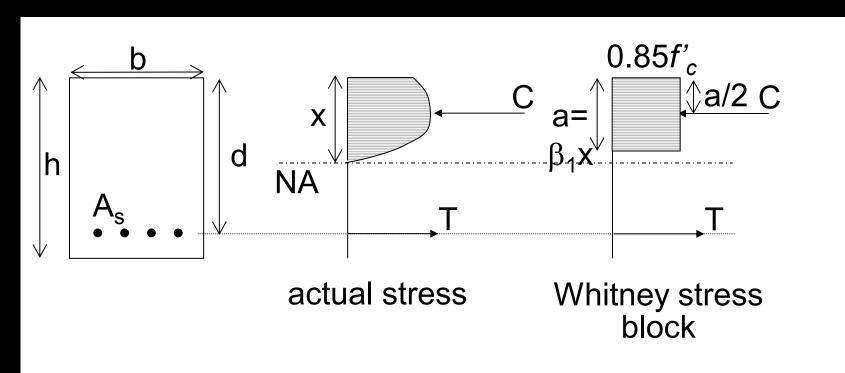
#### ACI Load Combinations\*

- 1.4D
- $1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
- $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$
- $1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$
- 1.2D + 1.0E + 1.0L + 0.2S
- 0.9D + 1.0W
- 0.9D + 1.0E

\*can also use old ACI factors

# Reinforced Concrete Design

stress distribution in bending



Wang & Salmon, Chapter 3

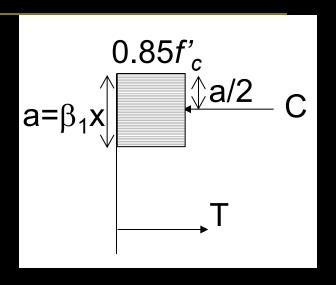
## Force Equations

- $C = 0.85 \, f'_c ba$
- $T = A_s f_v$
- where
  - $-f'_c$  = concrete compressive strength
  - a = height of stress block

$$-\beta_1$$
 = factor based on  $f_c$ 

$$- y = location to the n a$$

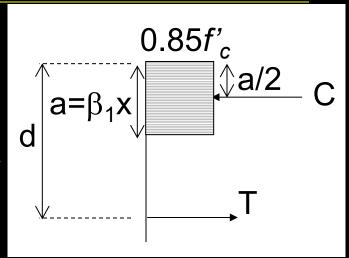
- -x = location to the n.a.
- -b = width of stress block
- $f_v = steel yield strength$
- $-A_s$  = area of steel reinforcement



 $\beta_1 = 0.85 - \left(\frac{f_c' - 4000}{1000}\right)(0.05) \ge 0.65$ 

## Equilibrium

- T = C
- $M_n = T(d-a/2)$ 
  - -d = depth to the steel n.a.
- with  $A_s$   $-a = \frac{A_s f_y}{0.85 f' b}$



\*
$$\phi = 0.65 + (\varepsilon_t - \varepsilon_y) \frac{0.25}{(0.005 - \varepsilon_y)} \ge 0.65$$

- $-M_u \le \phi M_n$   $\phi = 0.9$  for flexure\*
- $-\phi M_n = \phi T(d-a/2) = \phi A_s f_y (d-a/2)$

## Over and Under-reinforcement

- over-reinforced
  - steel won't yield
- under-reinforced
  - steel will yield
- reinforcement ratio





http://people.bath.ac.uk/abstji/concrete\_video/virtual\_lab.htm

$$- \rho = \frac{A_s}{hd}$$

- use as a design estimate to find A, b, d
- max  $\rho$  is found with  $\varepsilon_{\text{steel}} \ge 0.004$  (not  $\rho_{\text{bal}}$ )

— \*with 
$$arepsilon_{ ext{steel}} \geq 0.005$$
,  $\phi = 0.9$ 

# A<sub>s</sub> for a Given Section

- several methods
  - guess a and iterate
    - 1. guess a (less than n.a.)

2. 
$$A_s = \frac{0.85 f_c' ba}{f_v}$$

3. solve for a from  $M_u = \phi A_s f_v (d-a/2)$ 

$$a = 2 \left( d - \frac{M_u}{\phi A_s f_y} \right)$$

4. repeat from 2. until a from 3. matches a in 2.

# A<sub>s</sub> for a Given Section (cont)

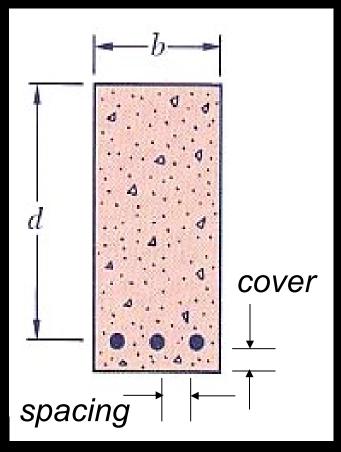
- chart method
  - Wang & Salmon Fig. 3.8.1  $R_n$  vs.  $\rho$

1. calculate 
$$R_n = \frac{M_n}{bd^2}$$

- 2. find curve for  $f'_c$  and  $f_v$  to get  $\rho$
- 3. calculate  $A_s$  and a
- simplify by setting h = 1.1d

## Reinforcement

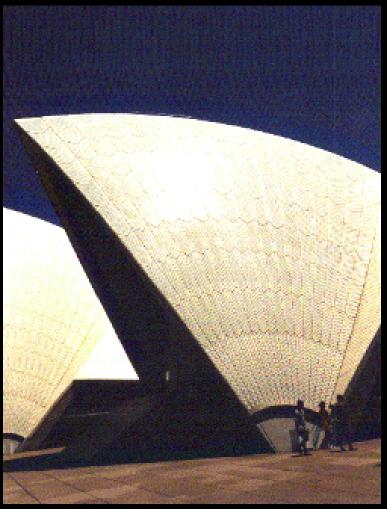
- min for crack control
- required  $A_s = \frac{3\sqrt{f_c'}}{f_v}(bd)$
- not less than  $A_s = \frac{200}{f_v} (bd)$
- $A_{s-max}$ :  $a = \beta_1(0.375d)$
- typical cover
  - 1.5 in, 3 in with soil
- bar spacing



# Shells







http:// nisee.berkeley.edu/godden Architectural Structures
Concrete Beams 27 ARCH 331

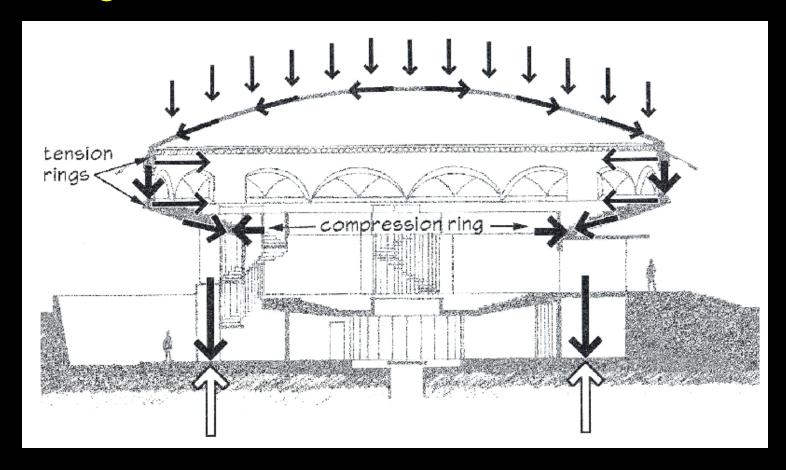
## Annunciation Greek Orthodox Church

• Wright, 1956



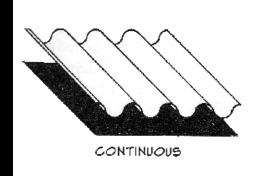
# Annunciation Greek Orthodox Church

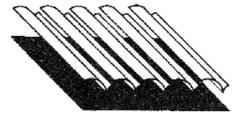
• Wright, 1956



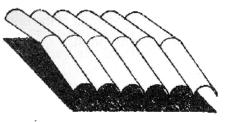
# Cylindrical Shells

- can resist tension
- shape adds "depth"

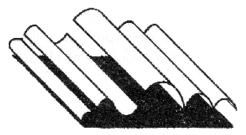




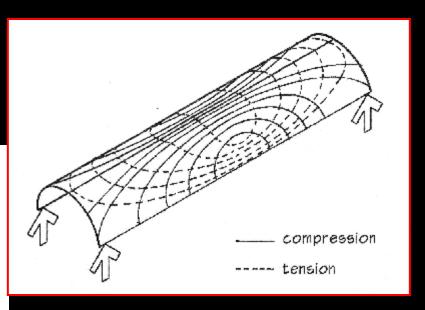
DISCONTINUOUS (to admit daylight)





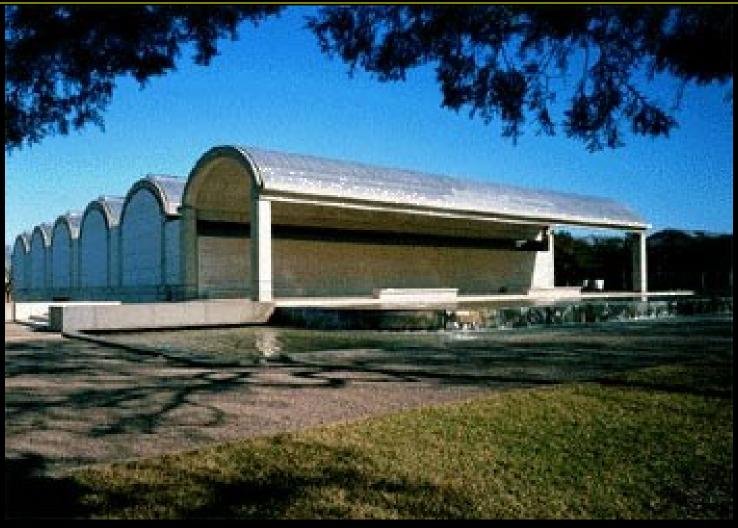


FREE FORM



- not vaults
- barrel shells

# Kimball Museum, Kahn 1972

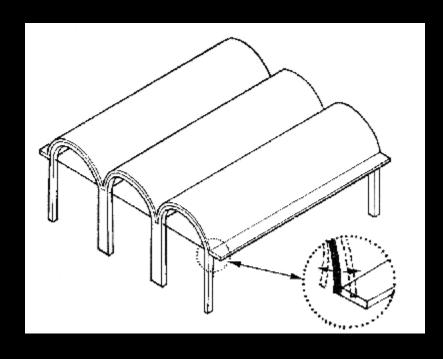


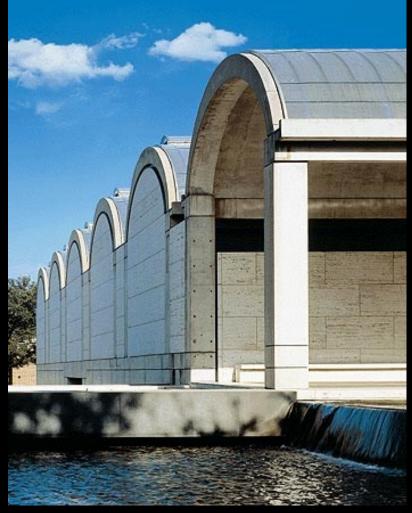
Concrete Beams 31 Lecture 19

Architectural Structures
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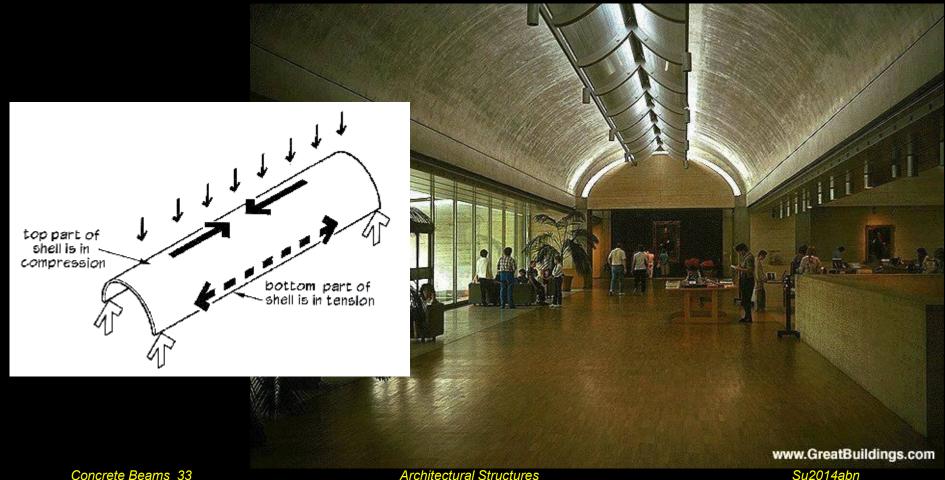
outer shell edges





# Kimball Museum, Kahn 1972

skylights at peak



# Approximate Depths

Concret Lecture

