ARCH 331

ARCHITECTURAL STRUCTURES

LECTURE NOTE SET
Spring 2014

by

Anne Nichols

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TEXAS A&M UNIVERSITY

DEPARTMENT OF ARCHITECTURE
SYLLABUS

Course title and number	ARCH 331 – Architectural Structures (section 500)
Term	Spring 2014
Meeting times and location	Lecture: 11:10am -12:25 pm T,R; Lab: 12:45-2 pm in 107A Langford A (1:40 total)

Course Description and Prerequisites

Architectural Structures. (2-2). Credit 3. Physical principles that govern statics and strength of materials through the design of architectural structures from a holistic view, in the context of architectural ideas and examples; introduction to construction, behavior of materials, and design considerations for simple and complex structural assemblies; computer applications. Prerequisites: Junior or senior classification in environmental design; MATH 142 or equivalent; PHYS 201.

Learning Outcomes or Course Objectives

- The student will be able to read a text or article about structural technology, identify the key concepts and related equations, and properly apply the concepts and equations to appropriate structural problems (**relevance)**. The student will also be able to define the answers to key questions in the reading material. The student will be able to evaluate their own skills, or lack thereof, with respect to reading and comprehension of structural concepts, **clarity** of written communication, reasonable determination of **precision** in numerical data, and **accuracy** of computations.
- The student will be able to read a problem statement, interpret the structural wording in order to identify the concepts and select equations necessary to solve the problem presented (**significance)**. The student will be able to identify common steps in solving structural problems regardless of the differences in the structural configuration and loads, and apply these steps in a clear and structured fashion (**logic)**. The student will draw upon existing mathematical and geometrical knowledge to gather information, typically related to locations and dimensions, provided by representational drawings or models of structural configurations, and to present information, typically in the form of plots that graph variable values. The student will be able to draw representational structural models and diagrams, and express information provided by the figures in equation form. The student will compare the computational results in a design problem to the requirements and properly decide if the requirements have been met. The student will take the corrective action to meet the requirements.
- The student will create a structural model with a computer application based on the concepts of the behavior and loading of the structural member or assemblage. The student will be able to interpret the modeling results and relate the results to the solution obtained by manual calculations.
- The student will be able to articulate the physical phenomena, behavior and design criteria which influence structural space and form. (**depth)** The student will be able to identify the structural purpose, label, behavior, advantages and disadvantages, and interaction of various types of structural members and assemblies. (**breadth)** The student will create a physical structure or structures using non-traditional building materials, considering material and structural behavior, in order to demonstrate the behavior and limitations of a variety of structural arrangements. The student will produce proper documentation and drawings of the size, spacing, location and connection of parts for the construction of the structure.
- The student will interact and participate in group settings to facilitate peer-learning and teaching. In addition, the student will be able to evaluate the comprehension of concepts, clarity of communication of these concepts or calculations, and the precision and accuracy of the data used in the computations in the work of their peers.

Instructor Information

Name	Dr, Anne Nichols, Associate Professor of the Practice
Telephone number	(979) 845-6540
Email address	anichols@tamu.edu
Office hours	12:30-2 pm MW, 1-2 pm TR (*and by appointment)*
Office location	A413 Langford

Textbook and/or Resource Material

Required Text:

- Statics and Strength of Materials –Foundations for Structural Design, Onouye, (2005) Pearson - Prentice Hall, ISBN 0-13-111837-4

Recommended Texts:

- A Structures Primer, Kaufman, (2010) Prentice Hall, ISBN 978-0-13-230256-3
- Understanding Structures, Moore, (1999) McGraw-Hill, ISBN 9780070432536

References:

- ACI 318-11 Code and Commentary
- AISC 14th ed. Steel Construction Manual
- Masonry Joint Structural Code
- National Design Specifications for Wood

Grading Policies

Students should refer to the Academic section in Student Rules and Regulations http://student-rules.tamu.edu.

Assignments:

- Due as stated on the assignment statements.
- Only *one* assignment without University excuse may be turned in for credit no later than one week after the due date **and** before final exams begin. All other assignments will receive *no credit* if late without a recognized excuse or after final exams have begun.
- Assignments with incorrect formatting will be penalized.

Format:

Date	Name	Course
Given:		
Find:		
Solution:		
:		

Quizzes:

- Quizzes will be given at any time during the class period. Make-up quizzes without an excuse will not be given.
- Practice quizzes will be posted electronically.
- No quiz scores will be "dropped".
- *Use of cell phones with a calculator application during quizzes and exams is prohibited.*

Final Exam:

- The final exam will be comprehensive and is officially scheduled for ***3-5 PM Friday, May 2***.

Teaching Assistant:

- Victoria Garcia (m2310_3@neo.tamu.edu)

Structures Help Desk:

- Miray Oktem (mrycan@neo.tamu.edu)
- ARCA129 845-6580 Posted Hours (link)

Other Resources:

- The Student Learning Center provides tutoring in math and physics. (http://slc.tamu.edu/tutoring.shtml) Other tutoring services are listed at http://scs.tamu.edu/sites/default/files/tutoring.pdf The Academic Success Center offers workshops at http://us.tamu.edu/Undergraduate-Studies/Academic-Success-Center

Grievances:

- For grievances other than those listed in Part III in Texas A&M University Student Rules: http://student-rules.tamu.edu/ the *instructor* must be the first point of contact.

Other Pertinent Grading Information (Rubric Included)

The levels listed for graded work (projects, quizzes, exams) and pass-fail work (assignments) ***must both be met*** to earn the course letter grade:

Letter Grade	*Graded work*	*Pass-Fail work*
A	A average (90-100%)	Pass for 90 to 100%of assignments
B	B average (80-89%)	Pass for 83 to 100% of assignments
C	C average (70-79%)	Pass for 75 to 100% of assignments
D	D average (60-69%)	Pass for 65 to 100% of assignments
F	F average (<59%)	Pass for 0% to 100% of assignments

Graded work: This typically constitutes 6 quizzes, a learning portfolio (worth 1.5 quizzes) and a final exam (worth 3 quizzes). This equates to proportions of approximately 57% to quizzes, 14% to the learning portfolio, and 29% to the final exam.

Pass/fail work: This constitutes all practice assignments and projects, each with a value of 1 unit. Criteria for passing is *at least* 75% completeness and correctness along with every problem attempted. Percent effort expected for a problem in a practice assignment is provided on the assignment statement. This is considered a lab course and the assignments **are required work** with credit given for competency. The work is necessary to apply the material and prepare for the quizzes and exam. It is expected that this work will be completed with assistance or group participation, but all *graded* work is only by the individual.

Attendance Policies

The University views class attendance as the responsibility of an individual student. Attendance is essential to complete the course successfully. University rules related to excused and unexcused absences are located on-line at http://student-rules.tamu.edu/rule07

Project due dates will be provided in the project statements. Students should contact the instructor if work is turned in late due to an absence that is excused under the University's attendance policy. In such cases the instructor will either provide the student an opportunity to make up any quiz, exam or other graded activities or provide a satisfactory alternative to be completed within 30 calendar days from the last day of the absence. There will be no opportunity for students to make up work missed because of an unexcused absence.

Other Pertinent Attendance Information

Absences related to illness or injury must be documented according to http://shs.tamu.edu/attendance.htm *including* the Explanatory Statement for Absence from class for 3 days or less. Doctor visits not related to immediate illness or injury are not excused absences.

Lecture, Lab, and Textbook:

- The lecture slides should be viewed prior to class. Class will be reserved for review of the lectures. Lab will consist of problem solving requiring the textbook. The lecture slide handouts are available on the class web page and eCampus.
- Attendance is required for both lecture and lab.
- *Use of electronic devices during lecture and lab is prohibited.*

Notes:

- The notes and related handouts are available on the class web page at http://faculty.arch.tamu.edu/anichols/331frame.html, or on eCampus. A bound set can be purchased from the Notes-n-Quotes at 701 W. University, directly across from the Mitchell Physics Building in the Northgate Neighborhood.

eCampus:

- eCampus is the on-line course system useful for downloading files, uploading assignments, reading messages and replying, as well as posting scores; and is accessed with your neo account. This will be used to post class materials, questions and responses by class members and the instructor, and scores. It can be accessed at http://ecampus.tamu.edu/

revised 3/26/14

Course Topics, Calendar of Activities, Major Assignment Dates

Tentative Schedule *(subject to change at any time throughout the semester)*
Note: Materials in the Class Note Set not specifically mentioned above are provided as references or aids.

Week	Topic	Required Reading/Problems
1	1. Design Loads and Structural Performance Requirements	**Read*:** Ch. 1, § 5.1 **Solve:** Assignment 1 (*start*)
	2. Structural Systems, Planning and Design	**Read**: Appendix B; note sets 2.1, 2.2, 2.3 & 2.4 **Reference:** *note set 2.5*
2	3. Forces and Moments	**Read:** Ch. 2; note sets 3.1 & 3.2 **Due:** Assignment 1
	4. Equilibrium of a Point & Analysis of Planar Trusses	**Read:** § 3.1, pg. 89-95; note set 4.1 **Reference:** *note set 4.2*
3	5. Rigid Body Equilibrium & Analysis of Planar Trusses	**Read:** § 3.2, 3.3, pg. 98-110; note sets 5.1 & 5.2 **Due:** Project
	6. Mechanics of Materials	**Read:** Ch. 6; note sets 6.1, 6.2 & 6.3 **Reference:** *note set 6.4* **Due:** Assignment 2
4	7. Beam Shear and Bending	**Read:** § 8.1-8.2, note set 7 **Quiz 1**
	8. Semi-graphical Method: Shear and Bending Moment Diagrams	**Read:** § 8.3-8.4; (note set 7) **Reference:** *note sets 8.1 & 8.2* **Due:** Assignment 3
5	9. Beam Section Properties	**Read:** § 7.1-7.4; note sets 9.1 & 9.2
	10. Beam Stresses	**Read:** § 9.1-9.4; note set 10.1 **Reference:** *note set 10.2* **Due:** Assignment 4
6	11. Other Beams and Pinned Frames	**Read:** § 4.2, pg. 73; note set 11 **Quiz 2**
	12. Rigid Frames - Compression & Buckling	**Read:** § 10.1,10.2 & 10.5; note sets 12.1 & 12.2 **Reference:** *note set 12.3* **Due:** Assignment 5
7	13. Design Loads, Codes and Methodology	**Read:** § 5.1; note set 13.1 **Reference:** *note sets 13.2, 13.3, 13.4, 13.5*
	14. System Assemblies and Load Tracing	**Read:** § 5.2, 5.3, 4.4; note set 14 **Due:** Assignment 6
8	15. Wood Construction Materials & Beam Design	**Read:** § 9.5-9.6; note sets 15.1 & 15.2 **Quiz 3**
	16. Column Design	**Read:** § 10.4; note set 15.1 **Due:** Assignment 7
9	17. Joints and Connection Stresses	**Read:** note set 15.1
	18. Steel Construction Materials & Beam Design	**Read:** note set 18 **Due:** Assignment 8
10	19. Trusses, Decks & Plate Girders	**Read:** pg. 98-110; note set 18 **Reference:** *note set 5.2* **Quiz 4**
	20. Column Design & Tension Members	**Read:** § 10.3; note set 18 **Due:** Assignment 9
11	21. Bolted Connections & Welds	**Read:** note set 18
	22. Concrete Construction Materials & Beam Design	**Read:** note set 22.1 **Reference:** *note set 22.2* **Due:** Assignment 10
12	23. T-beams & Slabs	**Read:** note set 22.1 **Quiz 5**
	24. Shear, Torsion, Reinforcement & Deflection	**Read:** note sets 22.1 & 24 **Due:** Assignment 11

Week	Topic		Required Reading/Problems
(13)	26.	Columns & Frames	**Read:** note set 22.1 **Due:** Assignment 12
14	27.	Foundation Design & Footings	**Read:** note sets 27.1 & 27.2 **Quiz 6**
	28. Masonry Construction	Beams & Columns	**Read:** note set 28.1 **Reference:** *note sets 28.2 & 28.3* **Due:** Assignment 13 & Learning Portfolio

FINAL: 3***-5 PM Friday, May 2***

Americans with Disabilities Act (ADA)

The Americans with Disabilities Act (ADA) is a federal anti-discrimination statute that provides comprehensive civil rights protection for persons with disabilities. Among other things, this legislation requires that all students with disabilities be guaranteed a learning environment that provides for reasonable accommodation of their disabilities. If you believe you have a disability requiring an accommodation, please contact Disability Services, in Cain Hall, Room B118, or call 845-1637. For additional information visit http://disability.tamu.edu

Academic Integrity

"An Aggie does not lie, cheat, or steal, or tolerate those who do."

Upon accepting admission to Texas A&M University, a student immediately assumes a commitment to uphold the Honor Code, to accept responsibility for learning, and to follow the philosophy and rules of the Honor System. Students will be required to state their commitment on examinations, research papers, and other academic work. Ignorance of the rules does not exclude any member of the TAMU community from the requirements or the processes of the Honor System. *For additional information please visit: http://aggiehonor.tamu.edu*

Care of Facilities

The use of spray paint or other surface-altering materials is not permitted in the Langford Complex, except in designated zones. Students who violate this rule will be liable for the expenses associated with repairing damaged building finishes and surfaces. At the end of the semester, your area must be clean of all trash.

Studio Policy (required of all studios)

All students, faculty, administration and staff of the Department of Architecture at Texas A&M University are dedicated to the principle that the Design Studio is the central component of an effective education in architecture. They are equally dedicated to the belief that students and faculty must lead balanced lives and use time wisely, including time outside the design studio, to gain from all aspects of a university education and world experiences. They also believe that design is the integration of many parts, that process is as important as product, and that the act of design and of professional practice is inherently interdisciplinary, requiring active and respectful collaboration with others.

Students and faculty in every design studio will embody the fundamental values of optimism, respect, sharing, engagement, and innovation. Every design studio will therefore encourage the rigorous exploration of ideas, diverse viewpoints, and the integration of all aspects of architecture (practical, theoretical, scientific, spiritual, and artistic), by providing a safe and supportive environment for thoughtful innovation. Every design studio will increase skills in professional communication, through drawing, modeling, writing and speaking.

Every design studio will, as part of the syllabus introduced at the start of each class, include a clear statement on time management, and recognition of the critical importance of academic and personal growth, inside and outside the studio environment. As such it will be expected that faculty members and students devote quality time to studio activities, while respecting the need to attend to the broad spectrum of the academic life. Every design studio will establish opportunities for timely and effective review of both process and products. Studio reviews will include student and faculty peer review. Where external reviewers are introduced, the design studio instructor will ensure that the visitors are aware of the Studio Culture Statement and recognize that the design critique is an integral part of the learning experience. The design studio will be recognized as place for open communication and movement, while respecting the needs of others, and of the facilities.

Important Links Below

Department of Architecture Website	http://dept.arch.tamu.edu/
Department Financial Assistance	http://dept.arch.tamu.edu/financial-assistance/
Academic Calendar	http://admissions.tamu.edu/registrar/general/calendar.aspx
Final Exam Schedule Online	http://admissions.tamu.edu/registrar/general/finalschedule.aspx
On-Line Catalog	http://catalog.tamu.edu
Student Rules	http://student-rules.tamu.edu/
Aggie Honor System Office	http://aggiehonor.tamu.edu/
American Institute of Architecture website	http://www.aia.org/index.htm

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
JANUARY	5	6	7	8	9	10 last day to register	11
	12	13 classes begin	14 **Lect 1**	15	16 **Lect 2**	17 last day to add	18
	19	20 King Holiday	21 **Lect 3** #1 due	22	23 **Lect 4**	24	25
	26	27	28 **Lect 5** project due	29	30 **Lect 6** #2 due	31	1
FEBRUARY	2	3	4 **Lect 7** Quiz 1	5	6 **Lect 8** #3 due	7	8
	9	10	11 **Lect 9**	12	13 **Lect 10** #4 due	14	15
	16	17	18 **Lect 11** Quiz 2	19	20 **Lect 12** #5 due	21	22
	23	24	25 **Lect 13**	26	27 **Lect 14** #6 due	28	1
MARCH	2	3 mid-term grades due	4 **Lect 15** Quiz 3	5	6 **Lect 16** #7 due	7	8
	9	10	11	12 Spring Break	13	14	15
	16	17	18 **Lect 17**	19	20 **Lect 18** #8 due	21	22
	23	24	25 **Lect 19** Quiz 4	26	27 **Lect 20** #9 due	28	29
	30	31	1 **Lect 21**	2	3 **Lect 22** #10 due	4	5
APRIL	6	7	8 **Lect 23** Quiz 5	9	10 **Lect 24** #11 due pre-registration begins	11	12
	13	14 last day to Q-drop	15 **Lect 25**	16	17 **Lect 26** #12 due	18 Reading Day	19
	20	21 Muster	22 **Lect 27** Quiz 6	23	24 **Lect 28** #13 & portfolio due	25	26
	27	28 (dead day) Monday classes	29 (dead day) Friday classes	30 Reading Days	1	2 Final exams 3-5 pm 331 FINAL	3
MAY	4	5	6	7	8	9 Commencement (and Saturday)	10
	11	12 Grades due	13	14	15	16	17
	18	19	20	21	22	23	24

ARCH 331. Student Understandings

1) I understand that there are intellectual standards in this course and that I am responsible for monitoring my own learning.___________

2) I understand that the class will focus on practice, not on lecture.___________

3) I understand that I am responsible for preparing for lecture with the assigned reading and lecture show by internalizing key concepts, recognizing key questions, and evaluating what makes sense and what doesn't make sense to me.___________

4) I understand that I will be held regularly responsible for assessing my own work using criteria and standards discussed in class.___________

5) I understand that if at any time in the semester I feel unsure about my "grade", I may request an assessment from the instructor.___________

6) I understand that there are **13 practice assignments**, one due every week during the bulk of the semester. ___________

7) I understand that I will occasionally be required to assess the work of my classmates in an objective manor using the same criteria and standards used to assess my own work.___________

8) I understand that there are **6 graded quizzes**, one given every other week during the bulk of the semester. ___________

9) I understand that there is a final exam in the course.___________

10) I understand that I must do a Learning Portfolio, which is a self-evaluation that makes my "case" for receiving a particular grade using criteria provided in class and citing evidence from my work across the semester.___________

11) I understand that the work of the course requires **Consistent classroom attendance** and active participation.___________

12) I understand that I will regularly be required to demonstrate that I have prepared for lecture.___________

13) I understand that the class will not be graded on a curve. I understand that it is theoretically possible for the whole class to get an A or an F.___________

14) I understand the basis of the final grade as outlined in the syllabus.___________

15) I understand that since the final grade is based on percentages from graded work and competency on assignments as outlined in the syllabus, that the minimum level of both must be satisfied to obtain the letter grade. The criteria for assignments that are considered "passing" is outlined in the syllabus section on Learning Objectives.___________

NAME ____________________________________ **DATE**_________

signature

____

printed name

List of Symbol Definitions

a long dimension for a section subjected to torsion (in, mm);
acceleration (ft/sec^2, m/sec^2);
width of the base of a retaining wall for pressure calculation (ft, m);
equivalent square column size in spread footing design (in, ft, mm, m);
distance used in beam formulas (ft, m);
depth of the effective compression block in a concrete beam (in, mm)

$\mathcal{a}$ area bounded by the centerline of a thin walled section subjected to torsion (in^2, mm^2)

A area, often cross-sectional (in^2, ft^2, mm^2, m^2)

A_b area of a bolt (in^2, mm^2)

A_e <u>effective</u> net area found from the product of the net area A_n by the shear lag factor U (in^2, ft^2, mm^2, m^2)

A_g gross area, equal to the total area ignoring any holes or reinforcement (in^2, ft^2, mm^2, m^2)

A_{gv} gross area subjected to shear for block shear rupture (in^2, ft^2, mm^2, m^2)

A_n net area, equal to the gross area subtracting any holes (in^2, ft^2, mm^2, m^2) *(see A_e)*

A_{net} net area, equal to the gross area subtracting any reinforcement (in^2, ft^2, mm^2, m^2)

A_{nt} net area subjected to tension for block shear rupture (in^2, ft^2, mm^2, m^2)

A_{nv} net area subjected to shear for block shear rupture (in^2, ft^2, mm^2, m^2)

A_p bearing area (in^2, ft^2, mm^2, m^2)

$A_{req'd}$ area required to satisfy allowable stress (in^2, ft^2, mm^2, m^2)

A_s area of steel reinforcement in concrete beam and masonry design (in^2, ft^2, mm^2, m^2)

A'_s area of steel compression reinforcement in concrete beam design (in^2, ft^2, mm^2, m^2)

A_{st} area of steel reinforcement in concrete and masonry column design (in^2, ft^2, mm^2, m^2)

A_{throat} area across the throat of a weld (in^2, ft^2, mm^2, m^2)

A_v area of concrete shear stirrup reinforcement (in^2, ft^2, mm^2, m^2)

A_{web} web area in a steel beam equal to the depth x web thickness (in^2, ft^2, mm^2, m^2)

A_1 area of column in spread footing design (in^2, ft^2, mm^2, m^2)

A_2 projected bearing area of column load in spread footing design (in^2, ft^2, mm^2, m^2)

ASD Allowable Stress Design

b width, often cross-sectional (in, ft, mm, m);
narrow dimension for a section subjected to torsion (in, mm);
number of truss members;
rectangular column dimension in concrete footing design (in, mm, m);
distance used in beam formulas (ft, m)

b_E effective width of the flange of a concrete T beam cross section (in, mm)

b_f width of the flange of a steel or concrete T beam cross section (in, mm)

b_o perimeter length for two-way shear in concrete footing design (in, ft, mm, m)

b_w width of the stem (web) of a concrete T beam cross section (in, mm)

B spread footing or retaining wall base dimension in concrete design (ft, m);
dimension of a steel base plate for concrete footing design (in, mm, m)

B_s width within the longer dimension of a rectangular spread footing that reinforcement must be concentrated within for concrete design (ft, m)

B_1 factor for determining M_u for combined bending and compression

c distance from the neutral axis to the top or bottom edge of a beam (in, mm, m);
distance from the center of a circular shape to the surface under torsional shear strain (in, mm, m); rectangular column dimension in concrete footing design (in, mm, m);
the distance from the top of a masonry beam to the neutral axis

c_i distance from the center of a circular shape to the inner surface under torsional shear strain (in, mm, m)

c_o distance from the center of a circular shape to the outer surface under torsional shear strain (in, mm, m)

c_1 coefficient for shear stress for a rectangular bar in torsion

c_2 coefficient for shear twist for a rectangular bar in torsion

CL, ℄ center line

C compression label;
compression force (lb, kips, N, kN):
dimension of a steel base plate for concrete footing design (in, mm, m)

C_b lateral torsional buckling modification factor for moment in ASD & LRFD steel beam design, C_b = 1 for simply supported beams (0 moments at the ends)

C_c column slenderness classification constant for steel column design;
compressive force in the concrete of a doubly reinforced concrete beam (lb, k, N, kN)

C_C curvature factor for laminated arch design

C_D load duration factor for wood design

C_f form factor for circular sections or or square sections loaded in plane of diagonal for wood design

C_{fu} flat use factor for other than decks in wood design

C_F size factor for wood design

C_H shear stress factor for wood design

C_i incising factor for wood design

C_L beam stability factor for wood design

C_m modification factor for combined stress in steel design;
compression force in the masonry for masonry design (lb, k, N, kN)

C_M wet service factor for wood design

C_p column stability factor for wood design

C_r repetitive member factor for wood design

C_v web shear coefficient for steel design

C_V volume factor for glue laminated timber design

C_s compressive force in the compression steel of a doubly reinforced concrete beam (lb, k, N, KN)

C_t temperature factor for wood design

d depth, often cross-sectional (in, mm, m);
diameter (in, mm, m);
perpendicular distance from a force to a point in a moment calculation (in, ft, mm, m);
effective depth from the top of a reinforced concrete or masonry beam to the centroid of the tensile steel (in, ft, mm, m):
critical cross section dimension of a rectangular timber column cross section related to the profile (axis) for buckling (in, mm, m);
symbol in calculus to represent a very small change (like the greek letters for d, *see* δ & Δ)

d' effective depth from the top of a reinforced concrete beam to the centroid of the compression steel (in, ft, mm, m)

d_b bar diameter of a reinforcing bar (in, mm);
nominal bolt diameter (in, mm)

d_f depth of a steel column flange (wide flange section) (in, mm)

d_x difference in the x direction between an area centroid ($\bar{x}$) and the centroid of the composite shape ($\hat{x}$) (in, mm)

d_y difference in the y direction between an area centroid ($\bar{y}$) and the centroid of the composite shape ($\hat{y}$) (in, mm)

D diameter of a circle (in, mm, m);
dead load for LRFD design

DL dead load

e eccentric distance of application of a force (P) from the centroid of a cross section (in, mm)

E modulus of elasticity (psi; ksi, kPa, MPa, GPa);
earthquake load for LRFD design

E_c modulus of elasticity of concrete (psi; ksi, kPa, MPa, GPa)

E_s modulus of elasticity of steel (psi; ksi, kPa, MPa, GPa)

f symbol for stress (psi, ksi, kPa, MPa)

f_a calculated axial stress (psi, ksi, kPa, MPa)

f_b calculated bending stress (psi, ksi, kPa, MPa)

f_c calculated compressive stress (psi, ksi, kPa, MPa)

f'_c concrete design compressive stress (psi, ksi, kPa, MPa)

f_{cr} calculated column stress based on the critical column load P_{cr} (psi, ksi, kPa, MPa)

f_m calculated compressive stress in masonry (psi, ksi, kPa, MPa)

f'_m masonry design compressive stress (psi, ksi, kPa, MPa)

f_p calculated bearing stress (psi, ksi, kPa, MPa)

f_s stress in the steel reinforcement for concrete or masonry design (psi, ksi, kPa, MPa)

f_s' compressive stress in the compression reinforcement for concrete beam design (psi, ksi, kPa, MPa)

f_t calculated tensile stress (psi, ksi, kPa, MPa)

f_v calculated shearing stress (psi, ksi, kPa, MPa)

f_x combined stress in the direction of the major axis of a column (psi, ksi, kPa, MPa)

f_y yield stress (psi, ksi, kPa, MPa)

F force (lb, kip, N, kN);
capacity of a nail in shear (lb, kip, N, kN);
symbol for allowable stress in design codes (psi, ksi, kPa, MPa);
fluid load for LRFD design

F_a allowable axial stress (psi, ksi, kPa, MPa)

F_b allowable bending stress (psi, ksi, kPa, MPa)

F_b' allowable bending stress for combined stress for wood design (psi, ksi, kPa, MPa)

F_c allowable compressive stress (psi, ksi, kPa, MPa)

$F_{c\perp}$ allowable compressive stress perpendicular to the wood grain (psi, ksi, kPa, MPa)

$F_{connector}$ resistance capacity of a connector (lb, kips, N, kN)

F_{cE} intermediate compressive stress for ASD wood column design dependant on material (psi, ksi, kPa, MPa)

F_{cr} flexural buckling (column) stress in ASD and LRFD (psi, ksi, kPa, MPa)

F_c' allowable compressive stress for ASD wood column design (psi, ksi, kPa, MPa)

F^*_c intermediate compressive stress for ASD wood column design dependant on load duration (psi, ksi, kPa, MPa)

F_e elastic critical buckling stress is steel design

F_{EXX} yield strength of weld material (psi, ksi, kPa, MPa)

$F_{horizontal\text{-}resist}$ resultant frictional force resisting sliding in a footing or retaining wall (lb, kip, N, kN)

F_n nominal strength in LRFD steel design (psi, ksi, kPa, MPa)
nominal tension or shear strength of a bolt (psi, ksi, kPa, MPa)

F_p allowable bearing stress parallel to the wood grain (psi, ksi, kPa, MPa)

F_s allowable tensile stress in reinforcement for masonry design (psi, ksi, kPa, MPa)

$F_{sliding}$ resultant force causing sliding in a footing or retaining wall (lb, kip, N, kN)

F_t allowable tensile stress (psi, ksi, kPa, MPa)

F_v allowable shear stress (psi, ksi, kPa, MPa);
allowable shear stress in a welded connection

F_x force component in the x coordinate direction (lb, kip, N, kN)

F_y force component in the y coordinate direction (lb, kip, N, kN);
yield stress (psi, ksi, kPa, MPa)

F_{yw} yield stress in the web of a steel wide flange section (psi, ksi, kPa, MPa)

F_u ultimate stress a material can sustain prior to failure (psi, ksi, kPa, MPa)

F.S. factor of safety

g acceleration due to gravity, 32.17 ft/sec^2, 9.807 m/sec^2;
gage spacing of staggered bolt holes (in, mm)

G shear modulus (psi; ksi, kPa, MPa, GPa);
gigaPascals (10^9 Pa or 1 kN/mm^2);
relative stiffness of columns to beams in a rigid connection *(see Ψ)*;
specific gravity (ie. factor multiplied by density of water to get density)

h depth, often cross-sectional (in, ft, mm, m);
height (in, ft, mm, m);
sag of a cable structure (ft, m);
effective height of a wall or column (*see* ℓ_e)

h_c height of the web of a wide flange steel section (in, ft, mm, m)

h_f depth of a flange in a T section (in, ft, mm, m);
height of a concrete spread footing (in, ft, mm, m)

H hydraulic soil load for LRFD design;
height of retaining wall (ft, m)

H_A horizontal force due to active soil pressure (lb, k, N, kN)

I moment of inertia (in^4, mm^4, m^4)

$\bar{I}$ moment of inertia about the centroid (in^4, mm^4, m^4)

I_c moment of inertia about the centroid (in^4, mm^4, m^4)

I_{min} minimum moment of inertia of I_x and I_y (in^4, mm^4, m^4)

$I_{transformed}$ moment of inertia of a multi-material section transformed to one material (in^4, mm^4, m^4)

I_x moment of inertia with respect to an x-axis (in^4, mm^4, m^4)

I_y moment of inertia with respect to a y-axis (in^4, mm^4, m^4)

j multiplier by effective depth of masonry section for moment arm, jd (*see d)*

J, J_o polar moment of inertia (in^4, mm^4, m^4)

k kips (1000 lb);
shape factor for plastic design of steel beams, M_p/M_y;
effective length factor for columns (*also K);*
distance from outer face of W flange to the web toe of fillet (in, mm);
multiplier by effective depth of masonry section for neutral axis, kd

kg kilograms

kN kiloNewtons (10^3 N)

kPa kiloPascals (10^3 Pa)

K effective length factor with respect to column end conditions *(also k)*;
masonry mortar strength designation

K_{cE} material factor for wood column design

ℓ	length (in, ft, mm, m); cable span (ft, m)
l_d	development length for reinforcing steel (in, ft, mm, m) *(also L_d)*
l_{dc}	development length for column dowels (in, ft, mm, m)
l_{dh}	development length for hooks (in, ft, mm, m)
ℓ_e	effective length that can buckle for wood column design (in, ft, mm, m) *(also L_e)*
l_n	clear span from face of support to face of support in concrete design (in, ft, mm, m)
l_s	lap splice length in concrete design (in, ft, mm, m)
lb	pound force
L	length (in, ft, mm, m); live load for LRFD design; spread footing dimension in concrete design (ft, m)
L_b	unbraced length of a steel beam in LRFD design (in, ft, mm, m)
L_c	clear distance between the edge of a hole and edge of next hole or edge of the connected steel plate in the direction of the load (in, ft, mm, m)
L_d	development length of reinforcement in concrete (ft, m) *(also l_d)*
L_e	effective length that can buckle for column design (in, ft, mm, m) *(also ℓ_e)*
L_m	projected length for bending in concrete footing design (ft, m)
L_p	maximum unbraced length of a steel beam in LRFD design for full plastic flexural strength (in, ft, mm, m)
L_r	roof live load in LRFD design; maximum unbraced length of a steel beam in LRFD design for inelastic lateral-torsional buckling (in, ft, mm, m)
L'	length of an angle in a connector with staggered holes (in, mm); length of the one-way shear area in concrete footing design (ft, m)
LL	live load
LRFD	Load and Resistance Factor Design
m	mass (lb-mass, g, kg); meters
mm	millimeters
M	moment of a force or couple (lb-ft, kip-ft, N-m, kN-m); bending moment (lb-ft, kip-ft, N-m, kN-m); masonry mortar strength designation
M_a	required bending moment in steel ASD beam design (unified) (lb-ft, kip-ft, N-m, kN-m)
M_A	moment value at quarter point of unbraced beam length for LRFD beam design (lb-ft, kip-ft, N-m, kN-m)
M_B	moment value at half point of unbraced beam length for LRFD beam design (lb-ft, kip-ft, N-m, kN-m)

M_C moment value at three quarter point of unbraced beam length for LRFD beam design (lb-ft, kip-ft, N-m, kN-m)

M_m moment capacity of a reinforced masonry beam (lb-ft, kip-ft, N-m, kN-m)

M_n nominal flexure strength with the full section at the yield stress for LRFD steel beam design (lb-ft, kip-ft, N-m, kN-m);
nominal flexure strength with the steel reinforcement at the yield stress and compressive stress at the concrete design strength for reinforced beam design (lb-ft, kip-ft, N-m, kN-m)

$M_{overturning}$ resulting moment from all forces on a footing or retaining wall causing overturning (lb-ft, kip-ft, N-m, kN-m)

M_p (also M_{ult}) internal bending moment when all fibers in a cross section reach the yield stress (lb-ft, kip-ft, N-m, kN-m)

M_{resist} resulting moment from all forces on a footing or retaining wall resisting overturning (lb-ft, kip-ft, N-m, kN-m)

M_u maximum moment from factored loads for LRFD beam design (lb-ft, kip-ft, N-m, kN-m)

M_{ult} (also M_p) internal bending moment when all fibers in a cross section reach the yield stress (lb-ft, kip-ft, N-m, kN-m)

M_y internal bending moment when the extreme fibers in a cross section reach the yield stress (lb-ft, kip-ft, N-m, kN-m)

M_1 smaller end moment used to calculate C_m for combined stresses in a beam-column (lb-ft, kip-ft, N-m, kN-m)

M_2 larger end moment used to calculate C_m for combined stresses in a beam-column (lb-ft, kip-ft, N-m, kN-m)

MPa megaPascals (10^6 Pa or 1 N/mm^2)

n number of truss joints, nails or bolts;
modulus of elasticity transformation coefficient for steel to concrete or masonry

n.a. neutral axis (axis connecting beam cross-section centroids)

N Newtons (kg-m/sec^2);
bearing-type connection with bolt threads included in shear plane;
normal load (lb, kip, N, kN);
masonry mortar strength designation;
bearing length on a wide flange steel section (in, mm);
number of stories

o point of overturning of a retaining wall, commonly at the "toe"

o.c. on-center

O point of origin;
masonry mortar strength designation

p pitch of nail or bolt spacing (in, ft, mm, m);
pressure (lb/ft^2, kips/ft^2, N/m^2, Pa, MPa)

p_A active soil pressure (lb/ft^2, kips/ft^2, N/m^2, Pa, MPa)

P force, concentrated (point) load (lb, kip, N, kN);
axial load in a column or beam-column (lb, kip, N, kN)

P_a allowable axial load (lb, kip, N, kN);
required axial force in ASD steel design (unified) (lb, kip, N, kN)

$P_{allowable}$ allowable axial load (lb, kip, N, kN)

P_c available axial strength for steel unified design (lb, kip, N, kN)

P_{cr} critical (failure) load in column calculations (lb, kip, N, kN)

P_{dowels} nominal capacity of dowels from concrete column to footing in concrete design ((lb, kip, N, kN)

P_{e1} Euler buckling strength in steel unified design (lb, kip, N, kN)

P_n nominal column or bearing load capacity in LRFD steel and concrete design (lb, kip, N, kN);
nominal axial load for a tensile member or connection in LRFD steel (lb, kip, N, kN)

P_o maximum axial force with no concurrent bending moment in a reinforced concrete column (lb, kip, N, kN)

P_r required axial force in steel unified design (lb, kip, N, kN)

P_u factored column load calculated from load factors in LRFD steel and concrete design (lb, kip, N, kN);
factored axial load for a tensile member or connection in LRFD steel (lb, kip, N, kN)

Pa Pascals (N/m^2)

q shear flow (lb/in, kips/ft, N/m, kN/m);
soil bearing pressure (lb/ft^2, $kips/ft^2$, N/m^2, Pa, MPa)

$q_{allowed}$ allowable soil bearing pressure (lb/ft^2, $kips/ft^2$, N/m^2, Pa, MPa)

q_g gross allowed soil pressure (lb/ft^2, $kips/ft^2$, N/m^2, Pa, MPa)

q_{net} net allowed soil bearing pressure (lb/ft^2, $kips/ft^2$, N/m^2, Pa, MPa)

q_u ultimate soil bearing strength in allowable stress design (lb/ft^2, $kips/ft^2$, N/m, Pa, MPa);
factored soil bearing pressure in concrete design from load factors (lb/ft^2, $kips/ft^2$, N/m, Pa, MPa)

Q first moment area used in shearing stress calculations (in^3, mm^3, m^3):
generic axial load quantity for LRFD design *(also see R)*

$Q_{connected}$ first moment area used in shearing stress calculations for built-up beams (in^3, mm^3, m^3)

Q_x first moment area about an x axis (using y distances) (in^3, mm^3, m^3)

Q_y first moment area about an y axis (using x distances) (in^3, mm^3, m^3)

r radius of a circle or arc (in, mm, m);
radius of gyration (in, mm, m)

r_o polar radius of gyration (in, mm, m)

r_x radius of gyration with respect to an x-axis (in, mm, m)

r_y radius of gyration with respect to a y-axis(in, mm, m)

R force, reaction or resultant (lb, kip, N, kN);
radius of curvature of a beam (ft, m);
rainwater or ice load for LRFD design;
generic load quantity (force, shear, moment, etc.) for LRFD design *(also see Q)*;
radius of curvature of a laminated arch (ft, m)

R_a required strength (ASD-unified) *(also see V_a, M_a)*

R_n concrete beam design ratio = M_u/bd^2 (lb/in^2, MPa)
nominal value for LRFD design to be multiplied by ϕ *(also see P_n, M_n)*
nominal value for ASD design to be divided by the safety factor Ω

R_u design value for LRFD design based on load factors *(also see P_u, M_u)*

R_x reaction or resultant component in the x coordinate direction (lb, kip, N, kN)

R_y reaction or resultant component in the y coordinate direction (lb, kip, N, kN)

s length of a segment of a thin walled section (in, mm);
spacing of stirrups in reinforced concrete beams (in, mm);
longitudinal center-to-center spacing of any two consecutive holes (in, mm)

s.w. self-weight

S section modulus (in^3, mm^3, m^3);
snow load for LRFD design;
allowable strength per length of a weld for a given size (lb/in, kips/in, N/mm, kN/m);
masonry mortar strength designation

$S_{required}$ section modulus required to not exceed allowable bending stress (in^3, mm^3, m^3)

S_x section modulus with respect to the x-centroidal axis (in^3, mm^3, m^3)

S_y section modulus with respect to the y-centroidal axis (in^3, mm^3, m^3)

SC slip critical bolted connection

S4S surface-four-sided

t thickness (in, mm, m)

t_f thickness of the flange of a steel beam cross section (in, mm, m)

t_w thickness of the web of a steel beam cross section (in, mm, m)

T tension label;
tensile force (lb, kip, N, kN);
torque (lb-ft, k-ft, N-m, kN-m);
throat size of a weld (in, mm);
effect of thermal load for LRFD design;
period of vibration (sec)

T_s tension force in the steel reinforcement for masonry design (lb, kip, N, kN)

U shear lag factor for steel tension member design *(see A_e and A_{net})*

U_{bs} reduction coefficient for block shear rupture

v shear force per unit length (lb/ft, k/ft, N/m, kN/m) *(see q)*

V volume (in^3, ft^3, mm^3, m^3);
shear force (lb, k, N, kN);
wind speed (mi/hr, m/hr)

V_a required shear in steel ASD design (unified) (lb, kip, N, kN)

V_c shear force capacity in concrete (lb, kip, N, kN)

V_n nominal shear strength capacity for LRFD beam design (lb, kip, N, kN)

V_s shear force capacity in steel shear stirrups(lb, kip, N, kN)

V_u maximum shear from factored loads for LRFD design (lb, kip, N, kN);
shear at a distance *d* away from the face of support for reinforced concrete beam design (lb, kip, N, kN)

V_{u1} maximum one-way shear from factored loads for LRFD beam design (lb, kip, N, kN)

V_{u2} maximum two-way shear from factored loads for LRFD beam design (lb, kip, N, kN)

w load per unit length on a beam (lb/ft, k/ft, N/m, kN/m) *(also* ω);
load per unit area (lb/ft^2, $kips/ft^2$, N/m^2, Pa, MPa);
width dimension (in, ft, mm, m)

$w_{adjusted}$ adjusted distributed load for equivalent live load deflection limit (lb/ft, kip/ft, N/m, kN/m)

w_c weight of reinforced concrete per unit volume (lb/ft^3, N/m^3)

$w_{equivalent}$ the equivalent distributed load derived from the maximum bending moment (lb/ft, kip/ft, N/m, kN/m)

w_u factored load per unit length on a beam from load factors (lb/ft, kip/ft, N/m, kN/m);
factored load per unit area on a surface from load factors (lb/ft^2, kip/ft^2, N/m^2, kN/m^2)

W weight (lb, kip, N, kN);
total load from a uniform distribution (lb, kip, N, kN);
wind load for LRFD design;
wide flange shape designation (i.e. W 21 x 68)

x a distance in the x direction (in, ft, mm, m);
the distance from the top of a concrete beam to the neutral axis

$\bar{x}$ the distance in the x direction from a reference axis to the centroid of a shape (in, mm)

$\hat{x}$ the distance in the x direction from a reference axis to the centroid of a composite shape (in, mm)

X bearing-type connection with bolt threads excluded from shear plane

y a distance in the y direction (in, ft, mm, m);
distance from the neutral axis to the y-level of a beam cross section (in, mm)

$\bar{y}$ the distance in the y direction from a reference axis to the centroid of a shape (in, mm)

$\hat{y}$ the distance in the y direction from a reference axis to the centroid of a composite shape (in, mm)

Z plastic section modulus of a steel beam (in^3, mm^3);
lateral design value for a single fastener in a timber connection (lb/nail, k/bolt)

Z_x plastic section modulus of a steel beam with respect to the x axis (in^3, mm^3)

' symbol for feet

" symbol for inches

symbol for pounds

= symbol for equal to

≈ symbol for approximately equal to

∝ symbol for proportional to

≤ symbol for less than or equal to

∫ symbol for integration

α coefficient of thermal expansion (/°C, /°F);
angle, in a math equation (degrees, radians)

β angle, in a math equation (degrees, radians)

β_c ratio of long side to short side of the column in concrete footing design

β_1 coefficient for determining stress block height, *a*, based on concrete strength, f'_c;
coefficient for determining stress block height, *c*, in masonry LRFD design

δ elongation (in, mm)

δ_P elongation due to axial load (in, mm)

δ_s shear deformation (in, mm)

δ_T elongation due to change in temperature (in, mm)

Δ beam deflection (in, mm);
an increment

Δ_{LL} beam deflection due to live load (in, mm)

Δ_{max} maximum calculated beam deflection (in, mm)

Δ_{TL} beam deflection due to total load (in, mm)

Δ_x beam deflection in beam diagrams and formulas (in, mm)

ΔT change in temperature (°C, °F)

ε strain (no units)

ε_t thermal strain (no units)

ε_y yield strain (no units)

ϕ diameter symbol;
angle of twist (degrees, radians);
resistance factor in LRFD steel design and reinforced concrete design

ϕ_b resistance factor for flexure in LRFD design

ϕ_c resistance factor for compression in LRFD design

ϕ_t resistance factor for tension in LRFD design

ϕ_v resistance factor for shear in LRFD design

μ Poisson's ratio;
coefficient of static friction

γ specific gravity of a material (lb/in^3, lb/ft^3, N/m^3,kN/m^3);
angle, in a math equation (degrees, radians);
shearing strain;
load factor in LRFD design

γ_D dead load factor in LRFD design

γ_L live load factor in LRFD design

θ angle, in a trig equation, ex. $\sin\theta$ (degrees, radians);
slope of the deflection of a beam at a point (degrees, radians)

π pi (180°)

ρ radial distance (in, mm);
radius of curvature in beam deflection relationships (ft, m);
reinforcement ratio in concrete beam design = A_s/bd

ρ_b balanced reinforcement ratio in masonry design

$\rho_{balanced}$ balanced reinforcement ratio in concrete beam design

ρ_g reinforcement ratio in concrete column design = A_{st}/A_g

ρ_{max} maximum reinforcement ratio allowed in concrete beam design for ductile behavior

σ engineering symbol for normal stress (axial or bending)

τ engineering symbol for shearing stress

υ_c shear strength in concrete design

ω load per unit length on a beam (lb/ft, kip/ft, N/m, kN/m) (*see w*);
load per unit area (lb/ft^2, kips/ft^2, N/m^2, Pa, MPa)

ω' load per unit volume (lb/ft, kip/ft, N/m, kN/m) (*see* γ)

Σ summation symbol

Ω safety factor for ASD of steel (unified)

Ψ relative stiffness of columns to beams in a rigid connection *(see G)*

Structural Glossary

Allowable strength: Nominal strength divided by the *safety factor.*

Allowable stress: Allowable strength divided by the appropriate section property, such as section modulus or cross section area.

Applicable building code: Building code und which the structure is designed.

ASD (Allowable Strength Design): Method of proportioning *structural components* such that the *allowable strength* equals or exceeds the *required strength* of the component under the action of the *ASD load combinations.*

ASD load combination: Load combination in the *applicable building code* intended for allowable strength design *(allowable stress design).*

ASTM standards: The American Society of Testing and Materials specifies standards for performance and testing of construction materials.

Axial force: A force that is acting along the longitudinal axis of a structural member.

Base shear: A lateral (wind or seismic) force acting at the base of a structure.

Beam: Structural member that has the primary function of resisting bending moments.

Beam-column: Structural member that resists both axial force and bending moment.

Bearing (local compressive yielding): Limit state of *local* compressive *yielding* due to the action of a member bearing against another member or surface.

Bending moment: A force rotating about a point; causes bending in beams, etc.

Block shear rupture: In a connection, *limit state* of tension fracture along one path and shear yielding or shear fracture along another path.

Bracing: Diagonal members that are used to stiffen a structure, by utilizing the inherent in-plane stiffness of a triangular framework.

Braced frame: An essentially vertical truss system that provides resistance to lateral forces and provides stability for the *structural system*.

Buckling: Limit state of sudden change in the geometry of a structure or any of its elements under a critical loading condition.

Buckling strength: Nominal strength for *buckling* or *instability limit states.*

Built-up member, cross-section, section, shape: Member, cross-section, section or shape fabricated from elements that are nailed, welded, glued or bolted together.

Camber: Curvature fabricated into a beam or truss so as to compensate for deflection induced by loads.

Cantilevers: Structural elements or systems that are supported only at one end.

Cement: The generic name for cementitious (binder) materials used in concrete, which is a commonly used building material.

Center of gravity: The location of resultant gravity forces on an object or objects.

Centroid: The center of mass of a shape or object.

Chord member: Primary member that extends, usually horizontally, through a truss *connection.*

Cold-rolled steel structural member: Shape manufactured by roll forming cold-or hot- rolled coils or sheets without manifest addition of heat such as would be required for hot forming.

Collector: An element that transfers load from a diaphragm to a resisting element.

Column: Structural member that has the primary function of resisting axial force.

Component (of vector): One of several vectors combined to a resultant vector.

Composite: Condition in which steel and concrete elements and members work as a unit in the distribution of internal forces.

Composite materials: Those consisting of a combination of two of more distinct materials, together yielding superior characteristics to those of the individual constituents.

Compression: A force that tends to shorten or crush a member or material.

Concentrated force: A force acting on a single point.

Concentrated load: An external concentrated force (also known as a point load).

Concrete: Material composed mainly of cement, crushed rock or gravel, sand and water.

Concrete crushing: Limit state of compressive failure in concrete having reached the ultimate strain.

Connection: A connection joins members to transfer forces or moments from one to the other.

Cope: Cutout made in a structural member to remove a flange and conform to the shape of an intersecting member.

Couple: A couple is a system of two equal forces of opposite direction offset by a distance. A couple causes a moment whose magnitude equals the magnitude of the force times the offset distance.

Cover plate: Plate welded or bolted to the flange of a member to increase cross-sectional area, section modulus or moment of inertia.

Creep: Plastic deformation that proceeds with time.

Curvature: The geometric quantity defined by the inverse of the radius of curvature, *1/R*

Damping: Reduces vibration amplitude, like amplitude seismic vibration.

Dead load: The weight of a structure or anything permanently attached to it.

Deflection: Deflection is the vertical moment under gravity load of beams for example, while lateral movement under wind of seismic load is called drift.

Deformation: A change of the shape of an object or material.

Design load: Applied *load* determined in accordance with either *LRFD load combinations* or *ASD load combinations,* whichever is applicable.

Design strength: Resistance factor multiplied by the *nominal strength,* $øR_n$.

Design stress range: Magnitude of change in stress due to the repeated application and removal of service live *loads*. For locations subject to stress reversal it is the algebraic difference of the peak stresses.

Design stress: Design strength divided by the appropriate section property, such as section modulus or cross section area.

Determinate structure: A structure with the number of reactions equal to the number of static equations (3).

Diagonal Bracing: Inclined structural member carrying primarily axial force in a *braced fame*.

Diaphragm plate: Plate possessing in-plane shear stiffness and strength, used to transfer forces to the supporting elements.

Diaphragm: Roof, floor or other membrane or bracing system that transfers in-plane forces to the lateral force resisting system.

Displacement: May be a translation, a rotation, or a combination of both.

Distributed load: An external force which acts over a length or an area.

Double curvature: Deformed shape of a beam with one or more inflection points within the span.

Double-concentrated forces: Two equal and opposite forces that form a couple on the same side of the loaded member.

Drift: Lateral deflection of structure due to lateral wind or seismic load.

Ductibility: The capacity of a material to deform without breaking; it is measured as the ratio of total strain at failure, divided by the strain at the elastic limit.

Durability: Ability of a material, element or structure to perform its intended function for its required life without the need for replacement or significant repair, but subject to normal maintenance.

Dynamic equilibrium: Equilibrium of a moving object without change of motion.

Dynamic load: Cyclic load, such as gusty wind or seismic loads.

Effective length factor, K: Ratio between the *effective length* and the unbraced length of the member.

Effective length: Length of an otherwise identical *column* with the same strength when analyzed with pinned end conditions.

Effective net area: Net area modified to account for the effect of shear lag.

Effective section modulus: Section modulus reduced to account for buckling of slender compression elements.

Effective width: Reduced width of a plate or slab with an assumed uniform stress distribution which produces the same effect on the behavior of a structural member as the actual plate or slab with its nonuniform stress distribution.

Elastic: A material or structure is elastic if it returns to its original geometry upon unloading.

Elastic/plastic: Materials that have both an elastic zone and a plastic zone (i.e. steel).

Elastic limit: The point of a stress/strain graph beyond which deformation of a material is plastic, i.e. remains permanently deformed.

Elastic modulus: The linear slope value relating material stress to strain.

End-bearing pile: A pile supported on firm soil or rock.

Energy: The work to move a body a distance; energy is the product of forces times distance.

Epicenter: The point on the Earth's surface above the hypocenter where an earthquake originates.

Equilibrium: An object is in equilibrium if the resultant of all forces acting on it has zero magnitude.

External force: A force acting on an object; external forces are also called applied forces.

Factored load: Product of a *load factor* and the *nominal load*.

Fatigue: Limit state of crack initiation and growth resulting from repeated application of live *loads*, usually by reversing the loading direction.

Fillet weld: Weld of generally triangular cross section made between intersecting surfaces of elements.

Fitted bearing stiffener: Stiffener used at a support or concentrated *load* that fits tightly against one or both flanges of a *beam* so as to transmit load through bearing.

Fixed connection: A connection that resists axial and shear forces and bending moments.

Flexure: Bending deformation (of increasing curvature).

Flexural buckling: Buckling mode in which a compression member deflects laterally without twist or change in cross-sectional shape.

Flexural-torsional buckling: Buckling mode in which a compression member bends and twists simultaneously without change in cross-sectional shape.

Force: Resultant of distribution of stress over a prescribed area, or an action that tends to change the shape of an object, move an object, or change the motion of an object.

Foundations: There are two basic types: 'shallow,' which includes pad footing, strip footings and rafts and 'deep' i.e. piles. The choice is a function of the strength and stiffness of the underlying strata and the load to be carried, the aim being to limit differential settlement on the structure and more importantly the finishes.

Fully restrained moment connection: Connection capable of transferring moment with negligible rotation between connected members.

Funicular: The shape of a chain or string suspended form two points under any load.

Gravity: An attractive force between objects; each object accelerates at the attractive force divided by its mass.

Groove weld: Weld in a groove between connection elements.

Gusset plate: Plate element connecting truss members of a strut or brace to a *beam* or *column*.

Hertz: Cycles per second.

Horizontal diaphragm: A floor or roof deck to resist lateral load.

Horizontal shear: Force at the interface between steel and concrete surfaces in a *composite beam*.

Indeterminate structure: A structure with more unknown reactions than static equations (3).

Inelastic: Inelastic (plastic) strain implies permanent deformation.

Inertia: Tendency of objects at rest to remain at rest and objects in motion to remain in motion.

In-plane instability: *Limit state* of a *beam-column* bent about its major axis while *lateral buckling* or *lateral-torsional buckling* is prevented by *lateral bracing*.

Instability: Limit state reached in the loading of a structural component, frame or structure in which a slight disturbance in the *loads* or geometry produces large displacements.

Internal force: The force within an object that resists external forces, also called resisting force.

Joint: Area where two or more ends, surfaces, or edges are attached. Categorized by type of *fastener* or weld used and method of force transfer.

Joist: A repetitive light beam.

K-connection: Connection in which forces in *branch members* or connecting elements transverse to the *main member* are primarily equilibrated by forces in other branch members or connecting elements on the same side of the main member.

Kern: The core of a post or other compression member which limits eccentric stresses being tensile.

Lacing: Plate, angle or other steel shape, in a lattice configuration, that connects two steel shapes together.

Lap joint: Joint between two overlapping connection elements in parallel planes.

Lateral bracing: Diagonal bracing, *shear walls* or equivalent means for providing in-plane lateral stability.

Lateral load resisting system: Structural system designed to resist lateral loads and provide stability for the structure as a whole.

Lateral load: Load, such as that produced by wind or earthquake effects, acting in a lateral direction.

Lateral-torsional buckling: Buckling mode of a flexural member involving deflection normal to the plane of bending occurring simultaneously with twist about the shear center of the cross-section.

Length effects: Consideration of the reduction in strength of a member based on its *unbraced length*.

Limit state: Condition in which a structure or component becomes unfit for service and is judged either to be no longer useful for its intended function (*serviceability limit state*) or to have reached its ultimate load-carrying capacity (*strength limit state*).

Linear: A structural or material behavior is linear if its deformation is directly proportional to the loading.

Line of action: The line of action defines the location and incline of a vector.

Linear elastic: A force-displacement relationship which is both linear and elastic.

Live load: Any load not permanently attached to the structure.

Load: Force or other action that results from the weight of building materials, occupants and their possessions, environmental effects, differential movement, or restrained dimensional changes.

Load effect: Forces, stresses and deformations produced in a *structural component* by the applied *loads*.

Load factor: Factor that accounts for deviations of the *nominal load* from the actual *load*, for uncertainties in the analysis that transforms the load into a *load effect* and for the probability that more than one extreme load will occur simultaneously.

Local bending: Limit state of large deformation of a flange under a concentrated tensile force.

Local buckling: Limit state of buckling of a compression element within a cross section.

Local crippling: Limit state of local failure of web plate in the immediate vicinity of a concentrated *load* or reaction.

Local yielding: Yielding that occurs in a local area of an element.

LRFD (Load and Resistance Factor Design): Method of proportioning *structural components* such that the *design strength* equals or exceeds the *required strength* of the component under the action of the *LRFD load combinations*.

LRFD load combination: Load combination in the *applicable building code* intended for strength design *(load and resistance factor design)*.

Main member: Chord member or column to which *branch members* or other connecting elements are attached.

Mass: Mass is the property of an object to resist acceleration.

Magnitude: a scalar value of physical units, such as force or displacement.

Modulus of elasticity: The proportional constant relating stress/strain of material in the linear elastic range: calculated as stress divided by strain. The modulus of elasticity is the slope of the stress-strain graph, usually denoted as E, also as Young's Modulus Y, or E-Modulus.

Moment: A force causing rotation without translation; defined as force times lever arm.

Moment of inertia: Moment of inertia is the capacity of an object to resist bending or buckling, defined as the sum of all parts of the object times the square of their distance from the centroid.

Moment connection: Connection that transmits bending moment between connected members.

Moment frame: Framing system that provides resistance to lateral loads and provides stability to the *structural system*, primarily by shear and flexure of the framing members and their connections.

Net area: Gross area reduced to account for removed material.

Nominal dimension: Designated or theoretical dimension, as in the tables of section properties.

Nominal load: Magnitude of the *load* specified by the *applicable building code*.

Nominal strength: Strength of a structure or component (without the *resistance factor* or *safety factor* applied) to resist *load effects*, as determined in accordance with this *Specification*.

Normal stress: Stress acting parallel to the axis of an object in compression and tension; normal stress is usually simply called stress.

Out-of-plane buckling: Limit state of a beam-column bent about its major axis while lateral buckling or *lateral-torsional buckling* is not prevented by lateral bracing.

Overlap connection: Connection in which intersecting *branch members* overlap.

Overturn: Topping, or tipping over under lateral load.

Permanent load: Load in which variations over time are rare or of small magnitude. All other *loads* are *variable loads*.

Pin connection: A pin connection transfers axial and shear forces but no bending moment.

Pin support: A pin support resists axial and shear forces but no bending moment.

Pitch: Longitudinal center-to-center spacing of fasteners. Center-to-center spacing bolt threads along axis of bolt.

Plastic: Material may be elastic or plastic. Plastic deformation of a structure or material under load remains after the load is removed.

Plastic analysis: Structural analysis based on the assumption of rigid-plastic behavior, in other words, that equilibrium is satisfied throughout the structure and the stress is at or below the yield stress.

Plastic hinge: Yielded zone that forms in a structural member when the *plastic moment* is attained. The member is assumed to rotate further as if hinged, except that such rotation is restrained by the *plastic moment*.

Plastic moment: Theoretical resisting moment developed within a fully yielded cross section.

Plastic stress distribution method: Method for determining the stresses in a composite member assuming that the steel section and the concrete in the cross section are fully plastic.

Plate girder: Built-up *beam*.

Plug weld: Weld made in a circular hole in one element of a joint fusing that element to another element.

Post-buckling strength: Load or force that can be carried by an element, member, or frame after initial buckling has occurred.

Pressure: Similar to stress, the force intensity at a point, except that pressure is acting on the surface of an object rather than within the object.

Prying action: Amplification of the tension force in a bolt caused by leverage between the point of applied load, the bolt and the reaction of the connected elements.

Punching load: Component of *branch member* force perpendicular to a *chord*.

P-δ effect: Effect of *loads* acting on the deflected shape of a member between joints or nodes.

P-Δ effect: Effect of *loads* acting on the displaced location of joints or nodes in a structure. In tiered building structures, this is the effect of *loads* acting on the laterally displaced location of floors and roofs.

Radius of gyration: A mathematical property, determining the stability of a cross section, defined as: $r = \sqrt{I/A}$, where I = moment of inertia and A = cross section area.

Reaction: The response of a structure to resist applied load.

Required strength: Forces, stresses and deformations acting on the *structural component*, determined by either *structural analysis*, for the *LRFD* or *ASD load combinations*, as appropriate, or as specified by the *Specification* or Standard.

Resilience: The property of structures to absorb energy without permanent deformation of fracture.

Resistance factor ϕ: Factor that accounts for unavoidable deviations of the *nominal strength* from the actual strength and for the manner and consequences of failure.

Resultant: The resultant of a system of forces is a single force or moment whose magnitude, direction, and location make it statically equivalent to the system of forces.

Retaining wall: Wall used to hold back soil or other materials.

Roller support: In two dimensions, a roller support restrains one translation degree of freedom.

Rupture strength: In a *connection*, strength limited by tension or shear rupture.

Safety factor: Factor that accounts for deviations of the actual strength from the nominal strength, deviations of the actual *load* from the *nominal load*, uncertainties in the analysis that transforms the load into a *load effect*, and for the manner and consequence of failure.

Scalar: A mathematical entity with a numeric value but no direction (in contrast to a vector).

Section modulus: The property of a cross section defined by its shape and orientation; section modulus is denoted S, and S = I/c, where I = moment of inertia about the centroid and c is the distance from the centroid to the edge of the section,

Service load combination: Load combination under which serviceability limit states are evaluated.

Service load: Load under which *serviceability limit states* are evaluated.

Serviceability limit state: Limiting condition affecting the ability of a structure to preserve its appearance, maintainability, durability or the comfort of its occupants or function of machinery, under normal usage.

Shear: A sliding force, pushing and pulling in opposite directions.

Shear buckling: Buckling mode in which a plate element, such as the web of a beam, deforms under pure shear applied in the plane of the plate.

Shear connector: Headed stud, cannel, plate or other shape welded to a steel member and embedded in concrete of a *composite member* to transmit shear forces at the interface between the two materials.

Shear connector strength: Limit state of reaching the strength of a *shear connector*, as governed by the connector bearing against the concrete in the slab or by the *tensile strength* of the connector.

Shear modulus: The ratio of shear stress divided by the shear strain in a linear elastic material.

Shear rupture: Limit state of *rupture (fracture) due to shear*.

Shear strain: Strain measuring the intensity of racking in a material. Shear strain is measured as the change in angle of a small square part of a material.

Shear stress: Stress acting parallel to an imaginary plane cut through an object.

Shear wall: Wall that provides resistance to lateral loads in the plane of the wall and provides stability for the structural system.

Shear yielding: Yielding that occurs due to shear.

Shear yielding (punching): In a connection, *limit state* based on out-of-plane shear strength of the *chord* wall to which *branch members* are attached.

Slip: In a bolted connection, *limit state* of relative motion of connected parts prior to the attainment of the *available strength* of the connection.

Slip-critical connection: Bolted *connection* designed to resist movement by friction on the faying surface of the connection under the clamping forces of the bolts.

Slot weld: Weld made in an elongated hole fusing an element to another element.

Splice: Connection between two structural elements joined at their ends to forma single, longer element.

Stability: Condition reached in the loading of a structural component, frame or structure in which a slight disturbance in the *loads* or geometry does not produce large displacements.

Static equilibrium: Equilibrium of an object at rest.

Stiffness: The capacity of a material to resist deformation.

Stiffened element: Flat compression element with adjoining out-of-plane elements along both edges parallel to the direction of loading.

Stiffener: Structural element, usually an angle or plate, attached to a *member* to distribute *load*, transfer shear or prevent buckling.

Stiffness: Resistance to deformation of a member or structure, measured by the ratio of the applied force (or moment) to the corresponding displacement (or rotation).

Strain: Change of length along an axis, calculated as $\varepsilon = \Delta L/L$, where L is the original length and ΔL is the change of length.

Strength: The capacity of a material to resist breaking.

Strength design: A design method based on factored load and ultimate strength for concrete design.

Strength limit state: Limiting condition affecting the safety of the structure, in which the ultimate load-carrying capacity is reached.

Stress: Force per unit area caused by axial force, moment, shear or torsion.

Stress concentration: Localized stress considerably higher than average (even in uniformly loaded cross sections of uniform thickness) due to abrupt changes in geometry or localized loading.

Stress resultant: A system of forces which is statically equivalent to a stress distribution over an area.

Stress: The internal reaction to an applied force, measured in force per unit area.

Structure: Composition of elements that define form and resist applied loads.

Structural Aluminum: Elements manufactured of aluminum for structural purposes, generally 50% larger than comparable steel elements due to the lower *modulus of elasticity*.

Structural Steel: Elements manufactured of steel with properties designated by *ASTM standards*, including A36, A992 & A572.

Strong axis: Major principal centroidal axis of a cross section.

Structural analysis: Determination of *load effects* on members and *connections* based on principles of structural mechanics.

Structural component: Member, connector, connecting element or assemblage.

Structural system: An assemblage of load-carrying components that are joined together to provide interaction or interdependence.

T-connection: Connection in which the *branch member* or connecting element is perpendicular to the *main member* and in which forces transverse to the main member are primarily equilibrated by shear in the main member.

Tensile rupture: Limit state of rupture (fracture) due to tension.

Tensile strength (of material): Maximum tensile stress that a material is capable of sustaining as defined by ASTM.

Tensile strength (of member): Maximum tension force that a member is capable of sustaining.

Tensile yielding: Yielding *that occurs due to tension*.

Tension: A force that tends to elongate or enlarge an object.

Tension and shear rupture: In a bolt, *limit state* of rupture (fracture) due to simultaneous tension and shear *force*.

Tie plate: Plate element used to join parallel components of a *built-up column*, girder or strut rigidly connected to the parallel components and designed to transmit shear between them.

Torsion: A twisting moment.

Torsional bracing: Bracing resisting twist of a *beam* or *column*.

Torsional buckling: Buckling mode in which a compression member twists about its shear center axis.

Torsional yielding: Yielding that occurs due to torsion.

Translation: Motion of an object along a straight line path without rotation.

Transverse reinforcement: Steel reinforcement in the form of closed ties or welded wire fabric providing confinement for the concrete surrounding the steel shape core in an *encased concrete composite column*.

Transverse stiffener: Web *stiffener* oriented perpendicular to the flanges, attached to the web.

Truss: A linear support system consisting of triangular panels usually with pin joints.

Ultimate strength: The utmost strength reached by a material before breaking.

Unbraced length: Distance between braced points of a member, measured between the centers of gravity of the bracing members.

Uneven load distribution: In a *connection*, condition in which the load is not distributed through the cross section of connected elements in a manner that can be readily determined.

Unframed end: The end of a member not restrained against rotation by stiffeners of connection elements.

Unstiffened elements: Flat compression element with an adjoining out-of-plane element along one edge parallel to the direction of loading.

Uplift: Upward force, usually wind uplift.

Variable load: Load not classified as *permanent load*.

Vector: A mathematical entity having a magnitude, line of action, and a direction in space.

Vertical bracing system: System of *shear walls, braced frames* or both, extending through one or more floors of a building.

Vertical diaphragm: A wall to resist lateral load.

Vibration: The cyclic motion of an object.

Wall: A vertical element to resist load and define space; shear walls also resist lateral loads.

Weak axis: Minor principal centroidal axis of a cross section.

Web buckling: Limit state of lateral instability of a web.

Web compression buckling: Limit state of out-of-plane compression buckling of the web due to a concentrated compression force.

Web sideway buckling: Limit state of lateral buckling of the tension flange opposite the location of a concentrated compression force.

Weld metal: Portion of a fusion weld that has been completely melted during welding. Weld metal has elements of filler metal and base metal melted in the weld thermal cycle.

Working stress: The same as allowable stress.

Yield moment: In a member subjected to bending, the moment at which the extreme outer fiber first attains the *yield stress*.

Yield point: First stress in a material at which an increase in strain occurs without an increase in stress as defined by ASTM.

Yield strength: Stress at which a material exhibits a specified limiting deviation from the proportionality of stress to strain as defined by ASTM.

Yield strain: The strain of a material which occurs at the level of yield stress.

Yield stress: Generic term to denote either *yield point* or *yield strength*, as appropriate for the material.

Yielding: Limit state of inelastic deformation that occurs after the *yield stress* is reached.

Yielding (plastic moment): Yielding throughout the cross section of a member as the bending moment reaches the *plastic moment*.

Yielding (yield moment): Yielding at the extreme fiber on the cross section of a member when the bending moment reached the *yield moment*.

References:
AISC, *Specifications for Structural Steel Buildings*, 13th ed. (2005)
Jacqueline Glass, *Encyclopaedia of Architectural Technology*, Wiley, Cornwall (2002)

Structural Systems

from Architectural Structures, Wayne Place, Wiley, 2007:

STRUCTURAL DESIGN PROCESS

1.1 Nature of the Process

Architects have a huge array of issues to address in architectural practice. Among these are the following: keeping rain out of a building, getting water off a site, thermal comfort, visual comfort, space planning, fire egress, fire resistance, corrosion and rot resistance, vermin resistance, marketing, client relations, the law, contracts, construction administration, the functional purposes of architecture, the role of the building in the larger cultural context, security, economy, resource management, codes and standards, and how to make a building withstand all the forces to which it will likely be subjected during its lifetime. This last subject area is referred to as *architectural structures.*

Because of the extraordinary range of demands on an architect's time and skills and the extraordinary number of subjects that architecture students must master, architectural structures are typically addressed in only two or three lecture courses in an accredited architectural curriculum in the United States. These two or three lecture courses must be contrasted with the ten or twelve courses that will normally be taken by a graduate of an accredited structural engineering curriculum. This contrast in level of focus makes it clear why a good structural engineering consultant is a very valuable asset to an architect. However, having a good structural consultant does not relieve the architect of serious responsibility in the structural domain. All architects must be well versed in matters related to structures. The architect has the primary responsibility for establishing the structural concept for a building, as part of the overall design concept, and must be able to speak the language of the structural consultant with sufficient skill and understanding to take full advantage of the consultant's capabilities.

1.2 General Comments Regarding Architectural Education

Structural design is one of the more rigorous aspects of architectural design. Much knowledge has been generated and codified over the centuries that human beings have been practicing in and developing this field. This book gives primary attention to those things that are known, quantified, and codified.

However, very few things in the realm of architecture yield a single solution. To any given design problem, there are many possible solutions, and picking the best solution is often the subject of intense debate. Therefore, no one should come to this subject matter assuming that this text, or any text, is going to serve up a single, optimized solution to any design problem, unless that design problem has been so narrowly defined as to be artificial.

In design, there is always a great deal of latitude for personal expression. Design is purposeful action. The designer must have an attitude to act. Architecture students develop an attitude through a chaotic learning process involving a lot of trial and error. In going through this process, an architecture student must remain aware of a fundamental premise: the process is more important than the product; that is, the student's learning and development are more important than the output. The student has a license to make mistakes. It is actually more efficient to plow forward and make mistakes than to spend too much time trying to figure out how to do it perfectly the first time. To paraphrase the immortal words of Thomas Edison: To have good ideas, you should have many ideas and then throw out the bad ones. Of course, throwing out the bad ones requires a lot of rigorous and critical thinking. No one should ever fall in love with any idea that has not been subjected to intense and prolonged critical evaluation and withstood the test with flying colors. Furthermore, important ideas should be subjected to periodic reevaluation. Times and conditions change. Ideas that once seemed unassailable may outlive their usefulness or, at the very least, need updating in the light of new knowledge and insights.

In pursuing this subject matter, it is valuable to have a frame of reference regarding the roles of the architect, as the leader of the design team, and the structural engineer, as a crucial contributor of expertise and hard work needed to execute the project safely and effectively. The diagram in Figure 1.1 will help provide that frame of reference.

In contemplating the diagram in Figure 1.1, keep in mind that design and analysis are two sides of the same coin and that the skills and points of view of architects and engineers, although distinctive, also overlap and sometimes blur together. The most effective design teams consist of individuals with strong foci who can play their respective roles while having enough overlap in understanding and purpose that they can see each other's point of view and cooperate in working toward mutually understood and shared goals. The most harmful poison to a design team is to have such a separation in points of view and understanding that a rift develops between the members of the team. Cooperation is the watchword in this process, as in all other team efforts.

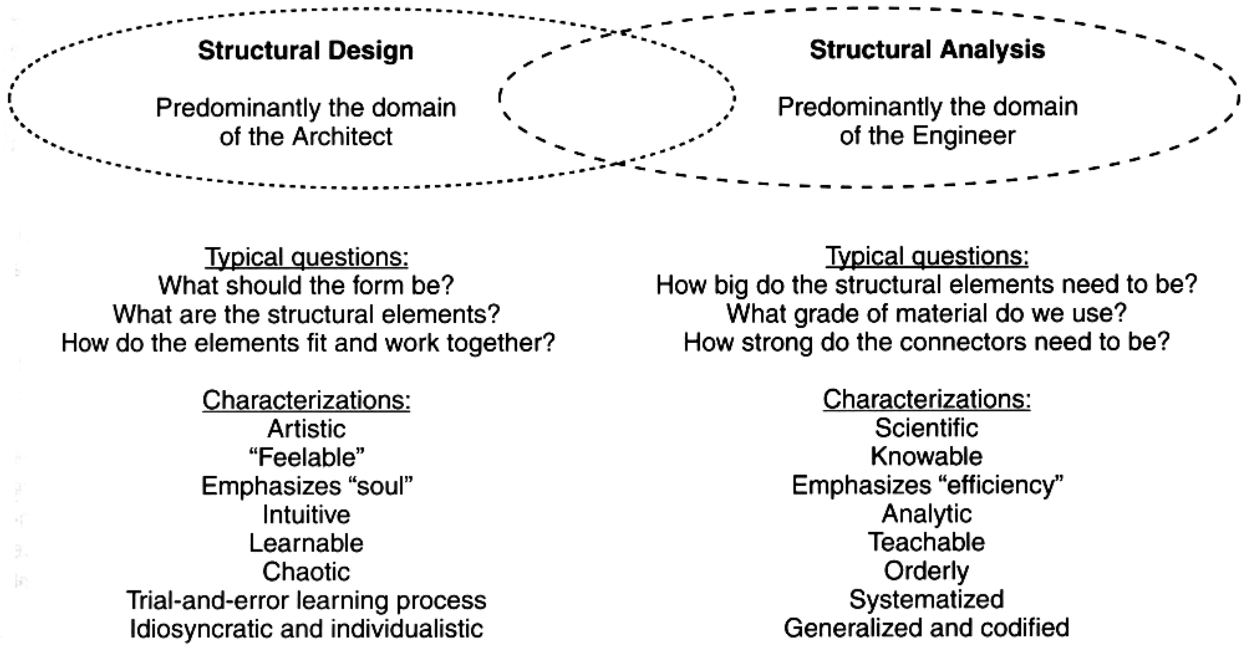
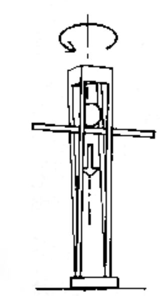


Figure 1.1 Nature of the design process and roles of the design participants.

Design Criteria for the Behavior of the Overall System

Components of a system consist of vertical and horizontal elements. Connections of the vertical to horizontal elements are also necessary. For the structural elements to behave and respond as designed, the system must have the following qualities:

- the components stay together
- the system resists overturning, sliding, twisting and excessive distortion
- the system has internal stability
- the system has overall strength and stiffness



Twisting

"Order" of Design

There is no set order to design of a structural system. But there are certain stages that can be recognized. These may be referred to as *preliminary, revised* and *final,* or more formally as:

<u>First order:</u> which can include determining structural type and organization, design intent, and contextual or programmatic emphasis. Preliminary member size charts are useful at this stage.

<u>Second order:</u> which can include evaluating structural strategies, choice of construction materials, and structural system options with those materials. System selection design aids are useful at this stage.

<u>Third order:</u> which, after the design has been narrowed down, is where analysis and design (shape and size) of individual structural elements (beams, columns, connections, etc.) is performed. The outcome here may direct further first order or second order investigations!!!

from Understanding Structures, Fuller Moore, McGraw-Hill, 1999:

DESIGN CRITERIA	Light-frame timber	Heavy-frame timber	Masonry bearing wall	Steel frame (hinge connections)	Steel frame (rigid connections)	Steel open-web joists	Steel space frame	Steel decking	Site-cast concrete: one-way slab	Site-cast concrete: two-way plate	Site-cast concrete: two-way slab	Site-cast concrete: one-way joists	Site-cast concrete: waffle slab	Precast concrete: solid slab	Precast concrete: hollow-core slab	Precast concrete: single tee	Precast concrete: double tee	RATIONALE
Exposed, fire-resiant construction		■	■						■	■	■	■	■	■	■	■	■	Inherently fire-resistive construction
Irregular building form	■		■						■	■	■							Simple, site-fabricated systems
Irregular column placement										■	■		■					Systems without beams in roof or floors
Minimize floor thickness									■	■				■	■			Precast-concrete systems without ribs
Allow for future renovations	■	■	■	■	■	■	■	■	■			■		■	■			Short-span, one-way, easily modified
Permit construction in poor weather	■	■		■	■	■	■	■						■	■	■	■	Quickly erected; avoid site-cast concrete
Minimize off-site fabrication time	■		■						■	■	■	■	■					Easily formed or built on site
Minimize on-site erection time		■		■	■	■		■						■	■	■	■	Highly prefabricated; modular components
Minimize low-rise construction time	■	■		■	■	■		■										Lightweight, easily formed or prefabricated
Minimize medium-rise construction time				■	■	■		■	■	■	■	■	■	■	■	■	■	Precast, site-cast concrete; steel frames
Minimize high-rise construction time				■	■	■		■										Strong; prefabricated; lightweight
Minimize shear walls or diagonal bracing			■		■				■	■	■	■	■					Capable of forming rigid joints
Minimize dead load on foundations	■	■		■	■	■	■	■										Lightweight, short-span systems
Minimize damage due to foundation settlement	■	■		■		■		■						■	■	■	■	Systems without rigid joints
Minimize the number of separate trades on job	■		■															Multipurpose components
Provide concealed space for mech. services	■					■	■								■			Systems that inherently provide voids
Minimize the number of supports							■											Two-way, long-span systems
Long spans						■	■									■		Long-span systems

Figure 18.6: Framing system selection chart.

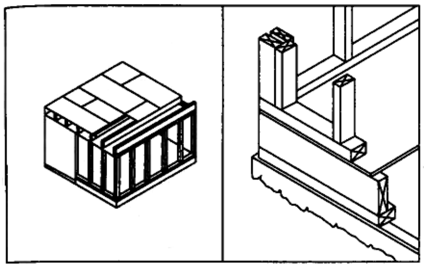
from The Architect's Studio Companion, 3rd ed., Allen & Iano, Wiley, 2002

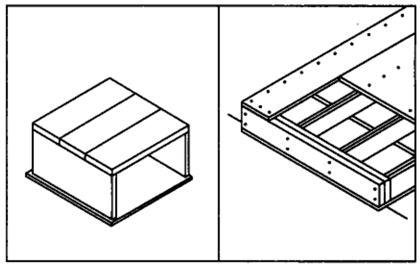
DESIGN CRITERIA: SUMMARY CHART

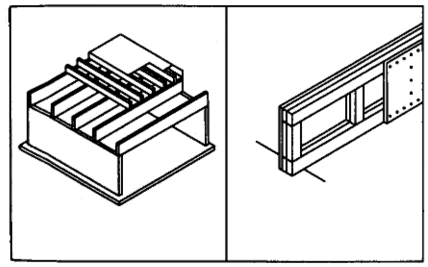
	WOOD AND MASONRY				STEEL				SITECAST CONCRETE										PRECAST CONCRETE			
	Pages 49–65	Pages 49–69	Pages 71–85	Pages 71–85	Pages 88–91	Pages 102–103	Pages 87–105	Pages 87–105	Pages 114–115	Pages 114–115	Pages 116–117	Pages 116–117	Pages 118–119	Pages 118–119	Pages 120–121	Pages 120–121	Pages 122–123	Pages 122–123	Pages 132–133	Pages 132–133	Pages 134–135	Pages 134–135
GIVE SPECIAL CONSIDERATION TO THE SYSTEMS INDICATED IF YOU WISH TO:	Platform Frame	Timber Frame	Ordinary Construction	Mill Constroction	Light Gauge Steel Framing	Single-Story Rigid Steel Frame	Steel Frame—Hinged Connections	Steel Frame—Rigid Connections	One-Way Solid Slab	Posttensioned One-Way Solid Slab	One-Way Joist	Posttensioned One-Way Joist	Two-Way Flat Plate	Posttensioned Two-Way Flat Plate	Two-Way Flat Slab	Posttensioned Two-Way Flat Slab	Waffle Slab	Posttensioned Waffle Slab	Solid Slab	Hollow Core Slab	Double Tee	Single Tee
Create a highly irregular building form	●		●		●				●	●			●	●	●	●						
Expose the structure while retaining a high fire-resistance rating		●		●					●	●	●	●	●	●	●	●	●	●	●	●	●	●
Allow column placements that deviate from a regular grid													●	●	●	●						
Minimize floor thickness										●			●	●	●	●			●	●		
Minimize the area occupied by columns or bearing walls						●	●	●									●	●			●	●
Allow for changes in the building over time	●	●	●	●	●		●	●	●		●								●	●		
Permit construction under adverse weather conditions	●	●			●	●	●	●											●	●	●	●
Minimize off-site fabrication time	●		●	●	●				●	●	●	●	●	●	●	●	●	●				
Minimize on-site erection time		●				●	●	●											●	●	●	●
Minimize construction time for a one- or two-story building	●	●			●	●	●	●														
Minimize construction time for a 4- to 20-story building							●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●
Minimize construction time for a building 30 stories or more in height							●	●														
Avoid the need for diagonal bracing or shear walls						●		●	●	●	●	●	●	●	●	●	●	●				
Minimize the dead load on a foundation	●	●			●	●	●	●														
Minimize structural distress due to unstable foundation conditions	●	●					●												●	●	●	●
Minimize the number of separate trades needed to complete a building			●	●																		
Provide concealed spaces for ducts, pipes, etc.	●		●		●																	

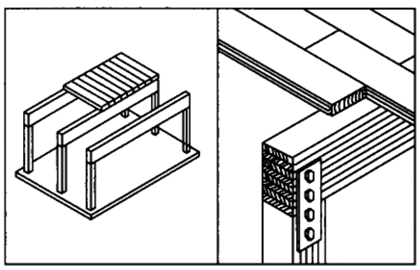
System Types by Material from Structures, Schodek & Bechthold, 6th ed.. Pearson, 2008:

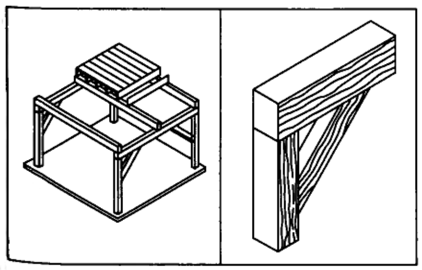
Timber Systems

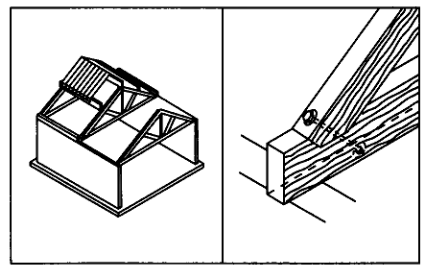

(a) Light frame construction.

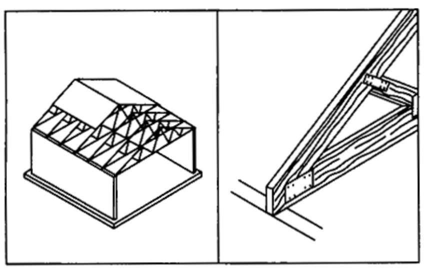

(b) Stressed-skin panels.

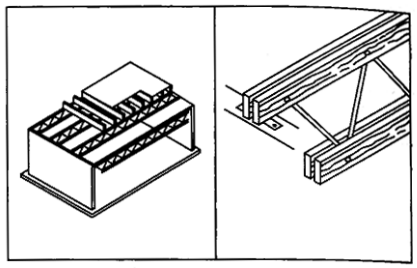

(c) Box beams.

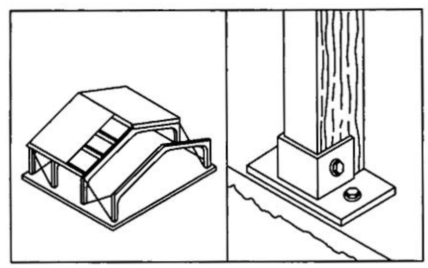

(b) Heavy timber construction: laminated beams.

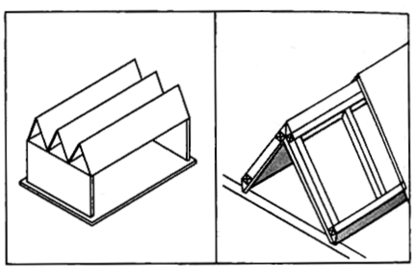

(e) Heavy timber construction: knee-braced frame.

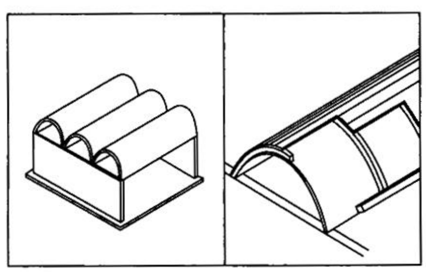

(f) Trusses: special designs.

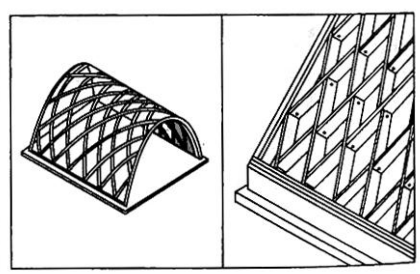

(g) Trusses: mass-produced "trussed rafters" for housing.


(h) Trusses: mass-produced open-web joists.

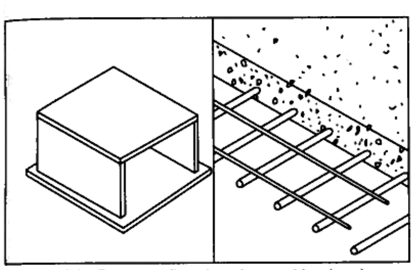

(i) Arches laminated timber members.

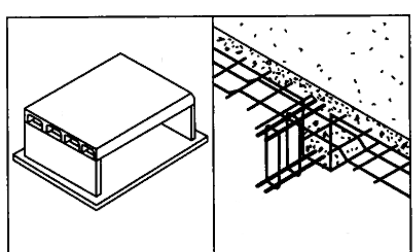

(j) Folded plates.

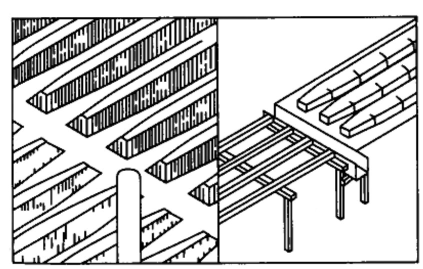

(k) Arch panels.


(l) Lamella construction.

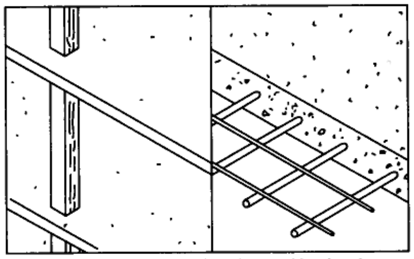
Reinforced Concrete Systems


(a) One-way flat plate (poured in place).

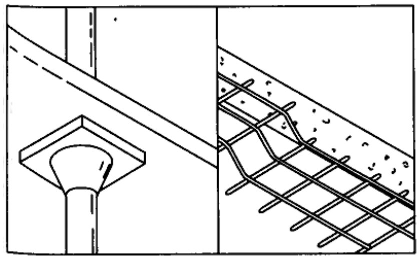

(b) One-way beam-and-slab system (poured in place).


(c) One-way pan joist system (poured in place).

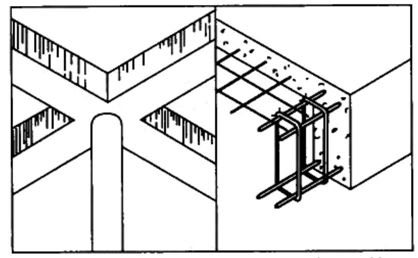
Reinforced Concrete Systems (continued)



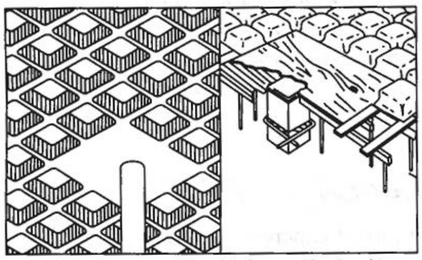
(d) Two-way flat plate (poured in place).



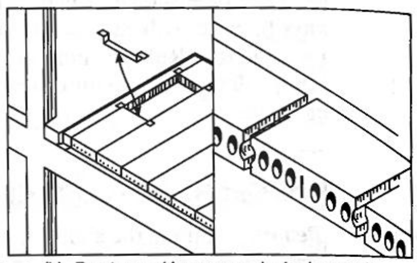
(e) Two-way flat slab (poured in place).



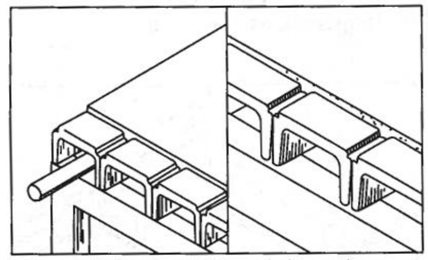
(f) Two-way beam-and-slab system (poured in place).



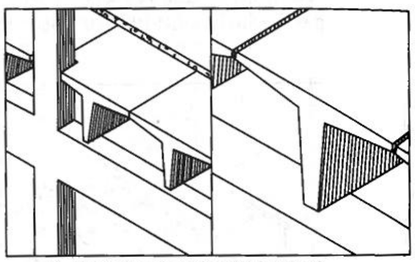
(g) Two-way waffle slab (poured in place).



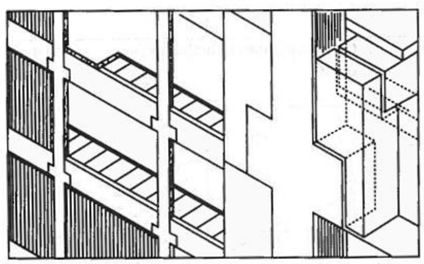
(h) Prestressed long-span planks (precast).



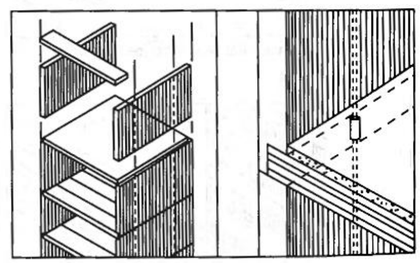
(i) Prestressed channels (precast).



(j) Prestressed single tees (precast).

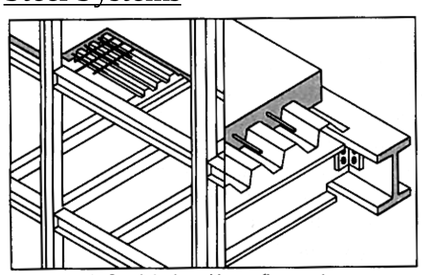


(k) Beam-and-column system (precast).

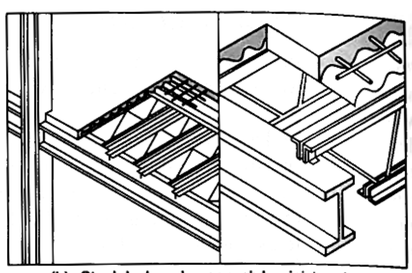


(l) Housing system (precast walls and planks post-tensioned together).

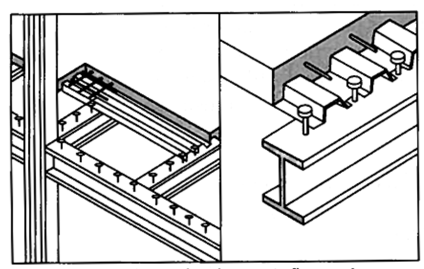
Steel Systems



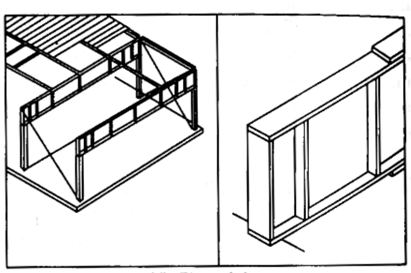
(a) Steel deck and beam floor system.



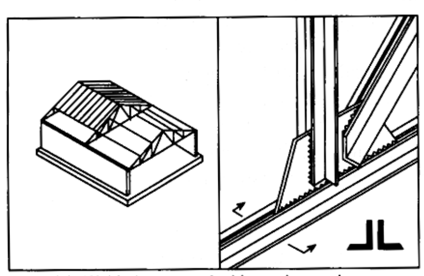
(b) Steel deck and open-web bar joist system.



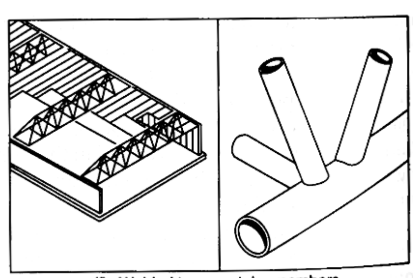
(c) Composite steel and concrete floor system.



(d) Plate girders.

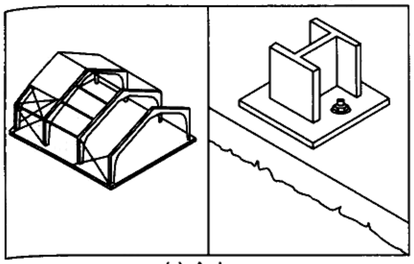


(e) Welded trusses: double-angle members.

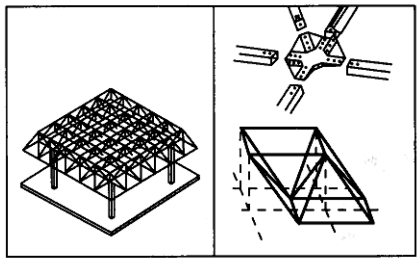


(f) Welded trusses: tube members.

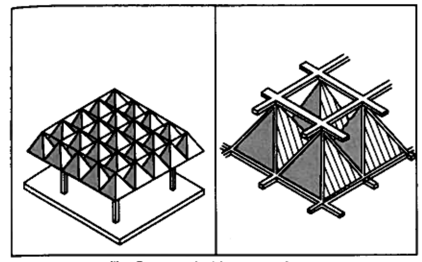
<u>Steel Systems(continued)</u>



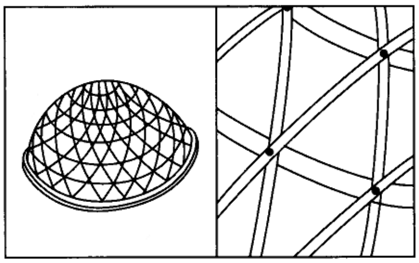
(g) Arches.



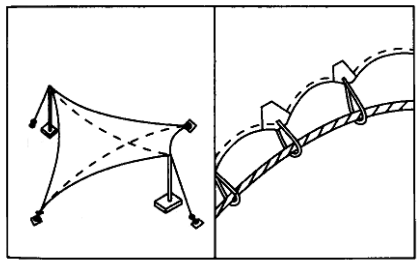
(h) Space frame.



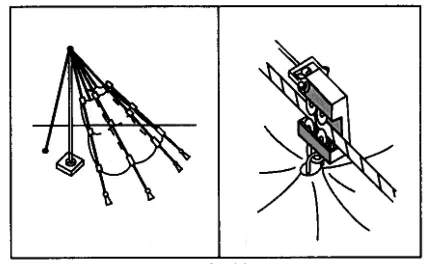
(i) Stressed-skin space frame.



(j) Ribbed dome.



(k) Prestressed membrane structure.



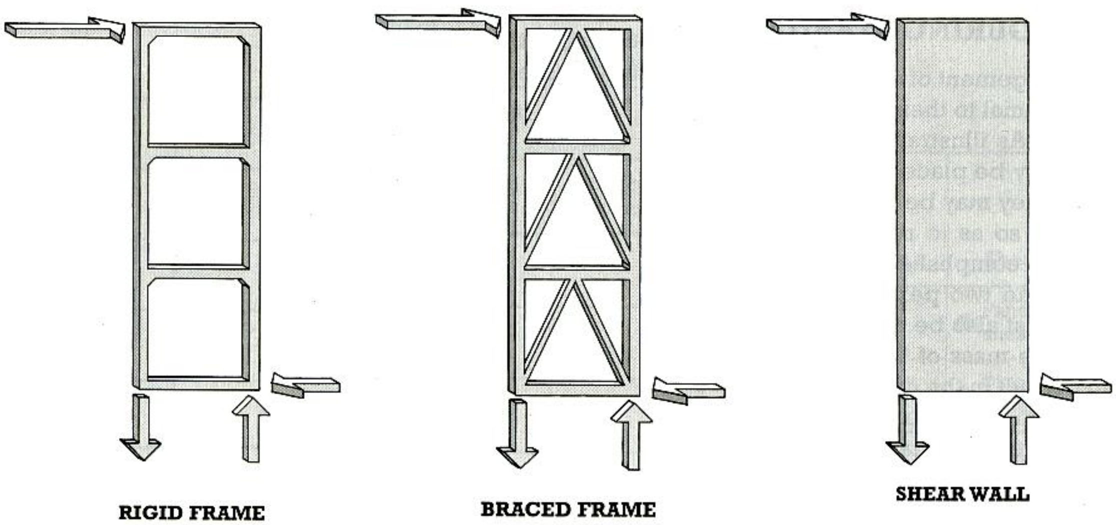
(l) Folding roof cable structure.

Structural Planning

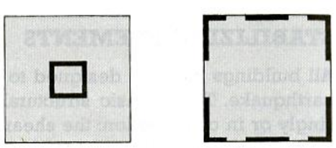
Design Issues (Reference: The Architect's Studio Companion, 3rd ed., Allen & Iano, Wiley, 2002)

Lateral Stability: Wind forces and inertial forces due to ground acceleration are two types of lateral loads buildings must be designed to resist. Without resisting elements or systems, the buildings will move a little, a lot, or suddenly. Stability is the ability to flex and not suddenly "snap" or in other words, the ability to remain in the configuration intended to transfer load.

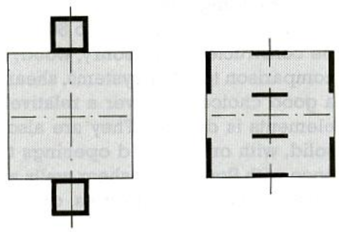
- Resisting systems include *shear walls, braced frames* and *rigid frames:*



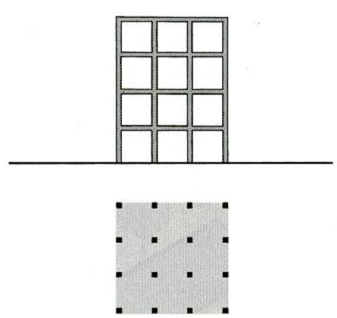
- Configurations are important for the systems to be effective. Symmetrical or balanced arrangements are the most effective for resisting the lateral forces from all directions.



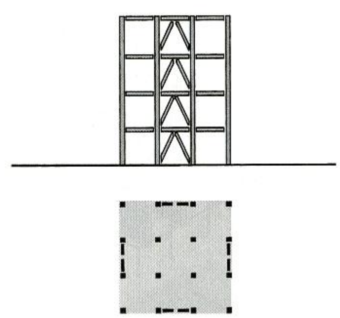
Stabilizing elements may be placed within the interior or at the perimeter of a building.



Stabilizing elements should be arranged in a balanced fashion.

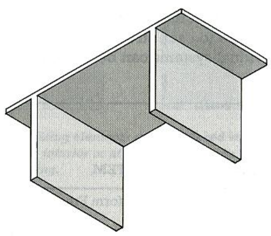


Rigid frame structures require no additional bracing or shear walls, as shown in this elevation and plan.

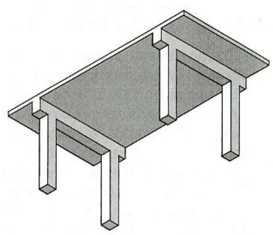


The locations of braced frames or shear walls must be considered in relation to the elevation and plan of the building.

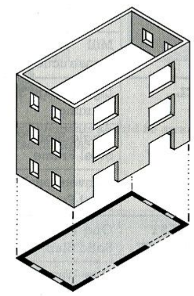
Vertical Load Resistance: Load bearing walls, columns and frames are examples of vertical load resisting elements. They can support a variety of horizontal spanning elements, such as beams and slabs. The order, or modular placement, becomes important, and uniform arrangements are economical. Load bearing walls can also function as shear walls to resist lateral loads. They are commonly constructed of reinforced concrete or masonry.



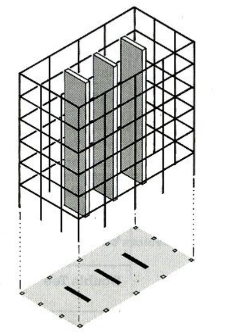
WALL AND SLAB SYSTEMS
(shown from below)



COLUMN AND BEAM SYSTEMS
(shown from below)

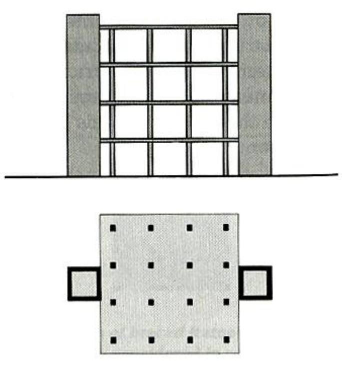


Shear walls may be arranged in a box form to resist lateral forces from all directions.

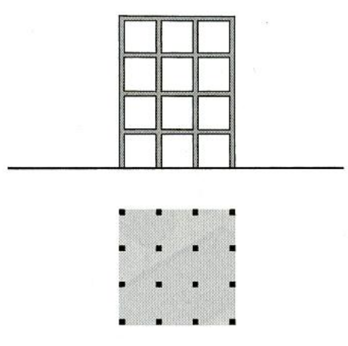


When combined with other stabilizing mechanisms, shear walls may be arranged so as to resist forces in only one direction of a building.

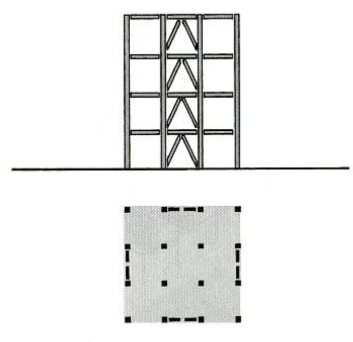
Horizontal Load Resistance: The combination of vertical and horizontal load resistance is dependant upon construction materials and size or utility of spaces. Slabs can act as diaphragms to transmit loads to the columns, shear wall or frames. They are commonly constructed of reinforced concrete. Rigid frames are commonly steel or monolithically cast reinforced concrete.



Shear walls are commonly used with column and slab systems. In this elevation and plan, the shear walls are shown incorporated into a pair of vertical cores.



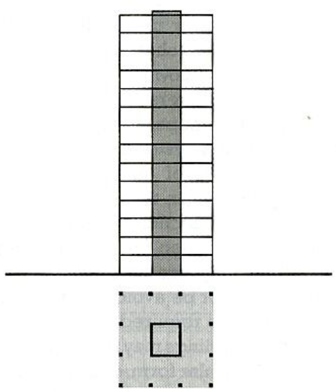
Rigid frame structures require no additional bracing or shear walls, as shown in this elevation and plan.



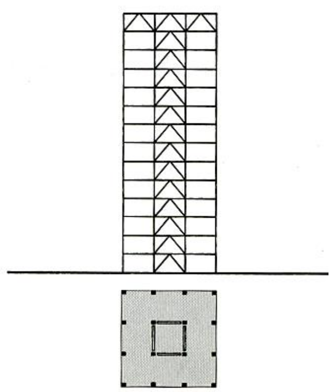
The locations of braced frames or shear walls must be considered in relation to the elevation and plan of the building.

<u>Multistory Design Issues</u>**:** As a building gets taller, it is exposed to more wind load that it must resist laterally. It also increases in mass at each story, which makes the inertial forces from ground acceleration very complex. The behavior of a structure under these types of loads is dependant upon the arrangement of the masses and the stiffness and placement of the horizontal and vertical load resisting elements.

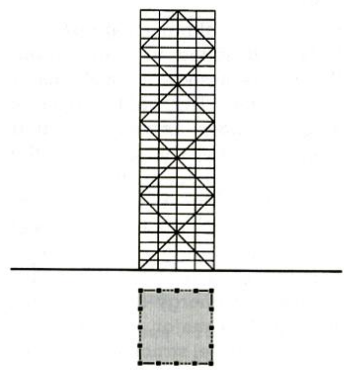
Cores are quite common to increase stiffness vertically. *Unfortunately, they can't provide effective horizontal load transfer, and should not be relied on as the sole lateral resistant mechanism!* Exterior bracing or tube formations, such as the Sears Tower in Chicago, are other multistory configurations to resist lateral loads.



RIGID CORE

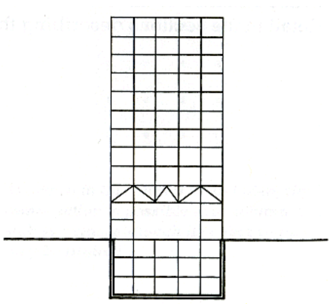


BRACED CORE WITH HAT TRUSS

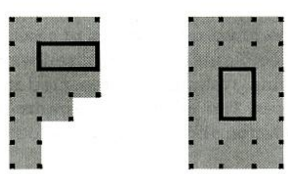


BRACED FRAME TUBE

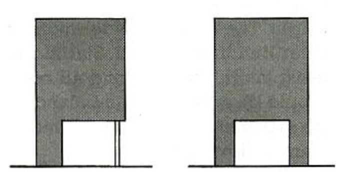
Vertical and horizontal "discontinuities" contribute to irregular or poor lateral response. Vertical discontinuities include "cut-outs" in stories, or changes in plan vertically, while horizontal continuities include problems such as "soft stories" which have different stiffness from the rest of the structure, and unbalanced placement of shear walls.



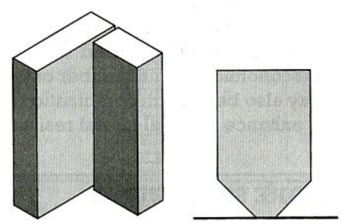
Transfer beams or trusses may be used to interrupt vertical loadbearing elements where necessary.



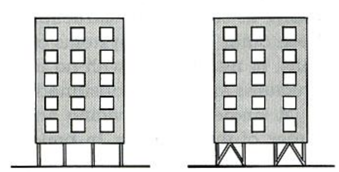
UNBALANCED PLAN **BALANCED PLAN**



UNBALANCED SECTION **BALANCED SECTION**



Discrete building masses should be structurally independent. Inherently unstable building masses should be avoided.



Discontinuities in the stiffness of structures at different levels should be avoided, or additional stabilizing elements may be required.

Structural Plans and Grids

(Reference: Construction Graphics, 1st ed., Bisharat, Wiley, 2004)

Foundation

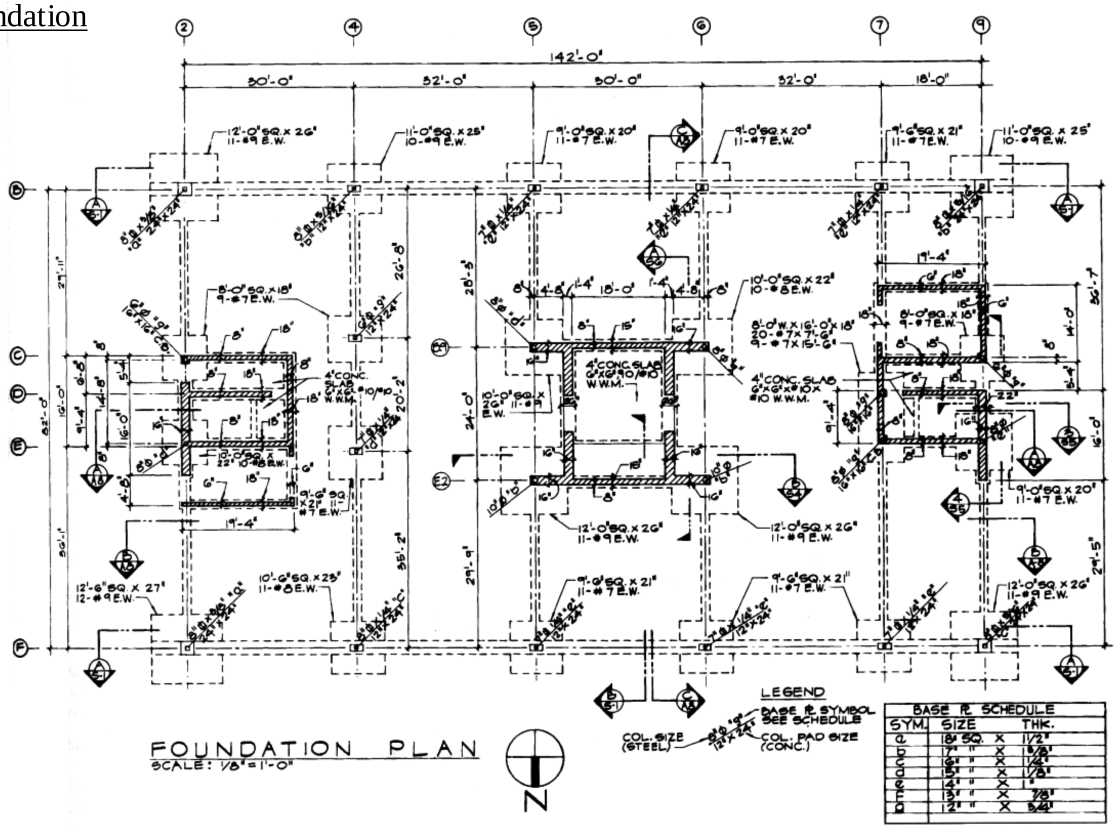


Figure 7.4 This foundation plan uses a grid referencing system, though not the one promoted by the National CAD Standard. Note the idiosyncrasies in this drawing: north is normally the top of the page. (From *The Professional Practice of Architectural Working Drawings*, 2nd edition, by Osamu Wakita and Linde, Richard, John Wiley & Sons, Inc., 1995. Used with permission of John Wiley & Sons, Inc.)

Footing Detail

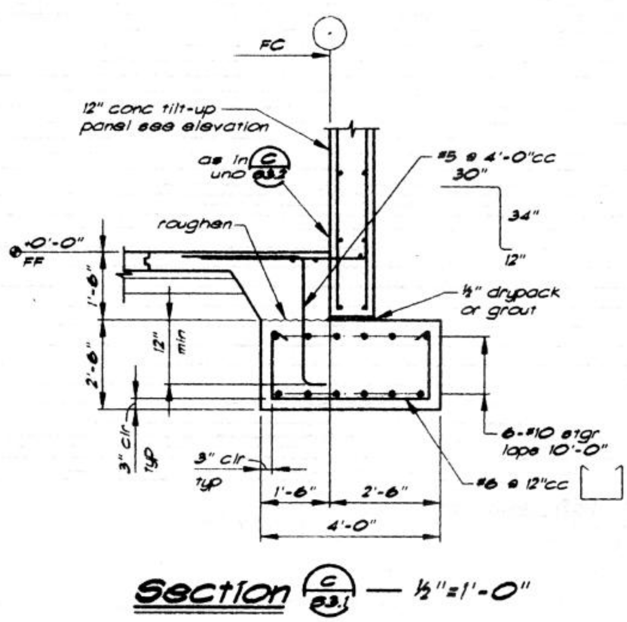


Figure 8.5b Footings are often depicted in wall sections on subsequent sheets, but in this instance the engineer is showing just a footing section, denoted C S3.1 on the plan in 8.5a.

Steel

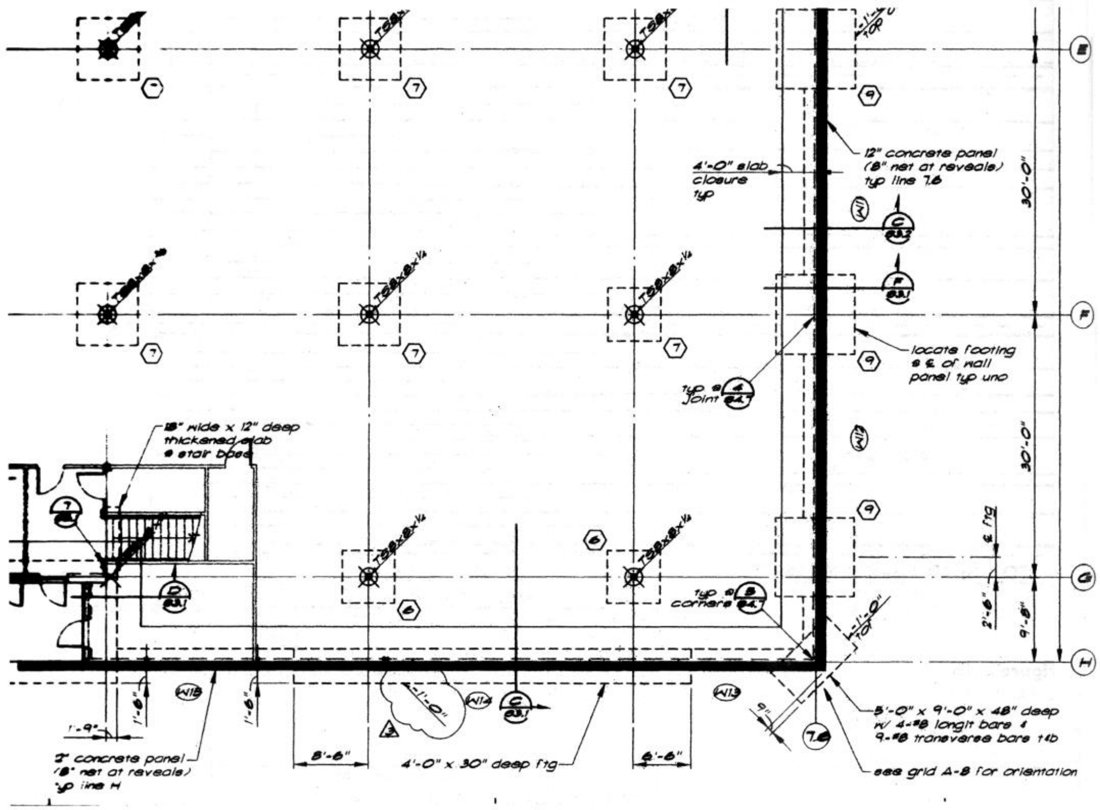


Figure 8.5a Drawings of structural steel framing systems begin with the foundation plan, which is where the columns and footings that carry the frame are described. (Drawing courtesy of Buehler and Buehler Structural Engineers.)

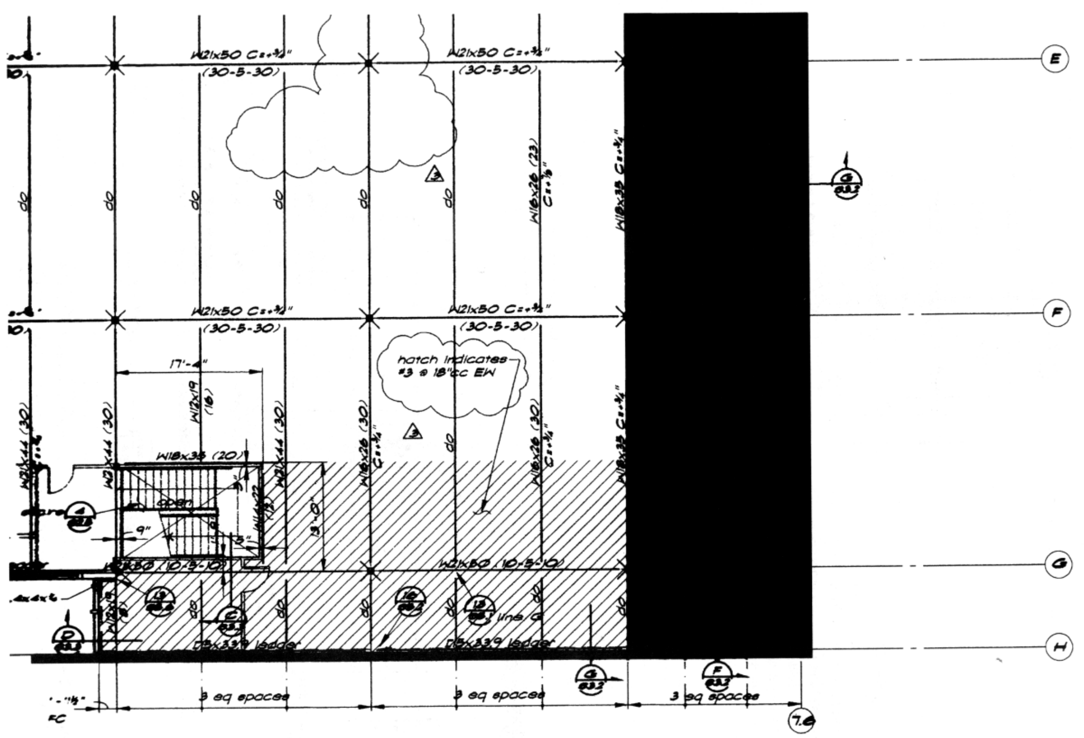
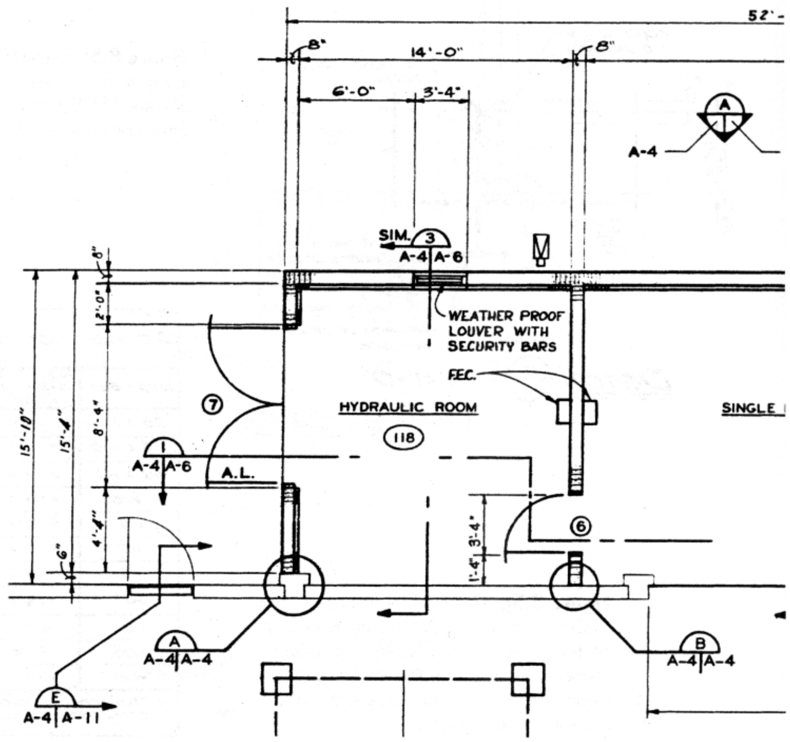


Figure 8.5c The first floor framing plan commonly shows column locations and lists girders and beams by size. The floor deck is also described on the plan. The girder designation W21 × 50 C = + ¾" (above gridline F) and 30-5-30 (below gridline F) is, respectively, the girder size and camber and number of headed stud anchors required in each third of the beam (left, center, right). The beam designation is slightly different (see lines perpendicular to girder lines): Above the beam line following the beam size is the number of headed stud anchors to be uniformly distributed between columns on the top of the beam, with the camber listed below the beam line. (Drawing courtesy of Buehler and Buehler Structural Engineers.)

Reinforced Masonry

Figure 8.6a In this partial floor plan for a reinforced masonry structure, the wall descriptions are very simple. Note the conservative use of the masonry symbol and the consequent uncluttered appearance of the drawing. The split-bubble referencing system used throughout these drawings directs the reader's attention to several details, depicted on other pages as well as the page on which they originate. Details 1 A-4/A-6 and 3 A-4/A-6 are building sections; details A and B A-4/A-4 are details of the connection to existing concrete columns; and detail E A-4/A-11 is a roof connection detail. In the upper right part of the drawing is the reference to an exterior elevation (A A-4/A-5).



Timber

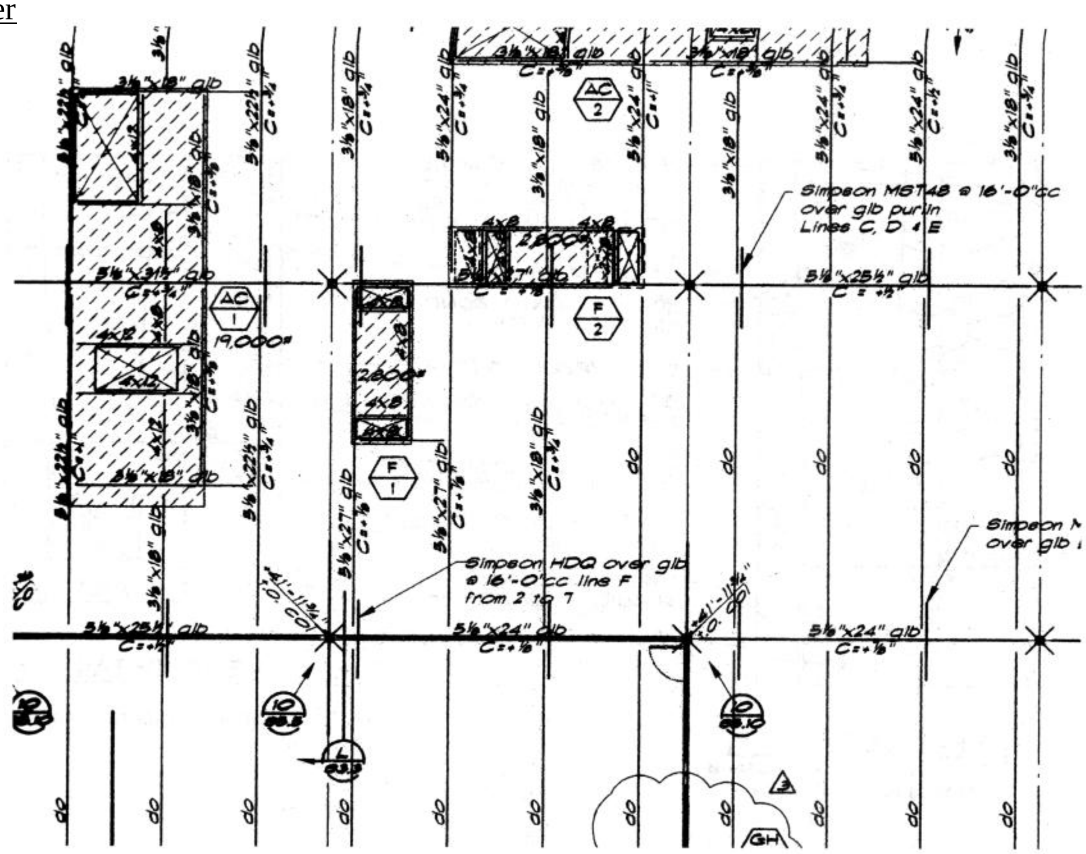


Figure 8.7a This partial roof framing plan shows the glued-laminated girder and beam system. Note the weight of AC unit 1 and how the structural engineer has addressed the additional loading where mechanical equipment is supported by the roof. (Drawing courtesy of Buehler and Buehler Structural Engineers.)

Common Span Lengths and Depths:

from Structures, 6th ed., Schodek & Bechthold, Pearson/Prentice Hall, 2007

Span Range by System

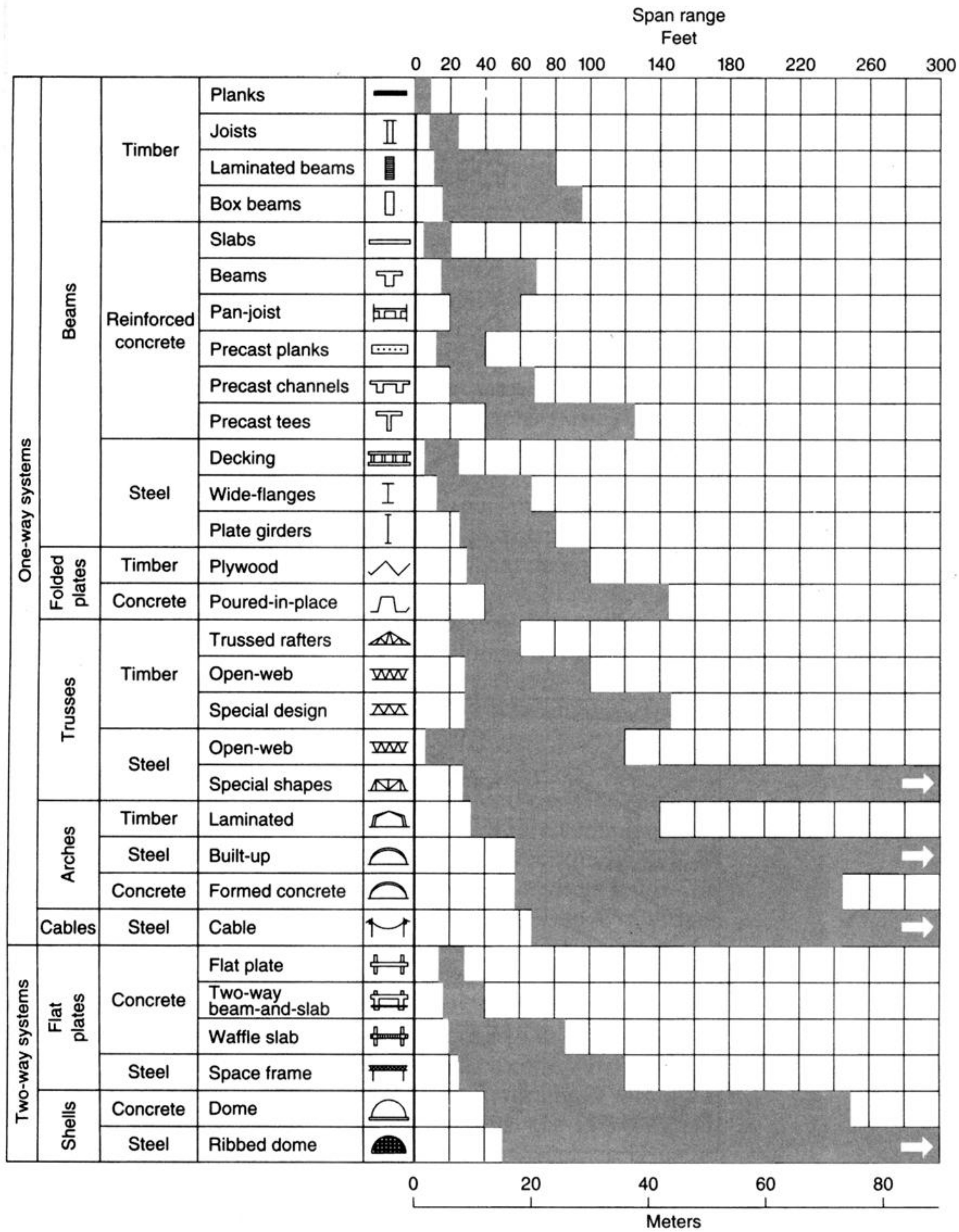


FIGURE 13.12 Approximate span ranges of different systems. (See also more detailed charts in Chapter 15.)

<u>Timber</u>

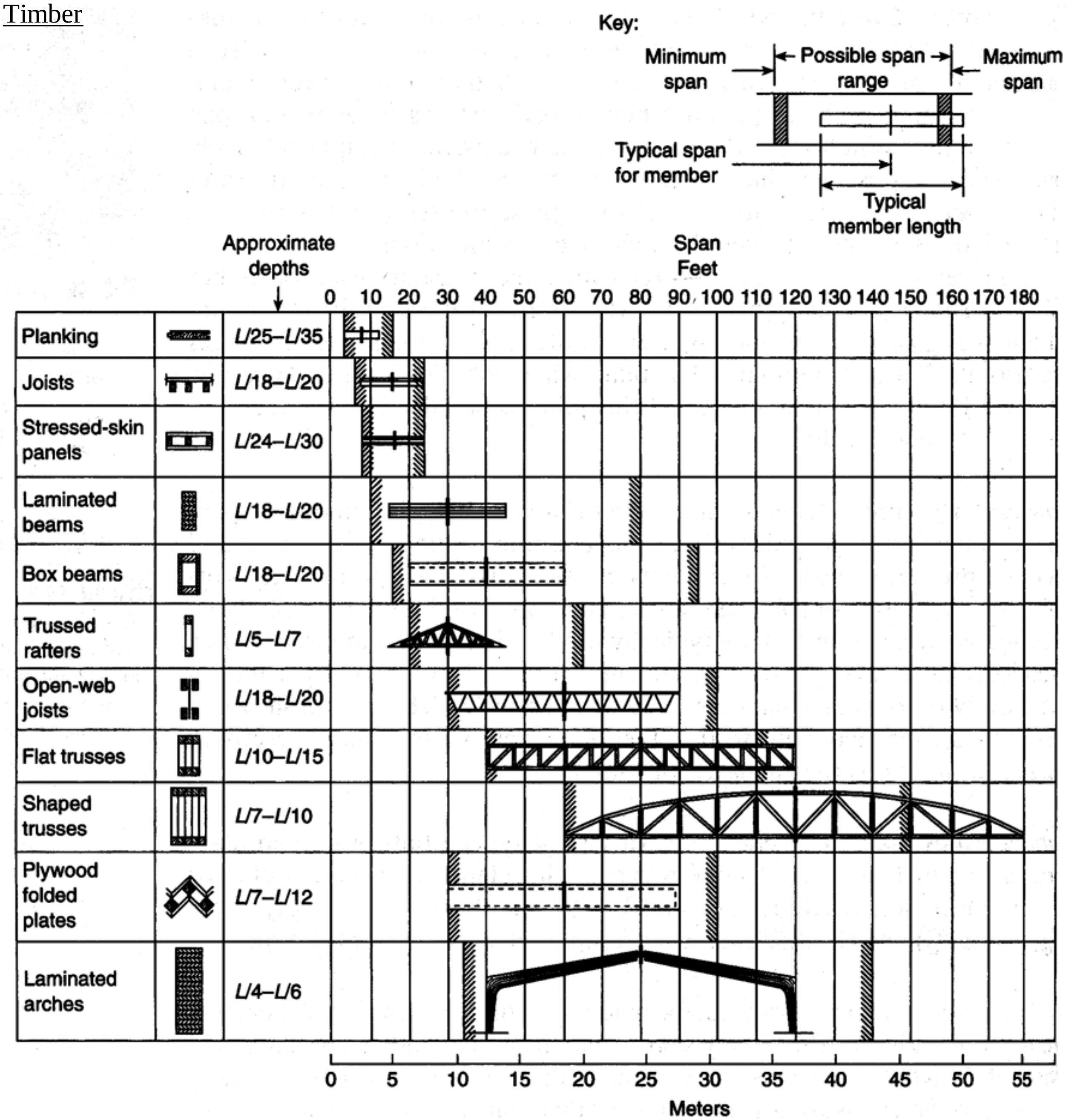


FIGURE 15.4 Approximate span ranges for timber systems. So that typical sizes of different timber members can be compared, the diagrams of the members are scaled to represent typical span lengths for each of the respective elements. The span lengths that are actually possible for each element are noted by the maximum and minimum span marks.

Reinforced Concrete

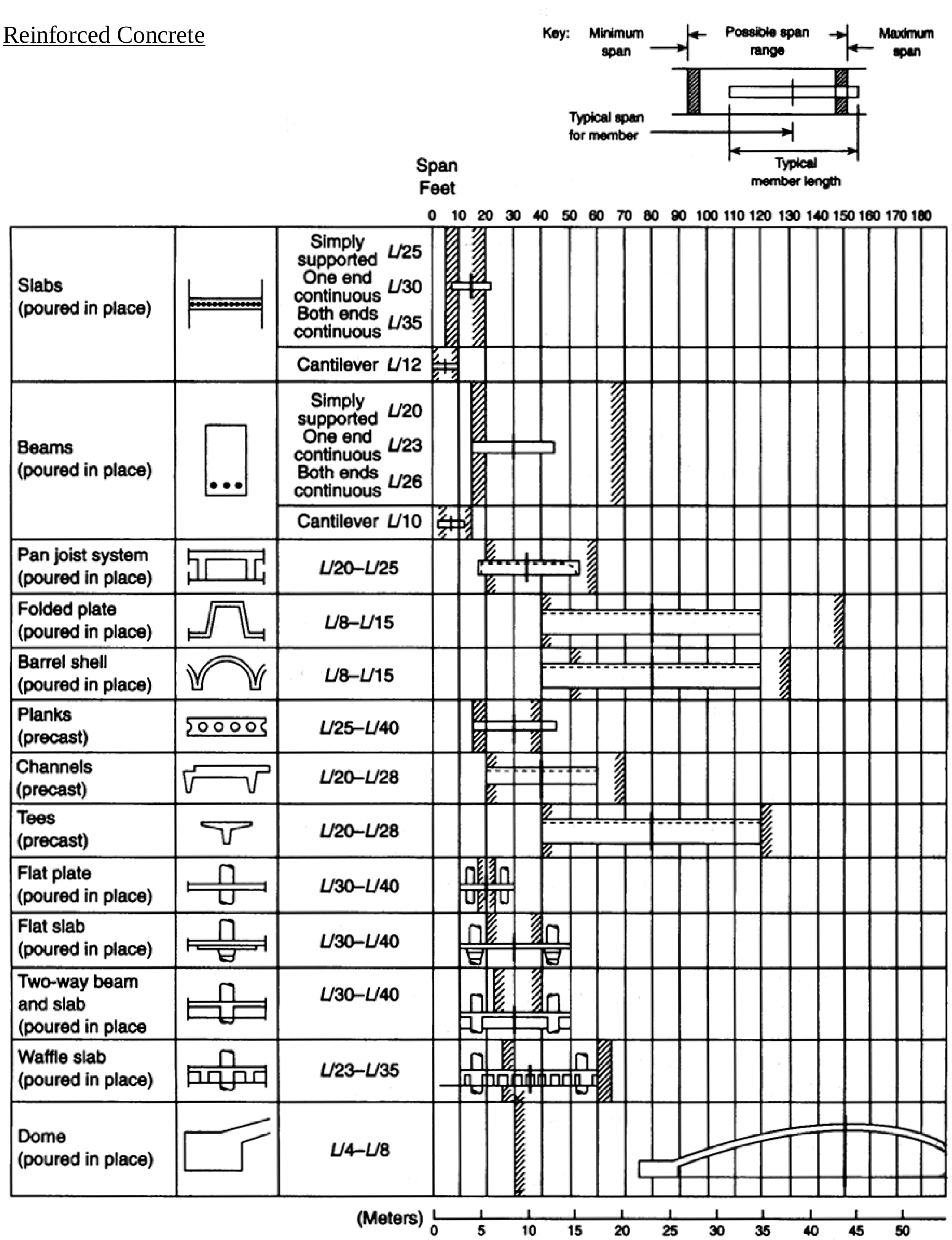


FIGURE 15.6 Approximate span ranges for reinforced-concrete systems. So that typical sizes of different members can be compared, the diagrams of the members are scaled to represent typical span lengths for each of the respective elements. The span lengths that are actually possible for each element are noted by the maximum and minimum span marks.

Steel

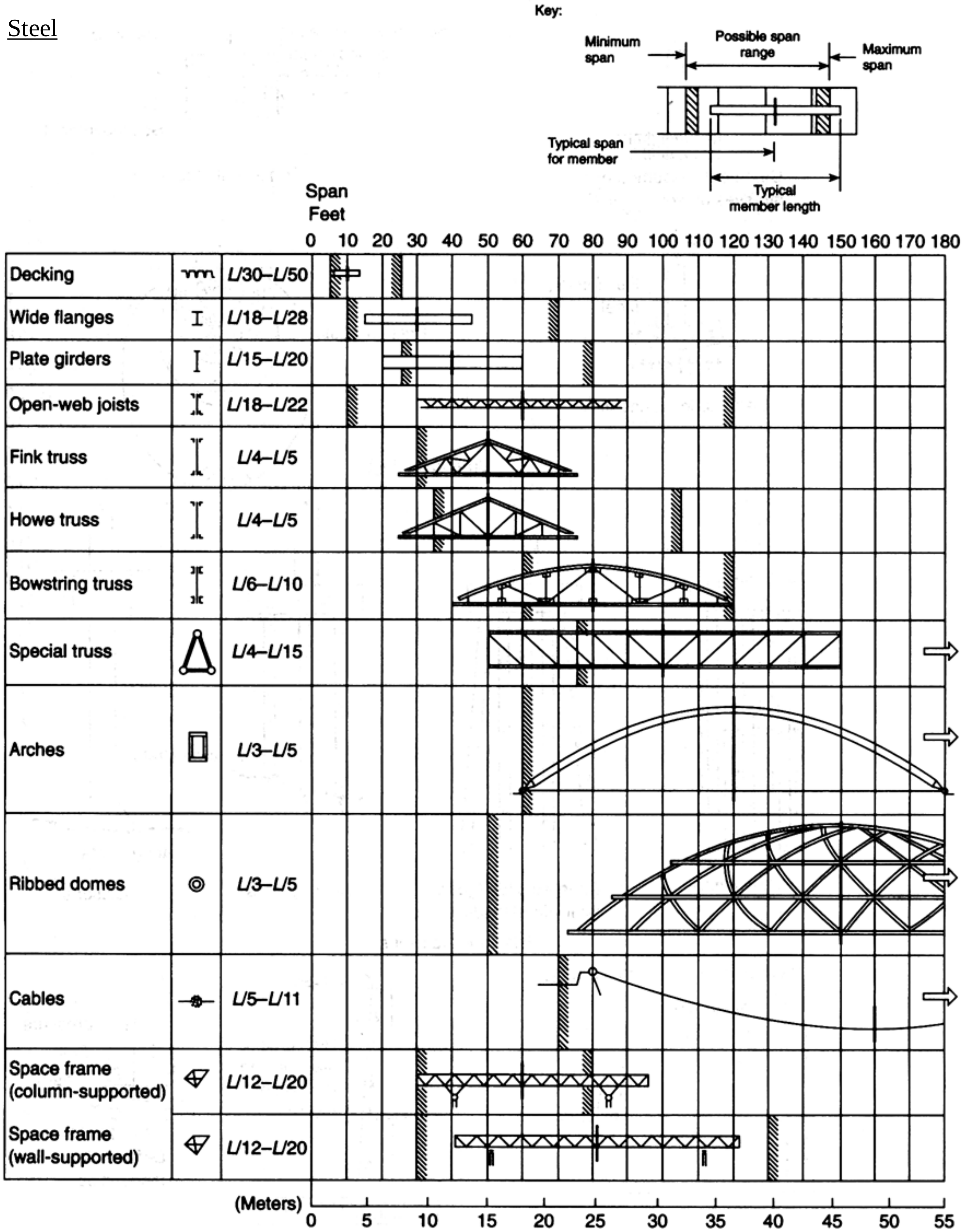


FIGURE 15.9 Approximate span ranges for steel systems. So that typical sizes of different members can be compared, the diagrams of the members are scaled to represent typical span lengths for each of the respective elements. The span lengths that are actually possible for each element are noted by the maximum and minimum span marks.

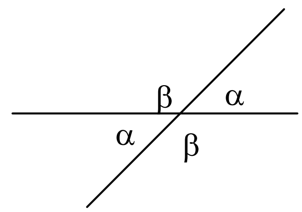
Math for Structures I

1. Parallel lines never intersect.

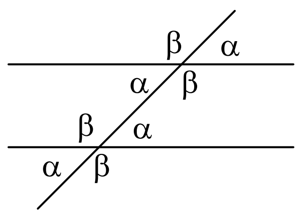
2. Two lines are *perpendicular* (or *normal*) when they intersect at a right angle = 90°.

3. *Intersecting* (or *concurrent)* lines cross or meet at a point.

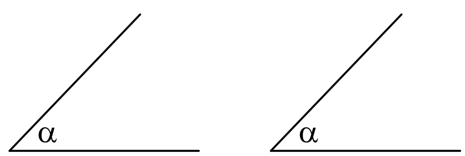
4. If two lines cross, the opposite angles are identical:



5. If a line crosses two parallel lines, the intersection angles with the same orientation are identical:

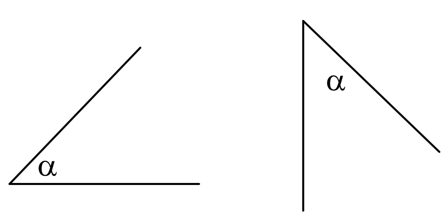


6. If the sides of two angles are parallel and intersect in the same fashion, the angles are identical.

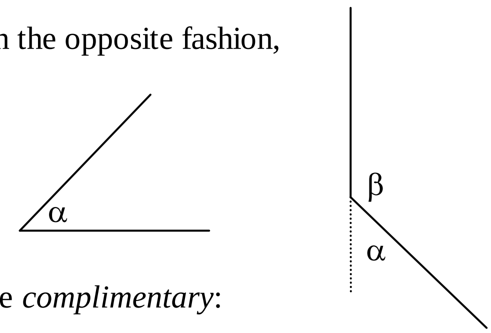


7. If the sides of two angles are parallel, but intersect in the opposite fashion, the angles are *supplementary*: $\alpha+\beta = 180°$.

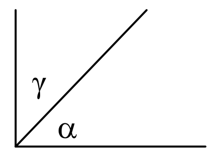
8. If the sides of two angles are perpendicular and intersect in the same fashion, the angles are identical.



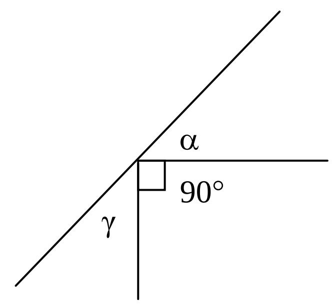
9. If the sides of two angles are perpendicular, but intersect in the opposite fashion, the angles are *supplementary*: $\alpha+\beta = 180°$.



10. If the side of two angles bisects a right angle, the angles are *complimentary*: $\alpha + \gamma = 90°$.

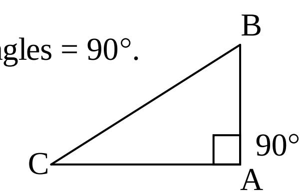


11. If a right angle bisects a straight line, the remaining angles are *complimentary*: $\alpha + \gamma = 90°$.



12. The sum of the interior angles of a triangle = 180°.

13. For a right triangle, that has one angle of 90°, the sum of the other angles = 90°.

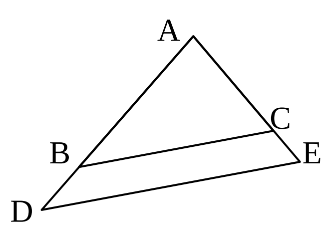


14. For a right triangle, the sum of the squares of the sides equals the square of the hypotenuse:

$$AB^2 + AC^2 = CB^2$$

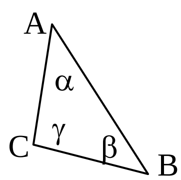
15. Similar triangles have identical angles in the same orientation. Their sides are related by:

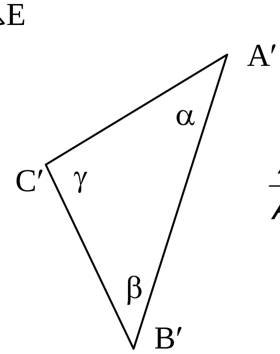
Case 1:



$$\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$$

Case 2:





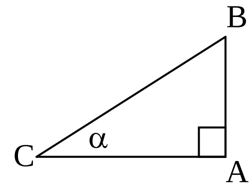
$$\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$$

16. For right triangles:

$$\sin = \frac{opposite side}{hypotenuse} = \sin\alpha = \frac{AB}{CB}$$

$$\cos = \frac{adjacent side}{hypotenuse} = \cos\alpha = \frac{AC}{CB}$$

$$\tan = \frac{opposite side}{adjacent side} = \tan\alpha = \frac{AB}{AC}$$

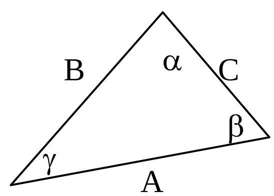


(SOHCAHTOA)

17. If an angle is greater than 180° and less than 360°, *sin* will be less than 0.
If an angle is greater than 90° and less than 270°, *cos* will be less than 0.
If an angle is greater than 90° and less than 180°, *tan* will be less than 0.
If an angle is greater than 270° and less than 360°, *tan* will be less than 0.

18. LAW of SINES (any triangle)

$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$



19. LAW of COSINES (any triangle)

$$A^2 = B^2 + C^2 - 2BC\cos\alpha$$

20. Surfaces or areas have dimensions of width and length and units of length *squared* (ex. in^2 or inches x inches).

21. Solids or volumes have dimension of width, length and height or thickness and units of length *cubed* (ex. m^3 or m x m x m)

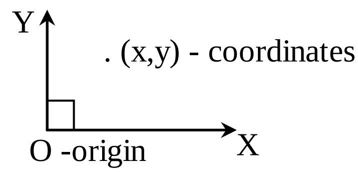
22. Force is defined as mass times acceleration. So a weight due to a mass is accelerated upon by gravity: F = m·g $\quad g = 9.81\, m/sec^2 = 32.17\, ft/sec^2$

23. Weight can be determined by volume if the unit weight or *density* is known: W = V·γ
where ·V is in units of $length^3$ and γ is in units of force/unit volume

24. Algebra: If a·b= c·d then it can be rewritten:

$a \cdot b + k = c \cdot d + k$	add a constant
$c \cdot d = a \cdot b$	switch sides
$a = \frac{c \cdot d}{b}$	divide both sides by *b*
$\frac{a}{c} = \frac{d}{b}$	divide both sides by *b·c*

25. Cartesian Coordinate System



26. Solving equations with one unknown:

1st order polynomial: $2x-1=0\cdots$ $2x=1\cdots$ $x=\frac{1}{2}$

$ax+b=0\cdots$ $x=\frac{-b}{a}$

2nd order polynomial $ax^2+bx+c=0\cdots$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ two answers (radical *cannot* be negative)

$x^2-1=0\cdots$ $(a=1, b=0, c=-1)$ $x=\frac{-0\pm\sqrt{0^2-4(-1)}}{2\cdot 1}\cdots$ $x=\pm 1$

27. Solving 2 linear equations simultaneously:
One equation consisting only of variables can be rearranged and then substituted into the second equation:

ex: $5x-3y=0$
$4x-y=2$

add 3y to both sides to rearrange	$5x=3y$
divide both sides by 5	$x=\frac{3}{5}y$
substitute x into the other equation	$4(\frac{3}{5}y)-y=2$
add like terms	$\frac{7}{5}y=2$
simplify	$\boxed{y=\frac{10}{7}=1.43}$

Equations can be added and factored to eliminate one variable:

ex: $2x+3y=8$
$4x-y=2$

	$2x+3y=8$
multiply both sides by 3	$12x-3y=6$
and add	$14x+0=14$
simplify	$\boxed{x=1}$
put x=1 in an equation for y	$2\cdot 1+3y=8$
simplify	$3y=6$
	$\boxed{y=2}$

28. Derivatives of polynomials

$$y = constant \qquad \frac{dy}{dx} = 0$$

$$y = x \qquad \frac{dy}{dx} = 1$$

$$y = ax \qquad \frac{dy}{dx} = a$$

$$y = x^2 \qquad \frac{dy}{dx} = 2x$$

$$y = x^3 \qquad \frac{dy}{dx} = 3x^2$$

29. The minimum and maximum of a function can be found by setting the derivative = 0 and solving for the unknown variable.

30. Calculators (and software) process equations by an "order of operations", which typically means they process functions like exponentials and square roots before simpler functions such as + or -. BE SURE to specify with parenthesis what order you want, or you'll get the wrong answers. It is also important to have degrees set in your calculator for trig functions.

For instance, Excel uses – for sign (like -1) first, then will process exponents and square roots, times and divide, followed by plus and minus. If you type 4x10^2 and really mean (4x10)^2 you will get an answer of 400 instead of 1600.

Numerical Computations

from Statics and Strength of Materials, 5th ed. Morrow & Kokernak, Prentice Hall, 2004

Accuracy

The accuracy of a numerical value is often expressed in terms of the number of *significant digits* that the value contains. What are significant digits? Any nonzero digit is considered significant; zeroes that appear to the left or right of a digit sequence are used to locate the decimal point and are not considered significant. Thus the numbers 0.00345, 3.45, 3450, and 3,450,000 all contain three significant digits represented by the sequence 3–4–5. Zeroes bounded on both sides by nonzero digits are also significant; 0.0005067, 5.067, 50.67, and 506,700 each contain four significant digits, as represented by the numerical sequence 5–0–6–7.

The accuracy of a solution can be no greater than the accuracy of the data on which the solution is based. For example, the length of one side of a right triangle may be given as 20 ft. Without knowing the possible error in the length measurement, it is impossible to determine the error in the answer obtained from it. We will usually assume that the data are known with an accuracy of 0.2 percent. The possible error in the 20-ft length would therefore be 0.04 ft.

To maintain an accuracy of approximately 0.2 percent in our calculations, we will use the following practical rule: use four digits to record numbers beginning with 1 and three digits to record numbers beginning with 2 through 9. Thus a length of 19 ft becomes 19.00 ft, a length of 20 ft becomes 20.0 ft, and a length of 43 ft becomes 43.0 ft.

You will notice one exception to this rule throughout the text: values of the trigonometric functions are traditionally written to four decimal places, and that practice will be followed here, not for increased accuracy, but to clarify the computations used in worked examples.

Rounding Off Numbers*

If the data are given with greater accuracy than we wish to maintain (see Fig. 1.1), the following rules may be used to round off their values:

1. When the digit dropped is greater than 5, increase the digit to the left by 1. *Example:* 23.56 ft becomes 23.6 ft.
2. When the digit dropped is less than 5, drop it without changing the digit to the left. *Example:* 23.34 ft becomes 23.3 ft.
3. When the digit dropped is 5 followed only by zeros, increase the digit to the left by 1 only if it becomes even. If the digit to the left becomes odd, drop the 5 without changing the digit to the left. *Example:* 23.5500 ft rounded to three numbers becomes 23.6 ft, and 23.4500 ft becomes 23.4 ft. (This practice is often referred to as the *round-even rule*.)

---

*American Society of Mechanical Engineers (ASME) *Orientation and Guide for Use of SI (Metric) Units*, 9th edition, 1982, p 11. By increasing the digit to the left for a final 5 followed by zeros only if the digit becomes even, we are dividing the rounding process evenly between increasing the digit to the left and leaving the digit to the left unchanged.

Calculators

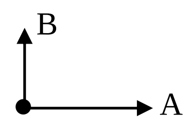
Electronic calculators and computers are widely available for use in engineering. Their speed and accuracy make it possible to do difficult numerical computations in a routine manner. However, because of the large number of digits appearing in solutions, their accuracy is often misleading. As pointed out previously, the accuracy of the solution can be no greater than the accuracy of the data on which the solution is based. Care should be taken to retain sufficient digits in the intermediate steps of the calculations to ensure the required accuracy of the final answer. Answers with more significant digits than are reasonable should not be recorded as the final answer. An accuracy greater than 0.2 percent is rarely justified.

Problem Solving, Units and Numerical Accuracy

Problem Solution Method:

1. Inputs / Outputs / "Critical Path" ⇨ GIVEN: / FIND: / SOLUTION } *on graph paper*
2. Draw simple diagram of body/bodies & forces acting on it/them.
3. Choose a reference system for the forces.
4. Identify key geometry and constraints.
5. Write the basic equations for force components.
6. Count the equations & unknowns.
7. SOLVE
8. "Feel" the validity of the answer. (Use common sense. Check units…)

Example: Two forces, A & B, act on a particle. What is the resultant?



1. GIVEN: Two forces on a particle and a diagram with size and orientation

   FIND: The "resultant" of the two forces

   SOLUTION:

2. Draw what you know (the diagram, any other numbers in the problem statement that could be put on the drawing….)
3. Choose a reference system. What would be the easiest? Cartesian, radian?
4. Key geometry: the location of the particle as the origin of all the forces
   Key constraints: the particle is "free" in space
5. Write equations:

$$sizeof\ A^2 + sizeof\ B^2 = sizeof\ resultant$$

$$\sin\alpha = \frac{sizeof\ B}{sizeof\ A + B}$$

6. Count: Unknowns: 2, magnitude and direction $\leq$ Equations: 2 $\therefore$ can solve
7. Solve: graphically or with equations
8. "Feel": Is the result bigger than A and bigger than B? Is it in the right direction? (like A & B)

Units

Units	Mass	Length	Time	Force
SI	kg	m	s	$N = \frac{kg \cdot m}{s^2}$
Absolute English	lb	ft	s	$Poundal = \frac{lb \cdot ft}{s^2}$
Technical English	$slug = \frac{lb_f \cdot s^2}{ft}$	ft	s	lb_{force}
Engineering English	lb	ft	s	lb_{force}

$$lb_{force} = lb_{(mass)} \times 32.17\, {}^{ft}/_{s^2}$$

gravitational constant	$g_c = 32.17\, {}^{ft}/_{s^2}$	(English)
	$g_c = 9.81\, {}^{m}/_{s^2}$	(SI)
conversions (pg. vii)	$1\,in = 25.4\,mm$	
	$1\,lb = 4.448\,N$	

Numerical Accuracy

Depends on 1) accuracy of data you are given

2) accuracy of the calculations performed

The solution CANNOT be more accurate than the less accurate of #1 and #2 above!

DEFINITIONS:		
	precision	the number of significant digits
	accuracy	the possible error

Relative error measures the degree of accuracy:

$$\frac{relative\ error}{measurement} \times 100 = degree\ of\ accuracy\ (\%)$$

For engineering problems, accuracy *rarely* is less than 0.2%.

Forces and Vectors

Notation:

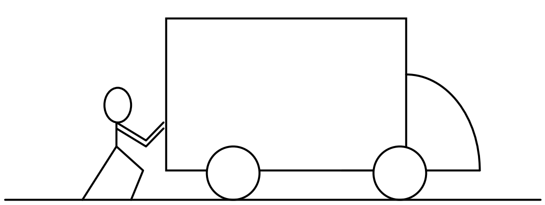
F = name for force vectors, as is *A, B, C, T and P*
F_x = force component in the x direction
F_y = force component in the y direction
R = name for resultant vectors
R_x = resultant component in the x direction
R_y = resultant component in the y direction
tail = start of a vector (without arrowhead)
tip = direction end of a vector (with arrowhead)
x = x axis direction
y = y axis direction
θ = angle, in a trig equation, ex. $\sin\theta$, that is measured between the x axis and *tail* of a vector

Characteristics

- Forces have *a point of application* – tail of vector
  size – units of lb, K, N, kN
  direction – to a reference system, sense indicated by an arrow
- Classifications include: *Static & Dynamic*
- Structural types separated primarily into *Dead Load* and *Live Load* with further identification as wind, earthquake (seismic), impact, etc.

Rigid Body

- *Ideal* material that doesn't deform
- Forces on rigid bodies can be *internal* – within or at connections
  or *external* – applied
- Rigid bodies can *translate* (move in a straight line)
  or *rotate* (change angle)

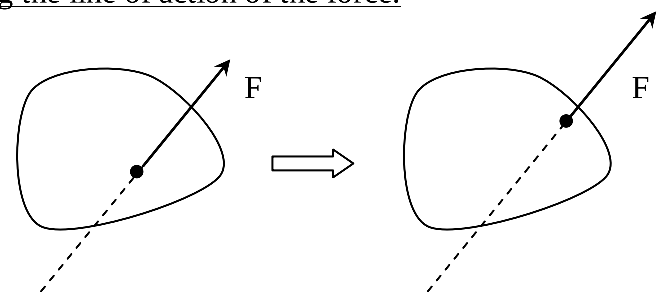


- Weight of truck is external (gravity)
- Push by driver is external
- Reaction of the ground on wheels is external

If the truck moves forward: *it translates*

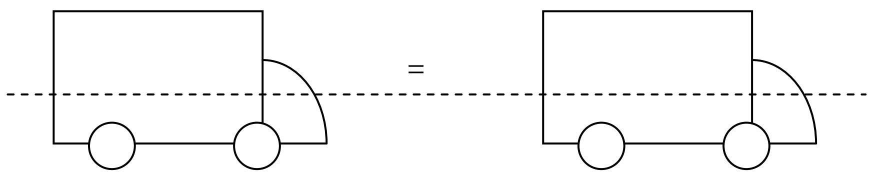
If the truck gets put up on a jack: *it rotates*

- *Transmissibility:* We can replace a force at a point on a body by that force on another point on the body along the line of action of the force.



External conditions haven't changed

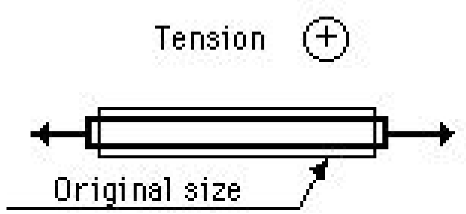
For the truck:



- The same external forces will result in the same conditions for motion
- Transmissibility applies to EXTERNAL forces. INTERNAL forces respond differently when an external force is moved.
- DEFINITION: *2D Structure* - A structure that is flat and may contain a plane of symmetry. All forces on this structure are in the same plane as the structure.

Internal and External

- *Internal forces* occur within a member or between bodies within a system
- *External forces* represent the action of other bodies or gravity on the rigid body



Force System Types

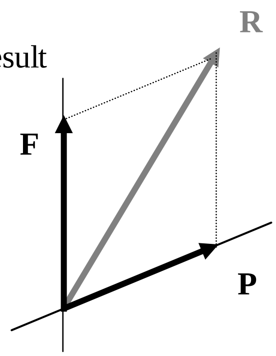
- *Collinear* – all forces along the same **line**
- *Coplanar* – all forces in the same **plane**
- *Space* – out there

Further classification as

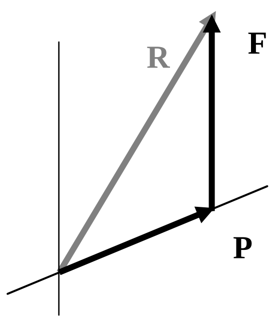
- *Concurrent* – all forces go through the same **point**
- *Parallel* – all forces are **parallel**

Graphical Addition

- *Parallelogram law:* when adding two vectors acting at a point, the result is the **diagonal** of the parallelogram



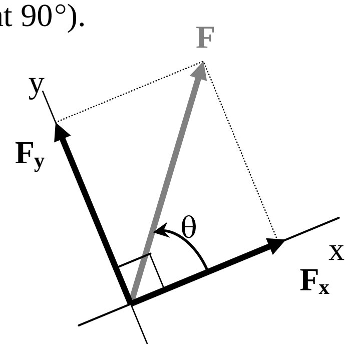
- The *tip-to-tail* method is another graphical way to add vectors.

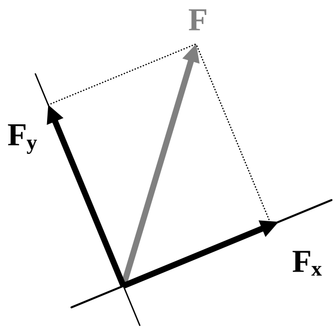


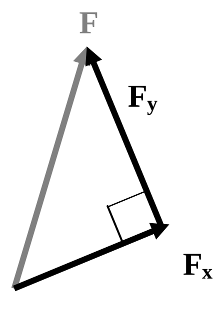
- With **3 (three)** or more vectors, successive application of the parallelogram law will find the resultant *OR* drawing all the vectors **tip-to-tail** in any order will find the resultant.

Rectangular Force Components and Addition

- It is convenient to resolve forces into perpendicular components (at 90°).
- Parallelogram law results in a rectangle.
- Triangle rule results in a right triangle.







θ is: *between x & F*

$F_x = F \cdot \cos\theta$

$F_y = F \cdot \sin\theta$

} magnitudes are *scalar* and can be negative; F_x & F_y are *vectors* in x and y direction

$F = \sqrt{F_x^2 + F_y^2}$

$\tan\theta = \dfrac{F_y}{F_x}$

When 90° < θ < 270°, F_x is *negative*

When 180° < θ < 360°, F_y is *negative*

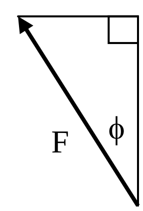
When 0° < θ < 90° and 180° < θ < 270°, tanθ is *positive*

When 90° < θ < 180° and 270° < θ < 360°, tanθ is *negative*

- Addition (analytically) can be done by adding all the *x* components for a **resultant** *x* component and adding all the **y** components for a resultant *y* component.

$$R_x = \sum F_x\,,\quad R_y = \sum F_y \quad \text{and} \quad R = \sqrt{R_x^2 + R_y^2} \qquad \tan\theta = \frac{R_y}{R_x}$$

<u>CAUTION:</u> An interior angle, ϕ, between a vector and *either* coordinate axis can be used in the trig functions. BUT <u>*No sign*</u> will be provided by the trig function, which means **you** must give a sign and determine if the component is in the x or y direction. *For example,* $F\sin\phi =$ *opposite side, which whould be negative in x!*

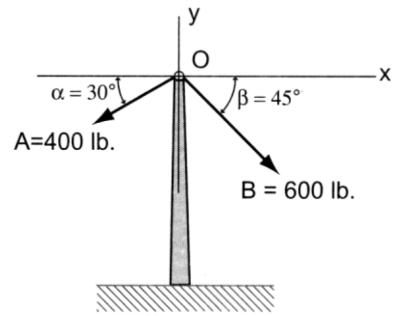


Example 1 (page 9)

Example Problem 2.2

A utility pole supports two tension forces *A* and *B* with the directions shown. Using the parallelogram law and the tip-to-tail methods, determine the resultant force for *A* and *B* (magnitude and direction).

Scale: 1" = 200 lb.



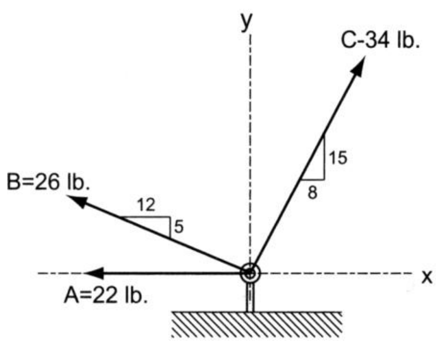
Steps:

1. GIVEN: Write down what's given (drawing and numbers).
2. FIND: Write down what you need to find. (resultant graphically)
3. SOLUTION:
4. Draw the 400 lb and 600 lb forces to scale with tails at 0. (If the scale isn't given, you must choose one that fits on your paper, ie. 1 inch = 200 lb.)
5. Draw parallel reference lines at the ends of the vectors.
6. Draw a line from O to the intersection of the reference lines
7. Measure the length of the line
8. Convert the line length by the scale into pounds (by multiplying by the force measure and dividing by the scale value, ie X inches * 200 lb / 1 inch).

Alternate solution:

4. Draw one vector
5. Draw the other vector at the TIP of the first one (away from the tip).
6. Draw a line from 0 to the tip of the final vector and continue at step 7

<u>Example 2 (pg 12)</u>

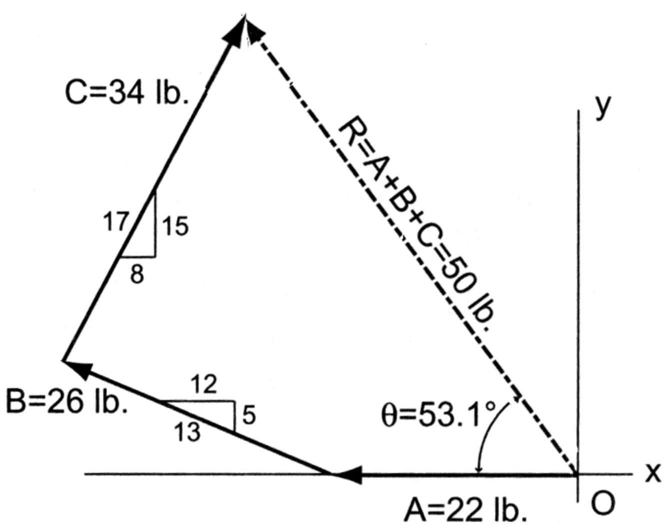


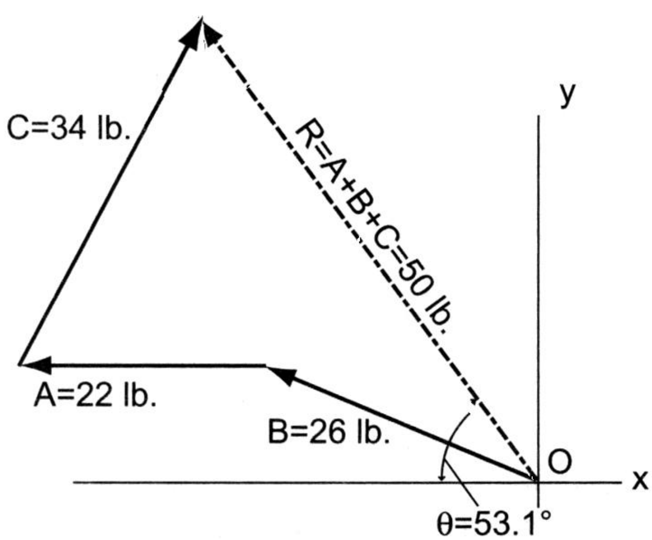
Example Problem 2.4

A tent stake is subjected to three pulling forces, as shown in Figure 2.18. Using the graphical tip-to-tail method, determine the resultant of forces ***A***, ***B***, and ***C*** (magnitude and direction).

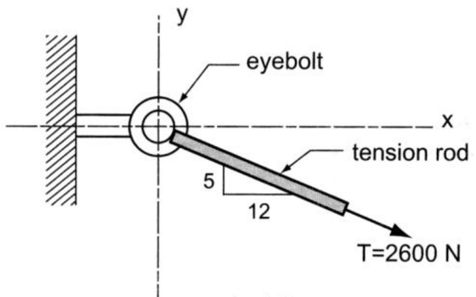
1.5 mm = 1 lb. or 1mm = 2/3 lb.

Suggested scale: ~~1/8" = 1 lb. or 1" = 8 lb.~~





Example 3 (pg 16)



(a)

Example Problem 2.7

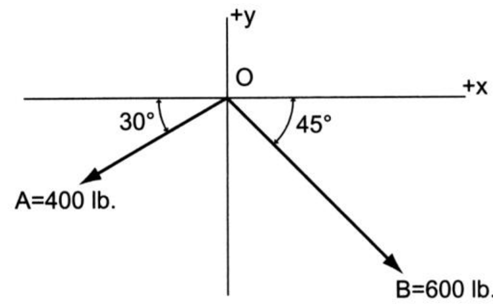
A large eyebolt (Figure 2.24) is used in supporting a canopy over the entry to an office building. The tension developed in the support rod is equal to 2600 newtons. Determine the rectangular components of the force if the rod is at a 5 in 12 slope.

Also determine the embedment length, L, if the anchor can resist 500 N for ever cm of embedment.

Example 4 (pg 19) Determine the resultant vector analytically with the component method.

Example Problem 2.9 (Figure 2.29)

This is the same problem as Example Problem 2.2, which was solved earlier using the graphical methods.



(a)

Moments

Notation:

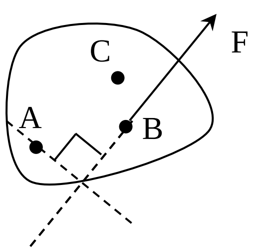
d	=	perpendicular distance to a force from a point
F	=	name for force vectors or magnitude of a force, as is *P, Q, R*
F_x	=	force component in the x direction
F_y	=	force component in the y direction
M	=	moment due to a force
W	=	name for force due to weight
x	=	horizontal distance
θ	=	angle, in a trig equation, ex. $\sin\theta$, that is measured between the x axis and *tail* of a vector

Moment of a Force About an Axis

- Two forces of the same size and direction acting at different points *are not equivalent.* They may cause the same **translation**, but they cause different **rotation**.

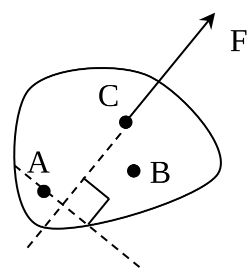
- DEFINITION: *Moment* – A moment is the tendency of a force to make a body rotate about an axis. It is measured by F×d, where d is the distance **perpendicular** to the line of action of the force and through the axis of rotation.

$M = F \cdot d$
(about A)

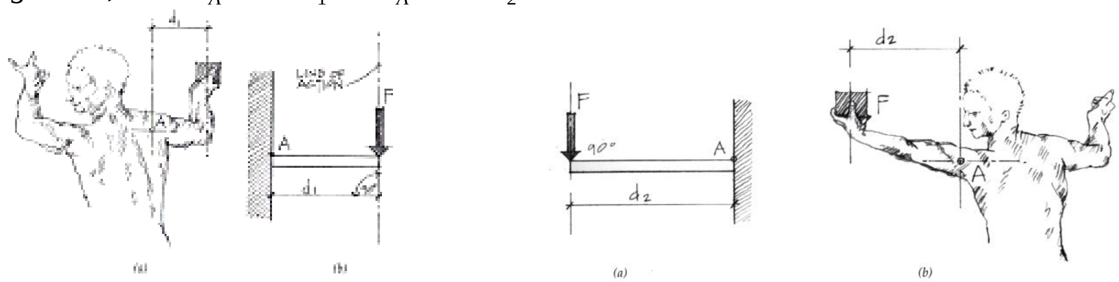


not equivalent

$M = F \cdot d'$
(force at C)



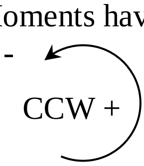
- For the same force, the bigger the **lever arm (or moment arm)**, the bigger the moment *magnitude,* i.e. $M_A = F \cdot d_1 < M_A = F \cdot d_2$



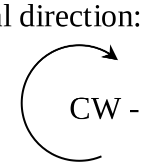
- Units: SI: N·m, KN·m
  Engr. English: lb-ft, kip-ft

- Sign conventions: Moments have magnitude *and* rotational direction:

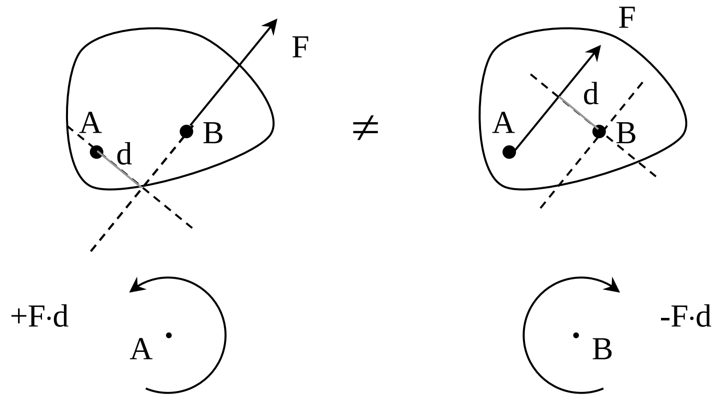
positive -



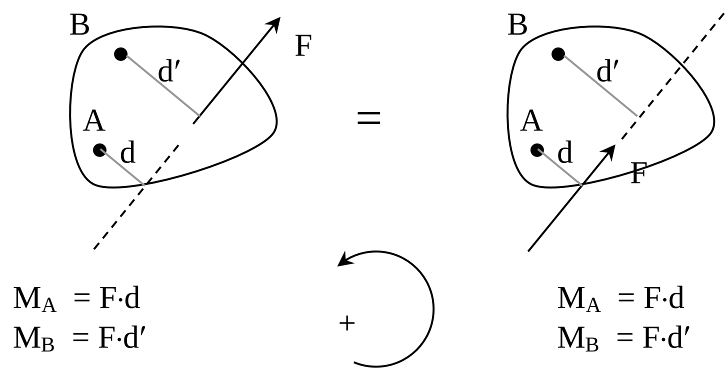
negative –



- Moments can be <u>added as scalar quantities</u> when there is a sign convention.

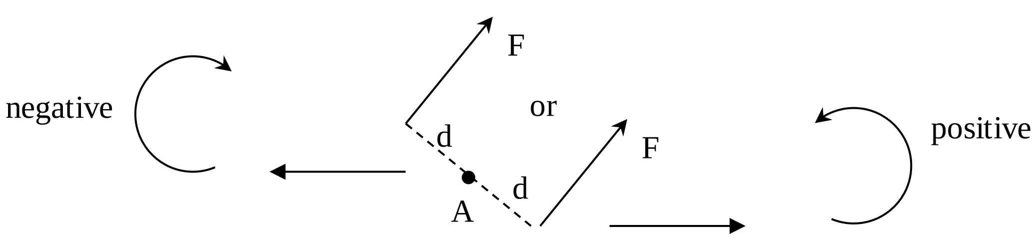


- Repositioning a force along its line of action results in the same moment about any axis.

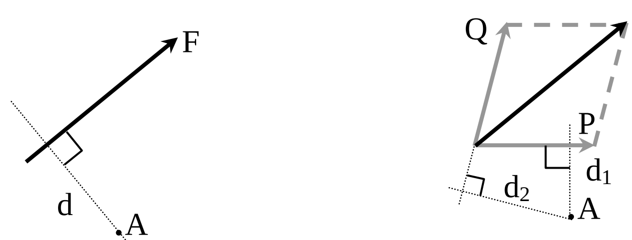


- A force is completely defined (except for its exact position on the line of action) by F_x, F_y, and M_A about A (size and direction).

- The *sign* of the moment is determined by which side of the axis the force is on.

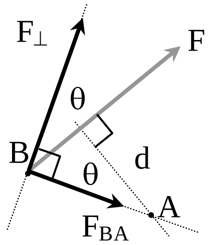


- *Varignon's Theorem:* The moment of a force about any axis is equal to the sum of moments of the components about that axis.



$$M = F \cdot d = P \cdot d_1 + Q \cdot d_2$$

- Proof 1: Resolve F into components along line BA and perpendicular to it (90°).



d from A to line AB = 0

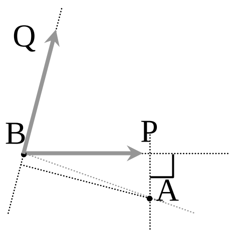
d from A to $F_{\perp}$ = d_{BA} = $\dfrac{d}{\cos\theta}$

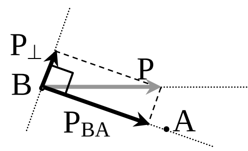
$F_{BA} = F\sin\theta$

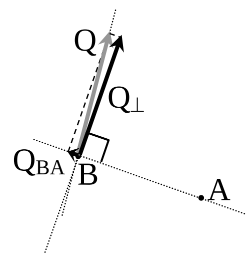
$F_{\perp} = F\cos\theta$

$$\sum M = -F\cdot d = -F_{BA}\cdot 0 - F_{\perp}\cdot d_{BA} = -F\cos\theta\cdot\frac{d}{\cos\theta} = -F\cdot d$$

- Proof 2: Resolve P and Q into P_{BA} & $P_{\perp}$, and Q_{BA} & $Q_{\perp}$.







d from A to line AB = 0

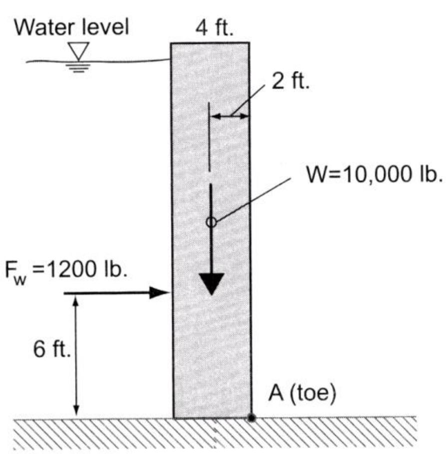
$M_{A\text{ by }P} = -P_{\perp}\cdot d_{BA}$ $M_{A\text{ by }Q} = -Q_{\perp}\cdot d_{BA}$

$\sum M = -P_{\perp}\cdot d_{BA} + (-Q_{\perp}\cdot d_{BA})$

and we know d_{BA} from Proof 1, and by definition: $P_{\perp} + Q_{\perp} = F_{\perp}$. We know d_{BA} and $F_{\perp}$ from above, so again $M = -F_{\perp}\cdot d_{BA} = -F\cdot d$

- By choosing component directions such that d = 0 to one of the components, we can simplify many problems.

Example 1 (pg 24)

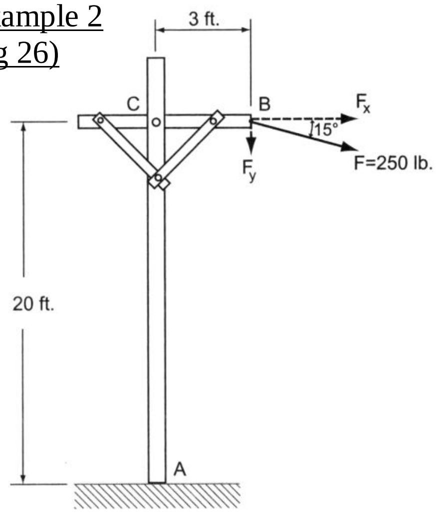


Example Problem 2.13 (Figure 2.35)

A 1-foot-wide slice of a 4-foot-thick concrete gravity dam weighs 10,000 pounds and the equivalent force due to water pressure behind the dam is equal to 1200 pounds. The stability of the dam against overturning is evaluated about the "toe" at *A*.

Determine the resultant moment at *A* due to the two forces shown. Is the dam stable?

<u>Example 2 (pg 26)</u>



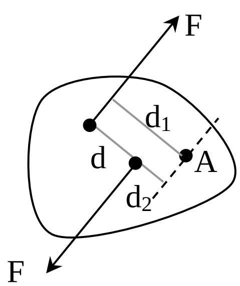
Example Problem 2.15 (Figure 2.38)

The same problem from Example Problem 2.14 will be solved using the principle of moments. (Figure 2.36)

(Moment at A)

Moment Couples

- *Moment Couple:* Two forces with equal magnitude, parallel lines of action and opposite sense tend to make our body rotate even though the sum of forces is 0. The sum of the moment of the forces about any axis *is not* zero.



$$\sum M = F \cdot d_2 - F \cdot d_1 = M$$

$$M = F(d_2 - d_1)$$

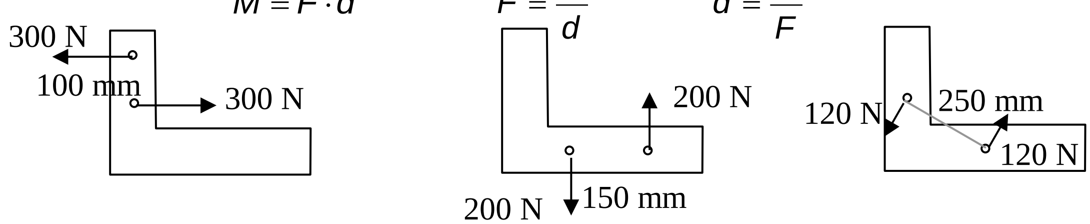
$M = -F \cdot d$: moment of the couple (CW)

- M does *not depend on where A is*. M depends on the perpendicular distance between the line of action of the parallel forces.

- M for a couple (defined by F and d) is a <u>constant.</u> And the sense (+/-) is obtained by observation.

- Just as there are equivalent moments (other values of F and d that result in M) there are equivalent couples. The magnitude is the same for different values of F and resulting d or different values of d and resulting F.

$M = F \cdot d$ $\quad F = \dfrac{M}{d}$ $\quad d = \dfrac{M}{F}$



Equivalent Force Systems

- Two systems of forces are equivalent if we can transform one of them into the other with:

  1.) replacing *two forces on a point* by their **resultant**
  2.) resolving a *force* into two components
  3.) canceling two equal and opposite forces on a point
  4.) attaching two equal and opposite forces to a point
  5.) moving a force along its line of action'
  6.) replacing a force and moment on a point with a force on another (specific) point
  7.) replacing a force on point with a force and moment on another (specific) point

  * based on the parallelogram rule and the principle of transmissibility

- The size and direction are important for a moment. The location on a body doesn't matter because couples with the same moment will have the same effect on the rigid body.

Addition of Couples

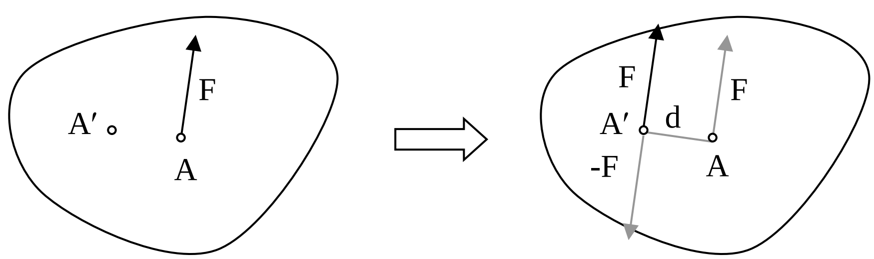
- Couples can be added as *scalars.*

- Two couples can be *replaced* by a single couple with the magnitude of the algebraic sum of the two couples.

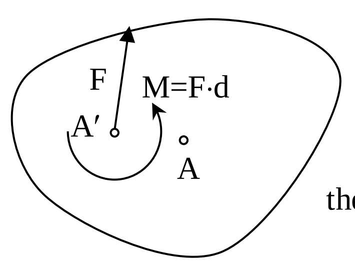
Resolution of a Force into a Force and a Couple

- The equivalent action of a force on a body can be reproduced by that force and a force couple:

  If we'd rather have F acting at A′ which isn't in the line of action, we can instead add F and –F at A′ with no change of action by F. Now it becomes a couple of F separated by d and the force F moved to A′. The size is F·d=M



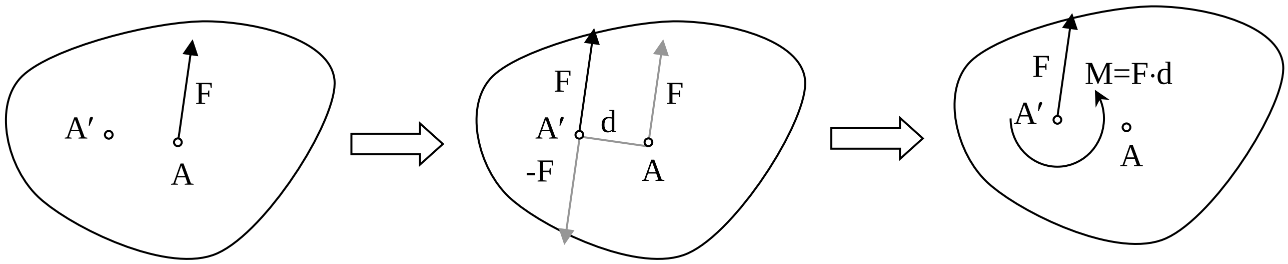
The couple can be represented by a moment symbol:



- Any force can be replaced by itself at another point and the moment equal to the force multiplied by the distance between the original line of action and *new* line of action.

Resolution of a Force into a Force and a Moment

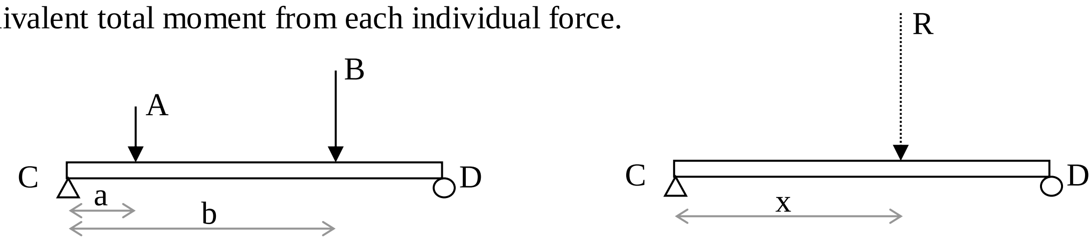
- *Principle:* Any force **F** acting on a rigid body (say the one at A) may be moved to any given point A′, provided that a couple **M** is added: the moment **M** of the couple must equal the moment of **F** (in its original position at A) about A′.



- IN REVERSE: A force **F** acting at A′ and a couple **M** may be combined into a single resultant force **F** acting at A (a distance *d* away) where the moment of **F** about A′ is equal to **M**.

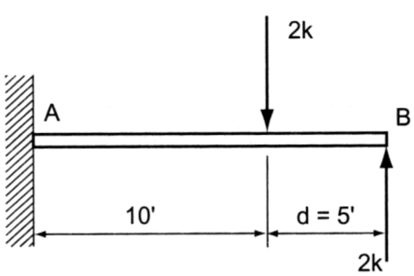
Resultant of Two Parallel Forces

- Gravity loads act in one direction, so we may have parallel forces on our structural elements. We know how to find the resultant **force**, but the *location* of the resultant must provide the equivalent total moment from each individual force.



$$R = A + B \qquad M_C = A \cdot a + B \cdot b = R \cdot x \Rightarrow x = \frac{A \cdot a + B \cdot b}{R}$$

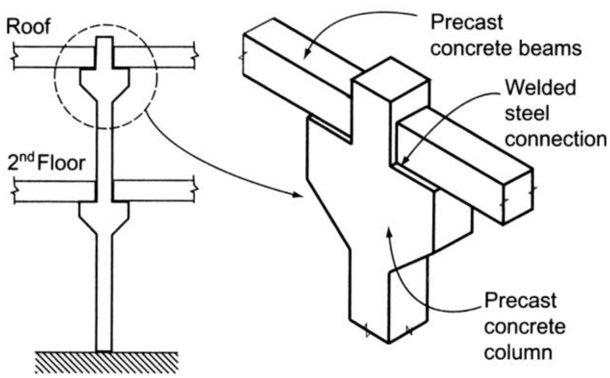
Example 3 (pg 19)



Example Problem 2.19

The cantilevered beam shown in Figure 2.43 is subjected to two equal and opposite forces as shown. Determine the resultant moment M_A at the beam support and the moment M_B at the free end.

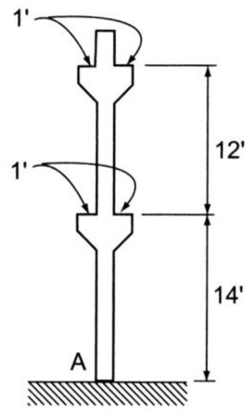
Example 4 (pg 34)



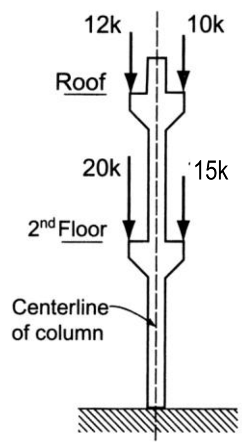
Example Problem 2.22 (Figures 2.49 and 2.50)

A major, precast-concrete column supports beam loads from the roof and second floor as shown. Beams are supported by seats projecting from the columns. Loads from the beams are assumed to be applied one foot from the column axis.

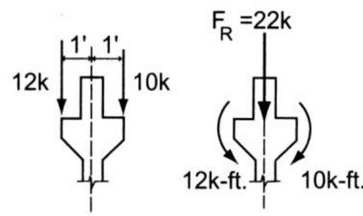
Determine the equivalent column load condition when all beam loads are shown acting through the column axis.



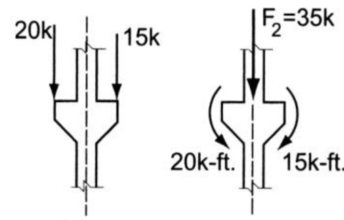
(a)



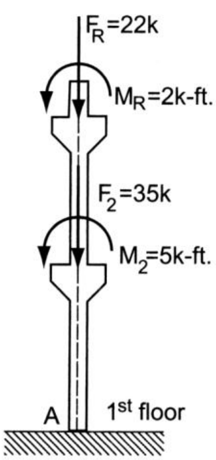
(b)



Roof level
(c)



2nd floor
(d)



(e)

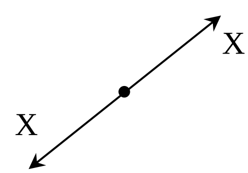
Equilibrium of a Particle & Truss Analysis

Notation:

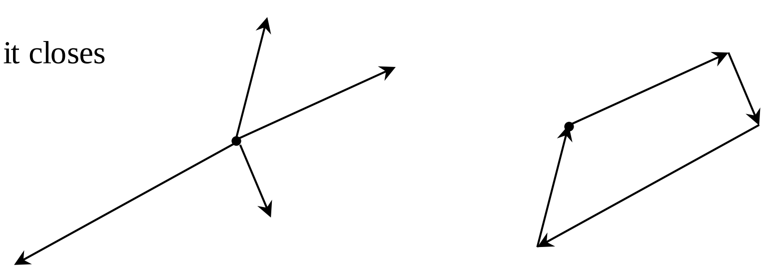
b = number of members in a truss
(C) = shorthand for *compression*
F = name for force vectors, as is X, *and* P
F_{AB} = name of a truss force between joints named A and B, *ex.*
FBD = free body diagram
F_x = force component in the x direction, as is T_x
F_y = force component in the y direction, as is T_y
h = cable sag height
L = span length
n = number of joints in a truss
N = normal force (perpendicular to something)
R = name for resultant vectors
R_x = resultant component in the x direction
R_y = resultant component in the y direction
T = name for a tension force
(T) = shorthand for *tension*
x = x axis direction, or horizontal dimension
y = y axis direction, or vertical dimension
W = name for force due to weight
μ = coefficient of static friction
θ = angle, in a trig equation, ex. $\sin\theta$, that is measured between the x axis and *tail* of a vector
Σ = summation symbol

- EQUILIBRIUM is the state where the resultant of the forces on a particle or a rigid body is *zero*. There will be no rotation or translation. The forces are referred to as balanced.

ex: 2 forces of same size, opposite direction



ex: 4 forces, polygon rule shows that it closes



- Analytically, for a point:

$$R_x = \sum F_x = 0 \qquad R_y = \sum F_y = 0 \quad \text{(scalar addition)}$$

- NEWTON'S FIRST LAW: If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

Collinear Force System

- *All forces act along the same line.* Only one equilibrium equation is needed: $\sum F_{(in-line)} = 0$

- Equivalently: $R_x = \sum F_x = \mathbf{0}$ and $R_y = \sum F_y = \mathbf{0}$

Concurrent Force System

- *All forces act through the same point.* Only two equilibrium equations are needed:

$$R_x = \sum F_x = \mathbf{0} \text{ and } R_y = \sum F_y = \mathbf{0}$$

- It is ABSOLUTELY NECESSARY to consider all the forces acting on a body (applied directly and indirectly) using a FREE BODY DIAGRAM. Omission of a force would ruin the conditions for equilibrium.

- FREE BODY DIAGRAM (aka FBD): Sketch of a significant isolated particle of a body or structure showing all the forces acting on it. Forces can be from

  - externally applied forces
  - weight of the rigid body
  - reaction forces or constraints
  - forces developed within a section member

- How to solve when there are more than three forces on a free body:

  1. *Resolve all forces into x and y components using known and unknown forces and angles. (Tables are helpful.)*
  2. *Determine if any unknown forces are related to other forces and write an equation.*
  3. *Write the two equilibrium equations (in x and y).*
  4. *Solve the equations simultaneously when there are the same number of equations as unknown quantities. (see math handout)*

- Common problems have unknowns of: 1) **magnitude of force**

  2) **direction of force**

FREE BODY DIAGRAM STEPS FOR A POINT;

1. Determine the point of interest. (What point is in equilibrium?)

2. Detach the point from and all other bodies (*"free" it).*

3. Indicate all external forces which include:

    - action <u>on</u> the point by the **supports & connections**

    - action on the point by other bodies

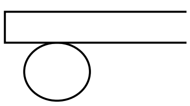
    - the weigh effect (=force) of any body attached to the point (force due to gravity)

4. All forces should be clearly marked with magnitudes and direction. The sense of forces should be those acting *on the point* not <u>from</u> the point.

5. Dimensions/angles should be included for force component computations.

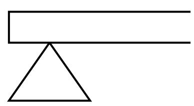
6. Indicate the <u>unknown</u> forces, such as those reactions or constraining forces where the body is supported or connected.

- *Force Reactions* can be categorized by the type of connections or supports. A force reaction is a force with known line of action, or a force of unknown direction. The line of action of the force is directly related to the motion that is prevented.



prevents motion:

up and down



prevents motion:

vertical & horizontal

- The line of action should be indicated on the FBD. The sense of direction is determined by the type of support. (Cables are in tension, etc…) *If the sense isn't obvious, assume a sense.* When the reaction value comes out positive, the assumption was correct. When the reaction value comes out negative, the assumption was *opposite* the actual sense. *DON'T CHANGE THE ARROWS ON YOUR FBD OR SIGNS IN YOUR EQUATIONS.*

- With the 2 equations of equilibrium for a point, <u>there can be no more than 2 unknowns</u>.

Friction

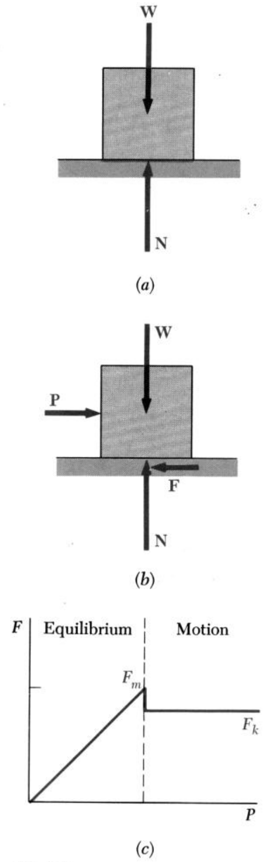
- There will be a force of resistance to movement developed at the contact face between objects when one is made to slide against the other. This is known as *static friction* and is determined from the reactive force, *N*, which is normal to the surface, and a coefficient of friction, μ, which is based on the materials in contact.

$$F = \mu N$$

- If the static friction force is exceeded by the pushing force, there will still be friction, but it is called *kinetic friction*, and it is smaller than static friction, so it is moving.

- The friction resistance is independent of the amount of contact area.

Materials	μ range
Metal on ice	0.03-0.05
Metal on metal	0.15-0.60
Metal on wood	0.20-0.60
Metal on stone	0.30-0.70
Wood on wood	0.30-0.70
Steel on steel	0.75
Stone on stone	0.40-0.70
Rubber on concrete	0.60-0.90
Aluminum on aluminum	1.10-1.70



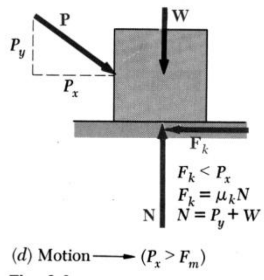


Fig. 8.1

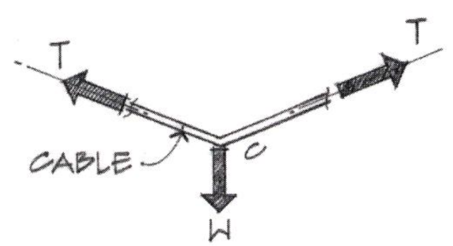
- CABLE STRUCTURES:

  Cables have the same tension all along the length if they are not cut. The force *magnitude* is the same everywhere in the cable even if it changes angles. Cables CANNOT be in compression. (They flex instead.)

  High-strength steel is the most common material used for cable structures because it has a high strength to weight ratio.

  Cables must be supported by vertical supports or towers and must be anchored at the ends. Flexing or unwanted movement should be resisted. (Remember the Tacoma Narrows Bridge?)

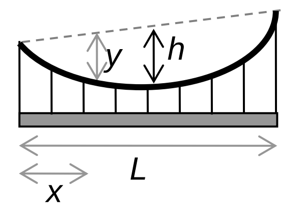
  Cables with a single load have a **concurrent** force system. It will only be in equilibrium if the cable is **symmetric**.



  The forces anywhere in a *straight segment* can be resolved into x and y components of $T_x = T\cos\theta$ and $T_y = T\sin\theta$.

The shape of a cable having a *uniform distributed load* is almost parabolic, which means the geometry and cable length can be found with:

$$y = 4h(Lx - x^2)/L^2$$



where y is the vertical distance from the straight line from cable start to end

h is the vertical sag (maximum y)

x is the distance from one end to the location of y

L is the horizontal span.

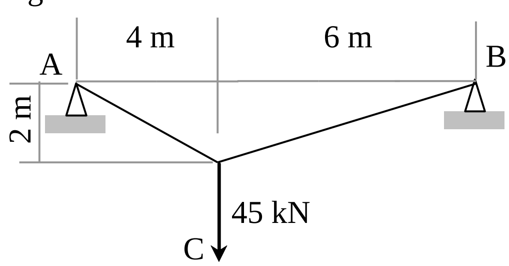
$$L_{total} = L\left(1 + \tfrac{8}{3}\frac{h^2}{L^2} - \tfrac{32}{5}\frac{h^4}{L^4}\right)$$

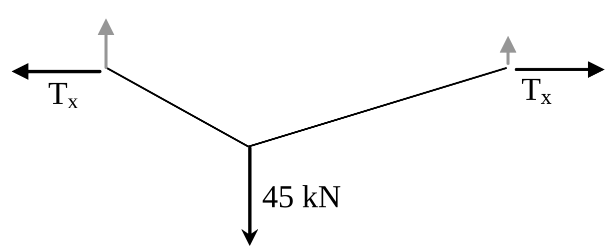
where L_{total} is the total length of parabolic cable

h and L are defined above.

Cables with Several Concentrated Loads or Fixed Geometry

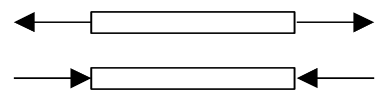
- In order to completely constrain cables, the number of unknown support reactions *will be more* than the available number of equilibrium equations. We can solve because we have additional equations from geometry due to the **slope** of the cable.
- The tension in the cable IS NOT the same everywhere, but the horizontal component in a cable segment WILL BE.





Truss Structures

- A truss is made up of straight two-force members connected at its ends. The triangular arrangement produces stable geometry. Loads on a truss are applied at the joints only.
- Joints are pin-type connections (resist translation, not rotation).
- Forces of action and reaction on a joint must be equal and opposite.
- Members in TENSION are being pulled.
- Members in COMPRESSION are being squeezed.



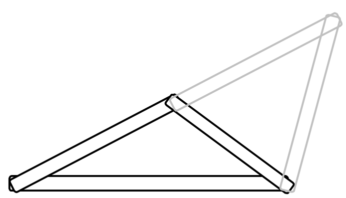
- External forces act on the joints.

- <u>Truss configuration</u>:

  Three members form a <u>rigid assembly</u> with **3 (three)** connections.

  To add members and still have a rigid assembly, **2 (two)** more must be added with one connection between.

  For rigidity: b = 2n – 3, where *b* is number of members and *n* is number of joints



<u>Method of Joints</u>

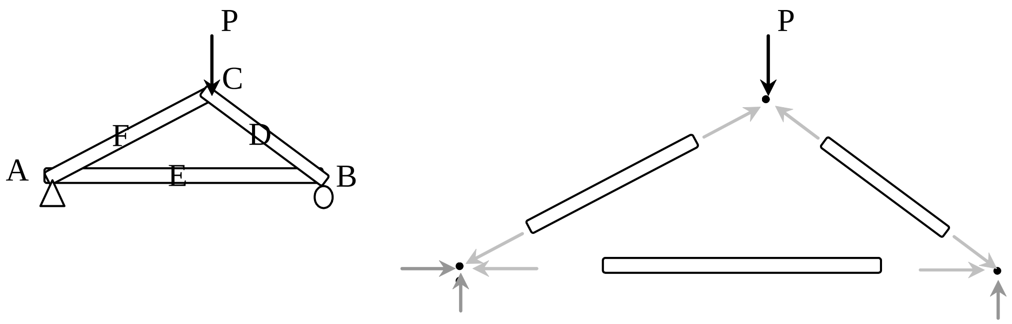
- The method takes advantage of the conditions of equilibrium at each joint.

1. Determine support reaction forces.
2. Draw a FBD of each member AND each joint
3. Identify geometry of angled members
4. Identify zero force members and other special (easy to solve) cases
5. Each pin is in equilibrium ($\sum F_x = 0$ and $\sum F_y = 0$ for a concurrent force system)
6. Total equations = 2n = b+3 (one force per member + 3 support reactions)

Advantages: Can find every member force
Disadvantages: Lots of equations, easy to lose track of forces found.

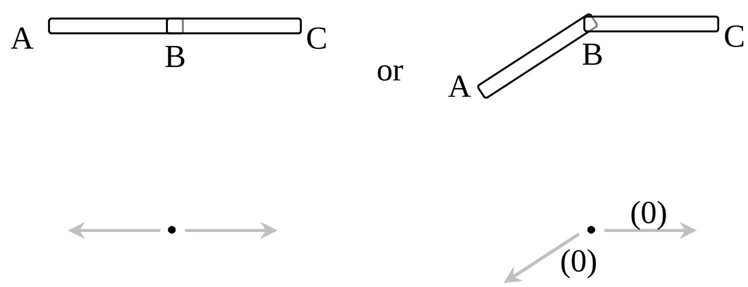
Tools available:

Tip-to-tail method for 3 joint forces must close
Analytically, there will be at most 2 unknowns with 2 equilibrium equations.



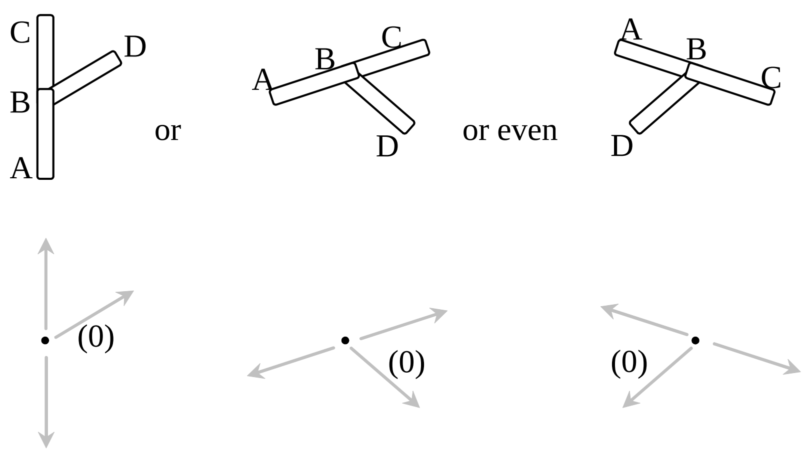
Joint Configurations (special cases to recognize for faster solutions)

Case 1) Two Bodies Connected



F_{AB} has to be **equal (=)** to F_{BC}

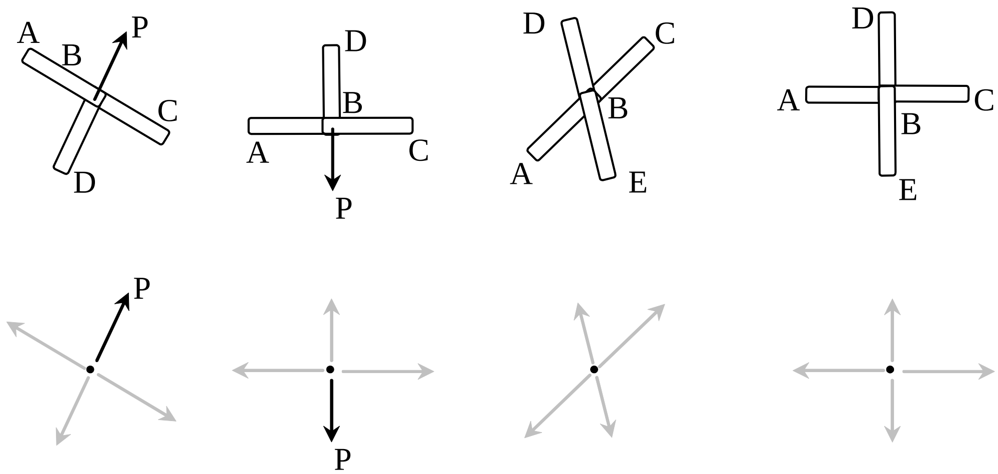
Case 2) Three Bodies Connected with Two Bodies in Line



F_{AB} and F_{BC} have to be equal, and F_{BD} has to be **0 (zero).**

Case 3) Three Bodies Connected and a Force – 2 Bodies aligned & 1 Body and a Force are Aligned

Four Bodies Connected - 2 Bodies Aligned and the Other 2 Bodies Aligned



F_{AB} has to equal F_{BC}, and [F_{BD} has to equal P] or [F_{BD} has to equal F_{BE}]

Graphical Analysis

The method utilizes what we know about force triangles and plotting force magnitudes *to scale*.

1. Draw an accurate form diagram of the truss at a convenient scale with the loads and support reaction forces.

2. Determine the support reaction forces.

3. Working clockwise and from left to right, apply interval notation to the diagram, assigning capital letters to the spaces between external forces and numbers to internal spaces.

4. Construct a load line to a convenient scale of length to force by using the interval notation and working clockwise around the truss from the upper left plotting the lengths of the vertical and horizontal loads.

5. Starting at a left joint where we know there are fewer than three forces, we draw reference lines in the direction of the unknown members so that they intersect. Label the intersection with the number of the internal space.

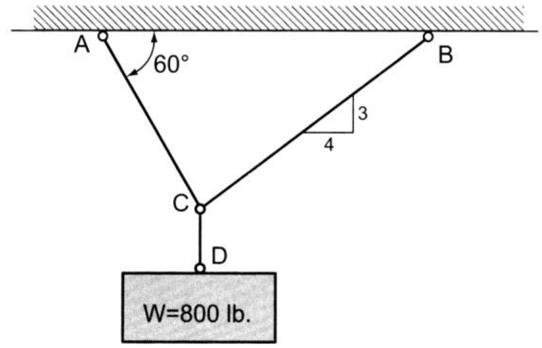
6. Go to the next joint (clockwise and left to right) with two unknown forces and repeat for all joints. The diagram should close.

7. Measure the line segments and apply interval notation to determine their sense: Proceeding clockwise around the joint, follow the notation. The direction toward the joint is compressive. The direction away from the joint is tensile.

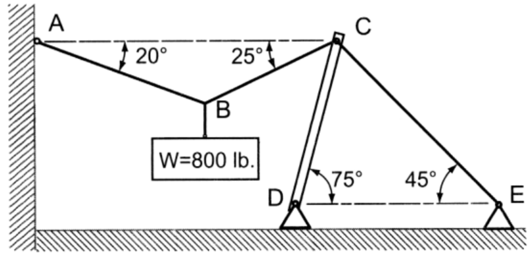
Example 1 (pg 49)

Example Problem 3.1: Equilibrium of a Particle

Two cables, shown in Figure 3.8, are used to support a weight W = 800 lb., suspended at concurrent point ***C***. Determine the tension developed in cables CA and CB for the system to be in equilibrium. Solve this problem analytically and check the answer graphically.



Example 2 (pg 56)



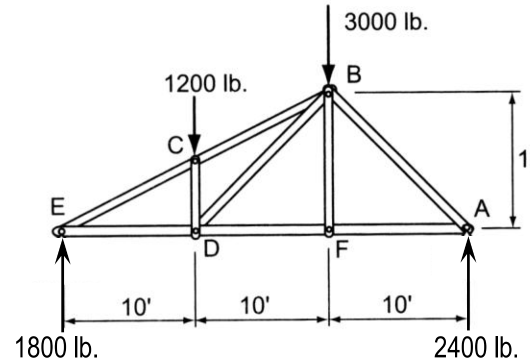
Example Problem 3.5

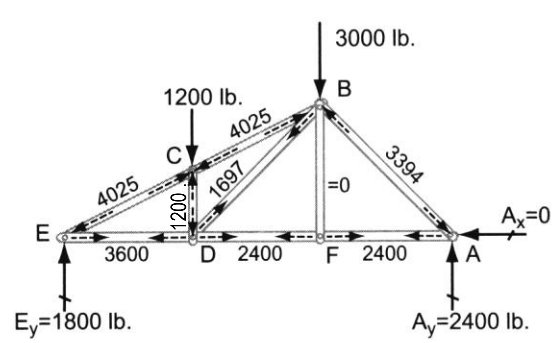
A compound cable system supports a weight W = 800 lb. at point ***B***, as shown in Figure 3.18. Cable *BA* is attached to a wall support at ***A*** and concurrent point ***C*** is supported by a compression strut *DC*. Determine all of the cable forces and the compression in strut *DC*.

Example 3 (pg 90)

Example Problem 4.1 (Method of Joints)

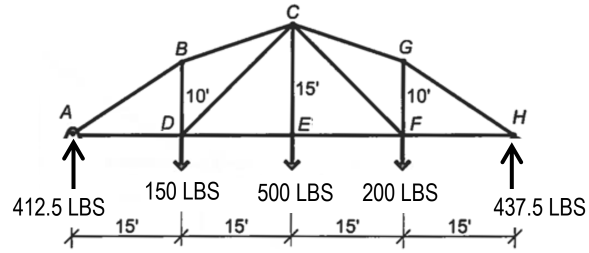
An asymmetrical roof truss, shown in Figure 4.4, supports two vertical roof loads. ~~Determine the support reactions at each end, then,~~ Using the method of joints, solve for all member forces. ~~Summarize the results of all member forces on a FBD (this diagram is referred to as a *force summation diagram*)~~.





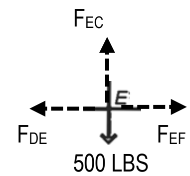
Example 4

Using the method of joint, determine all member forces.



SOLUTION:

Find the joints with 2 (or less unknowns) for FBD's : *A* and *H,* while looking for any special cases like *E,* which has "crossed" members and forces. $F_{DE} = F_{EF}$ and F_{EC} = 500 lb (tension).
(Check off members found:
AB, BD, AD, BC, DC, DE, ~~EC~~, EF, CG, CF, FG, GH, FH)



Let's use *A* first (but H is just as acceptable). Draw the point, adding the known force, and draw the unknown member forces ***away*** from the point, assuming tension (shown as dashed). Find the geometry of member AB from the rise of 10 ft and the run of 15 ft. The hypotenuse will be $\sqrt{10^2+15^2}$ = 18.03:

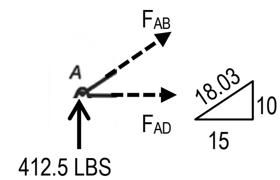
$$\Sigma F_x = F_{AD} + F_{AB}\frac{15}{18.03} = 0$$

$$\Sigma F_y = 412.5^{lb} + F_{AB}\frac{10}{18.03} = 0$$

F_{AB} =(-412.5)*18.03/10 = -743.7 lb (C)

and substituting the (negative) value of F_{AB} into the ΣF_x , F_{AD} = 618.75 lb (T)

(Check off members found: ~~AB~~, BD, ~~AD~~, BC, DC, DE, ~~EC~~, EF, CG, CF, FG, GH, FH)



Review which joints have 2 (or less) unknowns: *B* and *H.*
Let's use *B*. Because we know F_{AB} is in ***compression*** we draw the force ***into*** the point. We need the geometry of member BC with a rise of 5 (15-10) and a run of 15 with a hypotenuse of $\sqrt{5^2+15^2}$ = 15.81:

$$\Sigma F_x = 743.7^{lb}\frac{15}{18.03} + F_{BC}\frac{15}{15.81} = 0$$

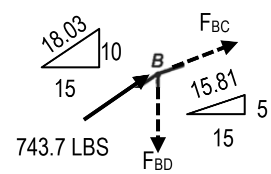
F_{BC} = -652.1 lb (C)

$$\Sigma F_y = 743.7^{lb}\frac{10}{18.03} + F_{BC}\frac{5}{15.81} - F_{BD} = 0$$

(substituting the *negative* value of F_{BC})

F_{BD} = 206.2 lb (T)

(Check off members found: ~~AB~~, ~~BD~~, ~~AD~~, ~~BC~~, DC, DE, ~~EC~~, EF, CG, CF, EF, FG, GH, FH)



Review which joints have 2 (or less) unknowns: *D* and *H.*
Let's use *D*. Both F_{AD} and F_{BD} are tensile, so we can draw them *away*. The geometry of DE is familiar with the rise the same as the run for an angle of 45°:

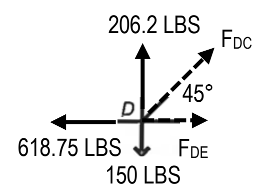
$$\Sigma F_x = -618.75^{lb} + F_{DC}\cos 45° + F_{DE} = 0$$

$$\Sigma F_y = -150^{lb} + 206.2^{lb} + F_{DC}\sin 45° = 0$$

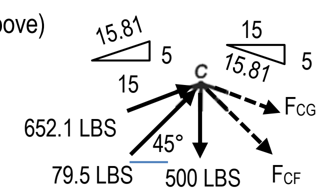
F_{DC} = -79.5 lb (C)

and substituting the (negative) value of F_{DC} into the ΣF_x , F_{DE} = 675.0 lb (T) = F_{EF} (! from above)

(Check off members found: ~~AB~~, ~~BD~~, ~~AD~~, ~~BC~~, ~~DC~~, ~~DE~~, ~~EC~~, ~~EF~~, CG, CF, FG, GH, FH)



Review which joints have 2 (or less) unknowns: *C* and *H.*
Let's use *C*. Draw F_{DC} and F_{BC} as compressive forces. And the geometry has been found due to symmetry, with the angle of F_{CF} a *negative* 45°:



$$F_x = 652.1^{lb}\frac{15}{15.81} + 79.5^{lb}\cos 45° + F_{CF}\cos(-45°) + F_{CG}\frac{15}{15.81} = 0$$

$$\Sigma F_y = 652.1^{lb}\frac{5}{15.81} + 79.5^{lb}\sin 45° - 500^{lb} + F_{CF}\sin(-45°) - F_{CG}\frac{5}{15.81} = 0$$

Solve simultaneously because there isn't an easy way to find one unknown equal to a value multiplied by the other unknown:

$$\Sigma F_x = 674.9^{lb} + 0.707F_{CF} + 0.949F_{CG} = 0$$

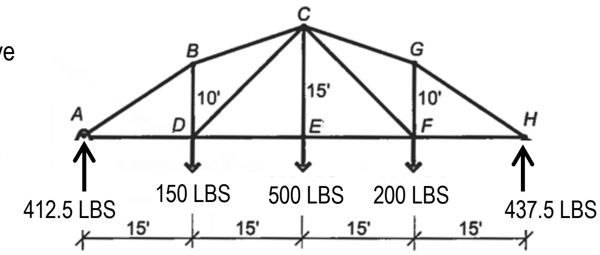
$$\Sigma F_y = -237.6^{lb} - 0.707F_{CF} - 0.316F_{CG} = 0$$

add: $$437.5^{lb} + 0F_{CF} + 0.633F_{CG} = 0$$

F_{CG} = -690.8 lb (C) and substituting, F_{CF} = -27.6 lb (C)

(Check off members found: ~~AB~~, ~~BD~~, ~~AD~~, ~~BC~~, ~~DC~~, ~~DE~~, ~~EC~~, ~~EF~~, ~~CG~~, ~~CF~~, FG, GH, FH)

Example 4 (continued)



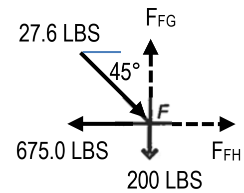
Review which joints have 2 (or less) unknowns: *G, F* and *H.*
Let's use *F* (because H really looks like *A* mirrored). Draw F_{CF} as compressive and F_{EF} in tension. The angle from for F_{CF} is negative 45°:

$$\Sigma F_x = -675.0^{lb} + 27.6^{lb}\cos(-45°) + F_{FH} = 0 \qquad F_{FH} = 655.5 \text{ lb (T)}$$

$$\Sigma F_y = 27.6^{lb}\sin(-45°) - 200^{lb} + F_{FG} = 0 \qquad F_{FG} = 219.5 \text{ lb (T)}$$

(Check off members found:
~~AB~~, ~~BD~~, ~~AD~~, ~~BC~~, ~~DC~~, ~~DE~~, ~~EC~~, ~~EF~~, ~~CG~~, ~~CF~~, ~~FG~~, GH, ~~FH~~)

Review which joints have 2 (or less) unknowns; which are *G* and *H.*
Let's use *G* and pretend that we have only found F_{GF} (and not F_{CG}) in order to show a set of equations that use substitution with the algebra. The geometry has been found due to symmetry:



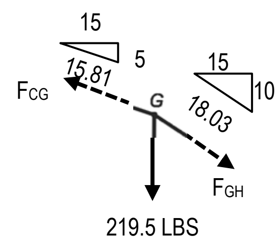
$$\Sigma F_x = -F_{CG}\frac{15}{15.81} + F_{GH}\frac{15}{18.03} = 0 \qquad \text{rearranging:} \quad F_{CG} = F_{GH}\frac{15}{18.03}\cdot\frac{15.81}{15} = F_{GH}\frac{15.81}{18.03}$$

$$\Sigma F_y = F_{CG}\frac{5}{15.81} - F_{GH}\frac{10}{18.03} - 219.5^{lb} = 0$$

Substituting:

$$\Sigma F_y = (F_{GH}\frac{15.81}{18.03})\frac{5}{15.81} - F_{GH}\frac{10}{18.03} - 219.5^{lb} = 0$$

Simplifying $\quad -0.277F_{GH} = 219.5^{lb} \qquad F_{GH} = -791.6 \text{ lb (C)}$

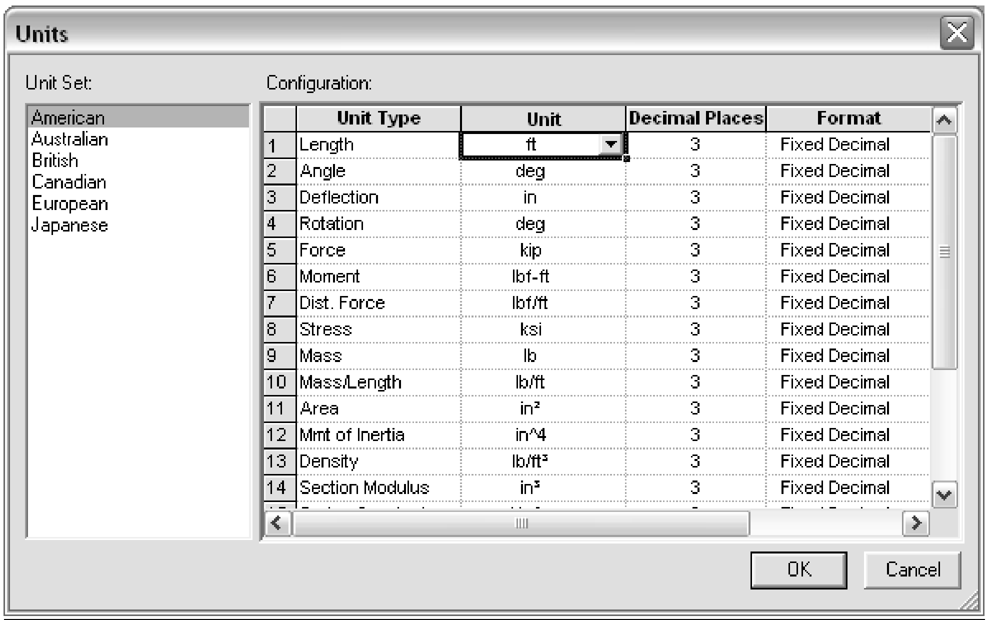


and F_{CG} = -694.1 lb (C) (*which validates the earlier answer found of 690.8 lb (C) with respect to rounding errors in all fractions and trig functions)*
(Check off members found: ~~AB~~, ~~BD~~, ~~AD~~, ~~BC~~, ~~DC~~, ~~DE~~, ~~EC~~, ~~EF~~, ~~CG~~, ~~CF~~, ~~FG~~, ~~GH~~, ~~FH~~)

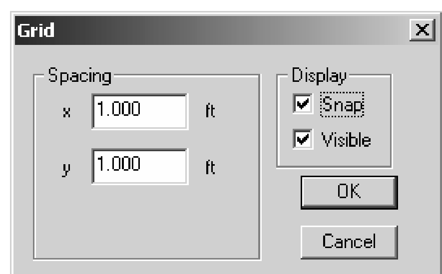
(Typically, the last joint left will verify that the joint is in equilibrium with values found, but in this exercise the last joint was used to show the algebraic method of substitution.)

Truss Analysis using Multiframe

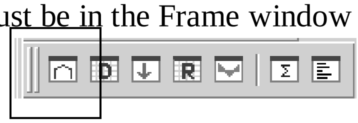
1. The software is on the computers in the College of Architecture in Programs under the Windows Start menu (see https://wikis.arch.tamu.edu/display/HELPDESK/Computer+Accounts for lab locations). Multiframe is under the Bentley Engineering menu.
2. There are tutorials available on line at http://www.formsys.com/mflearning that list the tasks and order in greater detail. The first task is to define the unit system:
   - Choose Units… from the View menu. Unit sets are available, but specific units can also be selected by double clicking on a unit or format and making a selection from the menu.



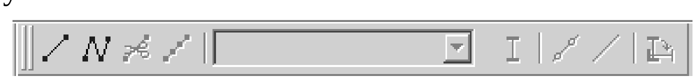
3. To see the scale of the geometry, a grid option is available:
   - Choose Grid… from the View menu



4. To create the geometry, you must be in the Frame window (default). The symbol is the frame in the window toolbar:

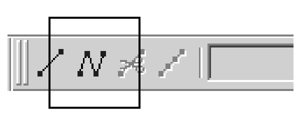


The Member toolbar shows ways to create members:



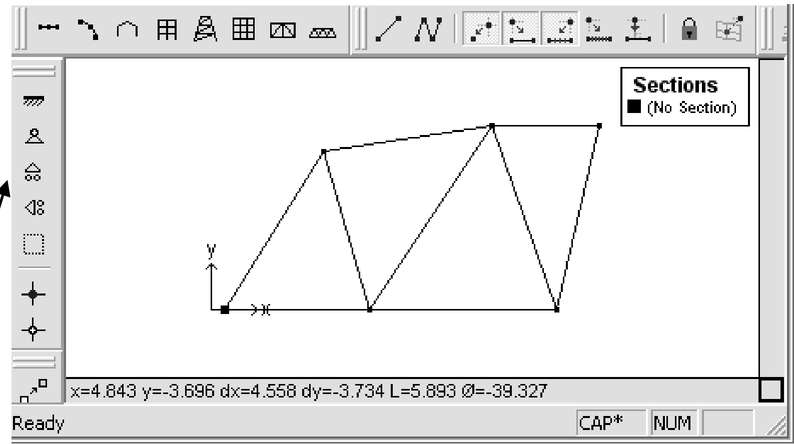
The Generate toolbar has convenient tools to create typical structural shapes.

   - To create a truss, use the add connected members button:



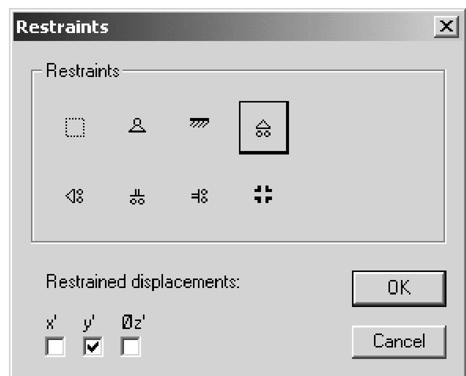
- Select a starting point and ending point with the cursor. The location of the cursor and the segment length is displayed at the bottom of the geometry window. The ESC button will end the segmented drawing. Continue to use the add connected members button. Any time the cursor is over an existing joint, the joint will be highlighted by a red circle.

- The geometry can be set precisely by selecting the joint (drag), and bringing up the joint properties menu (right click) to set the coordinates.



- The support types can be set by selecting the joint (drag) and using the Joint Toolbar (pin shown), or the Frame / Joint Restraint ... menu (right click).

NOTE: If the support appears at both ends of the member, you had the member selected rather than the joint. Select the joint to change support for and right click to select the joint restraints menu or select the correct support on the joint toolbar.



The support forces will be determined in the analysis.

5. All members must have sections assigned (see section 6.) in order to calculate reactions and deflections. To use a standard steel section **proceed to step 6.** For custom sections, the section information must be entered. To define a section:

- Choose Edit Sections / Add Section… from the Edit menu
- Type a name for your new section
- Choose group <u>Frame</u> from the group names provided so that the section will remain with the file data
- Choose a shape. The Flat Bar shape is a rectangular section.
- Enter the cross section data.

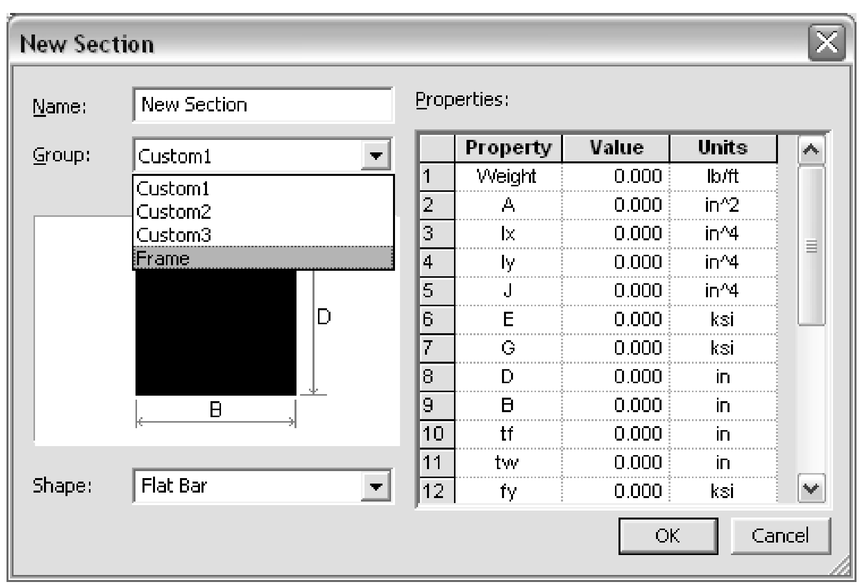
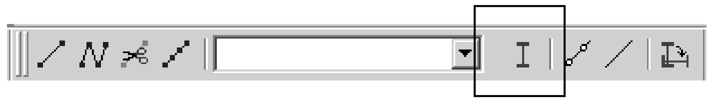


Table values 1-9 must have values for a Flat Bar, but not all are used for every analysis. A recommendation is to put the value of 1 for those properties you don't know or care about. Properties like t_f, t_w, etc. refer to wide flange sections.

- Answer any query. If the message says there is an error, the section will not be created until the error is corrected.

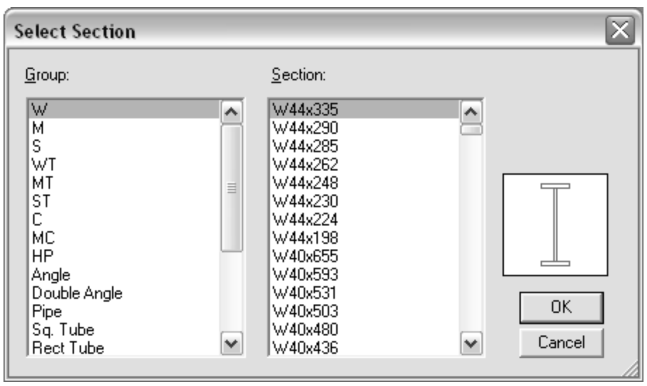
6. The standard sections library loaded is for the United States. If another section library is needed, use the Open Sections Library... command under the file menu, choose the library folder, and select the SectionsLibrary.slb file.

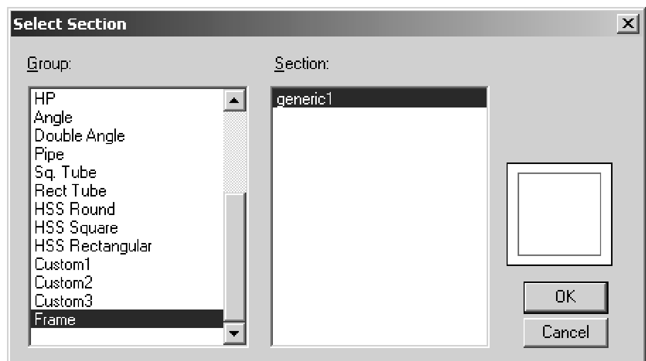
   Select the members (drag to make bold) and assign sections with the Section button on the Member toolbar:



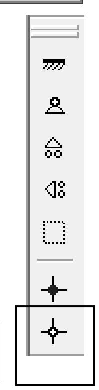
- Choose the group name and section name:

(STANDARD SHAPES) (CUSTOM)

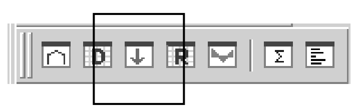




7. In order for Multiframe to recognize that the truss members are two-force bodies, all joints must be highlighted and assigned as pins with the Pinned Joints button on the Joint toolbar:

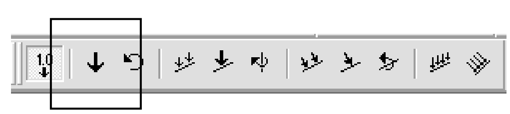


8. The truss geometry is complete, and in order to define the load conditions you must be in the Load window represented by the green arrow:

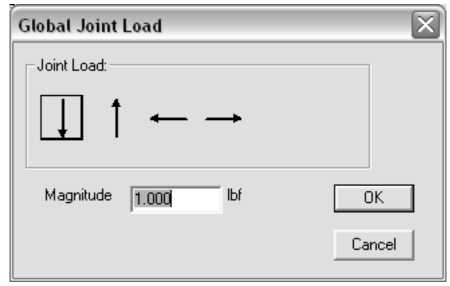


9. The Load toolbar allows a joint to be loaded with a force or a moment in global coordinates, shown by the first two buttons after the display numbers button. It allows a member to be loaded with a distributed load, concentrated load or moment (next three buttons) in global coordinates, as well as loading with distributed or single force or moment in the local coordinate system (next three buttons). It allows a load panel to be loaded with a distributed load in global or local coordinates (last two buttons).

- Choose the joint to be loaded (drag) and select the load type (here shown for point loading):.

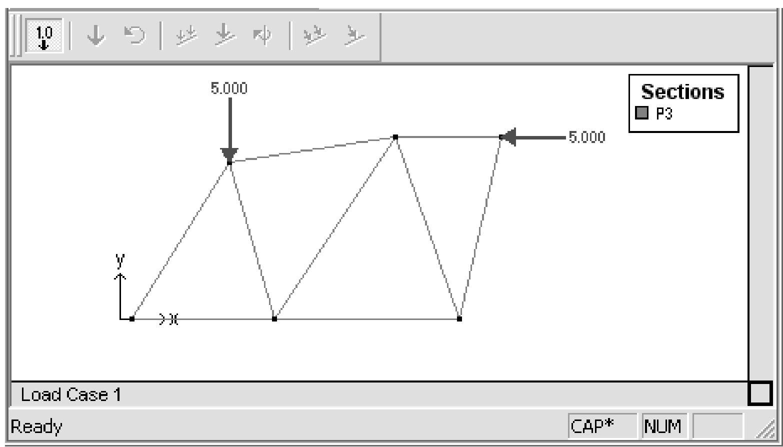


- Choose the direction by the arrow shown. There is no need to put in negative values for downward loading.
- Enter the values of the load

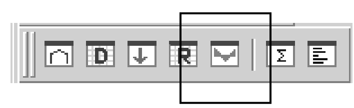


NOTE: <u>Do not</u> put support reactions as applied loads. The analysis will determine the reaction values

Multiframe will automatically generate a grouping called a Load Case named <u>Load Case 1</u> when a load is created. All additional loads will be added to this load case unless a new load case is defined (Add case under the Case menu).

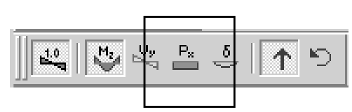


10. In order to run the analysis after the geometry, member properties and loading has been defined:
    - Choose Analyze Linear from the Case menu
11. If the analysis is successful, you can view the results in the Plot window represented by the red moment diagram:

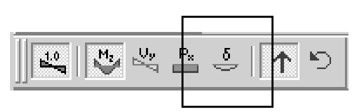


12. The Plot toolbar allows the numerical values to be shown (1.0 button), the reaction arrows to be shown (brown up arrow) and reaction moments to be shown (brown curved arrow):

- To show the axial force diagram, Choose the purple Axial Force button. Tensile members will have “T” by the value (if turned on), while compression members will have “C” by the value

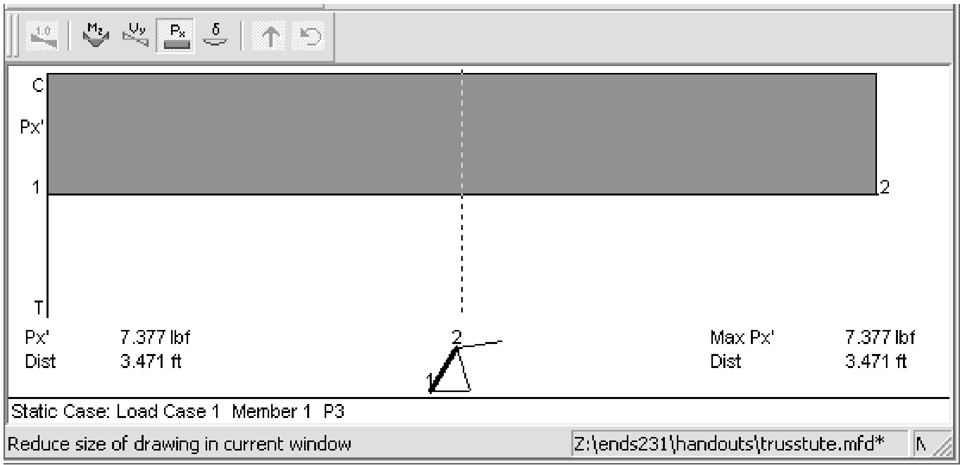


- To show the deflection diagram, Choose the blue Deflection button

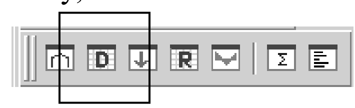


- To animate the deflection diagram, Choose Animate... from the Display menu. You can also save the animation to a .avi file by checking the box.

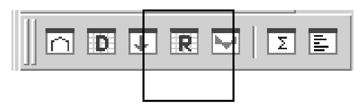
- To see exact values of axial load and deflection, double click on the member and move the vertical cross hair with the mouse. The ESC key will return you to the window.

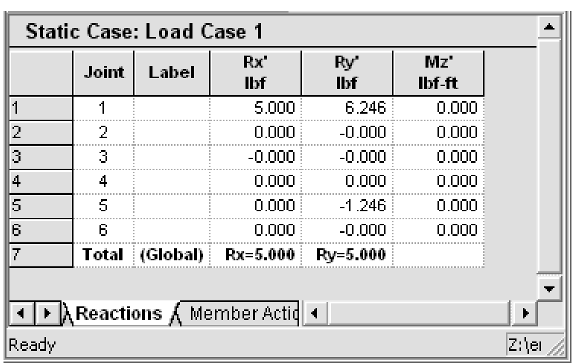


13. The Data window (D) allows you to view all data "entered" for the geometry, sections and loading. These values can be edited.



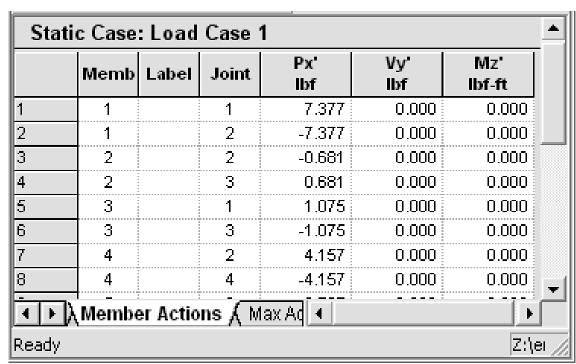
14. The Results window (R) allows you to view all results of the analysis including displacements, reactions, member forces (actions) and stresses. These values can be cut and pasted into other Windows programs such as Word or Excel.





Static Case: Load Case 1

	Joint	Label	Rx' lbf	Ry' lbf	Mz' lbf-ft
1	1		5.000	6.246	0.000
2	2		0.000	-0.000	0.000
3	3		-0.000	-0.000	0.000
4	4		0.000	0.000	0.000
5	5		0.000	-1.246	0.000
6	6		0.000	-0.000	0.000
7	Total	(Global)	Rx=5.000	Ry=5.000	



Static Case: Load Case 1

	Memb	Label	Joint	Px' lbf	Vy' lbf	Mz' lbf-ft
1	1		1	7.377	0.000	0.000
2	1		2	-7.377	0.000	0.000
3	2		2	-0.681	0.000	0.000
4	2		3	0.681	0.000	0.000
5	3		1	1.075	0.000	0.000
6	3		3	-1.075	0.000	0.000
7	4		2	4.157	0.000	0.000
8	4		4	-4.157	0.000	0.000

NOTE: Px' refers to the axial load (P) in the local axis x direction (x').

15. To save the file Choose Save from the File menu.
16. To load an existing file Choose Open... from the File menu.
17. To print a plot Choose Print Window... from the File menu. As an alternative, you may copy the plot (Ctrl+c) and paste it in a word processing document (Ctrl+v).

Equilibrium of Rigid Bodies

Notation:

k = spring constant
F = name for force vectors, as is P
F_x = force component in the x direction
F_y = force component in the y direction
FBD = free body diagram
L = beam span length
M = moment due to a force
x = horizontal distance
w = name for distributed load
W = name for total force due to distributed load
α = angle, in a math equation
θ = angle, in a trig equation, ex. $\sin\theta$, that is measured between the x axis and *tail* of a vector
Σ = summation symbol

- *Definition:* Equilibrium is the state when all the external forces acting on a rigid body form a system of forces equivalent to zero. There will be no rotation or translation. The forces are referred to as balanced.

$$R_x = \sum F_x = 0 \qquad R_y = \sum F_y = 0 \qquad \textbf{AND} \qquad \sum M = 0$$

- It is ABSOLUTELY NECESSARY to consider all the forces acting on a body (applied directly and indirectly) using a FREE BODY DIAGRAM. Omission of a force would ruin the conditions for equilibrium.

FREE BODY DIAGRAM STEPS;

1. Determine the free body of interest. (What body is in equilibrium?)

2. Detach the body from the ground and all other bodies (*"free" it).*

3. Indicate all external forces which include:

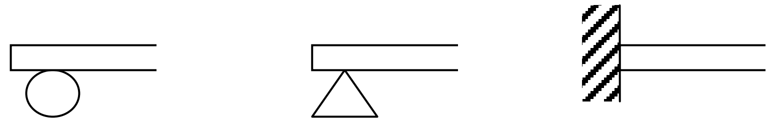
   - action on the free body by the **supports & connections**
   - action on the free body by other bodies
   - the weigh effect (=force) of the free body itself (force due to gravity)

4. All forces should be clearly marked with magnitudes and direction. The sense of forces should be those acting *on the body* not by the body.

5. Dimensions/angles should be included for moment computations and force computations.

6. Indicate the unknown angles, distances, forces or moments, such as those reactions or constraining forces where the body is supported or connected. (*Text uses hashes on the unknown forces to distinguish them.)*

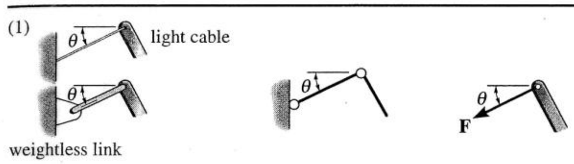
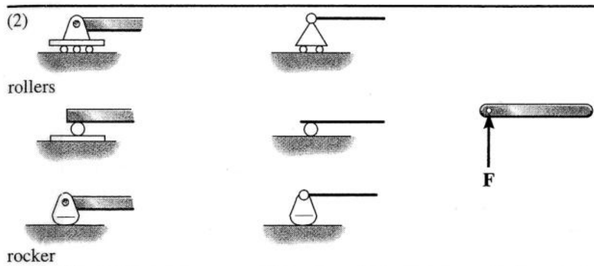
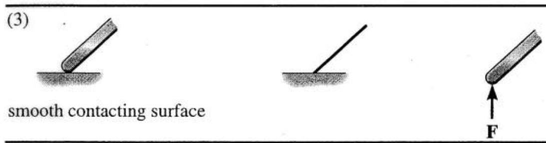
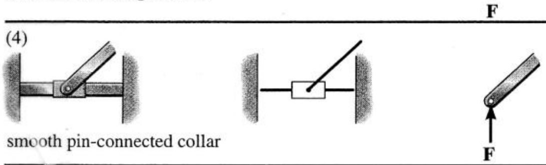
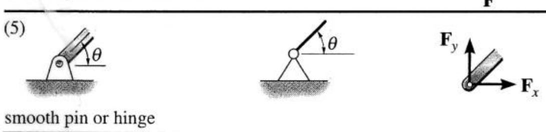
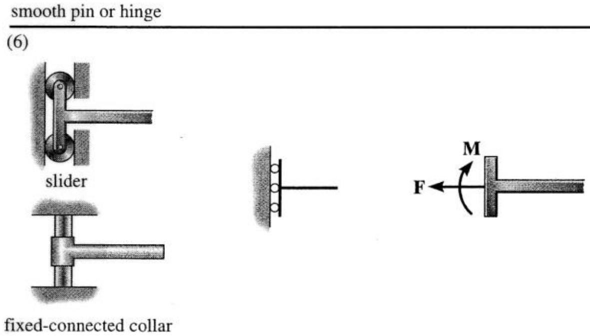
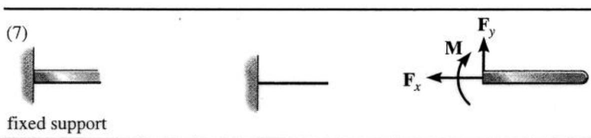
- *Reactions* can be categorized by the type of connections or supports. A reaction is a force with known line of action, or a force of unknown direction, or a moment. The line of action of the force or direction of the moment is directly related to the motion that is prevented.



prevents motion:	prevents motion:	prevents:
up and down	**vertical & horizontal**	**rotation & translation**

Reactions and Support Connections Structural Analysis, 4th ed., R.C. Hibbeler

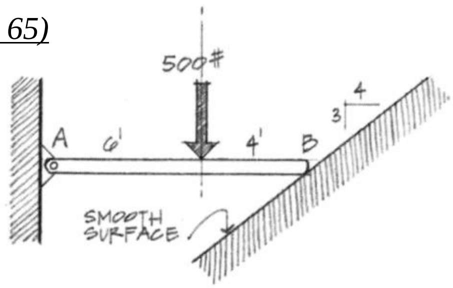
Table 2–1 Supports for Coplanar Structures

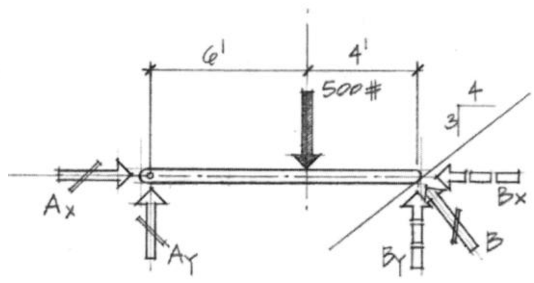
Type of Connection / Idealized Symbol / Reaction	Number of Unknowns
(1) 	One unknown. The reaction is a force that acts in the direction of the cable or link.
(2) 	One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(3) 	One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(4) 	One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(5) 	Two unknowns. The reactions are two force components.
(6) 	Two unknowns. The reactions are a force and a moment.
(7) 	Three unknowns. The reactions are the moment and the two force components.

The line of action should be indicated on the FBD. The sense of direction is determined by the type of support. (Cables are in tension, etc…) *If the sense isn't obvious, assume a sense.* When the reaction value comes out positive, the assumption was correct. When the reaction value comes out negative, the assumption was *opposite* the actual sense. *DON'T CHANGE THE ARROWS ON YOUR FBD OR SIGNS IN YOUR EQUATIONS.*

- With the 3 equations of equilibrium, there can be no more than 3 unknowns. *COUNT THE NUMBER OF UNKNOWN REACTIONS.*

Example 1
(*similar to ex. on pg 65*)





500 lb is *known*

check:

reactions for the pin-type support at A = $\mathbf{A_x}$ **&** $\mathbf{A_y}$

reactions and components for the smooth surface at B = **B (perpendicular to ground only)**

\# equations = **3**

procedure:

Write summation of forces in x and y and set = 0.

Choose a place to take a moment. Summing moments at **A** means that A_x, A_y and B_x have moment arms of *zero.*

- The general rule is to sum at point where there are the most unknown reactions which usually results in one unknown left in the equation. This “point” could also be where two lines of action intersect.

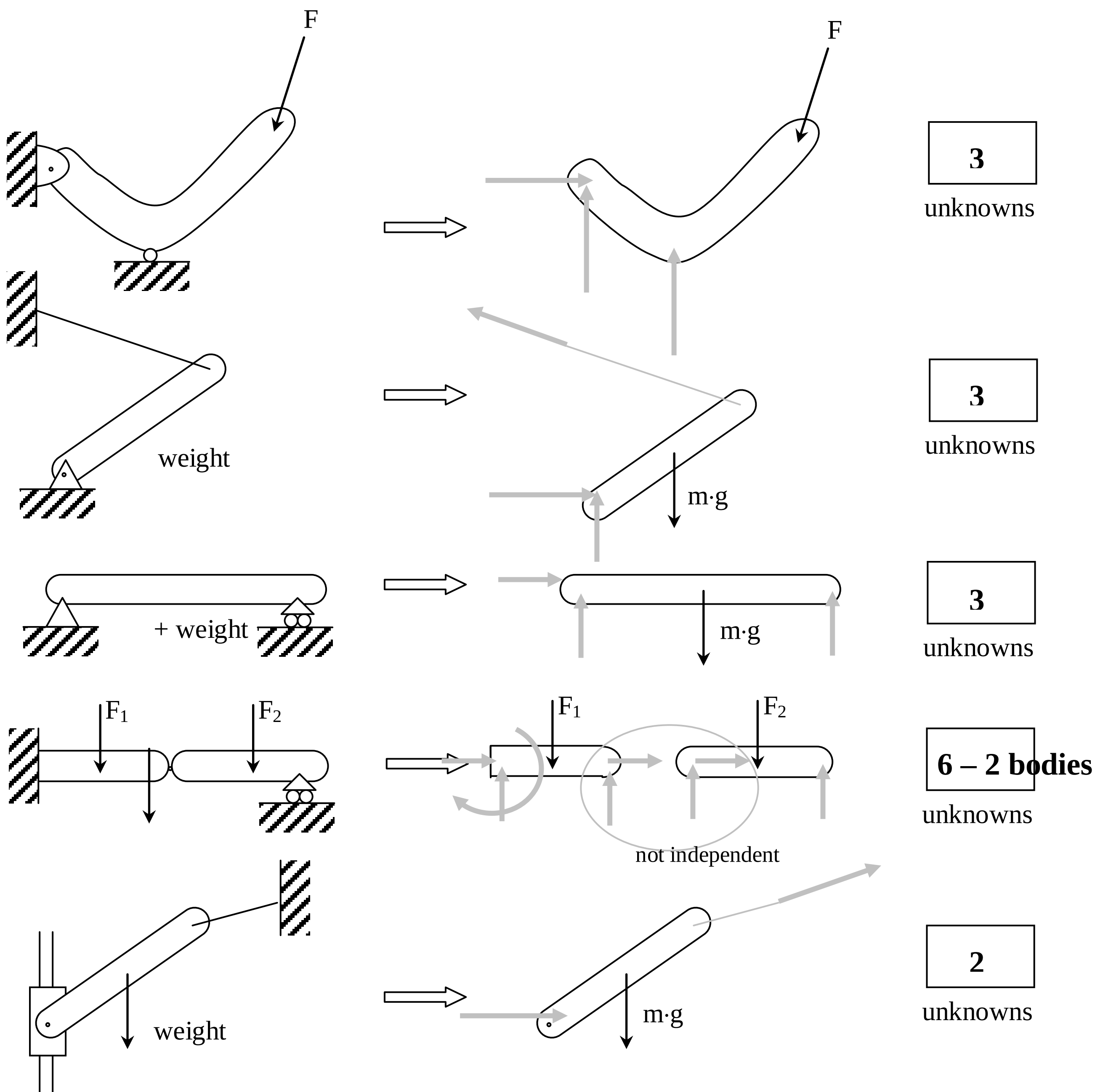
- More than one moment equation can be used, *but it will not be unique*. Only 3 equations are unique. Variations:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_1 = 0 \qquad \text{or}$$

$$\sum F_x = 0 \qquad \sum M_1 = 0 \qquad \sum M_2 = 0 \qquad \text{or}$$

$$\sum M_1 = 0 \qquad \sum M_2 = 0 \qquad \sum M_3 = 0$$

Recognizing support unknowns in FBD’s

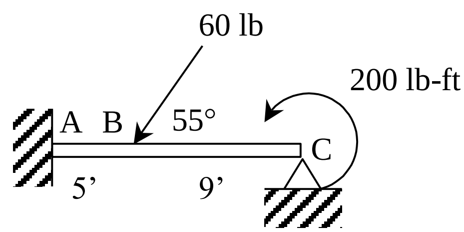


Statical Indeterminancy and Improper Constraints

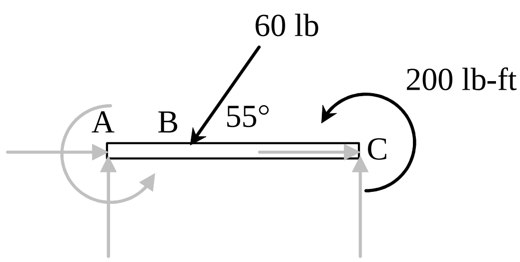
- *Definition:* A completely constrained rigid body has the same number of unknown reactions as number of equilibrium equations and cannot move under the loading conditions. The reactions are statically determinate.

- *Definition:* Statically indeterminate reactions appear on a rigid body when there are more unknown reactions than the number of equilibrium equations. The reactions that cannot be solved for are statically indeterminate. The degree of indeterminacy is the number of additional equations that would be needed to solve, i.e. one more = 1^{st} degree, 2 more = 2^{nd} degree...

Example of Static Indeterminancy:



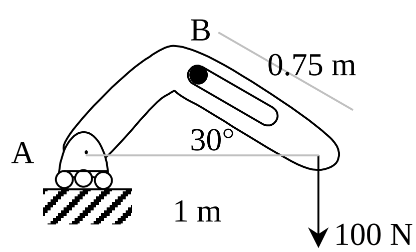
Find the reactions on the cantilever when a pin is added at C



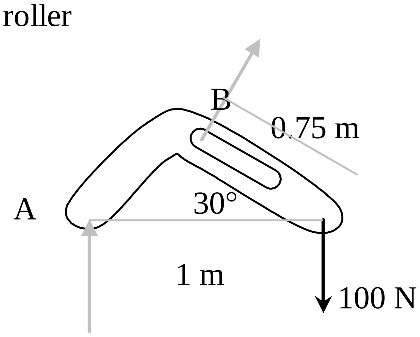
With 5 unknowns, two won't be solvable. (statically indeterminate to the 2^{nd} degree)

- *Definition:* When the support conditions provide the same or less unknown reactions as the equations of equilibrium *but allow the structure to move (not equilibrium)*, the structure is considered partially constrained. This occurs when the reactions must be either **concurrent** or **parallel**.

Example of Partial Constraints:



Find the reactions when the pin support at A changes to a roller

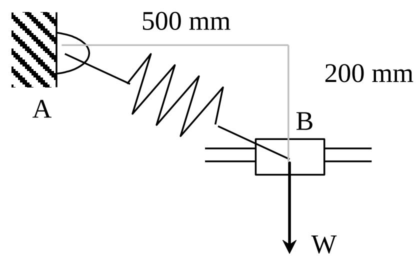


If ΣF has to equal 0, the x component must be 0, meaning B = 0.
A would have to equal 100 N, but then ΣM wouldn't be 0.

- The condition of at most as many unknown reactions as equilibrium equations is necessary for static determinacy, but isn't sufficient. *The supports must completely constrain the structure.*

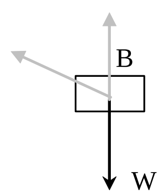
- We'd like to avoid partial or improper constraint in the design of our structures. However, some structures with these types of constraints may not collapse. They may move. Or they may require advanced analysis to find reaction forces.

Example of Partial Constraints and Static Indeterminacy:



Find the weight and reactions when the sleeve track is horizontal

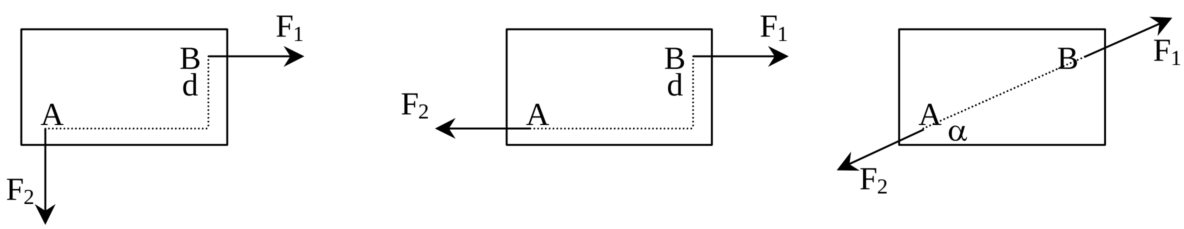
k = 5 N/mm
k(Δl) = F by spring
length of *unstretched* spring = 450 mm



For ΣF to equal 0, the spring force must be 0 (x component = 0) meaning it *can't* be stretched if there is no movement

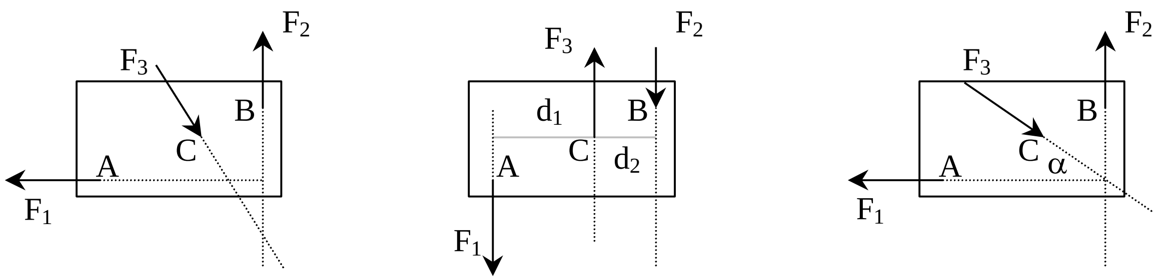
Rigid Body Cases:

1. Two-force body: Equilibrium of a body subjected to two forces on two points requires that those forces be **equal** and **opposite** and act in the same line of action.



(A) (B) (C)

2. Three-force body: Equilibrium of a body subjected to three forces on three points requires that the line of action of the forces be concurrent (intersect) or parallel AND that the resultant equal zero.

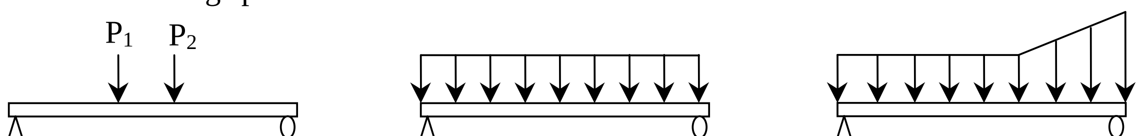


(A) -no (B) (C)

Loads, Support Conditions & Reactions for Beams

Types of Forces

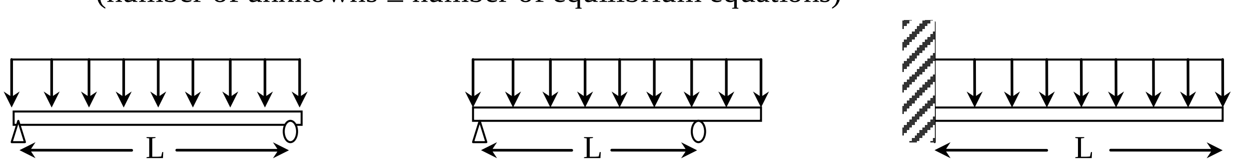
Concentrated – single load at one point
Distributed – loading spread over a distance or area



concentrated uniformly distributed distributed

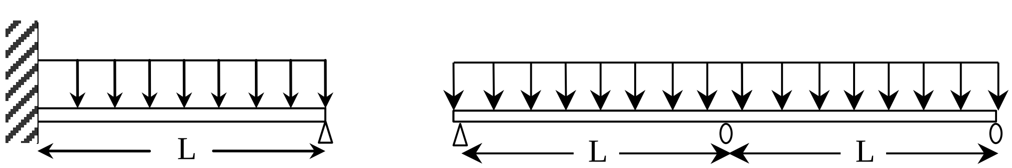
Types of supports:

- Statically determinate
  (number of unknowns ≤ number of equilibrium equations)



simply supported (most common) overhang cantilever

- Statically indeterminate: (need more equations from somewhere

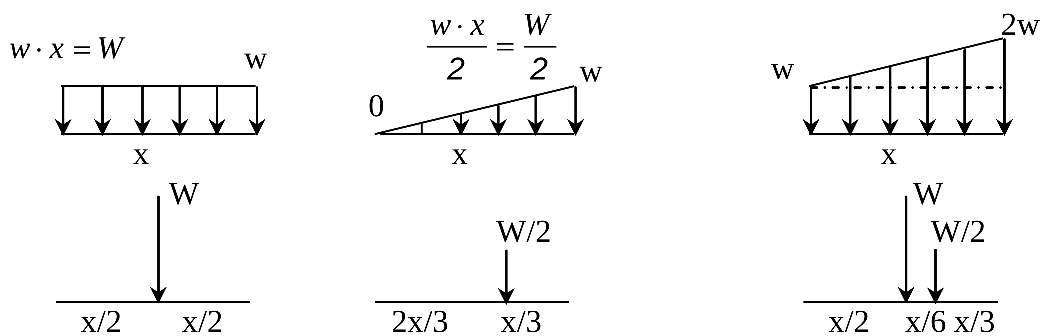


restrained, ex. continuous (most common case when L_1=L_2)

Distributed Loads

Distributed loads may be replaced by concentrated loads acting through the balance/center of the distribution or *load area*: THIS IS AN **EQUIVALENT** FORCE SYSTEM.

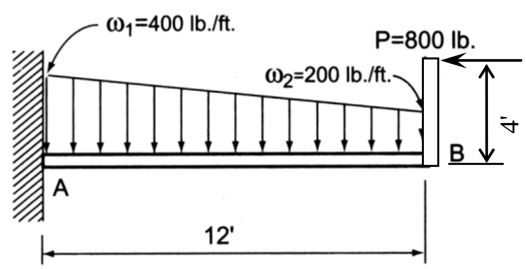
- *w* is the symbol used to describe the *load* per unit **length**.
- W is the symbol used to describe the *total load.*



Example 2 (*changed* from pg 72)

Example Problem 3.14—Cantilever (Figure 3.42)

Determine the support reactions developed at *A* for a cantilever beam supporting a trapezoidal load and a point load (horizontal) on the bar at the free end.



Method of Sections for Truss Analysis

Notation:

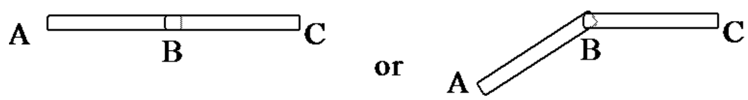
(C) = shorthand for *compression*

(T) = shorthand for *tension*

P = name for load or axial force vector

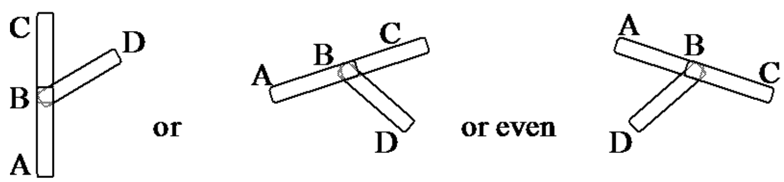
Joint Configurations (special cases to recognize for faster solutions)

Case 1) Two Bodies Connected



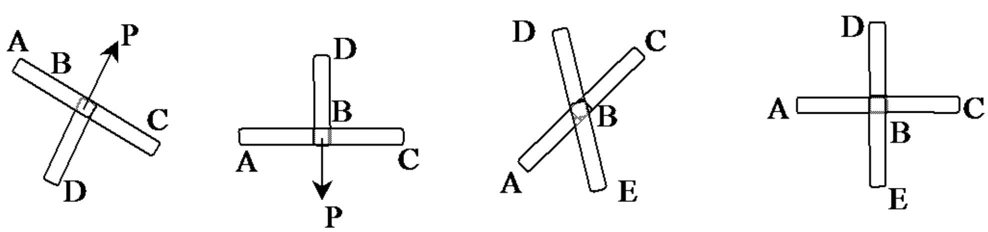
F_{AB} has to be equal and opposite to F_{BC}

Case 2) Three Bodies Connected with Two Bodies in Line



F_{AB} and F_{BC} have to be equal, and F_{BD} has to have zero force.

Case 3) Three Bodies Connected and a Force – 2 Bodies aligned & 1 Body and a Force are Aligned
Four Bodies Connected - 2 Bodies Aligned and the Other 2 Bodies Aligned

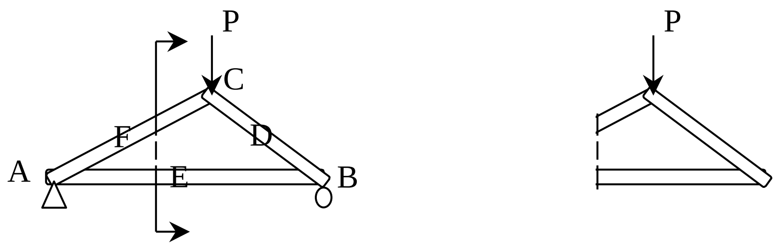


F_{AB} has to equal F_{BC}, and [F_{BD} has to equal P] or [F_{BD} has to equal F_{BE}]

Method of Sections (relies on internal forces being in equilibrium with external forces on a section)

1. Determine support reaction forces.
2. Cut a section in such a way that force action lines intersect.
3. Solve for equilibrium. Sum moments about an intersection of force lines of action

Advantages: Quick when you only need one or two forces (only 3 equations needed)
Disadvantages: Not always easy to find a place to cut a section or see where force lines intersect



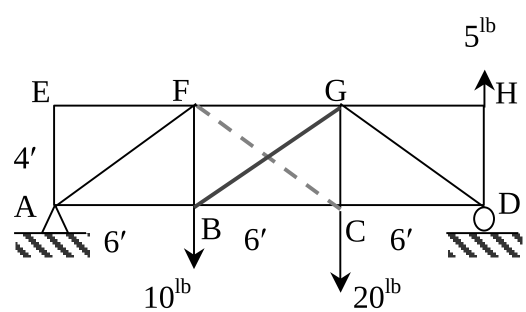
- Compound Truss: A truss assembled of simple trusses and additional links. It has b=2n-3, is statically determinate, rigid and completely constrained with a pin and roller. It can be identified by triangles with pins in the middle of some sides.

- Statically Indeterminate Trusses:

  Occur if there are more members than equations for all the joints

  OR if there are more reaction supports unknowns than 3

- Diagonal Tension Counters: Crossed bracing of cables or slender members is commonly used in bridge trusses, buildings and towers. These trusses look indeterminate, but can be solved statically because the bracing cannot hold a compressive force. The members are excluded in the analysis.



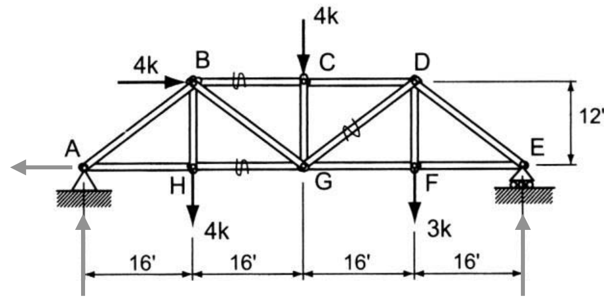
Method:

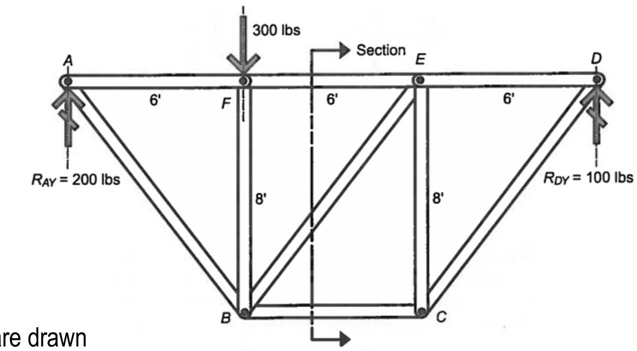
1. Determine support reaction forces.
2. Cut a section in such a way that the tension counters are exposed.
3. Solve for force equilibrium in *y* with one counter. If the value is positive (in tension), this is the solution.
4. Solve for force equilibrium in *y* with the other counter.

Example 1 (pg 99)

A 64-foot parallel chord truss (Figure 4.30) supports horizontal and vertical loads as shown. Using the method of sections, determine the member forces *BC*, *HG*, and *GD*.

(Support forces must be found as well).





Example 2

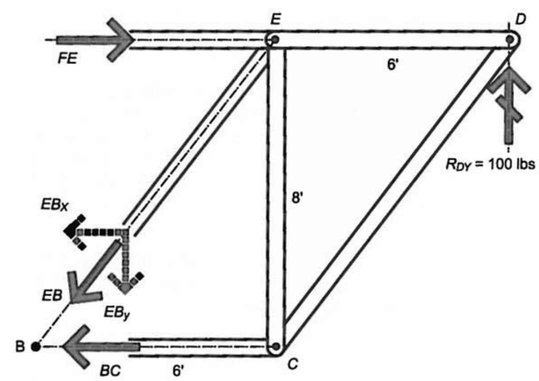
Using the method of sections, determine member forces in FE, EB, BC, AB and FB.

SOLUTION:

A section can't pass through 5 members, so there will have to be two sections. The first passes through FE, EB and BC.

FE is shown assumed to be in compression, while the other forces are drawn assumed to be in tension.

There can be only two intersections when two of the three forces are parallel – at E and B:



$$\Sigma M_E = 100^{lb}(6\,ft) - BC(8\,ft) = 0$$

BC = 75lb (T)

$$\Sigma M_B = 100^{lb}(12\,ft) - FE(8\,ft) = 0$$

FE = 150lb (C)

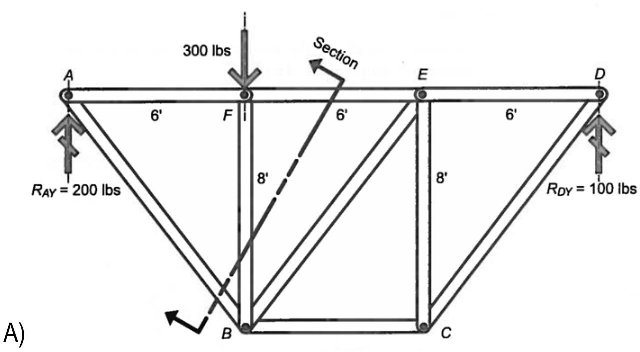
Because EB is the only unknown force with a y component, it is useful to sum forces in the y direction (although it also has the only remaining unknown x component):

$$\Sigma F_y = 100^{lb} - EB\left(\frac{8\,ft}{\sqrt{100}\,ft}\right) = 0$$

(or $\Sigma F_x = 150^{lb} - 75^{lb} - EB\left(\frac{6\,ft}{\sqrt{100}\,ft}\right) = 0$)

EB = 125lb (T)

A second section can be drawn through AB, FB and FE.



There are three points of intersection of the unknown forces - at A, F and B. B is not on the section, but we know where it is.

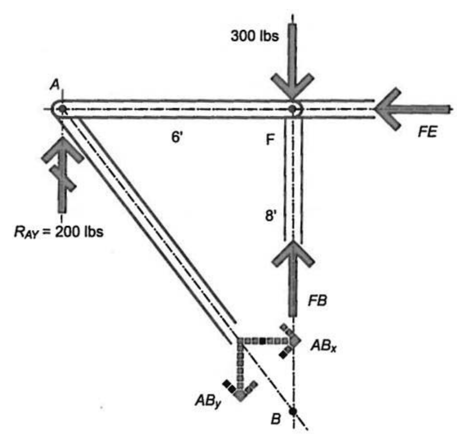
$\Sigma M_A = -300^{lb}(6\,ft) + FB(6\,ft) = 0$ FB = 300lb (C)

$\Sigma M_F = -200^{lb}(6\,ft) + AB_y(6\,ft) = 0$ (sliding AB components to A)

AB = $AB_y\left(\frac{\sqrt{100}}{8}\right)$ = 250lb (T)

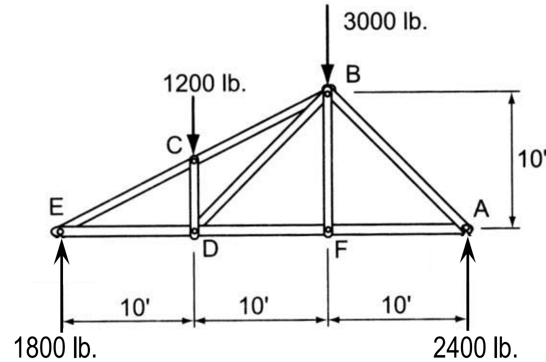
or $\Sigma M_F = -200^{lb}(6\,ft) + AB_x(8\,ft) = 0$ (sliding AB components to B)

AB = $AB_x\left(\frac{\sqrt{100}}{6}\right)$ = 250lb (T)



$$\Sigma M_B = -200^{lb}(6\,ft) + FE(8\,ft) = 0$$

FE = 150lb (C)

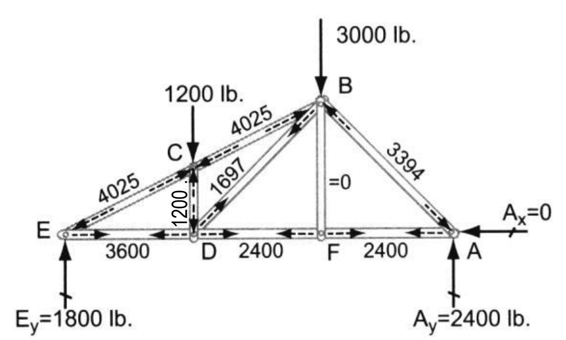


Example 3 (pg 90)

Example Problem 4.1 ~~(Method of Joints)~~

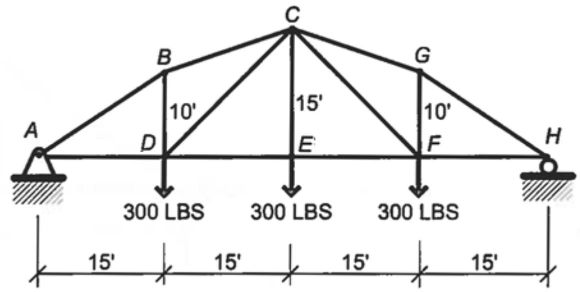
An asymmetrical roof truss, shown in Figure 4.4, supports two vertical roof loads. ~~Determine the support reactions at each end, then, using the method of joints, solve for all member forces. Summarize the results of all member forces on a FBD (this diagram is referred to as a *force summation diagram*).~~

Determine the member forces *CB*, *DB* and *DF*.



Example 4

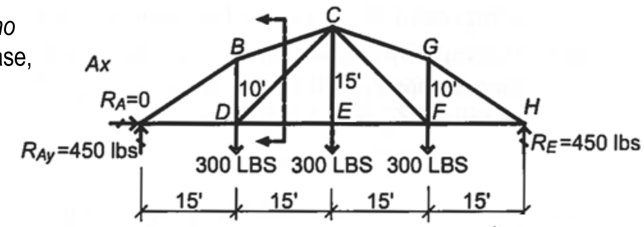
Using the method of sections, determine member forces in BC, CD and BD.



SOLUTION:

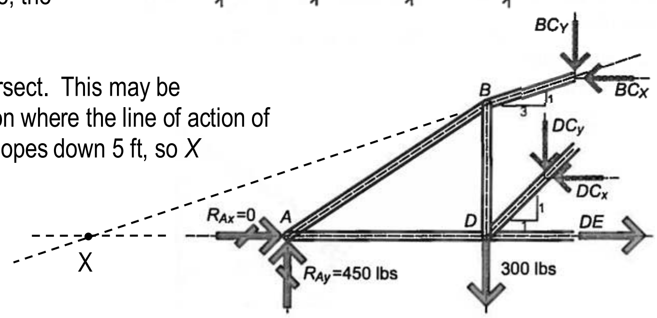
Find the support reactions from rigid body equilibrium, or in this case, from load tracing with symmetrical loads.

Draw a section line through the members of interest, *cutting through no more than 3 members* to separate the truss into two pieces. In this case, BC and CD can be cut through, while BD will need another section.



Draw one of the sections, exposing the member forces. Drawing them "out" or "away" from the cut assumes tension. BC is drawn in compression. So is DC, but because it has a 45 degree angle, the components will have the same magnitude.

Find a point to sum moments where two unknown forces intersect. This may be on a point of the section or *off* the section. *X* is such a location where the line of action of BC intersects that of DE. For every 15 ft to the left, the line slopes down 5 ft, so *X* is located (10 ft/ 5 ft)15 ft = 30 ft to the left of B.



$$\Sigma M_X = 450^{lb}(15\,ft) - 300^{lb}(30\,ft) - DC_y(30\,ft) = 0$$

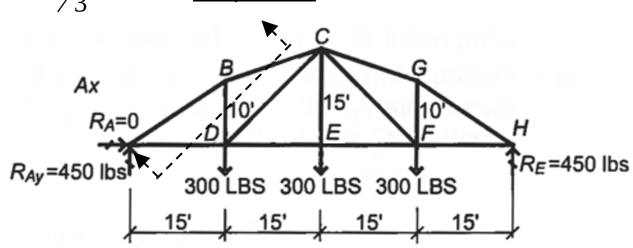
DCy = -75 lb, so DC = DC_y/sin45 = 106 ^{lb} tension

(compression was assumed, but the answer was negative indicating our assumption wasn't verified).

(*Notice that DC_x and DC_y "slid" down to D and then the lever arm for DC_x was 0. The components can also slide to the other end point of the member to locate the lever arms)*

Summing at D where DC and DE intersect means there will be no lever arms. Sliding the components of BC to B means there will be no lever arm for BC_y:

$$\Sigma M_D = -450^{lb}(15\,ft) + BC_x(10\,ft) = 0$$ $BC_x = 675^{lb}$, so BC = $BC_x \sqrt{10}/3$ = 711.5^{lb} compression

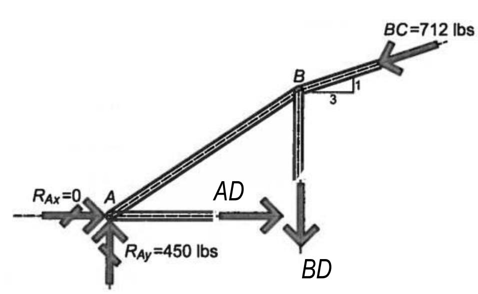


Draw a section line that passes through BD and cuts through no more than three members.

If we hadn't already found BC, we could sum moments at point *X* again to eliminate BC and AD from our equation, leaving BD.

But it is obvious that we have only one unknown y force, which is BD:

$$\Sigma F_y = 450^{lb} - BD - 711.5^{lb}\left(\frac{1}{\sqrt{10}}\right) = 0$$ BD = 225^{lb} tension



Mechanics of Materials

Notation:

A = area (net = with holes, bearing = in contact, etc...)
d = diameter of a hole
f = symbol for stress
$f_{allowable}$ = allowable stress
f_v = shear stress
f_p = bearing stress (see P)
$F_{allowed}$ = allowable stress (used by codes)
F_v = allowable shear stress
kPa = kilopascals or 1 kN/m^2
q = allowable soil bearing pressure
psi = pounds per square inch
P = name for axial force vector
P' = name for internal axial force vector
R = name for reaction force vector
t = thickness of a hole or member
x = horizontal dimension
y = vertical dimension
γ = density of a material (unit weight)
σ = engineering symbol for normal stress
τ = engineering symbol for shearing stress

Mechanics of Materials is a basic engineering science that deals with the relation between externally applied load and its effect on deformable bodies. The main purpose of Mechanics of Materials is to answer the question of which requirements have to be met to assure STRENGTH, RIGIDITY, AND STABILITY of engineering structures.

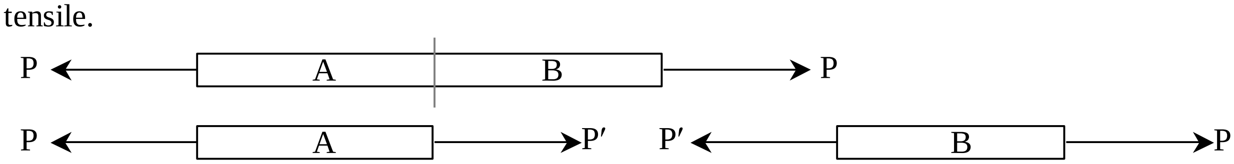
To solve a problem in Mechanics of Materials, one has to consider THREE ASPECTS OF THE PROBLEM:

1. **STATICS**: equilibrium of external forces, internal forces, stresses
2. **GEOMETRY**: deformations and conditions of geometric fit, strains
3. **MATERIAL PROPERTIES**: stress-strain relationship for each material, obtained from material testing.

- STRESS – The intensity of a force acting over an **area**.

Normal Stress

Stress that acts along an *axis* of a member; can be internal or external; can be compressive or tensile.



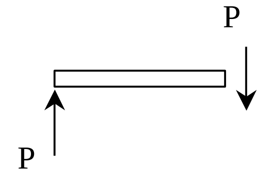
$$f = \sigma = \frac{P}{A_{net}}$$

Strength condition: $f = \frac{P}{A_{net}} < f_{allowable}\ or\ F_{allowed}$

Shear Stress

Stress that acts perpendicular to an *axis or length* of a member, or **parallel** to the cross section is called shear stress.

Shear stress cannot be assumed to be uniform, so we refer to *average shearing stress.*



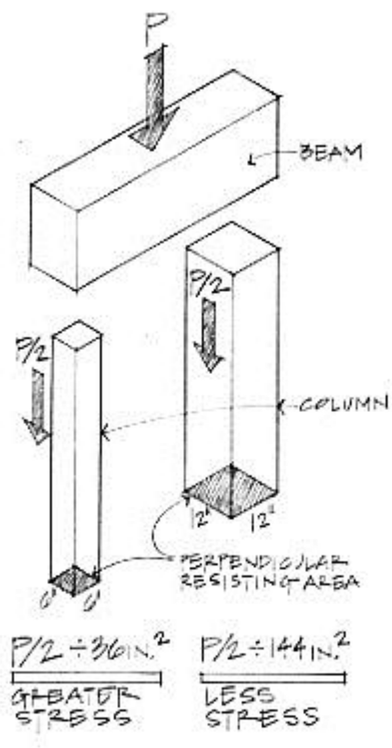
$$f_v = \tau = \frac{P}{A_{net}}$$

Strength condition: $f_v = \frac{P}{A_{net}} < \tau_{allowable}\, or\ F_{allowed}$

Bearing Stress

A compressive normal stress acting *between two bodies.*

$$f_p = \frac{P}{A_{bearing}}$$



Bending Stress

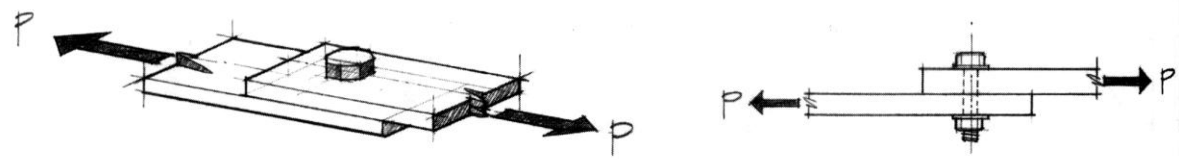
A normal stress caused by bending; can be compressive or tensile. (Discussed in Note Set on Beam Bending.)

Torsional Stress

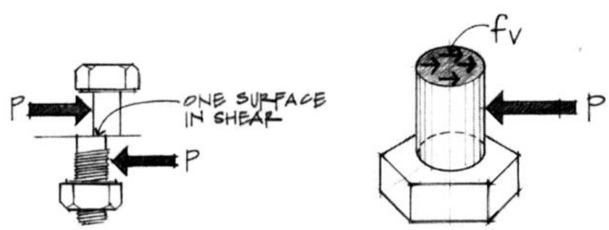
A shear stress caused by torsion (moment around the axis). (Discussed in Note Set on Torsion.)

Bolts in Shear and Bearing

Single shear - forces cause only one shear "drop" across the bolt.



(a) Two steel plates bolted using one bolt. *(b) Elevation showing the bolt in shear.*



(c) *(d)*

f_v = Average shear stress through bolt cross section

A = Bolt cross-sectional area

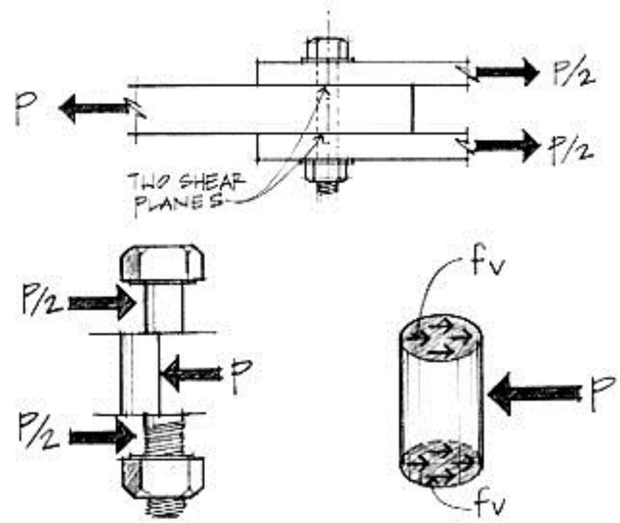
$$f_v = \frac{P}{A}$$

Figure 5.11 A bolted connection—single shear.

Double shear - forces cause two shear changes across the bolt.

$$f_v = \frac{P}{2A}$$

(two shear planes)

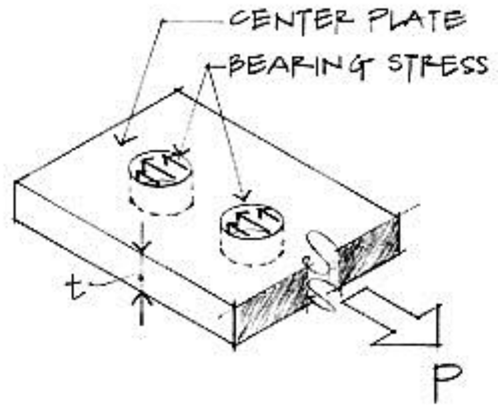


Free-body diagram of middle section of the bolt in shear.

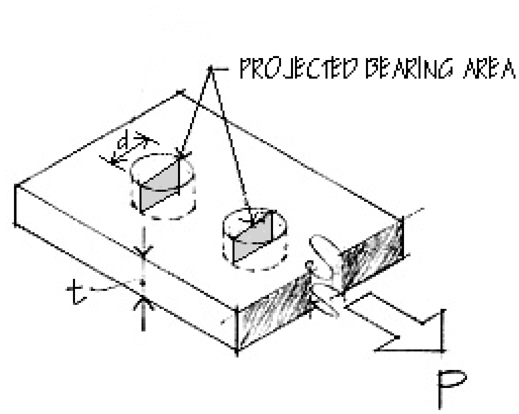
Figure 5.12 A bolted connection in double shear.

<u>Bearing of a bolt on a bolt hole</u> – The bearing surface can be represented by *projecting* the cross section of the bolt hole on a plane (into a rectangle).

$$f_p = \frac{P}{A} = \frac{P}{td}$$

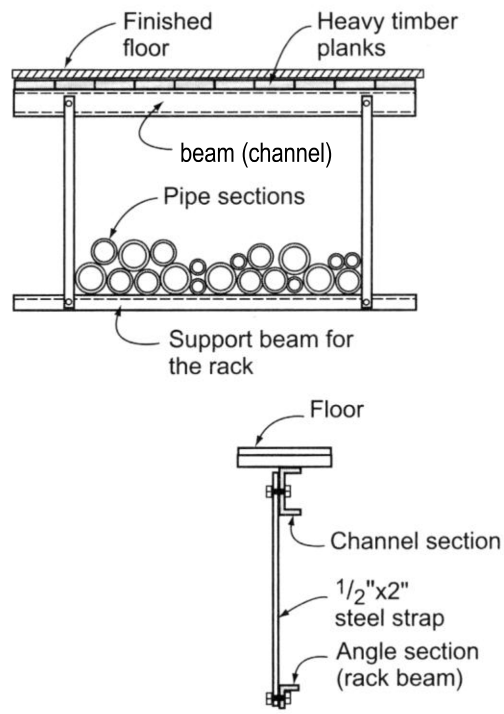


Bearing stress on plate.

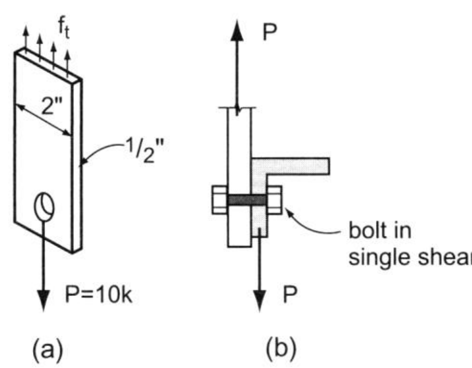


Example 1 (pg 201)*

Example Problem 6.8 (Figures 6.18 to 6.20)



A pipe storage rack is used for storing pipe in a shop. The support rack beam is fastened to the main floor beam using steel straps $\frac{1}{2}$" × 2" in dimension. Round bolts are used to fasten the strap to the floor beam in single shear. (a) If the weight of the pipes impose a maximum tension load of 10,000 pounds in each strap, determine the tension stress developed in the steel strap. (b) Also, what diameter bolt is necessary to fasten the strap to the floor beam if the allowable shear stress for the bolts equals $F_v = 15{,}000\ ^{\text{lb.}}/_{\text{in.}^2}$? Determine the bearing stress in the strap from the bolt diameter chosen. If the straps are 10 ft. in length, how much elongation would occur? What is the ultimate load capacity in each strap? Assume A36 steel: F_u= 58 ksi, E = 29 x 10^3 ksi.

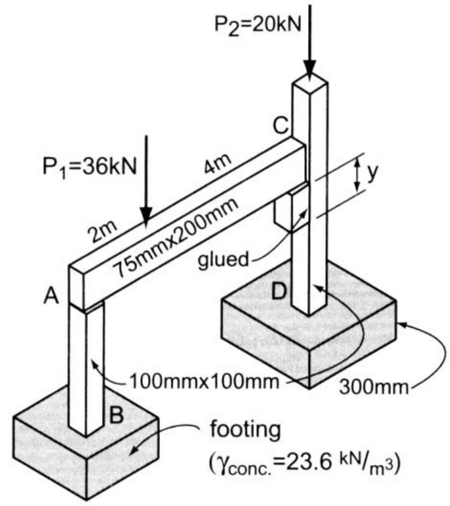


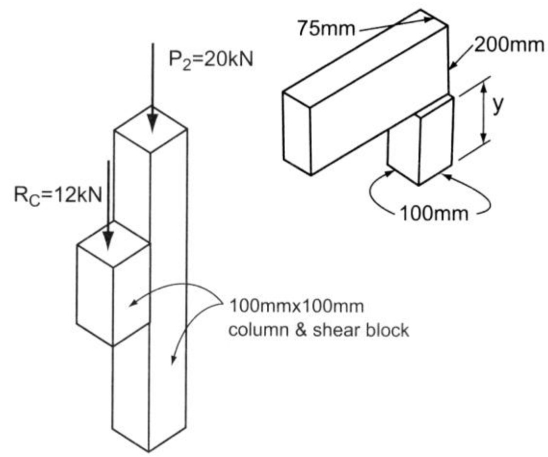
Example 2 (pg 202)

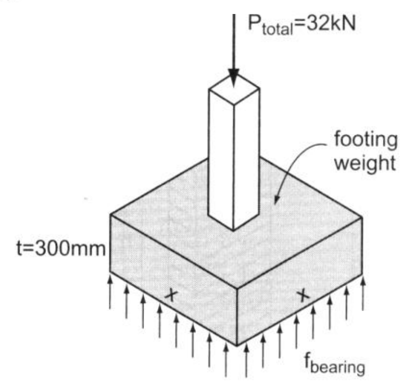
Example Problem 6.9 (Figures 6.21 to 6.26)

A 75 mm × 200 mm "rough cut" beam is supported by columns at both ends. Column *AB* supports the beam in bearing while column *CD* utilizes a shear block at *C*. Both columns bear on concrete footings on the ground.

a. What is the compressive stress developed in column *AB*? R_A = 24 kN
b. What is the bearing stress that develops at *C* between the beam and shear block made from a 100 mm × 100 mm block cut from a post?
c. What is the required depth *y* necessary to resist the shear force developed at the glued joint between the shear block and post? Assume that the glue is capable of safely resisting 500 kPa (72.5 psi) in shear.
d. Determine the size of square footing required to take the maximum column load if the allowable soil pressure $q = 73\,^{kN}/_{m^2} = 73$ kPa (1525 psf).





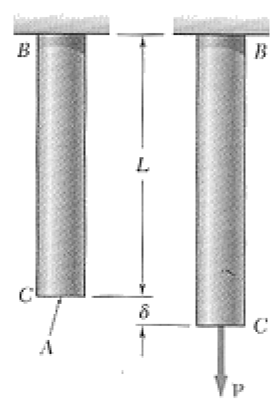


Stress and Strain – Elasticity

Notation:

A = area
D = diameter dimension
E = modulus of elasticity or Young's modulus
f = stress
$F_{allow.}$ = allowable stress
F_t = allowable tensile stress
$F.S.$ = factor of safety
h = height
kPa = kilopascals or 1 kN/m^2
ksi = kips per square inch
L = length
$LRFD$ = load and resistance factor design
MPa = megapascals or 10^6 N/m^2 or 1 N/mm^2
q = allowable soil bearing pressure
psf = pounds per square foot
P = name for axial force vector
R = name for design quantity (force or moment) for LRFD, ex. R_L, R_D, *or* R_n
t = thickness
δ = elongation or length change
ε = strain
ϕ = angle of twist
= resistance factor for LRFD
= diameter symbol
μ = lateral strain ratio or Poisson's ratio
γ = shear strain
= density or unit weight
γ_D = dead load factor for LRFD
γ_L = live load factor for LRFD
θ = angle of principle stress
ρ = radial distance
σ = engineering symbol for normal stress
τ = engineering symbol for shearing stress

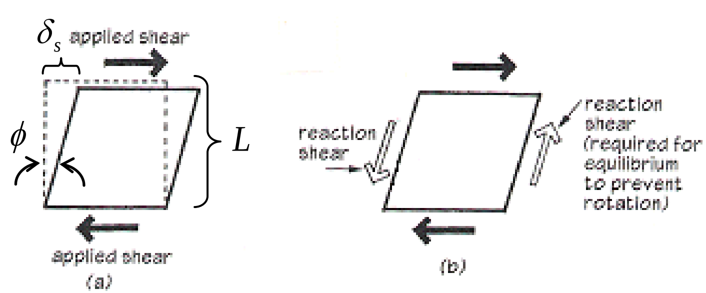
Normal Strain



In an axially loaded member, normal strain, ε is the change in the length, δ with respect to the original length, L.

$$\varepsilon = \frac{\delta}{L}$$

It is UNITLESS, but may be called strain or microstrain (μ).



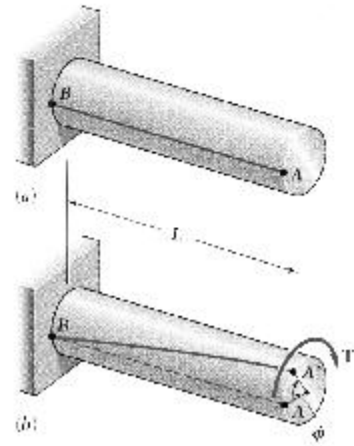
Shearing Strain

In a member loaded with shear forces, shear strain, γ is the change in the sheared side, δ_s with respect to the original height, L. For small angles: $\tan\phi \cong \phi$.

$$\gamma = \frac{\delta_s}{L} = \tan\phi \cong \phi$$

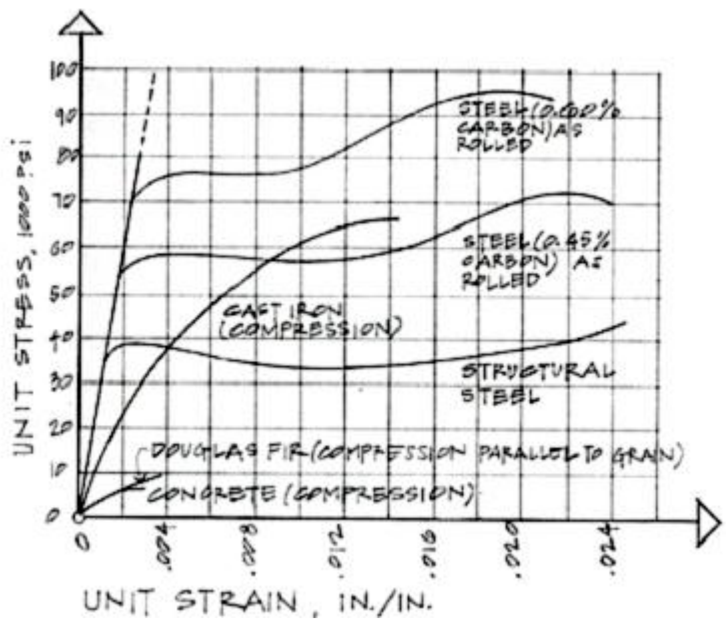
In a member subjected to twisting, the shearing strain is a measure of the angle of twist with respect to the length and distance from the center, ρ:

$$\gamma = \frac{\rho\phi}{L}$$



Testing of Load vs. Strain

Behavior of materials can be measured by recording deformation with respect to the size of the load. For members with constant cross section area, we can plot stress vs. strain.



<u>BRITTLE MATERIALS</u> - ceramics, glass, stone, cast iron; show abrupt fracture at small strains.

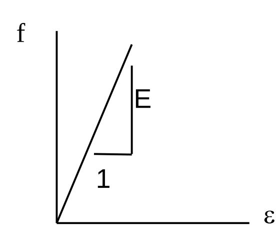
<u>DUCTILE MATERIALS</u> – plastics, steel; show a yield point and large strains (considered *plastic)* and "necking" (give warning of failure)

<u>SEMI-BRITTLE MATERIALS</u> – concrete; show no real yield point, small strains, but have some "strain-hardening".

<u>Linear-Elastic Behavior</u>

In the straight portion of the stress-strain diagram, the materials are *elastic*, which means if they are loaded and unloaded no permanent **deformation** occurs.

True Stress & Engineering Stress



True stress takes into account that the area of the cross section changes with loading.

Engineering stress uses the original area of the cross section.

Hooke's Law – Modulus of Elasticity

In the linear-elastic range, the slope of the stress-strain diagram is *constant*, and has a value of E, called Modulus of Elasticity or Young's Modulus.

$$f = E \cdot \varepsilon$$

<u>Isotropic Materials</u> – have the **same** E with any direction of loading.

<u>Anisotropic Materials</u> – have **different** E's with the direction of loading.

<u>Orthotropic Materials</u> – have **directionally based** E's

Table D-1 Elastic moduli of selected materials

Material	Modulus of elasticity E		Shear modulus G		Poisson's ratio ν
	10^6 psi	GPa	10^6 psi	GPa	
Aluminum	10	70	3.8	26	0.33
Aluminum alloys	10–12	70–80	3.8–4.4	26–30	0.33
2014-T6	10.6	73	4	28	0.33
6061-T6	10	70	3.8	26	0.33
7075-T6	10.4	72	3.9	27	0.33
Brick (compression)	1.5–3.5	10–24			
Cast iron	12–25	80–170	4.5–10	31–69	0.2–0.3
Gray cast iron	14	97	5.6	39	0.25
Concrete (compression)	2.6–4.4	18–30			0.1–0.2
Copper	17	115	6.2	43	0.35
Copper alloys	14–18	96–120	5.2–6.8	36–47	0.33–0.35
Brass	14–16	96–110	5.2–6	36–41	0.34
80% Cu, 20% Zn	15	100	5.5	38	0.33
Naval brass	15	100	5.5	38	0.33
Bronze	14–17	96–120	5.2–6.3	36–44	0.34
Manganese bronze	15	100	5.6	39	0.35
Glass	7–12	50–80	2.9–5	20–33	0.20–0.27
Magnesium	5.8	40	2.2	15	0.34
Nickel	30	210	11.4	80	0.31
Nylon	0.3–0.4	2–3			0.4
Rubber	0.0001–0.0006	0.001–0.004	0.00004–0.0002	0.0003–0.0014	0.44–0.50
Steel	28–32	190–220	10.8–12.3	75–85	0.28–0.30
Stone (compression)					
Granite	6–10	40–70			0.2–0.3
Marble	7–14	50–100			0.2–0.3
Titanium	16	110	5.8	40	0.33
Titanium alloys	15–18	100–124	5.6–6.8	39–47	0.33
Tungsten	52	360	22	150	0.2
Wood (bending)					
Ash	1.5–1.6	10–11			
Oak	1.6–1.8	11–12			
Southern pine	1.6–2	11–14			
Wrought iron	28	190	10.9	75	0.3

Plastic Behavior & Fatigue

Permanent deformations happen outside the linear-elastic range and are called *plastic* deformations. Fatigue is damage caused by reversal of loading.

- The proportional limit (at the end of the **elastic** range) is the greatest stress valid using Hooke's law.

- The elastic limit is the maximum stress that can be applied before permanent deformation would appear upon unloading.

- The yield point (at the ***yield stress***) is where a ductile material continues to elongate without an increase of load. (May not be well defined on the stress-strain plot.)

- The ultimate strength is the largest stress a material will see before rupturing, also called the *tensile strength.*

- The rupture strength is the stress at the point of rupture or failure. It may not coincide with the ultimate strength in ductile materials. In brittle materials, it will be the same as the ultimate strength.

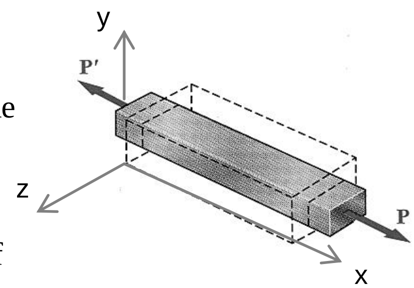
- The fatigue strength is the stress at failure when a member is subjected to reverse cycles of stress (up & down or compression & tension). This can happen at much lower values than the ultimate strength of a material.

- Toughness of a material is how much work (a combination of stress and strain) us used for fracture. It is the area under the stress-strain curve.

Concrete does not respond well to tension and is tested in compression. The strength at crushing is called the *compression strength.*

Materials that have time dependent elongations when loaded are said to have *creep.* Concrete and wood creep. Concrete also has the property of shrinking over time.

Poisson's Ratio



For an isometric material that is homogeneous, the properties are the same for the cross section:

$$\varepsilon_y = \varepsilon_z$$

There exists a linear relationship while in the linear-elastic range of the material between *longitudinal strain* and *lateral strain:*

$$\mu = -\frac{lateral\ strain}{axial\ strain} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x} \qquad \varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E}$$

Positive strain results from an increase in length with respect to overall length.

Negative strain results from a decrease in length with respect to overall length.

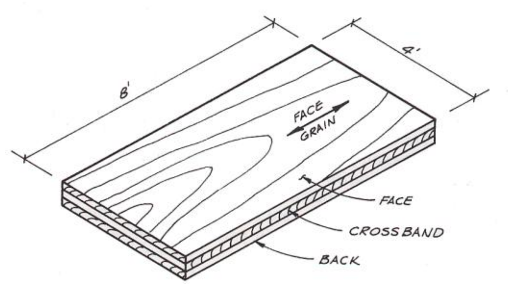
μ is the Poisson's ratio and has a value between 0 and ½, depending on the material

Relation of Stress to Strain

$$f = \frac{P}{A};\ \ \varepsilon = \frac{\delta}{L}\ \text{ and }\ E = \frac{f}{\varepsilon}\ \text{ so }\ E = \frac{P/A}{\delta/L}\ \text{ which rearranges to: } \qquad \delta = \frac{PL}{AE}$$

Orthotropic Materials

One class of non-isotropic materials is *orthotropic materials* that have directionally based values of modulus of elasticity and Poisson's ratio (E, μ).

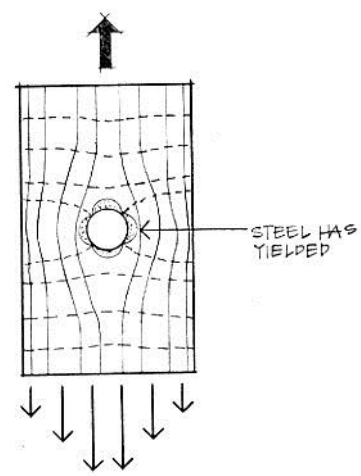


Ex: plywood, laminates, fiber reinforced polymers with direction fibers

Stress Concentrations

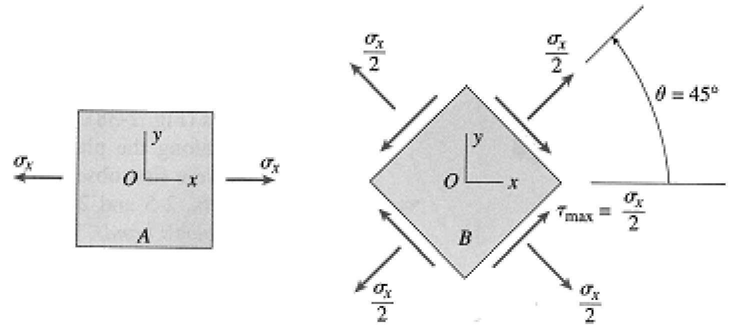
In some sudden changes of cross section, the stress concentration changes (and is why we used *average* normal stress). Examples are sharp notches, or holes or corners.

(*Think about airplane window shapes...)*



Maximum Stress

When both normal stress and shear stress occur in a structural member, the *maximum stresses can occur at some other planes* (angle of θ).



Maximum Normal Stress happens at $\theta = 0°$ AND

Maximum Shearing Stress happens at $\theta = 45°$ with only normal stress in the *x* direction.

Allowable Stress Design (ASD) and Factor of Safety (F.S.)

There are uncertainties in material strengths: $F.S = \frac{ultimate\ load}{allowable\ load} = \frac{ultimate\ stress}{allowable\ stress}$

Allowable stress design determines the allowable stress by: $allowable\ stress = \frac{ultimate\ stress}{F.S}$

Load and Resistance Factor Design – LRFD

There are uncertainties in material strengths *and* in structural loadings.

$$\gamma_D R_D + \gamma_L R_L \leq \phi R_n$$

where
γ = load factor for Dead and Live loads
R = load (dead or live)
ϕ = resistance factor
R_n = nominal load (capacity)

Thermal Effects and Indeterminacy

Notation:

A = area
E = modulus of elasticity or Young's modulus
f = stress
L = length
P = name for axial force vector
P' = name of reaction force
α = coefficient of thermal expansion for a material
ε_t = thermal strain (no units)
δ = elongation or length change
δ_P = elongation due to axial load
δ_{restr} = restrained length change
δ_T = elongation due or length change due to temperature
ΔT = change in temperature

Thermal Strains

Physical restraints limit deformations to be the same, or sum to **zero**, or be proportional with respect to the rotation of a rigid body.

We know axial stress relates to axial strain: $\delta = \frac{PL}{AE}$ which relates δ to P

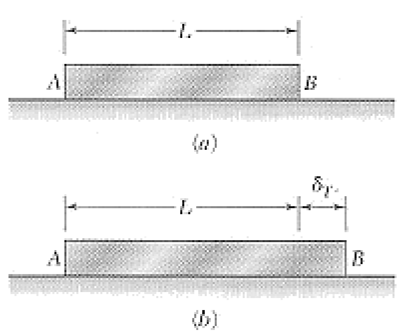
Deformations can be caused by the *material* reacting to a change in energy with temperature. In general (there are some exceptions):

- Solid materials can **contract** with a decrease in temperature.
- Solid materials can **expand** with an increase in temperature.

The change in length per unit temperature change is the *coefficient of thermal expansion*, α.

It has units of $/{}^\circ F$ or $/{}^\circ C$ and the deformation is related by: $\delta_T = \alpha(\Delta T)L$

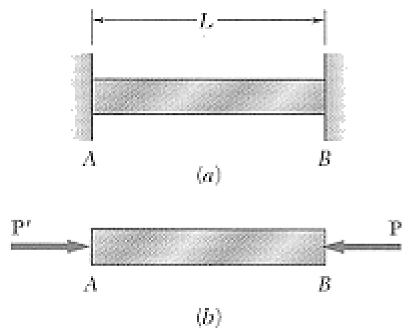
Thermal Strain: $\varepsilon_T = \alpha\Delta T$



There is **no stress** associated with the length change with free movement, BUT if there are restraints, thermal deformations or strains *can cause internal forces and stresses.*

How A Restrained Bar Feels with Thermal Strain

1. Bar pushes on supports because the material needs to expand with an increase in temperature.
2. Supports push *back*.
3. Bar is restrained, can't move and the reaction causes internal *stress*.



Superposition Method

If we want to solve a statically indeterminate problem that has extra support forces:

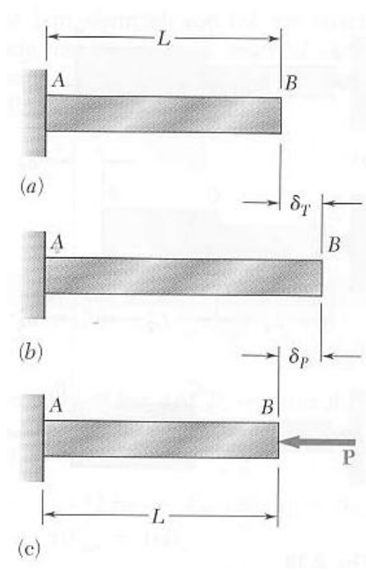
- We can remove a support or supports that *makes the problem look statically determinate*
- Replace it with a reaction and treat it like it is an applied force
- Impose geometry restrictions that the support imposes

For Example:

$$\delta_T = \alpha(\Delta T)L \qquad \delta_p = -\frac{PL}{AE}$$

$$\delta_P + \delta_T = 0 \qquad -\frac{PL}{AE} + \alpha(\Delta T)L = 0$$

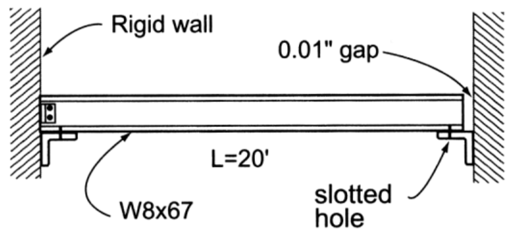
$$P = \alpha(\Delta T)L\frac{AE}{L} = \alpha(\Delta T)AE \qquad f = -\frac{P}{A} = -\alpha(\Delta T)E$$



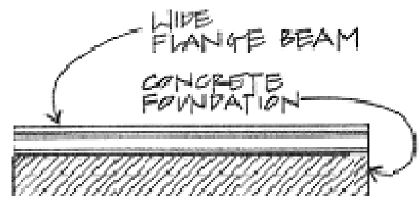
Example 1 (pg 228)

Example Problem 6.24 (Figures 6.58 and 6.59)

A W8×67 steel beam, 20 ft. in length, is rigidly attached at one end of a concrete wall. If a gap of 0.010 in. exists at the opposite end when the temperature is 45°F, what results when the temperature rises to 95°F?

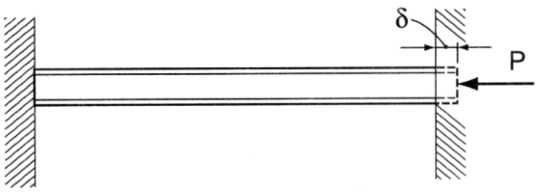


ALSO: If the beam is anchored to a concrete slab, and the steel sees a temperature change of 50° F while the concrete only sees a change of 30° F, determine the compressive stress in the beam.

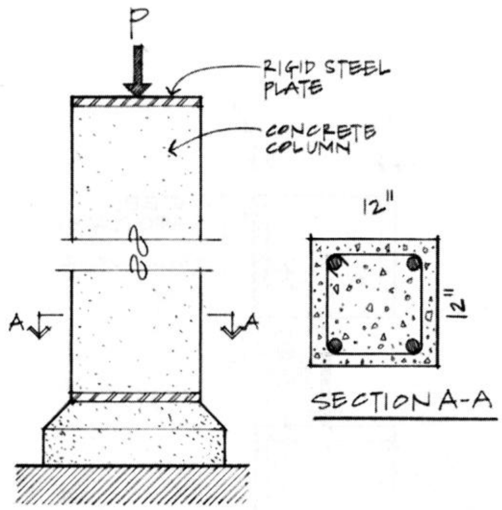


$\alpha_c = 5.5 \times 10^{-6}$ /° F $\quad E_c = 3 \times 10^6$ psi

$\alpha_s = 6.5 \times 10^{-6}$ /° F $\quad E_s = 29 \times 10^6$ psi



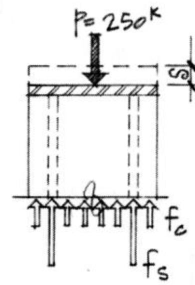
Example 2



5.21 A short concrete column measuring 12 in. square is reinforced with four #8 bars ($A_s = 4 \times 0.79$ in.^{2} = 3.14 in.^{2}) and supports an axial load of 250k. Steel bearing plates are used top and bottom to ensure equal deformations of steel and concrete. Calculate the stress developed in each material if:

$E_c = 3 \times 10^6$ psi and

$E_s = 29 \times 10^6$ psi



Solution:

From equilibrium:

$$[\Sigma F_y = 0] - 250\text{ k} + f_s A_s + f_c A_c = 0$$

$$A_s = 3.14\text{ in.}^2$$

$$A_c = (12'' \times 12'') - 3.14\text{ in.}^2 \cong 141\text{ in.}^2$$

$$3.14 f_s + 141 f_c = 250\text{ k}$$

From the deformation relationship:

$$\delta_s = \delta_c;\ \ L_s = L_c$$

$$\therefore \frac{\delta_s}{L} = \frac{\delta_c}{L}$$

and

$$\varepsilon_s = \varepsilon_c$$

Since

$$E = \frac{f}{\varepsilon}$$

and

$$\frac{f_s}{E_s} = \frac{f_c}{E_c}$$

$$f_s = f_c \frac{E_s}{E_c} = \frac{29 \times 10^3 (f_c)}{3 \times 10^3} = 9.67 f_c$$

Substituting into the equilibrium equation:

$$3.14\,(9.76 f_c) + 141 f_c = 250$$

$$30.4 f_c + 141 f_c = 250$$

$$171.4 f_c = 250$$

$$f_c = 1.46\text{ ksi}$$

$$\therefore f_s = 9.67\,(1.46)\text{ ksi}$$

$$f_s = 14.1\text{ ksi}$$

Sustainability Considerations in Materials

from Fundamentals of Building Construction Materials & Methods, 5th ed., Allen and Iano (2008)

Considerations of Sustainability in Wood Construction

Wood: A Renewable Resource

• Wood is the only major structural material that is renewable.

• In the United States and Canada, tree growth each year greatly exceeds the volume of harvested trees, though many timberlands are not managed in a sustainable manner.

• On other continents, many countries long ago felled the last of their forests, and many forests in other countries are being depleted by poor management practices and slash-and-burn agriculture. Particularly in the case of tropical hardwoods, it is wise to investigate sources and to ensure that the trees were grown in a sustainable manner.

• Some panel products can be manufactured from rapidly renewable vegetable fibers, recoverable and recycled wood fibers, or recycled cellulose fibers.

• Bamboo, a rapidly renewable grass, can replace wood in the manufacture of flooring, interior paneling, and other finish carpentry applications. In other parts of the world, bamboo is used for the construction of scaffolding, concrete formwork, and even as the source of fibrous material for structural panels analogous to wood-based oriented strand board (OSB), particleboard, and fiberboard.

Forestry Practices

• Two basic forms of forest management are practiced in North America: sustainable forestry, and clearcutting and replanting. The clearcutting forest manager attains sustainable production by cutting all the trees in an area, leaving the stumps, tops, and limbs to decay and become compost, setting out new trees, and tending them until they are ready for harvest. In sustainable forestry, trees are harvested more selectively from a forest in such a way as to minimize damage to the forest environment and maintain the biodiversity of its natural ecosystem.

• Environmental problems often associated with logging of forests include loss of wildlife habitat, soil erosion, pollution of waterways, and air pollution from machinery exhausts and burning of tree wastes. A recently clearcut forest is a shockingly ugly tangle of stumps, branches, tops, and substandard logs left to decay. It is crisscrossed by deeply rutted, muddy haul roads. Within a few years, decay of the waste wood and new tree growth largely heal the scars. Loss of forest area may raise levels of carbon dioxide, a greenhouse gas, in the atmosphere, because trees take up carbon dioxide from the air, utilize the carbon for growth, and give back pure oxygen to the atmosphere.

• The buyer of wood products can support sustainable forestry practices by specifying products certified as originating from sustainable forests, those that are managed in a socially responsible and environmentally sound manner. FSC-certified wood products, for example, satisfy the requirements of LEED and all other major green building assessment programs.

Mill Practices

• Skilled sawyers working with modern computerized systems can convert a high percentage of each log into marketable wood products. A measure of sawmill performance is the lumber recovery factor (LRF), which is the net volume of wood products produced from a cubic meter of log.

• Manufactured wood products such as oriented strand board, particleboard, I-joists, and laminated strand lumber efficiently utilize most of the wood fiber in a tree and can be produced from recycled or younger-growth, rapidly renewable materials; finger-jointed lumber is made by gluing end to end short pieces of lumber that might otherwise be treated as waste. The manufacturer of large, solid timbers generates more unused waste and yields fewer products from each log.

• Kiln drying uses large amounts of fuel but produces more stable, uniform lumber than air drying, which uses no fuel other than sunlight and wind.

• Mill wastes are voluminous: Bark may be shredded to sell as a landscape mulch, composted, burned, or buried in a landfill. Sawdust, chips, and wood scraps may be burned to generate steam to power the mill, used as livestock bedding, composted, burned, or buried in a landfill.

• Many wood products can be manufactured with significant percentages of recoverable or recycled wood, plant fiber, or paper materials.

Transportation

• Because the major commercial forests are located in concentrated regions of the United States and Canada, most lumber must be shipped considerable distances. Fuel consumption is minimized by planing and drying the lumber before it is shipped, which reduces both weight and volume.

- Some wood products can be harvested or manufactured locally or regionally.

Energy Content

- Solid lumber has an embodied energy of roughly 1000 to 3000 BTU per pound (2.3 to 7.0 MJ/kg). An average 8-foot-long 2 × 4 (2.4-m-long 38 × 89 mm) has an embodied energy of about 17,000 BTU (40 MJ). This includes the energy expended to fell the tree, transport the log, saw and surface the lumber, dry it in a kiln, and transport it to a building site.
- Manufactured wood products have higher embodied energy content than solid lumber, due to the glue and resin ingredients and the added energy required in their manufacture. The embodied energy of such products ranges from about 3000 to 7500 BTU per pound (7.0 to 17 MJ/kg).
- Wood construction involves large numbers of steel fasteners of various kinds. Because steel is produced by relatively energy-intensive processes, fasteners add considerably to the total energy embodied in a wood frame building.
- Wood does not have the lowest embodied energy of the major structural materials when measured on a pound-for-pound basis. However, when buildings of comparable size, but structured with either wood, light gauge steel studs, or concrete, are compared, most studies indicate that those of wood have the lowest total embodied energy of the three. This is due to wood's lighter weight (or, more precisely, its lesser density) in comparison to these other materials, as well as the relative efficiency of the wood light frame construction system.

Construction Process

- A significant fraction of the lumber delivered to a construction site is wasted: It is cut off when each piece is sawed to size and shape and ends up on the scrap heap, which is usually burned or taken to a landfill. On-site cutting of lumber also generates considerable quantities of sawdust. Construction site waste can be reduced by designing buildings that utilize full standard lengths of lumber and full sheets of wood panel materials.
- Wood construction lends itself to various types of prefabrication that can reduce waste and improve the efficiency of material usage in comparison to on-site building methods.

Indoor Air Quality (IAQ)

- Wood itself seldom causes IAQ problems. Very few people are sensitive to the odor of wood.
- Some of the adhesives and binders used in glue-laminated lumber, structural composite lumber, and wood panel products can cause serious IAQ problems by giving off volatile organic compounds such as formaldehyde. Alternative products with low-emitting binders and adhesives are also available.
- Some paints, varnishes, stains, and lacquers for wood also emit fumes that are unpleasant and/or unhealthful.
- In damp locations, molds and fungi may grow on wood members, creating unpleasant odors and releasing spores to which many people are allergic.

Building Life Cycle

- If the wood frame of a building is kept dry and away from fire, it will last indefinitely. However, if the building is poorly maintained and wood elements are frequently wet, wood components may decay and require replacement.
- Wood is combustible and gives off toxic gases when it burns. It is important to keep sources of ignition away from wood and to provide smoke alarms and easy escape routes to assist building occupants in escaping from burning buildings. Where justified by building size or type of occupancy, building codes require sprinkler systems to protect against the rapid spread of fire.
- When a building is demolished, wood framing members can be recycled directly into the frame of another building, sawn into new boards or timbers, or shredded as raw material for oriented-strand materials. There is a growing industry whose business is purchasing and demolishing old barns, mills, and factories and selling their timbers as *reclaimed lumber.*

A study commissioned by the Canadian Wood Council compares the full life cycle of three similar office buildings, one each framed with wood, steel, or concrete and all three operated in a typical Canadian climate. In this study, total embodied energy for the wood building is about half of that for the steel building and two-thirds of that for the concrete building. The wood building also outperforms the others in measures of greenhouse gas emissions, air pollution, solid waste generation, and ecological impact.

Considerations of Sustainability in Heavy Timber Construction

In addition to the issues of sustainability of wood production and use that were raised in the previous chapter, there are issues that pertain especially to heavy timber frame construction:

It is wasteful to saw large, solid timbers from logs: In most instances, only one or two timbers can be obtained from a log, and it is often difficult to saw smaller boards from the leftover slabs.

Glue-laminated timbers and composite timbers utilize wood fiber much more efficiently than solid timbers.

Recycled timbers from demolished mills, factories, and barns are often available. Most of these are from old-growth forests in which trees grew slowly, producing fine-grained, dense wood. As a result, many have structural properties that are superior to those of new-growth timbers. Recycled timbers may be used as is, resurfaced to give them a new appearance, or resawn into smaller members. However, they often contain old metal fasteners. Unless these are meticulously found and removed, they can damage saw blades and planer knives, causing expensive mill shutdowns while repairs are made.

Continuous bending action of beams may be created by splicing beams at points of inflection rather than over supports, as shown in Figures 4.15, 4.20 and 4.21. This reduces maximum bending moments, allowing timber sizes to be reduced substantially.

Timbers do not lose strength with age, although they do sag progressively if they are overloaded. When a heavy timber building is demolished at some time in the future, its timbers can be recycled, even if they were obtained as recycled material for the building that is being demolished.

A heavy timber frame enclosed with foam core sandwich or stressed-skin panels is relatively airtight and well insulated, with few thermal bridges. Heating and cooling of the building will consume relatively little energy.

The glues and finish coatings used with glue-laminated timbers may give off gases such as formaldehyde that can cause indoor air quality (IAQ) problems. It is wise to determine in advance what glues and coatings are to be used, and to avoid ones that may cause IAQ problems.

Considerations of Sustainability in Wood Light Frame Construction

In addition to the issues of sustainability of wood production and use that were raised in Chapter 3, there are issues that pertain especially to wood light frame construction:

- A wood light frame building can be designed to minimize waste in several ways. It can be dimensioned to utilize full sheets and lengths of wood products. Most small buildings can be framed with studs 24 inches (610 mm) o.c. rather than 16 inches (406 mm). A stud can be eliminated at each corner by using small, inexpensive metal clips to support the interior wall finish materials. If joists and rafters are aligned directly over studs, the top plate can be a single member rather than a double one. Floor joists can be spliced at points of inflection rather than over girders; this reduces bending moments and allows use of smaller joists. Roof trusses often use less wood than conventional rafters and ceiling joists.
- Laminated strand lumber and rim joists, wood I-joists, laminated veneer lumber beams and headers, glue-laminated girders, parallel strand lumber girders, and OSB sheathing are all materials that utilize trees more efficiently than solid lumber. Finger-jointed studs made up of short lengths of scrap lumber glued together may replace solid full-length wood studs.
- Framing carpenters can waste less lumber by saving cutoffs and reusing them rather than throwing them automatically on the scrap heap. In some localities, scrap lumber can be recycled by shredding it for use in OSB production. The burning of construction scrap should be discouraged because of the air pollution it generates.
- Although the thermal efficiency of wood light frame construction is inherently high, it can be improved substantially by various means, as shown in Figures 7.17–7.21. Wood framing is much less conductive of heat than light-gauge steel framing. Steel framing of exterior walls is not a satisfactory substitute for wood framing unless the heat flow path through the steel framing members can be broken with a substantial thickness of insulating foam.

Considerations of Sustainability in Brick Masonry

Brick Masonry Materials

- Mortar is made of minerals that are generally abundant in the earth. Portland cement and lime are energy-intensive products. (For more information about the sustainability of cement production, see Chapter 13.)
- Clay and shale, the raw materials for bricks, are plentiful. They are usually obtained from open pits, with the attendant disruption of drainage, vegetation, and wildlife habitat.
- Clay brick can include recycled brick dust, postindustrial wastes such as fly ash, and a variety of other waste products in their manufacture.

Brick Manufacturing

- Brick manufacturing plants are usually located close to the sources of their raw materials.
- Brick manufacturing produces few waste materials. Unfired clay is easily recycled into the production process. Fired bricks that are unusable are ground up and recycled into the production process or used as landscaping material.
- Brick manufacturing requires relatively large amounts of water. Water that doesn't evaporate can be reused many times. Little if any water need be discharged as waste.
- Because of the energy used in its firing, brick is a relatively energy-intensive product. Its embodied energy may range from about 1000 to 4000 BTU per pound (2.3-9.3 MJ/kg).
- The most common energy source for brick kilns is natural gas, although oil and coal are also used. Firing of clay masonry produces fluorine and chlorine emissions. Other types of air pollution can result from improperly regulated kilns.
- Most bricks are sold for use in regional markets close to their point of manufacture. This reduces the energy required for shipping and makes much brick eligible for credit as a regional material.

Brick Masonry Construction

- Relatively small amounts of waste are generated on a construction site during brick masonry work, including partial bricks, unsatisfactory bricks, and unused mortar. These wastes generally go into landfills or are buried on the site.
- Sealers applied to brick masonry to provide water repellency and protection from staining are potential sources of emissions. Solvent-based sealers generally have higher emissions than water-based products.

Brick Masonry Buildings

- Brick masonry is not normally associated with any indoor air quality problems, although in rare circumstances it can be a source of radon gas.
- The thermal mass effect of brick masonry can be a useful component of fuel-saving heating and cooling strategies such as solar heating and nighttime cooling.
- Brick masonry is a durable form of construction that requires relatively little maintenance and can last a very long time.
- Construction with brick masonry can reduce reliance on paint finishes, a source of volatile organic compounds.
- Brick masonry is resistant to moisture damage and mold growth.
- When a brick building is demolished, sound bricks may be cleaned of mortar and reused (once their physical properties have been verified as adequate for the new use). Brick waste can be crushed and used for landscaping. Brick and mortar waste can also be used as on-site fill. Much such waste, however, is disposed of off-site in landfills.

Considerations of Sustainability in Stone and Concrete Masonry

Stone and Concrete Masonry Materials

- Stone is a plentiful but finite resource. It is usually obtained from open pits, with the attendant disruption of drainage, vegetation, and wildlife habitat.
- The detrimental impacts of stone quarrying can long outlive the buildings for which the stone was extracted.
- Quarry reclamation practices, such as revegetation, land reshaping, and habitat restoration, can mitigate some of the adverse environmental impacts of stone quarrying and convert exhausted quarry sites to other beneficial uses.
- Concrete used in the manufacture of masonry units may include recycled materials such as fly ash, crushed glass, slag, and other postindustrial wastes. For more information regarding the sustainability of concrete, see Chapter 13.
- Mortar used for stone and concrete masonry is made from minerals that are generally abundant in the earth. However, portland cement and lime are energy-intensive products to manufacture. For more information about the sustainability of cement production, see Chapter 13.

Stone and Concrete Masonry Processing and Manufacturing

- Stone is heavy. It is expensive and energy intensive to transport. Stone may originate from local quarries or from sources in many places around the world. Fabrication may take place close to the source of the stone, close to the building site, or in some other location remote from both the stone's source and destination. Where uniquely sourced stones are desired or where specialized fabrication processes or skills are required, shipping over long distances may be required.
- The cutting, shaping, and polishing operations that take place during stone fabrication use large quantities of water that becomes contaminated with stone residue, lubricants, and abrasives. Water filtration and recycling systems can prevent contaminants from entering the wastewater stream and minimize water consumption.
- As much as one-half of quarried stone may become waste during fabrication. Depending on the type of stone, waste may be crushed and used as fill material on construction sites or as aggregate in concrete or asphalt. Stone with a strong color or other unique appearance qualities may be processed into aggregate for use in the manufacture of terrazzo, architectural concrete masonry units, or synthetic stone products. Much stone waste, however, is disposed of as landfill.
- The embodied energy of building stone can vary significantly with the source of the stone, fabrication processes, and distances and methods of shipping. Stone that is easily quarried and fabricated, and that is used locally, may have an embodied energy of as little as 300 to 400 BTU per pound (0.7–0.9 MJ/kg). On the other hand, stone that requires more effort and energy to extract and fabricate, and that is transported over long distances before arriving at the building site, may have an embodied energy 10 or even 20 times greater.
- Most concrete masonry units are manufactured in regional plants relatively close to their final end-use destinations.
- The use of lightweight concrete masonry reduces transportation-related costs and energy consumption.

• The embodied energy of concrete masonry units is slightly higher than that of the concrete from which they are made, due to the additional energy consumed in the curing of the units. Ordinary concrete masonry units have an embodied energy of approximately 250 BTU per pound (0.6 MJ/kg).

Stone and Concrete Masonry Construction

• Relatively small amounts of waste are generated on a construction site during stone and concrete masonry construction, including, for example, stone cutoffs, partial blocks, and unused mortar. These wastes generally go into landfills or are buried on the site.

• Sealers applied to stone and concrete masonry to provide water repellency and protection from staining are potential sources of emissions. Solvent-based sealers generally emit more air pollutants than water-based products.

Stone and Concrete Masonry Buildings

• Stone and concrete masonry are not normally associated with indoor air quality problems. In rare instances, stone aggregate in concrete or stone used in stone masonry has been found to be a source of radon gas emissions.

• The thermal mass effect of stone and concrete masonry can be a useful component of fuel-saving heating and cooling strategies such as solar heating and nighttime cooling.

• Stone and concrete masonry are dense materials that can effectively reduce sound transmission between adjacent spaces.

• Stone and concrete masonry construction are noncombustible. Lightweight concrete masonry units are especially effective for construction of fire resistance rated assemblies.

• Lightweight concrete masonry units have greater thermal resistance than more dense concrete units, stone, or brick.

• Construction with stone or concrete masonry can reduce reliance on paint finishes, a source of volatile organic compounds.

• Stone and concrete masonry are durable forms of construction that require relatively little maintenance and can last a very long time.

• Stone and concrete masonry are resistant to moisture damage and mold growth.

• When a building with stone or concrete masonry is demolished, the stone or masonry units can be crushed and recycled for use as on-site fill or as aggregate for paving. Some building stone can be salvaged for new construction.

Concrete Masonry Sitework

• Concrete masonry permeable pavers can facilitate on-site capture of storm water.

• Light-colored concrete pavers can lessen urban heat island effects.

• Interlocking concrete masonry units used in earth retaining walls are easily disassembled and reused

Considerations of Sustainability in Steel Frame Construction

Manufacture

• The raw materials for steel are iron ore, coal, limestone, air, and water. The ore, coal, and limestone are minerals whose mining and quarrying cause disruption of land and loss of wildlife habitat, often coupled with pollution of streams and rivers. Coal, limestone, and low-grade iron ore are plentiful, but high-grade iron ore has been depleted in many areas of the earth.

• The steel industry has worked hard to reduce pollution of air, water, and soil, but much work remains to be done.

• Supplies of some alloying metals, such as manganese, chromium, and nickel, are becoming depleted.

• The manufacture of a ton of steel from iron ore by the basic oxygen process consumes 3170 pounds (1440 kg) of ore, 300 pounds (140 kg) of limestone, 900 pounds (410 kg) of coke (made from coal), 80 pounds (36 kg) of oxygen, and 2575 pounds (1170 kg) of air. In the process, 4550 pounds (2070 kg) of gaseous emissions are given off, and 600 pounds (270 kg) of slag and 50 pounds (23) of dust are generated. Further emissions emanate from the process of converting coal to coke.

• The embodied energy of steel produced from ore by the basic oxygen process is about 14,000 BTU per pound (33 MJ/kg). In modern facilities, scrap steel is typically added as an ingredient during this process, resulting in recycled materials content of 25 to 35 percent.

• Today, most structural steel in North America is made from recycled scrap by the electric arc furnace process; its embodied energy is approximately 4000 BTU per pound (9.3 MJ/kg), less than one-third that of steel made from ore. The recycled materials content of steel made by this process is 90 percent or higher.

• In North America, virtually all hot-rolled structural steel shapes are manufactured by the electric arc furnace process. Steel plate and sheet, used in the manufacture, for example, of light gauge steel members, decking, and hollow structural sections, may be produced by either the electric arc furnace or basic oxygen processes.

• Ninety-five percent or more of all structural steel used in North American building construction is eventually recycled or reused, which is a very high rate. In a recent one-year period, 480 million tons (430 million metric tons) of scrap steel were consumed worldwide.

• Scrap used in the production of structural steel in mini-mills usually comes from sources within approximately 300 miles (500 km) of the mill. When the steel produced in such mills is then used for the construction of buildings not too far from the mill, the steel is potentially eligible for credit as a regionally extracted, processed, and produced material. This is most likely for the most commonly used steel alloys that are produced in the greatest number of mills. However, some less commonly produced steel alloys are only available from a limited number of mills or, in some cases, are produced solely overseas, and are not eligible for such a credit except for projects located fortuitously close to the mills where these particular types of steel are produced.

Construction

• Steel fabrication and erection are relatively clean, efficient processes, although the paints and oils used on steel members can cause air pollution.

• Steel frames are lighter in weight than concrete frames that would do the same job. This means that a steel building generally has smaller foundations and requires less excavation work.

• Some spray-on fireproofing materials can pollute the air with stray fibers.

In Service

• Steel framing, if protected from water and fire, will last for many generations with little or no maintenance.

• Steel exposed to weather needs to be repainted periodically unless it is galvanized, given a long-lasting polymer coating, or made of more expensive stainless steel.

• Steel framing members in building walls and roofs should be thermally broken or insulated in such a way that they do not conduct heat between indoors and outdoors.

• When a steel building frame is demolished, its material is almost always recycled.

• Steel seldom causes indoor air quality problems, although surface oils and protective coatings sometimes outgas and cause occupant discomfort.

Considerations of Sustainability in Light Gauge Steel Framing

In addition to the sustainability issues raised in the previous chapter, which also apply here, the largest issue concerning the sustainability of light gauge steel construction is the high thermal conductivity of the framing members. If a dwelling framed with light gauge steel members is framed, insulated, and finished as if it were framed with wood, it will lose heat in winter at about double the rate of the equivalent wood structure. To overcome this limitation, energy codes now require light gauge steel framed buildings constructed in cold regions, including most of the continental United States, to be sheathed with plastic foam insulation panels in order to eliminate the extensive thermal bridging that can otherwise occur through the steel framing members.

Even with insulating sheathing, careful attention must be given to avoid undesired thermal bridges. For example, on a building with a sloped roof, a significant thermal bridge may remain through the ceiling joist-rafter connections, as seen in Figure 12.4*b*. Foam sheathing on the inside wall and ceiling surfaces is one possible way to avoid this condition, but adding insulation to the inside of the metal framing exposes the studs and stud cavities to greater temperature extremes and increases the risk of condensation. It also still allows thermal bridging through the screws used to fasten interior gypsum wallboard to the framing. Though small in area, these thermal bridges can readily conduct heat and result in spots of condensation on interior finish surfaces in very cold weather.

Considerations of Sustainability in Concrete Construction

- Worldwide each year, the making of concrete consumes 1.6 billion tons (1.5 billion metric tons) of portland cement, 10 billion tons (9 billion metric tons) of sand and rock, and 1 billion tons (0.9 billion metric tons) of water, making the concrete industry the largest user of natural resources in the world.
- The quarrying of the raw materials for concrete in open pits can result in soil erosion, pollutant runoff, habitat loss, and ugly scars on the landscape.
- Concrete construction also uses large quantities of other materials—wood, wood panel products, steel, aluminum, plastics—for formwork and reinforcing.
- The total energy embodied in a pound of concrete varies, especially with the design strength. This is because higher-strength concrete relies on a greater proportion of portland cement in its mix, and the energy required to produce portland cement is very high in comparison to concrete's other ingredients. For average-strength concrete, the embodied energy ranges from about 200 to 300 BTU per pound (0.5-0.7 MJ/kg).
- There are various useful approaches to increasing the sustainability of concrete construction:
  - Use waste materials from other industries, such as fly ash from power plants, slag from iron furnaces, copper slag, foundry sand, mill scale, sandblasting grit, and others, as components of cement and concrete.
  - Use concrete made from locally extracted materials and local processing plants to reduce the transportation of construction materials over long distances.
  - Minimize the use of materials for formwork and reinforcing.
  - Reduce energy consumption, waste, and pollutant emissions from every step of the process of concrete construction, from quarrying of raw materials through the eventual demolition of a concrete building.
  - In regions where the quality of the construction materials is low, improve the quality of concrete so that concrete buildings will last longer, thus reducing the demand for concrete and the need to dispose of demolition waste.

Portland Cement

- The production of portland cement is by far the largest user of energy in the concrete construction process, accounting for about 85 percent of the total energy required. Portland cement production also accounts for roughly 5 percent of all carbon dioxide gas generated by human activities worldwide and about 1.5 percent of such emissions in North America.
- Since 1970, the North American cement industry has reduced the amount of energy expended in cement production by one-third, and the industry continues to work toward further reductions.
- The manufacture of cement produces large amounts of air pollutants and dust. For every ton of cement clinker produced, almost a ton of carbon dioxide, a greenhouse gas, is released into the atmosphere. Cement production accounts for approximately 1.5 percent of carbon dioxide emissions in the United States and 5 percent of carbon dioxide emissions worldwide.
- In the past 35 years, the emission of particulates from cement production has been reduced by more than 90 percent.
- The cement industry is committed to reducing greenhouse gas emissions per ton of product by 10 percent from 1990 levels by the year 2020. According to the Portland Cement Association, over concrete's lifetime, it reabsorbs roughly half of the carbon dioxide released during the original cement manufacturing process.
- The amount of portland cement used as an ingredient in concrete, and as a consequence, the energy required to produce the concrete, can be substantially reduced by the addition of certain industrial waste materials with cementing properties to the concrete mix. Substituting such supplementary cementitious materials, including fly ash, silica fume, and blast furnace slag, for up to half the portland cement in the concrete, can result in reductions in embodied energy of as great as one-third.
- When added to concrete, fly ash is most commonly substituted for portland cement at rates of between 15 and 25 percent. Mixes with even higher replacement rates, called *high-volume-fly-ash (HVFA) concrete*, are also finding increased acceptance. Concrete mixed with fly ash as an ingredient gains other benefits as well: It needs less water than normal concrete, its heat of hydration is lower, and it shrinks less, all characteristics that lead to a denser, more durable product. Research is underway to develop concrete mixes in which fly ash completely replaces all portland cement.
- Waste materials from other industries can also be used as cementing agents—wood ash and rice-husk ash are two examples. Used motor oil and used rubber vehicle tires can be employed as fuel in cement kilns. And while consuming waste products from other industries, a cement manufacturing plant can, if efficiently operated, generate virtually no solid waste itself.

Aggregates and Water

• Sand and crushed stone come from abundant sources in many parts of the world, but high-quality aggregates are becoming scarce in some countries.

• In rare instances, aggregate in concrete has been found to be a source of radon gas. Concrete itself is not associated with indoor air quality problems.

• Waste materials such as crushed, recycled glass, used foundry sand, and crushed, recycled concrete can substitute for a portion of the conventional aggregates in concrete.

• Water of a quality suitable for concrete is scarce in many developing countries. Concretes that use less water by using superplasticizers, air entrainment, and fly ash could be helpful.

Wastes

• A significant percentage of fresh concrete is not used because the truck that delivers it to the building site contains more than is needed for the job. This concrete is often dumped on the site, where it hardens and is later removed and taken to a landfill for disposal. An empty transit-mix truck must be washed out after transporting each batch, which produces a substantial volume of water that contains portland cement particles, admixtures, and aggregates. These wastes can be recovered and recycled as aggregates and mixing water, but more concrete suppliers need to implement schemes for doing this.

Formwork

• Formwork components that can be reused many times have a clear advantage over single-use forms, which represent a large waste of construction material.

• Form release compounds and curing compounds should be chosen for low volatile organic compound content and biodegradability.

• Insulating concrete forms eliminate most temporary formwork and produce concrete walls with high thermal insulating values.

Reinforcing

• In North America, reinforcing bars are made almost entirely from recycled steel scrap, primarily junked automobiles. This reduces resource depletion and energy consumption significantly.

Demolition and Recycling

• When a concrete building is demolished, its reinforcing steel can be recycled.

• In many if not most cases, fragments of demolished concrete can be crushed, sorted, and used as aggregates for new concrete. At present, however, most demolished concrete is buried on the site, used to fill other sites, or dumped in a landfill.

Green Uses of Concrete

• Pervious concrete, made with coarse aggregate only, can be used to make porous pavings that allow stormwater to filter into the ground, helping to recharge aquifers and reduce stormwater runoff.

• Concrete is a durable material that can be used to construct buildings that are long-lasting and suitable for adaptation and reuse, thereby reducing the environmental impacts of building demolition and new construction.

• In brownfield development, concrete fill materials can be used to stabilize soils and reduce leachate concentrations.

• Where structured parking garages (often constructed of concrete) replace surface parking, open space is preserved.

• Concrete's thermal mass can be exploited to reduce building heating and cooling costs by storing excess heat during overheated periods of the day or week and releasing it back to the interior of the building during underheated periods.

• Lighter-colored concrete paving reflects more solar radiation than darker asphalt paving, leading to lower paving surface temperatures and reduced urban heat island effects.

• Interior concrete slabs made with white concrete can improve illumination, visibility, and worker safety within interior spaces without the expense or added energy consumption of extra light fixtures or increasing the light output from existing fixtures. White concrete is made with white cement and white aggregates.

• Photocatalytic agents can be added to concrete used in the construction of roads and buildings. In the presence of sunlight, the concrete chemically breaks down carbon monoxide, nitrogen oxide, benzene, and other air pollutants.

Considerations of Sustainability in Precast Concrete Construction

In addition to the issues of sustainability of concrete construction that were raised in Chapter 13, there are issues that pertain especially to precast concrete construction:

- Because of the higher-strength concrete mixes typically used in the production of precast concrete, its embodied energy is higher on a pound-for-pound basis than that of conventional concrete, generally falling in the range of 500 to 600 BTU per pound (1.1-1.4 MJ/kg).
- Precast concrete production encourages the reuse of formwork, reducing waste. Wood and fiberglass forms can be used up to 50 times without major maintenance. Concrete and steel forms can be reused hundreds or thousands of times.
- Because precast concrete is manufactured in a controlled, factory-like setting, raw materials are used more efficiently and less waste is produced. Gray water used in various production processes, sand used in finishing, and large aggregate used to create voids in hollow planks can all be readily reused.
- In many cases, the optimized design of precast concrete results in elements that use less material than comparable sitecast concrete systems.
- Precast concrete elements with high-quality architectural finishes reduce the need for volatile organic compound–emitting paints or other finish coatings. Concrete is not easily damaged by moisture and does not support the growth of mold.
- Precast concrete wall panels with properly sealed joints have low permeability to air leakage, reducing building heating and cooling costs and contributing to good indoor air quality.
- Precast concrete wall panels can be reused when buildings are altered.

Considerations of Sustainability Relating to Glass

Glass Production

- The major raw materials for glass—sand, limestone, and sodium carbonate—are finite but abundant minerals.
- The high embodied energy of glass manufactured using traditional methods, roughly 7000 BTU per pound (16 MJ/kg), can be reduced by as much as 30 to 65 percent as new, more energy efficient manufacturing technologies are introduced.
- Some glass production involves the generation of potentially unhealthful or pollution-causing waste materials. Traditional mirror glass manufacturing, for example, generates an acidic waste effluent with high concentrations of copper or lead. However, recently, mirror glass manufactured with more environmentally friendly production techniques has become available.
- Although glass bottles and containers are recycled into new containers at a high rate, there is little recycling of flat glass at the present time. Most old glass goes to landfills.
- Efforts are underway to find new uses for waste glass. For example, vitrified glass aggregate (glass that has been melted and rapidly quenched to trap heavy metals and other contaminants) can be reused in asphalt, concrete, construction backfill, roofing shingles, and ceramic tiles.

Uses of Glass

- If it is not broken by accident or improper installation, glass lasts for a very long time with little degradation of quality, often much longer than most other building components.
- Glass is inert and does not affect indoor air quality. It is easily kept clean and free of molds and bacteria.
- The impact of glass on energy consumption can be very detrimental, very beneficial, or anything in between, depending on how intelligently it is used.
- If badly used, glass can contribute to summertime overheating from unwanted solar gain, excessive wintertime heat losses due to inherently low R-values, visual glare, wintertime discomfort caused by radiant heat loss from the body to cold glass surfaces, and condensation of moisture that can damage other building components.
- Well used, glass can bring solar heat into a building in winter and exclude it in summer, with attendant savings in heating and cooling energy. It can bring daylight into a building without glare, reducing both the use of electricity for lighting and the cooling load produced by that lighting.
- These benefits accrue over the entire life of the building, and the payoffs can be huge. Thus, glass is a key component of every energy-efficient building and a chief accomplice of the ill-informed designer in most energy-wasting buildings.

Considerations of Sustainability in Aluminum Cladding

Manufacture

- The ore from which aluminum is refined, bauxite, is finite but relatively plentiful. The richest deposits are generally found in tropical areas, often where rain forests must be clearcut to facilitate mining operations.
- Aluminum is refined from bauxite by an electrolytic process that uses huge quantities of electricity. Aluminum smelters are often located near plentiful supplies of inexpensive hydroelectric power for this reason.
- The embodied energy in aluminum is roughly 100,000 BTU per pound (230 MJ/kg), seven times that of steel, making it one of the most energy-intensive materials used in construction.
- Large volumes of water are required for smelting. Wastewater from aluminum manufacture contains cyanide, antimony, nickel, fluorides, and other pollutants.
- Aluminum is recycled at a very high rate, due largely to industry efforts. Recycled aluminum is produced using only a fraction of the energy, approximately 5000 BTU per pound (12 MJ/kg), required to convert ore to aluminum.
- Aluminum extrusions are easy to produce and to form into cladding components. Their light weight saves transportation energy.
- Powder coatings for aluminum, which release no solvents into the atmosphere, are preferable environmentally to solvent-based coatings.

Construction

- Aluminum cladding is easy to erect because of its light weight and simple connections. Little waste or pollution is associated with the process. Scrap is readily recycled.

In Service

- Aluminum cladding seldom needs maintenance, lasts for a very long time, and can be recycled when a building is demolished.
- Because aluminum is highly conductive of heat, cladding components must be thermally broken.
- Aluminum foils used as vapor retarders, components of insulation systems, and radiant heat barriers save large amounts of heating and cooling energy. They are so thin that they consume little metal relative to the energy they can save over the lifetime of the building.

Considerations of Sustainability in Gypsum Products

Sources of Gypsum

• Naturally occurring gypsum is not renewable, but it is plentiful and widely distributed geographically.

• The majority of newly extracted gypsum is quarried in surface mines, with attendant risks of loss of wildlife habitat, surface erosion, and water pollution, as well as the problem of disposing of overburden and mine tailings.

• There is increasing use of *synthetic gypsum*, material recovered from power plant flue gases that would otherwise be sent to landfills, in the manufacture of gypsum construction materials. According to the Gypsum Association, approximately 1.5 million tons (1.4 million metric tons) of synthetic gypsum is used annually to produce about 7 percent of the U.S. construction industry's calcined gypsum. Some synthetic gypsums, however, contain toxic byproducts from the manufacturing processes in which they are produced and cannot be safely recycled into new construction materials.

Gypsum Products Manufacturing

• The calcining of gypsum involves temperatures that are not much higher than the boiling point of water, which means that the embodied energy of gypsum is relatively low, about 1200 BTU per pound (2.8 MJ/kg) for plaster and 2600 BTU per pound (6.0 MJ/kg) for gypsum board.

• The calcining process emits particulates of calcium sulfate, an inert, benign chemical, as dust.

• The paper faces of gypsum board are composed primarily of recycled newspapers.

• Some manufacturers produce gypsum board products made with as much as 95 percent recycled materials, including synthetic gypsum and recycled postconsumer waste paper.

Gypsum Products on the Building Site

• Approximately 15 million tons (14 million metric tons) of gypsum board are manufactured annually in the United States. On a typical construction site, about 10 to 12 percent of this material becomes waste.

• Gypsum board waste generated during construction can be minimized by sizing walls and ceilings to make efficient use of whole boards or by ordering custom-sized boards for nonstandard-size surfaces.

• Gypsum board scrap can be permanently stored in the hollow cavities of finished walls, eliminating disposal and transportation costs and reducing the amount of material destined for landfills (though care must be taken not to create interference with the pulling of electrical wires at a later date).

• Some dust is generated by the cutting and sanding of gypsum board and plaster. This dust has not been tied to any specific illnesses, but it is a nuisance and a source of discomfort until the work is done and all the dust has been swept up and removed from the building. Remodeling and demolition also create large quantities of gypsum dust.

• Most installed gypsum products have extremely low emissions. Some joint compounds, however, may also be sources of emissions.

• Additives used in the manufacture of moisture-resistant and fire-resistant gypsum board are potential sources of volatile organic compound (VOC) emissions.

• Paints, wallcovering adhesives, and other products used to finish gypsum surfaces can be significant emitters of VOCs, and thus require care in selection and specification.

Gypsum Disposal and Recycling

• Gypsum board waste can be recycled back into the manufacture of new gypsum board products. Current efforts limit recycled content to no more than 15 or 20 percent, due to the amount of paper waste that can be safely introduced into the new gypsum without impairing its fire resistance.

• Gypsum board waste from the demolition of older buildings may be contaminated with nails, drywall tape, joint compound, and paint. Gypsum board demolished from buildings constructed prior to 1978 may be coated with lead-based paint. These foreign materials must be removed from the waste; their presence may limit the material's recycling potential.

• Gypsum board waste can be used as a soil amendment and plant nutrient. With the recent advent of mobile grinders, construction site recycling of gypsum board waste for use as a soil amendment on the same building site is now feasible.

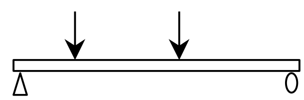
• Gypsum is an ingredient in many manufacturing and industrial processes. Studies and small-scale tests currently underway to identify potential uses of gypsum board waste in such processes are likely to lead to additional recycling opportunities in the future.

Beam Structures and Internal Forces

Notation:

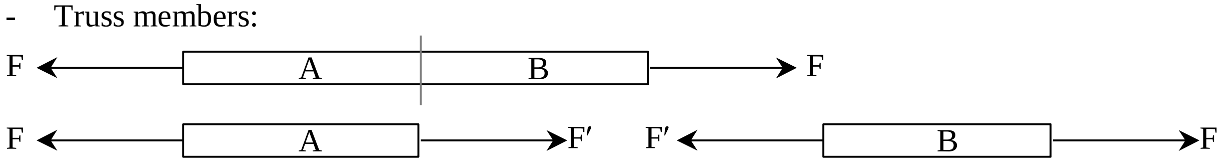
a = algebraic quantity, as is *b, c, d*
A = name for area
b = intercept of a straight line
d = calculus symbol for differentiation
(C) = shorthand for *compression*
F = name for force vectors, as is *P, F', P'*
= internal axial force
F_x = force component in the x direction
F_y = force component in the y direction
FBD = free body diagram
L = beam span length
m = slope of a straight line
M = internal bending moment
M(x) = internal bending moment as a function of distance *x*
R = name for reaction force vector
(T) = shorthand for *tension*
V = internal shear force
V(x) = internal shear force as a function of distance *x*
w = name for distributed load
W = name for total force due to distributed load
x = horizontal distance
y = vertical distance
^{o} = symbol for order of curve
$\int$ = symbol for integration
Δ = calculus symbol for small quantity
Σ = summation symbol

- BEAMS
  - Important type of structural members (floors, bridges, roofs)
  - Usually long, straight and rectangular
  - Have loads that are usually perpendicular applied at points along the length



Internal Forces 2

- *Internal forces* are those that hold the parts of the member together for equilibrium
  - Truss members:

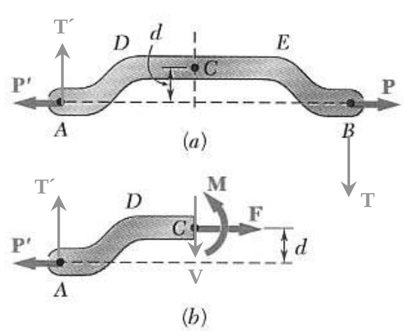


  - For any member:

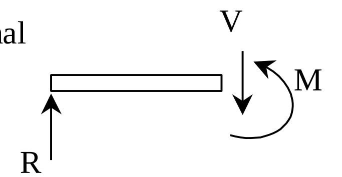
F = internal *axial force*
(perpendicular to cut across section)

V = internal *shear force*
(parallel to cut across section)

M = internal *bending moment*



Support Conditions & Loading



- Most often loads are perpendicular to the beam and cause <u>only</u> internal shear forces and bending moments

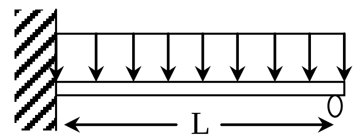
- Knowing the internal forces and moments is *necessary* when designing beam size & shape to resist those loads

- Types of loads

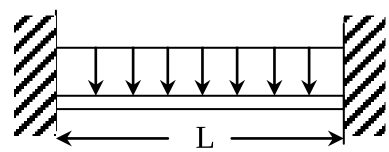
  - Concentrated – single load, single moment
  - Distributed – loading spread over a distance, uniform or **non-uniform**.

- Types of supports

  - *Statically determinate*: simply supported, cantilever, overhang (number of unknowns < number of equilibrium equations)
  - *Statically indeterminate*: continuous, fixed-roller, fixed-fixed (number of unknowns < number of equilibrium equations)

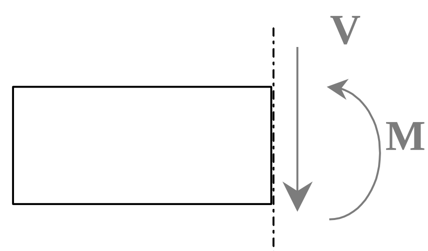


Propped



Restrained

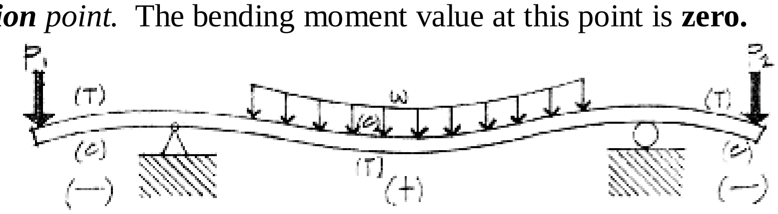
Sign Conventions for Internal Shear and Bending Moment
(<u>different</u> from statics and truss members!)



When ΣF_y **excluding V** on the left hand side (LHS) section is <u>positive</u>, V will direct <u>down</u> and is considered <u>POSITIVE.</u>

When ΣM **excluding M** about the cut on the left hand side (LHS) section causes a smile which could hold water (curl upward), M will be <u>counter clockwise</u> (+) and is considered <u>POSITIVE.</u>

On the deflected shape of a beam, the point where the shape changes from smile up to frown is called the ***inflection** point.* The bending moment value at this point is **zero.**



Shear And Bending Moment Diagrams

The plot of shear and bending moment as they vary across a beam length are *extremely important design tools:* V(x) is plotted on the y axis of the shear diagram, M(x) is plotted on the y axis of the moment diagram.

The *load* diagram is essentially the free body diagram of the beam *with the actual loading (not the equivalent of distributed loads.)*

Maximum Shear and Bending – The maximum *value,* regardless of sign, is important for design.

Method 1: *The Equilibrium Method*

Isolate FDB sections at significant points along the beam and determine V and M at the cut section. The values for V and M can also be written in equation format as functions of the distance to the cut section.

Important Places for FBD cuts
- at supports
- at concentrated loads
- at start and end of distributed loads
- at concentrated moments

Method 2: *The Semigraphical Method*

Relationships exist between the loading and shear diagrams, and between the shear and bending diagrams.

Knowing the *area* of the loading gives the *change in* ***shear (V)***.

Knowing the *area* of the shear gives the *change in* ***bending moment (M)***.

Concentrated loads and moments cause a vertical *jump* in the diagram.

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = \frac{dV}{dx} = -w$$ (the negative shows it is down because we give *w* a positive value)

$$V_D - V_C = -\int_{x_C}^{x_D} w\,dx = \text{the } \textbf{area} \text{ under the load curve between C \& D}$$

*<u>*These shear formulas are NOT VALID at discontinuities like concentrated loads*</u>

$$\frac{\Delta M}{\Delta x}\Big|_{\lim \to 0} = \frac{dM}{dx} = V$$

$$M_D - M_C = \int_{x_C}^{x_D} V dx =$$ the **area** under the shear curve between C & D

** These moment formulas ARE VALID even with concentrated loads.*

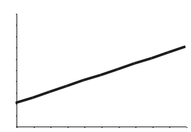
**These moment formulas are NOT VALID at discontinuities like applied moments.*

The MAXIMUM BENDING MOMENT from a curve that is continuous can be found when the slope is zero $\left(\frac{dM}{dx} = 0\right)$, which is when the value of the shear is 0.

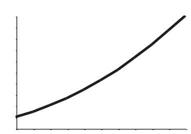
Basic Curve Relationships (from calculus) for y(x)

Horizontal Line: $y = b$ *(constant)* and the area (change in shear) = $b \cdot x$, resulting in a:

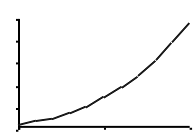
Sloped Line: $y = mx+b$ and the area (change in shear) = $\frac{\Delta y \cdot \Delta x}{2}$, resulting in a:



Parabolic Curve: $y = ax^2 + b$ and the area (change in shear) = $\frac{\Delta y \cdot \Delta x}{3}$, resulting in a:

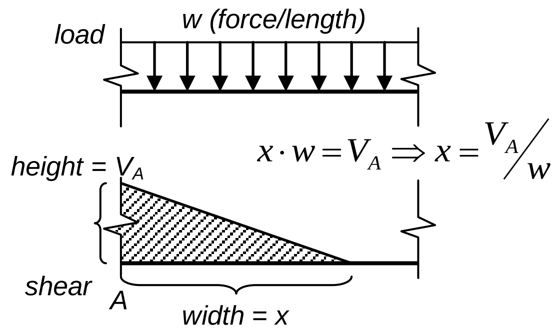


3rd Degree Curve: $y = ax^3 + bx^2 + cx + d$



Free Software Site: http://www.rekenwonder.com/atlas.htm

BASIC PROCEDURE:



1. Find all support forces.

V diagram:

2. At free ends and at simply supported ends, the shear will have a zero value.
3. At the left support, the shear will equal the reaction force.
4. The shear will not change in x until there is another load, where the shear is reduced if the load is negative. If there is a distributed load, the change in shear is the area under the loading.
5. At the right support, the reaction is treated just like the loads of step 4.
6. At the free end, the shear should go to zero.

M diagram:

7. At free ends and at simply supported ends, the moment will have a zero value.
8. At the left support, the moment will equal the reaction moment (if there is one).
9. The moment will not change in x until there is another load or applied moment, where the moment is reduced if the applied moment is negative. If there is a value for shear on the V diagram, the change in moment is the area under the shear diagram.

   For a triangle in the shear diagram, the width will equal the height ÷ w!

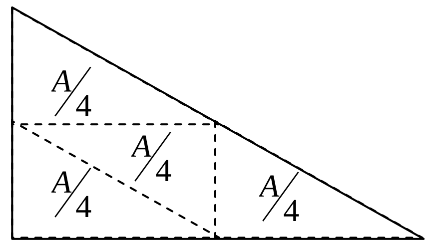
10. At the right support, the moment reaction is treated just like the moments of step 9.
11. At the free end, the moment should go to zero.

Parabolic Curve Shapes Based on Triangle Orientation

In order to tell if a parabola curves "up" or "down" from a triangular area in the preceding diagram, the orientation of the triangle is used as a reference.

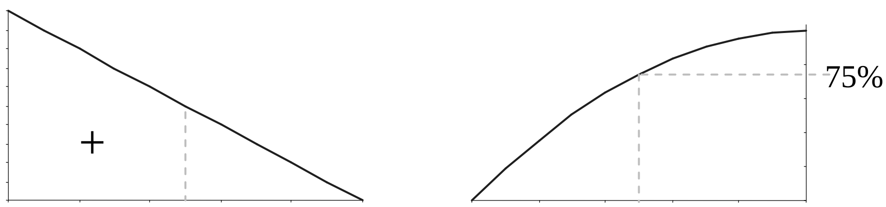
<u>Geometry of Right Triangles</u>

Similar triangles show that four triangles, each with ¼ the area of the large triangle, fit within the large triangle. This means that ¾ of the area is on one side of the triangle, if a line is drawn though the middle of the base, and ¼ of the area is on the other side.

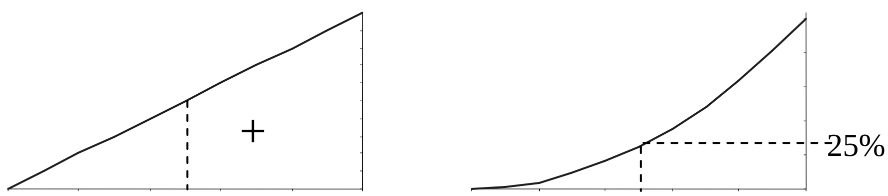


By how a triangle is oriented, we can determine the curve shape in the next diagram.

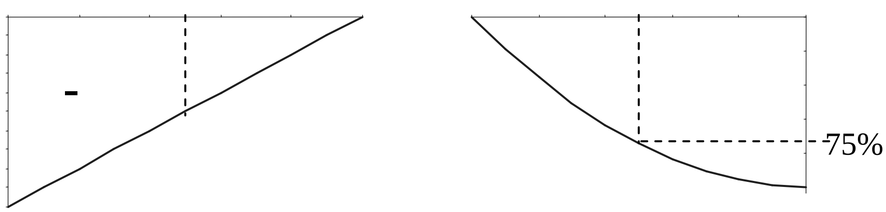
<u>CASE 1</u>: *Positive* triangle with fat side to the *left*.



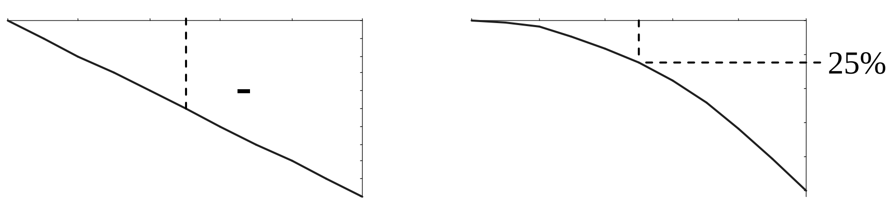
<u>CASE 2</u>: *Positive* triangle with fat side to the *right*.



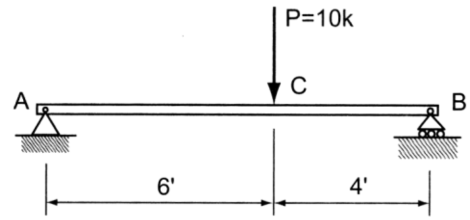
<u>CASE 3</u>: *Negative* triangle with fat side to the *left*.



<u>CASE 4</u>: *Negative* triangle with fat side to the *right*.

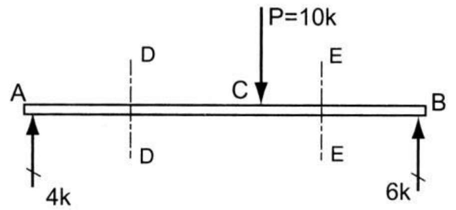


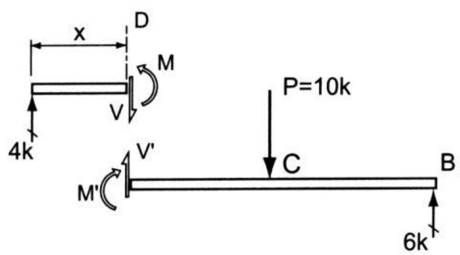
Example 1 (pg 273)

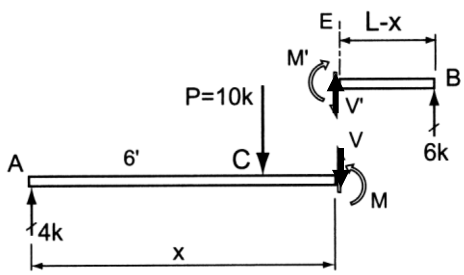


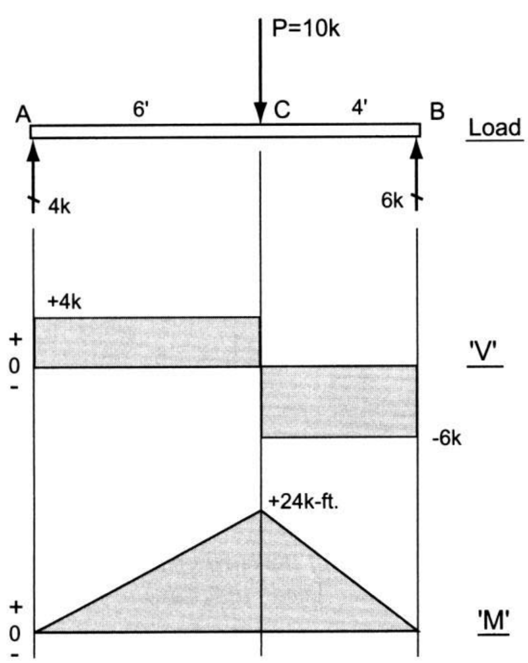
Example Problem 8.1 (Equilibrium Method)

Draw the shear and moment diagram for a simply supported beam with a single concentrated load (Figure 8.8), using the equilibrium method. Verify the general equation from Beam Diagrams & Formulas.





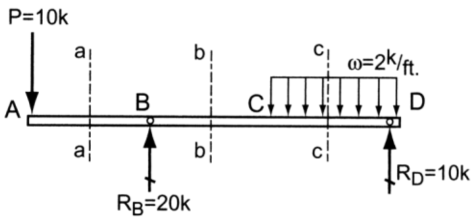


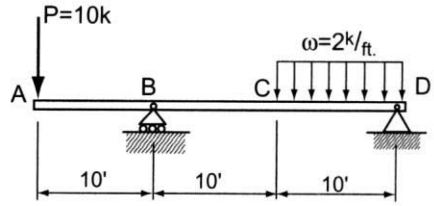


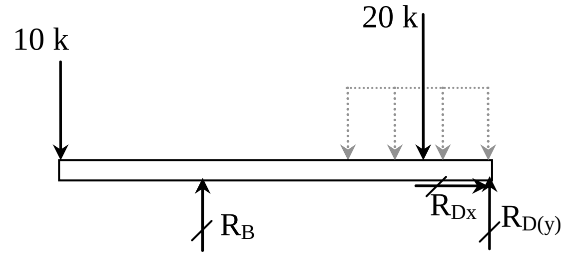
Example 2 (pg 275)

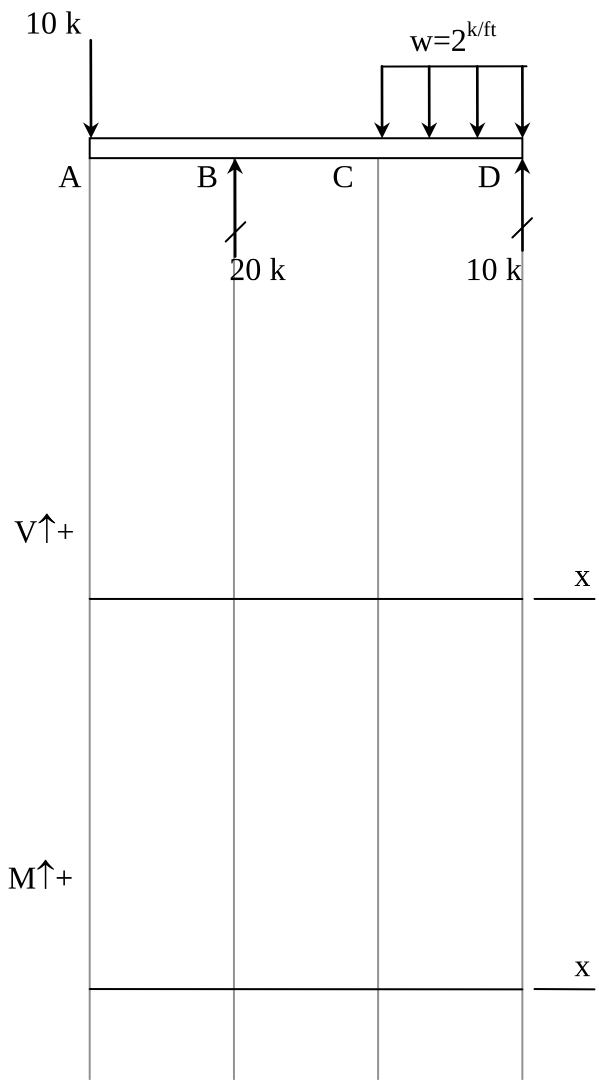
Example Problem 8.2 (Equilibrium Method)

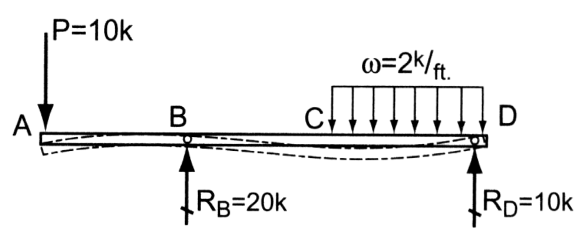
Draw V and M diagrams for an overhang beam (Figure 8.12) loaded as shown. Determine the critical V_{max} and M_{max} locations and magnitudes.







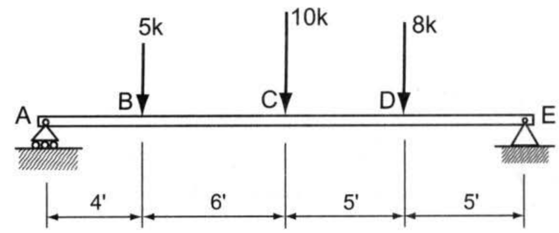


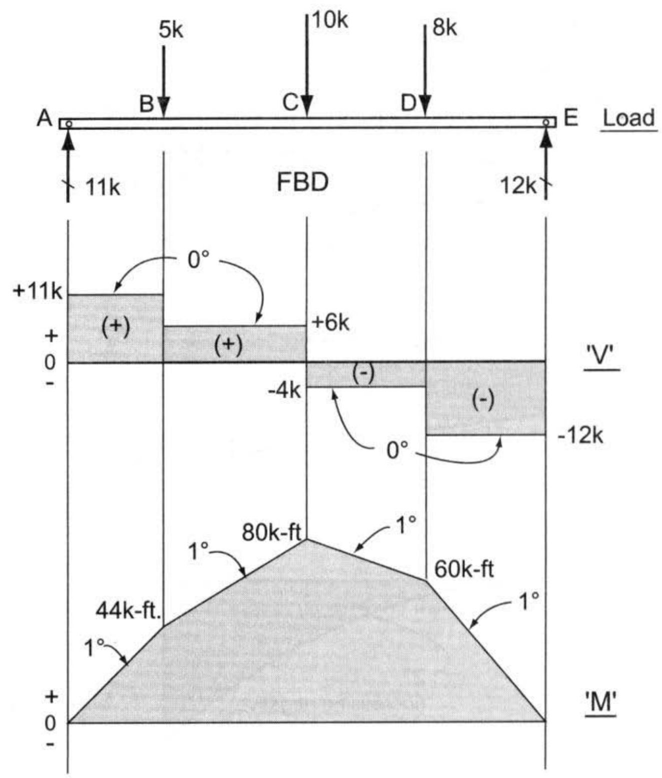


Example 3 (pg 283)

Example Problem 8.4

Construct the V and M diagrams for the girder that supports three concentrated loads as shown in Figure 8.28.

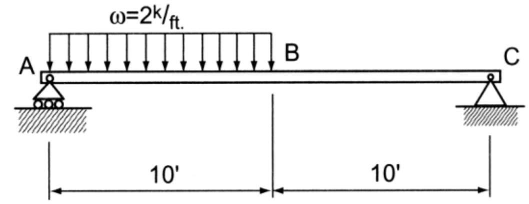


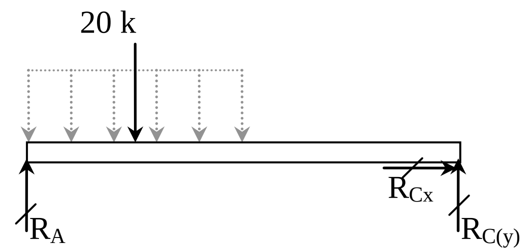


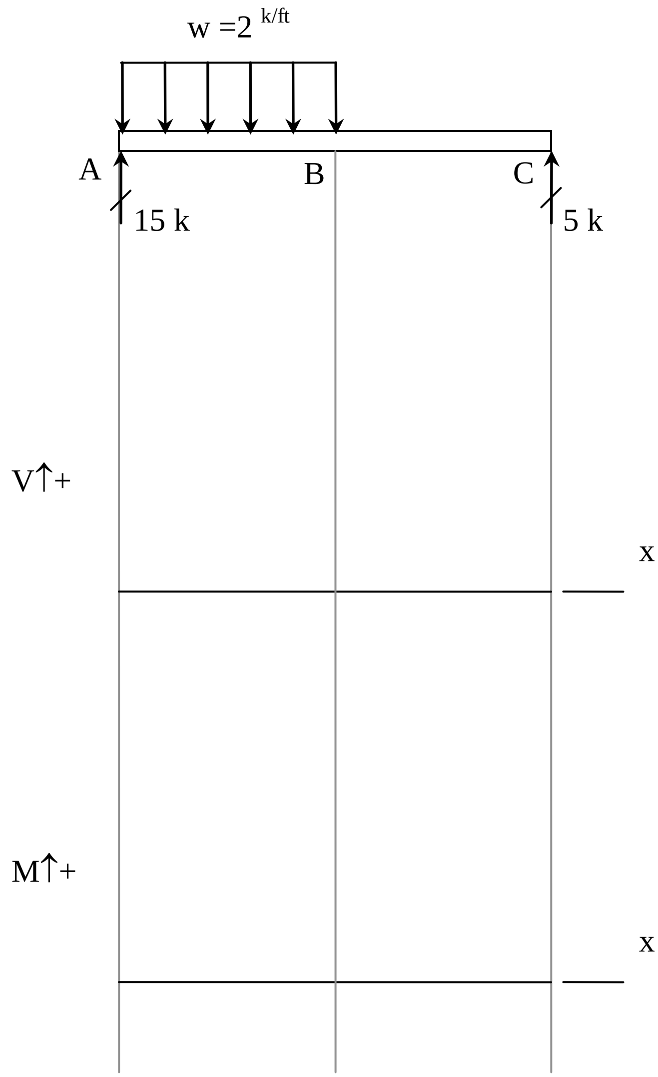
Example 4 (pg 285)

Example Problem 8.6 (Semi-Graphical Method)

Construct *V* and *M* diagrams for the simply supported beam *ABC*, which is subjected to a partial uniform load (Figure 8.30).



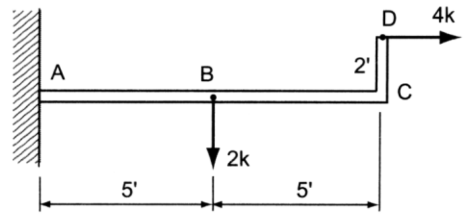


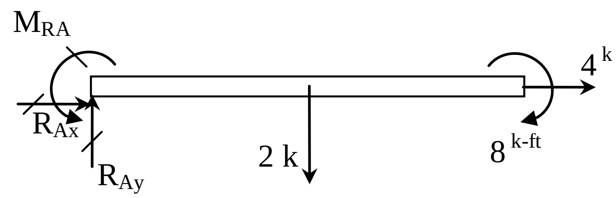


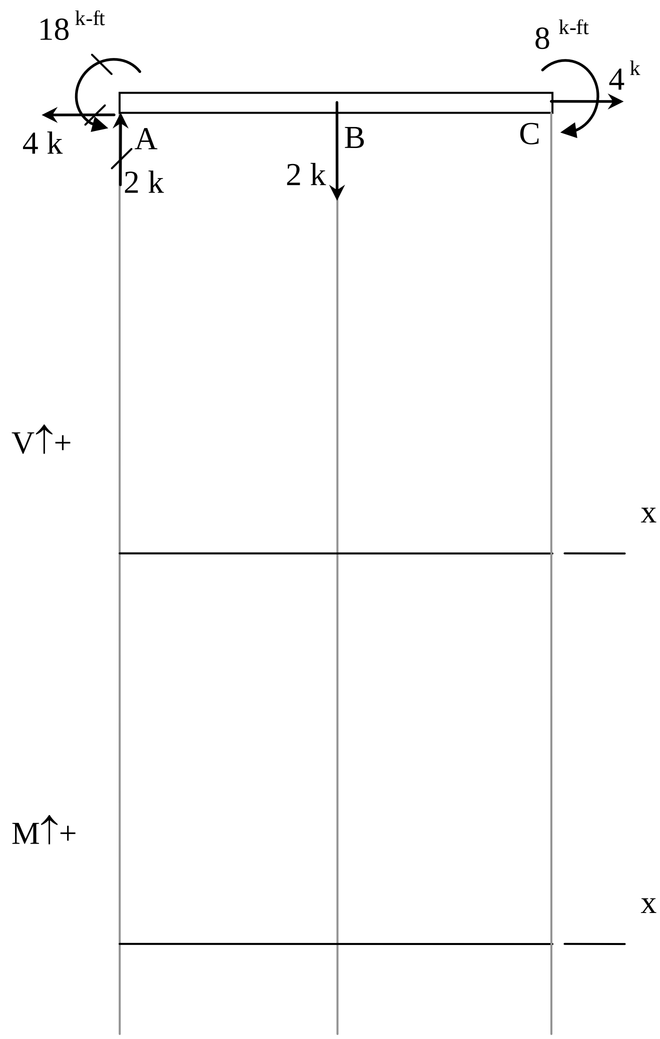
Example 5 (pg 286)

Example Problem 8.7 (Figure 8.31)

For a cantilever beam with an upturned end, draw the load, shear, and moment diagrams.



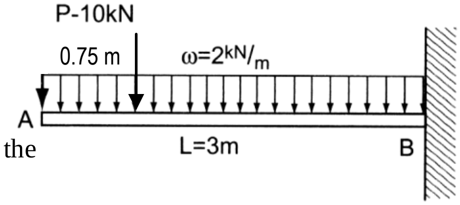




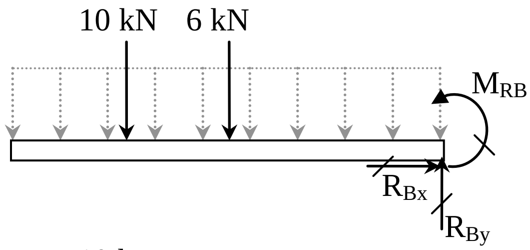
Example 6 (*changed from* pg 284)

Example Problem 8.5 (Semi-Graphical Method)

A cantilever beam supports a uniform load of $\omega = 2\,{}^{kN}/_{m}$ over its entire span, plus a concentrated load of 10 kN at 0.75 m from the free end. Construct the V and M diagrams (Figure 8.29).



SOLUTION:
Determine the reactions:

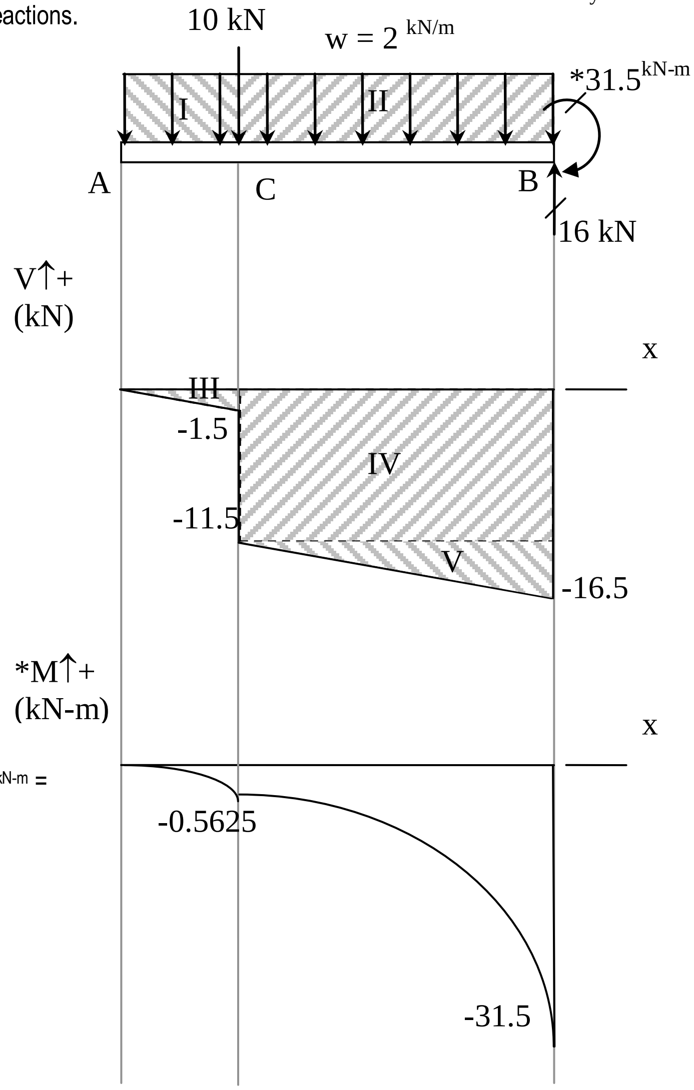


$\sum F_x = R_{Bx} = 0$ R_{Bx} = 0 kN

$\sum F_y = -10kN - (2\,{}^{kN}/_{m})(3m) + R_{By} = 0$ R_{By} = 16 kN

$\sum M_B = (10kN)(2.25m) + (6kN)(1.5m) + M_{RB} = 0$ M_{RB} = -31.5 kN-m

Draw the load diagram *with the distributed load* as given with the reactions.



Shear Diagram:

Label the load areas and calculate:

Area I = (-2 kN/m)(0.75 m) = -1.5 kN
Area II = (-2 kN/m)(2.25 m) = -4.5 kN

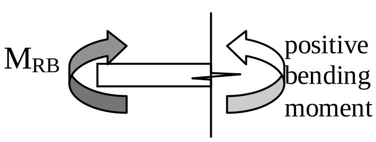
V_A = 0
V_C = V_A + Area I = 0 -1.5 kN = -1.5 kN *and*
V_C = V_C + force at C = -1.5 kN -10 kN= -11.5 kN
V_B = V_C + Area II = -11.5 kN -4.5 kN = -16 kN *and*
V_B = V_B + force at B = -16 kN +16 kN= 0 kN

Bending Moment Diagram:

Label the load areas and calculate:

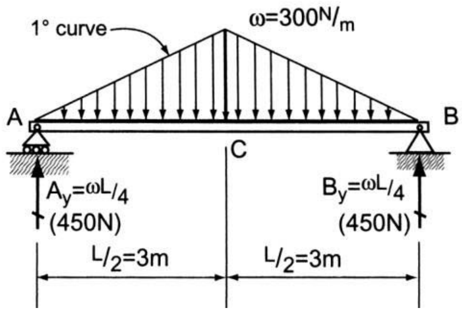
Area III = (-1.5 kN)(0.75 m)/2 = -0.5625 kN-m
Area IV = (-11.5 kN)(2.25m) = -25.875 kN-m
Area V = (-16 – 11.5 kN)(2.25m)/2 = -5.0625 kN-m

M_A = 0
M_C = M_A + Area III = 0 - 0.5625 kN-m = - 0.5625 kN-m
M_B = M_C + Area IV + Area V = - 0.5625 kN-m - 25.875 kN-m - 5.0625 kN-m =
= -31.5 kN-m *and*
M_B = M_B + moment at B = -31.5 kN-m + 31.5 kN-m = 0 kN-m



Example 7 (pg 287)

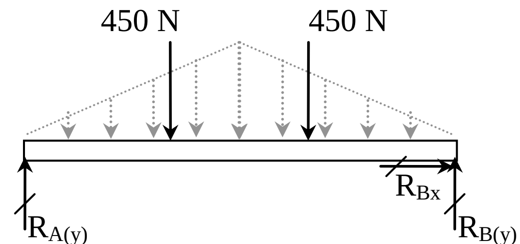
Example Problem 8.9 (Figure 8.33)



A header beam spanning a large opening in an industrial building supports a triangular load as shown. Construct the *V* and *M* diagrams and label the peak values.

SOLUTION:

Determine the reactions:



$\sum F_x = R_{Bx} = 0$ R_{Bx} = 0 kN

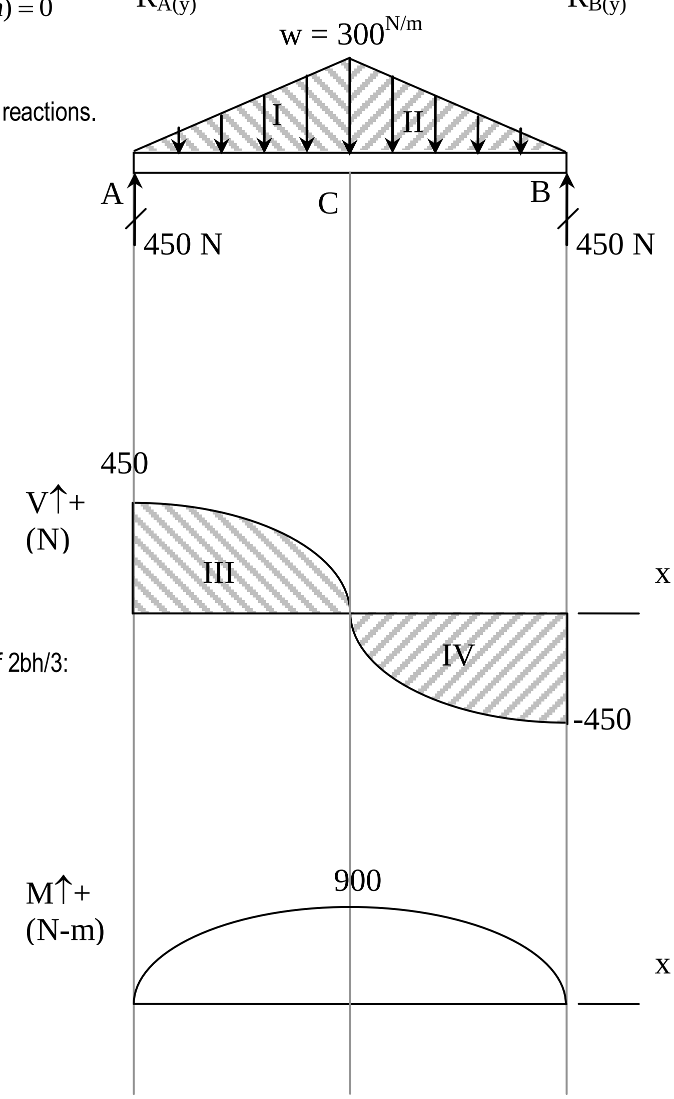
$\sum F_y = R_{Ay} - (300\,{}^{N}\!/_{m})(3m)\,{}^{1}\!/_{2} + -(300\,{}^{N}\!/_{m})(3m)\,{}^{1}\!/_{2} + R_{By} = 0$

or by load tracing R_{Ay} & R_{By} = (wL/2)/2 = (300 N/m)(6 m)/4 = 450 N

$\sum M_A = -(450N)({}^{2}\!/_{3} \times 3m) - (450N)(3 + {}^{1}\!/_{3} \times 3m) + R_{By}(6m) = 0$

R_{By} = 450 N

Draw the load diagram *with the distributed load* as given with the reactions.



Shear Diagram:

Label the load areas and calculate:

Area I = (-300 N/m)(3 m)/2 = -450 N
Area II = -300 N/m)(3 m)/2 = -450 N

V_A = 0 *and* V_A = V_A + force at A = 0 + 450 N = 450 N
V_C = V_A + Area I = 450 N -450 N = 0 N
V_B = V_C + Area II = 0 N – 450 N = -450 N *and*
V_B = V_B + force at B = -450 N + 450 N = 0 N

Bending Moment Diagram:

Label the load areas and calculate:

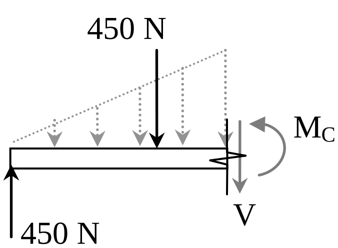
Areas III & IV happen to be parabolic segments with an area of 2bh/3:
Area III = 2(3 m)(450 N)/3 = 900 N-m
Area IV = -2(3 m)(450 N)/3 = -900 N-m

M_A = 0
M_C = M_A + Area III = 0 + 900 N-m = 900 N-m
M_B = M_C + Area IV = 900 N-m - 900 N-m = 0

We can prove that the area is a parabolic segment by using the equilibrium method at C:

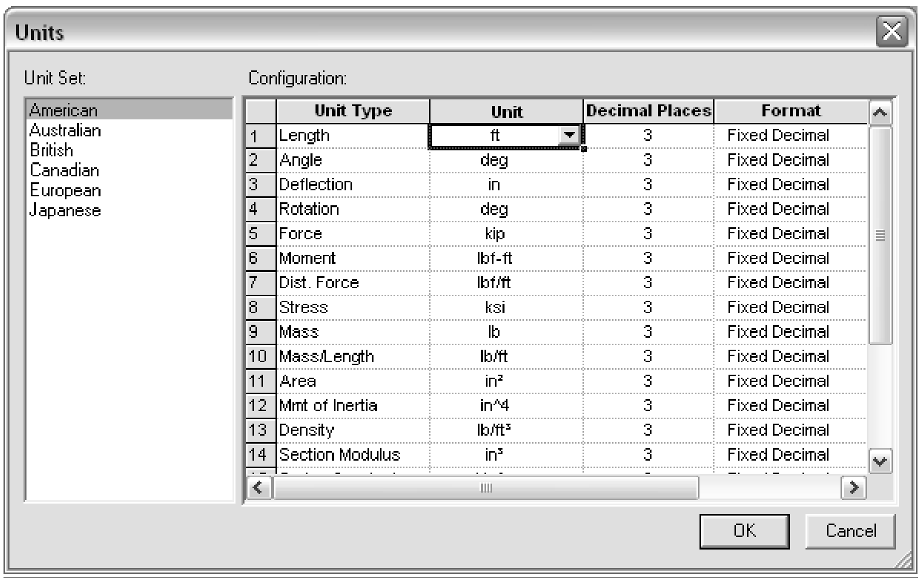
$\sum M_{section\,cut} = M_C - (450N)(3m) + (450N)({}^{1}\!/_{3} \times 3m) = 0$

so M_c = 900 N-m

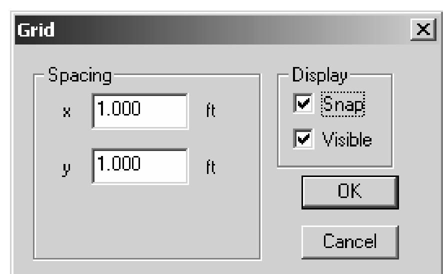


Beam Analysis Using Multiframe

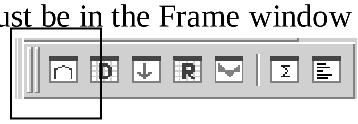
1. The software is on the computers in the College of Architecture in Programs under the Windows Start menu (see https://wikis.arch.tamu.edu/display/HELPDESK/Computer+Accounts for lab locations). Multiframe is under the Bentley Engineering menu.
2. There available on line at http://www.formsys.com/mflearning that list the tasks and order in greater detail. The first task is to define the unit system:
   - Choose Units… from the View menu. Unit sets are available, but specific units can also be selected by double clicking on a unit or format and making a selection from the menu.



3. To see the scale of the geometry, a grid option is available:
   - Choose Grid… from the View menu



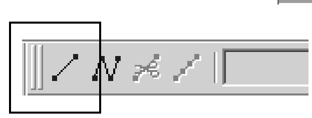
4. To create the geometry, you must be in the Frame window (default). The symbol is the frame in the window toolbar:



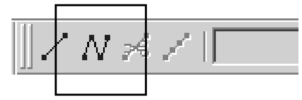
   The Member toolbar shows ways to create members:

   The Generate toolbar has convenient tools to create typical structural shapes.

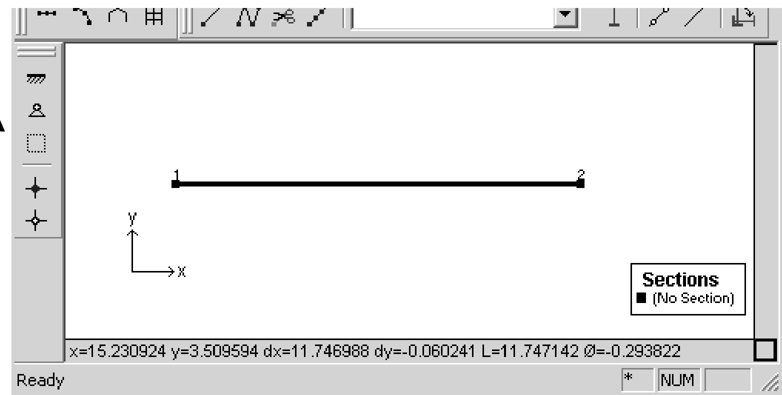
   - To create a beam with supports at one or both ends, use the add member button:



- Select a starting point and ending point with the cursor. The location of the cursor and the segment length is displayed at the bottom of the geometry window.
- To create a beam with supports NOT at the ends, use the add connected members button to create segments between supports and ends

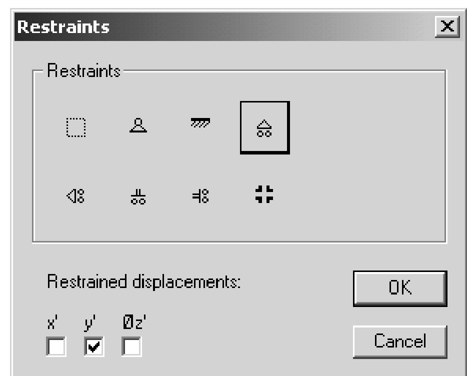


- Select a starting point and ending point with the cursor. The location of the cursor and the segment length is displayed at the bottom of the geometry window. The ESC button will end the segmented drawing.
- The geometry can be set precisely by selecting the beam member, bringing up the specific menu (right click), choosing Member Properties to set the length.



- The support types can be set by selecting the joint (drag) and using the Joint Toolbar (pin shown), or the Frame / Joint Restraint ... menu (right click).

  NOTE: If the support appears at both ends of the beam, you had the beam selected rather than the joint. Select the joint to change the support for and right click to select the joint restraints menu or select the correct support on the joint toolbar.



The support forces will be determined in the analysis.

5. All members must have sections assigned (see section 6.) in order to calculate reactions and deflections. To use a standard steel section **proceed to step 6.** For custom sections, the section information must be entered. To define a section:

- Choose Edit Sections / Add Section… from the Edit menu
- Type a name for your new section
- Choose group <u>Frame</u> from the group names provided so that the section will remain with the file data
- Choose a shape. The Flat Bar shape is a rectangular section.
- Enter the cross section data.

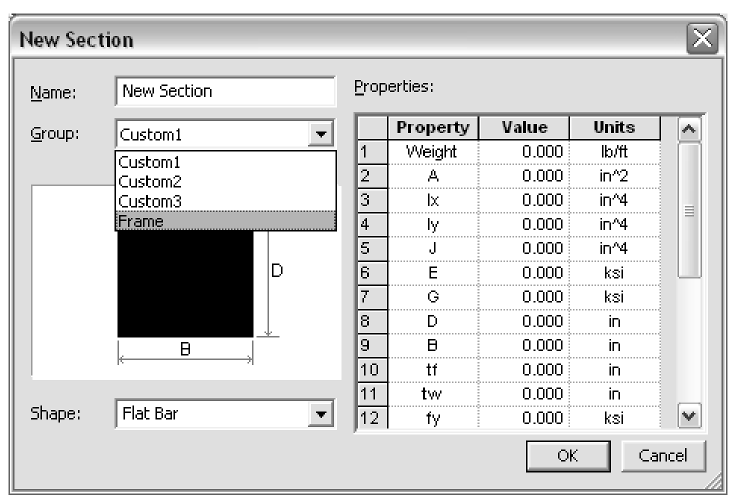
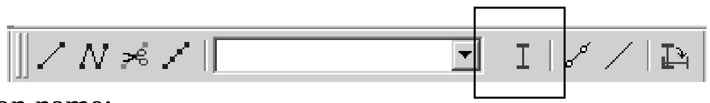


Table values 1-9 must have values for a Flat Bar, but not all are used for every analysis. A recommendation is to put the value of 1 for those properties you don't know or care about. Properties like t_f, t_w, etc. refer to wide flange sections.

- Answer any query. If the message says there is an error, the section will not be created until the error is corrected.

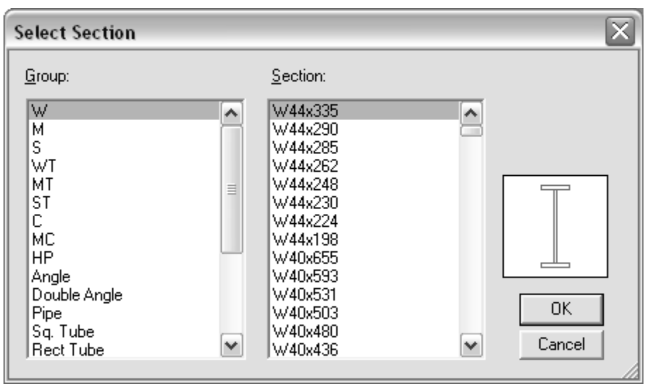
6. The standard sections library loaded is for the United States. If another section library is needed, use the Open Sections Library... command under the file menu, choose the library folder, and select the SectionsLibrary.slb file.

   Select the members (drag to make bold) and assign sections with the Section button on the Member toolbar:

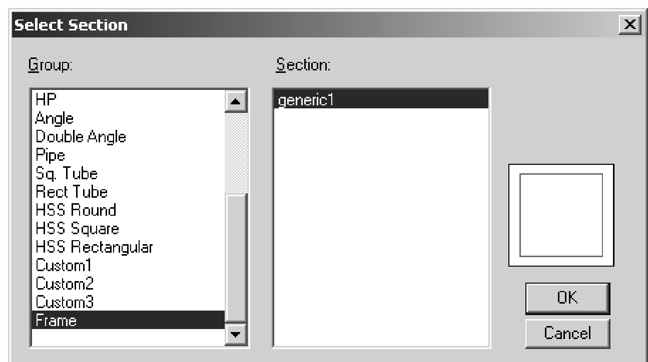


- Choose the group name and section name:

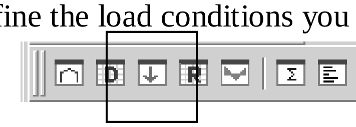
(STANDARD SHAPES)



(CUSTOM)

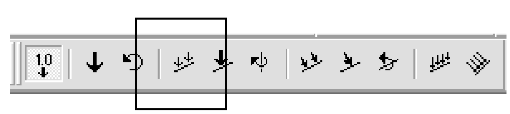


7. The beam geometry is complete, and in order to define the load conditions you must be in the Load window represented by the green arrow:

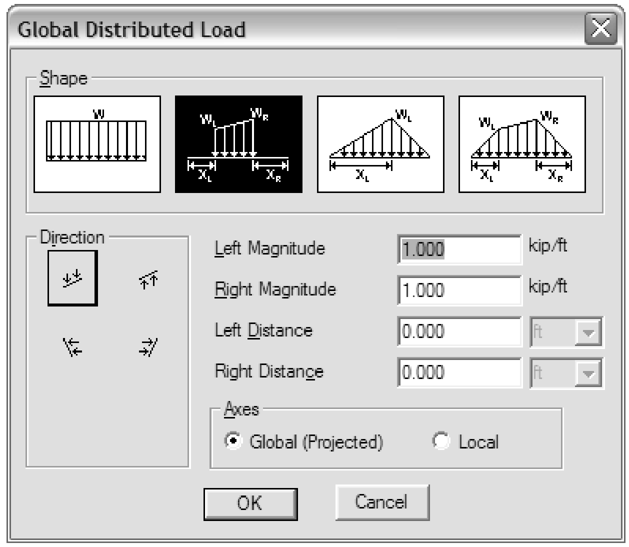


8. The Load toolbar allows a joint to be loaded with a force or a moment in global coordinates, shown by the first two buttons after the display numbers button. It allows a member to be loaded with a distributed load, concentrated load or moment (next three buttons) in global coordinates, as well as loading with distributed or single force or moment in the local coordinate system (next three buttons). It allows a load panel to be loaded with a distributed load in global or local coordinates (last two buttons).

- Choose the member to be loaded (drag) and select the load type (here shown for global distributed loading):

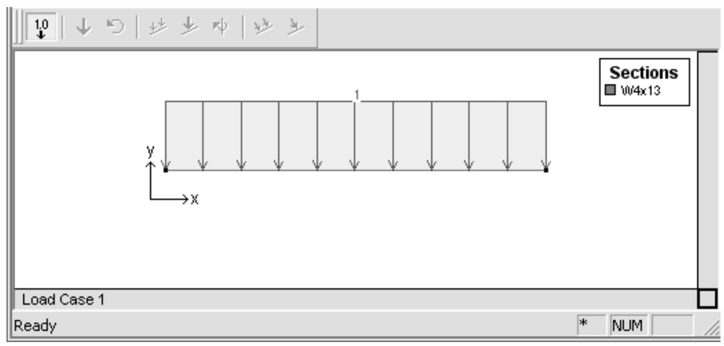


- Choose the distribution type and direction. Note that the arrow shown is the direction of the loading. There is no need to put in negative values for downward loading.



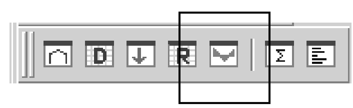
- Enter the values of the load and distances (if any). Distances can be entered as a function of the length , i.e. L/2, L/4...

NOTE: <u>Do not</u> put support reactions as applied loads. The analysis will determine the reaction values.



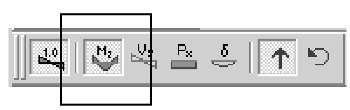
Multiframe will automatically generate a grouping called a Load Case named <u>Load Case 1</u> when a load is created. All additional loads will be added to this load case unless a new load case is defined (Add case under the Case menu).

9. In order to run the analysis after the geometry, member properties and loading has been defined:
   - Choose Linear from the Analyze menu
10. If the analysis is successful, you can view the results in the Plot window represented by the red moment diagram:

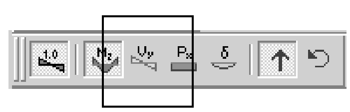


11. The Plot toolbar allows the numerical values to be shown (1.0 button), the reaction arrows to be shown (brown up arrow) and reaction moments to be shown (brown curved arrow):

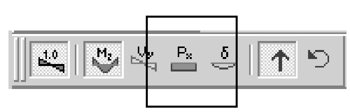
   - To show the moment diagram, Choose the red Moment button



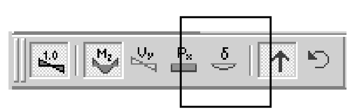
   - To show the shear diagram, Choose the green Shear button



   - To show the axial force diagram, Choose the purple Axial Force button



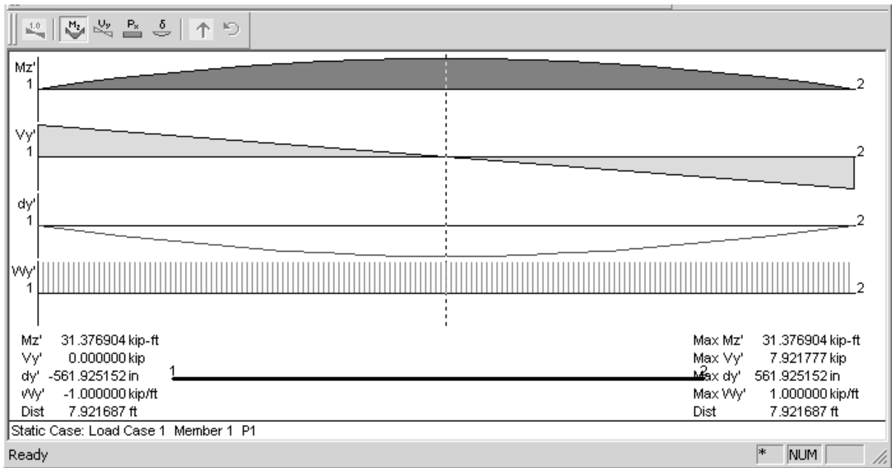
   - To show the deflection diagram, Choose the blue Deflection button



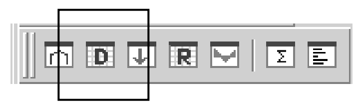
   - To animate the deflection diagram, Choose Animate... from the Display menu. You can also save the animation to a .avi file by checking the box.

- To plot the bending moment on the "top" choose Preferences from the Edit menu and under the Presentation tab Draw moments on the compression face

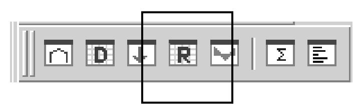
- To see exact values of shear, moment and deflection, double click on the member and move the vertical cross hair with the mouse. The ESC key will return you to the window.



12. The Data window (D) allows you to view all data "entered" for the geometry, sections and loading. These values can be edited.



13. The Results window (R) allows you to view all results of the analysis including displacements, reactions, member forces (actions) and stresses. These values can be cut and pasted into other Windows programs such as Word or Excel.



Static Case: Load Case 1

	Joint	Label	Rx' kip	Ry' kip	Mz' lbf-ft
1	1		0.000	7.922	0.000
2	2		0.000	7.922	0.000
3	Total	(Global)	Rx=0.000	Ry=15.844	

Reactions / Member Actio... — Ready

Static Case: Load Case 1

	Memb	Label	Joint	Px' kip	Vy' kip	Mz' lbf-ft
1	1		1	0.000	7.922	0.000
2	1		2	0.000	7.922	0.000

Member Actions / Max Ac... — Ready

NOTE: Px' refers to the axial load (P) in the local axis x direction (x'). Vy' refers to the shear perpendicular to the local x axis, and Mz' refers to the bending moment.

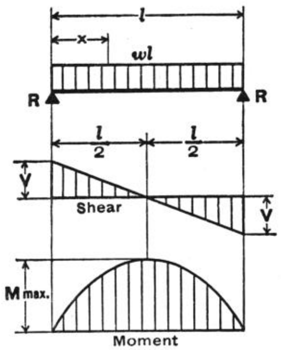
14. To save the file Choose Save from the File menu.

15. To load an existing file Choose Open... from the File menu.

16. To print a plot Choose Print Window... from the File menu. As an alternative, you may copy the plot (Ctrl+c) and paste it in a word processing document (Ctrl+v).

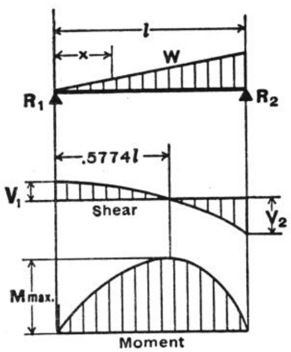
BEAM DIAGRAMS AND FORMULAS For Various Static Loading Conditions, AISC ASD 8th ed.

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



- Total Equiv. Uniform Load . . . . = wl
- $R = V$ . . . . = $\frac{wl}{2}$
- V_x . . . . = $w\left(\frac{l}{2} - x\right)$
- M max. (at center) . . . . = $\frac{wl^2}{8}$
- M_x . . . . = $\frac{wx}{2}(l - x)$
- Δmax. (at center) . . . . = $\frac{5wl^4}{384EI}$
- Δ_x . . . . = $\frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

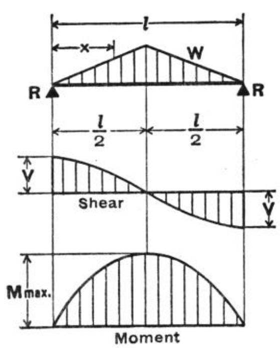
2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END



- Total Equiv. Uniform Load . . . . = $\frac{16W}{9\sqrt{3}} = 1.0264W$
- $R_1 = V_1$ . . . . = $\frac{W}{3}$
- $R_2 = V_2$ max. . . . . = $\frac{2W}{3}$
- V_x . . . . = $\frac{W}{3} - \frac{Wx^2}{l^2}$
- M max. $\left(\text{at } x = \frac{l}{\sqrt{3}} = .5774l\right)$ . . = $\frac{2Wl}{9\sqrt{3}} = .1283\,Wl$
- M_x . . . . = $\frac{Wx}{3l^2}(l^2 - x^2)$
- Δmax. $\left(\text{at } x = l\sqrt{1 - \sqrt{\frac{8}{15}}} = .5193l\right)$ = $.01304\,\frac{Wl^3}{EI}$
- Δ_x . . . . = $\frac{Wx}{180EIl^2}(3x^4 - 10l^2x^2 + 7l^4)$

$W = \frac{wl}{2}$

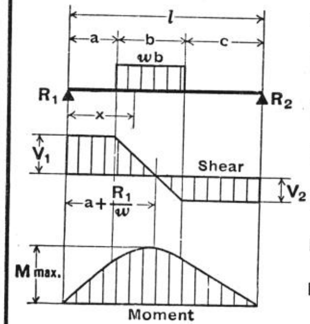
3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER



- Total Equiv. Uniform Load . . . . = $\frac{4W}{3}$
- $R = V$ . . . . = $\frac{W}{2}$
- V_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . = $\frac{W}{2l^2}(l^2 - 4x^2)$
- M max. (at center) . . . . = $\frac{Wl}{6}$
- M_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . = $Wx\left(\frac{1}{2} - \frac{2x^2}{3l^2}\right)$
- Δmax. (at center) . . . . = $\frac{Wl^3}{60EI}$
- Δ_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . = $\frac{Wx}{480EIl^2}(5l^2 - 4x^2)^2$

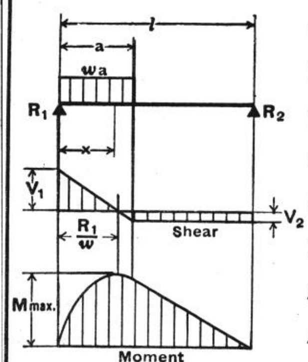
$W = \frac{wl}{2}$

4. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED



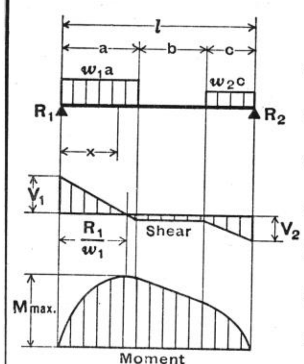
- $R_1 = V_1$ (max. when $a < c$) . . = $\frac{wb}{2l}(2c + b)$
- $R_2 = V_2$ (max. when $a > c$) . . = $\frac{wb}{2l}(2a + b)$
- V_x (when $x > a$ and $< (a + b)$) . = $R_1 - w(x - a)$
- M max. $\left(\text{at } x = a + \frac{R_1}{w}\right)$ . . . . = $R_1\left(a + \frac{R_1}{2w}\right)$
- M_x (when $x < a$) . . . . . = R_1x
- M_x (when $x > a$ and $< (a + b)$) . = $R_1x - \frac{w}{2}(x - a)^2$
- M_x (when $x > (a + b)$) . . . . = $R_2(l - x)$

5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END



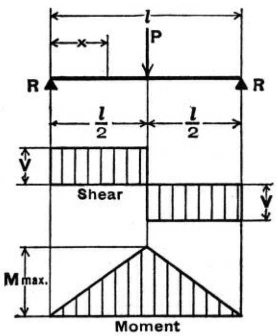
- $R_1 = V_1$ max. . . . . = $\frac{wa}{2l}(2l - a)$
- $R_2 = V_2$ . . . . = $\frac{wa^2}{2l}$
- V_x (when $x < a$) . . . . . = $R_1 - wx$
- M max. $\left(\text{at } x = \frac{R_1}{w}\right)$ . . . . . = $\frac{R_1^2}{2w}$
- M_x (when $x < a$) . . . . . = $R_1x - \frac{wx^2}{2}$
- M_x (when $x > a$) . . . . . = $R_2(l - x)$
- Δ_x (when $x < a$) . . . . . = $\frac{wx}{24EIl}\left(a^2(2l-a)^2 - 2ax^2(2l-a) + lx^3\right)$
- Δ_x (when $x > a$) . . . . . = $\frac{wa^2(l - x)}{24EIl}(4xl - 2x^2 - a^2)$

6. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END



- $R_1 = V_1$ . . . . = $\frac{w_1a(2l - a) + w_2c^2}{2l}$
- $R_2 = V_2$ . . . . = $\frac{w_2c(2l - c) + w_1a^2}{2l}$
- V_x (when $x < a$) . . . . . = $R_1 - w_1x$
- V_x (when $x > a$ and $< (a + b)$) . = $R_1 - w_1a$
- V_x (when $x > (a + b)$) . . . . = $R_2 - w_2(l - x)$
- M max. $\left(\text{at } x = \frac{R_1}{w_1} \text{ when } R_1 < w_1a\right)$. = $\frac{R_1^2}{2w_1}$
- M max. $\left(\text{at } x = l - \frac{R_2}{w_2} \text{ when } R_2 < w_2c\right)$ = $\frac{R_2^2}{2w_2}$
- M_x (when $x < a$) . . . . . = $R_1x - \frac{w_1x^2}{2}$
- M_x (when $x > a$ and $< (a + b)$) . = $R_1x - \frac{w_1a}{2}(2x - a)$
- M_x (when $x > (a + b)$) . . . . = $R_2(l - x) - \frac{w_2(l - x)^2}{2}$

7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER



Total Equiv. Uniform Load . . . . . . $= 2P$

$R = V$ . . . . $= \frac{P}{2}$

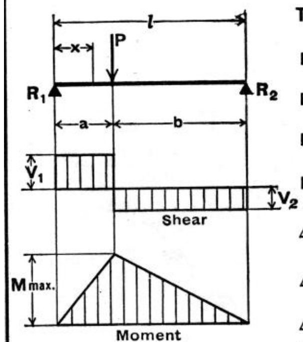
M max. (at point of load) . . . . . . $= \frac{Pl}{4}$

M_x (when $x < \frac{l}{2}$) . . . . . . $= \frac{Px}{2}$

Δmax. (at point of load) . . . . . . $= \frac{Pl^3}{48EI}$

Δ_x (when $x < \frac{l}{2}$) . . . . . . $= \frac{Px}{48EI}(3l^2 - 4x^2)$

8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load . . . . . . $= \frac{8\,Pab}{l^2}$

$R_1 = V_1$ (max. when $a < b$) . . . . $= \frac{Pb}{l}$

$R_2 = V_2$ (max. when $a > b$) . . . . $= \frac{Pa}{l}$

M max. (at point of load) . . . . $= \frac{Pab}{l}$

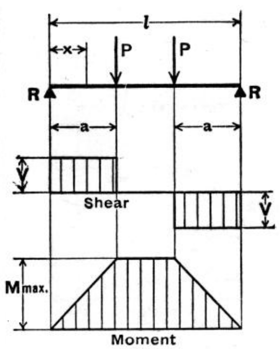
M_x (when $x < a$) . . . . $= \frac{Pbx}{l}$

Δmax. $\left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b\right)$ $= \frac{Pab\,(a+2b)\sqrt{3a\,(a+2b)}}{27\,EI\,l}$

Δ_a (at point of load) . . . . $= \frac{Pa^2b^2}{3EI\,l}$

Δ_x (when $x < a$) . . . . $= \frac{Pbx}{6EI\,l}(l^2 - b^2 - x^2)$

9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED



Total Equiv. Uniform Load . . . . . . $= \frac{8\,Pa}{l}$

$R = V$ . . . . $= P$

M max. (between loads) . . . . . . $= Pa$

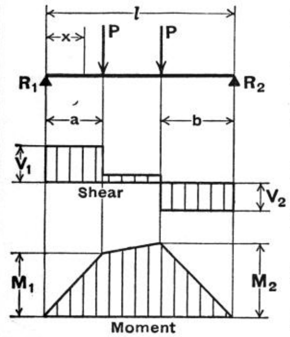
M_x (when $x < a$) . . . . . . $= Px$

Δmax. (at center) . . . . . . $= \frac{Pa}{24EI}(3l^2 - 4a^2)$

Δ_x (when $x < a$) . . . . . . $= \frac{Px}{6EI}(3la - 3a^2 - x^2)$

Δ_x (when $x > a$ and $< (l - a)$) . . $= \frac{Pa}{6EI}(3lx - 3x^2 - a^2)$

10. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



$R_1 = V_1$ (max. when $a < b$) . . . . . $= \frac{P}{l}(l - a + b)$

$R_2 = V_2$ (max. when $a > b$) . . . . . $= \frac{P}{l}(l - b + a)$

V_x (when $x > a$ and $< (l - b)$) . . $= \frac{P}{l}(b - a)$

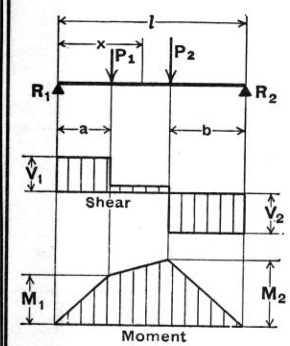
M_1 (max. when $a > b$) . . . . . $= R_1a$

M_2 (max. when $a < b$) . . . . . $= R_2b$

M_x (when $x < a$) . . . . $= R_1x$

M_x (when $x > a$ and $< (l - b)$) . . $= R_1x - P(x - a)$

11. SIMPLE BEAM—TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



$R_1 = V_1$ . . . . $= \frac{P_1(l - a) + P_2b}{l}$

$R_2 = V_2$ . . . . $= \frac{P_1a + P_2(l - b)}{l}$

V_x (when $x > a$ and $< (l - b)$) . . $= R_1 - P_1$

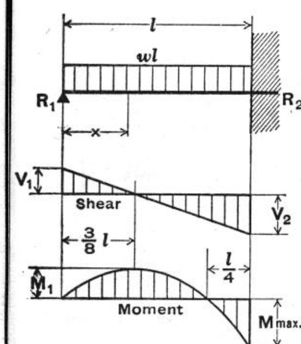
M_1 (max. when $R_1 < P_1$) . . . . $= R_1a$

M_2 (max. when $R_2 < P_2$) . . . $= R_2b$

M_x (when $x < a$) . . . . . . $= R_1x$

M_x (when $x > a$ and $< (l - b)$) . . $= R_1x - P_1(x - a)$

12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . . . $= wl$

$R_1 = V_1$ . . . . $= \frac{3wl}{8}$

$R_2 = V_2$ max. . . . . $= \frac{5wl}{8}$

V_x . . . . $= R_1 - wx$

M max. . . . . $= \frac{wl^2}{8}$

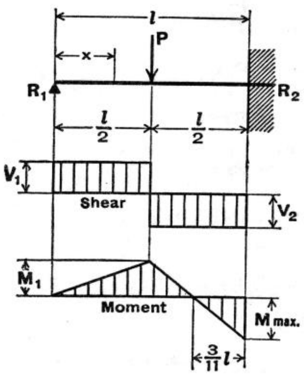
M_1 $\left(\text{at } x = \frac{3}{8}l\right)$ . . . . $= \frac{9}{128}wl^2$

M_x . . . . $= R_1x - \frac{wx^2}{2}$

Δmax. $\left(\text{at } x = \frac{l}{16}(1 + \sqrt{33}) = .4215l\right)$. $= \frac{wl^4}{185EI}$

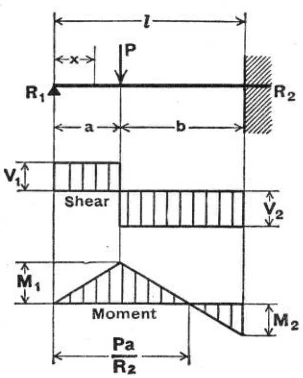
Δ_x . . . . $= \frac{wx}{48EI}(l^3 - 3lx^2 + 2x^3)$

13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT CENTER



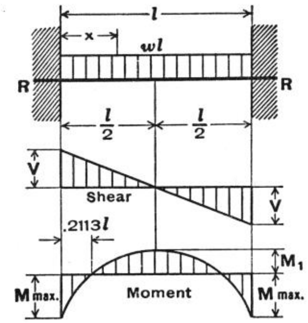
Total Equiv. Uniform Load	$= \frac{3P}{2}$
$R_1 = V_1$	$= \frac{5P}{16}$
$R_2 = V_2$ max.	$= \frac{11P}{16}$
M max. (at fixed end)	$= \frac{3Pl}{16}$
M_1 (at point of load)	$= \frac{5Pl}{32}$
M_x $\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{5Px}{16}$
M_x $\left(\text{when } x > \frac{l}{2}\right)$	$= P\left(\frac{l}{2} - \frac{11x}{16}\right)$
Δmax. $\left(\text{at } x = l\sqrt{\frac{1}{5}} = .4472l\right)$	$= \frac{Pl^3}{48EI\sqrt{5}} = .009317\,\frac{Pl^3}{EI}$
Δ_x (at point of load)	$= \frac{7Pl^3}{768EI}$
Δ_x $\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{Px}{96EI}(3l^2 - 5x^2)$
Δ_x $\left(\text{when } x > \frac{l}{2}\right)$	$= \frac{P}{96EI}(x - l)^2(11x - 2l)$

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT ANY POINT



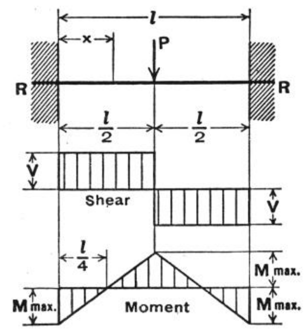
$R_1 = V_1$	$= \frac{Pb^2}{2l^3}(a + 2l)$
$R_2 = V_2$	$= \frac{Pa}{2l^3}(3l^2 - a^2)$
M_1 (at point of load)	$= R_1 a$
M_2 (at fixed end)	$= \frac{Pab}{2l^2}(a + l)$
M_x (when $x < a$)	$= R_1 x$
M_x (when $x > a$)	$= R_1 x - P(x - a)$
Δmax. $\left(\text{when } a < .414l \text{ at } x = l\,\frac{l^2 + a^2}{3l^2 - a^2}\right)$	$= \frac{Pa}{3EI}\,\frac{(l^2 - a^2)^3}{(3l^2 - a^2)^2}$
Δmax. $\left(\text{when } a > .414l \text{ at } x = l\sqrt{\frac{a}{2l + a}}\right)$	$= \frac{Pab^2}{6EI}\sqrt{\frac{a}{2l + a}}$
Δ_a (at point of load)	$= \frac{Pa^2b^3}{12EIl^3}(3l + a)$
Δ_x (when $x < a$)	$= \frac{Pb^2x}{12EIl^3}(3al^2 - 2lx^2 - ax^2)$
Δ_x (when $x > a$)	$= \frac{Pa}{12EIl^3}(l - x)^2(3l^2x - a^2x - 2a^2l)$

15. BEAM FIXED AT BOTH ENDS—UNIFORMLY DISTRIBUTED LOADS



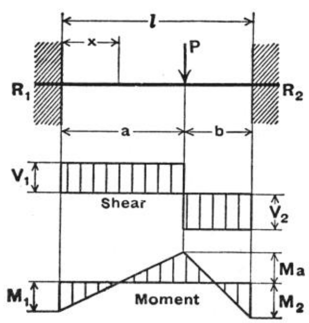
Total Equiv. Uniform Load	$= \frac{2wl}{3}$
R = V	$= \frac{wl}{2}$
V_x	$= w\left(\frac{l}{2} - x\right)$
M max. (at ends)	$= \frac{wl^2}{12}$
M_1 (at center)	$= \frac{wl^2}{24}$
M_x	$= \frac{w}{12}(6lx - l^2 - 6x^2)$
Δmax. (at center)	$= \frac{wl^4}{384EI}$
Δ_x	$= \frac{wx^2}{24EI}(l - x)^2$

16. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT CENTER



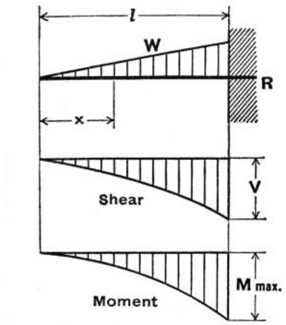
Total Equiv. Uniform Load	$= P$
R = V	$= \frac{P}{2}$
M max. (at center and ends)	$= \frac{Pl}{8}$
M_x $\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{P}{8}(4x - l)$
Δmax. (at center)	$= \frac{Pl^3}{192EI}$
Δ_x $\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{Px^2}{48EI}(3l - 4x)$

17. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$ (max. when $a < b$)	$= \frac{Pb^2}{l^3}(3a + b)$
$R_2 = V_2$ (max. when $a > b$)	$= \frac{Pa^2}{l^3}(a + 3b)$
M_1 (max. when $a < b$)	$= \frac{Pab^2}{l^2}$
M_2 (max. when $a > b$)	$= \frac{Pa^2b}{l^2}$
M_a (at point of load)	$= \frac{2Pa^2b^2}{l^3}$
M_x (when $x < a$)	$= R_1x - \frac{Pab^2}{l^2}$
Δmax. $\left(\text{when } a > b \text{ at } x = \frac{2al}{3a + b}\right)$	$= \frac{2Pa^3b^2}{3EI\,(3a + b)^2}$
Δ_a (at point of load)	$= \frac{Pa^3b^3}{3EIl^3}$
Δ_x (when $x < a$)	$= \frac{Pb^2x^2}{6EIl^3}(3al - 3ax - bx)$

18. CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END



Total Equiv. Uniform Load . . . . . = $\frac{8}{3}W$

R = V . . . . = W

$W = \frac{wl}{2}$

V_x . . . . = $W\frac{x^2}{l^2}$

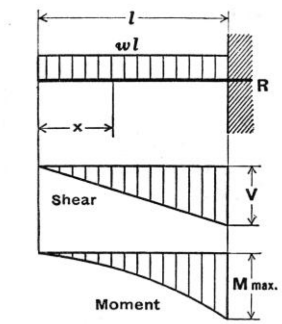
M max. (at fixed end) . . . . . . = $\frac{Wl}{3}$

M_x . . . . = $\frac{Wx^3}{3l^2}$

Δmax. (at free end) . . . . = $\frac{Wl^3}{15EI}$

Δx . . . . = $\frac{W}{60EIl^2}(x^5 - 5l^4x + 4l^5)$

19. CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . . = $4wl$

R = V . . . . = wl

V_x . . . . = wx

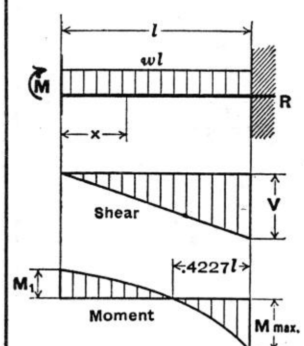
M max. (at fixed end) . . . . . . = $\frac{wl^2}{2}$

M_x . . . . = $\frac{wx^2}{2}$

Δmax. (at free end) . . . . = $\frac{wl^4}{8EI}$

Δx . . . . = $\frac{w}{24EI}(x^4 - 4l^3x + 3l^4)$

20. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . . = $\frac{8}{3}wl$

R = V . . . . = wl

V_x . . . . = wx

M max. (at fixed end) . . . . . . = $\frac{wl^2}{3}$

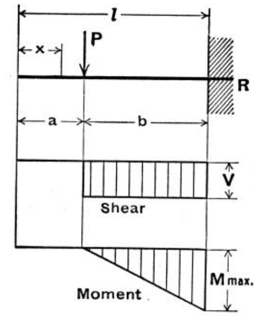
M_1 (at deflected end) . . . . . = $\frac{wl^2}{6}$

M_x . . . . = $\frac{w}{6}(l^2 - 3x^2)$

Δmax. (at deflected end) . . . . . = $\frac{wl^4}{24EI}$

Δx . . . . = $\frac{w(l^2 - x^2)^2}{24EI}$

21. CANTILEVER BEAM—CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load . . . . . = $\frac{8Pb}{l}$

R = V . . . . = P

M max. (at fixed end) . . . . . . = Pb

M_x (when x > a) . . . . . . = P (x − a)

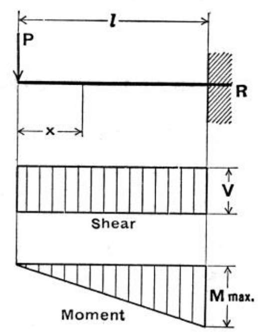
Δmax. (at free end) . . . . . . = $\frac{Pb^2}{6EI}(3l - b)$

Δa (at point of load) . . . . . = $\frac{Pb^3}{3EI}$

Δx (when x < a) . . . . . . = $\frac{Pb^2}{6EI}(3l - 3x - b)$

Δx (when x > a) . . . . . . = $\frac{P(l-x)^2}{6EI}(3b - l + x)$

22. CANTILEVER BEAM—CONCENTRATED LOAD AT FREE END



Total Equiv. Uniform Load . . . . . = 8P

R = V . . . . = P

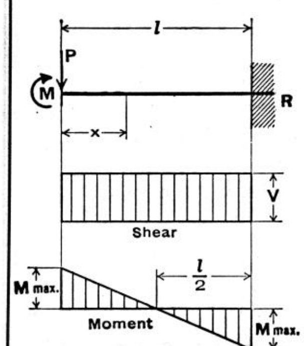
M max. (at fixed end) . . . . . . = Pl

M_x . . . . = Px

Δmax. (at free end) . . . . . = $\frac{Pl^3}{3EI}$

Δx . . . . = $\frac{P}{6EI}(2l^3 - 3l^2x + x^3)$

23. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—CONCENTRATED LOAD AT DEFLECTED END



Total Equiv. Uniform Load . . . . . = 4P

R = V . . . . = P

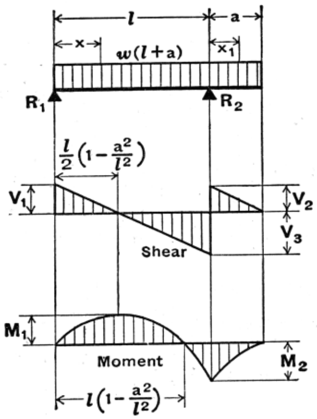
M max. (at both ends) . . . . . = $\frac{Pl}{2}$

M_x . . . . = $P\left(\frac{l}{2} - x\right)$

Δmax. (at deflected end) . . . . . = $\frac{Pl^3}{12EI}$

Δx . . . . = $\frac{P(l-x)^2}{12EI}(l + 2x)$

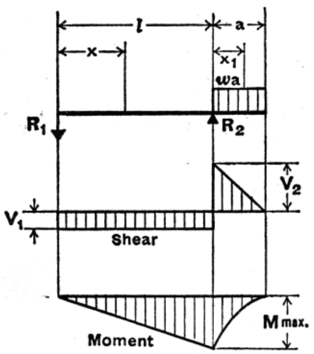
24. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD



- $R_1 = V_1 \dots = \frac{w}{2l}(l^2 - a^2)$
- $R_2 = V_2 + V_3 \dots = \frac{w}{2l}(l + a)^2$
- $V_2 \dots = wa$
- $V_3 \dots = \frac{w}{2l}(l^2 + a^2)$
- V_x (between supports) $\dots = R_1 - wx$
- V_{x_1} (for overhang) $\dots = w(a - x_1)$
- M_1 $\left(\text{at } x = \frac{l}{2}\left[1 - \frac{a^2}{l^2}\right]\right) \dots = \frac{w}{8l^2}(l+a)^2(l-a)^2$
- M_2 (at R_2) $\dots = \frac{wa^2}{2}$
- M_x (between supports) $\dots = \frac{wx}{2l}(l^2 - a^2 - xl)$
- M_{x_1} (for overhang) $\dots = \frac{w}{2}(a - x_1)^2$
- Δx (between supports) $\dots = \frac{wx}{24EIl}(l^4 - 2l^2x^2 + lx^3 - 2a^2l^2 + 2a^2x^2)$
- Δx_1 (for overhang) $\dots = \frac{wx_1}{24EI}(4a^2l - l^3 + 6a^2x_1 - 4ax_1^2 + x_1^3)$

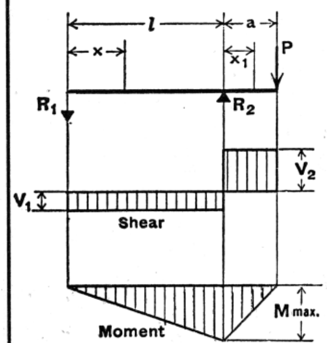
NOTE: For a negative value of Δ_x deflection is upward.

25. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD ON OVERHANG



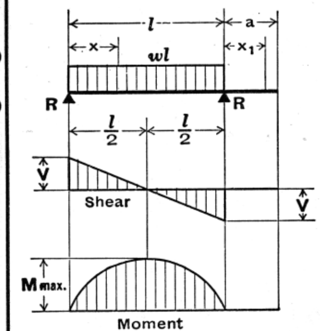
- $R_1 = V_1 \dots = \frac{wa^2}{2l}$
- $R_2 = V_1 + V_2 \dots = \frac{wa}{2l}(2l + a)$
- $V_2 \dots = wa$
- V_{x_1} (for overhang) $\dots = w(a - x_1)$
- M max. (at R_2) $\dots = \frac{wa^2}{2}$
- M_x (between supports) $\dots = \frac{wa^2x}{2l}$
- M_{x_1} (for overhang) $\dots = \frac{w}{2}(a - x_1)^2$
- Δmax. $\left(\text{between supports at } x = \frac{l}{\sqrt{3}}\right) = \frac{wa^2l^2}{18\sqrt{3}EI} = .03208\frac{wa^2l^2}{EI}$
- Δmax. (for overhang at $x_1 = a$) $\dots = \frac{wa^3}{24EI}(4l + 3a)$
- Δx (between supports) $\dots = \frac{wa^2x}{12EIl}(l^2 - x^2)$
- Δx_1 (for overhang) $\dots = \frac{wx_1}{24EI}(4a^2l + 6a^2x_1 - 4ax_1^2 + x_1^3)$

26. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT END OF OVERHANG



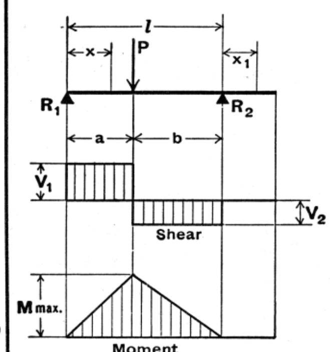
- $R_1 = V_1 \dots = \frac{Pa}{l}$
- $R_2 = V_1 + V_2 \dots = \frac{P}{l}(l + a)$
- $V_2 \dots = P$
- M max. (at R_2) $\dots = Pa$
- M_x (between supports) $\dots = \frac{Pax}{l}$
- M_{x_1} (for overhang) $\dots = P(a - x_1)$
- Δmax. $\left(\text{between supports at } x = \frac{l}{\sqrt{3}}\right) = \frac{Pal^2}{9\sqrt{3}EI} = .06415\frac{Pal^2}{EI}$
- Δmax. (for overhang at $x_1 = a$) $\dots = \frac{Pa^2}{3EI}(l + a)$
- Δx (between supports) $\dots = \frac{Pax}{6EIl}(l^2 - x^2)$
- Δx_1 (for overhang) $\dots = \frac{Px_1}{6EI}(2al + 3ax_1 - x_1^2)$

27. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD BETWEEN SUPPORTS



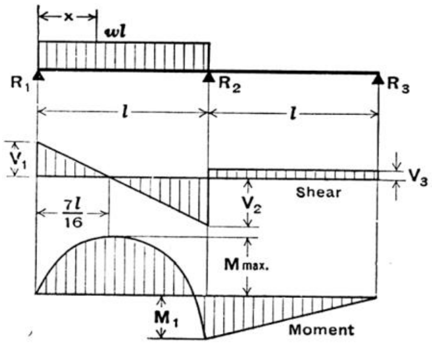
- Total Equiv. Uniform Load $\dots = wl$
- $R = V \dots = \frac{wl}{2}$
- $V_x \dots = w\left(\frac{l}{2} - x\right)$
- M max. (at center) $\dots = \frac{wl^2}{8}$
- $M_x \dots = \frac{wx}{2}(l - x)$
- Δmax. (at center) $\dots = \frac{5wl^4}{384EI}$
- $\Delta x \dots = \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$
- $\Delta x_1 \dots = \frac{wl^3x_1}{24EI}$

28. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT ANY POINT BETWEEN SUPPORTS



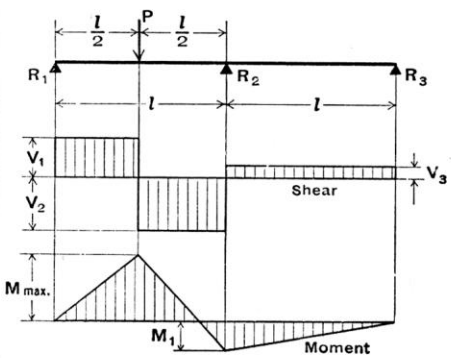
- Total Equiv. Uniform Load $\dots = \frac{8Pab}{l^2}$
- $R_1 = V_1$ (max. when $a < b$) $\dots = \frac{Pb}{l}$
- $R_2 = V_2$ (max. when $a > b$) $\dots = \frac{Pa}{l}$
- M max. (at point of load) $\dots = \frac{Pab}{l}$
- M_x (when $x < a$) $\dots = \frac{Pbx}{l}$
- Δmax. $\left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b\right) = \frac{Pab\,(a+2b)\sqrt{3a\,(a+2b)}}{27EIl}$
- Δa (at point of load) $\dots = \frac{Pa^2b^2}{3EIl}$
- Δx (when $x < a$) $\dots = \frac{Pbx}{6EIl}(l^2 - b^2 - x^2)$
- Δx (when $x > a$) $\dots = \frac{Pa\,(l - x)}{6EIl}(2lx - x^2 - a^2)$
- $\Delta x_1 \dots = \frac{Pabx_1}{6EIl}(l + a)$

29. CONTINUOUS BEAM—TWO EQUAL SPANS—UNIFORM LOAD ON ONE SPAN



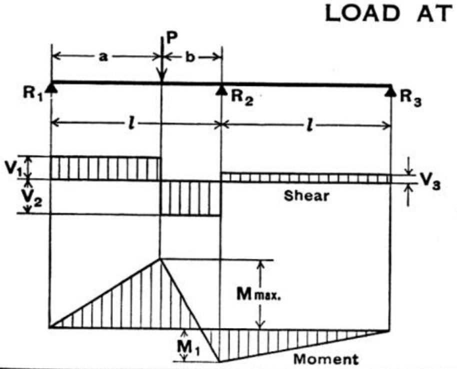
Total Equiv. Uniform Load	$= \frac{49}{64} wl$
$R_1 = V_1$	$= \frac{7}{16} wl$
$R_2 = V_2 + V_3$	$= \frac{5}{8} wl$
$R_3 = V_3$	$= -\frac{1}{16} wl$
V_2	$= \frac{9}{16} wl$
M max. $\left(\text{at } x = \frac{7}{16} l\right)$	$= \frac{49}{512} wl^2$
M_1 (at support R_2)	$= \frac{1}{16} wl^2$
M_x (when $x < l$)	$= \frac{wx}{16}(7l - 8x)$
Δ Max. (0.472 l from R_1)	$= 0.0092\ wl^4/EI$

30. CONTINUOUS BEAM—TWO EQUAL SPANS—CONCENTRATED LOAD AT CENTER OF ONE SPAN



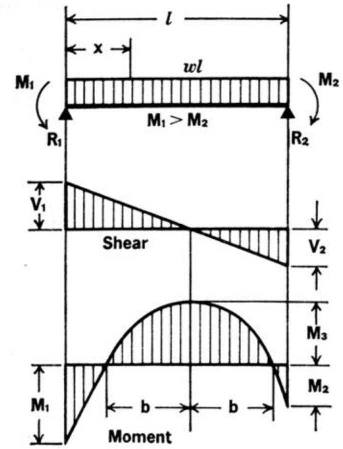
Total Equiv. Uniform Load	$= \frac{13}{8} P$
$R_1 = V_1$	$= \frac{13}{32} P$
$R_2 = V_2 + V_3$	$= \frac{11}{16} P$
$R_3 = V_3$	$= -\frac{3}{32} P$
V_2	$= \frac{19}{32} P$
M max. (at point of load)	$= \frac{13}{64} Pl$
M_1 (at support R_2)	$= \frac{3}{32} Pl$
Δ Max. (0.480 l from R_1)	$= 0.015\ Pl^3/EI$

31. CONTINUOUS BEAM—TWO EQUAL SPANS—CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$	$= \frac{Pb}{4l^3}\left(4l^2 - a(l+a)\right)$
$R_2 = V_2 + V_3$	$= \frac{Pa}{2l^3}\left(2l^2 + b(l+a)\right)$
$R_3 = V_3$	$= -\frac{Pab}{4l^3}(l+a)$
V_2	$= \frac{Pa}{4l^3}\left(4l^2 + b(l+a)\right)$
M max. (at point of load)	$= \frac{Pab}{4l^3}\left(4l^2 - a(l+a)\right)$
M_1 (at support R_2)	$= \frac{Pab}{4l^2}(l+a)$

32. BEAM—UNIFORMLY DISTRIBUTED LOAD AND VARIABLE END MOMENTS



$$R_1 = V_1 = \frac{wl}{2} + \frac{M_1 - M_2}{l}$$

$$R_2 = V_2 = \frac{wl}{2} - \frac{M_1 - M_2}{l}$$

$$V_x = w\left(\frac{l}{2} - x\right) + \frac{M_1 - M_2}{l}$$

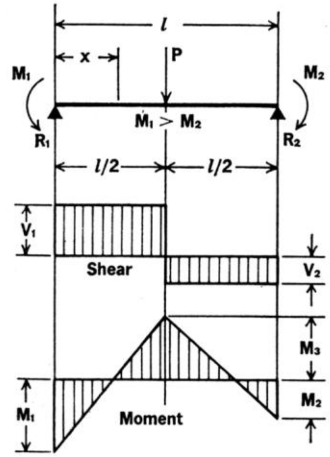
$$M_3 \left(\text{at } x = \frac{l}{2} + \frac{M_1 - M_2}{wl}\right) = \frac{wl^2}{8} - \frac{M_1 + M_2}{2} + \frac{(M_1 - M_2)^2}{2wl^2}$$

$$M_x = \frac{wx}{2}(l - x) + \left(\frac{M_1 - M_2}{l}\right)x - M_1$$

$$b\left(\text{To locate inflection points}\right) = \sqrt{\frac{l^2}{4} - \left(\frac{M_1 + M_2}{w}\right) + \left(\frac{M_1 - M_2}{wl}\right)^2}$$

$$\Delta_x = \frac{wx}{24EI}\left[x^3 - \left(2l + \frac{4M_1}{wl} - \frac{4M_2}{wl}\right)x^2 + \frac{12M_1}{w}x + l^3 - \frac{8M_1 l}{w} - \frac{4M_2 l}{w}\right]$$

33. BEAM—CONCENTRATED LOAD AT CENTER AND VARIABLE END MOMENTS



$$R_1 = V_1 = \frac{P}{2} + \frac{M_1 - M_2}{l}$$

$$R_2 = V_2 = \frac{P}{2} - \frac{M_1 - M_2}{l}$$

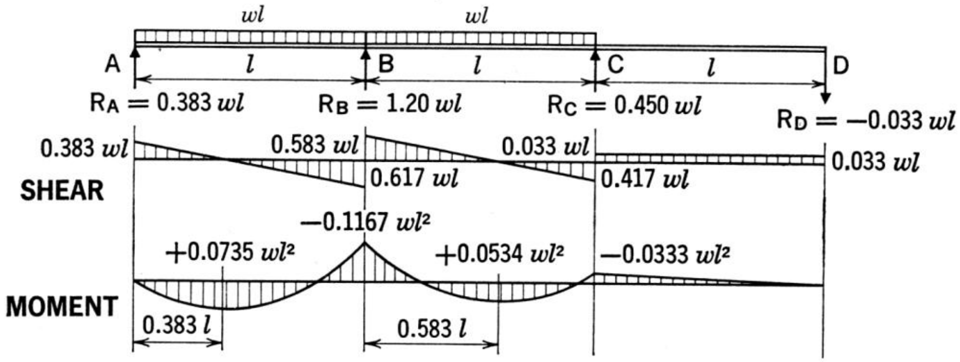
$$M_3 \text{ (At center)} = \frac{Pl}{4} - \frac{M_1 + M_2}{2}$$

$$M_x \left(\text{When } x < \frac{l}{2}\right) = \left(\frac{P}{2} + \frac{M_1 - M_2}{l}\right)x - M_1$$

$$M_x \left(\text{When } x > \frac{l}{2}\right) = \frac{P}{2}(l - x) + \frac{(M_1 - M_2)x}{l} - M_1$$

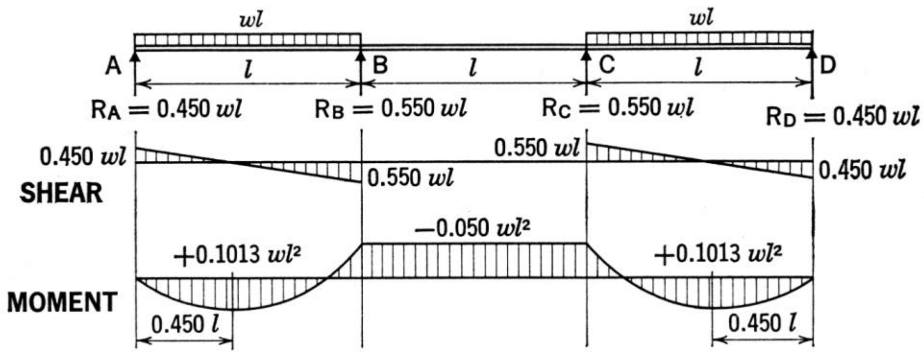
$$\Delta_x \left(\text{When } x < \frac{l}{2}\right) = \frac{Px}{48EI}\left(3l^2 - 4x^2 - \frac{8(l - x)}{Pl}\left[M_1(2l - x) + M_2(l + x)\right]\right)$$

34. CONTINUOUS BEAM—THREE EQUAL SPANS—ONE END SPAN UNLOADED



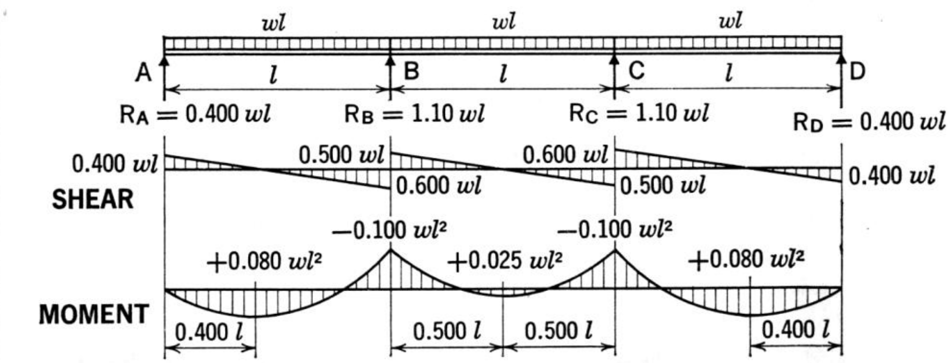
Δ Max. (0.430 l from A) = 0.0059 wl⁴/EI

35. CONTINUOUS BEAM—THREE EQUAL SPANS—END SPANS LOADED



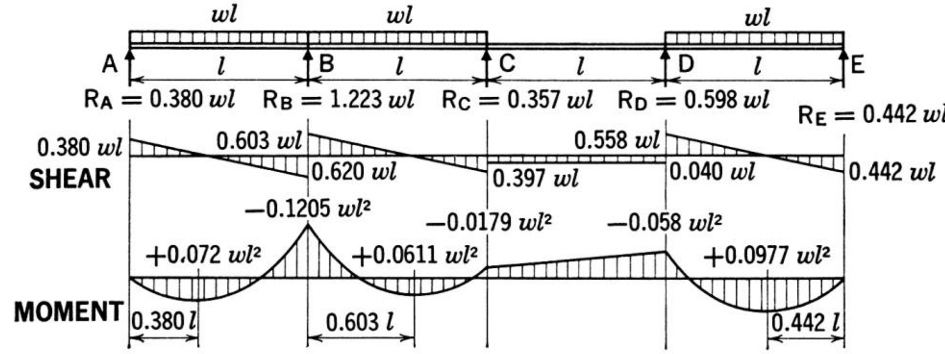
Δ Max. (0.479 l from A or D) = 0.0099 wl⁴/EI

36. CONTINUOUS BEAM—THREE EQUAL SPANS—ALL SPANS LOADED



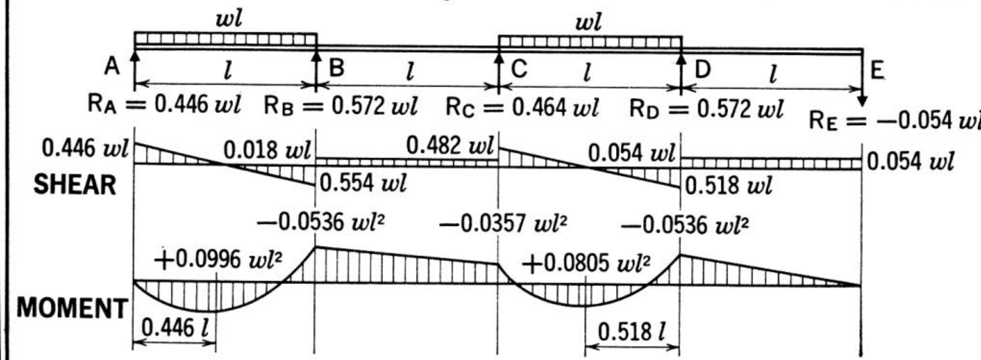
Δ Max. (0.446 l from A or D) = 0.0069 wl⁴/EI

37. CONTINUOUS BEAM—FOUR EQUAL SPANS—THIRD SPAN UNLOADED



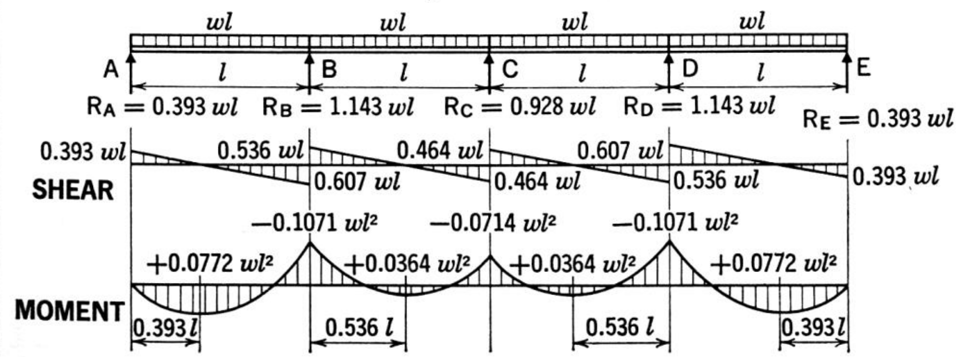
Δ Max. (0.475 l from E) = 0.0094 wl⁴/EI

38. CONTINUOUS BEAM—FOUR EQUAL SPANS—LOAD FIRST AND THIRD SPANS



Δ Max. (0.477 l from A) = 0.0097 wl⁴/EI

39. CONTINUOUS BEAM—FOUR EQUAL SPANS—ALL SPANS LOADED



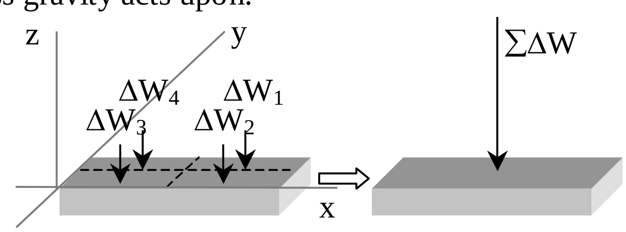
Δ Max. (0.440 l from A and E) = 0.0065 wl⁴/EI

Centers of Gravity - Centroids

Notation:

A = name for area
C = designation for channel section
= name for centroid
F_z = force component in the z direction
L = name for length
O = name for reference origin
Q_x = first moment area about an x axis (using y distances)
Q_y = first moment area about an y axis (using x distances)
t = name for thickness
t_w = thickness of web of wide flange
W = name for force due to weight
= designation for wide flange section
x = horizontal distance
$\bar{x}$ = the distance in the x direction from a reference axis to the centroid of a shape
$\hat{x}$ = the distance in the x direction from a reference axis to the centroid of a composite shape
y = vertical distance
$\bar{y}$ = the distance in the y direction from a reference axis to the centroid of a shape
$\hat{y}$ = the distance in the y direction from a reference axis to the centroid of a composite shape
z = distance perpendicular to *x-y* plane
$\int$ = symbol for integration
Δ = calculus symbol for small quantity
γ = density of a material (unit weight)
Σ = summation symbol

- The cross section shape and how it resists bending and twisting is important to understanding beam and column behavior.
- The *center of gravity* is the location of the equivalent force representing the total weight of a body comprised of particles that each have a mass gravity acts upon.



Resultant force: Over a body of constant thickness in x and y

$$\sum F_z = \sum_{i=1}^{n} \Delta W_i = \mathbf{W} \qquad W = \int dW$$

Location: $\bar{x}$, $\bar{y}$ is the equivalent location of the force W from all ΔW_i's over all x & y locations (with respect to the moment from each force) from:

$$\sum M_y = \sum_{i=1}^{n} x_i \Delta W_i = \bar{x}\mathbf{W} \qquad \bar{x}\mathbf{W} = \int x dW \Rightarrow \bar{x} = \frac{\int x dW}{\mathbf{W}} \text{ OR } \boxed{\bar{x} = \frac{\Sigma(x\Delta W)}{\mathbf{W}}}$$

$$\sum M_x = \sum_{i=1}^{n} y_i \Delta W_i = \bar{y}\mathbf{W} \qquad \bar{y}\mathbf{W} = \int y dW \Rightarrow \bar{y} = \frac{\int y dW}{\mathbf{W}} \text{ OR } \boxed{\bar{y} = \frac{\Sigma(y\Delta W)}{\mathbf{W}}}$$

- The *centroid of an area* is the average x and y locations of the area particles

For a discrete shape (ΔA_i) of a uniform thickness and material, the weight can be defined as:

$\Delta W_i = \gamma t \Delta A_i$ where:

γ is weight per unit **volume** (= specific weight) with units of <u>N/m^3</u> or <u>lb/ft^3</u>

$t\Delta A_i$ is the volume

So if $\boldsymbol{W} = \gamma t \boldsymbol{A}$:

$$\bar{x}\gamma t\boldsymbol{A} = \int x\gamma t dA \Rightarrow \bar{x}\boldsymbol{A} = \int xdA \text{ OR } \boxed{\bar{x} = \frac{\sum(x\Delta A)}{\boldsymbol{A}}} \text{ and similarly } \boxed{\bar{y} = \frac{\sum(y\Delta A)}{\boldsymbol{A}}}$$

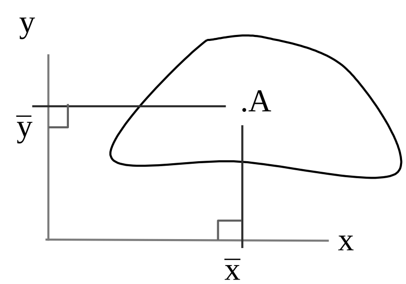
Similarly, for a line with constant cross section, *a* ($\Delta W_i = \gamma a \Delta L_i$):

$$\bar{x}\boldsymbol{L} = \int xdL \text{ OR } \boxed{\bar{x} = \frac{\sum(x\Delta L)}{\boldsymbol{L}}} \quad \text{and} \quad \bar{y}\boldsymbol{L} = \int ydL \text{ OR } \boxed{\bar{y} = \frac{\sum(y\Delta L)}{\boldsymbol{L}}}$$

- $\bar{x}$, $\bar{y}$ **with respect to an x, y coordinate system** is the centroid of an area AND the center of **gravity** for a body of uniform material and thickness.

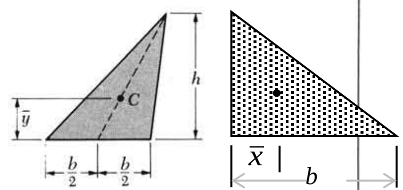
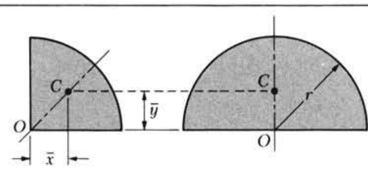
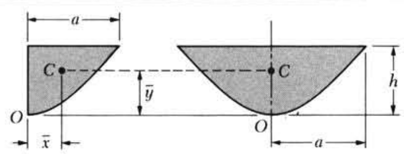
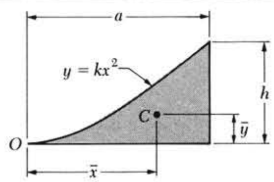
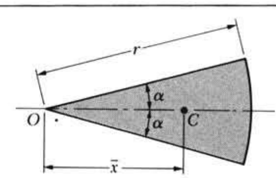
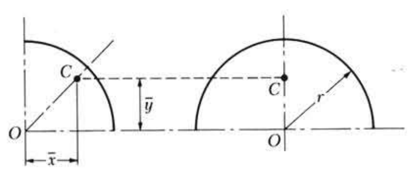
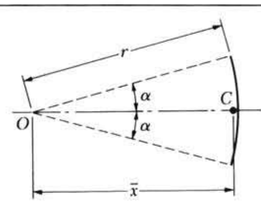
- The *first moment of the area* is like a force moment: and is the **area** multiplied by the perpendicular distance to an axis.

$$Q_x = \int ydA = \bar{y}A \qquad Q_y = \int xdA = \bar{x}A$$



- <u>Centroids of Common Shapes</u>

Centroids of Common Shapes of Areas and Lines

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic span-drel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

- Symmetric Areas

  - An area is symmetric with respect to a line when every point on one side is mirrored on the other. The line divides the area into equal parts and the centroid will be on that axis.

  - An area can be symmetric to a *center point* when every (x,y) point is matched by a (-x,-y) point. It does not necessarily have an axis of symmetry. The center point is the *centroid*.

  - If the symmetry line is on an axis, the centroid location is on that axis (value of 0). With double symmetry, the centroid is at the intersection.

  - Symmetry can also be defined by areas that match across a line, but are 180° to each other.

Basic Steps

1. Draw a reference origin.

2. Divide the area into basic shapes

3. Label the basic shapes (components)

4. Draw a table with headers of *Component*, *Area*, $\bar{x}$, $\bar{x}A$, $\bar{y}$, $\bar{y}A$

5. Fill in the table value

6. Draw a summation line. Sum all the areas, all the $\bar{x}A$ terms, and all the $\bar{y}A$ terms

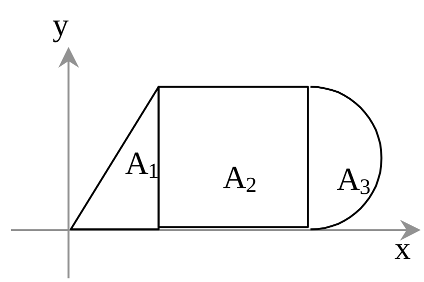
7. Calculate $\hat{x}$ and $\hat{y}$

- Composite Shapes

  If we have a shape made up of basic shapes that we know centroid locations for, we can find an "average" centroid of the areas.

$$\hat{x}\boldsymbol{A} = \hat{x}\sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \bar{x}_i A_i \qquad \hat{y}\boldsymbol{A} = \hat{y}\sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \bar{y}_i A_i$$

OR $$\hat{x} = \frac{\Sigma \bar{x}A}{\boldsymbol{A}} \qquad \hat{y} = \frac{\Sigma \bar{y}A}{\boldsymbol{A}}$$

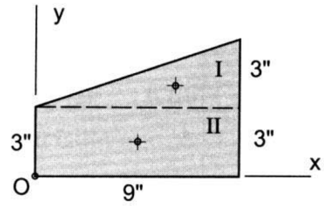


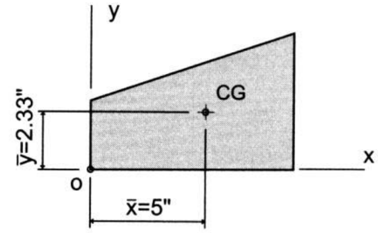
Centroid values can be negative.
Area values can be negative (holes)

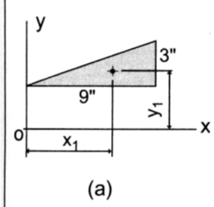
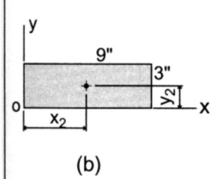
Example 1 (pg 243)

Example Problem 7.1: Centroids (Figures 7.5 and 7.6)

Determine the centroidal x and y distances for the composite area shown. Use the lower left corner of the trapezoid as the reference origin.





Component	Area (ΔA) (in.^{2})	$\bar{x}$ (in.)	$\bar{x}\Delta A$ (in.^{3})	$\bar{y}$ (in.)	$\bar{y}\Delta A$ (in.^{3})
 (a)	$\frac{9''(3'')}{2} = 13.5$ in.^{2}	6"	81 in.^{3}	4"	54 in.^{3}
 (b)	9" (3") = 27 in.^{2}	4.5"	121.5 in.^{3}	1.5"	40.5 in.^{3}
	$A = \sum \Delta A = 40.5$ in.^{2}		$\sum \bar{x}\Delta A = 202.5$ in.^{3}		$\sum \bar{y}\Delta A = 94.5$ in.^{3}

$$\hat{x} = \frac{202.5in^3}{40.5in^2} = 5in$$

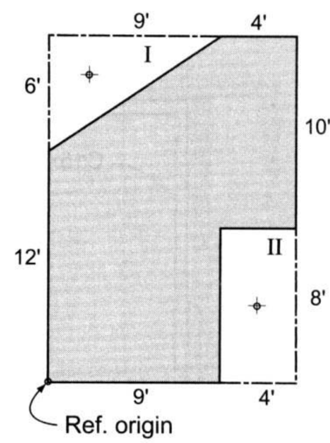
$$\hat{y} = \frac{94.5in^3}{40.5in^2} = 2.33in$$

Example 2 (pg 245)

Example Problem 7.3b (Figure 7.13)

An alternate method that can be employed in solving this problem is referred to as the *negative area method.*

A 6" thick concrete wall panel is precast to the dimensions as shown. Using the lower left corner as the reference origin, determine the center of gravity (centroid) of the panel.



Moments of Inertia

Notation:

A = name for area
b = name for a (base) width
d = calculus symbol for differentiation
= name for a difference
= name for a depth
d_x = difference in the x direction between an area centroid ($\bar{x}$) and the centroid of the composite shape ($\hat{x}$)
d_y = difference in the y direction between an area centroid ($\bar{y}$) and the centroid of the composite shape ($\hat{y}$)
h = name for a height
$\bar{I}$ = moment of inertia about the centroid
I_c = moment of inertia about the centroid
I_x = moment of inertia with respect to an x-axis
I_y = moment of inertia with respect to a y-axis
J_o = polar moment of inertia, as is J
O = name for reference origin
r_o = polar radius of gyration
r_x = radius of gyration with respect to an x-axis
r_y = radius of gyration with respect to a y-axis
t_f = thickness of a flange
t_w = thickness of web of wide flange
x = horizontal distance
$\bar{x}$ = the distance in the x direction from a reference axis to the centroid of a shape
$\hat{x}$ = the distance in the x direction from a reference axis to the centroid of a composite shape
y = vertical distance
$\bar{y}$ = the distance in the y direction from a reference axis to the centroid of a shape
$\hat{y}$ = the distance in the y direction from a reference axis to the centroid of a composite shape
P̲ = plate symbol
$\int$ = symbol for integration
Σ = summation symbol

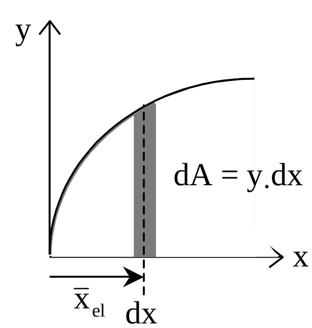
- The cross section shape and how it resists bending and twisting is important to understanding beam and column behavior.

- *Definition:* Moment of Inertia; the second area moment

$$I_y = \int x^2 dA \qquad I_x = \int y^2 dA$$

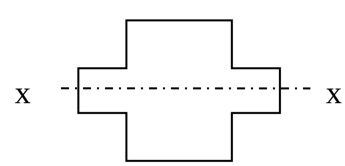
We can define a single integral using a narrow strip:

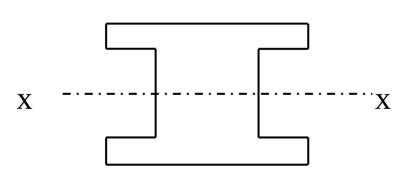
for I_x, strip is parallel to x for I_y, strip is parallel to y



**I can be negative if the area is negative (a hole or subtraction).*

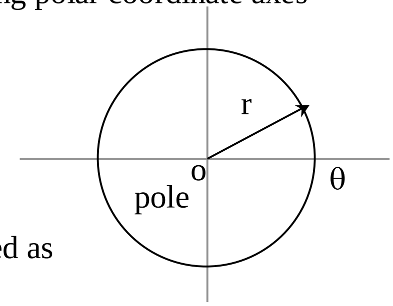
- A shape that has area at a greater distance away from an axis *through its centroid* will have a **larger** value of I.





- Just like for center of gravity of an area, the moment of inertia can be determined with respect to *any* reference **axis**.

- *Definition:* Polar Moment of Inertia; the second area moment using polar coordinate axes



$$J_o = \int r^2 dA = \int x^2 dA + \int y^2 dA$$
$$J_o = I_x + I_y$$

- *Definition*: Radius of Gyration; the distance from the moment of inertia axis for an area at which the entire area could be considered as being concentrated at.

$I_x = r_x{}^2 A \Rightarrow r_x = \sqrt{\frac{I_x}{A}}$ radius of gyration in x

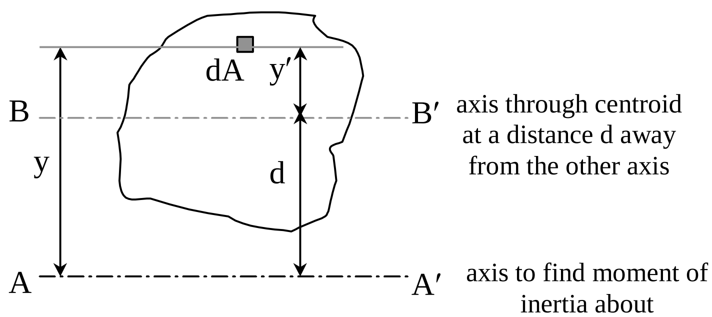
$r_y = \sqrt{\frac{I_y}{A}}$ radius of gyration in y

$r_o = \sqrt{\frac{J_o}{A}}$ polar radius of gyration, and $r_o^2 = r_x^2 + r_y^2$

The Parallel-Axis Theorem

- *The moment of inertia of an area with respect to any axis not through its centroid is equal to the moment of inertia of that area with respect to its own parallel centroidal axis plus the product of the area and the square of the distance between the two axes.*

$$I = \int y^2 dA = \int (y'\text{-}d)^2 dA$$
$$= \int y'^2 dA + 2d\int y' dA + d^2 \int dA$$



but $\int y' dA = 0$, because the centroid is on this axis, resulting in:

$I_x = I_{cx} + Ad_y{}^2$ (text notation) *or* $I_x = \bar{I}_x + Ad_y{}^2$

where I_{cx} (*or* $\bar{I}_x$) is the moment of inertia about the centroid of the area about an *x* axis and d_y is the y distance between the parallel axes

Similarly $I_y = \bar{I}_y + Ad_x{}^2$ Moment of inertia about a *y* axis

$J_o = \bar{J}_c + Ad^2$ Polar moment of Inertia

$r_o^2 = \bar{r}_c^2 + d^2$ Polar radius of gyration

$r^2 = \bar{r}^2 + d^2$ Radius of gyration

* *I can be negative again if the area is negative (a hole or subtraction).*
** *If* $\bar{I}$ is *not given in a chart, but* $\bar{x}$ *&* $\bar{y}$ *are: YOU MUST CALCULATE* $\bar{I}$ *WITH* $\bar{I} = I - Ad^2$

Composite Areas:

$I = \sum \bar{I} + \sum Ad^2$ where $\bar{I}$ is the moment of inertia about the centroid of the component area
d is the distance from the centroid of the component area to the centroid of the composite area (ie. $d_y = \hat{y} - \bar{y}$)

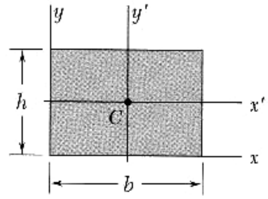
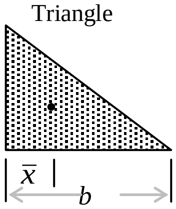
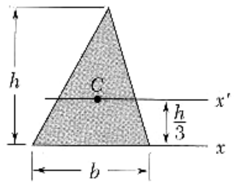
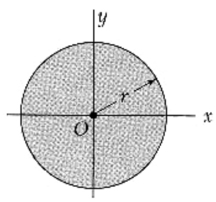
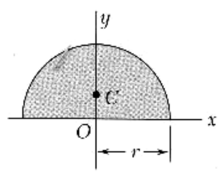
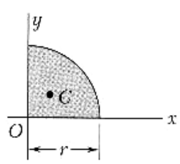
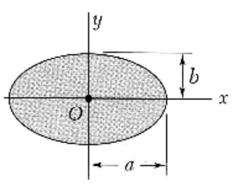
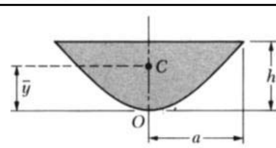
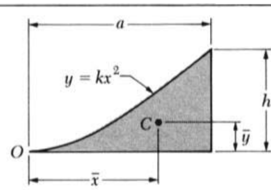
Basic Steps

1. Draw a reference origin.
2. Divide the area into basic shapes
3. Label the basic shapes (components)
4. Draw a table with headers of

   Component, Area, $\bar{x}$, $\bar{x}A$, $\bar{y}$, $\bar{y}A$, $\bar{I}_x$, d_y, Ad_y^2, $\bar{I}_y$, d_x, Ad_x^2

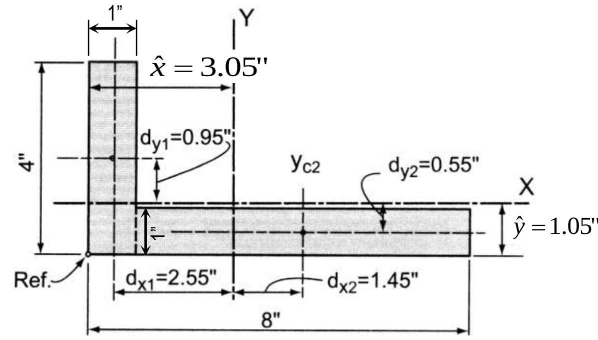
5. Fill in the table values needed to calculate $\hat{x}$ and $\hat{y}$ for the composite
6. Fill in the rest of the table values.
7. Sum the moment of inertia ($\bar{I}$'s) and Ad^2 columns and add together.

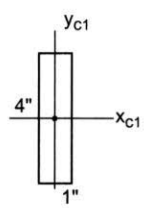
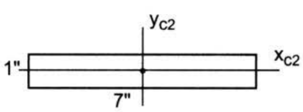
Geometric Properties of Areas

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$	Area = bh $\bar{x}$ = b/2 $\bar{y}$ = h/2
Triangle 		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{36}b^3h$	Area = $bh/2$ $\bar{x} = b/3$ $\bar{y} = h/3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$	Area = $\pi r^2 = \pi d^2/4$ $\bar{x}$ = 0 $\bar{y}$ = 0
Semicircle		$\bar{I}_x = 0.1098r^4$ $\bar{I}_y = \pi r^4/8$	Area = $\pi r^2/2 = \pi d^2/8$ $\bar{x}$ = 0 $\bar{y} = 4r/3\pi$
Quarter circle		$\bar{I}_x = 0.0549r^4$ $\bar{I}_y = 0.0549r^4$	Area = $\pi r^2/4 = \pi d^2/16$ $\bar{x} = 4r/3\pi$ $\bar{y} = 4r/3\pi$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$	Area = πab $\bar{x}$ = 0 $\bar{y}$ = 0
Semiparabolic area Parabolic area		$\bar{I}_x = 16ah^3/175$ $\bar{I}_y = 4a^3h/15$	Area = $4ah/3$ $\bar{x}$ = 0 $\bar{y} = 3h/5$
Parabolic spandrel		$\bar{I}_x = 37ah^3/2100$ $\bar{I}_y = a^3h/80$	Area = $ah/3$ $\bar{x} = 3a/4$ $\bar{y} = 3h/10$

Example 1 (pg 257)

Find the moments of inertia ($\hat{x}$ = 3.05", $\hat{y}$ = 1.05").

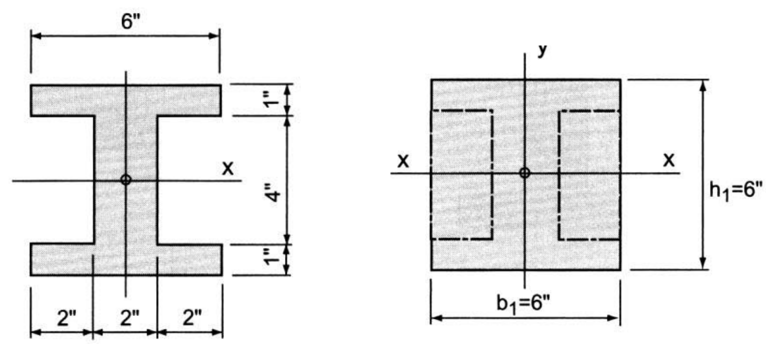


Component	I_{xc} (in.^{4})	d_y (in.)	Ad_y^2 (in.^{4})	I_{yc} (in.^{4})	d_x (in.)	Ad_x^2 (in.^{4})
	$\frac{(1)(4)^3}{12} = 5.33$	0.95	3.61	$\frac{(4)(1)^3}{12} = 0.33$	2.55	26.01
	$\frac{(7)(1)^3}{12} = 0.58$	0.55	2.12	$\frac{(1)(7)^3}{12} = 28.58$	1.45	14.72
	$\sum I_{xc} = 5.91$		$\sum Ad_y^2 = 5.73$	$\sum I_{yc} = 28.91$		$\sum Ad_x = 40.73$

Example 2 (pg 253)

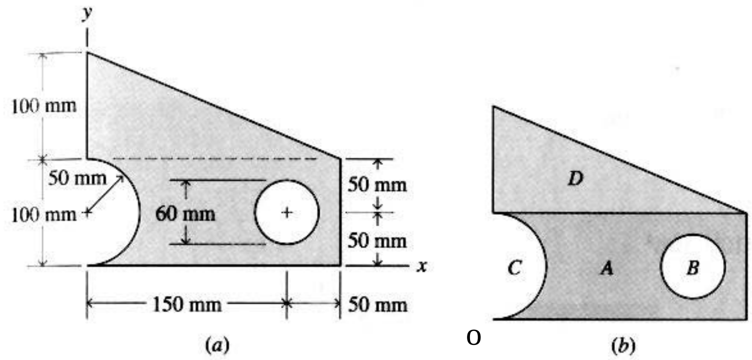
Example Problem 7.6 (Figures 7.24 to 7.26)

Determine the *I* about the centroidal *x*-axis.



Example 3

Determine the moments of inertia about the centroid of the shape.



Solution:

There is no reference origin suggested in figure (a), so the bottom left corner is good.

In figure (b) area A will be a complete rectangle, while areas C and A are "holes" with negative area and negative moment of inertias.

Area A = 200 mm x 100 mm = 20000 mm^2

I_x = (200 mm)(100 mm)^3/12 = 16.667 x 10^6 mm^4
I_y = (200 mm)^3(100 mm)/12 = 66.667 x 10^6 mm^4

Area B = -π(30 mm)^2 = -2827.4 mm^2

$I_x = I_y$ = -π (30 mm)^4/4 = -0.636 x 10^6 mm^4

Area C = -1/2π(50 mm)^2 = 3927.0 mm^2

I_x = -π (50 mm)^4/8 = -2.454 x 10^6 mm^4
I_y = -0.1098(50 mm)^4= -0.686 x 10^6 mm^4

Area D = 100 mm x 200 mm x 1/2 = 10000 mm^2

I_x = (200 mm)(100 mm)^3/36 = 5.556 x 10^6 mm^4
I_y = (200 mm)^3(100 mm)/36 = 22.222 x 10^6 mm^4

shape	A (mm^2)	$\bar{x}$ (mm)	$\bar{x}$A (mm^3)	$\bar{y}$ (mm)	$\bar{y}$A (mm^3)
A	20000	100	2000000	50	1000000
B	-2827.43	150	-424115	50	-141372
C	-3926.99	21.22066	-83333.3	50	-196350
D	10000	66.66667	666666.7	133.3333	1333333
	23245.58		2159218		1995612

$$\hat{x} = \frac{2159218\text{mm}^3}{23245.58\text{mm}^2} = 92.9 \text{ mm}$$

$$\hat{y} = \frac{1995612 \text{ mm}^3}{23245.58 \text{ mm}^2} = 85.8 \text{ mm}$$

shape	I_x(mm^4)	d_y (mm)	Ad_y^2 (mm^4)	I_y(mm^4)	d_x (mm)	Ad_x^2 (mm^4)
A	16666667	35.8	25632800	66666667	-7.1	1008200
B	-636173	35.8	-3623751.73	-636173	-57.1	-9218592.093
C	-2454369	35.8	-5032988.51	-686250	71.67934	-20176595.22
D	5555556	-47.5333	22594177.8	22222222	26.23333	6881876.029
	19131680		39570237.5	87566466		-21505111.29

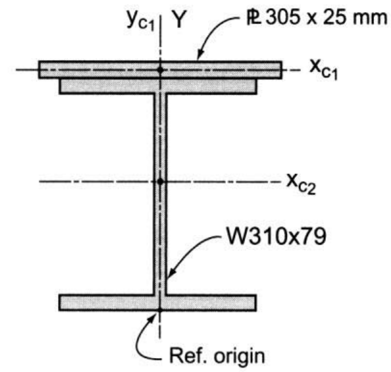
So, I_x = 19131680 + 39570237.5 = 58701918 = 58.7 x 10^6 mm^4

I_x = 87566466 +-21505111.3 = 43572025 = 66.1 x 10^6 mm^4

Example 4 (pg 258)

Example Problem 7.10 (Figures 7.35 and 7.36)

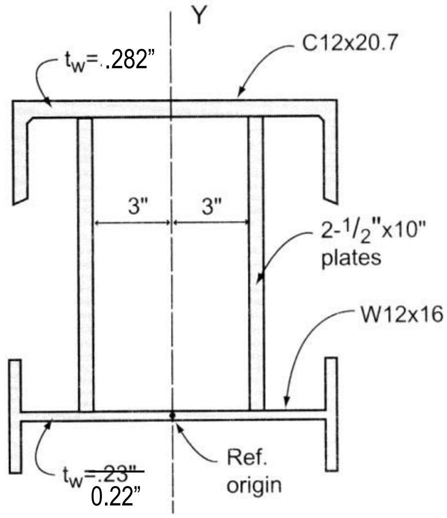
Locate the centroidal *x* and *y* axes for the cross-section shown. Use the reference origin indicated and assume that the steel plate is centered over the flange of the wide-flange section. Compute the I_x and I_y about the major centroidal axes.



Example 5 (pg 249)*

Example Problem 7.5 (Figures 7.16 and 7.17)

A composite or built-up cross-section for a beam is fabricated using two $\frac{1}{2}$" × 10" vertical plates with a C12 × 20.7 channel section welded to the top and a W12 × 16 section welded to the bottom as shown. Determine the location of the major x-axis using the center of the W12 × 16's web as the reference origin. Also determine the moment of inertia about both major centroidal axes.



shape	A (in²)	$\bar{x}$ (in)	$\bar{x}$A (in³)	$\bar{y}$ (in)	$\bar{y}$A (in³)
channel	6.09	0	0.00	9.694	59.04
left plate	5	-3.25	-16.25	5.11	25.55
right plate	5	3.25	16.25	5.11	25.55
wide flange	4.71	0	0.00	0	0.00
	20.80		0.00		110.14

$$\hat{x} = \frac{0\text{ in}^3}{20.8\text{ in}^2} = 0\text{ in}$$

$$\hat{y} = \frac{110.14\text{ in}^3}{20.8\text{ in}^2} = 5.295\text{ in}$$

shape	I_x (in⁴)	d_y (in)	Ad_y^2 (in⁴)	I_y (in⁴)	d_x (in)	Ad_x^2 (in⁴)
channel	3.880	-4.399	117.849	129.000	0.000	0.000
left plate	41.667	0.185	0.171	0.104	3.250	52.813
right plate	41.667	0.185	0.171	0.104	-3.250	52.813
wide flange	2.800	5.295	132.054	103.000	0.000	0.000
	90.013		250.245	232.208		105.625

I_x = 90.013 + 250.245 = 340.259 = 340.3 in^4

I_y = 232.208 + 105.625 = 337.833 = 337.8 in^4

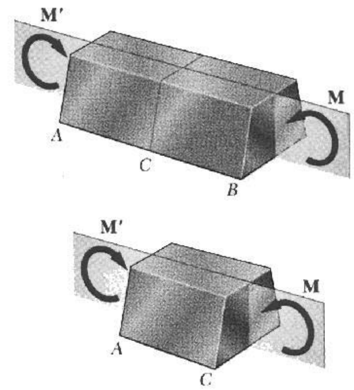
Beam Bending Stresses and Shear Stress

Notation:

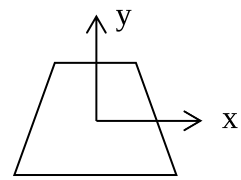
A = name for area
A_{web} = area of the web of a wide flange section
b = width of a rectangle
= total width of material at a horizontal section
c = largest distance from the neutral axis to the top or bottom edge of a beam
d = calculus symbol for differentiation
= depth of a wide flange section
d_y = difference in the y direction between an area centroid ($\bar{y}$) and the centroid of the composite shape ($\hat{y}$)
DL = shorthand for dead load
E = modulus of elasticity or Young's modulus
f_b = bending stress
f_c = compressive stress
f_{max} = maximum stress
f_t = tensile stress
f_v = shear stress
F_b = allowable bending stress
$F_{connector}$ = shear force capacity per connector
h = height of a rectangle
I = moment of inertia with respect to neutral axis bending
I_x = moment of inertia with respect to an x-axis
L = name for length
LL = shorthand for live load
M = internal bending moment
= name for a moment vector
n = number of connectors across a joint
$n.a.$ = shorthand for neutral axis (N.A.)
O = name for reference origin
p = pitch of connector spacing
P = name for a force vector
q = shear per length (shear flow)
Q = first moment area about a neutral axis
$Q_{connected}$ = first moment area about a neutral axis for the connected part
R = radius of curvature of a deformed beam
S = section modulus
$S_{req'd}$ = section modulus required at allowable stress
t_w = thickness of web of wide flange
V = internal shear force
$V_{longitudinal}$ = longitudinal shear force
V_T = transverse shear force
w = name for distributed load
x = horizontal distance
y = vertical distance
$\bar{y}$ = the distance in the y direction from a reference axis ($n.a$) to the centroid of a shape
$\hat{y}$ = the distance in the y direction from a reference axis to the centroid of a composite shape
Δ = calculus symbol for small quantity
δ = elongation or length change
ε = strain
θ = arc angle
Σ = summation symbol

Pure Bending in Beams

With bending moments along the axis of the member only, a beam is said to be in pure bending.



Normal stresses due to bending can be found for homogeneous materials having a plane of symmetry in the y axis that follow Hooke's law.



Maximum Moment and Stress Distribution

In a member of constant cross section, the maximum bending moment will govern the design of the section size when we know what kind of normal stress is caused by it.

For internal equilibrium to be maintained, the bending moment will be equal to the ΣM from the normal stresses $\times$ the areas $\times$ the moment arms. Geometric fit helps solve this statically indeterminate problem:

1. The normal planes remain normal for pure bending.
2. There is no net internal axial force.
3. Stress varies linearly over cross section.
4. Zero stress exists at the centroid and the line of centroids is the *neutral axis* (n. a)

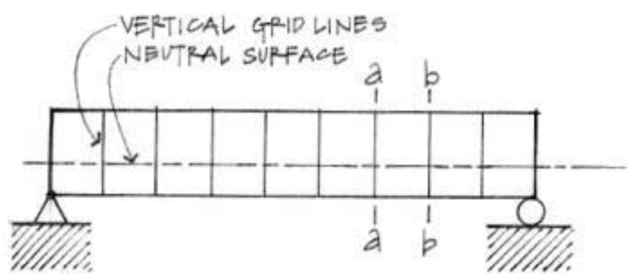
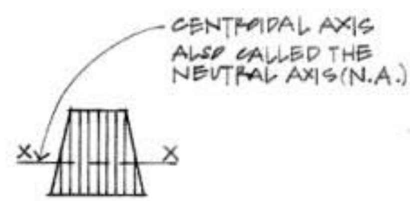


Figure 8.5(a) Beam elevation before loading.



Beam cross section.

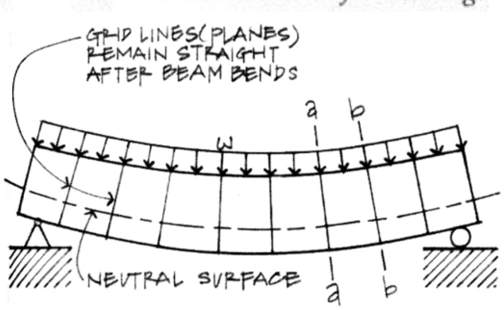


Figure 8.5(b) Beam bending under load.

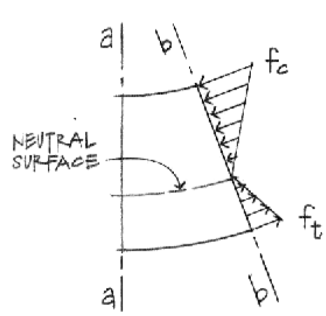
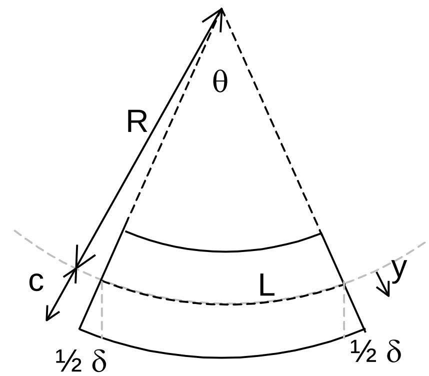


Figure 8.8 Bending stresses on section b-b.

Relations for Beam Geometry and Stress



Pure bending results in a circular arc deflection. R is the distance to the center of the arc; θ is the angle of the arc (radians); c is the distance from the n.a. to the *extreme fiber*; f_{max} is the maximum normal stress at the *extreme fiber*; y is a distance in y from the n.a.; M is the bending moment; I is the moment of inertia; S is the *section modulus.*

$$L = R\theta \qquad \varepsilon = \frac{\delta}{L} = R \qquad f = E\varepsilon = \frac{y}{c} f_{\max}$$

$$M = \Sigma f_i A_i \qquad M = \frac{f_{\max}}{c}\Sigma y_i^2 A_i \qquad I = \Sigma y^2 A \qquad S = \frac{I}{c} \qquad f_{\max} = \frac{Mc}{I} = \frac{M}{S}$$

Now: $f_b = \dfrac{My}{I}$ *for a rectangle of height h and width b:* $S = \dfrac{bh^3}{12\,h/2} = \dfrac{bh^2}{6}$

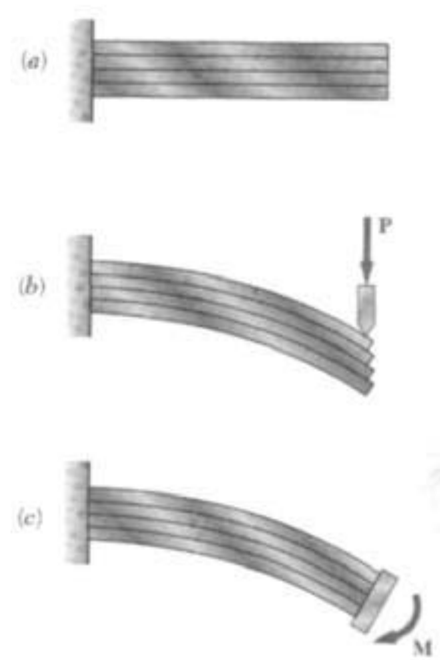
RELATIONS:

$$\boxed{\frac{1}{R} = \frac{M}{EI}} \qquad \boxed{f_b = \frac{My}{I}}^{*} \qquad \boxed{S = \frac{I}{c}}$$

$$\boxed{f_{b-\max} = \frac{Mc}{I} = \frac{M}{S}} \qquad \boxed{S_{required} \ge \frac{M}{F_b}}$$

**Note: y positive goes DOWN. With a positive M and y to the bottom fiber as positive, it results in a TENSION stress (we've called positive)*

Transverse Loading in Beams



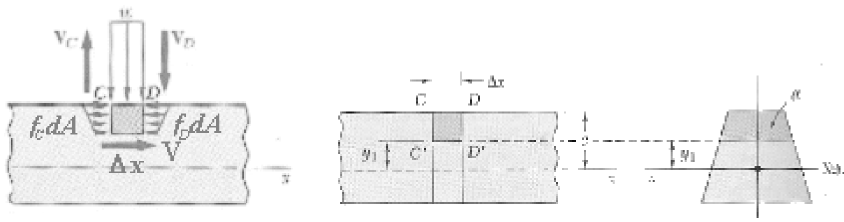
We are aware that transverse beam loadings result in internal shear and bending moments.

We designed sections based on bending stresses, since this stress dominates beam behavior.

There can be shear stresses *horizontally* within a beam member. It can be shown that $f_{horizontal} = f_{vertical}$

Equilibrium and Derivation

In order for equilibrium for any element CDD'C', there needs to be a horizontal force ΔH.



$$V = f_D dA - f_C dA$$

Q is a moment area with respect to the neutral axis of the area *above or below* the horizontal where the ΔH occurs.

$$V_{longitudinal} = \frac{V_T Q}{I}\Delta x$$

Q is a maximum when y = 0 (at the **neutral axis**).

q is a horizontal shear per unit length → *shear flow*

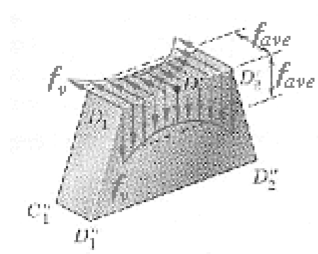
$$q = \frac{V_{longitudinal}}{\Delta x} = \frac{V_T Q}{I}$$

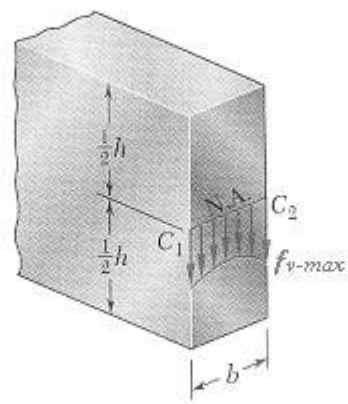
Shearing Stresses

f_{v-ave} = 0 on the beam's surface. Even if Q is a maximum at y = 0, we don't know that the thickness is a *minimum* there.

$$f_v = \frac{V}{\Delta A} = \frac{V}{b \cdot \Delta x}$$

$$\boxed{f_{v-ave} = \frac{VQ}{Ib}}$$





Rectangular Sections

$f_{v-\max}$ occurs at the neutral axis:

$$I = \frac{bh^3}{12} \qquad Q = A\bar{y} = b\,{}^{h}\!/_{2} \cdot {}^{1}\!/_{2}\,{}^{h}\!/_{2} = {}^{bh^2}\!/_{8}$$

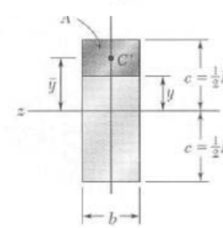
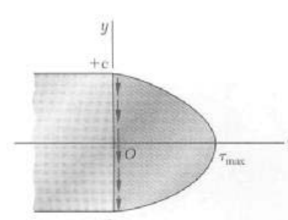


Fig. 6.15

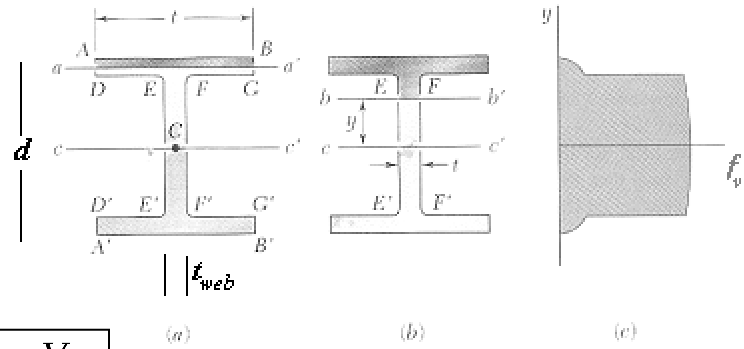
then:

$$f_v = \frac{VQ}{Ib} = \frac{V\,\frac{1}{8}bh^2}{\frac{1}{12}bh^3 b} = \frac{3V}{2bh}$$

$$\boxed{f_v = \frac{3V}{2A}}$$



Webs of Beams



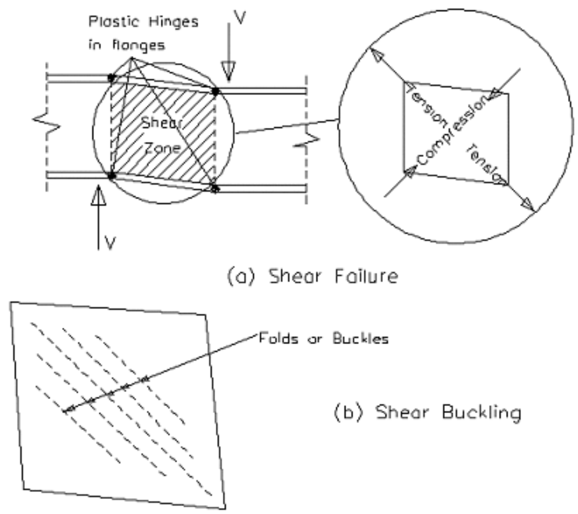
In steel W or S sections the thickness varies from the flange to the web.

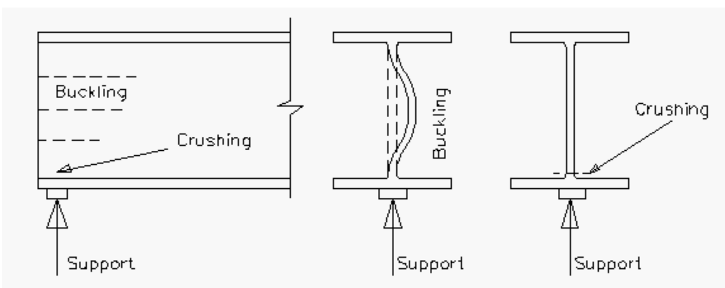
We neglect the shear stress in the flanges and consider the shear stress in the web to be constant:

$$f_{v-\max} = \frac{3V}{2A} \approx \frac{V}{A_{web}}$$

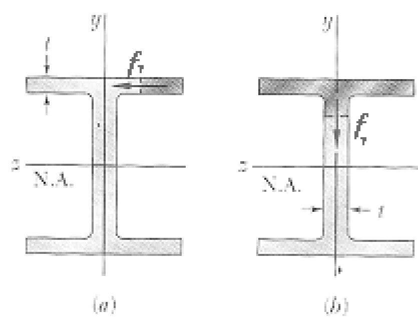
$$\boxed{f_{v-\max} = \frac{V}{t_{web}d}}$$

Webs of I beams can fail in tension shear across a panel with stiffeners or the web can buckle.



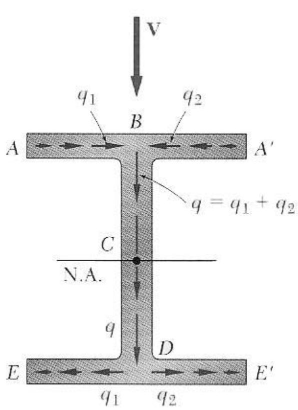


Shear Flow

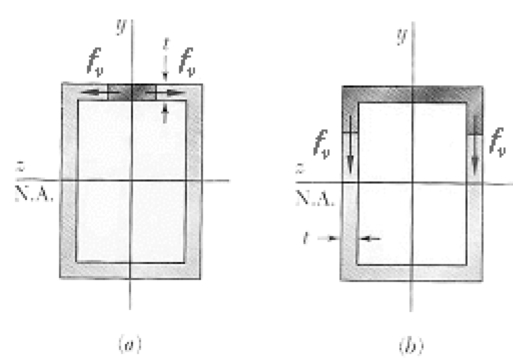


Even if the cut we make to find Q is not horizontal, but arbitrary, we can still find the shear flow, q, as long as the loads on thin-walled sections are applied in a plane of symmetry, and the cut is made *perpendicular* to the surface of the member.

$$q = \frac{VQ}{I}$$

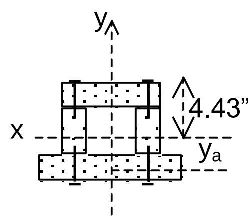


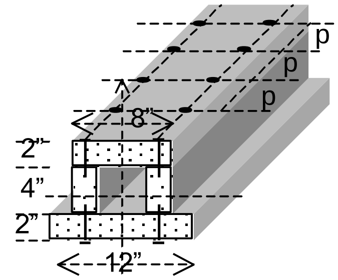
The shear flow magnitudes can be sketched by knowing Q.



Connectors to Resist Horizontal Shear in Composite Members

Typical connections needing to resist shear are plates with nails or rivets or bolts in composite sections or splices.





The pitch (spacing) can be determined by the capacity in shear of the connector(s) to the shear flow over the spacing interval, p.

$$\frac{V_{longitudinal}}{p} = \frac{VQ}{I} \qquad V_{longitudinal} = \frac{VQ}{I} \cdot p$$

where

p = pitch length

$$nF_{connector} \geq \frac{VQ_{connected\,area}}{I} \cdot p$$

n = number of connectors connecting the connected area to the rest of the cross section

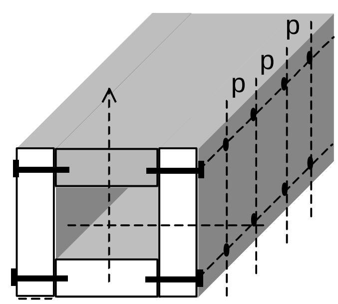
F = force capacity in one connector

$Q_{connected\ area} = A_{connected\ area} \times y_{connected\ area}$

$y_{connected\ area}$ = distance from the centroid of the connected area to the neutral axis

Connectors to Resist Horizontal Shear in Composite Members

Even vertical connectors have shear flow across them.



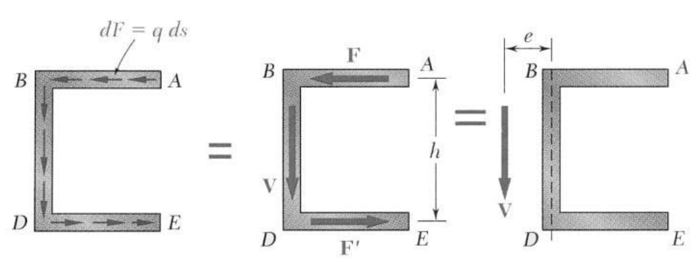
The spacing can be determined by the capacity in shear of the connector(s) to the shear flow over the spacing interval, p.

$$p \leq \frac{nF_{connector} I}{VQ_{connected\,area}}$$

Unsymmetrical Sections or Shear

If the section is <u>not</u> symmetric, or has a shear <u>not</u> in that plane, the member can bend and twist.

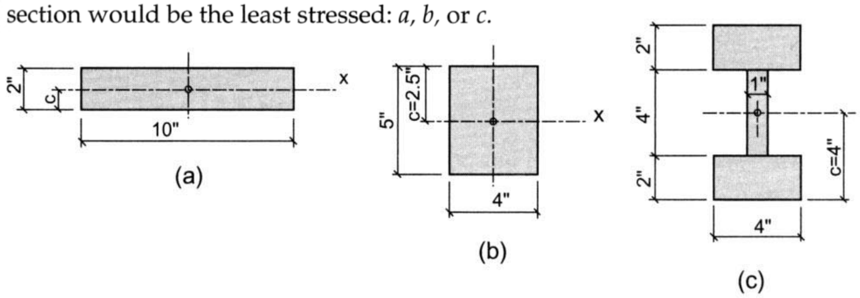
If the load is applied at the *shear center* there will not be twisting. This is the location where the moment caused by shear flow = the moment of the shear force about the shear center.

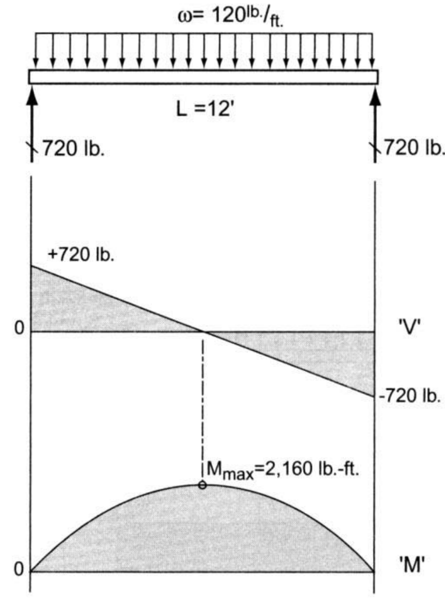


Example 1 (pg 303)

Example Problem 9.2 (Figures 9.15 to 9.18)

A beam must span a distance of 12' and carry a uniformly distributed load of 120 lb./ft. Determine which cross-section would be the least stressed: *a*, *b*, or *c*.





Example 2 (pg 309)

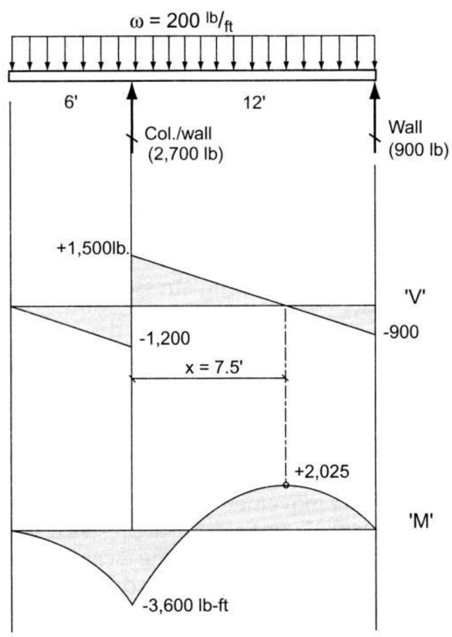
Example Problem 9.7 (Figures 9.31 to 9.33)

Design the roof and second-floor beams if F_b = 1550 psi (Southern pine No. 1), and evaluate the shear stress.

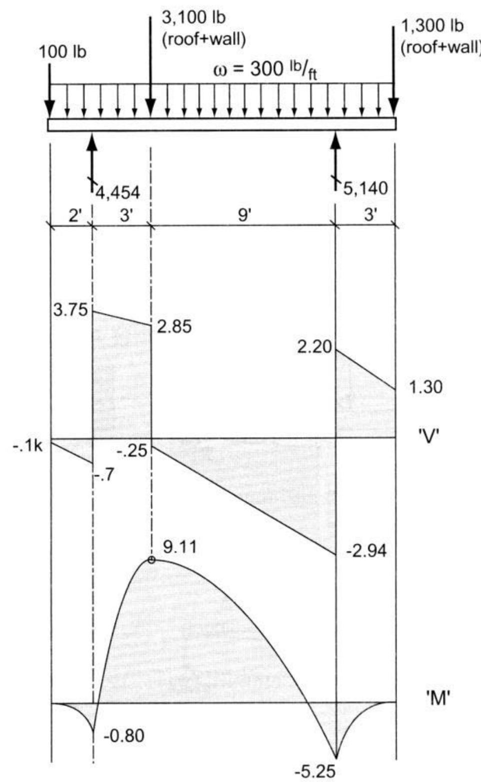
Roof: Snow +DL = 200 lb/ft
Walls: 400 lb on 2nd floor beams
Railing: 100 lb on beam overhang
Second Floor: DL + LL = 300 lb/ft
(including overhang)

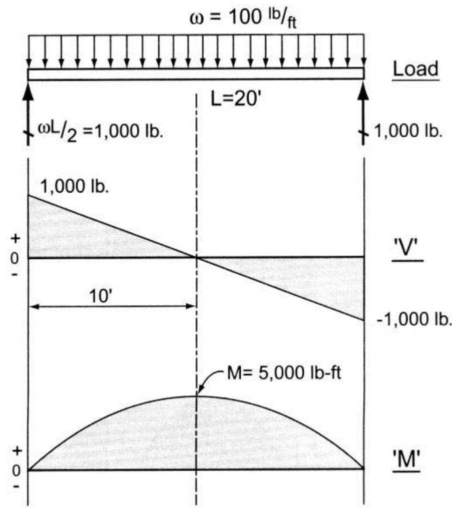
*Also select the most economical steel section for the second-floor when $S_{req'd} \geq 165$ in^3 and evaluate the shear stress when V = 60 k.

Roof:



Second Floor:



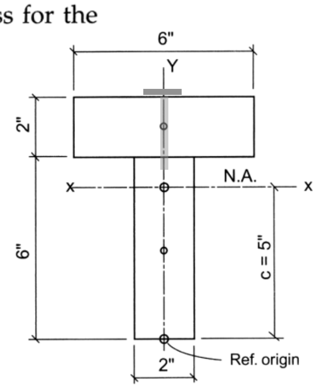


Example 3 (pg 313)

Example Problem 9.8: Shear Stress (Figures 9.43 to 9.47)

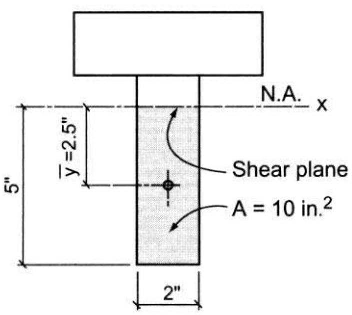
Calculate the maximum bending and shear stress for the beam shown.

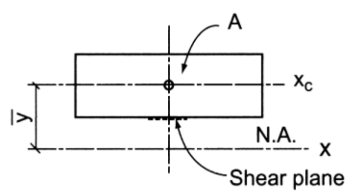
ALSO: Determine the minimum nail spacing required (pitch) if the shear capacity of a nail ($F_{connector}$) is 250 lb.



Component	A (in.^{2})	$\bar{y}$ (in.)	$\bar{y}\Delta A$ (in.^{3})
	12	7	84
	12	3	36

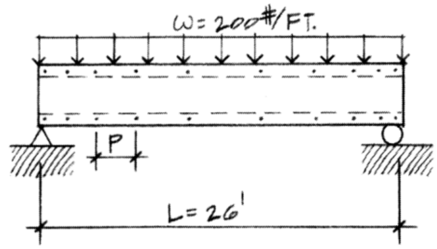
Component	I_{xc} (in.^{4})	A (in.^{2})	d_y (in.)	Ad_y^2 (in.^{4})
	4	12	2	48
	36	12	2	48





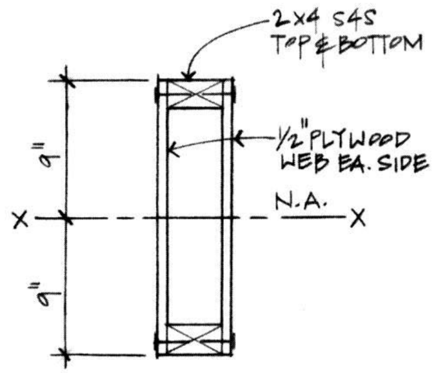
Example 4

8.11 A built-up plywood box beam with 2 × 4 S4S top and bottom flanges is held together by nails. Determine the pitch (spacing) of the nails if the beam supports a uniform load of 200 #/ft. along the 26-foot span. Assume the nails have a shear capacity of 80# each.



Solution:

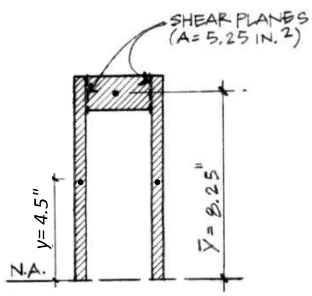
Construct the shear (V) diagram to obtain the critical shear condition and its location

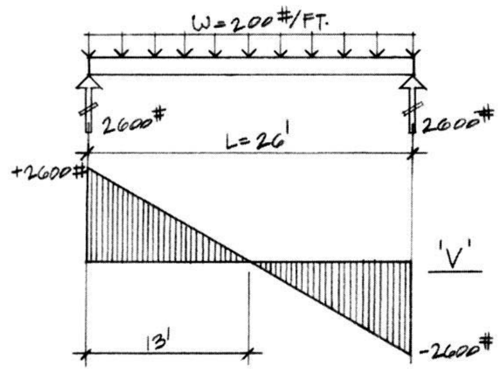


Note that the condition of shear is critical at the supports, and the shear intensity decreases as you approach the center line of the beam. This would indicate that the nail spacing P varies from the support to midspan. Nails are closely spaced at the support, but increasing spacing occurs toward midspan, following the shear diagram.

$$f_v = \frac{VQ}{Ib}$$

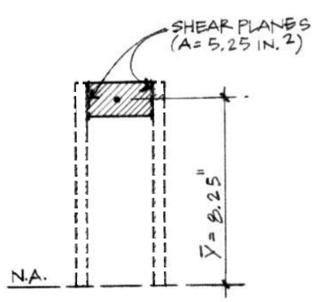
$$I_x = \frac{(4.5'')(18'')^3}{12} - \frac{(3.5'')(15'')^3}{12} = 1,202.6 \text{ in.}^4$$





$Q = \Sigma A\bar{y} = (9'')(½'')(4.5'')+(9'')(½'')(4.5'')+(1.5'')(3.5'')(8.25'') = 83.8 \text{ in}^3$

$$f_{v-\max} = \frac{(2{,}600\#)(83.3in.^3)}{(1{,}202.6in.^4)(\tfrac{1}{2}''+\tfrac{1}{2}'')} = 180.2\,psi$$



$Q = A\bar{y} = (5.25 \text{ in.}^2)(8.25'') = 43.3 \text{ in.}^3$

Shear force $= f_v \times A_v$

where:

A_v = shear area

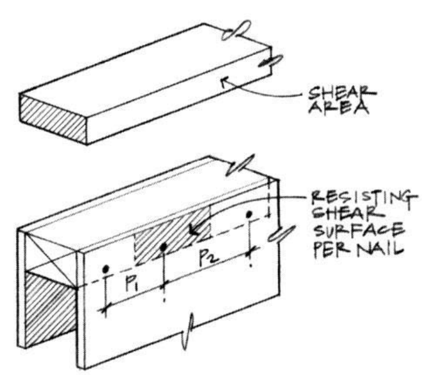
Assume:

(n)

(n)F = Capacity of two nails (one each side) at the flange; representing two shear surfaces

$$(n)F \geq f_v \times b \times p = \frac{VQ}{Ib} \times bp$$

$$\therefore\ (n)F \geq p \times \frac{VQ}{I}; \quad p \leq \frac{(n)FI}{VQ}$$



At the maximum shear location (support) where V = 2,600#

$$p = \frac{(2 \text{ nails} \times 80\ \#/\text{nail})(1{,}202.6 \text{ in.}^4)}{(2{,}600\#)(43.3 \text{ in.}^3)} = 1.71''$$

Introduction to Beam Stress Analysis and Preliminary Design

Beam Analysis

When the beam section is already known, beam analysis is used to calculate the maximum stresses. Beam design involves finding a trial section, recognizing that there is more load from the beam weight itself, performing analysis AND comparing stresses to some limits until the section satisfies all criteria.

Analysis Procedure

1. Solve for support forces and draw V & M diagrams to obtain V_{max} and M_{max} *(maximum magnitudes)*
2. Determine the critical section geometry properties:
   - centroid: $\hat{y}$ (*necessary to find the neutral axis, I_x, and to determine c – the distance from the neutral axis to the "extreme" fiber of the cross section)* (Note Set 9.1)
   - moment of inertia about axis of bending: I_x (Note Set 9.2)
   - section modulus S_x *($S_x = I_x/c$)*

   NOTE: if the section is a standard shape, the properties will be pre-determined and available in reference charts.
3. Calculate maximum bending stress using M_{max}: $f_{b-max} = \frac{Mc}{I_x} = \frac{M}{S_x}$
4. Calculate maximum shear stress using V_{max}:

   a. For a rectangular section ONLY: $f_v = \frac{3V}{2A}$

   - *A* is the area (*bh*)

   b. For a wide flange section ONLY: $f_v = \frac{V}{A_{web}}$

   - A_{web} is the area determined from the thickness of the web and depth of the W ($t_w d$). These values are available in reference charts.

   c. OTHERWISE: $f_{v-ave} = \frac{VQ}{I_x b}$ where:

   - *Q* is the first moment area of a section "cut" at the neutral axis. It is the sum of all the basic areas of the section multiplied by **y distances from the neutral axis** for each to their centroids: $Q = \sum A\bar{y}$. $\bar{y}$ *is always measured from the neutral axis as the origin (y=0)*. (Note Set 10.1)
   - *b* is the thickness of the section "cut" from the real material (voids aren't included).
   - I_x is the moment of inertia about the x axis (neutral axis)

5. If a section is built-up, and the shear force across an interface or the spacing for nails across that interface to resist the shear force is needed, then the form of the shear stress equation becomes:

$$nF_{connector} \geq \frac{VQ_{connected\,area}}{I_x} \cdot p$$

- *n* is the number of nails or bolts connecting the parts at the interface(s) of interest
- $F_{connector}$ is the shear force per nail or bolt that the connector can resists (capacity)
- $Q_{connected\ area}$ is the first moment of area a section "cut" at the interface(s) of interest to isolate the connected part. It is the sum of all the basic areas of the section multiplied by **y distances from the neutral axis** for each to their centroids: $Q = \sum A\bar{y}$. $\bar{y}$ *is always measured from the neutral axis as the origin (y=0).* (Note Set 10.1)
- *p* is the "pitch" spacing between connectors along the axis of the beam
- I_x is the moment of inertia about the x axis (neutral axis)

Beam Design

Design implies that the beam section has not yet been determined. Design involves choosing a trial section (preliminary design), then checking at every important computation of stress or deflection that the computed value does not exceed the acceptable limits. A finalized design means the section has been changed because of an unacceptable evaluation, but now meets all criteria.

Preliminary Design Procedure

The intent is to find the most light weight member satisfying the section modulus size.

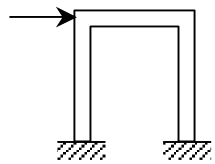
1. Know F_b (allowable stress) for the material or F_y & F_u for LRFD.
2. Draw V & M, finding M_{max}.
3. Calculate $S_{req'd}$ using M_{max}: $S_{required} \geq \frac{M}{F_b}$
   - This step is equivalent to evaluating if $f_b = \frac{M_{max}}{S_x} \leq F_b$
4. For rectangular beams $S_x = \frac{bh^2}{6}$
   - For steel or timber: use the section charts to find S that will work. And for steel, the design charts show the lightest section within a grouping of similar S's.
   - For any thing else, try a nice value for b, and calculate h or the other way around.

Pinned Frames and Arches

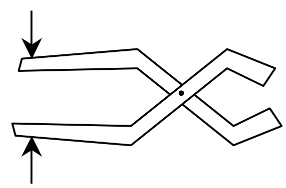
Notation:

F	= name for force vectors	*R*	= name for reaction force vector
F_x	= force component in the x direction	*w*	= name for distributed load
F_y	= force component in the y direction	*W*	= name for total force due to distributed load
FBD	= free body diagram		
M	= name for reaction moment, as is M_R	Σ	= summation symbol

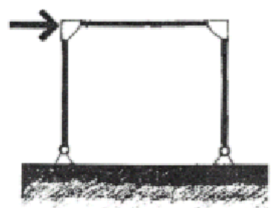
- A FRAME is made up of members where at least one member has more than 3 forces on it
  - Usually stationary and fully constrained

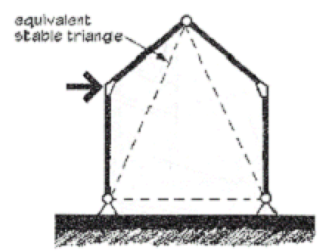


- A PINNED FRAME has member connected by pins
  - Considered *non-rigid* if it would collapse when the supports are removed
  - Considered *rigid* if it retains it's original shape when the supports are removed

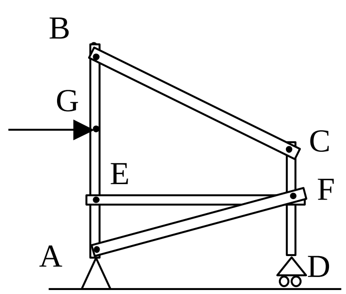


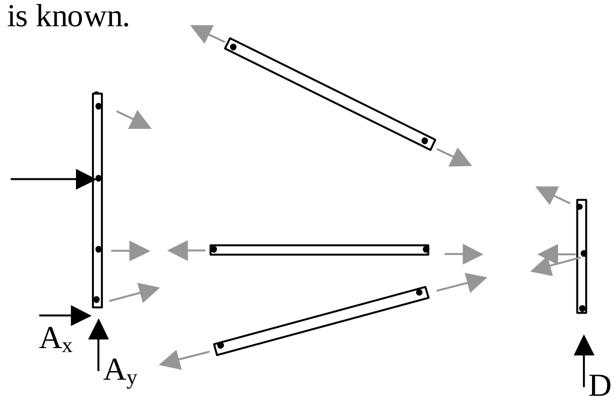
- A RIGID FRAME is all one member with no internal pins
  - Typically *statically indeterminate*
  - **Portal** frames look like door frames
  - Gable frames have a peak.



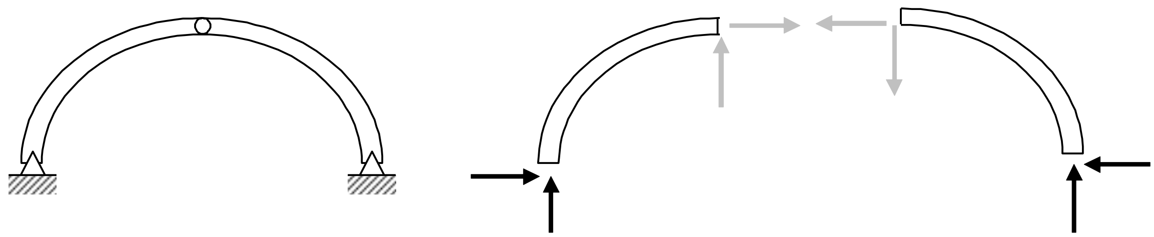


- INTERNAL PIN CONNECTIONS:
  - Pin connection forces are **equal** and **opposite** between the bodies they connect.
  - There are 2 unknown forces at a pin, but if we know a body is a **two-force** body, the direction of the *resultant* force is known.

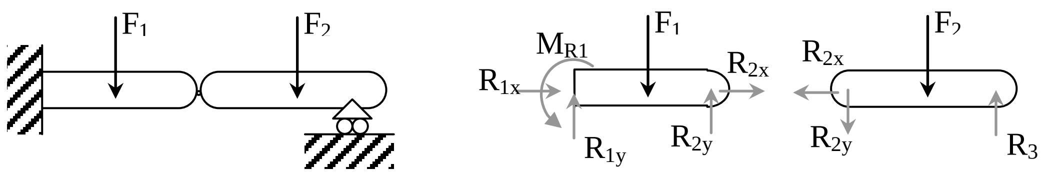




- AN ARCH is a structural shape that can span large distances and sees compression along its slope. It may have no hinges (or pins), two hinges at the supports, or two hinges at the supports with a hinge at the apex. The three-hinged arch types are statically determinate with 2 bodies and **6** unknown forces.



- CONTINUOUS BEAMS WITH PINS:
  - If pins within the span of a beam over multiple supports result in static determinacy (the right number of unknowns for the number of equilibrium equations), the internal forces at the pins are applied as reactions to the adjacent span.



  - The location of the internal pins can be chosen to increase or decrease the moments in order to make the section economical for both positive bending and negative bending (similar values for the moments).

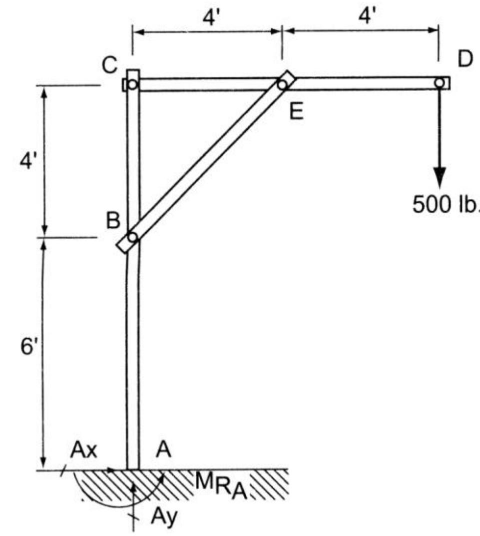
Solution Procedure

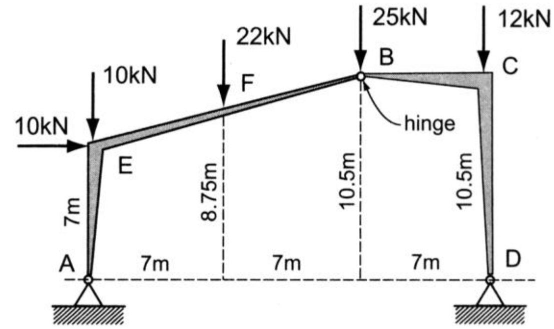
1. Solve for the support forces on the entire frame (FBD) if possible.
2. Draw a FBD of each member:
   - Consider all two-force bodies first.
   - Pins are integral with members
   - Pins with applied forces should belong to members with greater than two forces [Same if pins connect 3 or more members]
   - Draw forces on either side of a pin equal and opposite with arbitrary direction chosen for the first side
   - Consider all multi-force bodies
   - Represent connection forces not known by x & y components
   - There are still three equilibrium equations available, but the moment equations may be more helpful when the number of unknowns is greater than two.

Example 1 (pg 114)

Example Problem 4.12

A pinned frame with a fixed base at *A* supports a load at the overhang equal to 500 pounds, as shown in Figure 4.68. Draw free-body diagrams and solve for the support reactions and the pin reactions at *B*, *C*, and *E*.





Example 2 (pg 115)

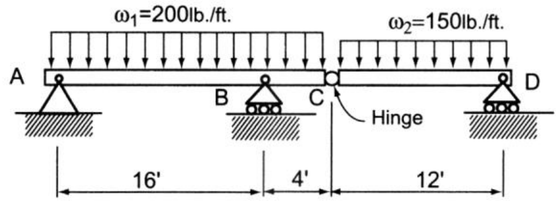
Example 4.13 (Three-Hinged Arch)

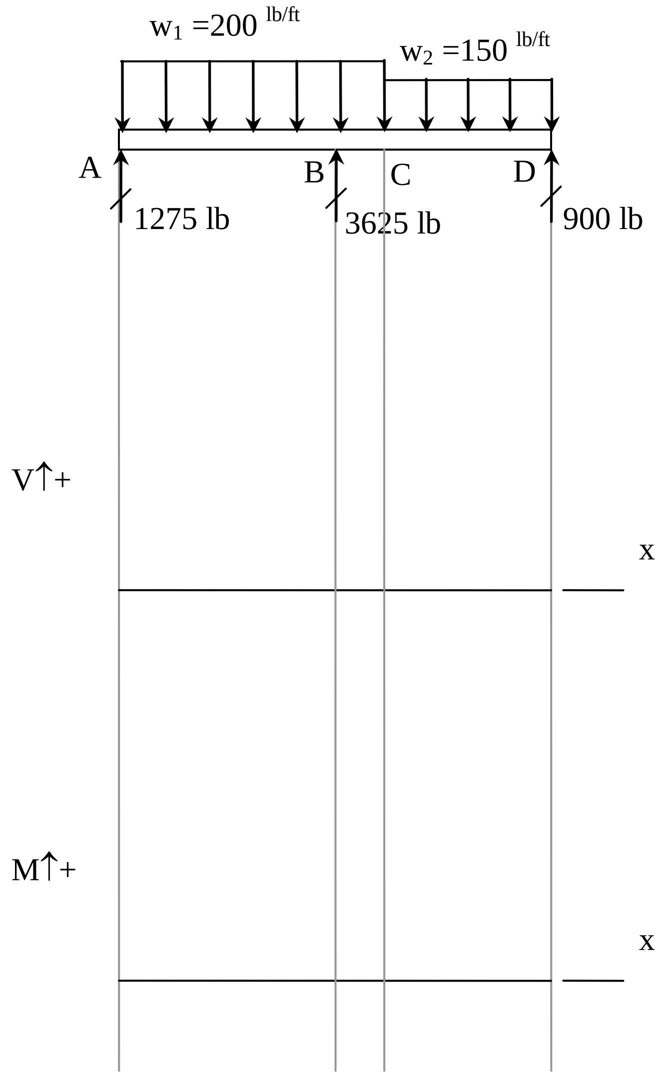
An industrial building is framed using tapered steel sections (haunches) and connected with three hinges (Figure 4.70). Assuming that the loads shown are from gravity loads and wind, determine the support reactions at *A* and *D* and the pin reactions at *B*.

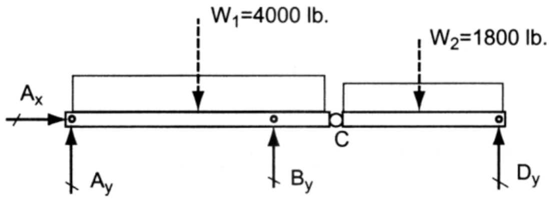
<u>Example 3 (pg 73)</u>

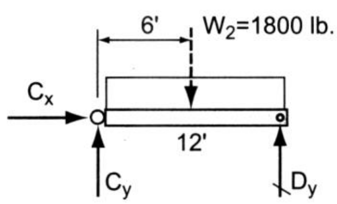
Example Problem 3.16 (Figures 3.44 and 3.45)

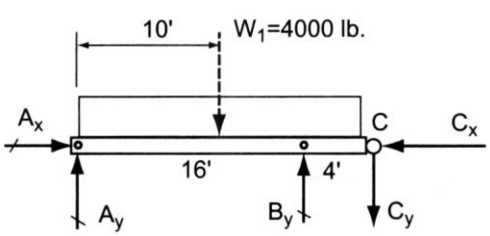
A compound beam has three supports at ***A***, ***B*** and ***D*** and an internal hinge at ***C***. Two uniformly distributed loads cover the entire length of the beams. Draw the appropriate FBDs and determine the reactions at the supports and the internal pin forces at ***C***.
Also construct the shear and bending moment diagrams.











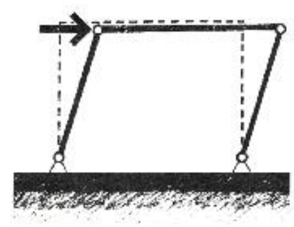
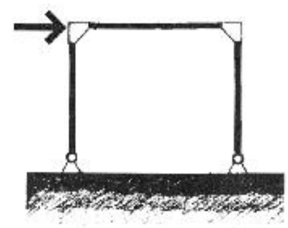
Rigid and Braced Frames

Notation:

E = modulus of elasticity or Young's modulus
F_x = force component in the x direction
F_y = force component in the y direction
FBD = free body diagram
G = relative stiffness of columns to beams in a rigid connection, as is Ψ
I = moment of inertia with respect to neutral axis bending
k = effective length factor for columns
ℓ_b = length of beam in rigid joint
ℓ_c = length of column in rigid joint
L = name for length
M = internal bending moment
= name for a moment vector
V = internal shear force
Σ = summation symbol

Rigid Frames

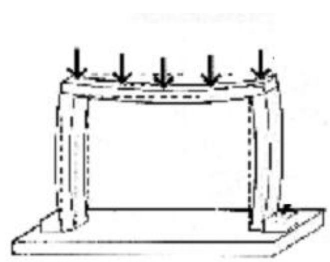
Rigid frames are identified by the lack of pinned joints within the frame. The joints are *rigid* and resist rotation. They may be supported by pins or fixed supports. They are typically statically indeterminate.

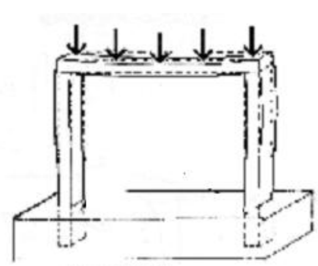



Frames are useful to resist **lateral** loads.

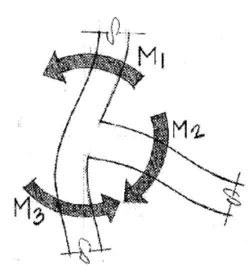
Frame members will see
- shear
- bending
- axial forces

and behave like *beam-columns.*





Behavior



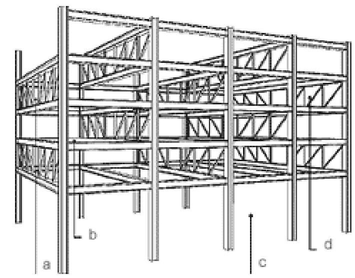
The relation between the joints has to be maintained, but the whole joint can *rotate*. The amount of rotation and distribution of moment depends on the *stiffness* (EI/L) of the members in the joint.

End restraints on columns reduce the effective length, allowing columns to be more slender. Because of the rigid joints, deflections and moments in beams are reduced as well.

Frames are sensitive to settlement because it induces strains and changes the stress distribution.

Types

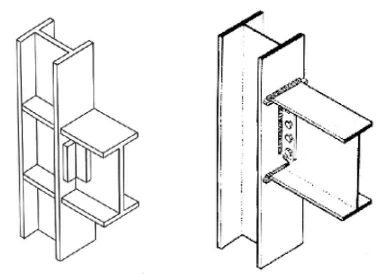
Gabled – has a peak


Staggered Truss

Portal – resembles a door. Multi-story, multiple bay portal frames are commonly used for commercial and industrial construction. The floor behavior is similar to that of continuous beams.

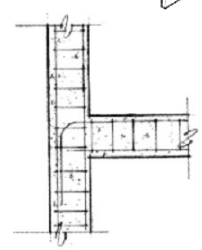
Staggered Truss – Full story trusses are staggered through the frame bays, allowing larger clear stories.

Connections



Steel – Flanges of members are fully attached to the flanges of the other member. This can be done with welding, or bolted plates.

Reinforced Concrete – Joints are monolithic with continuous reinforcement for bending. Shear is resisted with stirrups and ties.

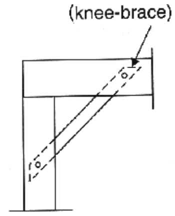


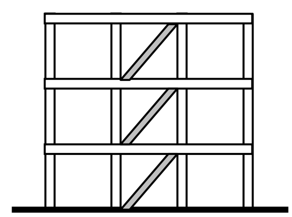
Braced Frames

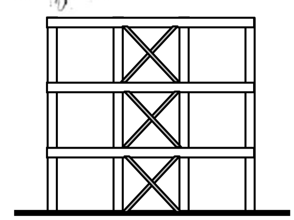
Braced frames have beams and columns that are "pin" connected with bracing to resist lateral loads.

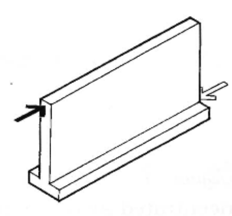
Types of Bracing

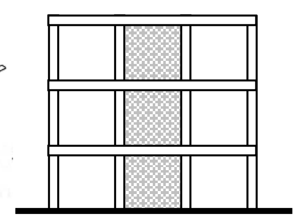
- knee-bracing
- diagonal (including eccentric)
- X
- K or chevron
- shear walls – which resist lateral forces in the plane of the wall

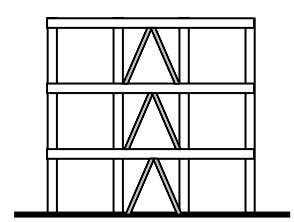



diagonal


X



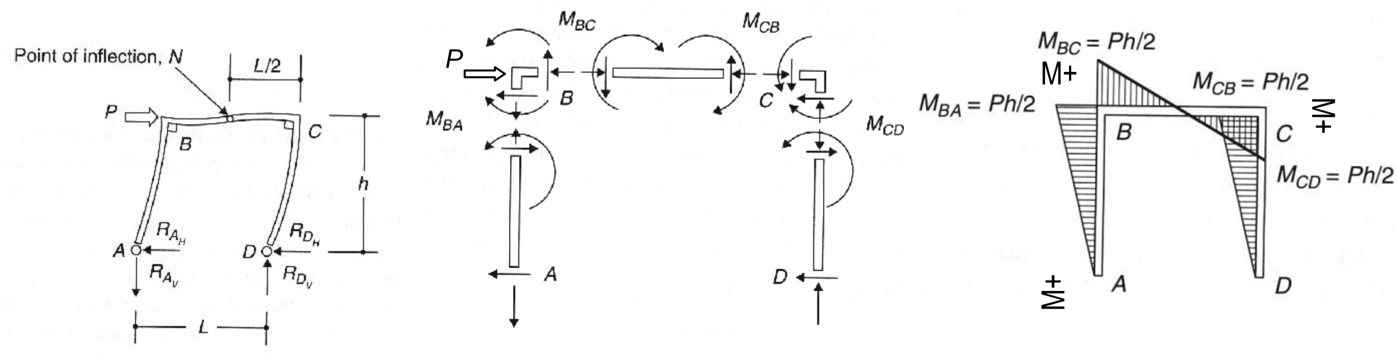

shear walls


K (chevron)

Rigid Frame Analysis

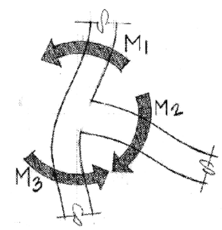
Structural analysis methods such as the *portal method* (approximate), the *method of virtual work, Castigliano's theorem*, the *force method*, the *slope-displacement method*, the *stiffness method*, and *matrix analysis*, can be used to solve for internal forces and moments and support reactions.

Shear and bending moment diagrams can be drawn for frame members by isolating the member from a joint and drawing a free body diagram. The internal forces at the end will be equal and opposite, just like for connections in *pinned frames*. Direction of the "beam-like" member is usually drawn by looking from the "inside" of the frame.



Frame Columns

Because joints can rotate in frames, the effective length of the column in a frame is harder to determine. The stiffness (EI/L) of each member in a joint determines how rigid or flexible it is. To find k, the relative stiffness, G or Ψ, must be found for both ends, plotted on the alignment charts, and connected by a line for braced and unbraced fames.



$$G = \Psi = \frac{\Sigma EI/l_c}{\Sigma EI/l_b}$$

where

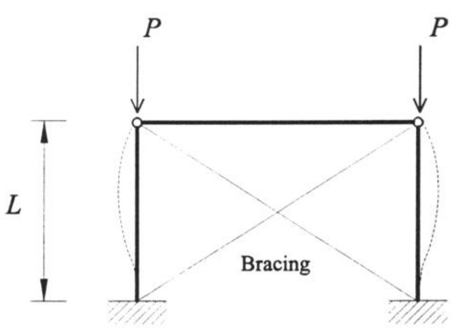
E = modulus of elasticity for a member

I = moment of inertia of for a member

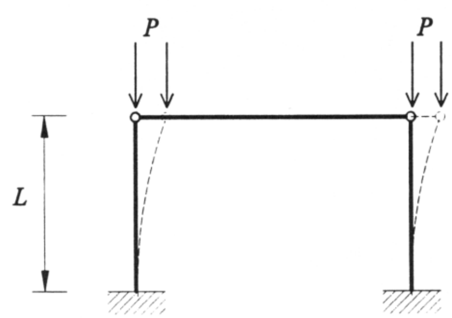
l_c = length of the column from center to center

l_b = length of the beam from center to center

- For pinned connections we typically use a value of 10 for Ψ.
- For fixed connections we typically use a value of 1 for Ψ.



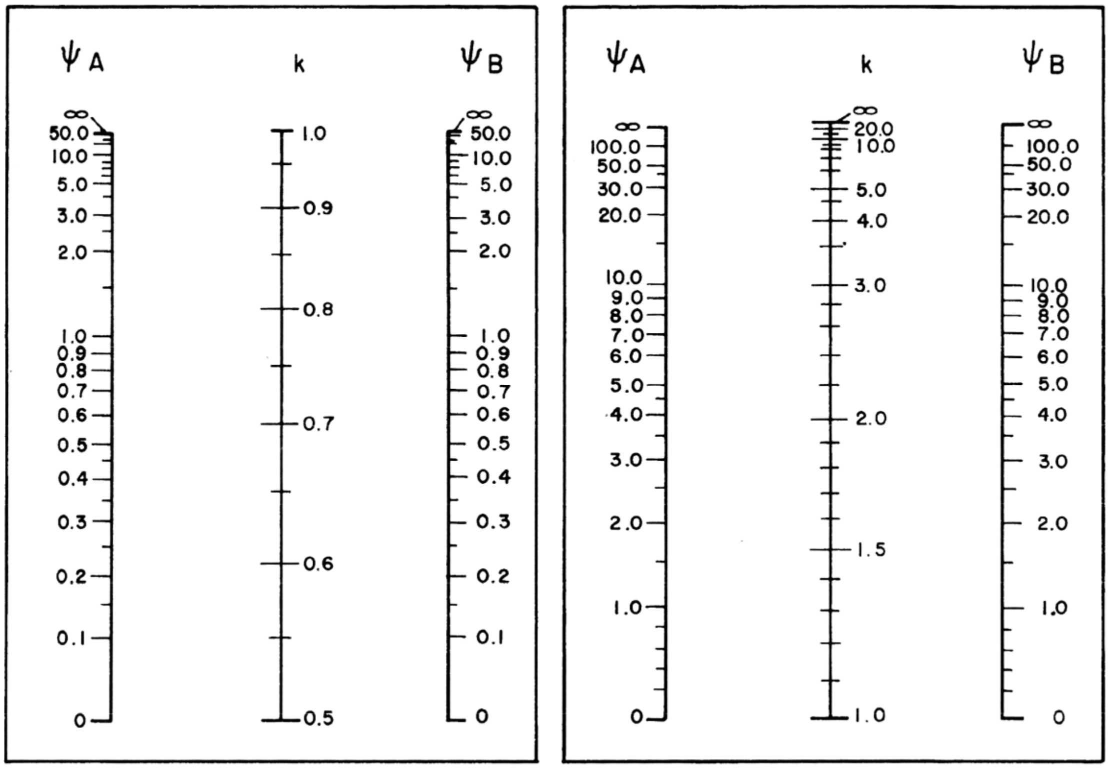
Braced – non-sway frame



Unbraced – sway frame

Frame Design

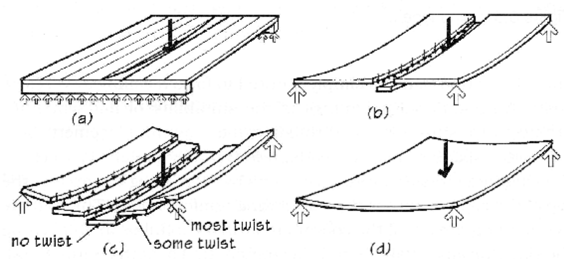
The possible load combinations for frames with dead load, live load, wind load, etc. is critical to the design. The maximum moments (positive and negative) may be found from different combinations and at different locations. Lateral wind loads can significantly affect the maximum moments.



(a) Nonsway Frames

(b) Sway Frames

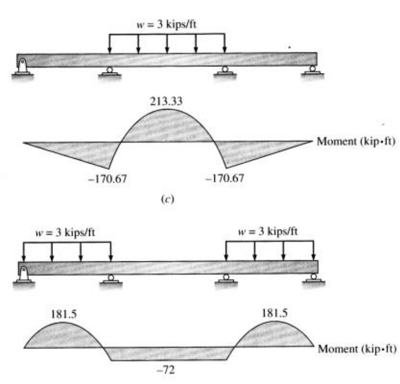
Plates and Slabs



If the frame is rigid or non-rigid, the floors can be a plate or slab (which has drop panels around columns). These elements behave differently depending on their supports and the ratio of the sides.

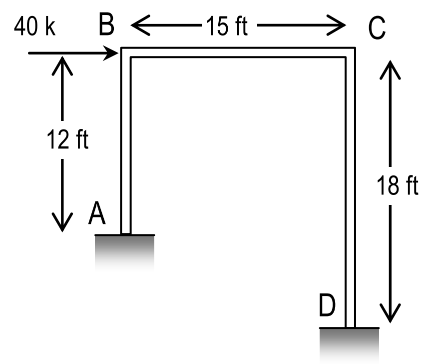
- one-way behavior: like a "wide" beam, when ratio of sides > 1.5
- two-way behavior: complex, non-determinate, look for handbook solutions

Floor Loading Patterns



With continuous beams or floors, the worst case loading typically occurs when alternate spans are loaded with live load (not every span). The maximum positive and negative moments may not be found for the same loading case! If you are designing with reinforced concrete, you must provide flexure reinforcement on the top and bottom and take into consideration that the maximum may move.

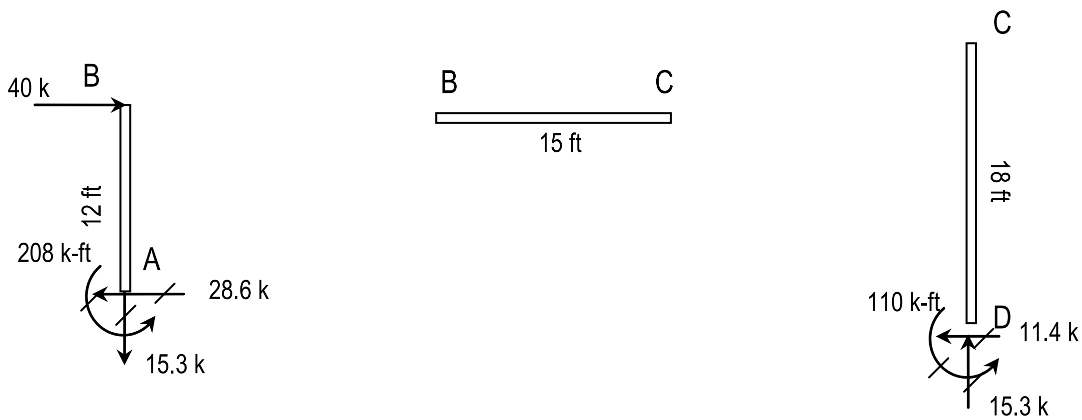
Example 1

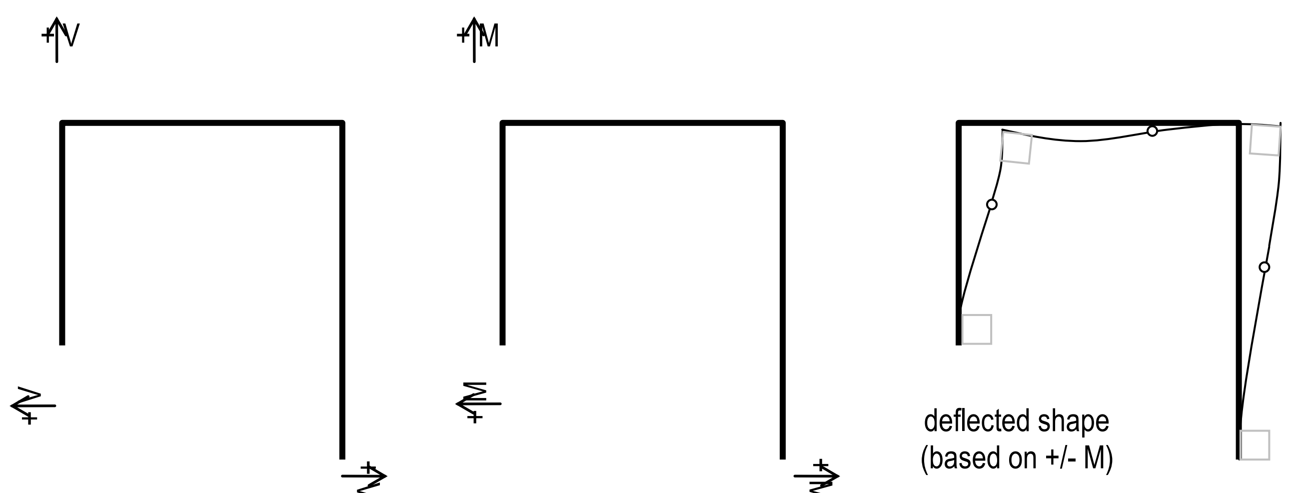


The rigid frame shown has been analyzed using an advanced structural analysis technique. The reactions at support A are: A_x = -28.6 k, A_y = -15.3 k, M_A = 208 k-ft. The reactions at support D are: D_x = -11.4 k, D_y = 15.3 k, M_D = 110 ft-k. Draw the shear and bending moment diagrams, and identify V_{max} & M_{max}.

Solution:

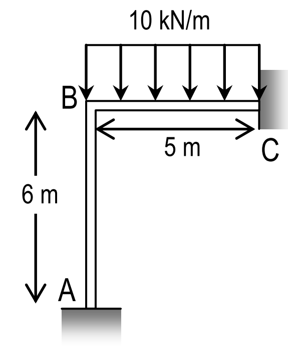
NOTE: The joints are not shown, and the load at joint B is put on <u>*only*</u> *one body.*



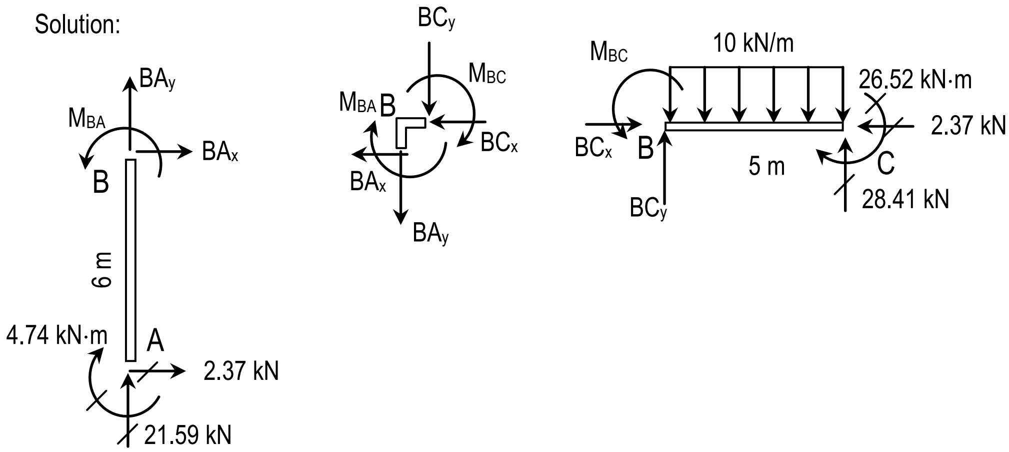


Example 2

The rigid frame shown has been analyzed using an advanced structural analysis technique. The reactions at support A are: A_x = 2.37 kN, A_y = 21.59 kN, M_A = -4.74 kN·m. The reactions at support C are: C_x = -2.37 kN, C_y = 28.4 kN, M_C = -26.52 kN·m. Draw the shear and bending moment diagrams, and identify V_{max} & M_{max}.



Solution:



Reactions These values must be given or found from non-static analysis techniques. The values are given with respect to the global coordinate system we defined for positive and negative forces and moments for equilibrium.

Member End Forces The free-body diagrams of all the members and joints of the frame are shown above. The unknowns on the members are drawn positive, and the opposite directions are drawn on the joint. We can begin the computation of internal forces with either member AB or BC, both of which have only three unknowns.

Member AB With the magnitudes of reaction forces at A know, the unknowns are at end B of BA_x, BA_y, and M_{BA}, which can get determined by applying $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma M_B = 0$. Thus,

$$\Sigma F_x = 2.37kN + BA_x = 0 \quad BA_x = -2.37\text{ kN}, \qquad \Sigma F_y = 21.59kN + BA_y = 0 \quad BA_y = -21.59\text{ kN}$$

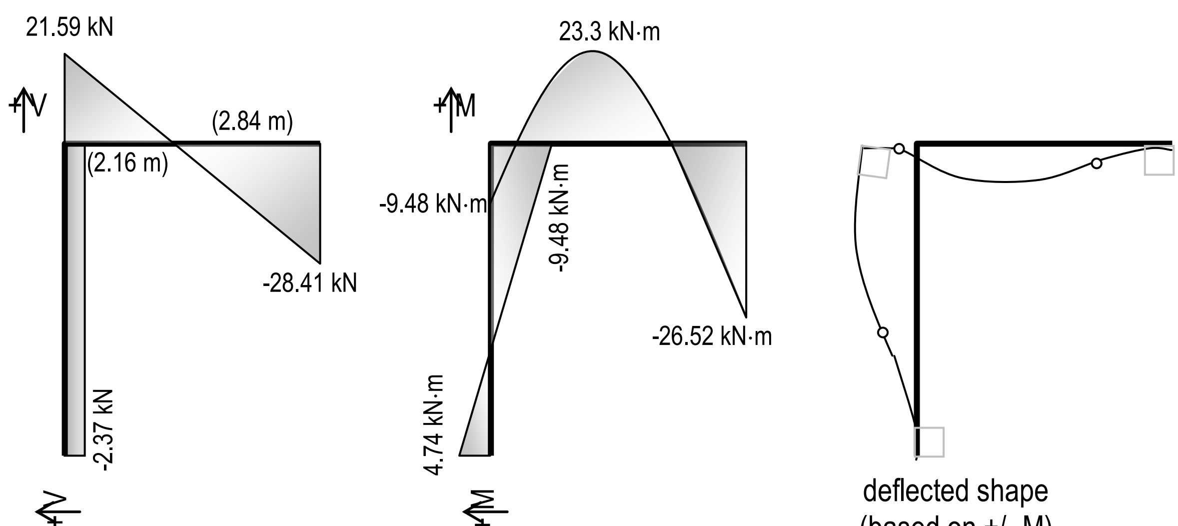
$$\Sigma M_B = 2.37kN(6m) - 4.74kN\cdot m + M_{BA} = 0 \quad M_{BA} = -9.48\text{ kN·m}$$

Joint B Because the forces and moments must be equal and opposite, BC_x = 2.37 kN, BC_y = 21.59 kN and M_{BC} = 9.48 kN·m

Member BC All forces are known, so equilibrium can be checked:

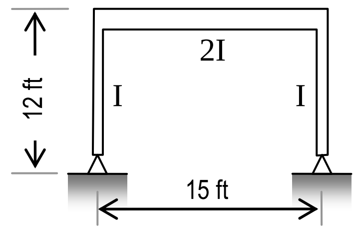
$$\Sigma F_x = 2.37kN - 2.37kN = 0 \qquad \Sigma F_y = 21.59kN + 28.49kN - (10kN/m)5m = 0$$

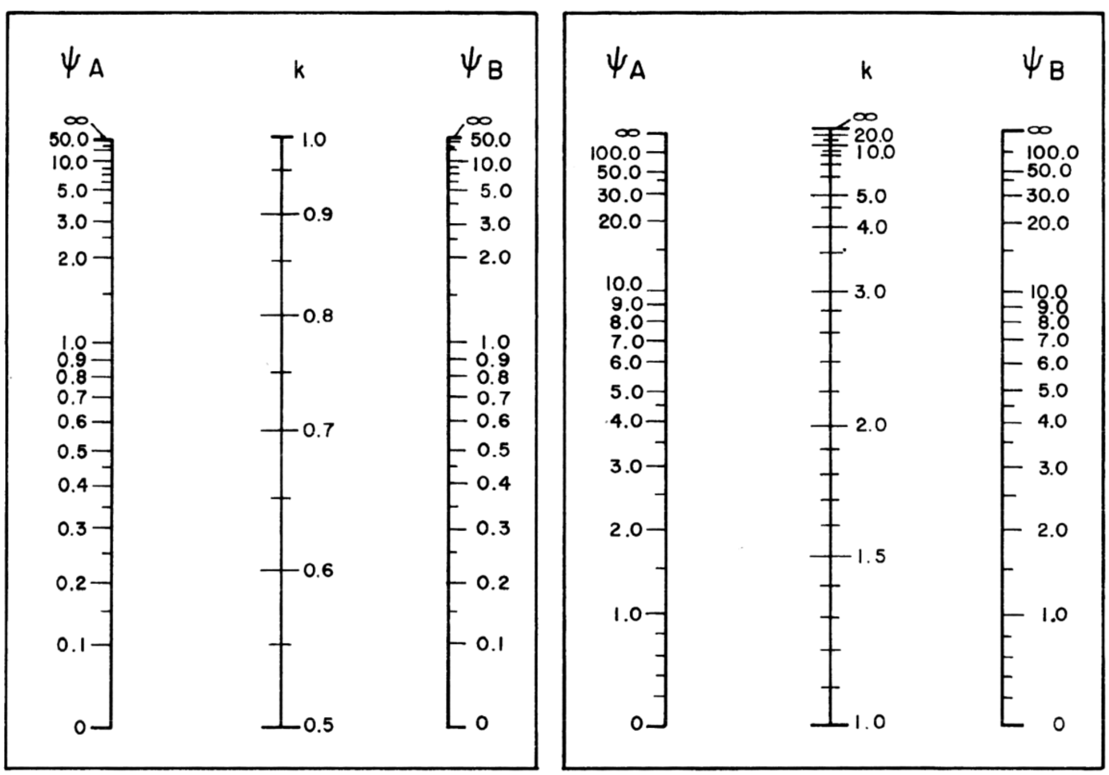
$$\Sigma M_B = 28.41kN(5m) - 10kN/m(5m)(2.5m) - 26.52kN\cdot m + 9.48kN\cdot m = 0$$



Example 3

Find the column effective lengths for a steel frame with 12 ft columns, a 15 ft beam when the support connections are pins for a) when it is braced and b) when it is allowed to sway. The relative stiffness of the beam is twice that of the columns (2I).





(a) Nonsway Frames

(b) Sway Frames

Columns and Stability

Notation:

A = name for area
$A36$ = designation of steel grade
b = name for width
C = symbol for compression
C_c = column slenderness classification constant for steel column design
d = name for dimension, or depth
e = eccentric distance of application of a force (P) from the centroid of a cross section
E = modulus of elasticity or Young's modulus
f_a = axial stress
f_b = bending stress
$f_{critical}$ = critical buckling stress in column calculations from $P_{critical}$
f_x = total stress in the x axis direction
F_a = allowable axial stress
F_b = allowable bending stress
F_y = yield stress
I = moment of inertia
I_{min} = moment of inertia that is critical to the calculation of slenderness ratio
K = effective length factor for columns
L = name for length
L_e = effective length that can buckle for column design, as is ℓ_e , $L_{effective}$
M = internal bending moment, as is M'
$N.A.$ = shorthand for neutral axis
P = name for axial force vector, as is P'
P_{crit} = critical buckling load in column calculations, as is $P_{critical}$, P_{cr}
r = radius of gyration
T = symbol for compression
W = designation for wide flange section
y = vertical distance
z = distance perpendicular to the x-y plane
Δ = calculus symbol for small quantity
= displacement due to bending
θ = angle
ϕ = diameter symbol
π = pi (3.1415 radians or 180°)
σ = engineering symbol for normal stress

Design Criteria

Including strength (stresses) and servicability (including deflections), another requirement is that the structure or structural member be *stable*.

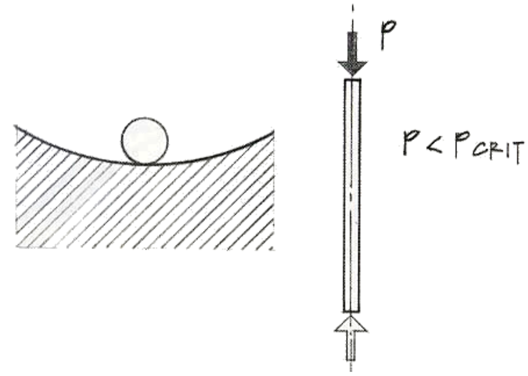
Stability is the ability of the structure to support a specified load without undergoing unacceptable (or sudden) deformations.

Physics

Recall that things like to be or *prefer* to be in their lowest energy state (potential energy). Examples include water in a water tank. The energy it took to put the water up there is stored until it is released and can flow due to gravity.

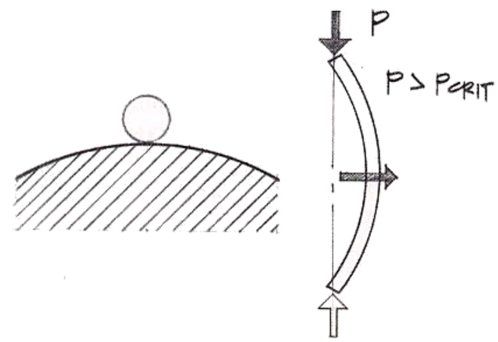
Stable Equilibrium

When energy is added to an object in the form of a push or disturbance, the object will return to it's original position. *Things don't change in the end.*



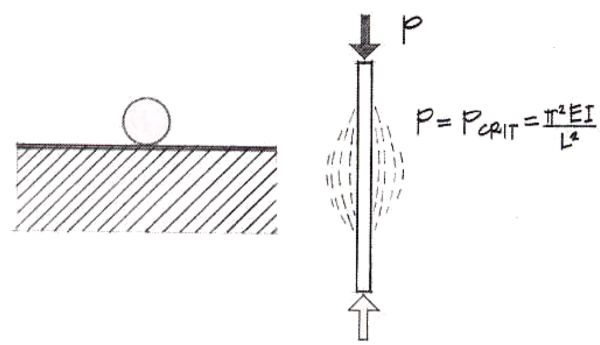
Unstable Equilibrium

When energy is added to an object, the object will move and get more "disturbed". *Things change rapidly* .



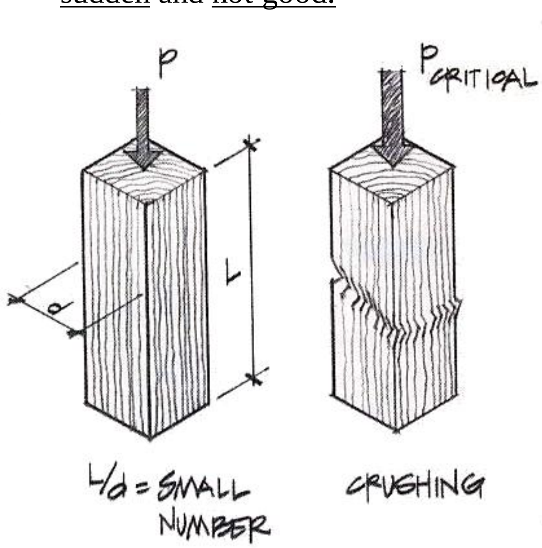
Neutral Equilibrium

When energy is added to an object, the object will move some then stop.. *Things change.*

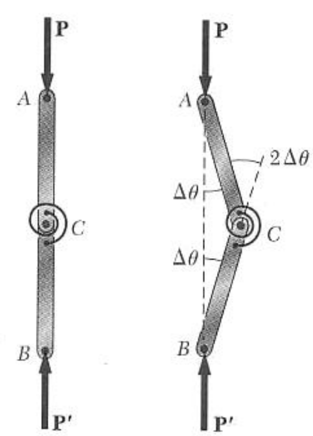


Column with Axial Loading

A column loaded centrically can experience unstable equilibrium, called *buckling*, because of how tall and slender they are. This instability is sudden and not good.



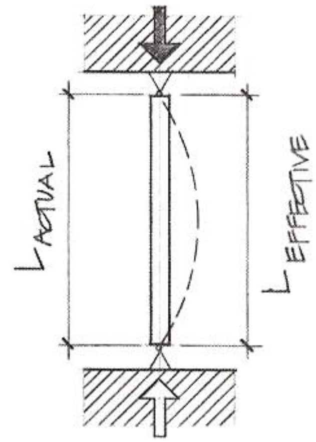
Buckling can occur in sheets (like my "memory metal" cookie sheet), pressure vessels or slender (narrow) beams not braced laterally.



Buckling can be thought of with the loads and motion of a column having a stiff spring at mid-height. There exists a load where the spring can't resist the moment in it any longer.

Short (stubby) columns will experience crushing before buckling.

Critical Buckling Load



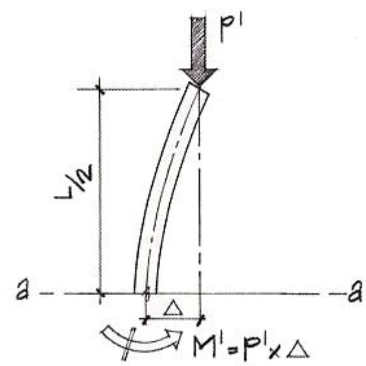
The critical axial load to cause buckling is related to the deflected shape we could get (or determine from bending moment of P·Δ).

The buckled shape will be in the form of a *sine wave*.

Euler Formula

Swiss mathematician Euler determined the relationship between the critical buckling load, the material, section and <u>effective length</u> (as long as the material stays in the elastic range):

$$P_{critical} = \frac{\pi^2 E I_{\min}}{(L)^2} \qquad \text{or} \qquad P_{cr} = \frac{\pi^2 EI}{(L_e)^2} = \frac{\pi^2 EA}{\left(L_e/r\right)^2}$$



and the critical stress (*if less than the normal stress)* is:

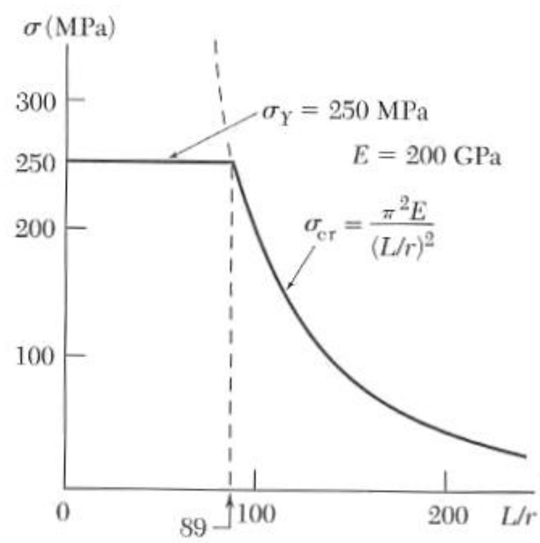
$$f_{critical} = \frac{P_{critical}}{A} = \frac{\pi^2 EAr^2}{A(L_e)^2} = \frac{\pi^2 E}{\left(L_e/r\right)^2}$$

where $I=Ar^2$ and L_e/r is called the <u>slenderness ratio</u>. The smallest I of the section will govern.

Yield Stress and Buckling Stress

The two design criteria for columns are that they do not buckle and the strength is not exceeded. Depending on slenderness, one will control over the other.

But, because in the real world, things are rarely perfect – and columns will not actually be loaded concentrically, but will see eccentricity – Euler's formula is used only if the critical stress is less than half of the yield point stress:



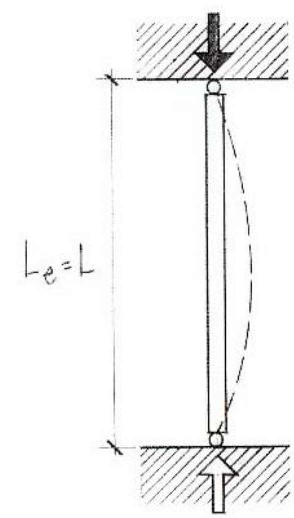
$$P_{critical} = \frac{\pi^2 E I_{\min}}{(L)^2}\ ; \quad f_{critical} = \frac{P_{critical}}{A} < \frac{F_y}{2}$$

to be used for $L_e/r > C_c = \sqrt{\dfrac{2\pi^2 E}{F_y}}$

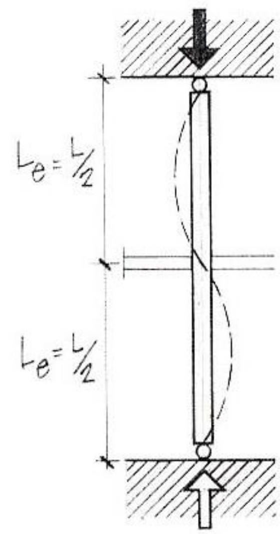
where C_c is the column slenderness classification constant and is the slenderness ratio of a column for which the critical stress is equal to half the yield point stress.

Effective Length and Bracing

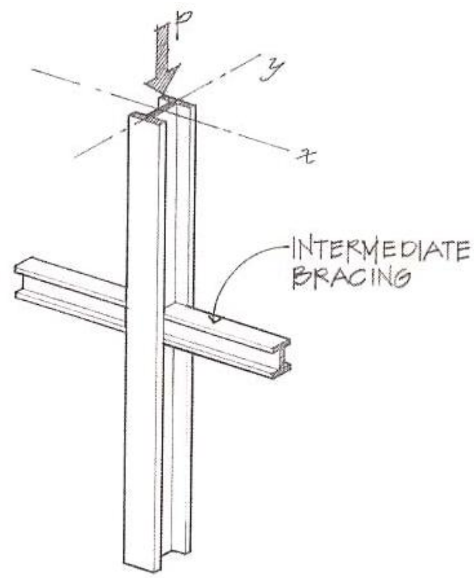
Depending on the end support conditions for a column, the effective length can be found from the deflected shape (elastic equations). If a very long column is braced intermittently along its length, the column length that will buckle can be determined. The effective length can be found by multiplying the column length by an effective length factor, K. $L_e = K \cdot L$

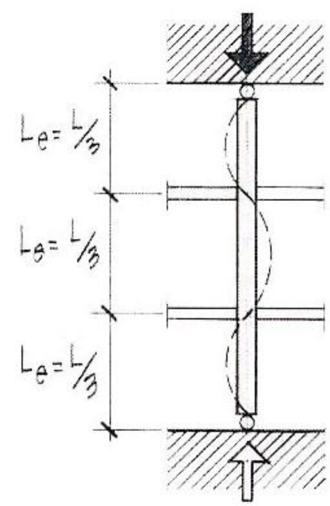


(a) No bracing.

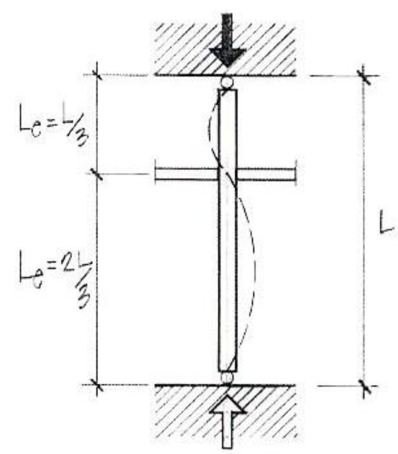


(b) Braced at midpoint.



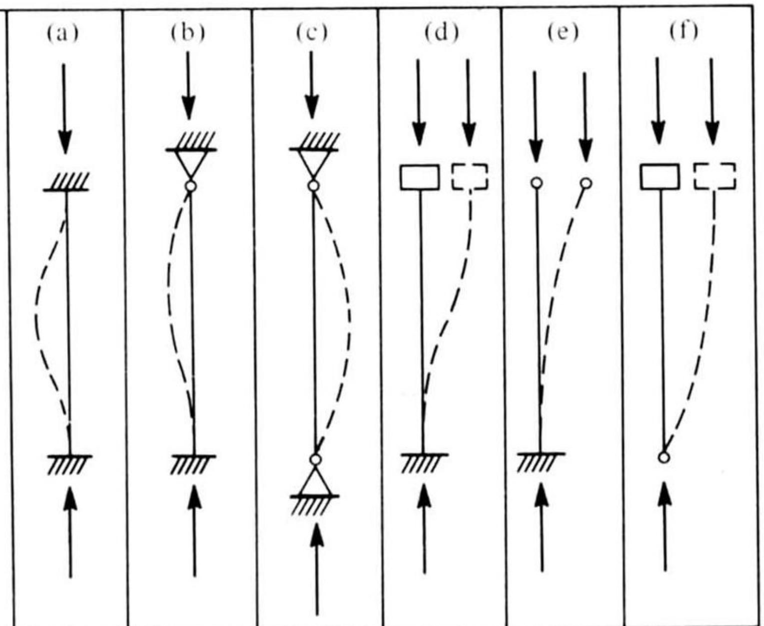


(c) Third-point bracing.



(d) Asymmetric bracing.

Buckled shape of column shown by dashed line	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design values when ideal conditions are approximated	0.65	0.80	1.0	1.2	2.10	2.0

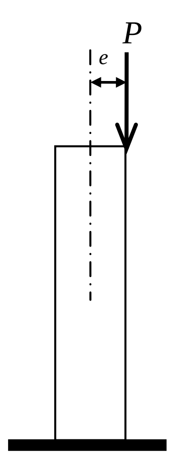


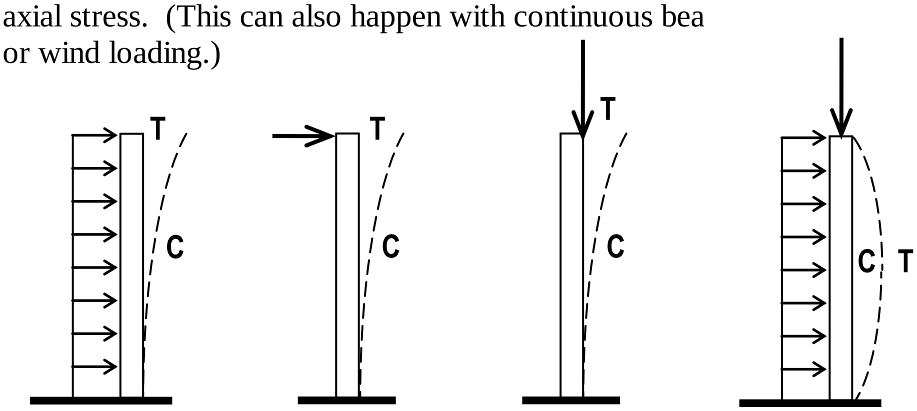
End conditions code

Loading Location

Centric loading: The load is applied at the centroid of the cross section. The limiting allowable stress is determined from strength (P/A) or buckling.

Eccentric loading: The load is offset from the centroid of the cross section because of how the beam load comes into the column. This offset introduces bending along with axial stress. (This can also happen with continuous bea
or wind loading.)





Wind Load Left Tension Left

Seismic Load Left Tension Left

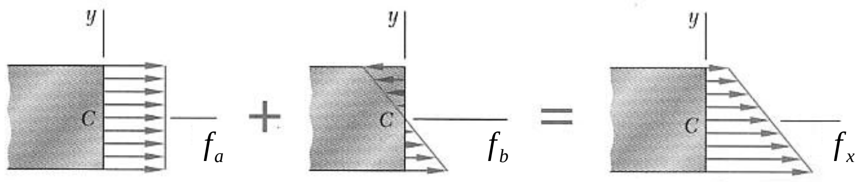
Eccentric Load Left Tension Left

Wind Load Left Braced at Top

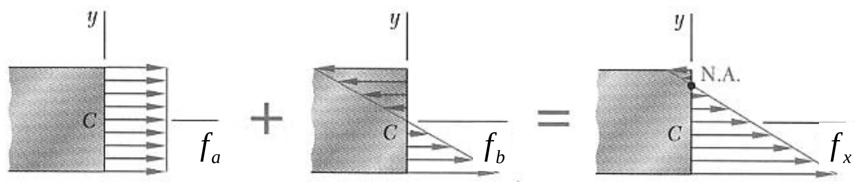
Eccentric Loading

The eccentricity causes bending stresses by a moment of value P x e. Within the elastic range (linear stresses) we can *superposition* or add up the normal and bending stresses:

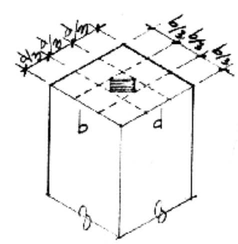
$$f_x = f_a + f_b = \frac{P}{A} + \frac{My}{I}$$



The resulting stress distribution is still *linear*. And the n.a. *moves* (if there is one).



The value of e (or location of P) that causes the stress at an edge to become zero is at the edge of the **kern**. As long as P stays within the kern, there will *not* be any tension stress.

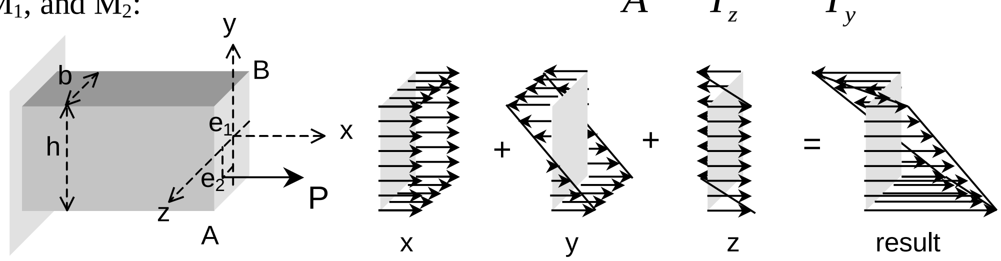


Kern

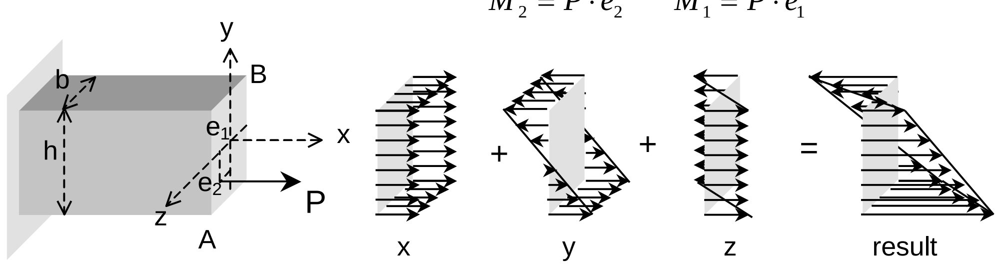
If there is bending in two directions (**bi-axial** bending), there will be one more bending stress added to the total:

$$f_x = f_a + f_{bx} + f_{by} = \frac{P}{A} + \frac{M_1 y}{I_z} + \frac{M_2 z}{I_y}$$

With P, M_1, and M_2:



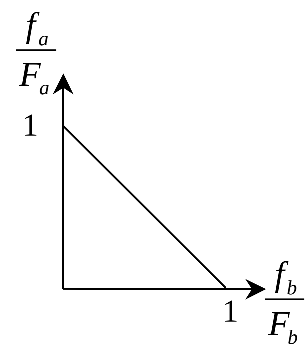
$M_2 = P \cdot e_2$ $\quad$ $M_1 = P \cdot e_1$



$M_2 = P \cdot e_2$ $\quad$ $M_1 = P \cdot e_1$

Eccentric Loading Design

Because there are combined stresses, we can't just compare the axial stress to a limit axial stress or a bending stress to a limit bending stress. We use a limit called the **interaction** diagram. The diagram can be simplified as a straight line from the ratio of axial stress to allowable stress= 1 (no bending) to the ratio of bending stress to allowable stress = 1 (no axial load).



The interaction diagram can be more sophisticated (represented by a curve instead of a straight line). These type of diagrams take the effect of the bending moment increasing because the beam deflects. This is called the **P-Δ (P-delta)** effect.

<u>Limit Criteria Methods</u>

1) $\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0$ interaction formula (bending in one direction)

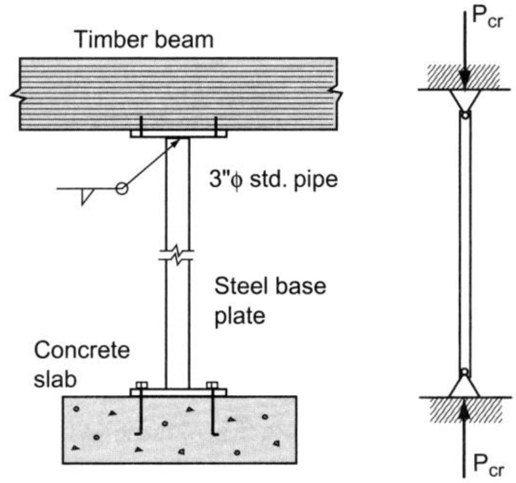
2) $\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0$ interaction formula (biaxial bending)

3) $\frac{f_a}{F_a} + \frac{f_b \times (Magnification\ factor)}{F_b} \leq 1.0$ interaction formula (P-Δ effect)

Example 1 (pg 346)

Example Problem 10.1: Short and Long Columns—Modes of Failure (Figures 10.11 and 10.12)

Determine the critical buckling load for a 3" ϕ standard weight steel pipe column that is 16 ft. tall and pin connected. Assume that $E = 29 \times 10^6$ psi

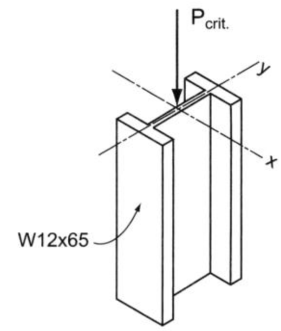


Example 2 (pg 346)

Example Problem 10.2 (Figure 10.13)

Determine the critical buckling stress for a 30-foot-long, W12×65 steel column. Assume simple pin connections at the top and bottom.

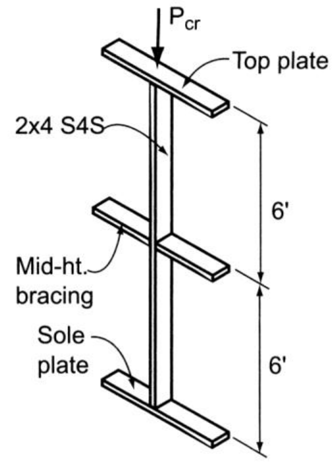
$F_y = 36$ ksi (A36 steel); $E = 29 \times 10^3$ ksi



Example 3 (pg357)

Example Problem 10.8 (Figures 10.33 and 10.34a, b)

Determine the buckling load capacity of a 2×4 stud 12 feet high if blocking is provided at midheight. Assume $E = 1.2 \times 10^6$ psi.



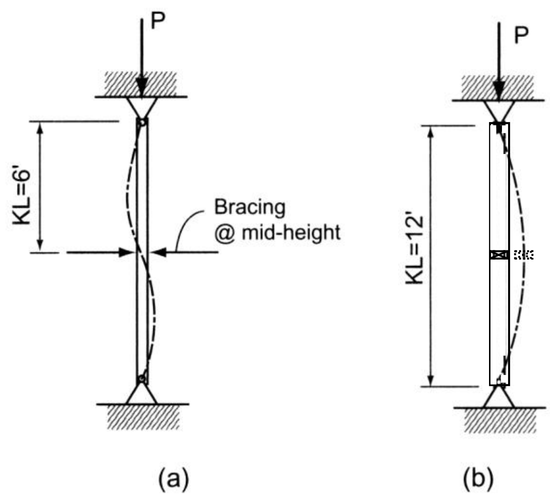


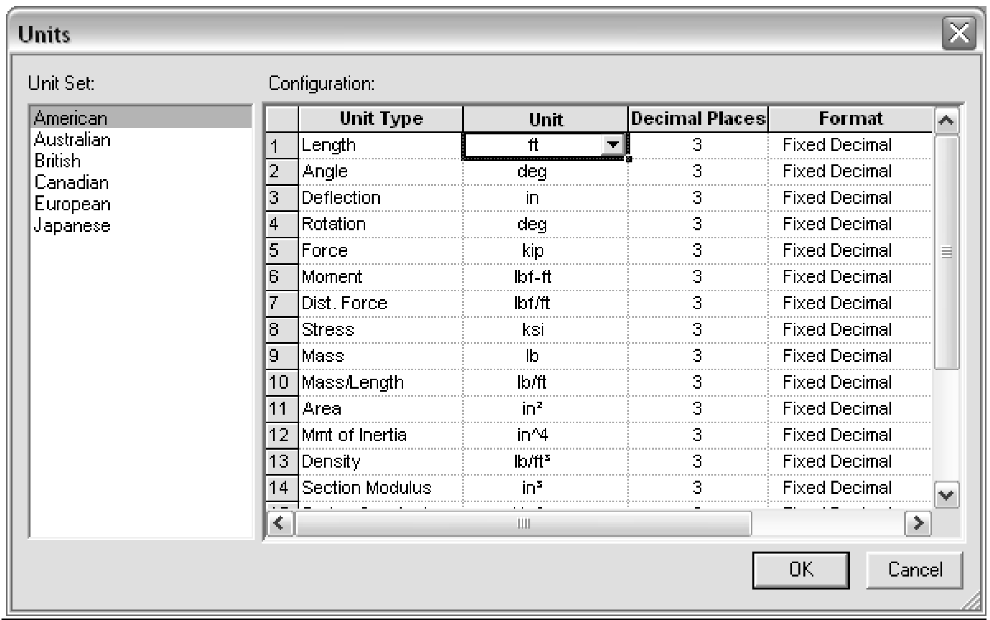
Figure 10.34 (a) Weak axis. (b) Strong axis.

Frame Analysis Using Multiframe

1. The software is on the computers in the College of Architecture in Programs under the Windows Start menu (see https://wikis.arch.tamu.edu/display/HELPDESK/Computer+Accounts for lab locations). Multiframe is under the Bentley Engineering menu.

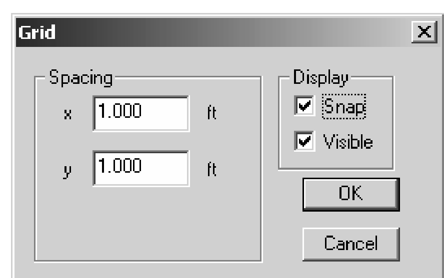
2. There are tutorials available on line at http://www.formsys.com/mflearning that list the tasks and order in greater detail. The first task is to define the unit system:

   - Choose Units… from the View menu. Unit sets are available, but specific units can also be selected by double clicking on a unit or format and making a selection from the menu. Pressure units are used for distributed area loads on load panels.

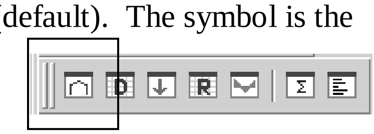


3. To see the scale of the geometry, a grid option is available:

   - Choose Grid… from the View menu



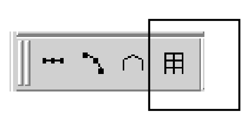
4. To create the geometry, you must be in the Frame window (default). The symbol is the frame in the window toolbar:



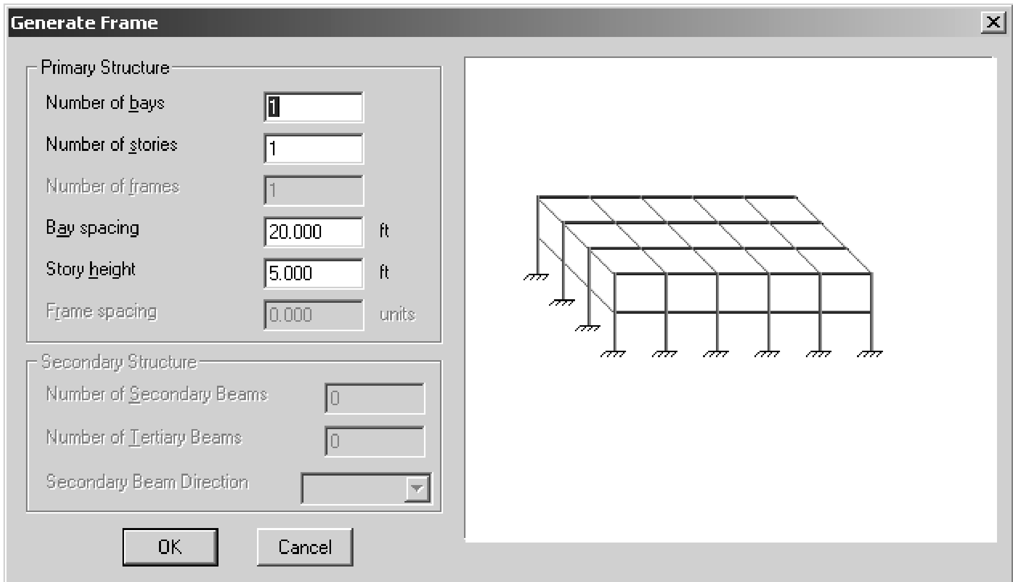
   The Member toolbar shows ways to create members:

   The Generate toolbar has convenient tools to create typical structural shapes.

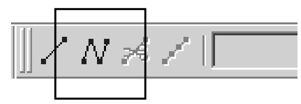
   - To create a frame, use the multi-bay frame button:



Enter the number of bays (horizontally), number of stories (vertically) and the corresponding spacings:



- If the frame does not have regular bays, use the add connected members button to create segments:

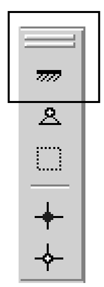


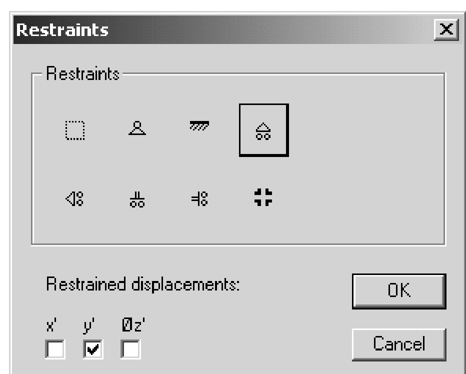
- Select a starting point and ending point with the cursor. The location of the cursor and the segment length is displayed at the bottom of the geometry window. The ESC button will end the segmented drawing.

- The geometry can be set precisely by selecting the joint (drag), and bringing up the joint properties menu (right click) to set the coordinates.

- The support types can be set by selecting the joint (drag) and using the Joint Toolbar (fixed shown), or the Frame / Joint Restraint ... menu (right click).

  NOTE: If the support appears at both ends of the member, you had the member selected rather than the joint. Select the joint to change support for and right click to select the joint restraints menu or select the correct support on the joint toolbar.





The support forces will be determined in the analysis.

5. All members must have sections assigned (see section 6.) in order to calculate reactions and deflections. To use a standard steel section **proceed to step 6.** For custom sections the section information must be entered. To define a section:

   - Choose Edit Sections / Add Section… from the Edit menu
   - Type a name for your new section
   - Choose group Frame from the group names provided so that the section will remain with the file data
   - Choose a shape. The Flat Bar shape is a rectangular section.
   - Enter the cross section data.

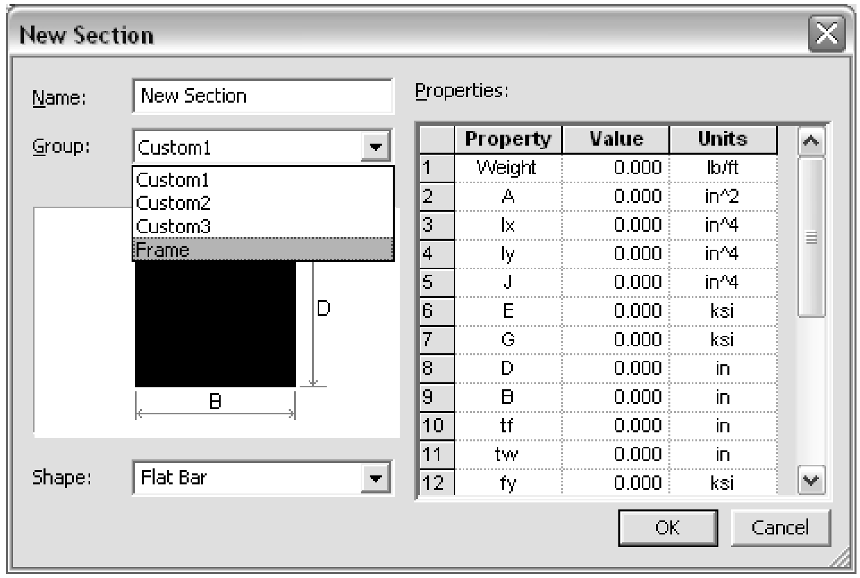
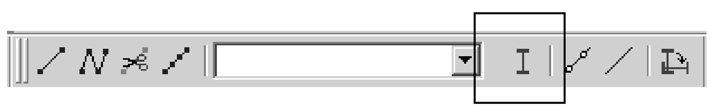


   Table values 1-9 must have values for a Flat Bar, but not all are used for every analysis. A recommendation is to put the value of 1 for those properties you don't know or care about. Properties like t_f, t_w, etc. refer to wide flange sections.

   - Answer any query. If the message says there is an error, the section will not be created until the error is corrected.

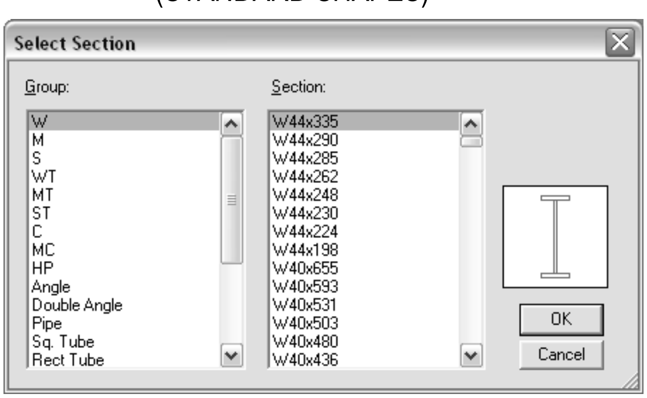
6. The standard sections library loaded is for the United States. If another section library is needed, use the Open Sections Library... command under the file menu, choose the library folder, and select the SectionsLibrary.slb file.

   Select the members (drag to make bold) and assign sections with the Section button on the Member toolbar:

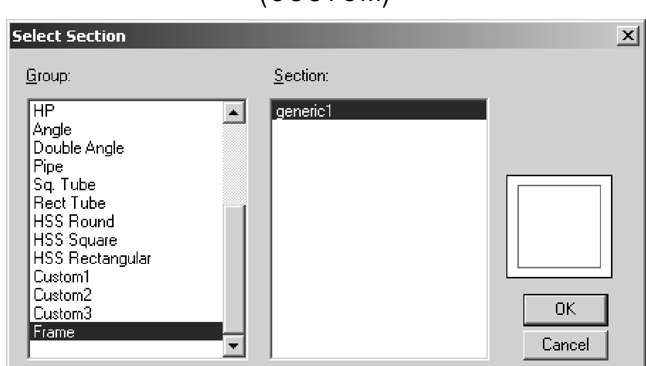


   - Choose the group name and section name:

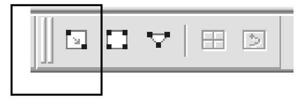
(STANDARD SHAPES)



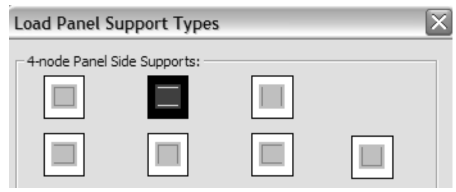
(CUSTOM)



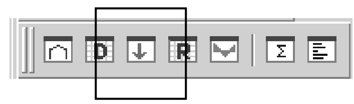
7. If there is an area that has a uniformly distributed load, load panels may be defined in the Frame window. Because the loaded area may not be visible in the current view, choose the View button at the lower left of the Frame window. The options for view are shown. (See 3D Frames, last page.)
   - Choose the panel type (rectangular, 4-node, or 3-node) from the menu and select the corners. If the area is rectangular, only the opposite corners need to be selected.



   - Select the panel and from the pop-up menu, or the Frame menu, specify the load panel supports. The default supports are on all sides. If the panel is one way, chose the corresponding picture

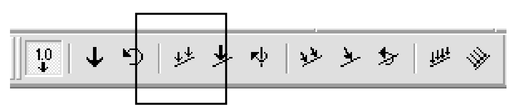


8. The frame geometry is complete, and in order to define the load conditions you must be in the Load window represented by the green arrow:

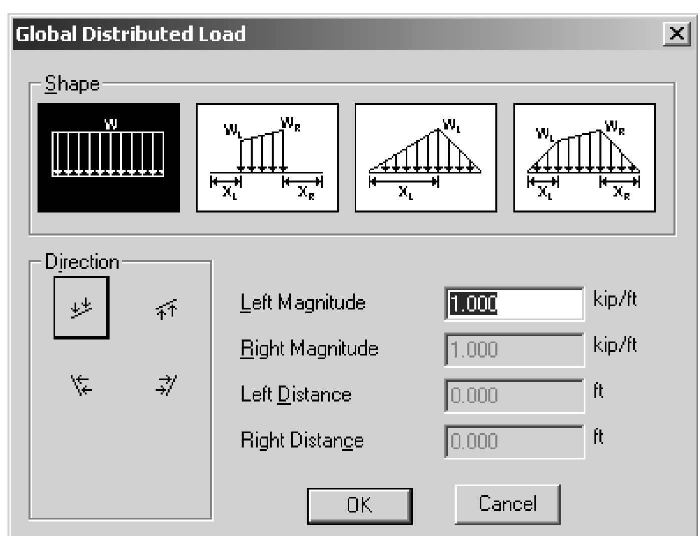


9. The Load toolbar allows a joint to be loaded with a force or a moment in global coordinates, shown by the first two buttons after the display numbers button. It allows a member to be loaded with a distributed load, concentrated load or moment (next three buttons) in global coordinates, as well as loading with distributed or single force or moment in the local coordinate system (next three buttons). It allows a load panel to be loaded with a distributed load in global or local coordinates (last two buttons).

   - Choose the member to be loaded (drag) and select the load type (here shown for global distributed loading):

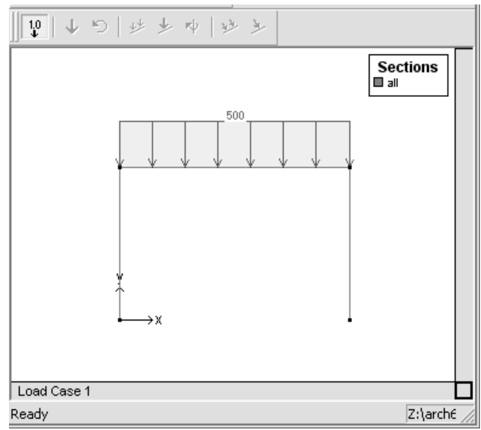


   - Choose the distribution type and direction. Note that the arrow shown is the direction of the loading. There is no need to put in negative values for downward loading.
   - Enter the values of the load and distances (if any). Distances can be entered as a function of the length , i.e. L/2, L/4...
   - Area load units may have to be changed in the View Units dialogue.

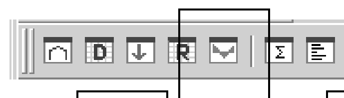


NOTE: <u>Do not</u> put support reactions as applied loads. The analysis will determine the reaction values.

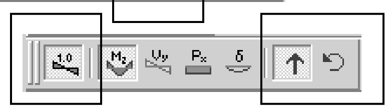
Multiframe will automatically generate a grouping called a Load Case named Load Case 1 when a load is created. All additional loads will be added to this load case unless a new load case is defined (Add case under the Case menu).



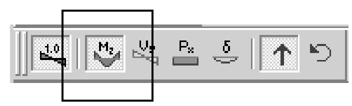
10. In order to run the analysis after the geometry, member properties and loading has been defined:
    - Choose Linear from the Analyze menu
11. If the analysis is successful, you can view the results in the Plot window represented by the red moment diagram:



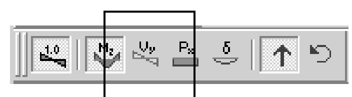
12. The Plot toolbar allows the numerical values to be shown (1.0 button), the reaction arrows to be shown (brown up arrow) and reaction moments to be shown (brown curved arrow):



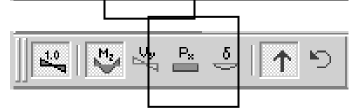
- To show the moment diagram, Choose the red Moment button



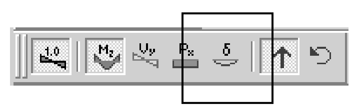
- To show the shear diagram, Choose the green Shear button



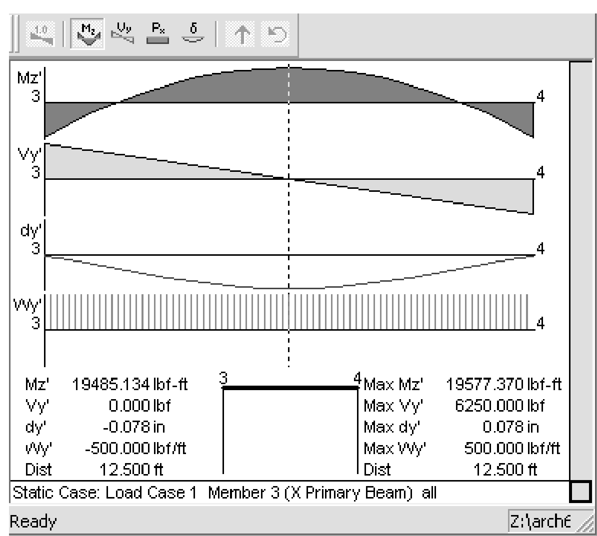
- To show the axial force diagram, Choose the purple Axial Force button



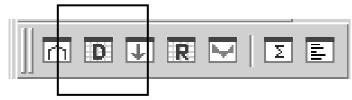
- To show the deflection diagram, Choose the blue Deflection button

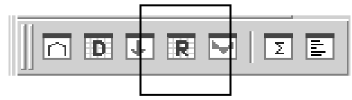


- To animate the deflection diagram, Choose Animate... from the Display menu. You can also save the animation to a .avi file by checking the box.
- To plot the bending moment on the "top" choose Preferences from the Edit menu and under the Presentation tab Draw moments on the compression face
- To see exact values of shear, moment and deflection, double click on the member and move the vertical cross hair with the mouse. The ESC key will return you to the window.



13. The Data window (D) allows you to view all data "entered" for the geometry, sections and loading. These values can be edited.





14. The Results window (R) allows you to view all results of the analysis including displacements, reactions, member forces (actions) and stresses. These values can be cut and pasted into other Windows programs such as Word or Excel.

    NOTE: Px' refers to the axial load (P) in the local axis x direction (x'). Vy' refers to the shear perpendicular to the local x axis, and Mz' refers to the bending moment.

Static Case: Load Case 1

	Memb	Label	Joint	Px' lbf	Vy' lbf	Mz' lbf-ft
1	1	Column	1	6250.000	-1786.320	-9725.784
2	1	Column	3	-6250.000	1786.320	-19577.371
3	2	Column	2	6250.000	1786.320	9725.784
4	2	Column	4	-6250.000	-1786.320	19577.371
5	3	X Prima	3	1786.320	6250.000	19577.371
6	3	X Prima	4	-1786.320	6250.000	-19577.371

Member Actions / Max Acti

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15. To save the file Choose Save from the File menu.
16. To load an existing file Choose Open... from the File menu.
17. To print a plot Choose Print Window... from the File menu. As an alternative, you may copy the plot (Ctrl+c) and paste it in a word processing document (Ctrl+v).

Example of Combined Stresses:

for member 3: M_{max} = 19.6 k-ft, P = 1.76 k

knowing A = 21.46 in^2, I = 796.0 in^4, c = 7.08 in

$$f_{\max} = \frac{1.76k}{21.46in^2} + \frac{19.6^{k-ft}\cdot 7.08in}{796in^4}\cdot\frac{12in}{ft} = 0.082ksi + 2.092ksi = 2.174ksi$$

Results window:

Static Case: Load Case 1

	Memb	Label	Joint	Sbz' top ksi	Sbz' bot ksi	Sy' ksi	Sx' ksi	Sx'+Sbz' top ksi	Sx'+Sbz' bot ksi
1	1	Column	1	1.039	-1.039	-1152.461	0.286	1.325	-0.753
2	1	Column	3	-2.092	2.092	-1152.461	0.286	-1.806	2.378
3	2	Column	2	-1.039	1.039	1152.461	0.286	-0.753	1.325
4	2	Column	4	2.092	-2.092	1152.461	0.286	2.378	-1.806
5	3	X Prima	3	-2.092	2.092	4032.245	0.082	-2.011	2.174
6	3	X Prima	4	-2.092	2.092	-4032.245	0.082	-2.011	2.174

Member Stresses / Max Stresses / Member [

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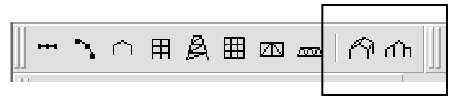
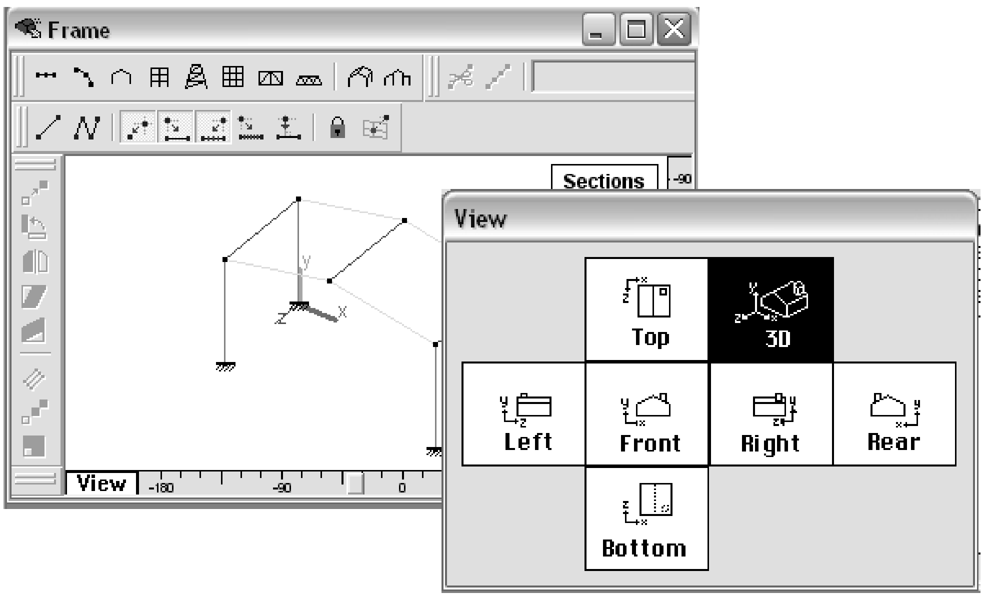
where Sx' refers to the axial stress, Sy' refers to the bending stress around the local vertical axis and Sz' refers to the bending stress around the local horizontal axis.

For 3D Frames:

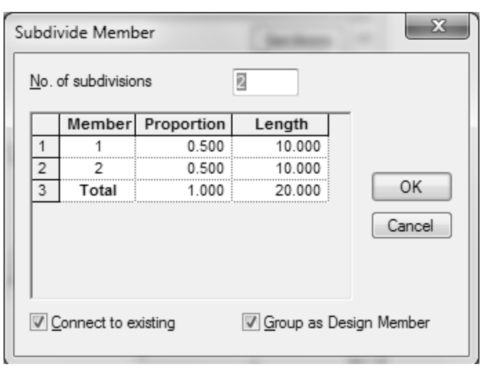
- There are tutorials available on line at http://www.formsys.com/mflearning that list the tasks and order in greater detail. It expects that you have been through the 2D tutorial to build on the steps already mastered.
- There are standard 3D frame shapes on the frame toolbar.

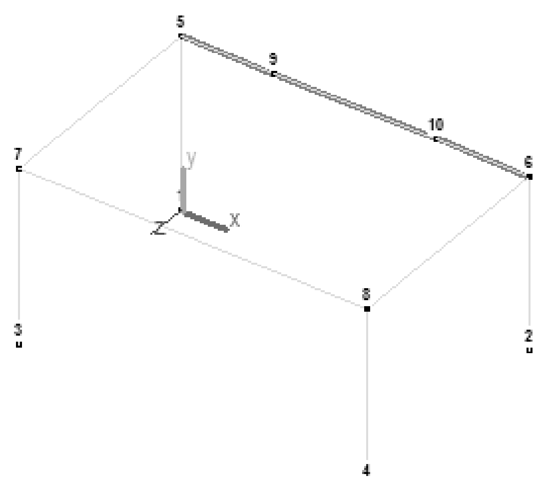


- It is very useful to change the view to isometric with the View Button

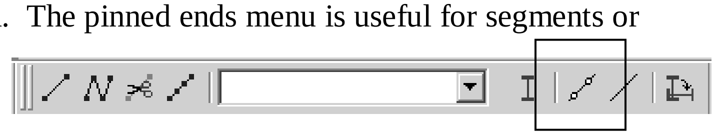


- If you wish to have additional beams supported by the beams of your frame, choose the beam and use the Subdivide Member menu under Geometry. This will make additional joints, but keep the segments together.

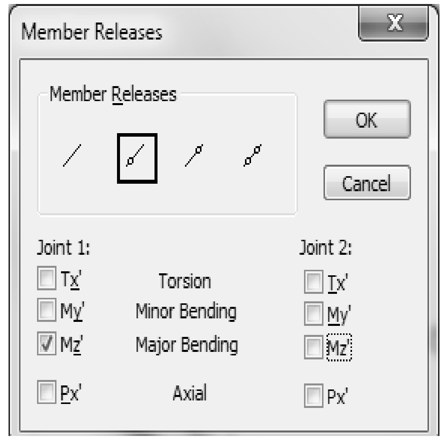




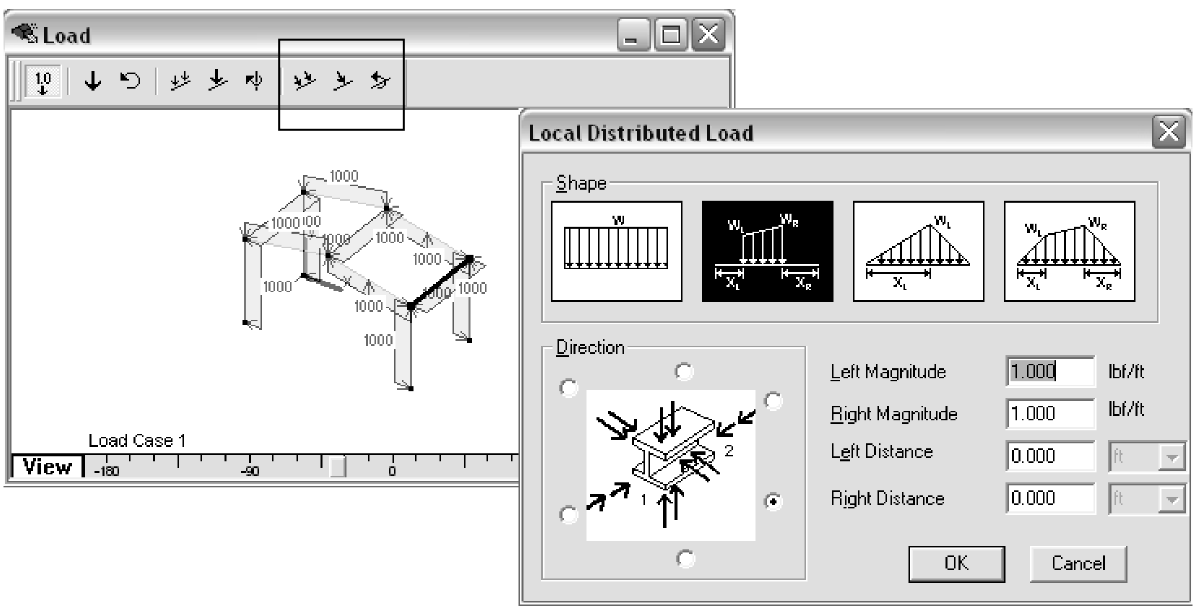
- In order to model a beam end as simply supported, you must release the restraint preventing rotation about the x-x axis of the beam. The pinned ends menu is useful for segments or subdivided members.



Or, by selecting a segment and right clicking for a menu, you can use Member Releases (also under the Frame menu) to release the Major Bending (M'_Z) for one end or both.



- It is necessary to understand the local member axes to assign the correct load direction. Choosing the *local* loading types will show the member orientation with respect to the load direction.



Common Design Loads in Building Codes

Notation:

A = name for area
AASHTO = American Association of State Highway and Transportation Officials
ASCE = American Society of Civil Engineers
ASD = allowable stress design
D = dead load symbol
E = earthquake load symbol
F = hydraulic loads from fluids symbol
H = hydraulic loads from soil symbol
L = live load symbol
L_r = live roof load symbol
LRFD = load and resistance factor design
R = rainwater load or ice water load symbol
S = snow load symbol
SEI = Structural Engineering Institute
t = name for thickness
T = effect of material & temperature symbol
V = name for volume
w = name for distributed load
W = wind load symbol
= force due to a weight
= name for total force due to distributed load
γ = density or unit weight

Design Codes in General

Design codes are issued by a professional organization interested in insuring safety and standards. They are legally backed by the engineering profession. Different design methods are used, but they typically defined the *load cases or combination*, stress or strength limits, and deflection limits.

Load Types

Loads used in design load equations are given letters by *type:*

D = dead load

L = live load

L_r = live roof load

W = wind load

S = snow load

E = earthquake load

R = rainwater load or ice water load

T = effect of material & temperature

H = hydraulic loads from soil

F = hydraulic loads from fluids

Determining Dead Load from Material Weights

Material density is a measure of how much mass in a unit volume causes a force due to gravity. The common symbol for density is γ. When volume, V, is multiplied by density, a force value results:

$$W = \gamma \cdot V$$

Materials "weight" can also be presented as a weight per unit area or length. This takes into account that the volume is a thickness times an area: $V = t{\cdot}A$; so the calculation becomes:

$W = (weight/unit\ area){\cdot}A$

$w = (weight/unit\ volume){\cdot}t$ which is a weight per unit area

$w = (weight/unit\ volume){\cdot}A$ which is a weight per unit length

Minimum Concentrated Loads

adapted from SEI/ASCE 7-10: Minimum Design Loads for Buildings and Other Structures

Location	*Concentrated load lb (kN)*
Catwalks for maintenance access	300 (1.33)
Elevator machine room grating (on area of 2 in. by 2 in. (50 mm by 50 mm))	300 (1.33)
Finish light floor plate construction (on area of 1 in. by 1 in. (25 mm by25 mm))	200 (0.89)
Hospital floors	1,000 (4.45)
Library floors	1,000 (4.45)
Manufacturing	
Light	2,000 (8.90)
Heavy	3,000 (13.40)
Office floors	2,000 (8.90)
Awnings and canopies	
Screen enclosure support frame	200 (0.89)
Roofs – primary members and subject to maintenance workers	300 (1.33)
School floors	1,000 (4.45)
Sidewalks, vehicular driveways, and yards subject to trucking (over wheel area of 4.5 in. by 4.5 in. (114 mm x 114 mm)	8,000 (35.60)
Stairs and exit ways on area of 2 in. by 2 in. (50 mm by 50 mm) non-concurrent with uniform load	300 (1.33)
Store floors	1,000 (4.45)

Allowable Stress Design (ASD)

Combinations of service (also referred to as *working)* loads are evaluated for maximum stresses and compared to allowable stresses. The allowed stresses are some fraction of limit stresses.

ASCE-7 (2010) combinations of loads:

1. D
2. $D + L$
3. $D + 0.75(L_r \text{ or } S \text{ or } R)$
4. $D + 0.75L + 0.75(L_r \text{ or } S \text{ or } R)$
5. $D + (0.6W \text{ or } 0.7E)$

6a. $D + 0.75L + 0.75(0.6W) + 0.75(L_r \text{ or } S \text{ or } R)$

6b. $D + 0.75L + 0.75(0.7E) + 0.75S$

7. $0.6D + 0.6W$
8. $0.6D + 0.7E$

When F loads are present, they shall be included with the same load factor as dead load D in 1 through 6 and 8.

When H loads are present, they shall have a load factor of 1.0 when adding to load effect, or 0.6 when resisting the load when permanent.

Load and Resistance Factor Design – LRFD

Combinations of loads that have been *factored* are evaluated for maximum loads, moments or stresses. These factors take into consideration how likely the load is to happen and how often. This "imaginary" worse case load, moment or stress is compared to a limit value that has been modified by a *resistance* factor. The resistance factor is a function of how "comfortable" the design community is with the type of limit, ie. yielding or rupture...

ASCE-7 (2010) combinations of factored nominal loads:

1. $1.4D$
2. $1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
3. $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.5W)$
4. $1.2D + 1.0W + L + 0.5(L_r \text{ or } S \text{ or } R)$
5. $1.2D + 1.0E + L + 0.2S$
6. $0.9D + 1.0W$
7. $0.9D + 1.0E$

When F loads are present, they shall be included with the same load factor as dead load D in 1 through 5 and 7.

When H loads are present, they shall have a load factor of 1.6 when adding to load effect, or 0.9 when resisting the load when permanent.

Minimum Uniformly Distributed Live Loads

adapted from SEI/ASCE 7-10: Minimum Design Loads for Buildings and Other Structures

Location	*Uniform load psf (kN/m^2)*
Apartments (see Residential)	
Access floor systems	
Office use	50 (2.4)
Computer use	100 (4.79)
Armories and drill rooms	150 (7.18)
Assembly areas	
Fixed seats (fastened to floor)	60 (2.87)
Lobbies	100 (4.79)
Movable seats	100 (4.79)
Platforms (assembly)	100 (4.79)
Stage floors	150 (7.18)
Assembly areas (other)	100 (4.79)

Location	*Uniform load psf (kN/m^2)*
Balconies and decks	1.5 times the live load for the area served. Not required`to exceed 100 psf (4.79 kN/m2)
Catwalks for maintenance access	40 (1.92)
Corridors	
First floor	100 (4.79)
Other floors	same as occupancy served except as indicated
Dining rooms and restaurants	100 (4.79)
Dwellings (see Residential)	
Elevator machine room grating (on area of 2 in. by 2 in. (50 mm by 50 mm))	300 (1.33)
Finish light floor plate construction (on area of 1 in. by 1 in. (25 mm by 25 mm))	200 (0.89)
Fire escapes	100 (4.79)
On single-family dwellings only	40 (1.92)
Garages	
Passenger vehicles only	40 (1.92)
Helipads	60 (2.87)
Hospitals	
Operating rooms, laboratories	60 (2.87)
Patient rooms	40 (1.92)
Corridors above first floor	80 (3.83)
Hotels (see Residential)	
Libraries	
Reading rooms	60 (2.87)
Stack rooms	150 (7.18)
Corridors above first floor	80 (3.83)
Manufacturing	
Light	125 (6.00)
Heavy	250 (11.97)
Office buildings	
File and computer rooms shall be designed for heavier loads based on anticipated occupancy	
Lobbies and first floor corridors	100 (4.79)
Offices	50 (2.40)
Corridors above first floor	80 (3.83)
Penal institutions	
Cell blocks	40 (1.92)
Corridors	100 (4.79)
Recreational uses	
Bowling alleys, poolrooms, and similar uses	75 (3.59)
Dance halls and ballrooms	100 (4.79)
Gymnasiums	100 (4.79)
Reviewing stands, grandstands, and bleachers	100 (4.79)
Stadiums and arenas with fixed seats (fastened to the floor)	60 (2.87)
Residential	
One- and two-family dwellings	
Uninhabitable attics without storage	10 (0.48)
Uninhabitable attics with storage	20 (0.96)
Habitable attics and sleeping areas	30 (1.44)
All other areas except stairs	40 (1.92)
All other residential occupancies	
Private rooms and corridors serving them	40 (1.92)
Public rooms and corridors serving them	100 (4.79)

Location	*Uniform load psf (kN/m^2)*
Roofs	
Ordinary flat, pitched, and curved roofs	20 (0.96n
Roofs used for roof gardens	100 (4.79)
Roofs used for other occupancies	Same as occupancy served
Roofs used for other special purposes	As approved by authority having jurisdiction
Awnings and canopies	
Fabric construction supported by a skeleton structure	5 (0.24) nonreducible
Screen enclosure support frame	5 (0.24) nonreducible and based on the tributary area of the roof supported by the frame
All other construction	20 (0.96)
Schools	
Classrooms	40 (1.92)
Corridors above first floor	80 (3.83)
First-floor corridors	100 (4.79)
Scuttles, skylight ribs, and accessible ceilings	200 (0.89)
Sidewalks, vehicular driveways, and yards subject to trucking	250 (11.97)
Stairs and exit ways	100 (4.79)
One- and two-family dwellings only	40 (1.92)
Storage areas above ceilings	20 (0.96)
Storage warehouses (shall be designed for heavier loads if required for anticipated storage)	
Light	125 (6.00)
Heavy	250 (11.97)
Stores	
Retail	
First floor	100 (4.79)
Upper floors	75 (3.59)
Wholesale, all floors	125 (6.00)
Walkways and elevated platforms (other than exit ways)	60 (2.87)
Yards and terraces, pedestrian	100 (4.79)

Live load reductions are not permitted for specific types (see code).
Some occupancies must be designed for appropriate loads as approved by the authority having jurisdiction.
Library stack room floors have specified limitations (see code)
AASHTO lane loads should also be considered where appropriate.

Building Material Weights-AISC Manual of Load and Resistance Factor Design, 3rd ed.

Table 17-12. Weights and Specific Gravities

Substance	Weight lb per cu ft	Specific Gravity
ASHLAR, MASONRY		
Granite, syenite, gneiss	165	2.3–3.0
Limestone, marble	160	2.3–2.8
Sandstone, bluestone	140	2.1–2.4
MORTAR RUBBLE MASONRY		
Granite, syenite, gneiss	155	2.2–2.8
Limestone, marble	150	2.2–2.6
Sandstone, bluestone	130	2.0–2.2
DRY RUBBLE MASONRY		
Granite, syenite, gneiss	130	1.9–2.3
Limestone, marble	125	1.9–2.1
Sandstone, bluestone	110	1.8–1.9
BRICK MASONRY		
Pressed brick	140	2.2–2.3
Common brick	120	1.8–2.0
Soft brick	100	1.5–1.7
CONCRETE MASONRY		
Cement, stone, sand	144	2.2–2.4
Cement, slag, etc.	130	1.9–2.3
Cement, cinder, etc.	100	1.5–1.7
VARIOUS BUILDING MATERIALS		
Ashes, cinders	40–45	–
Cement, portland, loose	90	–
Cement, portland, set	183	2.7–3.2
Lime, gypsum, loose	53–64	–
Mortar, set	103	1.4–1.9
Slags, bank slag	67–72	–
Slags, bank screenings	98–117	–
Slags, machine slag	96	–
Slags, slag sand	49–55	–
EARTH, ETC., EXCAVATED		
Clay, dry	63	–
Clay, damp, plastic	110	–
Clay and gravel, dry	100	–
Earth, dry, loose	76	–
Earth, dry, packed	95	–
Earth, moist, loose	78	–
Earth, moist, packed	96	–
Earth, mud, flowing	108	–
Earth, mud, packed	115	–
Riprap, limestone	80–85	–
Riprap, sandstone	90	–
Riprap, shale	105	–
Sand, gravel, dry, loose	90–105	–
Sand, gravel, dry, packed	100–120	–
Sand, gravel, wet	118–120	–
EXCAVATIONS IN WATER		
Sand or gravel	60	–
Sand or gravel and clay	65	–
Clay	80	–
River mud	90	–
Soil	70	–
Stone riprap	65	–

Substance	Weight lb per cu ft	Specific Gravity
MINERALS		
Asbestos	153	2.1–2.8
Barytes	281	4.50
Basalt	184	2.7–3.2
Bauxite	159	2.55
Borax	109	1.7–1.8
Chalk	137	1.8–2.6
Clay, marl	137	1.8–2.6
Dolomite	181	2.9
Feldspar, orthoclase	159	2.5–2.6
Gneiss, serpentine	159	2.4–2.7
Granite, syenite	175	2.5–3.1
Greenstone, trap	187	2.8–3.2
Gypsum, alabaster	159	2.3–2.8
Hornblende	187	3.0
Limestone, marble	165	2.5–2.8
Magnesite	187	3.0
Phosphate rock, apatite	200	3.2
Porphyry	172	2.6–2.9
Pumice, natural	40	0.37–0.90
Quartz, flint	165	2.5–2.8
Sandstone, bluestone	147	2.2–2.5
Shale, slate	175	2.7–2.9
Soapstone, talc	169	2.6–2.8
STONE, QUARRIED, PILED		
Basalt, granite, gneiss	96	–
Limestone, marble, quartz	95	–
Sandstone	82	–
Shale	92	–
Greenstone, hornblende	107	–
BITUMINOUS SUBSTANCES		
Asphaltum	81	1.1–1.5
Coal, anthracite	97	1.4–1.7
Coal, bituminous	84	1.2–1.5
Coal, lignite	78	1.1–1.4
Coal, peat, turf, dry	47	0.65–0.85
Coal, charcoal, pine	23	0.28–0.44
Coal, charcoal, oak	33	0.47–0.57
Coal, coke	75	1.0–1.4
Graphite	131	1.9–2.3
Paraffin	56	0.87–0.91
Petroleum	54	0.87
Petroleum, refined	50	0.79–0.82
Petroleum, benzine	46	0.73–0.75
Petroleum, gasoline	42	0.66–0.69
Pitch	69	1.07–1.15
Tar, bituminous	75	1.20
COAL AND COKE, PILED		
Coal, anthracite	47–58	–
Coal, bituminous, lignite	40–54	–
Coal, peat, turf	20–26	–
Coal charcoal	10–14	–
Coal coke	23–32	–

The specific gravities of solids and liquids refer to water at 4°C, those of gases to air at 0°C and 760 mm pressure. The weights per cubic foot are derived from average specific gravities, except where stated that weights are for bulk, heaped, or loose material, etc.

Table 17-12 (cont.). Weights and Specific Gravities

Substance	Weight lb per cu ft	Specific Gravity
METALS, ALLOYS, ORES		
Aluminum, cast, hammered	165	2.55–2.75
Brass, cast, rolled	534	8.4–8.7
Bronze, 7.9 to 14% Sn	509	7.4–8.9
Bronze, aluminum	481	7.7
Copper, cast, rolled	556	8.8–9.0
Copper ore, pyrites	262	4.1–4.3
Gold, cast, hammered	1205	19.25–19.3
Iron, cast, pig	450	7.2
Iron, wrought	485	7.6–7.9
Iron, speigel-eisen	468	7.5
Iron, ferro-silicon	437	6.7–7.3
Iron ore, hematite	325	5.2
Iron ore, hematite in bank	160–180	–
Iron ore, hematite loose	130–160	–
Iron ore, limonite	237	3.6–4.0
Iron ore, magnetite	315	4.9–5.2
Iron slag	172	2.5–3.0
Lead	710	11.37
Lead ore, galena	465	7.3–7.6
Magnesium, alloys	112	1.74–1.83
Manganese	475	7.2–8.0
Manganese ore, pyrolusite	259	3.7–4.6
Mercury	849	13.6
Monel Metal	556	8.8–9.0
Nickel	565	8.9–9.2
Platinum, cast, hammered	1330	21.1–21.5
Silver, cast, hammered	656	10.4–10.6
Steel, rolled	490	7.85
Tin, cast, hammered	459	7.2–7.5
Tin ore, cassiterite	418	6.4–7.0
Zinc, cast, rolled	440	6.9–7.2
Zinc ore, blende	253	3.9–4.2
VARIOUS SOLIDS		
Cereals, oats ... bulk	32	–
Cereals, barley ... bulk	39	–
Cereals, corn, rye ... bulk	48	–
Cereals, wheat ... bulk	48	–
Hay and Straw ... bales	20	–
Cotton, Flax, Hemp	93	1.47–1.50
Fats	58	0.90–0.97
Flour, loose	28	0.40–0.50
Flour, pressed	47	0.70–0.80
Glass, common	156	2.40–2.60
Glass, plate or crown	161	2.45–2.72
Glass, crystal	184	2.90–3.00
Leather	59	0.86–1.02
Paper	58	0.70–1.15
Potatoes, piled	42	–
Rubber, caoutchouc	59	0.92–0.96
Rubber goods	94	1.0–2.0
Salt, granulated, piled	48	–
Saltpeter	67	–
Starch	96	1.53
Sulphur	125	1.93–2.07
Wool	82	1.32

Substance	Weight lb per cu ft	Specific Gravity
TIMBER, U.S. SEASONED		
Moisture content by weight:		
Seasoned timber 15 to 20%		
Green timber up to 50%		
Ash, white, red	40	0.62–0.65
Cedar, white, red	22	0.32–.038
Chestnut	41	0.66
Cypress	30	0.48
Fir, Douglas spruce	32	0.51
Fir, eastern	25	0.40
Elm, white	45	0.72
Hemlock	29	0.42–0.52
Hickory	49	0.74–0.84
Locust	46	0.73
Maple, hard	43	0.68
Maple, white	33	0.53
Oak, chestnut	54	0.86
Oak, live	59	0.95
Oak, red, black	41	0.65
Oak, white	46	0.74
Pine, Oregon	32	0.51
Pine, red	30	0.48
Pine, white	26	0.41
Pine, yellow, long-leaf	44	0.70
Pine, yellow, short-leaf	38	0.61
Poplar	30	0.48
Redwood, California	26	0.42
Spruce, white, black	27	0.40–0.46
Walnut, black	38	0.61
Walnut, white	26	0.41
VARIOUS LIQUIDS		
Alcohol, 100%	49	0.79
Acids, muriatic 40%	75	1.20
Acids, nitric 91%	94	1.50
Acids, sulphuric 87%	112	1.80
Lye, soda 66%	106	1.70
Oils, vegetable	58	0.91–0.94
Oils, mineral, lubricants	57	0.90–0.93
Water, 4°C max. density	62.428	1.0
Water, 100°C	59.830	0.9584
Water, ice	56	0.88–0.92
Water, snow, fresh fallen	8	.125
Water, sea water	64	1.02–1.03
GASES		
Air, 0°C 760 mm	.08071	1.0
Ammonia	.0478	0.5920
Carbon dioxide	.1234	1.5291
Carbon monoxide	.0781	0.9673
Gas, illuminating	.028–.036	0.35–0.45
Gas, natural	.038–.039	0.47–0.48
Hydrogen	.00559	0.0693
Nitrogen	.0784	0.9714
Oxygen	.0892	1.1056

The specific gravities of solids and liquids refer to water at 4°C, those of gases to air at 0°C and 760 mm pressure. The weights per cubic foot are derived from average specific gravities, except where stated that weights are for bulk, heaped, or loose material, etc.

Table 17-13. Weights of Building Materials

Materials	Weight lb per sq ft
CEILINGS	
Channel suspended system	1
Lathing and plastering	See Partitions
Acoustical fiber tile	1
FLOORS	
Steel deck	See Manufacturer
Concrete-Reinforced 1 in.	
Stone	12 1/2
Slag	11 1/2
Lightweight	6 to 10
Concrete-Plain 1 in.	
Stone	12
Slag	11
Lightweight	3 to 9
Fills 1 inch	
Gypsum	6
Sand	8
Cinders	4
Finishes	
Terrazzo 1 in.	13
Ceramic or Quarry Tile 3/4-in.	10
Linoleum 1/4-in.	1
Mastic 3/4-in.	9
Hardwood 7/8-in.	4
Softwood 3/4-in.	2 1/2
ROOFS	
Copper or tin	1
Corrugated steel	See Manufactuer
3-ply ready roofing	1
3-ply felt and gravel	5 1/2
5-ply felt and gravel	6
Shingles	
Wood	2
Asphalt	3
Clay tile	9 to 14
Slate 1/4 in.	10
Sheathing	
Wood 3/4 in.	3
Gypsum 1 in.	4
Insulation 1 in.	
Loose	1/2
Poured	2
Rigid	1 1/2

Materials	Weight lb per sq ft
PARTITIONS	
Clay tile	
3 in.	17
4 in.	18
6 in.	28
8 in.	34
10 in.	40
Gypsum block	
2 in.	9 1/2
3 in.	10 1/2
4 in.	12 1/2
5 in.	14
6 in.	18 1/2
Wood studs 2×4	
12–16 in. o.c.	2
Steel partitions	4
Plaster 1 in.	
Cement	10
Gypsum	5
Lathing	
Metal	1/2
Gypsum board 1/2 in.	2
WALLS	
Brick	
4 in.	40
8 in.	80
12 in.	120
Hollow concrete block	
(Heavy aggregate)	
4 in.	30
6 in.	43
8 in.	55
12 1/2 in.	80
Hollow concrete block	
(Light aggregate)	
4 in.	21
6 in.	30
8 in.	38
12 in.	55
Clay tile (Load bearing)	
4 in.	25
6 in.	30
8 in.	33
12 in.	45
Stone 4 in.	55
Glass block 4 in.	18
Window, Glass, Frame, & Sash	8
Curtain walls	See Manufacturer
Structural glass 1 in.	15
Corrugated Cement Asbestos 1/4 in.	3

For weights of other materials used in building construction, see Table 17-12.

AMERICAN INSTITUTE OF STEEL CONSTRUCTION

Table 17-14. Weights and Measures United States System

LINEAR MEASURE

Inches	Feet	Yards	Rods	Furlongs	Miles
1.0 =	.08333 =	.02778 =	.0050505 =	.00012626 =	.00001578
12.0 =	1.0 =	.33333 =	.0606061 =	.00151515 =	.00018939
36.0 =	3.0 =	1.0 =	.1818182 =	.00454545 =	.00056818
198.0 =	16.5 =	5.5 =	1.0 =	.025 =	.003125
7,920.0 =	660.0 =	220.0 =	40.0 =	1.0 =	.125
63,360.0 =	5,280.0 =	1,760.0 =	320.0 =	8.0 =	1.0

SQUARE AND LAND MEASURE

Sq. Inches	Square Feet	Square Yards	Square Rods	Acres	Sq. Miles
1.0 =	.006944 =	.000772			
144.0 =	1.0 =	.111111			
1,296.0 =	9.0 =	1.0 =	.03306 =	.000207	
39,204.0 =	272.25 =	30.25 =	1.0 =	.00625 =	.0000098
	43,560.0 =	4,840.0 =	160.0 =	1.0 =	.0015625
		3,097,600.0 =	102,400.0 =	640.0 =	1.0

AVOIRDUPOIS WEIGHTS

Grains	Drams	Ounces	Pounds	Tons
1.0 =	.03657 =	.002286 =	.000143 =	.0000000714
27.34375 =	1.0 =	.0625 =	.003906 =	.00000195
437.5 =	16.0 =	1.0 =	.0625 =	.00003125
7,000.0 =	256.0 =	16.0 =	1.0 =	.0005
14,000,000.0 =	512,000.0 =	32,000.0 =	2,000.0 =	1.0

DRY MEASURE

Pints	Quarts	Pecks	Cubic Feet	Bushels
1.0 =	.5 =	.0625 =	.01945 =	.01563
2.0 =	1.0 =	.125 =	.03891 =	.03125
16.0 =	8.0 =	1.0 =	.31112 =	.25
51.42627 =	25.71314 =	3.21414 =	1.0 =	.80354
64.0 =	32.0 =	4.0 =	1.2445 =	1.0

LIQUID MEASURE

Gills	Pints	Quarts	U.S. Gallons	Cubic Feet
1.0 =	.25 =	.125 =	.03125 =	.00418
4.0 =	1.0 =	.5 =	.125 =	.01671
8.0 =	2.0 =	1.0 =	.250 =	.03342
32.0 =	8.0 =	4.0 =	1.0 =	.1337
			7.48052 =	1.0

AMERICAN INSTITUTE OF STEEL CONSTRUCTION

Example 1

Determine the controlling load combinations(s) using AISC-LRFD for a building column subject to the following service or nominal (unfactored) axial compressive loads: D = 30 k, L = 50 k, L_r = 10 k, W = 25 k, E = 40 k

Using a spreadsheet analysis:

LRFD (ASCE-7)		FACTORED LOAD
$1.4D$		
$1.4D$	=	42 kips
$1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$		
$1.2D + 1.6L + 0.5L_r$	=	121
$1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.5W)$		
$1.2D + 1.6L_r + L$	=	102
$1.2D + 1.6L_r + 0.5W$	=	64.5
$1.2D + 1.6L_r - 0.5W$	=	39.5
$1.2D + 1.0W + L + 0.5(L_r \text{ or } S \text{ or } R)$		
$1.2D + 1.0W + L + 0.5L_r$	=	116
$1.2D - 1.0W + L + 0.5L_r$	=	66
$1.2D + 1.0E + L + 0.2S$		
$1.2D + 1.0E + L$	=	126
$1.2D - 1.0E + L$	=	46
$0.9D + 1.0W$		
$0.9D + 1.0W$	=	52
$0.9D - 1.0W$	=	2
$0.9D + 1.0E$		
$0.9D + 1.0E$	=	67
$0.9D - 1.0E$	=	-13
Critical Factored Load		126 kips (C) -13 kips (T)

Example 2

EXAMPLE 2–4

Determine factored loads for the beam shown in Figure 2–16.

Solution

For the left half of the beam:

$$w_{u1} = 1.2w_D + 1.6w_L$$
$$w_{u1} = 1.2 \times 1.0 + 1.6 \times 2.0 = 4.4 \text{ kip/ft}$$

For the right half of the beam:

$$w_{u2} = 1.2w_D + 1.6w_L$$
$$w_{u2} = 1.2 \times 1.0 + 1.6 \times 0 = 1.2 \text{ kip/ft}$$

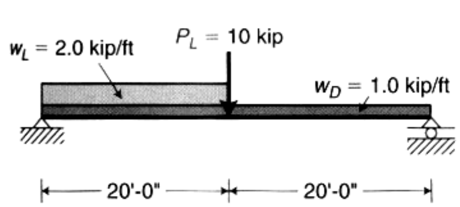


FIGURE 2–16 Example 2–4 (service loads).

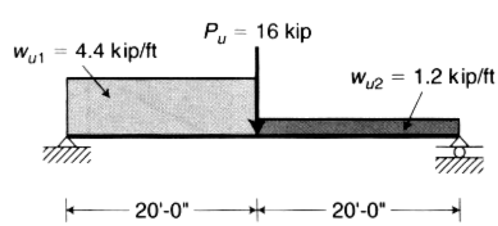


FIGURE 2–17 Example 2–4 (factored loads).

The concentrated load is a live load only:

$$P_u = 1.2P_D + 1.6P_L$$
$$P_u = 1.2 \times 0 + 1.6 \times 10 = 16 \text{ kip}$$

The factored loads on the beam are shown in Figure 2–17.

Structural Load Requirements
International Building Code (2012)

TABLE 1607.1
MINIMUM UNIFORMLY DISTRIBUTED LIVE LOADS, L_o, AND MINIMUM CONCENTRATED LIVE LOADS[g]

OCCUPANCY OR USE	UNIFORM (psf)	CONCENTRATED (lbs.)
1. Apartments (see residential)	—	—
2. Access floor systems		
Office use	50	2,000
Computer use	100	2,000
3. Armories and drill rooms	150[m]	—
4. Assembly areas		—
Fixed seats (fastened to floor)	60[m]	
Follow spot, projections and control rooms	50	
Lobbies	100[m]	
Movable seats	100[m]	
Stage floors	150[m]	
Platforms (assembly)	100[m]	
Other assembly areas	100[m]	
5. Balconies and decks[h]	Same as occupancy served	—
6. Catwalks	40	300
7. Cornices	60	—
8. Corridors		—
First floor	100	
Other floors	Same as occupancy served except as indicated	
9. Dining rooms and restaurants	100[m]	—
10. Dwellings (see residential)	—	—
11. Elevator machine room grating (on area of 2 inches by 2 inches)	—	300
12. Finish light floor plate construction (on area of 1 inch by 1 inch)	—	200
13. Fire escapes	100	—
On single-family dwellings only	40	
14. Garages (passenger vehicles only)	40[m]	Note a
Trucks and buses	See Section 1607.7	
15. Handrails, guards and grab bars	See Section 1607.8	
16. Helipads	See Section 1607.6	
17. Hospitals		
Corridors above first floor	80	1,000
Operating rooms, laboratories	60	1,000
Patient rooms	40	1,000
18. Hotels (see residential)	—	—
19. Libraries		
Corridors above first floor	80	1,000
Reading rooms	60	1,000
Stack rooms	150[b, m]	1,000
20. Manufacturing		
Heavy	250[m]	3,000
Light	125[m]	2,000
21. Marquees	75	—
22. Office buildings		
Corridors above first floor	80	2,000
File and computer rooms shall be designed for heavier loads based on anticipated occupancy	—	—
Lobbies and first-floor corridors	100	2,000
Offices	50	2,000

TABLE 1607.1—continued
MINIMUM UNIFORMLY DISTRIBUTED LIVE LOADS, L_o, AND MINIMUM CONCENTRATED LIVE LOADS[g]

OCCUPANCY OR USE	UNIFORM (psf)	CONCENTRATED (lbs.)
23. Penal institutions		—
Cell blocks	40	
Corridors	100	
24. Recreational uses:		—
Bowling alleys, poolrooms and similar uses	75[m]	
Dance halls and ballrooms	100[m]	
Gymnasiums	100[m]	
Reviewing stands, grandstands and bleachers	100[c, m]	
Stadiums and arenas with fixed seats (fastened to floor)	60[c, m]	
25. Residential		—
One- and two-family dwellings		
Uninhabitable attics without storage[i]	10	
Uninhabitable attics with storage[i, j, k]	20	
Habitable attics and sleeping areas[k]	30	
All other areas	40	
Hotels and multifamily dwellings		
Private rooms and corridors serving them	40	
Public rooms[m] and corridors serving them	100	
26. Roofs		
All roof surfaces subject to maintenance workers		300
Awnings and canopies:		
Fabric construction supported by a skeleton structure	5 nonreducible	
All other construction	20	
Ordinary flat, pitched, and curved roofs (that are not occupiable)	20	
Where primary roof members are exposed to a work floor, at single panel point of lower chord of roof trusses or any point along primary structural members supporting roofs:		
Over manufacturing, storage warehouses, and repair garages		2,000
All other primary roof members		300
Occupiable roofs:		
Roof gardens	100	
Assembly areas	100[m]	
All other similar areas	Note 1	Note 1
27. Schools		
Classrooms	40	1,000
Corridors above first floor	80	1,000
First-floor corridors	100	1,000
28. Scuttles, skylight ribs and accessible ceilings	—	200
29. Sidewalks, vehicular drive ways and yards, subject to trucking	250[d, m]	8,000[e]

TABLE 1607.1—continued
MINIMUM UNIFORMLY DISTRIBUTED LIVE LOADS, L_o, AND MINIMUM CONCENTRATED LIVE LOADS[g]

OCCUPANCY OR USE	UNIFORM (psf)	CONCENTRATED (lbs.)
30. Stairs and exits		
One- and two-family dwellings	40	300[f]
All other	100	300[f]
31. Storage warehouses (shall be designed for heavier loads if required for anticipated storage)		—
Heavy	250[m]	
Light	125[m]	
32. Stores		
Retail		
First floor	100	1,000
Upper floors	75	1,000
Wholesale, all floors	125[m]	1,000
33. Vehicle barriers	See Section 1607.8.3	
34. Walkways and elevated platforms (other than exitways)	60	—
35. Yards and terraces, pedestrians	100[m]	—

For SI: 1 inch = 25.4 mm, 1 square inch = 645.16 mm^2,
1 square foot = 0.0929 m^2,
1 pound per square foot = 0.0479 kN/m^2, 1 pound = 0.004448 kN,
1 pound per cubic foot = 16 kg/m^3.

a. Floors in garages or portions of buildings used for the storage of motor vehicles shall be designed for the uniformly distributed live loads of Table 1607.1 or the following concentrated loads: (1) for garages restricted to passenger vehicles accommodating not more than nine passengers, 3,000 pounds acting on an area of 4.5 inches by 4.5 inches; (2) for mechanical parking structures without slab or deck that are used for storing passenger vehicles only, 2,250 pounds per wheel.

b. The loading applies to stack room floors that support nonmobile, double-faced library book stacks, subject to the following limitations:
1. The nominal bookstack unit height shall not exceed 90 inches;
2. The nominal shelf depth shall not exceed 12 inches for each face; and
3. Parallel rows of double-faced book stacks shall be separated by aisles not less than 36 inches wide.

c. Design in accordance with ICC 300.

d. Other uniform loads in accordance with an approved method containing provisions for truck loadings shall also be considered where appropriate.

e. The concentrated wheel load shall be applied on an area of 4.5 inches by 4.5 inches.

f. The minimum concentrated load on stair treads shall be applied on an area of 2 inches by 2 inches. This load need not be assumed to act concurrently with the uniform load.

g. Where snow loads occur that are in excess of the design conditions, the structure shall be designed to support the loads due to the increased loads caused by drift buildup or a greater snow design determined by the building official (see Section 1608).

h. See Section 1604.8.3 for decks attached to exterior walls.

i. Uninhabitable attics without storage are those where the maximum clear height between the joists and rafters is less than 42 inches, or where there are not two or more adjacent trusses with web configurations capable of accommodating an assumed rectangle 42 inches in height by 24 inches in width, or greater, within the plane of the trusses. This live load need not be assumed to act concurrently with any other live load requirements.

Live Loads & Allowed Reductions

1607.10 Reduction in uniform live loads. Except for uniform live loads at roofs, all other minimum uniformly distributed live loads, L_o, in Table 1607.1 are permitted to be reduced in accordance with Section 1607.10.1 or 1607.10.2. Uniform live loads at roofs are permitted to be reduced in accordance with Section 1607.12.2.

1607.10.1 Basic uniform live load reduction. Subject to the limitations of Sections 1607.10.1.1 through 1607.10.1.3 and Table 1607.1, members for which a value of $K_{LL}A_T$ is 400 square feet (37.16 m²) or more are permitted to be designed for a reduced uniformly distributed live load, L, in accordance with the following equation:

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right) \qquad \textbf{(Equation 16-23)}$$

For SI: $L = L_o\left(0.25 + \dfrac{4.57}{\sqrt{K_{LL}A_T}}\right)$

where:

L = Reduced design live load per square foot (m²) of area supported by the member.

L_o = Unreduced design live load per square foot (m²) of area supported by the member (see Table 1607.1).

K_{LL} = Live load element factor (see Table 1607.10.1).

A_T = Tributary area, in square feet (m²).

L shall not be less than $0.50L_o$ for members supporting one floor and L shall not be less than $0.40L_o$ for members supporting two or more floors.

TABLE 1607.10.1
LIVE LOAD ELEMENT FACTOR, K_{LL}

ELEMENT	K_{LL}
Interior columns Exterior columns without cantilever slabs	4 4
Edge columns with cantilever slabs	3
Corner columns with cantilever slabs Edge beams without cantilever slabs Interior beams	2 2 2
All other members not identified above including: Edge beams with cantilever slabs Cantilever beams One-way slabs Two-way slabs Members without provisions for continuous shear transfer normal to their span	1

1607.10.1.1 One-way slabs. The tributary area, A_T, for use in Equation 16-23 for one-way slabs shall not exceed an area defined by the slab span times a width normal to the span of 1.5 times the slab span.

1607.10.1.2 Heavy live loads. Live loads that exceed 100 psf (4.79 kN/m²) shall not be reduced.

Exceptions:

1. The live loads for members supporting two or more floors are permitted to be reduced by a maximum of 20 percent, but the live load shall not be less than L as calculated in Section 1607.10.1.
2. For uses other than storage, where *approved*, additional live load reductions shall be permitted where shown by the *registered design professional* that a rational approach has been used and that such reductions are warranted.

1607.10.1.3 Passenger vehicle garages. The live loads shall not be reduced in passenger vehicle garages.

Exception: The live loads for members supporting two or more floors are permitted to be reduced by a maximum of 20 percent, but the live load shall not be less than L as calculated in Section 1607.10.1.

1607.10.2 Alternative uniform live load reduction. As an alternative to Section 1607.10.1 and subject to the limitations of Table 1607.1, uniformly distributed live loads are permitted to be reduced in accordance with the following provisions. Such reductions shall apply to slab systems, beams, girders, columns, piers, walls and foundations.

1. A reduction shall not be permitted where the live load exceeds 100 psf (4.79 kN/m²) except that the design live load for members supporting two or more floors is permitted to be reduced by a maximum of 20 percent.

   Exception: For uses other than storage, where *approved*, additional live load reductions shall be permitted where shown by the *registered design professional* that a rational approach has been used and that such reductions are warranted.

2. A reduction shall not be permitted in passenger vehicle parking garages except that the live loads for members supporting two or more floors are permitted to be reduced by a maximum of 20 percent.
3. For live loads not exceeding 100 psf (4.79 kN/m²), the design live load for any structural member supporting 150 square feet (13.94 m²) or more is permitted to be reduced in accordance with Equation 16-24.
4. For one-way slabs, the area, A, for use in Equation 16-24 shall not exceed the product of the slab span and a width normal to the span of 0.5 times the slab span.

$$R = 0.08(A - 150) \qquad \textbf{(Equation 16-24)}$$

For SI: $R = 0.861(A - 13.94)$

Such reduction shall not exceed the smallest of:

1. 40 percent for horizontal members;
2. 60 percent for vertical members; or
3. R as determined by the following equation.

$$R = 23.1(1 + D/L_o) \qquad \textbf{(Equation 16-25)}$$

where:

A = Area of floor supported by the member, square feet (m²).

D = Dead load per square foot (m²) of area supported.

L_o = Unreduced live load per square foot (m²) of area supported.

R = Reduction in percent.

1607.11 Distribution of floor loads. Where uniform floor live loads are involved in the design of structural members arranged so as to create continuity, the minimum applied loads shall be the full dead loads on all spans in combination with the floor live loads on spans selected to produce the greatest *load effect* at each location under consideration. Floor live loads are permitted to be reduced in accordance with Section 1607.10.

Minimum Roof Loads

1607.12 Roof loads. The structural supports of roofs and marquees shall be designed to resist wind and, where applicable, snow and earthquake loads, in addition to the dead load of construction and the appropriate live loads as prescribed in this section, or as set forth in Table 1607.1. The live loads acting on a sloping surface shall be assumed to act vertically on the horizontal projection of that surface.

1607.12.1 Distribution of roof loads. Where uniform roof live loads are reduced to less than 20 psf (0.96 kN/m^2) in accordance with Section 1607.12.2.1 and are applied to the design of structural members arranged so as to create continuity, the reduced roof live load shall be applied to adjacent spans or to alternate spans, whichever produces the most unfavorable *load effect*. See Section 1607.12.2 for reductions in minimum roof live loads and Section 7.5 of ASCE 7 for partial snow loading.

1607.12.2 General. The minimum uniformly distributed live loads of roofs and marquees, L_o, in Table 1607.1 are permitted to be reduced in accordance with Section 1607.12.2.1.

1607.12.2.1 Ordinary roofs, awnings and canopies. Ordinary flat, pitched and curved roofs, and awnings and canopies other than of fabric construction supported by a skeleton structure, are permitted to be designed for a reduced uniformly distributed roof live load, L_r, as specified in the following equations or other controlling combinations of loads as specified in Section 1605, whichever produces the greater *load effect*.

In structures such as greenhouses, where special scaffolding is used as a work surface for workers and materials during maintenance and repair operations, a lower roof load than specified in the following equations shall not be used unless *approved* by the *building official*. Such structures shall be designed for a minimum roof live load of 12 psf (0.58 kN/m^2).

$L_r = L_o R_1 R_2$ **(Equation 16-26)**

where: $12 \le L_r \le 20$

For SI: $L_r = L_o R_1 R_2$

where: $0.58 \le L_r \le 0.96$

L_o = Unreduced roof live load per square foot (m^2) of horizontal projection supported by the member (see Table 1607.1).

L_r = Reduced roof live load per square foot (m^2) of horizontal projection supported by the member.

The reduction factors R_1 and R_2 shall be determined as follows:

$R_1 = 1$ for $A_t \le 200$ square feet (18.58 m^2) **(Equation 16-27)**

$R_1 = 1.2 - 0.001A_t$ for 200 square feet $< A_t < 600$ square feet **(Equation 16-28)**

For SI: $1.2 - 0.011A_t$ for 18.58 square meters $< A_t <$ 55.74 square meters

$R_1 = 0.6$ for $A_t \ge 600$ square feet (55.74 m^2) **(Equation 16-29)**

where:

A_t = Tributary area (span length multiplied by effective width) in square feet (m^2) supported by the member, and

$R_2 = 1$ for $F \le 4$ **(Equation 16-30)**

$R_2 = 1.2 - 0.05F$ for $4 < F < 12$ **(Equation 16-31)**

$R_2 = 0.6$ for $F \ge 12$ **(Equation 16-32)**

where:

F = For a sloped roof, the number of inches of rise per foot (for SI: $F = 0.12 \times$ slope, with slope expressed as a percentage), or for an arch or dome, the rise-to-span ratio multiplied by 32.

1607.12.3 Occupiable roofs. Areas of roofs that are occupiable, such as roof gardens, or for assembly or other similar purposes, and marquees are permitted to have their uniformly distributed live loads reduced in accordance with Section 1607.10.

1607.12.3.1 Landscaped roofs. The uniform design live load in unoccupied landscaped areas on roofs shall be 20 psf (0.958 kN/m^2). The weight of all landscaping materials shall be considered as dead load and shall be computed on the basis of saturation of the soil.

1607.12.4 Awnings and canopies. Awnings and canopies shall be designed for uniform live loads as required in Table 1607.1 as well as for snow loads and wind loads as specified in Sections 1608 and 1609.

Minimum Snow Loads

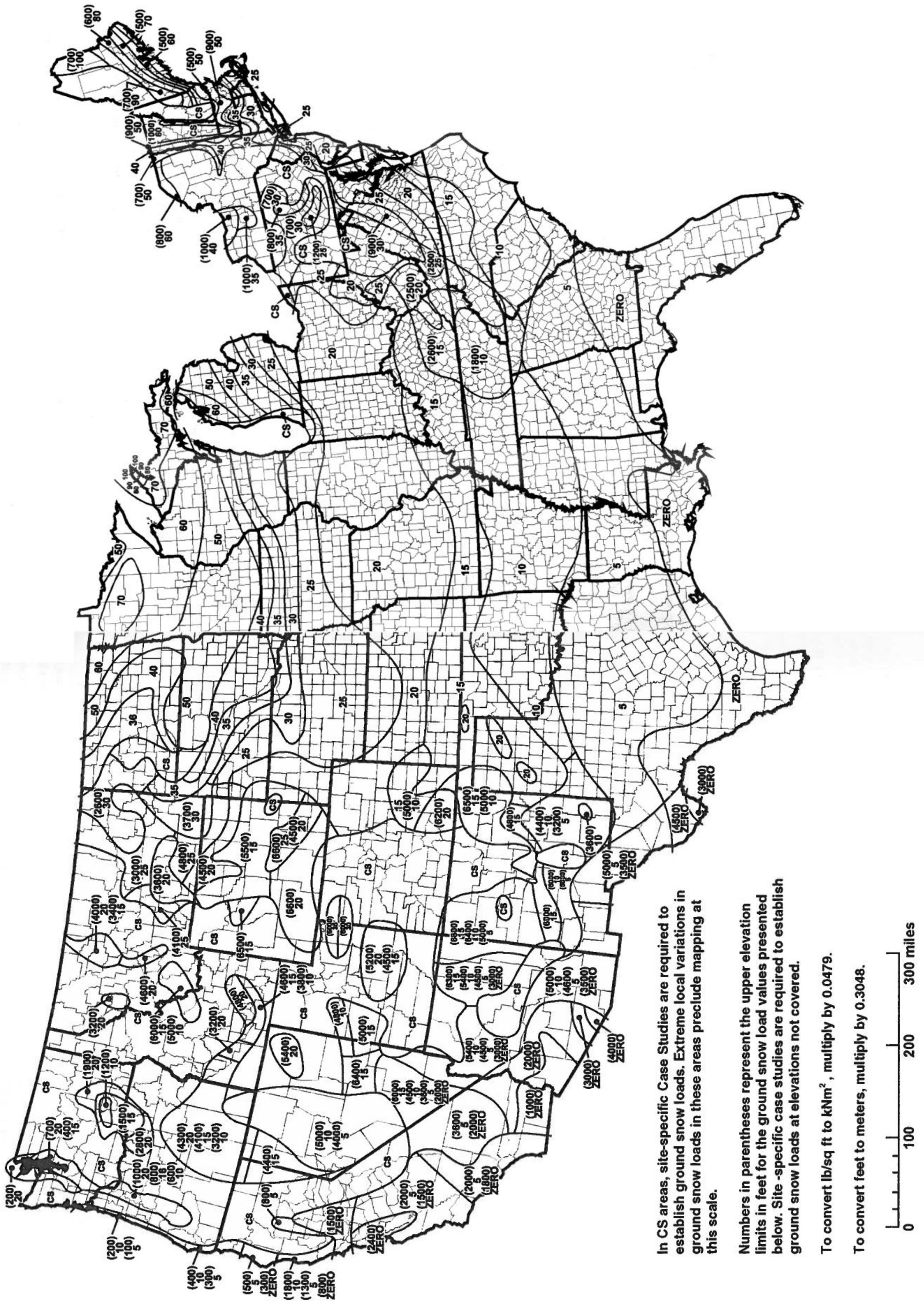


FIGURE 1608.2—continued
GROUND SNOW LOADS, p_g, FOR THE UNITED STATES (psf)

Documentation of Loads

SECTION 1603
CONSTRUCTION DOCUMENTS

1603.1 General. *Construction documents* shall show the size, section and relative locations of structural members with floor levels, column centers and offsets dimensioned. The design loads and other information pertinent to the structural design required by Sections 1603.1.1 through 1603.1.9 shall be indicated on the *construction documents*.

Exception: *Construction documents* for buildings constructed in accordance with the *conventional light-frame construction* provisions of Section 2308 shall indicate the following structural design information:

1. Floor and roof live loads.
2. Ground snow load, P_g.
3. Ultimate design wind speed, V_{ult}, (3-second gust), miles per hour (mph) (km/hr) and nominal design wind speed, V_{asd}, as determined in accordance with Section 1609.3.1 and wind exposure.
4. *Seismic design category* and *site class*.
5. Flood design data, if located in *flood hazard areas* established in Section 1612.3.
6. Design load-bearing values of soils.

1603.1.1 Floor live load. The uniformly distributed, concentrated and impact floor live load used in the design shall be indicated for floor areas. Use of live load reduction in accordance with Section 1607.10 shall be indicated for each type of live load used in the design.

1603.1.2 Roof live load. The roof live load used in the design shall be indicated for roof areas (Section 1607.12).

1603.1.3 Roof snow load data. The ground snow load, P_g, shall be indicated. In areas where the ground snow load, P_g, exceeds 10 pounds per square foot (psf) (0.479 kN/m^2), the following additional information shall also be provided, regardless of whether snow loads govern the design of the roof:

1. Flat-roof snow load, P_f.
2. Snow exposure factor, C_e.
3. Snow load importance factor, *I*.
4. Thermal factor, C_t.

1603.1.4 Wind design data. The following information related to wind loads shall be shown, regardless of whether wind loads govern the design of the lateral force-resisting system of the structure:

1. Ultimate design wind speed, V_{ult}, (3-second gust), miles per hour (km/hr) and nominal design wind speed, V_{asd}, as determined in accordance with Section 1609.3.1.
2. *Risk category*.
3. Wind exposure.Where more than one wind exposure is utilized, the wind exposure and applicable wind direction shall be indicated.
4. The applicable internal pressure coefficient.
5. Components and cladding. The design wind pressures in terms of psf (kN/m^2) to be used for the design of exterior component and cladding materials not specifically designed by the *registered design professional*.

1603.1.5 Earthquake design data. The following information related to seismic loads shall be shown, regardless of whether seismic loads govern the design of the lateral force-resisting system of the structure:

1. *Risk category*.
2. Seismic importance factor, I_e.
3. Mapped spectral response acceleration parameters, S_S and S_1.
4. *Site class*.
5. Design spectral response acceleration parameters, S_{DS} and S_{D1}.
6. *Seismic design category*.
7. Basic seismic force-resisting system(s).
8. Design base shear(s).
9. Seismic response coefficient(s), C_S.
10. Response modification coefficient(s), *R*.
11. Analysis procedure used.

1603.1.6 Geotechnical information. The design load-bearing values of soils shall be shown on the *construction documents*.

1603.1.7 Flood design data. For buildings located in whole or in part in *flood hazard areas* as established in Section 1612.3, the documentation pertaining to design, if required in Section 1612.5, shall be included and the following information, referenced to the datum on the community's Flood Insurance Rate Map (FIRM), shall be shown, regardless of whether flood loads govern the design of the building:

1. In *flood hazard areas* not subject to high-velocity wave action, the elevation of the proposed lowest floor, including the basement.
2. In *flood hazard areas* not subject to high-velocity wave action, the elevation to which any nonresidential building will be dry flood proofed.
3. In *flood hazard areas* subject to high-velocity wave action, the proposed elevation of the bottom of the lowest horizontal structural member of the lowest floor, including the basement.

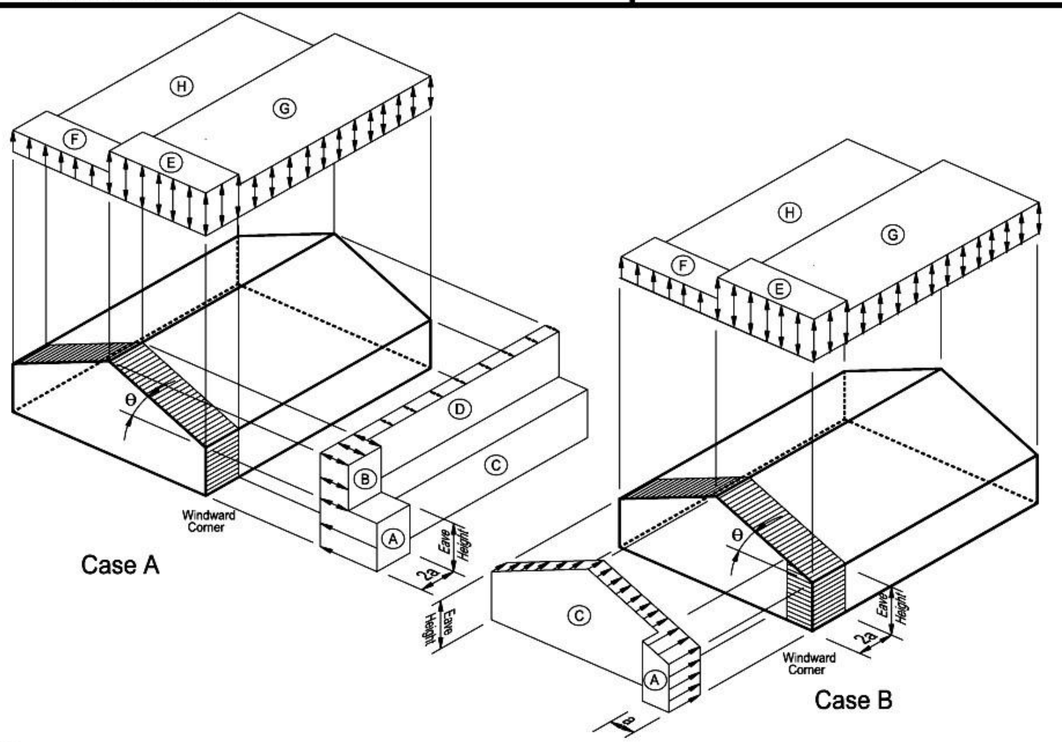
1603.1.8 Special loads. Special loads that are applicable to the design of the building, structure or portions thereof shall be indicated along with the specified section of this code that addresses the special loading condition.

1603.1.9 Systems and components requiring special inspections for seismic resistance. *Construction documents* or specifications shall be prepared for those systems and components requiring *special inspection* for seismic resistance as specified in Section 1705.11 by the *registered design professional* responsible for their design and shall be submitted for approval in accordance with Section 107.1. Reference to seismic standards in lieu of detailed drawings is acceptable.

Design Wind Pressures – Envelope Procedure
SEI/ASCE 7-10:

Velocity pressure, *p*, irrespective of terrain and height above ground or recurrence probability is related to the wind speed, *V*, by $p = 0.00256V^2$. Wind codes also consider the effect of the geometry of the building and location on the surface, wind gusts or turbulence, the local terrain, and annual probability of exceeding the design wind speed.

Main Wind Force Resisting System – Method 2		**h ≤ 60 ft.**
Figure 28.6-1	**Design Wind Pressures**	**Walls & Roofs**
Enclosed Buildings		



Notes:

1. Pressures shown are applied to the horizontal and vertical projections, for exposure B, at h=30 ft (9.1m). Adjust to other exposures and heights with adjustment factor λ.
2. The load patterns shown shall be applied to each corner of the building in turn as the reference corner. (See Figure 28.4-1)
3. For Case B use $\theta = 0°$.
4. Load cases 1 and 2 must be checked for $25° < \theta \le 45°$. Load case 2 at 25° is provided only for interpolation between 25° and 30°.
5. Plus and minus signs signify pressures acting toward and away from the projected surfaces, respectively.
6. For roof slopes other than those shown, linear interpolation is permitted.
7. The total horizontal load shall not be less than that determined by assuming $p_S = 0$ in zones B & D.
8. Where zone E or G falls on a roof overhang on the windward side of the building, use E_{OH} and G_{OH} for the pressure on the horizontal projection of the overhang. Overhangs on the leeward and side edges shall have the basic zone pressure applied.
9. Notation:
   a: 10 percent of least horizontal dimension or 0.4h, whichever is smaller, but not less than either 4% of least horizontal dimension or 3 ft (0.9 m).
   h: Mean roof height, in feet (meters), except that eave height shall be used for roof angles <10°.
   θ: Angle of plane of roof from horizontal, in degrees.

Main Wind Force Resisting System – Method 2	$h \le 60$ ft.
Figure 28.6-1 (cont'd) Design Wind Pressures	Walls & Roofs
Enclosed Buildings	

Simplified Design Wind Pressure, p_{S30} (psf) *(Exposure B at h = 30 ft. with I = 1.0)*

Basic Wind Speed (mph)	Roof Angle (degrees)	Load Case	Zones									
			Horizontal Pressures				Vertical Pressures				Overhangs	
			A	B	C	D	E	F	G	H	E_{OH}	G_{OH}
110	0 to 5°	1	19.2	-10.0	12.7	-5.9	-23.1	-13.1	-16.0	-10.1	-32.3	-25.3
	10°	1	21.6	-9.0	14.4	-5.2	-23.1	-14.1	-16.0	-10.8	-32.3	-25.3
	15°	1	24.1	-8.0	16.0	-4.6	-23.1	-15.1	-16.0	-11.5	-32.3	-25.3
	20°	1	26.6	-7.0	17.7	-3.9	-23.1	-16.0	-16.0	-12.2	-32.3	-25.3
	25°	1	24.1	3.9	17.4	4.0	-10.7	-14.6	-7.7	-11.7	-19.9	-17.0
		2	-------	-------	-------	-------	-4.1	-7.9	-1.1	-5.1	-------	-------
	30 to 45	1	21.6	14.8	17.2	11.8	1.7	-13.1	0.6	-11.3	-7.6	-8.7
		2	21.6	14.8	17.2	11.8	8.3	-6.5	7.2	-4.6	-7.6	-8.7
115	0 to 5°	1	21.0	-10.9	13.9	-6.5	-25.2	-14.3	-17.5	-11.1	-35.3	-27.6
	10°	1	23.7	-9.8	15.7	-5.7	-25.2	-15.4	-17.5	-11.8	-35.3	-27.6
	15°	1	26.3	-8.7	17.5	-5.0	-25.2	-16.5	-17.5	-12.6	-35.3	-27.6
	20°	1	29.0	-7.7	19.4	-4.2	-25.2	-17.5	-17.5	-13.3	-35.3	-27.6
	25°	1	26.3	4.2	19.1	4.3	-11.7	-15.9	-8.5	-12.8	-21.8	-18.5
		2	-------	-------	-------	-------	-4.4	-8.7	-1.2	-5.5	-------	-------
	30 to 45	1	23.6	16.1	18.8	12.9	1.8	-14.3	0.6	-12.3	-8.3	-9.5
		2	23.6	16.1	18.8	12.9	9.1	-7.1	7.9	-5.0	-8.3	-9.5
120	0 to 5°	1	22.8	-11.9	15.1	-7.0	-27.4	-15.6	-19.1	-12.1	-38.4	-30.1
	10°	1	25.8	-10.7	17.1	-6.2	-27.4	-16.8	-19.1	-12.9	-38.4	-30.1
	15°	1	28.7	-9.5	19.1	-5.4	-27.4	-17.9	-19.1	-13.7	-38.4	-30.1
	20°	1	31.6	-8.3	21.1	-4.6	-27.4	-19.1	-19.1	-14.5	-38.4	-30.1
	25°	1	28.6	4.6	20.7	4.7	-12.7	-17.3	-9.2	-13.9	-23.7	-20.2
		2	-------	-------	-------	-------	-4.8	-9.4	-1.3	-6.0	-------	-------
	30 to 45	1	25.7	17.6	20.4	14.0	2.0	-15.6	0.7	-13.4	-9.0	-10.3
		2	25.7	17.6	20.4	14.0	9.9	-7.7	8.6	-5.5	-9.0	-10.3
130	0 to 5°	1	26.8	-13.9	17.8	-8.2	-32.2	-18.3	-22.4	-14.2	-45.1	-35.3
	10°	1	30.2	-12.5	20.1	-7.3	-32.2	-19.7	-22.4	-15.1	-45.1	-35.3
	15°	1	33.7	-11.2	22.4	-6.4	-32.2	-21.0	-22.4	-16.1	-45.1	-35.3
	20°	1	37.1	-9.8	24.7	-5.4	-32.2	-22.4	-22.4	-17.0	-45.1	-35.3
	25°	1	33.6	5.4	24.3	5.5	-14.9	-20.4	-10.8	-16.4	-27.8	-23.7
		2	-------	-------	-------	-------	-5.7	-11.1	-1.5	-7.1	-------	-------
	30 to 45	1	30.1	20.6	24.0	16.5	2.3	-18.3	0.8	-15.7	-10.6	-12.1
		2	30.1	20.6	24.0	16.5	11.6	-9.0	10.0	-6.4	-10.6	-12.1
140	0 to 5°	1	31.1	-16.1	20.6	-9.6	-37.3	-21.2	-26.0	-16.4	-52.3	-40.9
	10°	1	35.1	-14.5	23.3	-8.5	-37.3	-22.8	-26.0	-17.5	-52.3	-40.9
	15°	1	39.0	-12.9	26.0	-7.4	-37.3	-24.4	-26.0	-18.6	-52.3	-40.9
	20°	1	43.0	-11.4	28.7	-6.3	-37.3	-26.0	-26.0	-19.7	-52.3	-40.9
	25°	1	39.0	6.3	28.2	6.4	-17.3	-23.6	-12.5	-19.0	-32.3	-27.5
		2	-------	-------	-------	-------	-6.6	-12.8	-1.8	-8.2	-------	-------
	30 to 45	1	35.0	23.9	27.8	19.1	2.7	-21.2	0.9	-18.2	-12.3	-14.0
		2	35.0	23.9	27.8	19.1	13.4	-10.5	11.7	-7.5	-12.3	-14.0
150	0 to 5°	1	35.7	-18.5	23.7	-11.0	-42.9	-24.4	-29.8	-18.9	-60.0	-47.0
	10°	1	40.2	-16.7	26.8	-9.7	-42.9	-26.2	-29.8	-20.1	-60.0	-47.0
	15°	1	44.8	-14.9	29.8	-8.5	-42.9	-28.0	-29.8	-21.4	-60.0	-47.0
	20°	1	49.4	-13.0	32.9	-7.2	-42.9	-29.8	-29.8	-22.6	-60.0	-47.0
	25°	1	44.8	7.2	32.4	7.4	-19.9	-27.1	-14.4	-21.8	-37.0	-31.6
		2	-------	-------	-------	-------	-7.5	-14.7	-2.1	-9.4	-------	-------
	30 to 45	1	40.1	27.4	31.9	22.0	3.1	-24.4	1.0	-20.9	-14.1	-16.1
		2	40.1	27.4	31.9	22.0	15.4	-12.0	13.4	-8.6	-14.1	-16.1

Unit Conversions – 1.0 ft = 0.3048 m; 1.0 psf = 0.0479 kN/m^2

Main Wind Force Resisting System – Method 2		h ≤ 60 ft.
Figure 28.6-1 (cont'd)	Design Wind Pressures	Walls & Roofs
Enclosed Buildings		

Simplified Design Wind Pressure , p_{S30} (psf) *(Exposure B at h = 30 ft.)*

Basic Wind Speed (mph)	Roof Angle (degrees)	Load Case	Zones									
			Horizontal Pressures				Vertical Pressures				Overhangs	
			A	B	C	D	E	F	G	H	E_{OH}	G_{OH}
160	0 to 5°	1	40.6	-21.1	26.9	-12.5	-48.8	-27.7	-34.0	-21.5	-68.3	-53.5
	10°	1	45.8	-19.0	30.4	-11.1	-48.8	-29.8	-34.0	-22.9	-68.3	-53.5
	15°	1	51.0	-16.9	34.0	-9.6	-48.8	-31.9	-34.0	-24.3	-68.3	-53.5
	20°	1	56.2	-14.8	37.5	-8.2	-48.8	-34.0	-34.0	-25.8	-68.3	-53.5
	25°	1	50.9	8.2	36.9	8.4	-22.6	-30.8	-16.4	-24.8	-42.1	-35.9
		2	-------	------	-------	-------	-8.6	-16.8	-2.3	-10.7	-------	-------
	30 to 45	1	45.7	31.2	36.3	25.0	3.5	-27.7	1.2	-23.8	-16.0	-18.3
		2	45.7	31.2	36.3	25.0	17.6	-13.7	15.2	-9.8	-16.0	-18.3
180	0 to 5°	1	51.4	-26.7	34.1	-15.8	-61.7	-35.1	-43.0	-27.2	-86.4	-67.7
	10°	1	58.0	-24.0	38.5	-14.0	-61.7	-37.7	-43.0	-29.0	-86.4	-67.7
	15°	1	64.5	-21.4	43.0	-12.2	-61.7	-40.3	-43.0	-30.8	-86.4	-67.7
	20°	1	71.1	-18.8	47.4	-10.4	-61.7	-43.0	-43.0	-32.6	-86.4	-67.7
	25°	1	64.5	10.4	46.7	10.6	-28.6	-39.0	-20.7	-31.4	-53.3	-45.4
		2	-------	------	-------	-------	-10.9	-21.2	-3.0	-13.6	-------	-------
	30 to 45	1	57.8	39.5	45.9	31.6	4.4	-35.1	1.5	-30.1	-20.3	-23.2
		2	57.8	39.5	45.9	31.6	22.2	-17.3	19.3	-12.3	-20.3	-23.2
200	0 to 5°	1	63.4	-32.9	42.1	-19.5	-76.2	-43.3	-53.1	-33.5	-106.7	-83.5
	10°	1	71.5	-29.7	47.6	-17.3	-76.2	-46.5	-53.1	-35.8	-106.7	-83.5
	15°	1	79.7	-26.4	53.1	-15.0	-76.2	-49.8	-53.1	-38.0	-106.7	-83.5
	20°	1	87.8	-23.2	58.5	-12.8	-76.2	-53.1	-53.1	-40.2	-106.7	-83.5
	25°	1	79.6	12.8	57.6	13.1	-35.4	-48.2	-25.6	-38.7	-65.9	-56.1
		2	-------	------	-------	-------	-13.4	-26.2	-3.7	-16.8	-------	-------
	30 to 45	1	71.3	48.8	56.7	39.0	5.5	-43.3	1.8	-37.2	-25.0	-28.7
		2	71.3	48.8	56.7	39.0	27.4	-21.3	23.8	-15.2	-25.0	-28.7

Adjustment Factor
for Building Height and Exposure, λ

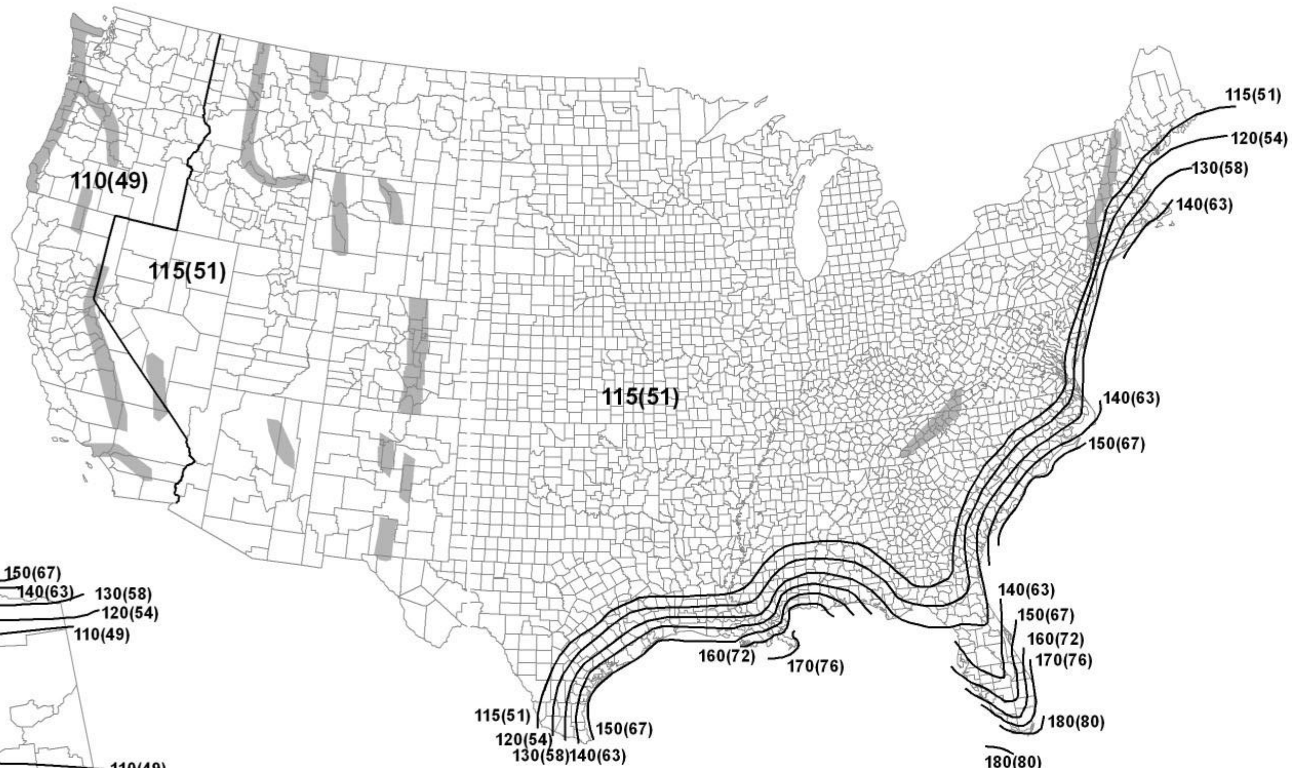
Mean roof height (ft)	Exposure		
	B	C	D
15	1.00	1.21	1.47
20	1.00	1.29	1.55
25	1.00	1.35	1.61
30	1.00	1.40	1.66
35	1.05	1.45	1.70
40	1.09	1.49	1.74
45	1.12	1.53	1.78
50	1.16	1.56	1.81
55	1.19	1.59	1.84
60	1.22	1.62	1.87

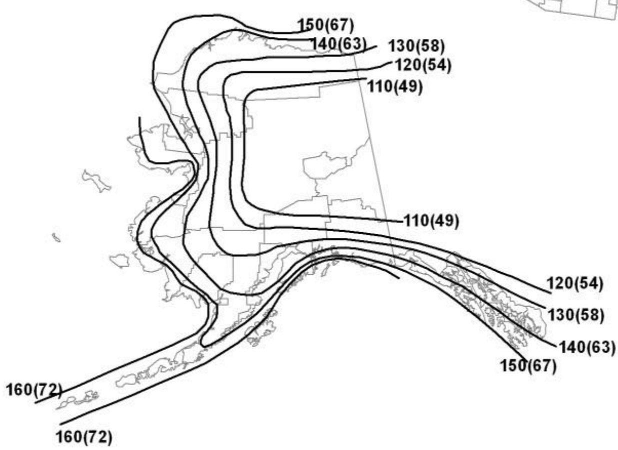
Unit Conversions – 1.0 ft = 0.3048 m; 1.0 psf = 0.0479 kN/m^2

Table 1.5-1 Risk Category of Buildings and Other Structures for Flood, Wind, Snow, Earthquake, and Ice Loads

Use or Occupancy of Buildings and Structures	Risk Category
Buildings and other structures that represent a low risk to human life in the event of failure	I
All buildings and other structures except those listed in Risk Categories I, III, and IV	II
Buildings and other structures, the failure of which could pose a substantial risk to human life.	III
Buildings and other structures, not included in Risk Category IV, with potential to cause a substantial economic impact and/or mass disruption of day-to-day civilian life in the event of failure.	
Buildings and other structures not included in Risk Category IV (including, but not limited to, facilities that manufacture, process, handle, store, use, or dispose of such substances as hazardous fuels, hazardous chemicals, hazardous waste, or explosives) containing toxic or explosive substances where their quantity exceeds a threshold quantity established by the authority having jurisdiction and is sufficient to pose a threat to the public if released.	
Buildings and other structures designated as essential facilities.	IV
Buildings and other structures, the failure of which could pose a substantial hazard to the community.	
Buildings and other structures (including, but not limited to, facilities that manufacture, process, handle, store, use, or dispose of such substances as hazardous fuels, hazardous chemicals, or hazardous waste) containing sufficient quantities of highly toxic substances where the quantity exceeds a threshold quantity established by the authority having jurisdiction to be dangerous to the public if released and is sufficient to pose a threat to the public if released.[a]	
Buildings and other structures required to maintain the functionality of other Risk Category IV structures.	

[a]Buildings and other structures containing toxic, highly toxic, or explosive substances shall be eligible for classification to a lower Risk Category if it can be demonstrated to the satisfaction of the authority having jurisdiction by a hazard assessment as described in Section 1.5.2 that a release of the substances is commensurate with the risk associated with that Risk Category.





Special Wind Region

Location	Vmph	(m/s)
Guam	195	(87)
Virgin Islands	165	(74)
American Samoa	160	(72)
Hawaii – Special Wind Region Statewide	130	(58)

Puerto Rico

Figure 26.5-1A Basic Wind Speeds for Occupancy Category II Buildings and Other Structures.

Figure 26.5-1A (*Continued*)

Notes:

1. Values are nominal design 3-second gust wind speeds in miles per hour (m/s) at 33 ft (10m) above ground for Exposure C category.
2. Linear interpolation between contours is permitted.
3. Islands and coastal areas outside the last contour shall use the last wind speed contour of the coastal area.
4. Mountainous terrain, gorges, ocean promontories, and special wind regions shall be examined for unusual wind conditions.
5. Wind speeds correspond to approximately a 7% probability of exceedance in 50 years (Annual Exceedance Probability = 0.00143, MRI = 700 Years).

Earthquake Ground Motion, 0.2 Second Spectral Response

International Building Code 2012:

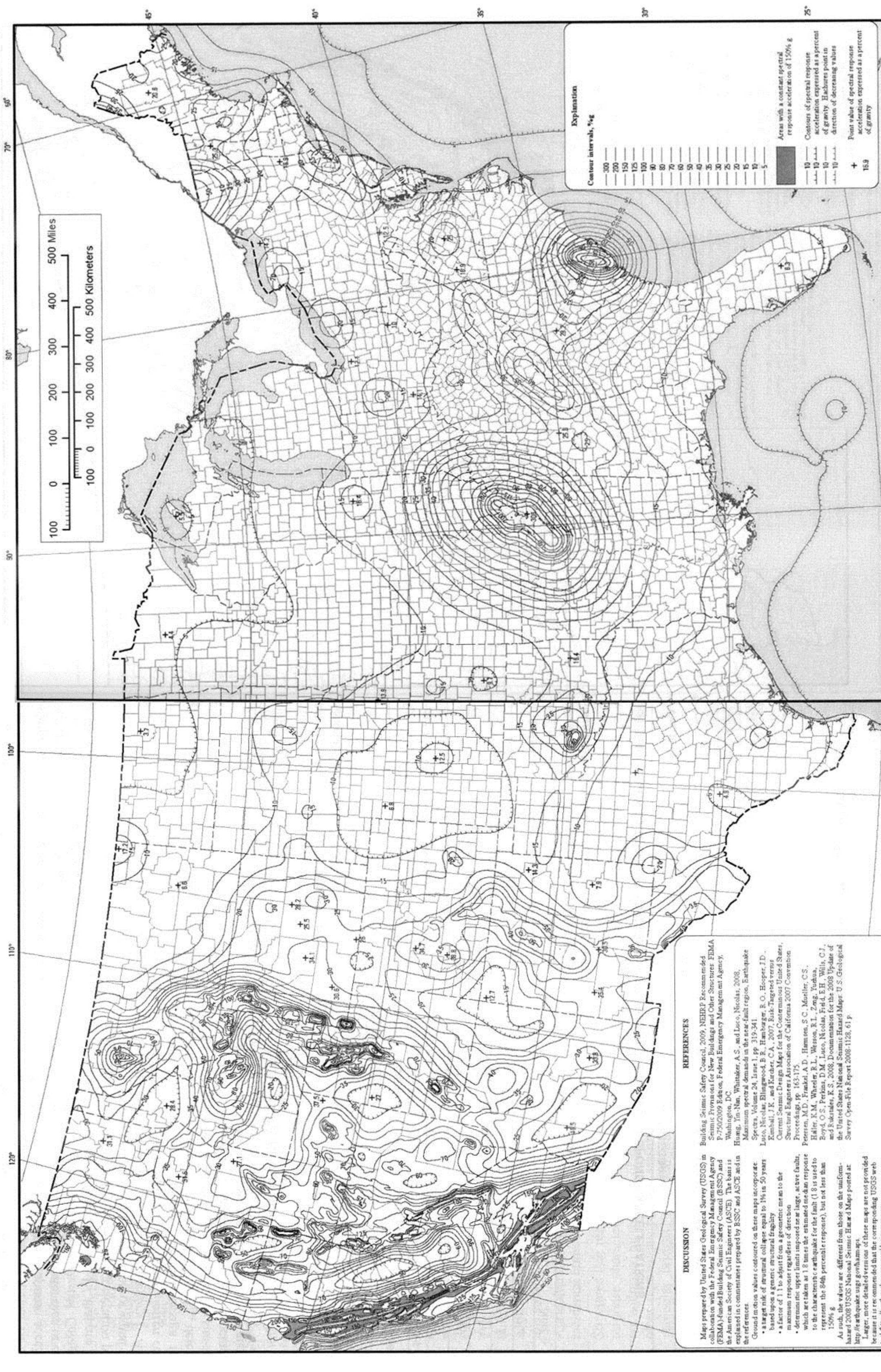


FIGURE 1613.3.1(1)
RISK-TARGETED MAXIMUM CONSIDERED EARTHQUAKE (MCE_R) GROUND MOTION RESPONSE ACCELERATIONS FOR THE CONTERMINOUS UNITED STATES OF 0.2-SECOND SPECTRAL RESPONSE ACCELERATION (5% OF CRITICAL DAMPING), SITE CLASS B

US Geological Survey, Earthquake Hazards Program, ShakeMap Scientific Background at http://earthquake.usgs.gov/eqcenter/shakemap/background.php

Spectral Response Maps

Following earthquakes larger than magnitude 5.5, spectral response maps are made. Response spectra portray the response of a damped, single-degree-of-freedom oscillator to the recorded ground motions. This data representation is useful for engineers determining how a structure will react to ground motions. The response is calculated for a range of periods. Within that range, the International Building Code (IBC) refers to particular reference periods that help define the shape of the "design spectra" that reflects the building code.

Building Code Requirements for Masonry Structures (2011)

CHAPTER 2
ALLOWABLE STRESS DESIGN OF MASONRY

CODE

2.1 — General

2.1.1 *Scope*

This chapter provides requirements for allowable stress design of masonry. Masonry design in accordance with this chapter shall comply with the requirements of Chapter 1, Sections 2.1.2 through 2.1.7, and either Section 2.2 or 2.3.

2.1.2 *Load combinations*

When the legally adopted building code does not provide allowable stress load combinations, structures and members shall be designed to resist the combinations of load specified by the building official.

2.1.3 *Design strength*

2.1.3.1 Project drawings shall show the specified compressive strength of masonry, f'_m, for each part of the structure.

2.1.3.2 Each portion of the structure shall be designed based on the specified compressive strength of masonry, f'_m, for that part of the work.

2.1.3.3 Computed stresses shall not exceed the allowable stress requirements of this Chapter.

2.1.4 *Anchor bolts embedded in grout*

2.1.4.1 *Design requirements* — Anchor bolts shall be designed using either the provisions of Section 2.1.4.2 or, for headed and bent-bar anchor bolts, by the

COMMENTARY

2.1 — General

2.1.1 *Scope*

Historically, a one-third increase in allowable stress has been permitted for load combinations that include wind or seismic loads. The origin and the reason for the one-third stress increase are unclear [2.1]. From a structural reliability standpoint, the one-third stress increase is a poor way to handle load combination effects. Therefore, the one-third stress increase is no longer permitted in this Code. The allowable stresses of this Chapter should not be increased by one-third for wind and load combinations.

2.1.2 *Load combinations*

When there is no legally adopted building code or the legally adopted building code does not have allowable stress load combinations, possible sources of allowable stress load combinations are ASCE 7[2.2] and IBC[2.3].

2.1.3 *Design strength*

The structural adequacy of masonry construction requires that the compressive strength of masonry equal or exceed the specified strength. The specified compressive strength f'_m on which design is based for each part of the structure must be shown on the project drawings.

The 1995, 1999, 2002, and 2005 editions of the Code contained provisions to permit use of strength-level load combinations in allowable stress design, to compensate for lack of service-level load combinations in previously referenced load standards. This procedure, which enabled the calculation of 'pseudo-strengths' on the basis of allowable stresses, is no longer included in the Code because recent editions of ASCE 7 include both service-level and strength-level load combinations. The 2005 edition of the Code provides guidance for using strength-level load combinations whenever the legally adopted building code does not provide service-level load combinations.

2.1.4 *Anchor bolts embedded in grout*

Allowable Stress Design anchor bolt provisions were obtained by calibrating corresponding Strength Design provisions to produce similar results. See Code

Code Requirements for Steel Construction, AISC 14th ed.

CHAPTER B

DESIGN REQUIREMENTS

The general requirements for the analysis and design of steel structures that are applicable to all chapters of the specification are given in this chapter.

The chapter is organized as follows:

- B1. General Provisions
- B2. Loads and Load Combinations
- B3. Design Basis
- B4. Classification of Sections for Local Buckling
- B5. Fabrication, Erection and Quality Control
- B6. Evaluation of Existing Structures

B1. GENERAL PROVISIONS

The design of members and *connections* shall be consistent with the intended behavior of the framing system and the assumptions made in the *structural analysis*. Unless restricted by the *applicable building code*, *lateral load* resistance and *stability* may be provided by any combination of members and connections.

B2. LOADS AND LOAD COMBINATIONS

The loads and load combinations shall be as stipulated by the *applicable building code*. In the absence of a building code, the loads and load combinations shall be those stipulated in SEI/ASCE 7. For design purposes, the *nominal loads* shall be taken as the *loads* stipulated by the applicable building code.

> **User Note:** For LRFD designs, the load combinations in SEI/ASCE 7, Section 2.3 apply. For ASD designs, the load combinations in SEI/ASCE 7, Section 2.4 apply.

B3. DESIGN BASIS

Designs shall be made according to the provisions for *Load and Resistance Factor Design* (LRFD) or to the provisions for *Allowable Strength Design* (ASD).

1. Required Strength

The *required strength* of structural members and *connections* shall be determined by *structural analysis* for the appropriate load combinations as stipulated in Section B2.

Design by *elastic*, *inelastic* or *plastic analysis* is permitted. Provisions for inelastic and plastic analysis are as stipulated in Appendix 1, Inelastic Analysis and Design. The provisions for moment redistribution in continuous beams in Appendix 1, Section 1.3 are permitted for elastic analysis only.

2. Limit States

Design shall be based on the principle that no applicable strength or serviceability *limit state* shall be exceeded when the structure is subjected to all appropriate load combinations.

3. Design for Strength Using Load and Resistance Factor Design (LRFD)

Design according to the provisions for *Load and Resistance Factor Design* (LRFD) satisfies the requirements of this Specification when the *design strength* of each *structural component* equals or exceeds the *required strength* determined on the basis of the *LRFD load combinations*. All provisions of this Specification, except for those in Section B3.4, shall apply.

Design shall be performed in accordance with Equation B3-1:

$$R_u \le \phi R_n \qquad \text{(B3-1)}$$

where

R_u = required strength (LRFD)
R_n = *nominal strength*, specified in Chapters B through K
ϕ = *resistance factor*, specified in Chapters B through K
ϕR_n = design strength

4. Design for Strength Using Allowable Strength Design (ASD)

Design according to the provisions for *Allowable Strength Design* (ASD) satisfies the requirements of this Specification when the *allowable strength* of each *structural component* equals or exceeds the *required strength* determined on the basis of the *ASD load combinations*. All provisions of this Specification, except those of Section B3.3, shall apply.

Design shall be performed in accordance with Equation B3-2:

$$R_a \le R_n/\Omega \qquad \text{(B3-2)}$$

where

R_a = required strength (ASD)
R_n = *nominal strength*, specified in Chapters B through K
Ω = safety factor, specified in Chapters B through K
R_n/Ω = allowable strength

Code Requirements for Structural Concrete, ACI 318-11

CHAPTER 9 — STRENGTH AND SERVICEABILITY REQUIREMENTS

CODE

9.1 — General

9.1.1 — Structures and structural members shall be designed to have design strengths at all sections at least equal to the required strengths calculated for the factored loads and forces in such combinations as are stipulated in this Code.

9.1.2 — Members also shall meet all other requirements of this Code to ensure adequate performance at service load levels.

9.1.3 — Design of structures and structural members using the load factor combinations and strength reduction factors of Appendix C shall be permitted. Use of load factor combinations from this chapter in conjunction with strength reduction factors of Appendix C shall not be permitted.

9.2 — Required strength

9.2.1 — Required strength $\boldsymbol{U}$ shall be at least equal to the effects of factored loads in Eq. (9-1) through (9-7). The effect of one or more loads not acting simultaneously shall be investigated.

$$U = 1.4D \quad \text{(9-1)}$$

$$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R) \quad \text{(9-2)}$$

$$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W) \quad \text{(9-3)}$$

$$U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R) \quad \text{(9-4)}$$

$$U = 1.2D + 1.0E + 1.0L + 0.2S \quad \text{(9-5)}$$

COMMENTARY

R9.1 — General

In the 2002 Code, the factored load combinations and strength reduction factors of the 1999 Code were revised and moved to Appendix C. The 1999 combinations were replaced with those of SEI/ASCE 7-02.[9.1] The strength reduction factors were replaced with those of the 1999 Appendix C, except that the factor for flexure was increased. In the 2011 Code, the factored load combinations were revised for consistency with ASCE/SEI 7-10.[9.2]

The changes were made to further unify the design profession on one set of load factors and combinations, and to facilitate the proportioning of concrete building structures that include members of materials other than concrete. When used with the strength reduction factors in 9.3, the designs for gravity loads will be comparable to those obtained using the strength reduction and load factors of the 1999 and earlier Codes. For combinations with lateral loads, some designs will be different, but the results of either set of load factors are considered acceptable.

Chapter 9 defines the basic strength and serviceability conditions for proportioning structural concrete members.

The basic requirement for strength design may be expressed as follows:

Design Strength ≥ Required Strength

ϕ (Nominal Strength) ≥ $\boldsymbol{U}$

In the strength design procedure, the margin of safety is provided by multiplying the service load by a load factor and the nominal strength by a strength reduction factor.

R9.2 — Required strength

The required strength U is expressed in terms of factored loads, or related internal moments and forces. Factored loads are the loads specified in the general building code multiplied by appropriate load factors.

The factor assigned to each load is influenced by the degree of accuracy to which the load effect usually can be calculated and the variation that might be expected in the load during the lifetime of the structure. Dead loads, because they are more accurately determined and less variable, are assigned a lower load factor than live loads. Load factors also account for variability in the structural analysis used to compute moments and shears.

Code Requirements for Structural Concrete, ACI 318-11 (continued)

CODE

$$U = 0.9D + 1.0W \quad (9\text{-}6)$$

$$U = 0.9D + 1.0E \quad (9\text{-}7)$$

except as follows:

(a) The load factor on the live load L in Eq. (9-3) to (9-5) shall be permitted to be reduced to 0.5 except for garages, areas occupied as places of public assembly, and all areas where L is greater than 100 lb/ft^2.

(b) Where W is based on service-level wind loads, $1.6W$ shall be used in place of $1.0W$ in Eq. (9-4) and (9-6), and $0.8W$ shall be used in place of $0.5W$ in Eq. (9-3).

(c) Where E is based on service-level forces, $1.4E$ shall be used in place of $1.0E$ in Eq. (9-5) and (9-7).

COMMENTARY

The Code gives load factors for specific combinations of loads. In assigning factors to combinations of loading, some consideration is given to the probability of simultaneous occurrence. While most of the usual combinations of loadings are included, it should not be assumed that all cases are covered.

Due regard is to be given to sign in determining U for combinations of loadings, as one type of loading may produce effects of opposite sense to that produced by another type. The load combinations with **0.9*D*** are specifically included for the case where a higher dead load reduces the effects of other loads. The loading case may also be critical for tension-controlled column sections. In such a case, a reduction in axial load and an increase in moment may result in a critical load combination.

Consideration should be given to various combinations of loading to determine the most critical design condition. This is particularly true when strength is dependent on more than one load effect, such as strength for combined flexure and axial load or shear strength in members with axial load.

If unusual circumstances require greater reliance on the strength of particular members than encountered in usual practice, some reduction in the stipulated strength reduction factors ϕ or increase in the stipulated load factors may be appropriate for such members.

In 2011, the Code removed the weight of soil and other fill materials as part of the definition of H. Consistent with ASCE/SEI 7-10, the weight of these materials is part of dead load, D. The load factors for D are appropriate provided the unit weight and thickness of earth or other fill materials are well controlled. If the weight of earth stabilizes the structure, a load factor of zero may be appropriate.

R9.2.1(a) — The load modification factor of 9.2.1(a) is different than the live load reductions based on the loaded area that may be allowed in the legally adopted general building code. The live load reduction, based on loaded area, adjusts the nominal live load (L_0 in ASCE/SEI 7) to L. The live load reduction as specified in the legally adopted general building code can be used in combination with the 0.5 load factor specified in 9.2.1(a).

R9.2.1(b) — ASCE/SEI 7-10 has converted wind loads to strength level, and reduced the wind load factor to 1.0. ACI 318 requires use of the previous load factor for wind loads, 1.6, when service-level wind loads are used. For serviceability checks, the commentary to Appendix C of ASCE/SEI 7-10 provides service-level wind loads, W_a.

R9.2.1(c) — In 1993, ASCE 7[9.3] converted earthquake forces to strength level, and reduced the earthquake load factor to 1.0. Model building codes[9.4-9.6] followed. ACI 318 requires use of the previous load factor for earthquake effects, approximately 1.4, when service-level earthquake effects are used.

SEI/ASCE 7-10: Minimum Design Loads for Buildings and Other Structures

Chapter 1 GENERAL

1.1 SCOPE

This standard provides minimum load requirements for the design of buildings and other structures that are subject to building code requirements. Loads and appropriate load combinations, which have been developed to be used together, are set forth for strength design and allowable stress design. For design strengths and allowable stress limits, design specifications for conventional structural materials used in buildings and modifications contained in this standard shall be followed.

1.2 DEFINITIONS AND NOTATIONS

1.2.1 Definitions

The following definitions apply to the provisions of the entire standard.

ALLOWABLE STRESS DESIGN: A method of proportioning structural members such that elastically computed stresses produced in the members by nominal loads do not exceed specified allowable stresses (also called "working stress design").

AUTHORITY HAVING JURISDICTION: The organization, political subdivision, office, or individual charged with the responsibility of administering and enforcing the provisions of this standard.

BUILDINGS: Structures, usually enclosed by walls and a roof, constructed to provide support or shelter for an intended occupancy.

DESIGN STRENGTH: The product of the nominal strength and a resistance factor.

ESSENTIAL FACILITIES: Buildings and other structures that are intended to remain operational in the event of extreme environmental loading from flood, wind, snow, or earthquakes.

FACTORED LOAD: The product of the nominal load and a load factor.

HIGHLY TOXIC SUBSTANCE: As defined in 29 CFR 1910.1200 Appendix A with Amendments as of February 1, 2000.

IMPORTANCE FACTOR: A factor that accounts for the degree of risk to human life, health, and welfare associated with damage to property or loss of use or functionality.

LIMIT STATE: A condition beyond which a structure or member becomes unfit for service and is judged either to be no longer useful for its intended function (serviceability limit state) or to be unsafe (strength limit state).

LOAD EFFECTS: Forces and deformations produced in structural members by the applied loads.

LOAD FACTOR: A factor that accounts for deviations of the actual load from the nominal load, for uncertainties in the analysis that transforms the load into a load effect, and for the probability that more than one extreme load will occur simultaneously.

LOADS: Forces or other actions that result from the weight of all building materials, occupants and their possessions, environmental effects, differential movement, and restrained dimensional changes. Permanent loads are those loads in which variations over time are rare or of small magnitude. All other loads are variable loads (see also "nominal loads").

NOMINAL LOADS: The magnitudes of the loads specified in this standard for dead, live, soil, wind, snow, rain, flood, and earthquake.

NOMINAL STRENGTH: The capacity of a structure or member to resist the effects of loads, as determined by computations using specified material strengths and dimensions and formulas derived from accepted principles of structural mechanics or by field tests or laboratory tests of scaled models, allowing for modeling effects and differences between laboratory and field conditions.

OCCUPANCY: The purpose for which a building or other structure, or part thereof, is used or intended to be used.

OTHER STRUCTURES: Structures, other than buildings, for which loads are specified in this standard.

P-DELTA EFFECT: The second order effect on shears and moments of frame members induced by axial loads on a laterally displaced building frame.

RESISTANCE FACTOR: A factor that accounts for deviations of the actual strength from the nominal strength and the manner and consequences of failure (also called "strength reduction factor").

RISK CATEGORY: A categorization of buildings and other structures for determination of flood, wind, snow, ice, and earthquake loads based on the risk associated with unacceptable performance. See Table 1.5-1.

STRENGTH DESIGN: A method of proportioning structural members such that the computed forces produced in the members by the factored loads do not

Table 1.5-1 Risk Category of Buildings and Other Structures for Flood, Wind, Snow, Earthquake, and Ice Loads

Use or Occupancy of Buildings and Structures	Risk Category
Buildings and other structures that represent a low risk to human life in the event of failure	I
All buildings and other structures except those listed in Risk Categories I, III, and IV	II
Buildings and other structures, the failure of which could pose a substantial risk to human life.	III
Buildings and other structures, not included in Risk Category IV, with potential to cause a substantial economic impact and/or mass disruption of day-to-day civilian life in the event of failure.	
Buildings and other structures not included in Risk Category IV (including, but not limited to, facilities that manufacture, process, handle, store, use, or dispose of such substances as hazardous fuels, hazardous chemicals, hazardous waste, or explosives) containing toxic or explosive substances where their quantity exceeds a threshold quantity established by the authority having jurisdiction and is sufficient to pose a threat to the public if released.	
Buildings and other structures designated as essential facilities.	IV
Buildings and other structures, the failure of which could pose a substantial hazard to the community.	
Buildings and other structures (including, but not limited to, facilities that manufacture, process, handle, store, use, or dispose of such substances as hazardous fuels, hazardous chemicals, or hazardous waste) containing sufficient quantities of highly toxic substances where the quantity exceeds a threshold quantity established by the authority having jurisdiction to be dangerous to the public if released and is sufficient to pose a threat to the public if released.[a]	
Buildings and other structures required to maintain the functionality of other Risk Category IV structures.	

[a]Buildings and other structures containing toxic, highly toxic, or explosive substances shall be eligible for classification to a lower Risk Category if it can be demonstrated to the satisfaction of the authority having jurisdiction by a hazard assessment as described in Section 1.5.2 that a release of the substances is commensurate with the risk associated with that Risk Category.

exceed the member design strength (also called "load and resistance factor design").

TEMPORARY FACILITIES: Buildings or other structures that are to be in service for a limited time and have a limited exposure period for environmental loadings.

TOXIC SUBSTANCE: As defined in 29 CFR 1910.1200 Appendix A with Amendments as of February 1, 2000.

1.1.2 Symbols and Notations

F_x A minimum design lateral force applied to level x of the structure and used for purposes of evaluating structural integrity in accordance with Section 1.4.2.

W_x The portion of the total dead load of the structure, D, located or assigned to Level x.

D Dead load.

L Live load.

L_r Roof live load.

N Notional load used to evaluate conformance with minimum structural integrity criteria.

R Rain load.

S Snow load.

1.3 BASIC REQUIREMENTS

1.3.1 Strength and Stiffness

Buildings and other structures, and all parts thereof, shall be designed and constructed with adequate strength and stiffness to provide structural stability, protect nonstructural components and systems from unacceptable damage, and meet the serviceability requirements of Section 1.3.2.

Acceptable strength shall be demonstrated using one or more of the following procedures:

a. the Strength Procedures of Section 1.3.1.1,
b. the Allowable Stress Procedures of Section 1.3.1.2, or
c. subject to the approval of the authority having jurisdiction for individual projects, the Performance-Based Procedures of Section 1.3.1.3.

It shall be permitted to use alternative procedures for different parts of a structure and for different load combinations, subject to the limitations of Chapter 2. Where resistance to extraordinary events is considered, the procedures of Section 2.5 shall be used.

1.3.1.1 Strength Procedures

Structural and nonstructural components and their connections shall have adequate strength to resist the applicable load combinations of Section 2.3 of this Standard without exceeding the applicable strength limit states for the materials of construction.

1.3.1.2 Allowable Stress Procedures

Structural and nonstructural components and their connections shall have adequate strength to resist the applicable load combinations of Section 2.4 of this Standard without exceeding the applicable allowable stresses for the materials of construction.

1.3.1.3 Performance-Based Procedures

Structural and nonstructural components and their connections shall be demonstrated by analysis or by a combination of analysis and testing to provide a reliability not less than that expected for similar components designed in accordance with the Strength Procedures of Section 1.3.1.1 when subject to the influence of dead, live, environmental, and other loads. Consideration shall be given to uncertainties in loading and resistance.

1.3.1.3.1 Analysis Analysis shall employ rational methods based on accepted principles of engineering mechanics and shall consider all significant sources of deformation and resistance. Assumptions of stiffness, strength, damping, and other properties of components and connections incorporated in the analysis shall be based on approved test data or referenced Standards.

1.3.1.3.2 Testing Testing used to substantiate the performance capability of structural and nonstructural components and their connections under load shall accurately represent the materials, configuration, construction, loading intensity, and boundary conditions anticipated in the structure. Where an approved industry standard or practice that governs the testing of similar components exists, the test program and determination of design values from the test program shall be in accordance with those industry standards and practices. Where such standards or practices do not exist, specimens shall be constructed to a scale similar to that of the intended application unless it can be demonstrated that scale effects are not significant to the indicated performance. Evaluation of test results shall be made on the basis of the values obtained from not less than 3 tests, provided that the deviation of any value obtained from any single test does not vary from the average value for all tests by more than 15%. If such deviaton from the average value for any test exceeds 15%, then additional tests shall be performed until the deviation of any test from the average value does not exceed 15% or a minimum of 6 tests have been performed. No test shall be eliminated unless a rationale for its exclusion is given. Test reports shall document the location, the time and date of the test, the characteristics of the tested specimen, the laboratory facilities, the test configuration, the applied loading and deformation under load, and the occurrence of any damage sustained by the specimen, together with the loading and deformation at which such damage occurred.

1.3.1.3.3 Documentation The procedures used to demonstrate compliance with this section and the results of analysis and testing shall be documented in one or more reports submitted to the authority having jurisdiction and to an independent peer review.

1.3.1.3.4 Peer Review The procedures and results of analysis, testing, and calculation used to demonstrate compliance with the requirements of this section shall be subject to an independent peer review approved by the authority having jurisdiction. The peer review shall comprise one or more persons having the necessary expertise and knowledge to evaluate compliance, including knowledge of the expected performance, the structural and component behavior, the particular loads considered, structural analysis of the type performed, the materials of construction, and laboratory testing of elements and components to determine structural resistance and performance characteristics. The review shall include the assumptions, criteria, procedures, calculations, analytical models, test setup, test data, final drawings, and reports. Upon satisfactory completion, the peer review shall submit a letter to the authority having jurisdiction indicating the scope of their review and their findings.

1.3.2 Serviceability

Structural systems, and members thereof, shall be designed to have adequate stiffness to limit deflections, lateral drift, vibration, or any other deformations that adversely affect the intended use and performance of buildings and other structures.

Chapter 2
COMBINATIONS OF LOADS

2.1 GENERAL

Buildings and other structures shall be designed using the provisions of either Section 2.3 or 2.4. Where elements of a structure are designed by a particular material standard or specification, they shall be designed exclusively by either Section 2.3 or 2.4.

2.2 SYMBOLS

A_k = load or load effect arising from extra ordinary event A
D = dead load
D_i = weight of ice
E = earthquake load
F = load due to fluids with well-defined pressures and maximum heights
F_a = flood load
H = load due to lateral earth pressure, ground water pressure, or pressure of bulk materials
L = live load
L_r = roof live load
R = rain load
S = snow load
T = self-straining load
W = wind load
W_i = wind-on-ice determined in accordance with Chapter 10

2.3 COMBINING FACTORED LOADS USING STRENGTH DESIGN

2.3.1 Applicability

The load combinations and load factors given in Section 2.3.2 shall be used only in those cases in which they are specifically authorized by the applicable material design standard.

2.3.2 Basic Combinations

Structures, components, and foundations shall be designed so that their design strength equals or exceeds the effects of the factored loads in the following combinations:

1. $1.4D$
2. $1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
3. $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.5W)$
4. $1.2D + 1.0W + L + 0.5(L_r \text{ or } S \text{ or } R)$
5. $1.2D + 1.0E + L + 0.2S$
6. $0.9D + 1.0W$
7. $0.9D + 1.0E$

EXCEPTIONS:

1. The load factor on L in combinations 3, 4, and 5 is permitted to equal 0.5 for all occupancies in which L_o in Table 4-1 is less than or equal to 100 psf, with the exception of garages or areas occupied as places of public assembly.
2. In combinations 2, 4, and 5, the companion load S shall be taken as either the flat roof snow load (p_f) or the sloped roof snow load (p_s).

Where fluid loads F are present, they shall be included with the same load factor as dead load D in combinations 1 through 5 and 7.

Where load H are present, they shall be included as follows:

1. where the effect of H adds to the primary variable load effect, include H with a load factor of 1.6;
2. where the effect of H resists the primary variable load effect, include H with a load factor of 0.9 where the load is permanent or a load factor of 0 for all other conditions.

Effects of one or more loads not acting shall be investigated. The most unfavorable effects from both wind and earthquake loads shall be investigated, where appropriate, but they need not be considered to act simultaneously. Refer to Section 12.4 for specific definition of the earthquake load effect E.[1]

Each relevant strength limit state shall be investigated.

2.3.3 Load Combinations Including Flood Load

When a structure is located in a flood zone (Section 5.3.1), the following load combinations shall be considered in addition to the basic combinations in Section 2.3.2:

1. In V-Zones or Coastal A-Zones, $1.0W$ in combinations 4 and 6 shall be replaced by $1.0W + 2.0F_a$.
2. In noncoastal A-Zones, $1.0W$ in combinations 4 and 6 shall be replaced by $0.5W + 1.0F_a$.

[1]The same E from Sections 1.4 and 12.4 is used for both Sections 2.3.2 and 2.4.1. Refer to the Chapter 11 Commentary for the Seismic Provisions.

2.3.4. Load Combinations Including Atmospheric Ice Loads

When a structure is subjected to atmospheric ice and wind-on-ice loads, the following load combinations shall be considered:

1. $0.5(L_r$ or S or $R)$ in combination 2 shall be replaced by $0.2D_i + 0.5S$.
2. $1.0W + 0.5(L_r$ or S or $R)$ in combination 4 shall be replaced by $D_i + W_i + 0.5S$.
3. $1.0W$ in combination 6 shall be replaced by $D_i + W_i$.

2.3.5 Load Combinations Including Self-Straining Loads

Where applicable, the structural effects of load T shall be considered in combination with other loads. The load factor on load T shall be established considering the uncertainty associated with the likely magnitude of the load, the probability that the maximum effect of T will occur simultaneously with other applied loadings, and the potential adverse consequences if the effect of T is greater than assumed. The load factor on T shall not have a value less than 1.0.

2.3.6 Load Combinations for Nonspecified Loads

Where approved by the Authority Having Jurisdiction, the Responsible Design Professional is permitted to determine the combined load effect for strength design using a method that is consistent with the method on which the load combination requirements in Section 2.3.2 are based. Such a method must be probability-based and must be accompanied by documentation regarding the analysis and collection of supporting data that is acceptable to the Authority Having Jurisdiction.

2.4 COMBINING NOMINAL LOADS USING ALLOWABLE STRESS DESIGN

2.4.1 Basic Combinations

Loads listed herein shall be considered to act in the following combinations; whichever produces the most unfavorable effect in the building, foundation, or structural member being considered. Effects of one or more loads not acting shall be considered.

1. D
2. $D + L$
3. $D + (L_r$ or S or $R)$
4. $D + 0.75L + 0.75(L_r$ or S or $R)$
5. $D + (0.6W$ or $0.7E)$

6a. $D + 0.75L + 0.75(0.6W) + 0.75(L_r$ or S or $R)$

6b. $D + 0.75L + 0.75(0.7E) + 0.75S$

7. $0.6D + 0.6W$
8. $0.6D + 0.7E$

EXCEPTIONS:

1. In combinations 4 and 6, the companion load S shall be taken as either the flat roof snow load (p_f) or the sloped roof snow load (p_s).
2. For nonbuilding structures, in which the wind load is determined from force coefficients, C_f, identified in Figures 29.5-1, 29.5-2 and 29.5-3 and the projected area contributing wind force to a foundation element exceeds 1,000 square feet on either a vertical or a horizontal plane, it shall be permitted to replace W with $0.9W$ in combination 7 for design of the foundation, excluding anchorage of the structure to the foundation.
3. It shall be permitted to replace $0.6D$ with $0.9D$ in combination 8 for the design of Special Reinforced Masonry Shear Walls, where the walls satisfy the requirement of Section 14.4.2.

Where fluid loads F are present, they shall be included in combinations 1 through 6 and 8 with the same factor as that used for dead load D.

Where load H is present, it shall be included as follows:

1. where the effect of H adds to the primary variable load effect, include H with a load factor of 1.0;
2. where the effect of H resists the primary variable load effect, include H with a load factor of 0.6 where the load is permanent or a load factor of 0 for all other conditions.

The most unfavorable effects from both wind and earthquake loads shall be considered, where appropriate, but they need not be assumed to act simultaneously. Refer to Section 1.4 and 12.4 for the specific definition of the earthquake load effect E.[2]

Increases in allowable stress shall not be used with the loads or load combinations given in this standard unless it can be demonstrated that such an increase is justified by structural behavior caused by rate or duration of load.

[2]The same E from Sections 1.4 and 12.4 is used for both Sections 2.3.2 and 2.4.1. Refer to the Chapter 11 Commentary for the Seismic Provisions.

2.4.2 Load Combinations Including Flood Load

When a structure is located in a flood zone, the following load combinations shall be considered in addition to the basic combinations in Section 2.4.1:

1. In V-Zones or Coastal A-Zones (Section 5.3.1), $1.5F_a$ shall be added to other loads in combinations 5, 6, and 7, and E shall be set equal to zero in 5 and 6.
2. In non-coastal A-Zones, $0.75F_a$ shall be added to combinations 5, 6, and 7, and E shall be set equal to zero in 5 and 6.

2.4.3 Load Combinations Including Atmospheric Ice Loads

When a structure is subjected to atmospheric ice and wind-on-ice loads, the following load combinations shall be considered:

1. $0.7D_i$ shall be added to combination 2.
2. (L_r or S or R) in combination 3 shall be replaced by $0.7D_i + 0.7W_i + S$.
3. $0.6W$ in combination 7 shall be replaced by $0.7D_i + 0.7W_i$.

2.4.4 Load Combinations Including Self-Straining Loads

Where applicable, the structural effects of load T shall be considered in combination with other loads. Where the maximum effect of load T is unlikely to occur simultaneously with the maximum effects of other variable loads, it shall be permitted to reduce the magnitude of T considered in combination with these other loads. The fraction of T considered in combination with other loads shall not be less than 0.75.

2.5 LOAD COMBINATIONS FOR EXTRAORDINARY EVENTS

2.5.1 Applicability

Where required by the owner or applicable code, strength and stability shall be checked to ensure that structures are capable of withstanding the effects of extraordinary (i.e., low-probability) events, such as fires, explosions, and vehicular impact without disproportionate collapse.

2.5.2 Load Combinations

2.5.2.1 Capacity

For checking the capacity of a structure or structural element to withstand the effect of an extraordinary event, the following gravity load combination shall be considered:

$$(0.9 \text{ or } 1.2)D + A_k + 0.5L + 0.2S \qquad (2.5\text{-}1)$$

in which A_k = the load or load effect resulting from extraordinary event A.

2.5.2.2 Residual Capacity

For checking the residual load-carrying capacity of a structure or structural element following the occurrence of a damaging event, selected load-bearing elements identified by the Responsible Design Professional shall be notionally removed, and the capacity of the damaged structure shall be evaluated using the following gravity load combination:

$$(0.9 \text{ or } 1.2)D + 0.5L + 0.2(L_r \text{ or } S \text{ or } R) \qquad (2.5\text{-}2)$$

2.5.3 Stability Requirements

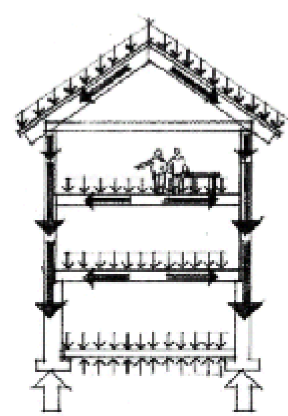
Stability shall be provided for the structure as a whole and for each of its elements. Any method that considers the influence of second-order effects is permitted.

System Assemblies & Load Tracing

Notation:

a = name for a dimension
C = symbol for compression
DL = shorthand for dead load
$F_{horizontal\text{-}resisting}$ = total force resisting horizontal sliding
$F_{sliding}$ = total sliding force
F_y = force component in the y direction
FBD = free body diagram
h = name for height
L = name for length
LL = shorthand for live load
M = moment due to a force
$M_{overturning}$ = total overturning moment
$M_{resisting}$ = total moment resisting overturning about a point
N = name for normal force to a surface
$o.c.$ = shorthand for on center
p = pressure
P = force due to a pressure
SF = shorthand for factor of safety
R = name for reaction force
T = symbol for tension
= name of a tension force
v = distributed shear
V = name for shear (horizontal) force
w = name for distributed load/length, as is ω
= name for distributed load/area
$w_{self\ wt}$ = name for distributed load from self weight of member
$w_{selfwt\ equiv}$ = name for equivalent distributed vertical load from self weight of slanted member
W = name for total force due to distributed load
= force due to a weight
x = horizontal distance
μ = coefficient of static friction
γ = density or unit weight
ω' = equivalent fluid density of a soil
Σ = summation symbol

Load Tracing



- LOAD TRACING is the term used to describe how the loads on and in the structure are transferred through the members (*load paths)* to the foundation, and ultimately supported by the ground.

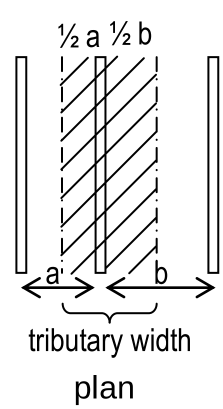
- It is a sequence of **actions**, NOT reactions. Reactions in statically determinate members (using FBD's) can be solved for to determine the actions on the next member in the hierarchy.

- The *tributary area* is a loaded area that contributes to the load on the member supporting that area, *ex.* the area from the center between two beams to the center of the next two beams for the full span is the load on the center beam

- The *tributary load* on the member is found by **concentrating (or consolidating)** the load into the center.

$$w = \left(\frac{load}{area}\right) x (tributary\ width)$$

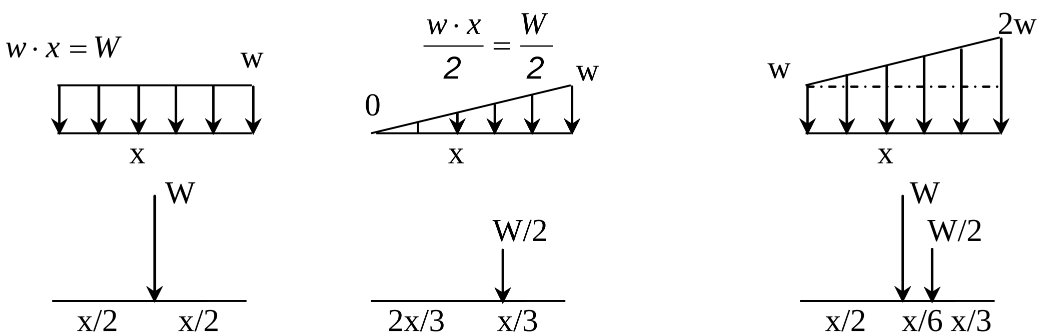
where w = distributed load in units of load/length.



Distributed Loads

Distributed loads may be replaced by concentrated loads acting through the balance/center of the distribution or *load area*: THIS IS AN **EQUIVALENT** FORCE SYSTEM.

- *w* is the symbol used to describe the *load* per unit **length**.
  Note: It can also represent a load per unit <u>area</u>.
- W is the symbol used to describe the *total load.*



Framing Systems

Horizontal levels must transfer loads to vertical elements. There are many ways to configure the systems. The horizontal levels can be classified by how many elements transfer loads in the plane. Decking is not usually considered a level in a multiple level system because it isn't significantly load-bearing. It is considered a level when it is the only horizontal element and must resist loads.

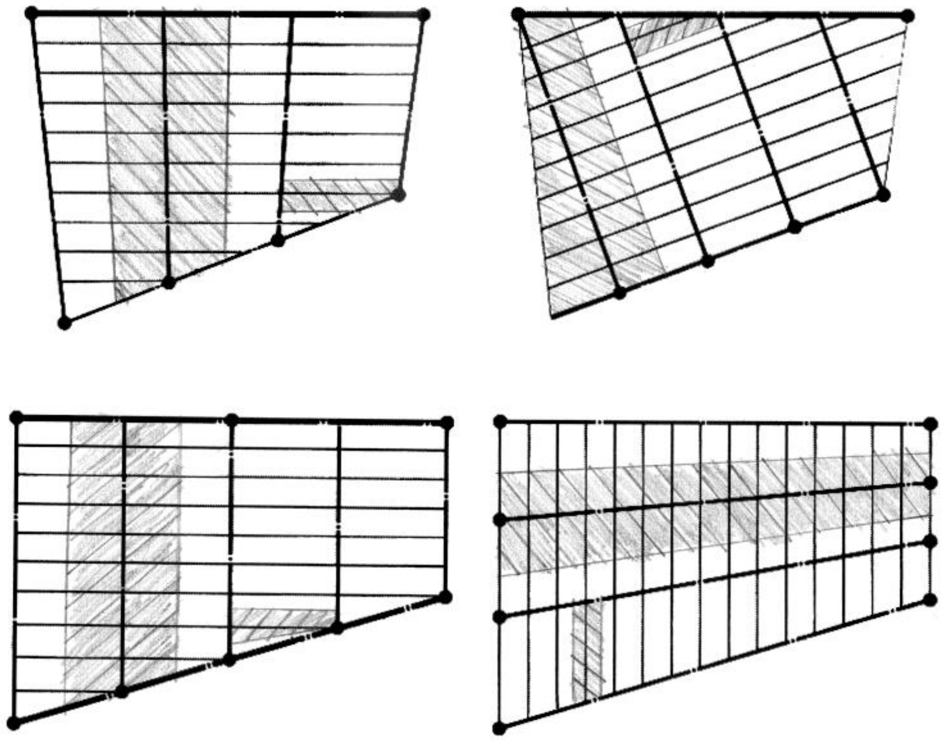
Foundations

The final path of the load for the structure is to the foundation. The foundation must transfer the loads to the soil, which is a "natural" structural material. The soil conditions will determine if a shallow foundation (most economical an easy to construct) can be used, or a deep foundation (for larger loads or poorer soil capacities) must be considered.

Distribution of Loads with Irregular Configurations

When a bay (defined by the area bounded by vertical supports) is not rectangular, it is commonly constructed with parallel or non-parallel spanning members of non-uniform lengths. With parallel spanning members, the tributary width is uniform. With non-parallel members, the tributary width at each end is different, but still defined as half the distance (each side) to the next member. The resulting distribution will be linear (and not uniform).

The most efficient one-way systems have regular, rectangular bays. Two way systems are most efficient when they are square. With irregular bays, attempts are made to get as many parallel members as possible with similar lengths, resulting in an economy of scale.



Distribution of Loads on Edge Supported Slabs

Distributed loads on two-way slabs (i.e. not one-way like beams) do not have obvious tributary "widths". The distribution is modeled using a 45 degree tributary "boundary" in addition to the tributary boundary that is half way between supporting elements, in this case, edge beams.

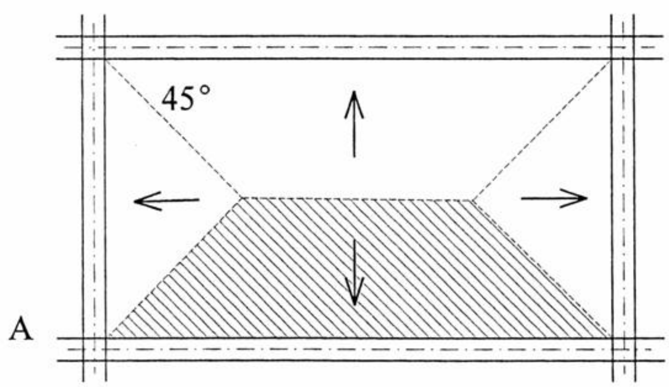


Figure 2-16: Supporting beams' contributing areas for reinforced concrete floor system.

The tributary distribution *from the area loads* result in a trapezoidal distribution. Self weight will be a uniform distributed load, and will also have to be included for design of beam AB.

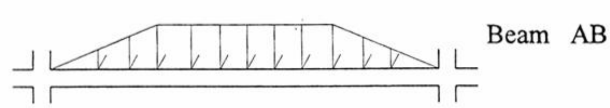


Figure 2-17: Trapezoidal distributed load for Beam AB of Fig. 2-16.

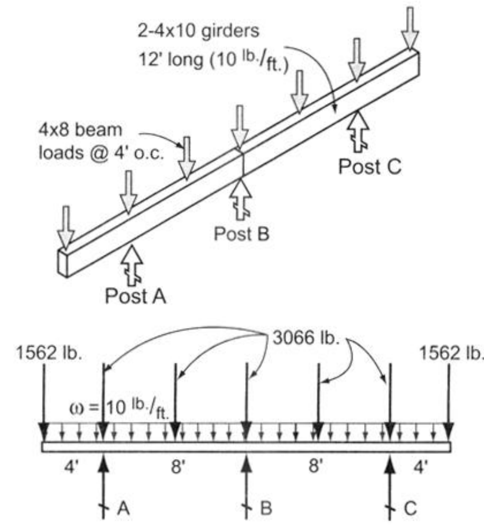
Openings in Floor/Roof Plans

Openings in a horizontal system usually are framed on all sides. This provides for stiffness and limiting the deflection. The edge beams may not be supporting the flooring, however, so care needs to be taken to determine if an opening edge beam must support tributary area, or just itself.

- *Any edge beam supporting a load has load on only one side to the next supporting element.*

Beams Supported by Other Beams

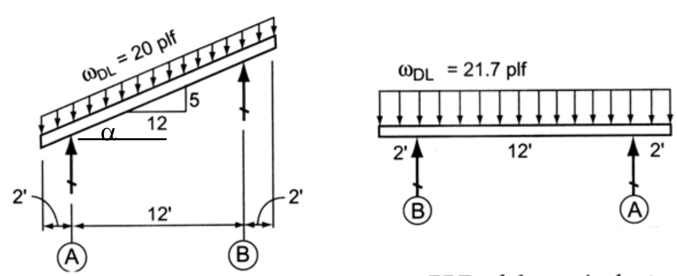
Joists are commenly supported by beams with beam hangers. The reaction at the support is transferred to the beam as a single force. A beam, in turn, can be supported by a larger beam or girder, and the reaction from this beam having a uniform distributed self weight, and the forces, will be an action on the girder.



Horizontal Projection of Gravity Load on a Rafter

When an angled member, such as a rafter has a self weight per unit length, that weight is usually converted to a weight per horizontal length:

$$w_{selfwt.equiv.} = w_{selfwt.}\left(\frac{length}{horizontal\ distance}\right) or\ {w_{self\ wt.}}/{\cos\alpha}$$



FBD of the inclined rafter. *FBD of the equivalent horizontally projected rafter.*

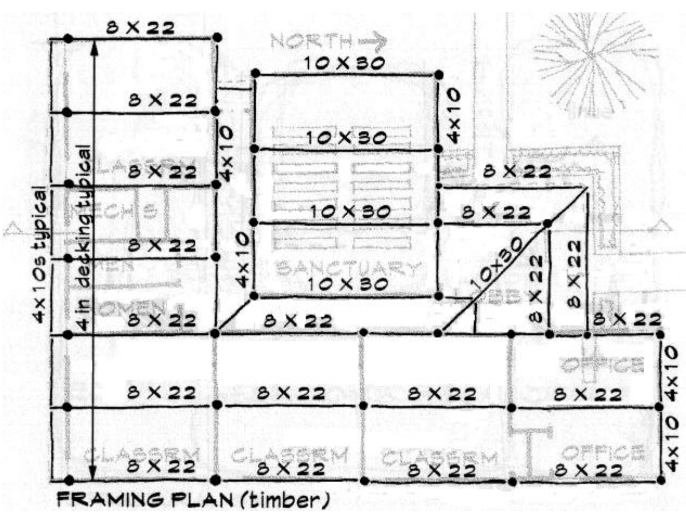
Framing Plans

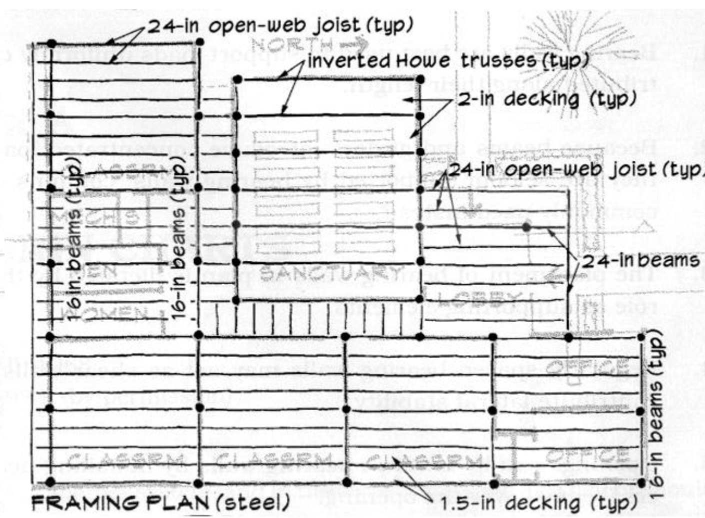
Framing plans are diagrams representing the placement and organization of structural members. Until the final architecture has been determined, framing plans are often drawn freehand with respect to the floor plans, and quite often use the formal conventions for structural construction drawings.

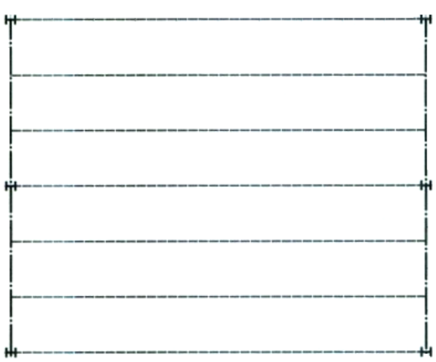
Parts of the building are identified by letter symbols:

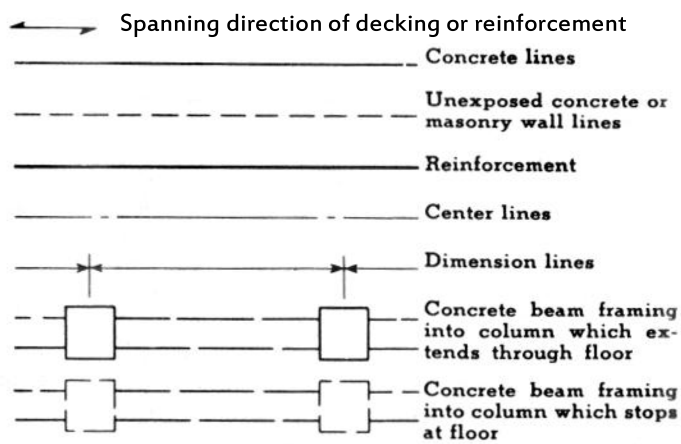
B – Beams	*F* – Footings	*L* – Lintels	*U* – Stirrups
C – Columns	*G* – Girders	*S* – Slabs	*W* – Walls
D – Dowels	*J* – Joists	*T* – Ties	

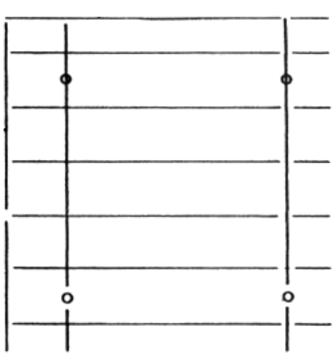
Other parts are represented with lines (beams and joists), dots, squares, rectangles or wide-flange shapes for columns. Column and footing locations in structural drawings are referred to by letters and numbers, with vertical lines at column centers given letters – *A, B, C,* etc., and horizontal lines at columns given numbers – *1, 2, 3,* etc. The designation *do* may be used to show like members (like *ditto*).







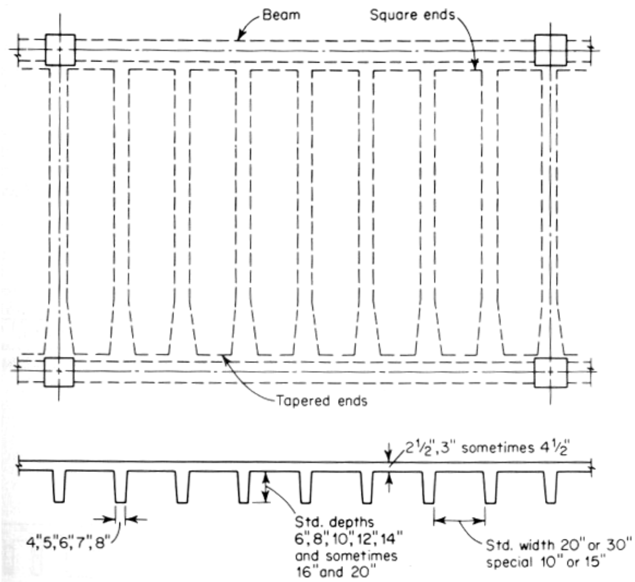




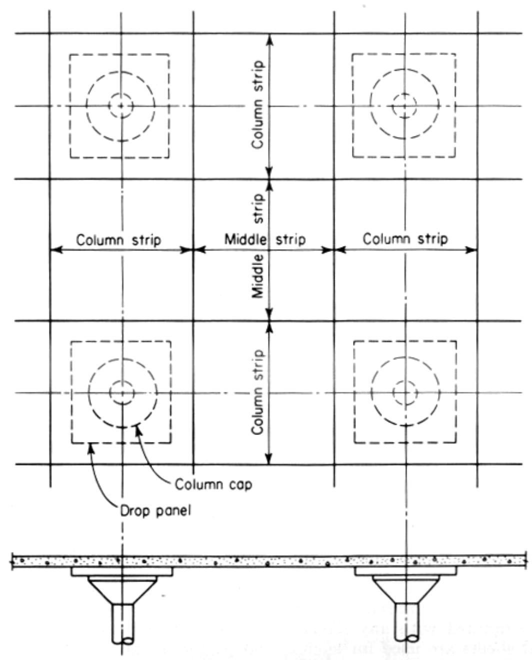
Breaks in the lines are commonly used to indicate the *end* of a beam that is supported by another member, such as a girder or column. Beams can span over a support (as a continuous beam) and therefore, there is no break shown at the column.

Joists can span over a supporting beam, and the lines will cross. (Looking for the ends of the crossing members give information about which is below and which is above.)

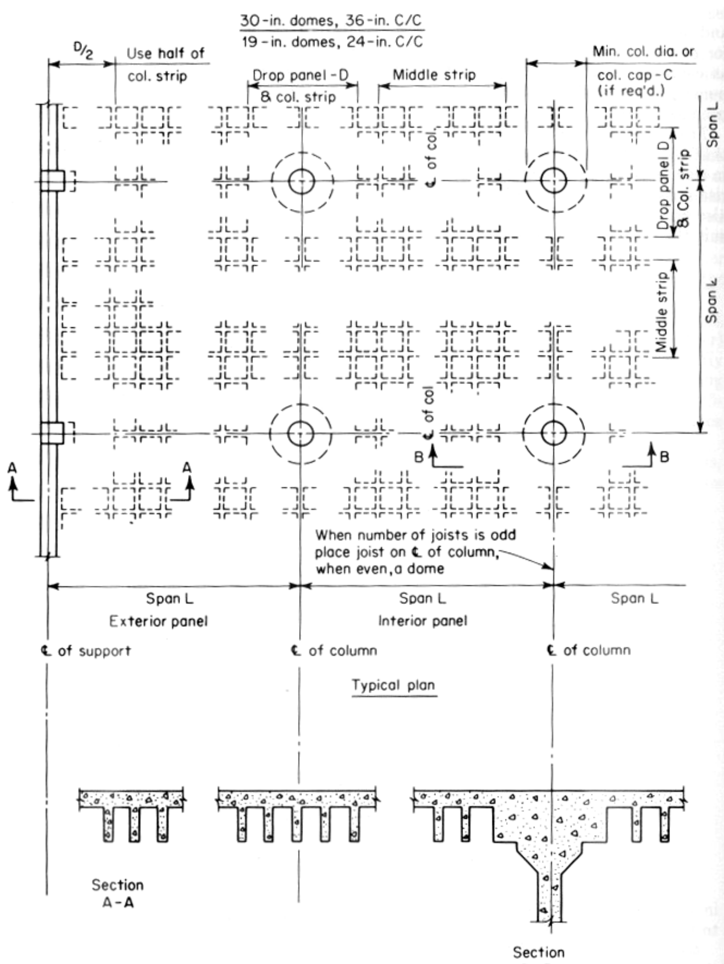
Concrete systems often have slabs, ribs or drop panels or strips, which aren't easily represented by centerlines, so hidden lines represent the edges. Commonly isolated "patches" of repeated geometry are used for brevity.



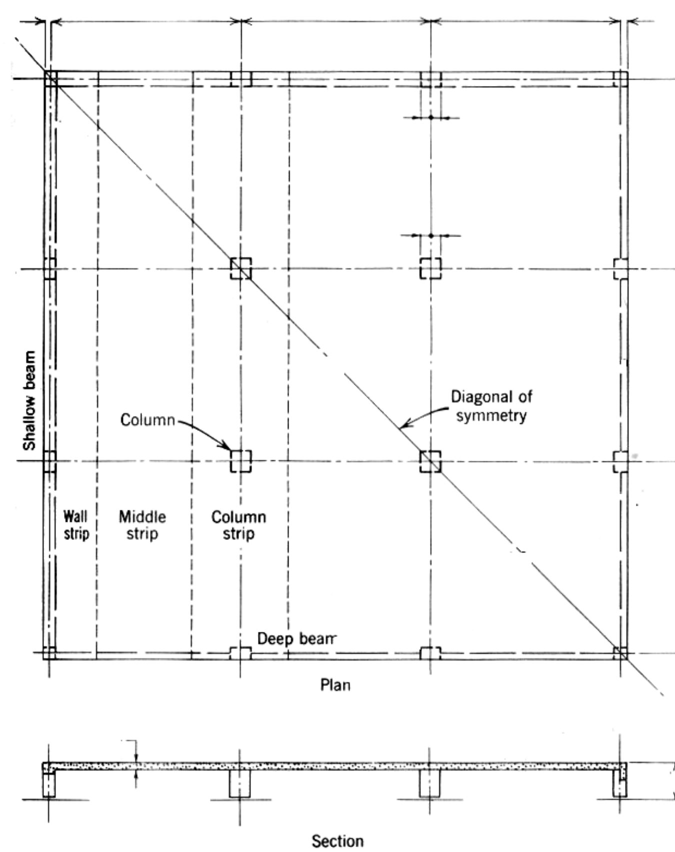
One-way concrete ribbed slab.



Two-way flat slab.



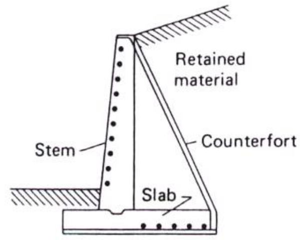
Two-way joist construction.



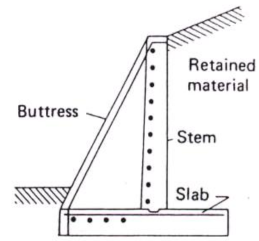
Layout of flat plate test structure.

Retaining Walls

Retaining walls are used to hold back soil or other material with the wall. The other key components include bases, counterforts, buttresses or keys. Gravity loads help provide resistance to movement, while the walls with lateral loads behave like cantilever beams.



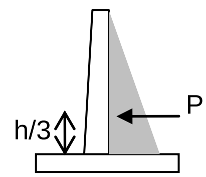
(c) Counterfort wall



(d) Buttress wall

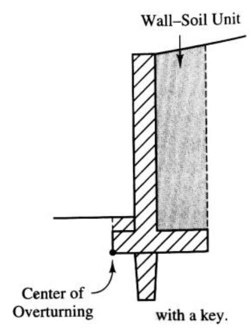
Loads

The design of retaining walls must consider overturning, settlement, sliding and bearing pressure. The water in the retained soil can significantly affect the loading and the active pressure of the soil. The lateral force, P, acting at a height of h/3 is determined from the equivalent fluid weight (density), ω', (in force/cubic area) as:



$$P = \frac{\omega' h^2}{2} \; or \; \frac{ph}{2}$$

where p is the maximum pressure at the base: $p = \omega' \cdot h$



Overturning is considered the same as for eccentric footings:

$$SF = \frac{M_{resist}}{M_{overturning}} \geq 1.5 - 2$$

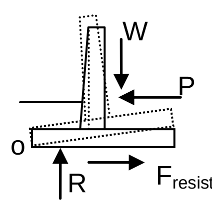
where

M_{resist} = summation of moments about "o" to resist rotation, typically including the moment due to the weight of the stem and base and the moment due to the passive pressure.

$M_{overturning}$ = moment due to the active pressure about the toe "o".

Sliding must also be avoided:

$$SF = \frac{F_{horizontal\text{-}resist}}{F_{sliding}} \geq 1.25 - 2$$



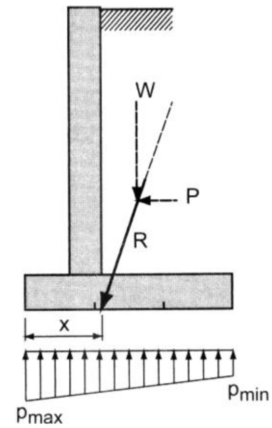
where

$F_{horizontal\text{-}resist}$ = summation of forces to resist sliding, typically including the force from the passive pressure and friction (F=μ·N where . μ is a constant for the materials in contact and N is the normal force to the ground acting down and is shown as R).

$F_{sliding}$ = sliding force as a result of active pressure.

Pressure Distribution

Because the resultant force from the gravity loads and pressure is not vertical, the vertical pressure distribution under the footing will not be uniform, but will be linearly distributed. The vertical component of the resultant must be in the same horizontal location as the pressure reaction force.



- *There can never be a tensile pressure because the footing will not be in contact with* the *soil.*
- To make certain all the area under the footing is used to distributed the load, the vertical resultant needs to be within the middle third of the base width. This area is called the *kern.*
- Soil pressure is most commonly called ***q*** in the design texts and codes.

To determine the size of the maximim pressure we find the equivalent location of the pressure reaction, P, at *x* using moment calculations when $R_x = W$:

$$W = P = 1/2p(3x)$$

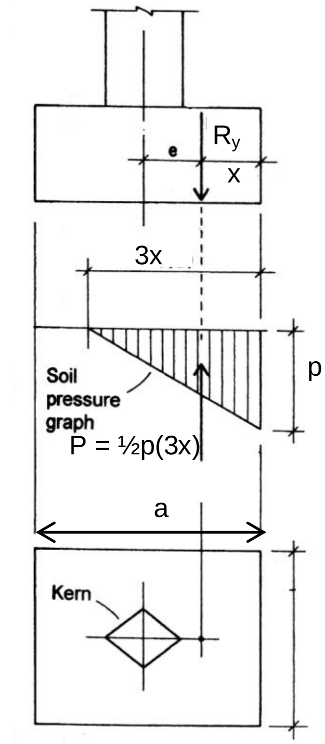
so $p = 2W/3x$ when $x < a/3$

$p = 2W/a$ when $x = a/3$

$$x = \frac{M_{resisting} - M_{overturning}}{W_{total}}$$

and $p = \frac{W}{a^2}(4a - 6x)$ when $a/3 < x < 2a/3$

where *x* is the location of the resultant force and *a* is the width of the base.



Wind Load Tracing

For design purposes, wind loads are treated as static pressure distributions over the walls and roof. In the case of walls, the loads are traced just like those for horizontal surfaces. If there is a roof diaphragm, it is the "top" supporting element and the tributary boundary is half way "up" to the diaphragm. If the supporting elements are the side walls the tributary boundary is vertical and half way between sides. In either case, the traced action force at the top of the walls is a lateral *shear* force (V) that must be resisted. The shear over the width of a shear wall, *v*, is a *unit shear* used for determining the connection and framing capacity required.

Lateral Resisting Systems

- Shear Walls
- Braced Frames
- Rigid Frames
- Diaphragms
- Cores
- Tubes

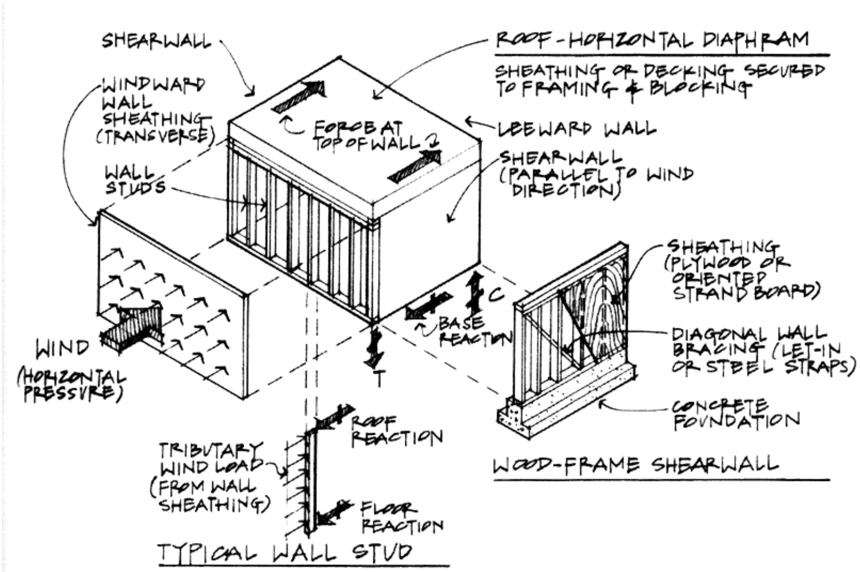


Figure 4.48 Exploded view of a light-framed wood building showing the various lateral resisting components.

Bracing Configurations

Without proper arrangement of the lateral resisiting components, the system cannot transfer lateral loads that may come *from any direction.*

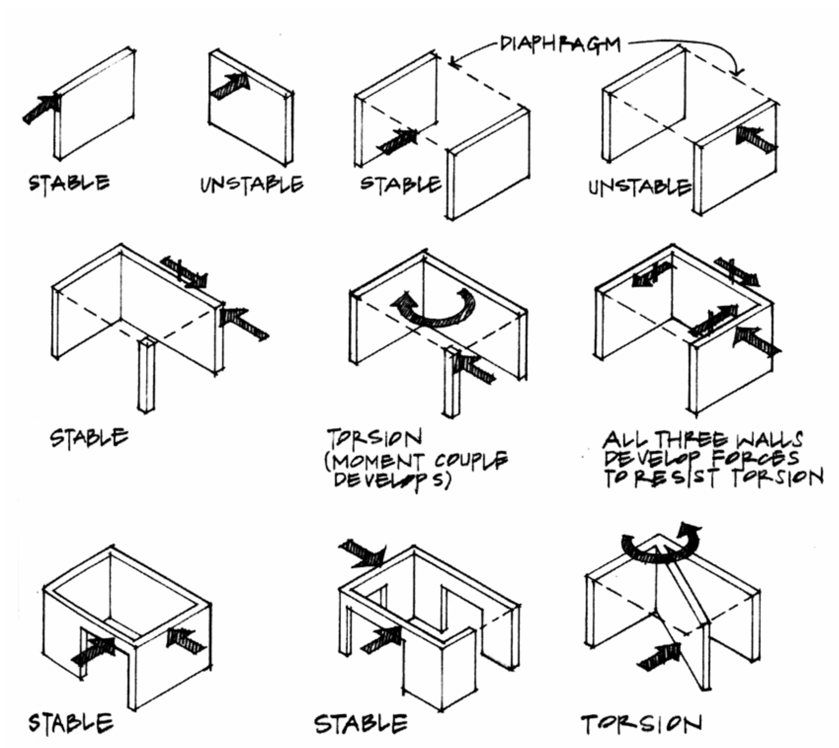
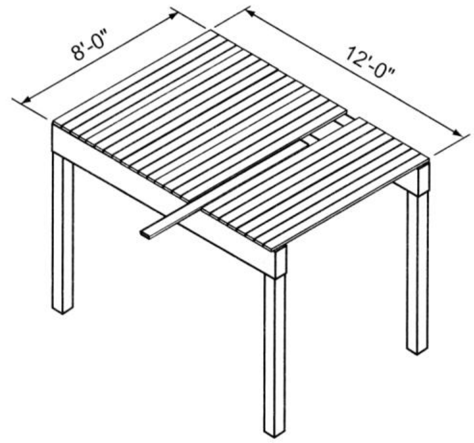


Figure 4.54 Various shearwall arrangements—some stable, others unstable.

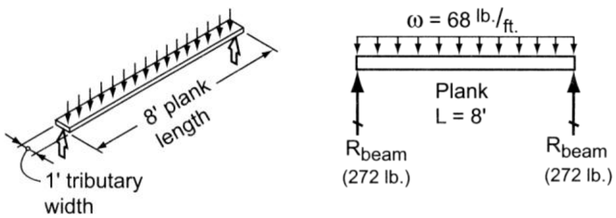
Example 1 (pg 168)

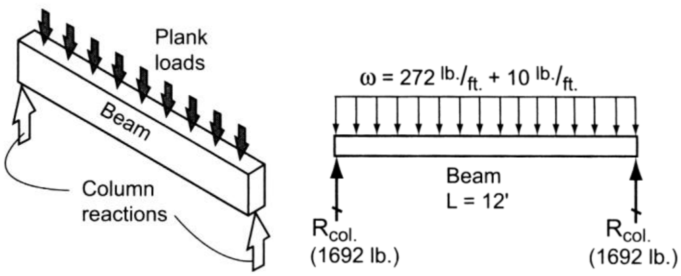
Example Problem 5.2

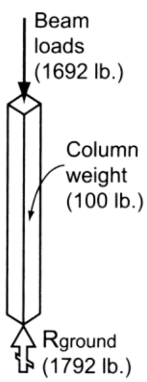


In the single-bay, post-and-beam deck illustrated, planks typically are available in nominal widths of 4" or 6", but for the purposes of analysis it is permissible to assume a unit width equal to one foot. Determine the plank, beam, and column reactions.

The loads are: 60 lb/ft^2 live load, 8 lb/ft^2 dead load, 10 lb/ft self weight of 12' beams, and 100 lb self weight of columns.







Example 2

EXAMPLE

Assume that the average dead plus live load on the structure shown in Figure 3.15 is 60 lbs/ft^2. **Determine** the reactions for Beam D. This is the same structure as shown in Figure 3.1.
^ E, B and A *Assuming all beams are weightless!*

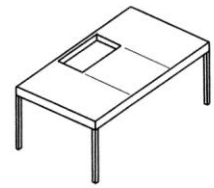
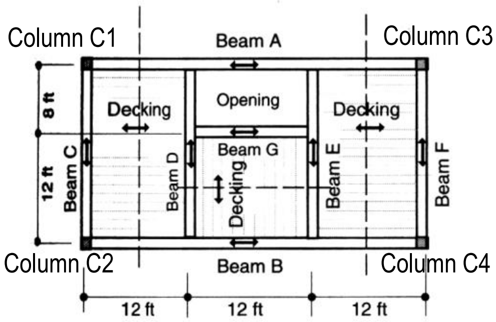


Figure 3.1

Solution:

Note carefully the directions of the decking span. Beam D carries floor loads from the decking to the left (see the contributory area and load strip), but not to the right, since the



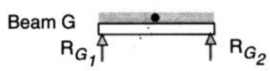
Live and dead load

Assume $w_{DL+LL} = 60$ lbs/ft^2

Beam G carries distributed loads only

Find reactions for Beam G

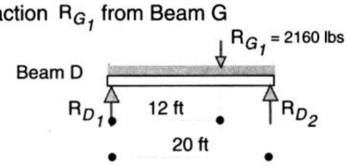
$w = 6 \text{ ft}(60 \text{ lbs/ft}^2) = 360 \text{ lb/ft}$



$R_{G_1} = wL/2 = (360 \text{ lb/ft})(12 \text{ ft})/2 = 2160 \text{ lbs}$

$R_{G_2} = wL/2 = (360 \text{ lb/ft})(12 \text{ ft})/2 = 2160 \text{ lbs}$

Beam D (and E) carries both distributed loads and the reaction R_{G_1} from Beam G



$\Sigma M_{D_1} = 0$

$-(12 \text{ ft})(2160 \text{ lb}) - (360 \text{ lb/ft})(20 \text{ ft})(20 \text{ ft}/2) + 20 R_{D_2} = 0$

$R_{D_2} = 4896 \text{ lb} = R_{E2}$

$\Sigma F_y = 0$

$R_{D_1} + R_{D_2} = (360 \text{ lb/ft})(20 \text{ ft}) + 2160 \text{ lb}$

$R_{D_1} = 4464 \text{ lb} = R_{E1}$

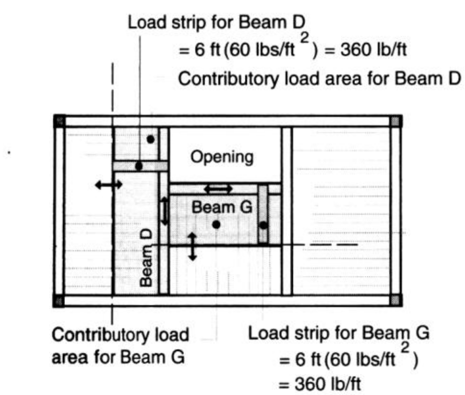
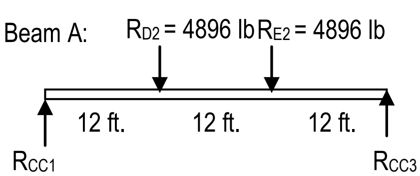
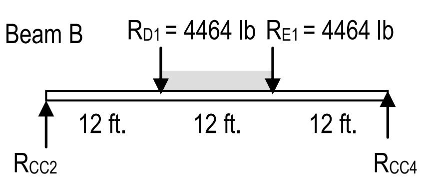


FIGURE 3.15 Load modeling and reaction determination.



By symmetry; $R_{CC1} = R_{CC3} = (4896 \text{ lb} + 4896 \text{ lb})/2 = 4896 \text{ lb}$



By symmetry; $R_{CC2} = R_{CC4} = (4464 \text{ lb} + 4464 \text{ lb})/2 + (6 \text{ ft})(60 \text{ lb/ft}^2)(12 \text{ ft})/2 = 6624 \text{ lb}$

Additional loads are transferred to the column from the reactions on Beams C and F:

$R_{C1} = R_{C2} = R_{F1} = R_{F2} = wL/2 = (6 \text{ ft})(60 \text{ lb/ft}^2)(20 \text{ ft})/2 = 3600 \text{ lb}$

center decking runs parallel to Beam D and is not carried by it. Beam D also picks up the end of Beam G and thus also "carries" the reactive force from Beam G. It is therefore necessary to analyze Beam G first to determine the magnitude of this force. The analysis appears in Figure 3.15. The reactive force from Beam G of 2160 lbs is then treated as a downward force acting on Beam D. The load model for Beam D thus consists of distributed forces from the decking plus the 2160-lb force. It is then analyzed by means of the equations of statics to obtain reactive forces of 4896 lbs and 4464 lbs at its ends.

C1 = 4896 lb + 3600 lb = 8,496 lb

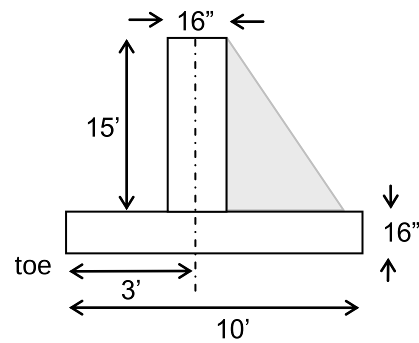
C2 = 6624 lb + 3600 lb = 10,224 lb

C3 = 4896 lb + 3600 lb = 8,496 lb

C4 = 6624 lb + 3600 lb = 10,224 lb

Example 3

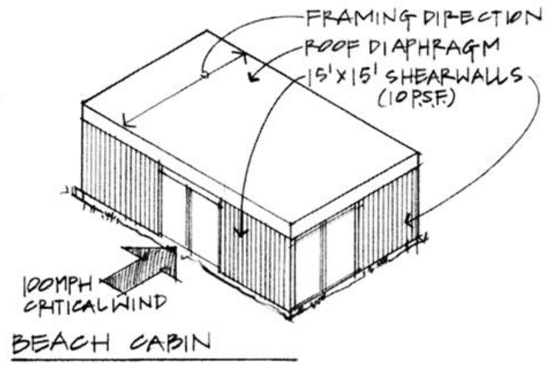
Determine the factor of safety for overturning and sliding on the 15 ft. retaining wall, 16 in. wide stem, 10 ft. wide x 16 in. high base, when the equivalent fluid pressure is 30 lb/ft^3, the weight of the stem of the footing is 4 kips, the weight of the pad is 5 kips, the passive pressure is ignored for this design, and the friction coefficient for sliding is 0.58. The center of the stem is located 3 ft. from the toe.
Also find the maximum bearing pressure.

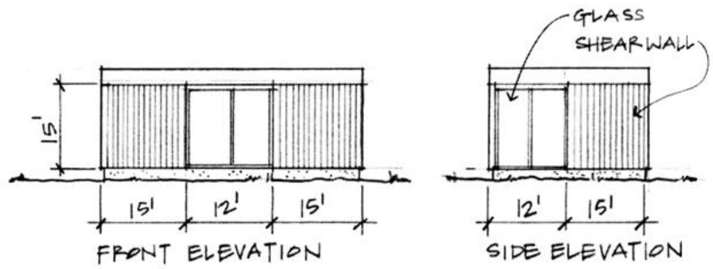


Example 4

4.10 A beach cabin on the Washington coast (100 mph wind velocity) is required to resist a wind pressure of 35 psf. Assuming wood-frame construction, the cabin utilizes a roof diaphragm and four exterior shearwalls for its lateral resisting strategy.

Draw an exploded view of the building and perform a lateral load trace in the N-S direction. Show the magnitude of shear (V) and intensity of shear (v) for the roof and critical shearwall. Also, determine the theoretical tie-down force necessary to establish equilibrium of the shearwall. Note that the dead weight of the wall can be used to aid in the stabilizing of the wall.



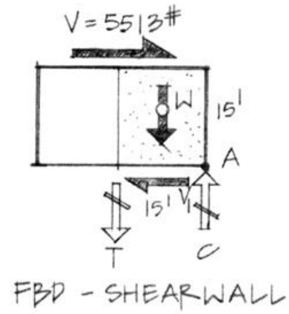


Solution:

$\omega = 35 \text{ psf} \times 7.5' = 262.5 \text{ \#/ft.}$

Examining the roof diaphragm as a deep beam spanning 42′ between shearwalls:

$$V = \frac{\omega L}{2} = \frac{262.5 \text{ \#/ft. } (42')}{2} = 5{,}513\#$$



An FBD of the shearwall shows a shear V' developing at the base (foundation) to equilibrate the shear V at the top of the wall. In addition to equilibrium in the horizontal direction, rotational equilibrium must be maintained by the development of a force couple T and C at the edges of the solid portion of wall.

$v = V/\text{shearwall length} = 5{,}513 \text{ \#}/15' = 368 \text{ \#/ft.}$

W = dead load of the wall

$W = 10 \text{ psf} \times 15' \times 15' = 2{,}250\#$

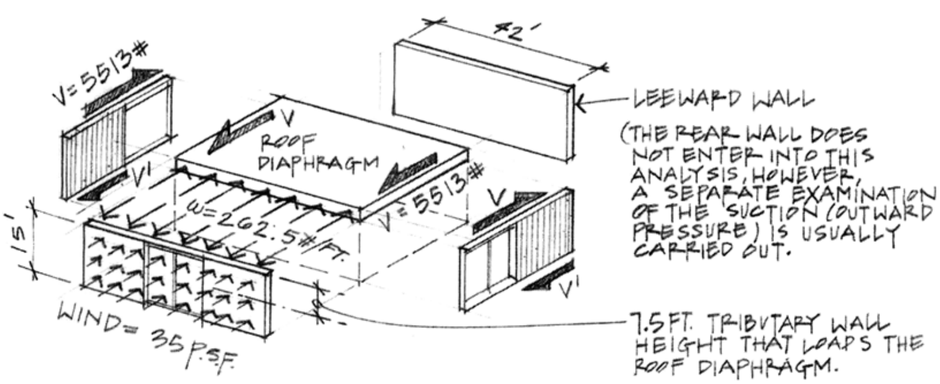
Tie-down force T is determined by writing a moment equation of equilibrium. Summing moments about point A:

$[\Sigma M_A = 0] - V\,(15') + W\,(15'/2) + T\,(15') = 0$

$15\,T = 5{,}513\#(15') - 2{,}250\#(7.5')$

$$T = \frac{(82{,}695 \text{ \#-ft.}) - (16{,}875 \text{ \#-ft.})}{15}$$

$T = 4{,}390\#$



Wood Design

Notation:

a = name for width dimension
A = name for area
$A_{req'd\text{-}adj}$ = area required at allowable stress when shear is adjusted to include self weight
b = width of a rectangle
= name for height dimension
c = largest distance from the neutral axis to the top or bottom edge of a beam
c_1 = coefficient for shear stress for a rectangular bar in torsion
C_C = curvature factor for laminated arches
C_D = load duration factor
C_{fu} = flat use factor for other than decks
C_F = size factor
C_H = shear stress factor
C_i = incising factor
C_L = beam stability factor
C_M = wet service factor
C_p = column stability factor for wood design
C_r = repetitive member factor for wood design
C_V = volume factor for glue laminated timber design
C_t = temperature factor for wood design
d = name for depth
= calculus symbol for differentiation
d_{min} = dimension of timber critical for buckling
D = shorthand for dead load
= name for diameter
DL = shorthand for dead load
E = modulus of elasticity
f = stress (strength is a stress limit)
f_b = bending stress
$f_{from\ table}$ = tabular strength (from table)
f_p = bearing stress
f_v = shear stress
$f_{v\text{-}max}$ = maximum shear stress
F_{allow} = allowable stress
F_b = tabular bending strength
= allowable bending stress
F_b' = allowable bending stress (adjusted)
F_c = tabular compression strength parallel to the grain
F_c' = allowable compressive stress (adjusted)
F^*_c intermediate compressive stress for dependant on load duration
F_{cE} = theoretical allowed buckling stress
$F_{c\perp}$ = tabular compression strength perpendicular to the grain
$F_{connector}$ = shear force capacity per connector
F_p = tabular bearing strength parallel to the grain
= allowable bearing stress
F_t = tabular tensile strength
F_u = ultimate strength
F_v = tabular bending strength
= allowable shear stress
F_y = yield strength
h = height of a rectangle
H = name for a horizontal force
I = moment of inertia with respect to neutral axis bending
I_{trial} = moment of inertia of trial section
$I_{req'd}$ = moment of inertia required at limiting deflection
I_y = moment of inertia with respect to an y-axis
J = polar moment of inertia
K = effective length factor for columns
L_e = effective length that can buckle for column design, as is ℓ_e
L = name for length or span length
LL = shorthand for live load
$LRFD$ = load and resistance factor design
M = internal bending moment
M_{max} = maximum internal bending moment
$M_{max\text{-}adj}$ = maximum bending moment adjusted to include self weight
n = number of connectors across a joint, as is N

p = pitch of connector spacing
= safe connector load parallel to the grain
P = name for axial force vector
$P_{allowable}$ = allowable axial force
q = safe connector load perpendicular to the grain
$Q_{connected}$ = first moment area about a neutral axis for the connected part
r = radius of gyration
= interior radius of a laminated arch
R = radius of curvature of a deformed beam
= radius of curvature of a laminated arch
= name for a reaction force
S = section modulus
$S_{req'd}$ = section modulus required at allowable stress
$S_{req'd\text{-}adj}$ = section modulus required at allowable stress when moment is adjusted to include self weight
T = torque (axial moment)
V = internal shear force
V_{max} = maximum internal shear force
$V_{max\text{-}adj}$ = maximum internal shear force adjusted to include self weight
w = name for distributed load
$w_{self\ wt}$ = name for distributed load from self weight of member
W = shorthand for wind load
x = horizontal distance
y = vertical distance
Z = force capacity of a connector
Δ_{actual} = actual beam deflection
$\Delta_{allowable}$ = allowable beam deflection
Δ_{limit} = allowable beam deflection limit
Δ_{max} = maximum beam deflection
γ = density or unit weight
θ = slope of the beam deflection curve
ρ = radial distance
$\int$ = symbol for integration
Σ = summation symbol

Wood or Timber Design

Structural design standards for wood are established by the *National Design Specification (NDS)* published by the National Forest Products Association. There is a combined specification (from 2005) for **Allowable** Stress Design and limit state design (LRFD).

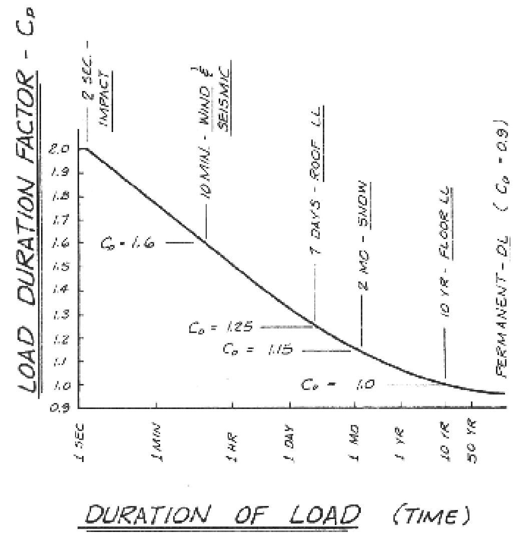
Tabulated wood strength values are used as the base allowable strength and modified by appropriate **adjustment** factors:

$$F' = C_D C_M C_F \ldots \times F_{from\,table}$$

Size and Use Categories

Boards:	1 to 1½ in. thick	2 in. and wider
Dimension lumber	2 to 4 in. thick	2 in. and wider
Timbers	5 in. and thicker	5 in. and wider

Adjustment Factors
(partial list)



- C_D load duration factor
- C_M wet service factor (1.0 dry < 16% moisture content)
- C_F size factor for visually graded sawn lumber and round timber > 12" depth

$$C_F = (12/d)^{1/9} \le 1.0$$

- C_{fu} flat use factor (excluding decking)
- C_i incising factor (from increasing the depth of pressure treatment)
- C_t temperature factor (at high temperatures **strength** decreases)
- C_r repetitive member factor
- C_H shear stress factor (amount of splitting)
- C_V volume factor for glued laminated timber (similar to C_F)
- C_L beam stability factor (for beams without full lateral support)
- C_C curvature factor for laminated arches

Tabular Design Values

- F_b: bending stress
- F_t: tensile stress
- F_v: horizontal shear stress
- $F_{c\perp}$: compression stress (perpendicular to grain)
- F_c: compression stress (parallel to grain)
- E: modulus of elasticity
- F_p: bearing stress (parallel to grain)

Wood is significantly weakest in **shear** and strongest along the direction of the grain (tension and compression).

Load Combinations and Deflection

The critical load combination is determined by the largest of either:

$$\frac{dead\ load}{0.9} \ or \ \frac{(dead\ load + any\ combination\ of\ live\ load\ )}{C_D}$$

The deflection limits may be increased for less stiffness with total load: LL + 0.5(DL)

Criteria for Design of Beams

Allowable normal stress or normal stress from LRFD should not be exceeded:

$$F_b' \geq f_b = \frac{Mc}{I}$$

Knowing M and F_b, the minimum section modulus fitting the limit is:

$$S_{req'd} \geq \frac{M}{F_b'}$$

Besides strength, we also need to be concerned about *serviceability*. This involves things like limiting deflections & cracking, controlling noise and vibrations, preventing excessive settlements of foundations and durability. When we know about a beam section and its material, we can determine beam deformations.

Determining Maximum Bending Moment

Drawing V and M diagrams will show us the maximum values for design. Remember:

$$V = \Sigma(-w)dx \qquad \frac{dV}{dx} = -w \qquad \frac{dM}{dx} = V$$
$$M = \Sigma(V)dx$$

Determining Maximum Bending Stress

For a prismatic member (constant cross section), the maximum normal stress will occur at the maximum moment.

For a *non-prismatic* member, the stress varies with the cross section AND the moment.

Deflections

If the bending moment changes, M(x) across a beam of constant material and cross section then the curvature will change:

$$\frac{1}{R} = \frac{M(x)}{EI}$$

The slope of the n.a. of a beam, θ, will be tangent to the radius of curvature, R:

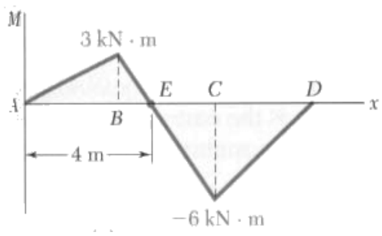
$$\theta = slope = \frac{1}{EI}\int M(x)dx$$

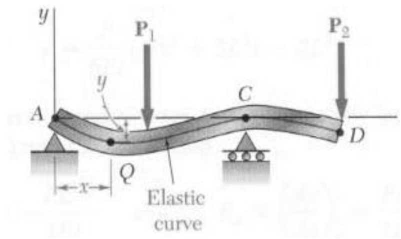
The equation for deflection, y, along a beam is:

$$y = \frac{1}{EI}\int \theta dx = \frac{1}{EI}\iint M(x)dx$$

Elastic curve equations can be found in handbooks, textbooks, design manuals, etc.. Computer programs can be used as well (like *Multiframe*).

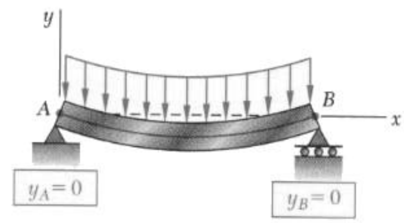
Elastic curve equations can be **superpositioned** ONLY if the stresses are in the elastic range. *The deflected shape is roughly the same shape flipped as the bending moment diagram but is constrained by supports and geometry.*



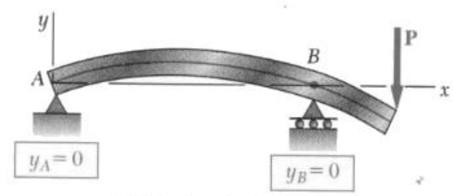


Boundary Conditions

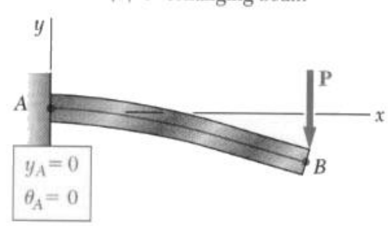
The boundary conditions are geometrical values that we know – slope or deflection – which may be restrained by supports or symmetry.



(a) Simply supported beam



(b) Overhanging beam



(c) Cantilever beam

At Pins, Rollers, Fixed Supports: y = 0

At Fixed Supports: θ = 0

At Inflection Points From Symmetry: θ = 0

The Slope Is Zero At The Maximum Deflection y_{max}:

$$\theta = \frac{dy}{dx} = slope = 0$$

Allowable Deflection Limits

All building codes and design codes limit deflection for beam types and damage that could happen based on service condition and severity.

$$y_{max}(x) = \Delta_{actual} \leq \Delta_{allowable} = L/value$$

Use	LL only	DL+LL
Roof beams:		
Industrial	L/180	L/120
Commercial		
plaster ceiling	L/240	L/180
no plaster	L/360	L/240
Floor beams:		
Ordinary Usage	L/360	L/240
Roof or floor (damageable elements)		L/480

Lateral Buckling

With compression stresses in the top of a beam, a sudden "popping" or **buckling** can happen even at low stresses. In order to prevent it, we need to brace it along the top, or laterally brace it, or provide a bigger I_y.

Beam Loads & Load Tracing

In order to determine the loads on a beam (or girder, joist, column, frame, foundation...) we can start at the top of a structure and determine the *tributary area* that a load acts over and the beam needs to support. Loads come from material weights, people, and the environment. This area is assumed to be from half the distance to the next beam over to halfway to the next beam.

The reactions must be supported by the next lower structural element *ad infinitum*, to the ground.

Design Procedure

The intent is to find the most light weight member satisfying the section modulus size.

1. Know F' for the material or F_U for LRFD.
2. Draw V & M, finding M_{max}.
3. Calculate $S_{req'd}$. This step is equivalent to determining $f_b = \frac{M_{max}}{S} \le F_b'$
4. For rectangular beams $S = \frac{bh^2}{6}$
   - For timber: use the section charts to find S that will work *and remember that the beam self weight will increase* $S_{req'd.}$ $w_{self\ wt} = \gamma A$

*****Determine the "updated"* V_{max} *and* M_{max} *including the beam self weight, and verify that the updated* $S_{req'd}$ *has been met.*******

5. Consider lateral stability.
6. Evaluate horizontal shear stresses using V_{max} to determine if $f_v \le F_v'$ *or* find $A_{req'd}$

   For rectangular beams $f_{v-max} = \frac{3V}{2A} = 1.5\frac{V}{A}$ $\quad \therefore A_{req'd} \le \frac{3V}{2F_v'}$

7. Provide adequate bearing area at supports: $f_p = \frac{P}{A} \le F_c' \text{ or } F_{c\perp}'$
8. Evaluate shear due to torsion $f_v = \frac{T\rho}{J} \text{ or } \frac{T}{c_1 ab^2} \le F_v'$

   (circular section or rectangular)

9. Evaluate the deflection to determine if $\Delta_{max LL} \le \Delta_{LL-allowed}$ and/or $\Delta_{max\,Total} \le \Delta_{Total-allowed}$

***** note: when* $\Delta_{calculated} > \Delta_{limit}$, $I_{required}$ *can be found with:* $I_{req'd} \ge \frac{\Delta_{too\,big}}{\Delta_{limit}} I_{trial}$
and $S_{req'd}$ *will be satisfied for similar self weight ******

FOR ANY EVALUATION:

Redesign (with a new section) at any point that a stress or serviceability criteria is NOT satisfied and re-evaluate each condition until it is satisfactory.

Load Tables for Uniformly Loaded Joists & Rafters

Tables exists for the common loading situation for joists and rafters – that of uniformly distributed load. The tables either provide the safe distributed load based on bending and deflection limits, they give the allowable span for specific live and dead loads. If the load is *not uniform*, an *equivalent distributed load* can be calculated from the maximum moment equation.

Decking

Flat panels or planks that span several joists or evenly spaced support behave as continuous beams. Design tables consider a "1 unit" wide strip across the supports and determine maximum bending moment and deflections in order to provide allowable loads depending on the depth of the material.

The other structural use of decking is to construct what is called a *diaphragm*, which is a horizontal or vertical (if the panels are used in a shear wall) unit tying the sheathing to the joists or studs that resists forces parallel to the surface of the diaphragm.

Criteria for Design of Columns

If we know the loads, we can select a section that is adequate for strength & buckling.

If we know the length, we can find the limiting load satisfying strength & buckling.

Any slenderness ratio, $L_e/d \leq 50$:

$$f_c = \frac{P}{A} \leq F_c' \qquad\qquad F_c' = F_c(C_D)(C_M)(C_t)(C_F)(C_p)$$

The allowable stress equation uses factors to replicate the combination crushing-buckling curve:

where:

- F_c' = allowable compressive stress parallel to the grain
- F_c = compressive strength parallel to the grain
- C_D = load duration factor
- C_M = wet service factor (1.0 for dry)
- C_t = temperature factor
- C_F = size factor
- C_p = column stability factor off chart or equation:

$$C_p = \frac{1 + (F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1 + F_{cE}/F_c^*}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}}$$

For preliminary column design:

$$F'_c = F^*_c C_p = (F_c C_D) C_p$$

Procedure for Analysis

1. Calculate L_e/d_{min} (KL/d for each axis and chose largest)
2. Obtain F'_c

   compute $F_{cE} = \dfrac{K_{cE}E}{\left(\frac{l_e}{d}\right)^2}$ with K_{cE} =0.3 for sawn, = 0.418 for glu-lam
3. Compute $F^*_c \cong F_c C_D$ with C_D = 1, normal, C_D =1.25 for 7 day roof, etc....
4. Calculate F_{cE}/F^*_c and get C_p from table or calculation
5. Calculate $F'_c = F^*_c C_p$
6. Compute $P_{allowable} = F'_c \cdot A$ *or alternatively* compute $f_{actual} = P/A$
7. Is the design satisfactory?

   Is $P \le P_{allowable}$? $\Rightarrow$ yes, it is; no, it is no good

   or Is $f_{actual} \le F'_c$? $\Rightarrow$ yes, it is; no, it is no good

Procedure for Design

1. Guess a size by picking a section
2. Calculate L_e/d_{min} (KL/d for each axis and choose largest)
3. Obtain F'_c

   compute $F_{cE} = \dfrac{K_{cE}E}{\left(\frac{l_e}{d}\right)^2}$ with K_{cE} =0.3 for sawn, = 0.418 for glu-lam
4. Compute $F^*_c \cong F_c C_D$ with C_D = 1, normal, C_D =1.25 for 7 day roof...
5. Calculate F_{cE}/F^*_c and get C_p from table or calculation
6. Calculate $F'_c = F^*_c C_p$
7. Compute $P_{allowable} = F'_c \cdot A$ *or alternatively* compute $f_{actual} = P/A$
8. Is the design satisfactory?

   Is $P \le P_{allowable}$? $\Rightarrow$ yes, it is; *no, pick a bigger section and go back to step 2.*

   or Is $f_{actual} \le F'_c$? $\Rightarrow$ yes, it is; *no, pick a bigger section and go back to step 2.*

Trusses

Timber trusses are commonly manufactured with continuous top or bottom chords, but the members are still design as compression and tension members (without the effect of bending.)

Stud Walls

Stud wall construction is often used in *light frame construction* together with joist and rafters. Studs are typically 2-in. nominal thickness and must be braced in the weak axis. Most wall coverings provide this function. Stud spacing is determined by the width of the panel material, and is usually 16 in. The lumber grade can be relatively low. The walls must be designed for a combination of wind load and bending, which means beam-column analysis.

Columns with Bending (Beam-Columns)

The modification factors are included in the form: $\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_{bx}}{F'_{bx}\left[1-\frac{f_c}{F_{cEx}}\right]} \leq 1.0$

where:

$1-\frac{f_c}{F_{cEx}}$ = magnification factor accounting for P-Δ

F'_{bx} = allowable bending stress

f_{bx} = working stress from bending about x-x axis

In order to *design* an adequate section for allowable stress, we have to start somewhere:

1. Make assumptions about the limiting stress from:
   - buckling
   - axial stress
   - combined stress

2. See if we can find values for r or A or S ($=I/c_{max}$)

3. Pick a trial section based on if we think r or A is going to govern the section size.

4. Analyze the stresses and compare to allowable using the allowable stress method or interaction formula for eccentric columns.

5. Did the section pass the stress test?
   - If not, do you *increase* r or A or S?
   - If so, is the difference really big so that you could *decrease* r or A or S to make it more efficient (economical)?

6. Change the section choice and go back to step 4. Repeat until the section meets the stress criteria.

Laminated Arches

The radius of curvature, *R*, is limited because of residual bending stresses between lams of thickness *t* to 100t for Southern pine and hardwoods and 250t for softwoods.

The allowable bending stress for combined stresses is $F'_b = F_b(C_F C_C)$

where $C_C = 1 - 2000\left(\frac{t}{r}\right)^2$

and r is the radius to the inside of the lamination.

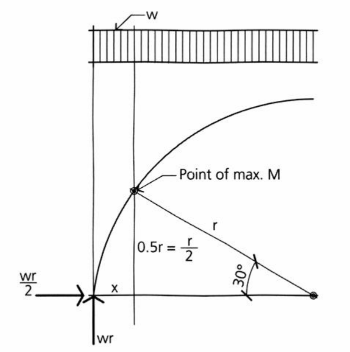


Fig. 24.6 Circular arch moment analysis

Criteria for Design of Connections

Connections for wood are typically mechanical fasteners. Shear plates and split ring connectors are common in trusses. Bolts of metal bear on holes in wood, and nails rely on shear resistance transverse and parallel to the nail shaft. Timber rivets with steel side plates are allowed with glue laminated timber.

Connections must be able to transfer any axial force, shear, or moment from member to member or from beam to column.

Bolted Joints

Stress must be evaluated in the member being connected using the load being transferred and the reduced cross section area called *net area*. Bolt capacities are usually provided in tables and take into account the allowable shearing stress across the diameter for *single* and *double shear*, and the allowable bearing stress of the connected material based on the direction of the load with respect to the grain. Problems, such as ripping of the bolt hole at the end of the member, are avoided by following code guidelines on minimum edge distances and spacing.

Nailed Joints

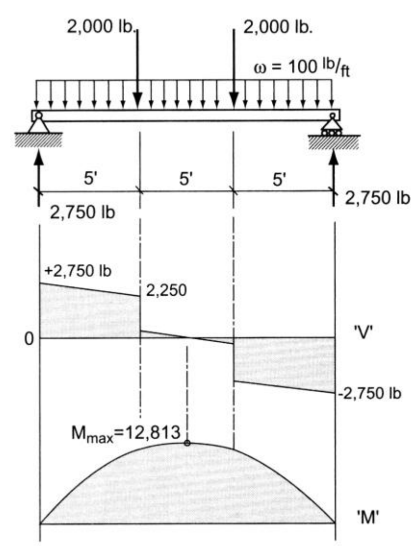
Because nails rely on shear resistance, a common problem when nailing is splitting of the wood at the end of the member, which is a shear failure. Tables list the shear force capacity per unit length of embedment per nail. Jointed members used for beams will have shear stress across the connector, and the pitch spacing, *p*, can be determined from the shear stress equation when the capacity, *F*, is known:

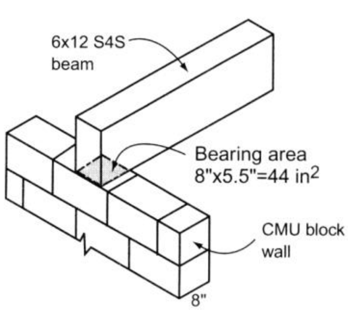
$$nF_{connector} \geq \frac{VQ_{connected\ area}}{I} \cdot p$$

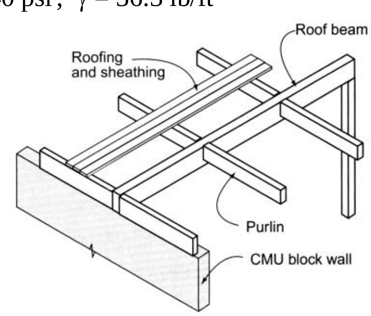
Example 1 (pg 328)

Example Problem 9.15 (Figures 9.73 to 9.75)

Design a Southern pine No. 1 beam to carry the loads shown (roof beam, no plaster). Assume the beam is supported at each end by an 8" block wall. F_b = 1550 psi; F_v = 110 psi; $E = 1.6 \times 10^6$ psi. $F_{c\perp}$ = 440 psi , γ = 36.3 lb/ft^3







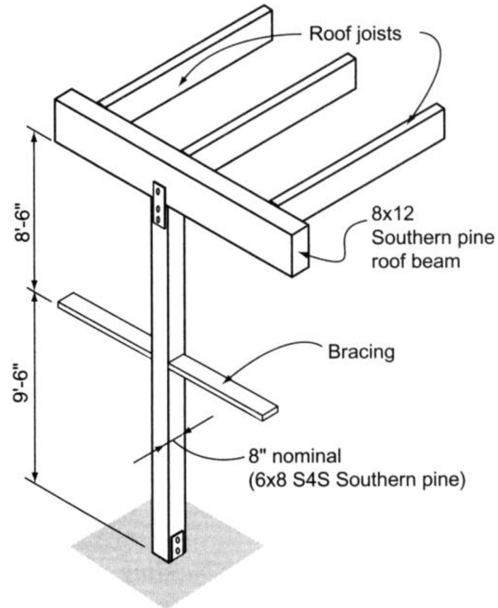
Example 1 (continued)

Example 2 (pg 379)

Example Problem 10.18 (Figures 10.60 and 10.61)

An 18' tall 6×8 Southern pine column supports a roof load (dead load plus a 7-day live load) equal to 16 kips. The weak axis of buckling is braced at a point 9'6" from the bottom support. Determine the adequacy of the column.

$$F_c = 975 \text{ psi}, \; E = 1.6 \times 10^6 \text{ psi}$$



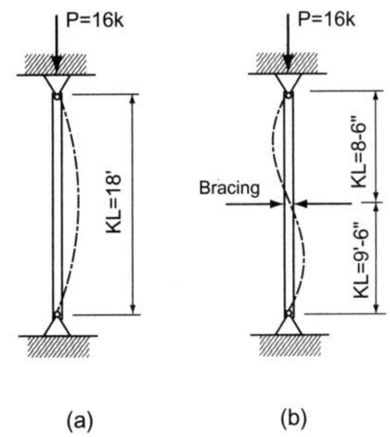


Figure 10.61 (a) Strong axis. (b) Weak axis.

Example 3 (pg 381)

Example Problem 10.20: Design of Wood Columns(Figure 10.66)

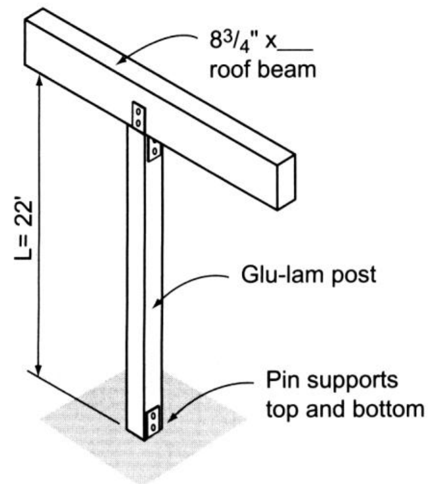
A 22'-tall glu-lam column is required to support a roof load (including snow) of 40 kips. Assuming 8 3/4" in one dimension (to match the beam width above), determine the minimum column size if the top and bottom are pin supported.

Select from the following sizes:

- 8 3/4" × 9" (A = 78.75 in.^{2})
- 8 3/4" × 10 1/2" (A = 91.88 in.^{2})
- 8 3/4" × 12" (A = 105.00 in.^{2})

F_c = 1650 psi , E = 1.8 x 10^6 psi

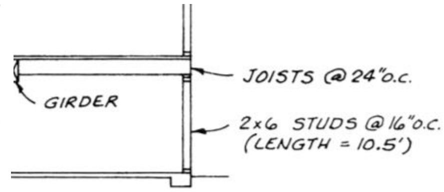
Also verify with allowable load tables



Example 4

EXAMPLE 7.16 Combined Bending and Compression in a Stud Wall

Check the 2 × 6 stud in the first-floor bearing wall in the building shown in Fig. 7.20*a*. Consider the given vertical loads and lateral forces. Lumber is No. 2 DF-L. MC ≤ 19 percent and normal temperatures apply. Allowable stresses are to be in accordance with the NDS. F'_b = 2152 psi F_c = 1350 psi



A = 8.25 in^2
S_x^* = 7.56 in^3

COLUMN CAPACITY:

Sheathing provides lateral support about the weak axis of the stud. Therefore, check column buckling about the x axis only (L = 10.5 ft and d_x = 5.5 in.):

$$\left(\frac{l_e}{d}\right)_y = 0 \quad \text{because of sheathing}$$

$$\left(\frac{l_e}{d}\right)_{max} = \left(\frac{l_e}{d}\right)_x = \frac{10.5\text{ ft} \times 12\text{ in./ft}}{5.5\text{ in.}} = 22.9$$

$$E = 1{,}600{,}000\text{ psi}$$

For visually graded sawn lumber:

$$K_{cE} = 0.3$$

$$c = 0.8$$

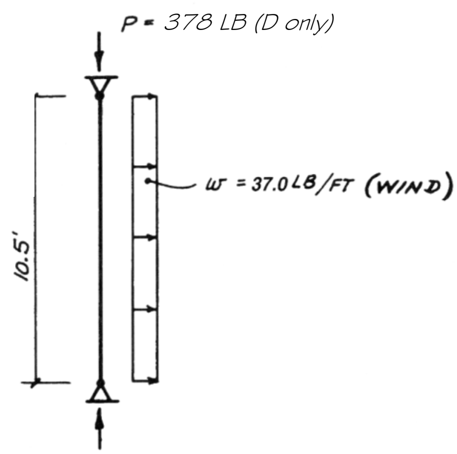
$$F_{cE} = \frac{K_{cE}E'}{(l_e/d)^2} = \frac{0.3(1{,}600{,}000)}{(22.9)^2} = 915\text{ psi}$$

$$F_c^* = F_c(C_D) \quad C_D = 1.6 \text{ from wind loading}$$

$$= 1350(1.6) = 2376\text{ psi}$$

$$\frac{F_{cE}}{F_c^*} = \frac{915}{2376} = 0.385 \quad C_P = 0.35$$

$$F'_c = F_c(C_D)(C_P) = 2376(0.35) = 832\text{ psi}$$



Load Case 2: Gravity Loads + Lateral Forces

BENDING:

Wind governs over seismic. Force to one stud:

$$\text{Wind} = 27.8\text{ psf}$$

$$w = 27.8\text{ psf} \times \frac{16in}{12^{in/ft}} = 37.0\text{ lb/ft}$$

$$M = \frac{wL^2}{8} = \frac{37.0(10.5)^2}{8} = 510\text{ ft-lb} = 6115\text{ in.-lb}$$

$$f_b = \frac{M}{S} = \frac{6115}{7.56} = 809\text{ psi} \qquad F'_b = 2152\text{ psi}$$

AXIAL:

$$\text{D + W:} \quad f_c = \frac{P}{A} = \frac{378}{8.25} = 46\text{ psi}$$

COMBINED STRESS:

The simplified interaction formula from Example 7.13 (Sec. 7.12) applies:

$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_{bx}}{F'_{bx}(1 - f_c/F_{cEx})} \le 1.0$$

$$F_{cEx} = F_{cE} = 915\text{ psi}$$

D + W:

In this load combination, D produces the axial stress f_c and W results in the bending stress f_{bx}.

$$\left(\frac{f_c}{F'_c}\right)^2 + \left(\frac{1}{1 - f_c/F_{cEx}}\right)\frac{f_{bx}}{F'_{bx}} =$$

$$\left(\frac{46}{832}\right)^2 + \left(\frac{1}{1 - 46/915}\right)\frac{809}{2152} = 0.399 < 1.0$$

2 × 6 No. 2 DF-L exterior bearing wall *OK*

Example 5

Example 2. The truss heel joint shown in Figure 7.5 is made with 2-in. nominal thickness lumber and gusset plates of ½-in.-thick plywood. Nails are 6d common wire with the nail layout shown occurring in both sides of the joint. Find the tension load capacity for the bottom chord member (load 3 in the figure).

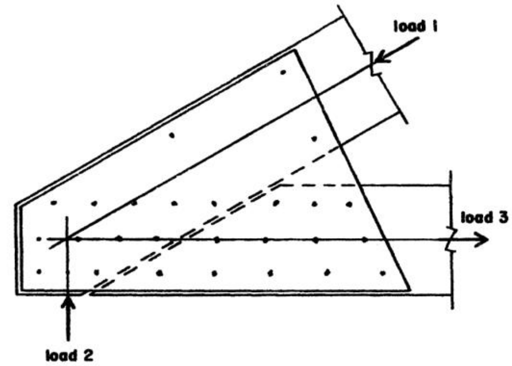


TABLE 7.1 Reference Lateral Load Values for Common Wire Nails (lb/in.)

Side Member Thickness, t_s (in.)	Nail Length, L (in.)	Nail Diameter, D (in.)	Nail Pennyweight	Load per Nail, Z (lb)
Part 1 — With Wood Structural Panel Side Members[a] (G = 0.42)				
	2	0.113	6d	48
3/8	2½	0.131	8d	63
	3	0.148	10d	76
	2	0.113	6d	50
15/32	2½	0.131	8d	65
	3	0.148	10d	78
	3½	0.162	16d	92

Example 6

A nominal 4 x 6 in. redwood beam is to be supported by two 2 x 6 in. members acting as a spaced column. The minimum spacing and edge distances for the ½ inch bolts are shown. How many ½ in. bolts will be required to safely carry a load of 1500 lb? Use the chart provided.

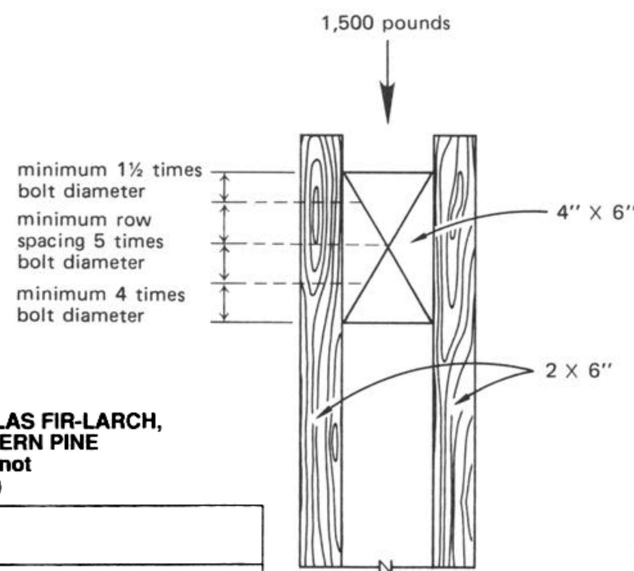


TABLE 23-I-F—HOLDING POWER OF BOLTS[1,2,3] FOR DOUGLAS FIR-LARCH, CALIFORNIA REDWOOD (CLOSE GRAIN) AND SOUTHERN PINE
(See U.B.C. Standard 23-17 where members are not of equal size and for values in other species.)

p = safe loads parallel to grain, in pounds.
q = safe loads perpendicular to grain, in pounds.

× 4.45 for N

LENGTH OF BOLT IN MAIN WOOD MEMBER[4] (inches)		DIAMETER OF BOLT (inches) 3/8	1/2	5/8	3/4	7/8	1
× 25.4 for mm							
2½	Single p		630	910	1,155	1,370	1,575
	Shear q		360	405	450	495	540
	Double p	710	1,260	1,820	2,310	2,740	3,150
	Shear q	620	720	810	900	990	1,080
3½	Single p			990	1,400	1,790	2,135
	Shear q			565	630	695	760
	Double p	710	1,270	1,980	2,800	3,580	4,270
	Shear q	640	980	1,130	1,260	1,390	1,520

[1]Tabulated values are on a normal load-duration basis and apply to joints made of seasoned lumber used in dry locations. See Division III for other service conditions.
[2]Double shear values are for joints consisting of three wood members in which the side members are one half the thickness of the main member. Single shear values are for joints consisting of two wood members having a minimum thickness not less than that specified.
[3]See Division III for wood-to-metal bolted joints.
[4]The length specified is the length of the bolt in the main member of double shear joints or the length of the bolt in the thinner member of single shear joints.

Example 7

EXAMPLE 12.8 Knee Brace Connection

The carport shown in Fig. 12.13a uses 2 × 6 knee braces to resist the longitudinal seismic force. Determine the number of 16d common nails required for the connection of the brace to the 4 × 4 post. Material is Southern Pine lumber that is dry at the time of construction. Normal temperatures apply.

Force to one row of braces:

$$R = \frac{wL}{2} = 76\left(\frac{22}{2}\right) = 836 \text{ lb}$$

Assume the force is shared equally by all braces.

$$\Sigma M_0 = 0$$

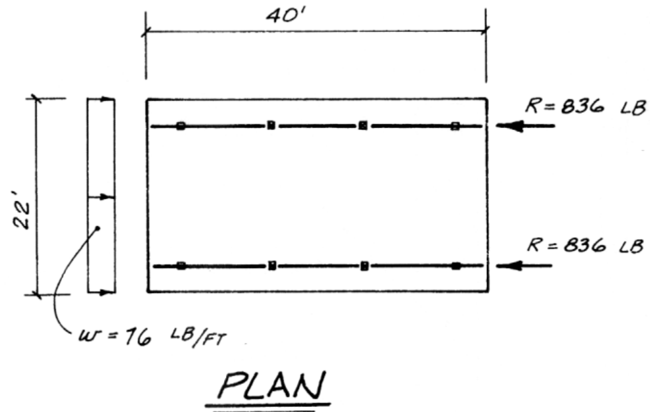
$$3H - 209(10) = 0$$

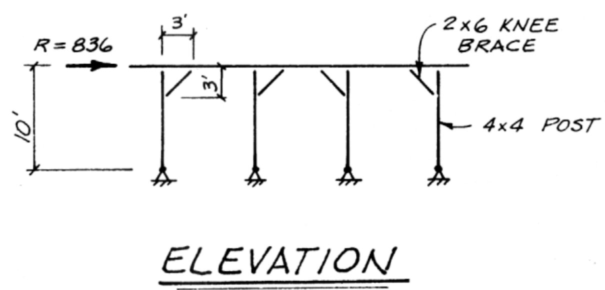
$$H = 697 \text{ lb}$$

$$B = \sqrt{2}H = \sqrt{2}(697)$$

$$= 985 \text{ lb axial force in knee brace}$$

$$= \text{force on nailed connection}$$

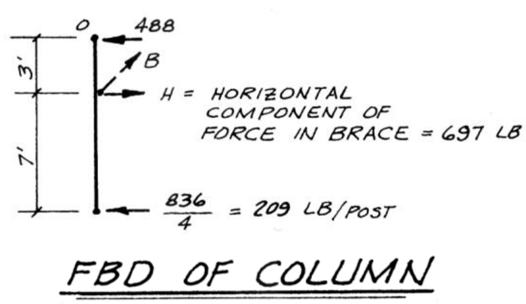


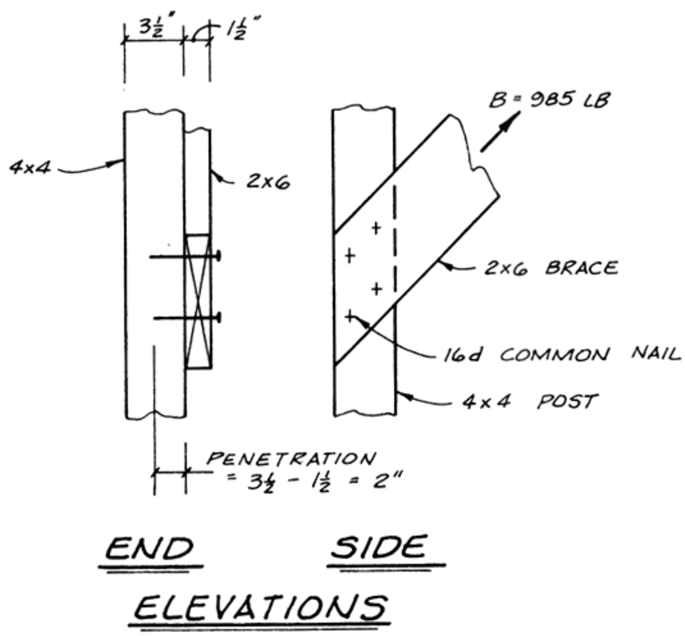


The nominal design value for a 16d common nail in Southern Pine can be evaluated using the yield equations (Sec. 12.4), or it can be obtained from NDS Table 12.3B.

Nominal design value from NDS Table 12.3B

$$Z = 154 \text{ lb/nail}$$





Example 7 (continued)

Adjustment Factors

Penetration

Required penetration to use the full value of Z

$$12D = 12(0.162) = 1.94 \text{ in.} < 2.0$$

$\therefore$ Penetration depth factor is

$$C_d = 1.0$$

Moisture content

Because the building is "unenclosed," the brace connection may be exposed to the weather, and the severity of this exposure must be judged by the designer. Assume that a reduction for high moisture content is deemed appropriate, and the wet service factor C_M is obtained from NDS Table 7.3.3.

$$C_M = 0.7$$

Load duration

The load duration factor recommended in the NDS for seismic forces is $C_D = 1.6$. The designer is cautioned to verify local code acceptance before using this value in practice.

Other adjustment factors

All other adjustment factors for allowable nail capacity do not apply to the given problem, and each can be set equal to unity:

$C_t = 1.0$	because normal temperature range is assumed
$C_{eg} = 1.0$	because nails are driven into side grain of holding member
$C_{di} = 1.0$	because connection is not part of nailing for diaphragm or shearwall
$C_{tn} = 1.0$	because nails are not toenailed

Allowable load for 16d common nail in Southern Pine:

$$Z' = Z(C_D C_M C_t C_d C_{eg} C_{di} C_{tn})$$

$$= 154(1.6)(0.7)(1.0)(1.0)(1.0)(1.0)(1.0) = 172 \text{ lb/nail}$$

Required number of nails: $N = \dfrac{B}{Z'} = \dfrac{985}{172} = 5.73$

Use six 16d common nails each end of knee brace for high-moisture conditions.*

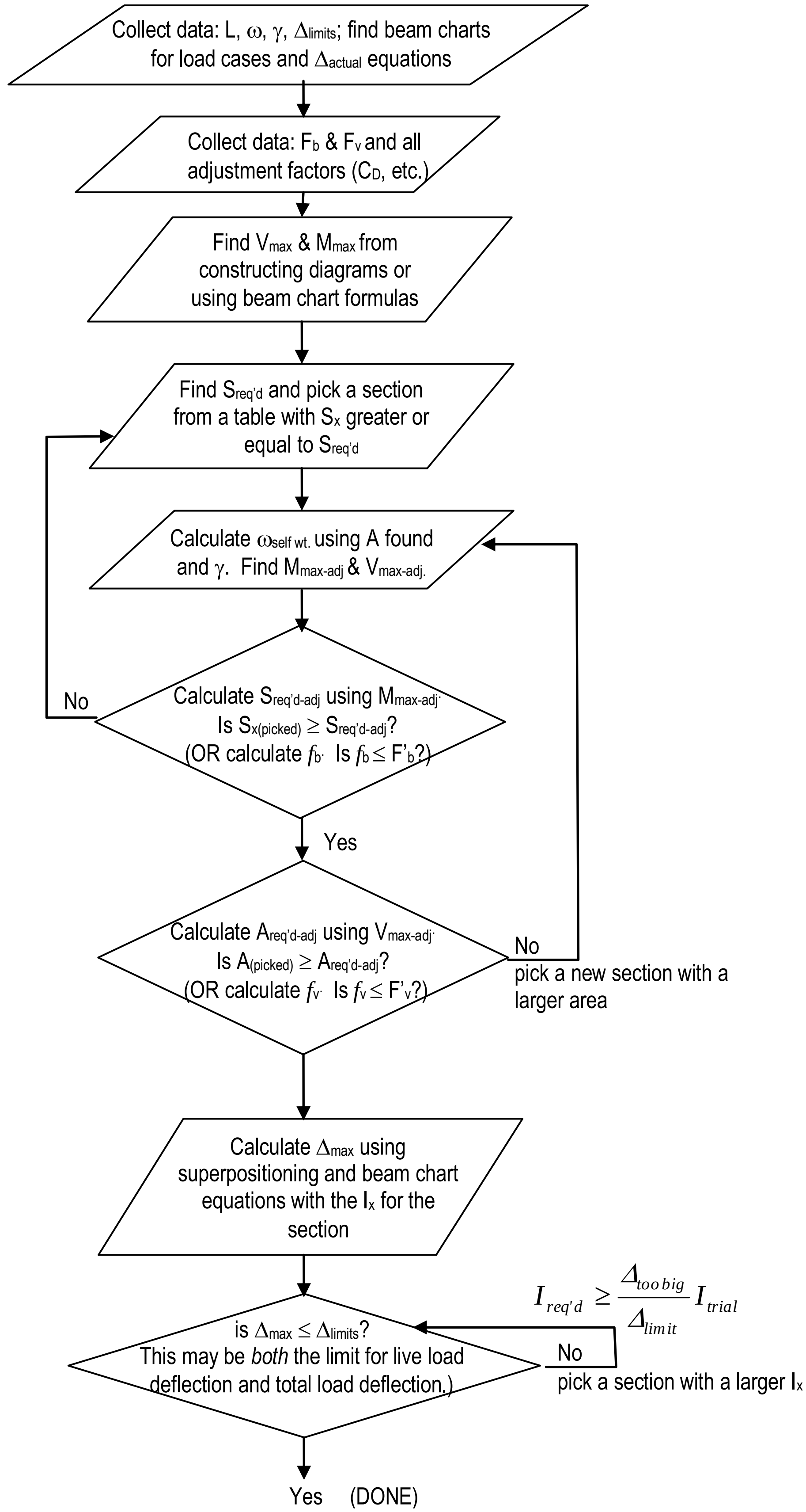
If the reduction for wet service is not required, $C_M = 1.0$. The revised connection is

$$Z' = 154(1.6)(1.0) = 246 \text{ lb/nail}$$

$$N = \frac{985}{246} = 4.00$$

Use four 16d common nails each end of knee brace if moisture is not a concern.

ASD Beam Design Flow Chart



Beam Design and Deflections

Notation:

a = name for width dimension
A = name for area
$A_{req'd\text{-}adj}$ = area required at allowable stress when shear is adjusted to include self weight
A_{web} = area of the web of a wide flange section
b = width of a rectangle
= total width of material at a horizontal section
= name for height dimension
c = largest distance from the neutral axis to the top or bottom edge of a beam
c_1 = coefficient for shear stress for a rectangular bar in torsion
d = calculus symbol for differentiation
DL = shorthand for dead load
E = modulus of elasticity
f_b = bending stress
f_p = bearing stress (see P)
f_v = shear stress
$f_{v\text{-}max}$ = maximum shear stress
F_b = allowable bending stress
F_v = allowable shear stress
F_p = allowable bearing stress
F_y = yield strength
F_{yweb} = yield strength of the web material
h = height of a rectangle
I = moment of inertia with respect to neutral axis bending
I_{trial} = moment of inertia of trial section
$I_{req'd}$ = moment of inertia required at limiting deflection
J = polar moment of inertia
L = name for span length
LL = shorthand for live load
$LRFD$ = load and resistance factor design
M = internal bending moment
M_{max} = maximum internal bending moment
$M_{max\text{-}adj}$ = maximum bending moment adjusted to include self weight
M_n nominal flexure strength with the full section at the yield stress for LRFD
M_u = maximum moment from factored loads for LRFD
P = name for axial force vector
Q = first moment area about a neutral axis
R = radius of curvature of a deformed beam
S = section modulus
$S_{req'd}$ = section modulus required at allowable stress
T = torque (axial moment)
V = internal shear force
V_{max} = maximum internal shear force
$V_{max\text{-}adj}$ = maximum internal shear force adjusted to include self weight
V_u = maximum shear from factored loads for LRFD
w = name for distributed load
$w_{self\ wt}$ = name for distributed load from self weight of member
x = horizontal distance
y = vertical distance
Δ_{actual} = actual beam deflection
$\Delta_{allowable}$ = allowable beam deflection
Δ_{limit} = allowable beam deflection limit
Δ_{max} = maximum beam deflection
ϕ_b = resistance factor for flexure in LRFD design
ϕ_v = resistance factor for shear for LRFD
γ = density or unit weight
θ = slope of the beam deflection curve
ρ = radial distance
$\int$ = symbol for integration
Σ = summation symbol

Criteria for Design

Allowable bending stress or bending stress from LRFD should not be exceeded:

$$F_b \geq f_b = \frac{Mc}{I}$$

Knowing M and F_b, the minimum section modulus fitting the limit is:

$$S_{req'd} \geq \frac{M}{F_b}$$

Besides strength, we also need to be concerned about *serviceability*. This involves things like limiting deflections & cracking, controlling noise and vibrations, preventing excessive settlements of foundations and durability. When we know about a beam section and its material, we can determine beam deformations.

Determining Maximum Bending Moment

Drawing V and M diagrams will show us the maximum values for design. Remember:

$$V = \Sigma(-w)dx \qquad \frac{dV}{dx} = -w \qquad \frac{dM}{dx} = V$$
$$M = \Sigma(V)dx$$

Determining Maximum Bending Stress

For a prismatic member (constant cross section), the maximum normal stress will occur at the maximum moment.

For a *non-prismatic* member, the stress varies with the cross section AND the moment.

Deflections

If the bending moment changes, M(x) across a beam of constant material and cross section then the curvature will change:

$$\frac{1}{R} = \frac{M(x)}{EI}$$

The slope of the n.a. of a beam, θ, will be tangent to the radius of curvature, R:

$$\theta = slope = \frac{1}{EI}\int M(x)dx$$

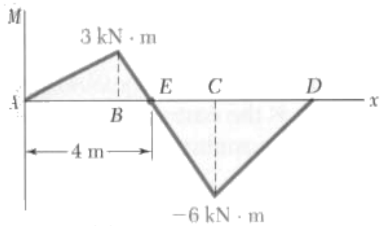
The equation for deflection, y, along a beam is:

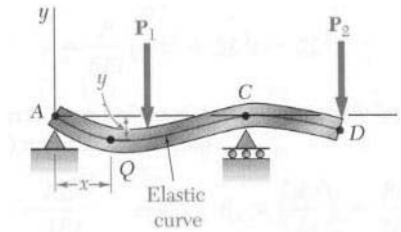
$$y = \frac{1}{EI}\int \theta dx = \frac{1}{EI}\iint M(x)dx$$

Elastic curve equations can be found in handbooks, textbooks, design manuals, etc...Computer programs can be used as well. (BigBoy Beam freeware: http://forum.simtel.net/pub/pd/33994.html)

Elastic curve equations can be **superpositioned** ONLY if the stresses are in the elastic range.

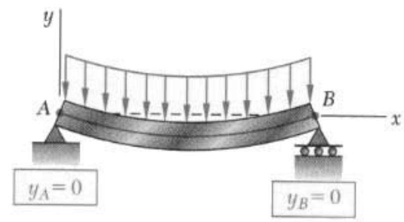
The deflected shape is roughly the shame shape as the bending moment diagram flipped but is constrained by supports and geometry.



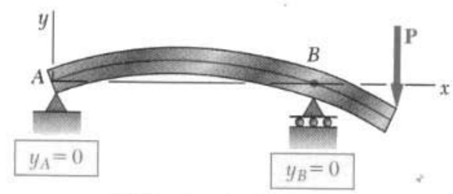


Boundary Conditions

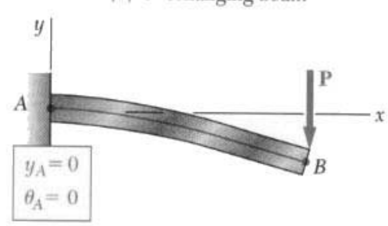
The boundary conditions are geometrical values that we know – slope or deflection – which may be restrained by supports or symmetry.



(a) Simply supported beam



(b) Overhanging beam



(c) Cantilever beam

At Pins, Rollers, Fixed Supports: $y = 0$

At Fixed Supports: $\theta = 0$

At Inflection Points From Symmetry: $\theta = 0$

The Slope Is Zero At The Maximum Deflection y_{max}:.

$$\theta = \frac{dy}{dx} = slope = 0$$

Allowable Deflection Limits

All building codes and design codes limit deflection for beam types and damage that could happen based on service condition and severity.

$$y_{\max}(x) = \Delta_{actual} \le \Delta_{allowable} = L/_{value}$$

Use	LL only	DL+LL
Roof beams:		
Industrial	L/180	L/120
Commercial		
plaster ceiling	L/240	L/180
no plaster	L/360	L/240
Floor beams:		
Ordinary Usage	L/360	L/240
Roof or floor (damageable elements)		L/480

Beam Loads & Load Tracing

In order to determine the loads on a beam (or girder, joist, column, frame, foundation...) we can start at the top of a structure and determine the *tributary area* that a load acts over and the beam needs to support. Loads come from material weights, people, and the environment. This area is assumed to be from half the distance to the next beam over to halfway to the next beam.

The reactions must be supported by the next lower structural element *ad infinitum*, to the ground.

Design Procedure

The intent is to find the most light weight member satisfying the section modulus size.

1. Know F_b (allowable stress) for the material or F_y & F_u for LRFD.
2. Draw V & M, finding M_{max}.
3. Calculate $S_{req'd}$. This step is equivalent to determining $f_b = \frac{M_{max}}{S} \leq F_b$
4. For rectangular beams $S = \frac{bh^2}{6}$
   - For steel or timber: use the section charts to find S that will work *and remember that the beam self weight will increase $S_{req'd}$.* And for steel, the design charts show the lightest section within a grouping of similar S's. $w_{self\ wt} = \gamma A$
   - For any thing else, try a nice value for b, and calculate h or the other way around.

*****Determine the "updated" V_{max} and M_{max} including the beam self weight, and verify that the updated $S_{req'd}$ has been met.*******

5. Consider lateral stability
6. Evaluate horizontal shear stresses using V_{max} to determine if $f_v \leq F_v$

   For rectangular beams, W's, and others: $$f_{v-max} = \frac{3V}{2A} \approx \frac{V}{A_{web}} \ or\ \frac{VQ}{Ib}$$

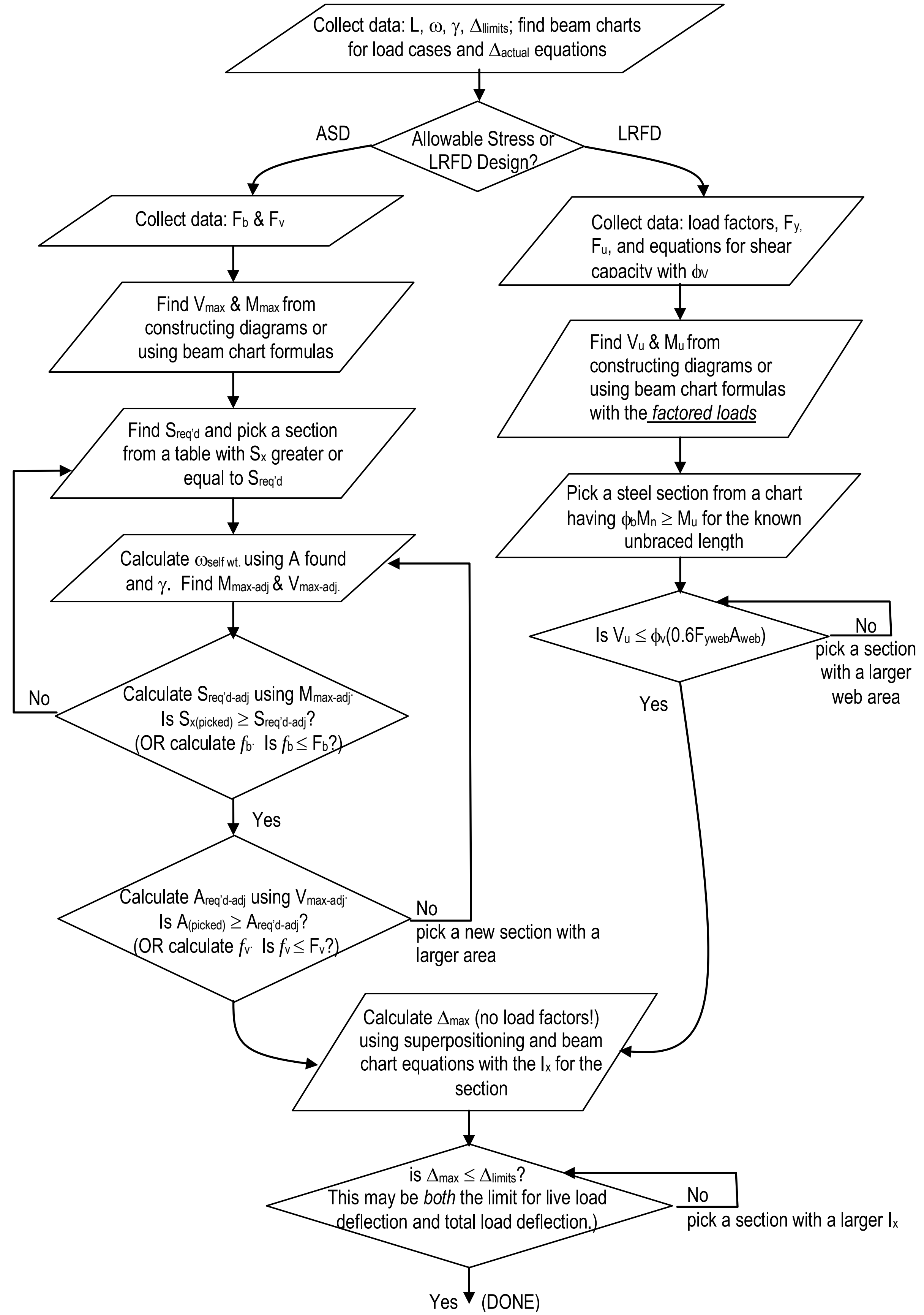
7. Provide adequate bearing area at supports: $$f_p = \frac{P}{A} \leq F_p$$
8. Evaluate shear due to torsion $$f_v = \frac{T\rho}{J} \ or\ \frac{T}{c_1 ab^2} \leq F_v$$
   (circular section or rectangular)
9. Evaluate the deflection to determine if $\Delta_{max LL} \leq \Delta_{LL-allowed}$ and/or $\Delta_{max Total} \leq \Delta_{T-allowed}$

**** *note: when $\Delta_{calculated} > \Delta_{limit}$, $I_{required}$ can be found with: and $S_{req'd}$ will be satisfied for similar self weight* *****
$$I_{req'd} \geq \frac{\Delta_{too\ big}}{\Delta_{limit}} I_{trial}$$

FOR ANY EVALUATION:

Redesign (with a new section) at any point that a stress or serviceability criteria is NOT satisfied and re-evaluate each condition until it is satisfactory.

Beam Design Flow Chart



Steel Design

Notation:

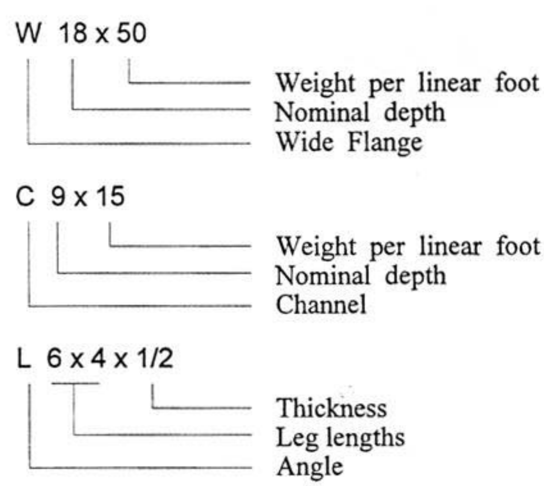
a = name for width dimension
A = name for area
A_b = area of a bolt
A_e = effective net area found from the product of the net area A_n by the shear lag factor U
A_g = gross area, equal to the total area ignoring any holes
A_{gv} = gross area subjected to shear for block shear rupture
A_n = net area, equal to the gross area subtracting any holes, as is A_{net}
A_{nt} = net area subjected to tension for block shear rupture
A_{nv} = net area subjected to shear for block shear rupture
A_w = area of the web of a wide flange section
$AISC$ = American Institute of Steel Construction
ASD = allowable stress design
b = name for a (base) width
= total width of material at a horizontal section
= name for height dimension
b_f = width of the flange of a steel beam cross section
B_1 = factor for determining M_u for combined bending and compression
c = largest distance from the neutral axis to the top or bottom edge of a beam
c_1 = coefficient for shear stress for a rectangular bar in torsion
C_b = lateral torsional buckling modification factor for moment in ASD & LRFD steel beam design
C_c = column slenderness classification constant for steel column design
C_m = modification factor accounting for combined stress in steel design
C_v = web shear coefficient
d = calculus symbol for differentiation
= depth of a wide flange section
= nominal bolt diameter
d_b = nominal bolt diameter
D = shorthand for dead load
DL = shorthand for dead load
e = eccentricity
E = shorthand for earthquake load
= modulus of elasticity
f_c = axial compressive stress
f_b = bending stress
f_p = bearing stress
f_v = shear stress
$f_{v\text{-}max}$ = maximum shear stress
f_y = yield stress
F = shorthand for fluid load
$F_{allow(able)}$ = allowable stress
F_a = allowable axial (compressive) stress
F_b = allowable bending stress
F_{cr} = flexural buckling stress
F_e = elastic critical buckling stress
F_{EXX} = yield strength of weld material
F_n = nominal strength in LRFD
= nominal tension or shear strength of a bolt
F_p = allowable bearing stress
F_t = allowable tensile stress
F_u = ultimate stress prior to failure
F_v = allowable shear stress
F_y = yield strength
F_{yw} = yield strength of web material
$F.S.$ = factor of safety
g = gage spacing of staggered bolt holes
G = relative stiffness of columns to beams in a rigid connection, as is Ψ
h = name for a height
h_c = height of the web of a wide flange steel section
H = shorthand for lateral pressure load
I = moment of inertia with respect to neutral axis bending
I_{trial} = moment of inertia of trial section
$I_{req'd}$ = moment of inertia required at limiting deflection
I_y = moment of inertia about the y axis
J = polar moment of inertia

k = distance from outer face of W flange to the web toe of fillet
= shape factor for plastic design of steel beams
K = effective length factor for columns, as is k
l = name for length
ℓ_b = length of beam in rigid joint
ℓ_c = length of column in rigid joint
L = name for length or span length
= shorthand for live load
L_b = unbraced length of a steel beam
L_c = clear distance between the edge of a hole and edge of next hole or edge of the connected steel plate in the direction of the load
L_e = effective length that can buckle for column design, as is ℓ_e
L_r = shorthand for live roof load
= maximum unbraced length of a steel beam in LRFD design for inelastic lateral-torsional buckling
L_p = maximum unbraced length of a steel beam in LRFD design for full plastic flexural strength
L' = length of an angle in a connector with staggered holes
LL = shorthand for live load
$LRFD$ = load and resistance factor design
M = internal bending moment
M_a = required bending moment (ASD)
M_n = nominal flexure strength with the full section at the yield stress for LRFD beam design
M_{max} = maximum internal bending moment
$M_{max\text{-}adj}$ = maximum bending moment adjusted to include self weight
M_p = internal bending moment when all fibers in a cross section reach the yield stress
M_u = maximum moment from factored loads for LRFD beam design
M_y = internal bending moment when the extreme fibers in a cross section reach the yield stress
n = number of bolts
$n.a.$ = shorthand for neutral axis
N = bearing length on a wide flange steel section
= bearing type connection with threads included in shear plane
p = bolt hole spacing (pitch)
P = name for load or axial force vector
P_a = allowable axial force
= required axial force (ASD)
$P_{allowable}$ = allowable axial force
P_c = available axial strength
P_{e1} = Euler buckling strength
P_n = nominal column load capacity in LRFD steel design
P_r = required axial force
P_u = factored column load calculated from load factors in LRFD steel design
Q = first moment area about a neutral axis
= generic axial load quantity for LRFD design
r = radius of gyration
r_y = radius of gyration with respect to a y-axis
R = generic load quantity (force, shear, moment, etc.) for LRFD design
= shorthand for rain or ice load
= radius of curvature of a deformed beam
R_a = required strength (ASD)
R_n = nominal value (capacity) to be multiplied by ϕ in LRFD and divided by the safety factor Ω in ASD
R_u = factored design value for LRFD design
s = longitudinal center-to-center spacing of any two consecutive holes
S = shorthand for snow load
= section modulus
= allowable strength per length of a weld for a given size
$S_{req'd}$ = section modulus required at allowable stress
$S_{req'd\text{-}adj}$ = section modulus required at allowable stress when moment is adjusted to include self weight
SC = slip critical bolted connection

t = thickness of the connected material
t_f = thickness of flange of wide flange
t_w = thickness of web of wide flange
T = torque (axial moment)
= shorthand for thermal load
= throat size of a weld
U = shear lag factor for steel tension member design
U_{bs} = reduction coefficient for block shear rupture
V = internal shear force
V_a = required shear (ASD)
V_{max} = maximum internal shear force
$V_{max\text{-}adj}$ = maximum internal shear force adjusted to include self weight
V_n = nominal shear strength capacity for LRFD beam design
V_u = maximum shear from factored loads for LRFD beam design
w = name for distributed load
$w_{adjusted}$ = adjusted distributed load for equivalent live load deflection limit
$w_{equivalent}$ = the equivalent distributed load derived from the maximum bending moment
$w_{self\ wt}$ = name for distributed load from self weight of member
W = shorthand for wind load
x = horizontal distance
X = bearing type connection with threads excluded from the shear plane
y = vertical distance
Z = plastic section modulus of a steel beam
$Z_{req'd}$ = plastic section modulus required
Z_x = plastic section modulus of a steel beam with respect to the x axis
α = method factor for B_1 equation
Δ_{actual} = actual beam deflection
$\Delta_{allowable}$ = allowable beam deflection
Δ_{limit} = allowable beam deflection limit
Δ_{max} = maximum beam deflection
ε_y = yield strain (no units)
ϕ = resistance factor
= diameter symbol
ϕ_b = resistance factor for bending for LRFD
ϕ_c = resistance factor for compression for LRFD
ϕ_t = resistance factor for tension for LRFD
ϕ_v = resistance factor for shear for LRFD
γ = load factor in LRFD design
π = pi (3.1415 radians or 180°)
θ = slope of the beam deflection curve
ρ = radial distance
Ω = safety factor for ASD
$\int$ = symbol for integration
Σ = summation symbol

Steel Design

Structural design standards for steel are established by the *Manual of Steel Construction* published by the American Institute of Steel Construction, and uses **Allowable Stress Design** and **Load and Factor Resistance Design**. With the 13th edition, both methods are combined in one volume which provides common requirements for analyses and design and requires the application of the same set of specifications.



Materials

American Society for Testing Materials (ASTM) is the organization responsible for material and other standards related to manufacturing. Materials meeting their standards are guaranteed to have the published strength and material properties for a designation.

A36 – carbon steel used for plates, angles F_y = 36 ksi, F_u = 58 ksi, E = 29,000 ksi
A572 – high strength low-alloy use for some beams F_y = 60 ksi, F_u = 75 ksi, E = 29,000 ksi
A992 – for building framing used for most beams F_y = 50 ksi, F_u = 65 ksi, E = 29,000 ksi
(A572 Grade 50 has the same properties as A992)

ASD $$R_a \leq R_n / \Omega$$

where R_a = required strength (dead or live; force, moment or stress)
R_n = nominal strength specified for ASD
Ω = safety factor

Factors of Safety are applied to the limit stresses for allowable stress values:

bending (braced, $L_b < L_p$)	Ω = 1.67
bending (unbraced, $L_p < L_b$ and $L_b > L_r$)	Ω = 1.67 (nominal moment reduces)
shear (beams)	Ω = 1.5 or 1.67
shear (bolts)	Ω = 2.00 (tabular nominal strength)
shear (welds)	Ω = 2.00

- L_b is the unbraced length between bracing points, laterally
- L_p is the limiting laterally unbraced length for the limit state of yielding
- L_r is the limiting laterally unbraced length for the limit state of inelastic lateral-torsional buckling

LRFD $$R_u \leq \phi R_n \qquad where \cdots R_u = \Sigma \gamma_i R_i$$

where ϕ = resistance factor
γ = load factor for the type of load
R = load (dead or live; force, moment or stress)
R_u = factored load (moment or stress)
R_n = nominal load (ultimate capacity; force, moment or stress)

Nominal strength is defined as the

> capacity of a structure or component to resist the effects of loads, as determined by computations using specified material strengths (such as yield strength, F_y, or ultimate strength, F_u) and dimensions and formulas derived from accepted principles of structural mechanics or by field tests or laboratory tests of scaled models, allowing for modeling effects and differences between laboratory and field conditions

Factored Load Combinations

The design strength, ϕR_n, of each structural element or structural assembly must equal or exceed the design strength based on the ASCE-7 (2010) combinations of factored nominal loads:

$1.4D$
$1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
$1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.5W)$
$1.2D + 1.0W + L + 0.5(L_r \text{ or } S \text{ or } R)$
$1.2D + 1.0E + L + 0.2S$
$0.9D + 1.0W$
$0.9D + 1.0E$

Criteria for Design of Beams

Allowable normal stress or normal stress from LRFD should not be exceeded:

$$F_b \text{ or } \phi F_n \geq f_b = \frac{Mc}{I}$$
$$(M_a \leq M_n / \Omega \text{ or } M_u \leq \phi_b M_n)$$

Knowing M and F_y, the minimum plastic section modulus fitting the limit is:

$$Z_{req'd} \geq \frac{M_a}{F_y \Omega} \qquad \left(S_{req'd} \geq \frac{M}{F_b} \right)$$

Determining Maximum Bending Moment

Drawing V and M diagrams will show us the maximum values for design. Remember:

$$V = \Sigma(-w)dx \qquad \frac{dV}{dx} = -w \qquad \frac{dM}{dx} = V$$
$$M = \Sigma(V)dx$$

Determining Maximum Bending Stress

For a prismatic member (constant cross section), the maximum normal stress will occur at the maximum moment.

For a *non-prismatic* member, the stress varies with the cross section AND the moment.

Deflections

If the bending moment changes, M(x) across a beam of constant material and cross section then the curvature will change:

$$\frac{1}{R} = \frac{M(x)}{EI}$$

The slope of the n.a. of a beam, θ, will be tangent to the radius of curvature, R:

$$\theta = slope = \frac{1}{EI}\int M(x)dx$$

The equation for deflection, y, along a beam is:

$$y = \frac{1}{EI}\int \theta dx = \frac{1}{EI}\iint M(x)dx$$

Elastic curve equations can be found in handbooks, textbooks, design manuals, etc...Computer programs can be used as well. Elastic curve equations can be superimposed ONLY if the stresses are in the elastic range.

The deflected shape is roughly the same shape flipped as the bending moment diagram but is constrained by supports and geometry.

Allowable Deflection Limits

All building codes and design codes limit deflection for beam types and damage that could happen based on service condition and severity.

$$y_{\max}(x) = \Delta_{actual} \le \Delta_{allowable} = L/value$$

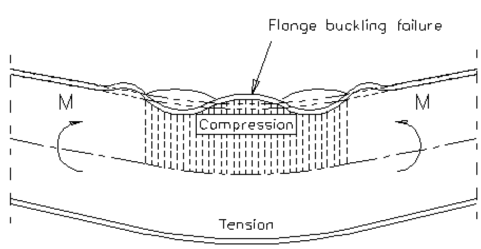
Use	LL only	DL+LL
Roof beams:		
Industrial	L/180	L/120
Commercial		
plaster ceiling	L/240	L/180
no plaster	L/360	L/240
Floor beams:		
Ordinary Usage	L/360	L/240
Roof or floor (damageable elements)		L/480

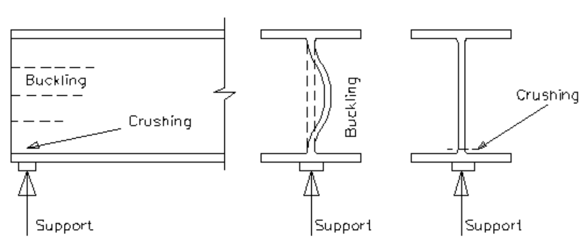
Lateral Buckling

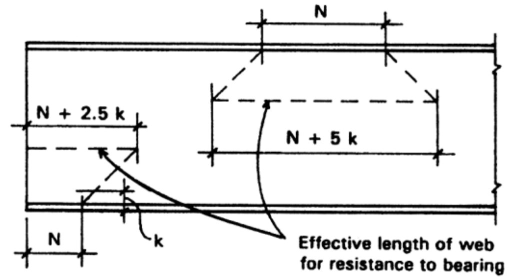
With compression stresses in the top of a beam, a sudden "popping" or buckling can happen even at low stresses. In order to prevent it, we need to brace it along the top, or laterally brace it, or provide a bigger I_y.

Local Buckling in Steel Wide-flange Beams– Web Crippling or Flange Buckling

Concentrated forces on a steel beam can cause the web to buckle (called **web crippling**). Web stiffeners under the beam loads and bearing plates at the supports reduce that tendency. Web stiffeners also prevent the web from shearing in plate girders.







The maximum support load and interior load can be determined from:

$$P_{n(\text{max}-\text{end})} = (2.5k + N)F_{yw}t_w$$

$$P_{n(\text{interior})} = (5k + N)F_{yw}t_w$$

where t_w = thickness of the web
F_{yw} = yield strength of the web
N = bearing length
k = dimension to fillet found in beam section tables

ϕ = 1.00 (LRFD) Ω = 1.50 (ASD)

Beam Loads & Load Tracing

In order to determine the loads on a beam (or girder, joist, column, frame, foundation...) we can start at the top of a structure and determine the *tributary area* that a load acts over and the beam needs to support. Loads come from material weights, people, and the environment. This area is assumed to be from half the distance to the next beam over to halfway to the next beam.

The reactions must be supported by the next lower structural element *ad infinitum*, to the ground.

LRFD - Bending or Flexure

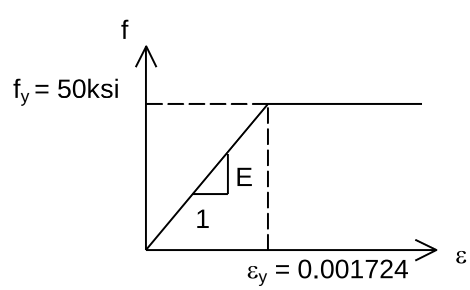
For determining the flexural design strength, $\phi_b M_n$, for resistance to pure bending (no axial load) in most flexural members where the following conditions exist, a single calculation will suffice:

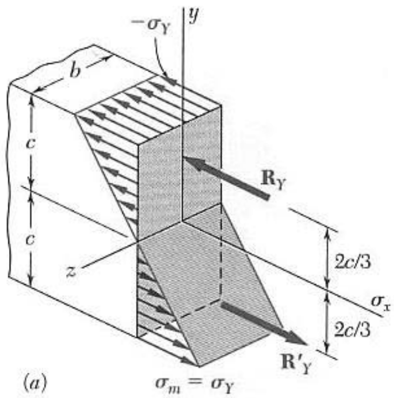
$$\Sigma\gamma_i R_i = M_u \le \phi_b M_n = 0.9F_y Z$$

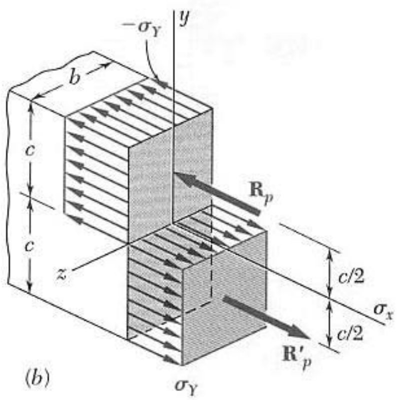
where M_u = maximum moment from factored loads
ϕ_b = resistance factor for bending = 0.9
M_n = nominal moment (ultimate capacity)
F_y = yield strength of the steel
Z = plastic section modulus

Plastic Section Modulus

Plastic behavior is characterized by a yield point and an increase in strain with no increase in stress.







Internal Moments and Plastic Hinges

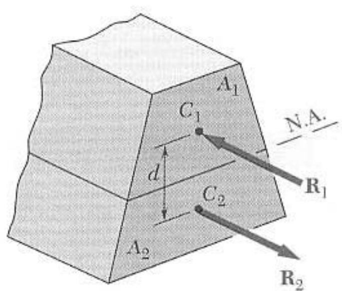
Plastic hinges can develop when all of the material in a cross section sees the yield stress. Because all the material at that section can strain without any additional load, the member segments on either side of the hinge can rotate, possibly causing instability.

For a rectangular section:

Elastic to f_y: $$M_y = \frac{I}{c} f_y = \frac{bh^2}{6} f_y = \frac{b(2c)^2}{6} f_y = \frac{2bc^2}{3} f_y$$

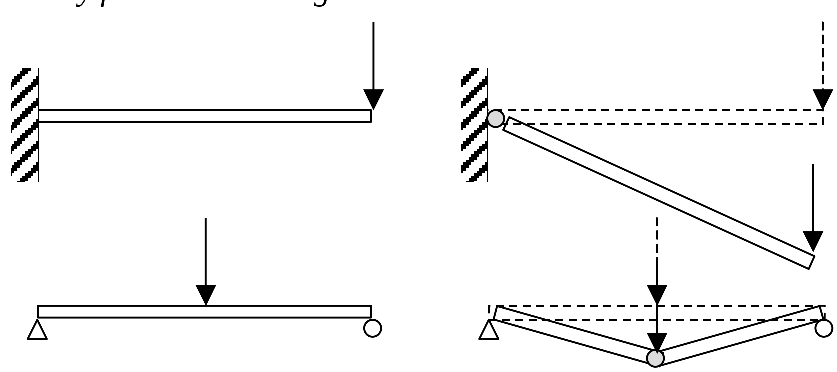
Fully Plastic: $$M_{ult} \text{ or } M_p = bc^2 f_y = \tfrac{3}{2} M_y$$

For a non-rectangular section and internal equilibrium at σ_y, the n.a. *will not necessarily be at the centroid.* The n.a. occurs where the $A_{tension} = A_{compression}$. The reactions occur at the centroids of the tension and compression areas.



$A_{tension} = A_{compression}$

Instability from Plastic Hinges



Shape Factor:

The ratio of the plastic moment to the elastic moment at yield:

$$k = M_p / M_y$$

k = 3/2 for a rectangle
k ≈ 1.1 for an I beam

Plastic Section Modulus

$$Z = \frac{M_p}{f_y} \qquad \text{and} \qquad k = Z/S$$

Design for Shear

$$V_a \leq V_n / \Omega \text{ or } V_u \leq \phi_v V_n$$

The nominal shear strength is dependent on the cross section shape. Case 1: With a thick or stiff web, the shear stress is resisted by the web of a wide flange shape (with the exception of a handful of W's). Case 2: When the web is not stiff for doubly symmetric shapes, singly symmetric shapes (like channels) (excluding round high strength steel shapes), inelastic web buckling occurs. When the web is very slender, elastic web buckling occurs, reducing the capacity even more:

Case 1) For $h/t_w \leq 2.24\sqrt{\frac{E}{F_y}}$ $\quad V_n = 0.6F_{yw}A_w$ $\quad \phi_v = 1.00$ (LRFD) $\quad \Omega = 1.50$ (ASD)

where h equals the clear distance between flanges less the fillet or corner radius for rolled shapes
V_n = nominal shear strength
F_{yw} = yield strength of the steel in the web
$A_w = t_w d$ = area of the web

Case 2) For $h/t_w > 2.24\sqrt{\frac{E}{F_y}}$ $\quad V_n = 0.6F_{yw}A_w C_v$ $\quad \phi_v = 0.9$ (LRFD) $\quad \Omega = 1.67$ (ASD)

where C_v is a reduction factor (1.0 or less by equation)

Design for Flexure

$M_a \leq M_n / \Omega$ or $M_u \leq \phi_b M_n$ $\qquad \phi_b = 0.90$ (LRFD) $\qquad \Omega = 1.67$ (ASD)

The nominal flexural strength M_n is the *lowest* value obtained according to the limit states of

1. yielding, limited at length $L_p = 1.76 r_y \sqrt{\frac{E}{F_y}}$, where r_y is the radius of gyration in y
2. lateral-torsional buckling limited at length L_r
3. flange local buckling
4. web local buckling

Beam design charts show available moment, M_n/Ω and $\phi_b M_n$, for unbraced length, L_b, of the compression flange in one-foot increments from 1 to 50 ft. for values of the bending coefficient $C_b = 1$. For values of $1<C_b\leq 2.3$, the required flexural strength M_u can be reduced by dividing it by C_b. ($C_b = 1$ when the bending moment at any point within an unbraced length is larger than that at both ends of the length. C_b of 1 is conservative and permitted to be used in any case. When the free end is unbraced in a cantilever or overhang, $C_b = 1$. The full formula is provided below.)

NOTE: the self weight <u>is not</u> included in determination of M_n/Ω $\phi_b M_n$

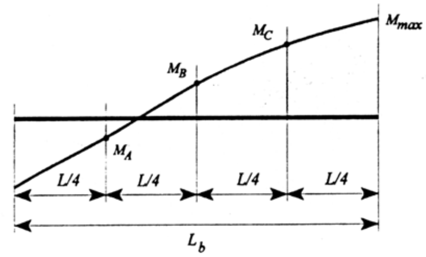
Compact Sections

For a laterally braced *compact* section (one for which the plastic moment can be reached before local buckling) only the limit state of yielding is applicable. For unbraced compact beams and non-compact tees and double angles, only the limit states of yielding and lateral-torsional buckling are applicable.

Compact sections meet the following criteria: $\frac{b_f}{2t_f} \le 0.38\sqrt{\frac{E}{F_y}}$ and $\frac{h_c}{t_w} \le 3.76\sqrt{\frac{E}{F_y}}$

where:

b_f = flange width in inches
t_f = flange thickness in inches
E = modulus of elasticity in ksi
F_y = minimum yield stress in ksi
h_c = height of the web in inches
t_w = web thickness in inches



With lateral-torsional buckling the nominal flexural strength is

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

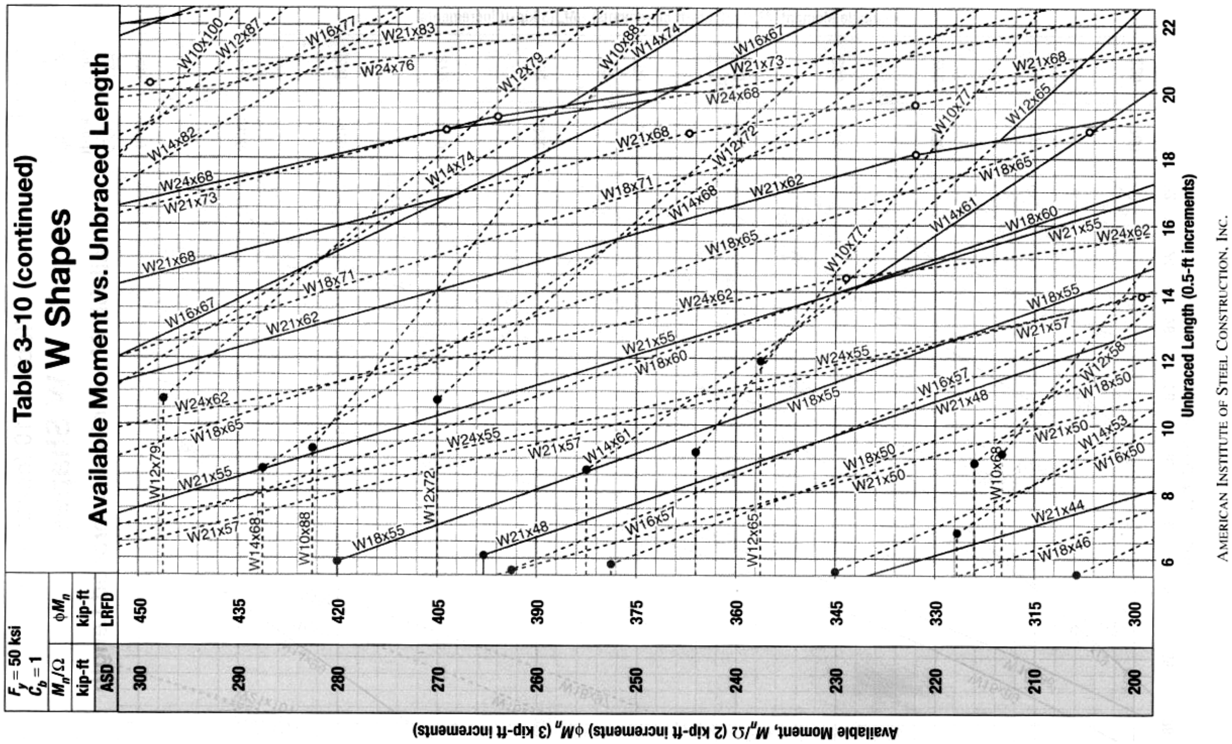
where $M_p = M_n = F_yZ_x$
and C_b is a modification factor for non-uniform moment diagrams where, when both ends of the beam segment are braced:

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

M_{max} = absolute value of the maximum moment in the unbraced beam segment
M_A = absolute value of the moment at the quarter point of the unbraced beam segment
M_B = absolute value of the moment at the center point of the unbraced beam segment
M_C = absolute value of the moment at the three quarter point of the unbraced beam segment length.

Available Flexural Strength Plots

Plots of the available moment for the unbraced length for wide flange sections are useful to find sections to satisfy the design criteria of $M_a \le M_n/\Omega$ *or* $M_u \le \phi_b M_n$. The maximum moment that can be applied on a beam (taking self weight into account), M_a or M_u, can be plotted against the unbraced length, L_b. The limiting length, L_p (fully plastic), is indicated by a solid dot (•), while the limiting length, L_r (for lateral torsional buckling), is indicated by an open dot (○). Solid lines indicate the most economical, while dashed lines indicate there is a lighter section that could be used. C_b, which is a lateral torsional buckling modification factor for non-zero moments at the ends, is 1 for simply supported beams (0 moments at the ends). (see *figure)*



<u>Design Procedure</u>

The intent is to find the most light weight member (which is economical) satisfying the section modulus size.

1. Determine the unbraced length to choose the limit state (yielding, lateral torsional buckling or more extreme) and the factor of safety and limiting moments. Determine the material.
2. Draw V & M, finding V_{max} and M_{max}.for unfactored loads (ASD, V_a & M_a) or from factored loads (LRFD, V_u & M_u)
3. Calculate $Z_{req'd}$ when yielding is the limit state. This step is equivalent to determining if $f_b = \frac{M_{max}}{S} \le F_b$, $Z_{req'd} \ge \frac{M_{max}}{F_b} = \frac{M_{max}}{F_y/\Omega}$ and $Z_{req'd} \ge \frac{M_u}{\phi_b F_y}$ to meet the design criteria that

   $$M_a \le M_n/\Omega \text{ or } M_u \le \phi_b M_n$$

   If the limit state *is something other than yielding*, determine the nominal moment, M_n, or use plots of available moment to unbraced length, L_b.

4. For steel: use the section charts to find a trial Z *and remember that the beam self weight (the second number in the section designation) will increase $Z_{req'd.}$* The design charts show the lightest section within a grouping of similar Z's.

TABLE 9.1 Load Factor Resistance Design Selection

		F_y = 36 ksi			
Designation	Z_x in.^{3}	L_p ft	L_r ft	M_p kip-ft	M_r kip-ft
W 33 × 141	**514**	**10.1**	**30.1**	**1,542**	**971**
W 30 × 148	500	9.50	30.6	1,500	945
W 24 × 162	468	12.7	45.2	1,404	897
W 24 × 146	418	12.5	42.0	1,254	804
W 33 × 118	**415**	**9.67**	**27.8**	**1,245**	**778**
W 30 × 124	408	9.29	28.2	1,224	769
W 21 × 147	373	12.3	46.4	1,119	713
W 24 × 131	370	12.4	39.3	1,110	713
W 18 × 158	356	11.4	56.5	1,068	672

**** *Determine the "updated" V_{max} and M_{max} including the beam self weight, and verify that the updated $Z_{req'd}$ has been met.* ******

5. Consider lateral stability.

6. Evaluate horizontal shear using V_{max}. This step is equivalent to determining if $f_v \le F_v$ is satisfied to meet the design criteria that $V_a \le V_n / \Omega$ or $V_u \le \phi_v V_n$

   For I beams: $$f_{v-\max} = \frac{3V}{2A} \approx \frac{V}{A_{web}} = \frac{V}{t_w d} \qquad V_n = 0.6F_{yw}A_w \quad or\ V_n = 0.6F_{yw}A_w C_v$$

   Others: $$f_{v-max} = \frac{VQ}{Ib}$$

7. Provide adequate bearing area at supports. This step is equivalent to determining if $f_p = \frac{P}{A} \le F_p$ is satisfied to meet the design criteria that $P_a \le P_n / \Omega$ or $P_u \le \phi P_n$

8. Evaluate shear due to torsion $\quad f_v = \frac{T\rho}{J}\ or\ \frac{T}{c_1 ab^2} \le F_v$ (circular section or rectangular)

9. Evaluate the deflection to determine if $\Delta_{maxLL} \le \Delta_{LL-allowed}$ and/or $\Delta_{maxTotal} \le \Delta_{Total\,allowed}$

**** *note: when $\Delta_{calculated} > \Delta_{limit}$, $I_{req'd}$ can be found with: and $Z_{req'd}$ will be satisfied for similar self weight* *****
$$I_{req'd} \ge \frac{\Delta_{too\,big}}{\Delta_{limit}} I_{trial}$$

FOR ANY EVALUATION:

Redesign (with a new section) at any point that a stress or serviceability criteria is NOT satisfied and re-evaluate each condition until it is satisfactory.

Load Tables for Uniformly Loaded Joists & Beams

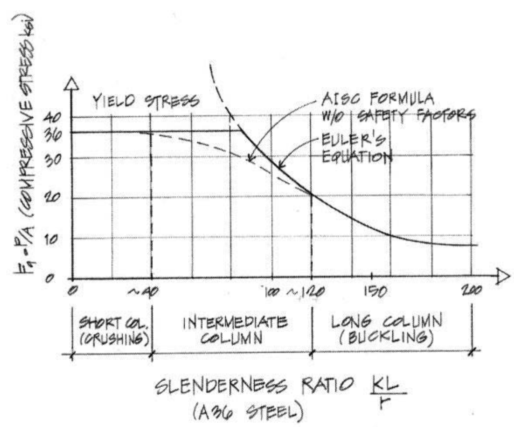
Tables exist for the common loading situation of uniformly distributed load. The tables either provide the safe distributed load based on bending and deflection limits, they give the allowable span for specific live and dead loads including live load deflection limits.
If the load is *not uniform*, an *equivalent uniform load* can be calculated from the maximum moment equation:
$$M_{max} = \frac{w_{equivalent} L^2}{8}$$

If the deflection limit is less, the design live load to check against allowable must be increased, ex.
$$w_{adjusted} = w_{ll-have}\left(\frac{L/360}{L/400}\right) \begin{matrix} \textit{table limit} \\ \textit{wanted} \end{matrix}$$

Criteria for Design of Columns

If we know the loads, we can select a section that is adequate for strength & buckling.

If we know the length, we can find the limiting load satisfying strength & buckling.



Allowable Stress Design

American Institute of Steel Construction (AISC) Manual of ASD, 9th ed:

<u>Long and slender:</u> [$L_e/r \geq C_c$, preferably < 200]

$$F_{allowable} = \frac{F_{cr}}{F.S.} = \frac{12\pi^2 E}{23\left(\frac{Kl}{r}\right)^2}$$

The yield limit is idealized into a parabolic curve that blends into the Euler's Formula at C_c.

With F_y = 36 ksi, C_c = 126.1

With F_y = 50 ksi, C_c = 107.0

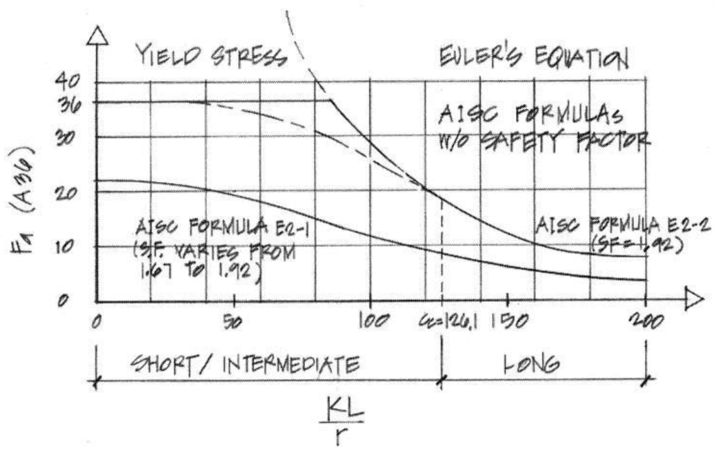
$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$

<u>Short and stubby:</u> [$L_e/r < C_c$]

$$F_a = \left[1 - \frac{\left(\frac{Kl}{r}\right)^2}{2C_c^2}\right]\frac{F_y}{F.S.}$$

with:

$$F.S. = \frac{5}{3} + \frac{3\left(\frac{Kl}{r}\right)}{8C_c} - \frac{\left(\frac{Kl}{r}\right)^3}{8C_c^3}$$



Design for Compression

American Institute of Steel Construction (AISC) Manual 14th ed:

$P_a \leq P_n / \Omega$ or $P_u \leq \phi_c P_n$ where $P_u = \Sigma \gamma_i P_i$

γ is a <u>load factor</u>
P is a <u>load</u> type
ϕ is a <u>resistance factor</u>
P_n is the <u>nominal load capacity (strength)</u>

ϕ = 0.90 (LRFD) Ω = 1.67 (ASD)

For compression $P_n = F_{cr} A_g$

where : A_g is the cross section area and F_{cr} is the flexural buckling stress

The flexural buckling stress, F_{cr}, is determined as follows:

when $\frac{KL}{r} \le 4.71\sqrt{\frac{E}{F_y}}$ or ($F_e \ge 0.44F_y$):

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}} \right] F_y$$

when $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}$ or ($F_e < 0.44F_y$):

$$F_{cr} = 0.877F_e$$

where F_e is the elastic critical buckling stress: $$F_e = \frac{\pi^2 E}{\left(KL/r\right)^2}$$

Design Aids

Tables exist for the value of the flexural buckling stress based on slenderness ratio. In addition, tables are provided in the AISC Manual for Available Strength in Axial Compression based on the effective length with respect to least radius of gyration, r_y. If the critical effective length is about the largest radius of gyration, r_x, it can be turned into an effective length about the y axis by dividing by the fraction r_x/r_y.

Sample AISC Table for Available Strength in Axial Compression

Table 4-1 (continued) Available Strength in Axial Compression, kips — W Shapes

F_y = 50 ksi — W12

Shape	W12×									
Wt/ft	96		87		79		72		65	
Design	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
Effective length KL (ft) with respect to least radius of gyration r_y										
0	844	1270	766	1150	694	1040	633	951	571	859
6	811	1220	735	1110	667	1000	607	913	548	824
7	800	1200	725	1090	657	987	598	899	540	811
8	787	1180	713	1070	646	971	588	884	531	798
9	772	1160	699	1050	634	952	577	867	520	782
10	756	1140	685	1030	620	932	565	849	509	765
11	739	1110	669	1010	606	910	551	828	497	747
12	720	1080	652	980	590	887	537	807	484	727
13	701	1050	634	953	573	862	522	784	470	706
14	680	1020	615	924	556	836	506	761	456	685
15	659	990	595	895	538	809	490	736	441	662
16	637	957	575	864	520	781	473	710	425	639
17	614	923	554	833	501	752	455	684	409	615
18	591	888	533	801	481	723	437	657	393	591
19	567	852	511	769	461	694	419	630	377	566
20	543	816	490	736	442	664	401	603	360	541
22	495	744	446	670	402	603	365	548	327	491
24	447	672	402	605	362	544	328	493	294	442
26	401	602	360	541	323	486	293	440	262	393
28	356	534	319	479	286	430	259	389	231	347
30	312	469	279	420	250	376	226	340	202	303
32	274	412	246	369	220	331	199	299	177	267
34	243	365	218	327	195	293	176	265	157	236
36	217	326	194	292	174	261	157	236	140	211
38	195	292	174	262	156	234	141	212	126	189
40	176	264	157	236	141	212	127	191	114	171
Properties										
P_{wo} (kips)	137	206	121	181	104	157	90.9	136	78.2	117
P_{wi} (kips/in.)	18.3	27.5	17.2	25.8	15.7	23.5	14.3	21.5	13.0	19.5
P_{wb} (kips)	296	445	243	366	185	278	142	213	106	159
P_{fb} (kips)	152	228	123	185	101	152	84.0	126	68.5	103
L_p (ft)	10.9		10.8		10.8		10.7		11.9	
L_r (ft)	46.6		43.0		39.9		37.4		35.1	
A_g (in.^{2})	28.2		25.6		23.2		21.1		19.1	
I_x (in.^{4})	833		740		662		597		533	
I_y (in.^{4})	270		241		216		195		174	
r_y (in.)	3.09		3.07		3.05		3.04		3.02	
Ratio r_x/r_y	1.76		1.75		1.75		1.75		1.75	
$P_{ex}(KL^2)/10^4$ (k-in.^{2})	23800		21200		18900		17100		15300	
$P_{ey}(KL^2)/10^4$ (k-in.^{2})	7730		6900		6180		5580		4980	

ASD	LRFD
$\Omega_c = 1.67$	$\phi_c = 0.90$

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Procedure for Analysis

1. Calculate KL/r for each axis (if necessary). The largest will govern the buckling load.
2. Find F_a ***or*** F_{cr} as a function of KL/r from the appropriate equation (above) or table.
3. Compute $P_{allowable} = F_a \cdot A$ ***or*** $P_n = F_{cr} \cdot A_g$

   or alternatively compute $f_c = P/A$ or P_u/A
4. Is the design satisfactory?

   Is $P \le P_{allowable}$ *(or $P_a \le P_n/\Omega$)* or $P_u \le \phi_c P_n$? $\Rightarrow$ yes, it is; no, it is no good

   or Is $f_c \le F_a$ *(or $\le F_{cr}/\Omega$)* or $\phi_c F_{cr}$? $\Rightarrow$ yes, it is; no, it is no good

Procedure for Design

1. Guess a size by picking a section.
2. Calculate KL/r for each axis (if necessary). The largest will govern the buckling load.
3. Find F_a ***or*** F_{cr} as a function of KL/r from appropriate equation (above) or table.
4. Compute $P_{allowable} = F_a \cdot A$ ***or*** $P_n = F_{cr} \cdot A_g$

   or alternatively compute $f_c = P/A$ or P_u/A
5. Is the design satisfactory?

   Is $P \le P_{allowable}$ *($P_a \le P_n/\Omega$)* or $P_u \le \phi_c P_n$? yes, it is; *no, pick a bigger section and go back to step 2.*

   Is $f_c \le F_a$ *($\le F_{cr}/\Omega$)* or $\phi_c F_{cr}$? $\Rightarrow$ yes, it is; *no, pick a bigger section and go back to step 2.*
6. Check design efficiency by calculating percentage of stress used:

$$\frac{P}{P_{allowable}} \cdot 100\% \left(\frac{P_a}{P_n/\Omega} \cdot 100\% \right) or \ \frac{P_u}{\phi_c P_n} \cdot 100\%$$

   If value is between 90-100%, it is efficient.

   If values is less than 90%, *pick a smaller section and go back to step 2.*

Columns with Bending (Beam-Columns)

In order to *design* an adequate section for allowable stress, we have to start somewhere:

1. Make assumptions about the limiting stress from:
   - buckling
   - axial stress
   - combined stress

2. See if we can find values for r or A or Z.

3. Pick a trial section based on if we think r or A is going to govern the section size.

4. Analyze the stresses and compare to allowable using the allowable stress method or interaction formula for eccentric columns.

5. Did the section pass the stress test?
   - If not, do you *increase* r or A or Z?
   - If so, is the difference really big so that you could *decrease* r or A or Z to make it more efficient (economical)?

6. Change the section choice and go back to step 4. Repeat until the section meets the stress criteria.

Design for Combined Compression and Flexure:

The interaction of compression and bending are included in the form for two conditions based on the size of the required axial force to the available axial strength. This is notated as P_r (either P from ASD or P_u from LRFD) for the axial force being supported, and P_c (either P_n/Ω for ASD or $\phi_c P_n$ for LRFD). The increased bending moment due to the P-Δ effect must be determined and used as the moment to resist.

For $\frac{P_r}{P_c} \geq 0.2$:

$$\frac{P}{P_n/\Omega} + \frac{8}{9}\left(\frac{M_x}{M_{nx}/\Omega} + \frac{M_y}{M_{ny}/\Omega}\right) \leq 1.0 \quad \text{(ASD)}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0 \quad \text{(LRFD)}$$

For $\frac{P_r}{P_c} < 0.2$:

$$\frac{P}{2P_n/\Omega} + \left(\frac{M_x}{M_{nx}/\Omega} + \frac{M_y}{M_{ny}/\Omega}\right) \leq 1.0 \quad \text{(ASD)}$$

$$\frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0 \quad \text{(LRFD)}$$

where:

for compression	$\phi_c = 0.90$ (LRFD)	$\Omega = 1.67$ (ASD)
for bending	$\phi_b = 0.90$ (LRFD)	$\Omega = 1.67$ (ASD)

For a <u>braced</u> condition, the moment magnification factor B_1 is determined by $B_1 = \frac{C_m}{1-\alpha(P_u/P_{e1})} \geq 1.0$

where C_m is a modification factor accounting for end conditions

When not subject to transverse loading between supports in plane of bending:
= 0.6 – 0.4 (M_1/M_2) where M_1 and M_2 are the end moments and $M_1<M_2$. M_1/M_2 is positive when the member is bent in reverse curvature (same direction), negative when bent in single curvature.

When there is transverse loading between the two ends of a member:
= 0.85, members with restrained (fixed) ends
= 1.00, members with unrestrained ends

$$P_{e1} = \frac{\pi^2 EA}{\left(Kl/r\right)^2}$$

α = 1.00 (LRFD), 1.60 (ASD)

P_{e1} = Euler buckling strength

Criteria for Design of Connections

Connections must be able to transfer any axial force, shear, or moment from member to member or from beam to column.

Connections for steel are typically high strength bolts and electric arc welds. Recommended practice for ease of construction is to specified *shop welding* and *field bolting.*

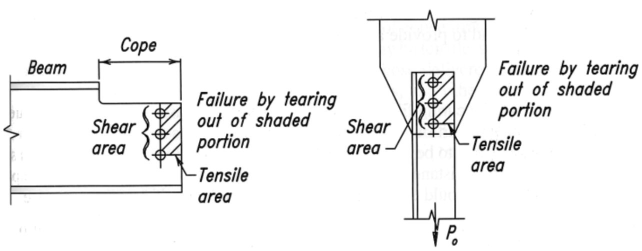


Fig. C-J4.1. Failure for block shear rupture limit state.

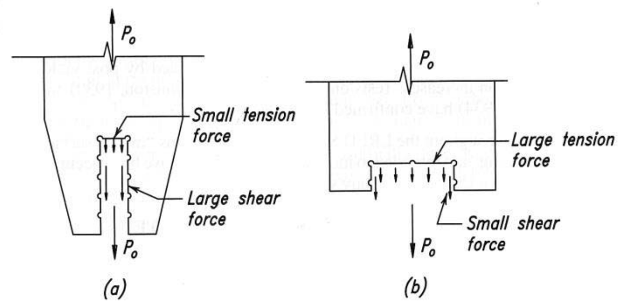
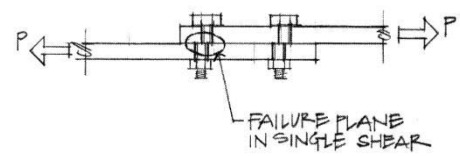


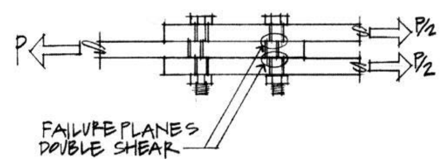
Fig. C-J4.2. Block shear rupture in tension.



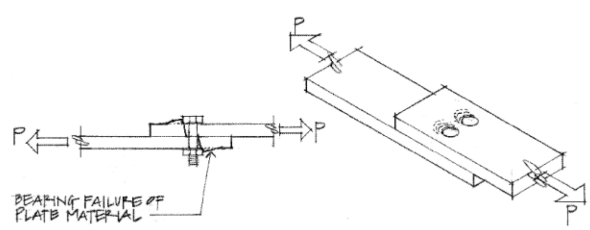
<u>Bolted and Welded Connections</u>

The limit state for connections depends on the loads:

1. tension yielding
2. shear yielding
3. bearing yielding
4. bending yielding due to eccentric loads
5. rupture



Welds must resist shear stress. The design strengths depend on the weld materials.



<u>Bolted Connection Design</u>

Bolt designations signify material and type of connection where

SC: slip critical
N: bearing-type connection with bolt threads *included* in shear plane
X: bearing-type connection with bolt threads *excluded* from shear plane

A307: similar in strength to A36 steel (also known as ordinary, common or unfinished bolts)
A325: high strength bolts (Group A)
A490: high strength bolts (higher than A325) (Group B)

Bearing-type connection: no frictional resistance in the contact surfaces is assumed and slip between members occurs as the load is applied. (Load transfer through bolt only).

Slip-critical connections: bolts are torqued to a high tensile stress in the shank, resulting in a clamping force on the connected parts. (Shear resisted by clamping force). Requires inspections and is useful for structures seeing dynamic or fatigue loading. Class A indicates the *faying* (contact) surfaces are clean mill scale or adequate paint system, while Class B indicates blast cleaning or paint for $\mu = 0.50$.

Bolts rarely fail in **bearing**. The material with the hole will more likely yield first.

For the determination of the net area of a bolt hole the width is taken as *1/16" greater* than the nominal dimension of the hole. Standard diameters for bolt holes are *1/16"* larger than the bolt diameter. (This means the net width will be *1/8"* larger than the bolt.)

Design for Bolts in Bearing, Shear and Tension

Available shear values are given by bolt type, diameter, and loading (Single or Double shear) in AISC manual tables. Available shear value for slip-critical connections are given for limit states of serviceability or strength by bolt type, hole type (standard, short-slotted, long-slotted or oversized), diameter, and loading. Available tension values are given by bolt type and diameter in AISC manual tables.

Available bearing force values are given by bolt diameter, ultimate tensile strength, F_u, of the connected part, and thickness of the connected part in AISC manual tables.

For shear OR tension (same equation) in bolts: $R_a \le R_n / \Omega$ or $R_u \le \phi R_n$

where $R_u = \Sigma \gamma_i R_i$

- single shear (or tension) $R_n = F_n A_b$
- double shear $R_n = F_n 2A_b$

where ϕ = the resistance factor
F_n = the nominal tension or shear strength of the bolt
A_b = the cross section area of the bolt

$\phi = 0.75$ (LRFD) $\Omega = 2.00$ (ASD)

For bearing of plate material at bolt holes: $R_a \le R_n / \Omega$ or $R_u \le \phi R_n$

where $R_u = \Sigma \gamma_i R_i$

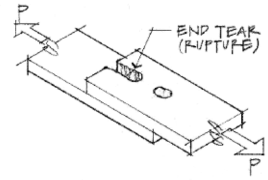


Figure 10.11 End tear-out.

- deformation at bolt hole is a concern

$$R_n = 1.2 L_c t F_u \le 2.4 dt F_u$$

- deformation at bolt hole is not a concern

$$R_n = 1.5 L_c t F_u \le 3.0 dt F_u$$

- long slotted holes with the slot perpendicular to the load

$$R_n = 1.0 L_c t F_u \le 2.0 dt F_u$$

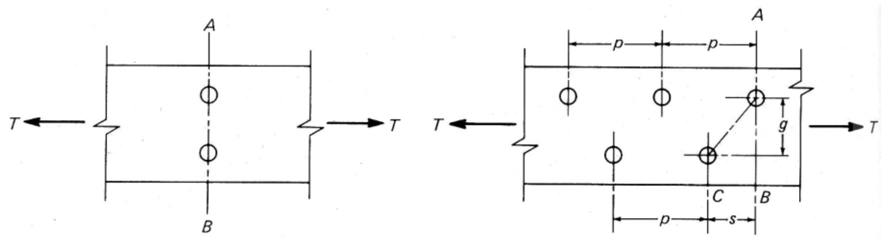
where R_n = the nominal bearing strength
F_u = specified minimum tensile strength
L_c = clear distance between the edges of the hole and the next hole or edge in the direction of the load
d = nominal bolt diameter
t = thickness of connected material

$\phi = 0.75$ (LRFD) $\Omega = 2.00$ (ASD)

The *minimum* edge desistance from the center of the outer most bolt to the edge of a member is generally 1¾ times the bolt diameter for the sheared edge and 1¼ times the bolt diameter for the rolled or gas cut edges.

The *maximum* edge distance should not exceed 12 times the thickness of thinner member or 6 in.

Standard bolt hole spacing is 3 in. with the minimum spacing of $2\frac{2}{3}$ times the diameter of the bolt, d_b. Common edge distance from the center of last hole to the edge is 1¼ in..



Tension Member Design

In steel tension members, there may be bolt holes that reduce the size of the cross section.

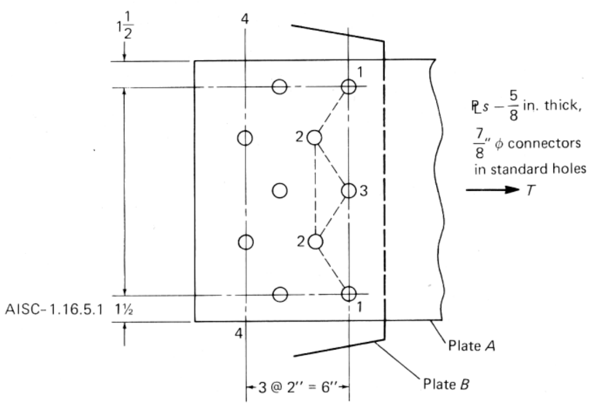
g refers to the row spacing or *gage*
p refers to the bolt spacing or *pitch*
s refers to the longitudinal spacing of two consecutive holes

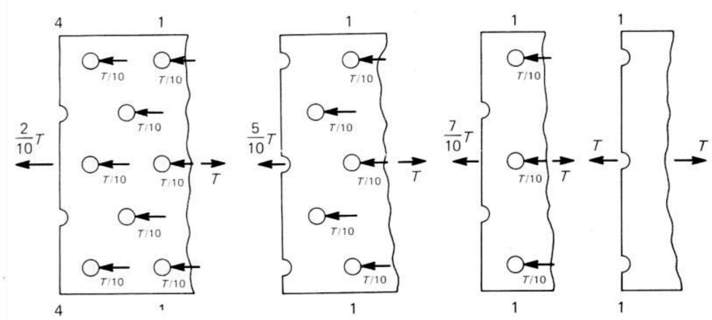
Effective Net Area:

The smallest effective are must be determined by subtracting the bolt hole areas. With staggered holes, the shortest length must be evaluated.

A series of bolts can also transfer a portion of the tensile force, and some of the effective net areas see reduced stress.

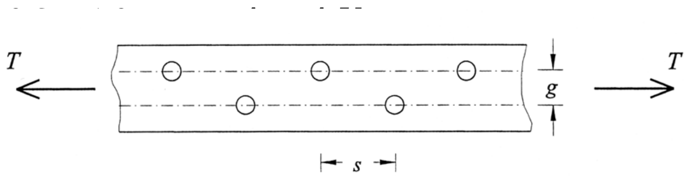
The effective net area, A_e, is determined from the net area, A_n, multiplied by a shear lag factor, U, which depends on the element type and connection configuration. If a portion of a connected member is not fully connected (like the leg of an angle), the unconnected part is not subject to the





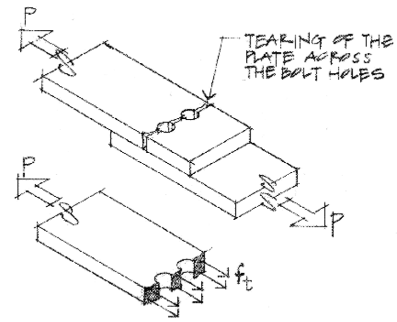
full stress and the shear lag factor can range fro

The staggered hole path area is determined by:



$$A_n = A_g - A_{of\ all\ holes} + t\Sigma\frac{s^2}{4g}$$

where t is the plate thickness, s is each stagger spacing, and g is the gage spacing.



For tension elements: $R_a \le R_n/\Omega$ or $R_u \le \phi R_n$

where $R_u = \Sigma\gamma_i R_i$

1. yielding $R_n = F_y A_g$

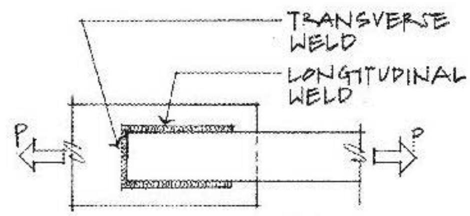
   ϕ = 0.90 (LRFD) Ω = 1.67 (ASD)

2. rupture $R_n = F_u A_e$

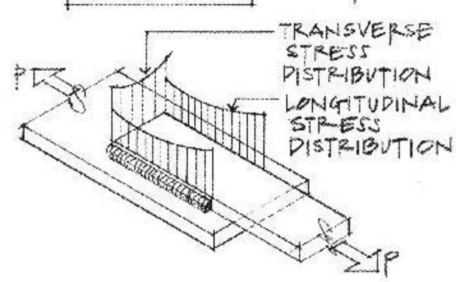
   ϕ = 0.75 (LRFD) Ω = 2.00 (ASD)

where A_g = the gross area of the member (excluding holes)
A_e = the effective net area (with holes, etc.)
F_y = the yield strength of the steel
F_u = the tensile strength of the steel (ultimate)

Welded Connections



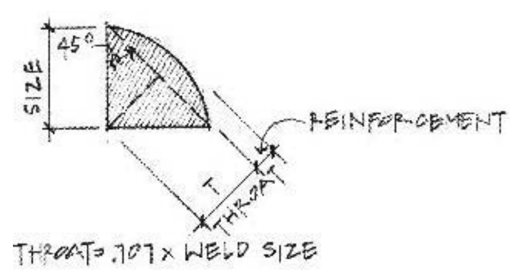
Weld designations include the strength in the name, i.e. E70XX has Fy = 70 ksi. Welds are weakest in shear and are assumed to always fail in the shear mode.



The throat size, T, of a fillet weld is determined trigonometry by: T = 0.707 ×weld size*

* When the submerged arc weld process is used, welds over 3/8" will have a throat thickness of 0.11 in. larger than the formula.

Weld sizes are limited by the size of the parts being put together and are given in AISC manual table J2.4 along with the allowable strength per length of fillet weld, referred to as *S*.



The *maximum* size of a fillet weld:

a) can't be greater than the material thickness if it is ¼" or less
b) is permitted to be 1/16" less than the thickness of the material if it is over ¼"

The *minimum length* of a fillet weld is 4 times the nominal size. If it is not, then the weld size used for design is ¼ the length.

Intermittent fillet welds cannot be less than four times the weld size, not to be less than 1 ½".

TABLE J2.4
Minimum Size of Fillet Welds

Material Thickness of Thicker Part Joined (in.)	Minimum Size of Fillet Weld[a] (in.)
To ¼ inclusive	⅛
Over ¼ to ½	3/16
Over ½ to ¾	¼
Over ¾	5/16

[a] Leg dimension of fillet welds. Single-pass welds must be used.

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<u>*For fillet welds:*</u> $R_a \le R_n / \Omega$ or $R_u \le \phi R_n$

where $R_u = \Sigma \gamma_i R_i$

for the weld metal: $R_n = 0.6 F_{EXX} Tl = Sl$

$\phi = 0.75$ (LRFD) $\Omega = 2.00$ (ASD)

where:

T is throat thickness
l is length of the weld

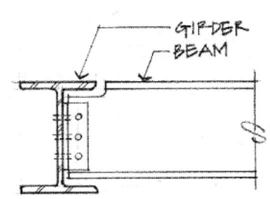
For a connected part, the other limit states for the base metal, such as tension yield, tension rupture, shear yield, or shear rupture **must** be considered.

Available Strength of Fillet Welds per inch of weld (ϕS)

Weld Size (in.)	E60XX (k/in.)	E70XX (k/in.)
3/16	3.58	4.18
¼	4.77	5.57
5/16	5.97	6.96
3/8	7.16	8.35
7/16	8.35	9.74
½	9.55	11.14
5/8	11.93	13.92
¾	14.32	16.70

(not considering increase in throat with submerged arc weld process)

Framed Beam Connections

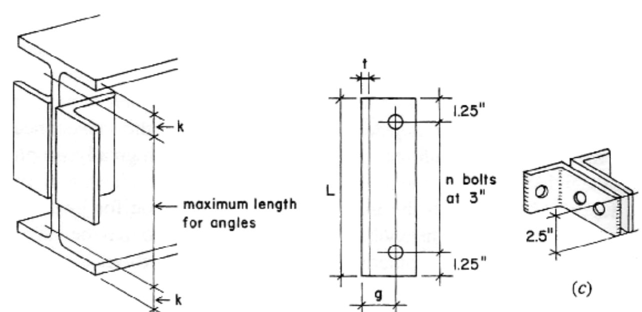


Coping is the term for cutting away part of the flange to connect a beam to another beam using welded or bolted angles.

AISC provides tables that give bolt and angle available strength knowing number of bolts, bolt type, bolt diameter, angle leg thickness, hole type and coping, *and* the wide flange beam being connected. For the connections the limit-state of bolt shear, bolts bearing on the angles, shear yielding of the angles, shear rupture of the angles, and block shear rupture of the angles, and bolt bearing on the beam web are considered.

Group A bolts include A325, while Group B includes A490.

There are also tables for bolted/welded double-angle connections and all-welded double-angle connections.



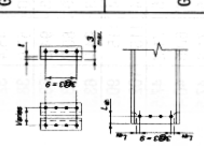
(a) (b) (c)

Sample AISC Table for Bolt and Angle Available Strength in All-Bolted Double-Angle Connections

Table 10-1 (continued)
All-Bolted Double-Angle Connections — **3/4-in. Bolts**

Beam: F_y = 50 ksi, F_u = 65 ksi
Angle: F_y = 36 ksi, F_u = 58 ksi

4 Rows — W24, 21, 18, 16



Bolt and Angle Available Strength, kips

Bolt Group	Thread Cond.	Hole Type	Angle Thickness 1/4 in. ASD	1/4 LRFD	5/16 ASD	5/16 LRFD	3/8 ASD	3/8 LRFD	1/2 ASD	1/2 LRFD
Group A	N	STD	67.1	101	83.9	126	95.5	143	95.5	143
	X	STD	67.1	101	83.9	126	101	151	120	180
	SC Class A	STD	50.6	75.9	50.6	75.9	50.6	75.9	50.6	75.9
		OVS	43.1	64.5	43.1	64.5	43.1	64.5	43.1	64.5
		SSLT	50.6	75.9	50.6	75.9	50.6	75.9	50.6	75.9
	SC Class B	STD	67.1	101	83.9	126	84.4	127	84.4	127
		OVS	65.3	97.9	71.9	108	71.9	108	71.9	108
		SSLT	65.8	98.7	82.2	123	84.4	127	84.4	127
Group B	N	STD	67.1	101	83.9	126	101	151	120	180
	X	STD	67.1	101	83.9	126	101	151	134	201
	SC Class A	STD	63.3	94.9	63.3	94.9	63.3	94.9	63.3	94.9
		OVS	53.9	80.7	53.9	80.7	53.9	80.7	53.9	80.7
		SSLT	63.3	94.9	63.3	94.9	63.3	94.9	63.3	94.9
	SC Class B	STD	67.1	101	83.9	126	101	151	105	158
		OVS	65.3	97.9	81.6	122	89.9	134	89.9	134
		SSLT	65.8	98.7	82.2	123	98.7	148	105	158

Beam Web Available Strength per Inch Thickness, kips/in.

Hole Type		STD				OVS				SSLT			
	L_{eh}*, in.	1 1/2		1 3/4		1 1/2		1 3/4		1 1/2		1 3/4	
	L_{ev}, in.	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Coped at Top Flange Only	1 1/4	167	250	175	262	156	234	164	246	164	245	172	257
	1 3/8	169	254	177	266	158	238	167	250	166	249	174	261
	1 1/2	171	257	180	269	161	241	169	254	168	253	177	265
	1 5/8	174	261	182	273	163	245	171	257	171	256	179	268
	2	181	272	189	284	171	256	179	268	178	267	186	279
	3	201	301	209	313	190	285	198	297	198	296	206	309
Coped at Both Flanges	1 1/4	156	234	156	234	146	219	146	219	156	234	156	234
	1 3/8	161	241	161	241	151	227	151	227	161	241	161	241
	1 1/2	166	249	166	249	156	234	156	234	166	249	166	249
	1 5/8	171	256	171	256	161	241	161	241	171	256	171	256
	2	181	272	185	278	171	256	176	263	178	267	185	278
	3	201	301	209	313	190	285	198	297	198	296	206	309
Uncoped		234	351	234	351	234	351	234	351	234	351	234	351

Support Available Strength per Inch Thickness, kips/in.

Hole Type	ASD	LRFD
STD/ OVS/ SSLT	468	702

Notes:
STD = Standard holes
OVS = Oversized holes
SSLT = Short-slotted holes transverse to direction of load
N = Threads included
X = Threads excluded
SC = Slip critical

* Tabulated values include 1/4-in. reduction in end distance, L_{eh}, to account for possible underrun in beam length.

Note: Slip-critical bolt values assume no more than one filler has been provided or bolts have been added to distribute loads in the fillers.

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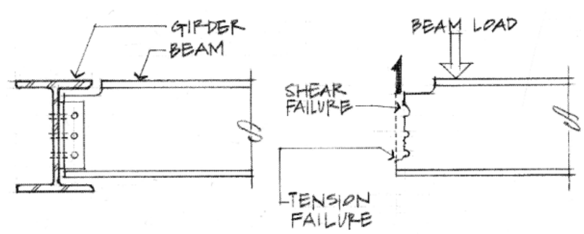
Limiting Strength or Stability States

In addition to resisting shear and tension in bolts and shear in welds, the connected materials may be subjected to shear, bearing, tension, flexure and even prying action. Coping can significantly reduce design strengths and may require web reinforcement. All the following must be considered:

- shear yielding
- shear rupture
- block shear rupture -
  failure of a block at a beam as a result of shear and tension
- tension yielding
- tension rupture
- local web buckling
- lateral torsional buckling



Block Shear Strength (or Rupture): $R_a \le R_n / \Omega$ or $R_u \le \phi R_n$

where $R_u = \Sigma \gamma_i R_i$

$$R_n = 0.6F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.6F_y A_{gv} + U_{bs} F_u A_{nt}$$

ϕ = 0.75 (LRFD) Ω = 2.00 (ASD) where:

A_{nv} is the net area subjected to shear
A_{nt} is the net area subjected to tension
A_{gv} is the gross area subjected to shear
U_{bs} = 1.0 when the tensile stress is uniform (most cases)
= 0.5 when the tensile stress is non-uniform

Gusset Plates

Gusset plates are used for truss member connections where the geometry prevents the members from coming together at the joint "point". Members being joined are typically double angles.

Decking

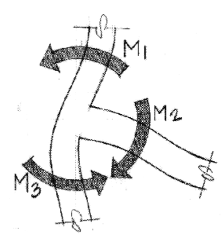
Shaped, thin sheet-steel panels that span several joists or evenly spaced support behave as continuous beams. Design tables consider a "1 unit" wide strip across the supports and determine maximum bending moment and deflections in order to provide allowable loads depending on the depth of the material.

The other structural use of decking is to construct what is called a *diaphragm*, which is a horizontal unit tying the decking to the joists that resists forces parallel to the surface of the diaphragm.

When decking supports a concrete topping or floor, the steel-concrete construction is called *composite*.

Frame Columns

Because joints can rotate in frames, the effective length of the column in a frame is harder to determine. The stiffness (EI/L) of each member in a joint determines how rigid or flexible it is. To find k, the relative stiffness, G or Ψ, must be found for both ends, plotted on the alignment charts, and connected by a line for braced and unbraced fames.

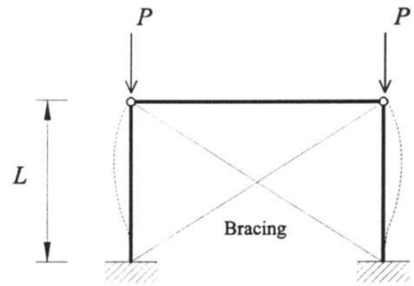


$$G = \Psi = \frac{\Sigma EI/l_c}{\Sigma EI/l_b}$$

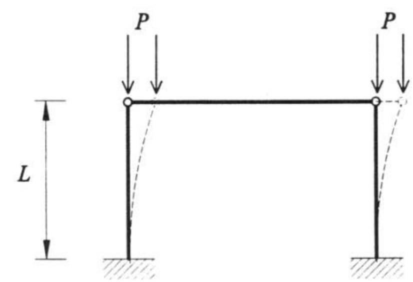
where

E = modulus of elasticity for a member
I = moment of inertia of for a member
l_c = length of the column from center to center
l_b = length of the beam from center to center

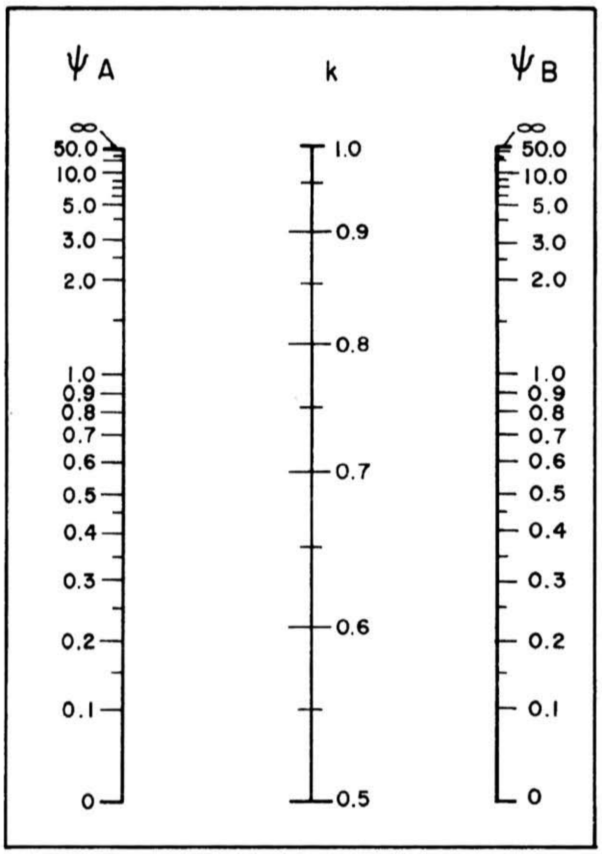
- For pinned connections we typically use a value of 10 for Ψ.
- For fixed connections we typically use a value of 1 for Ψ.



Braced – non-sway frame

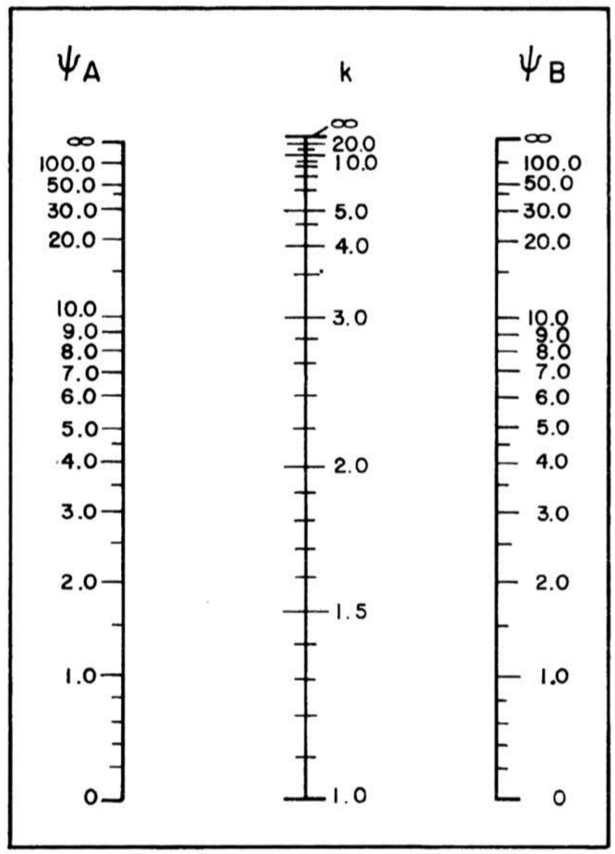


Unbraced – sway frame



(a)

Nonsway Frames

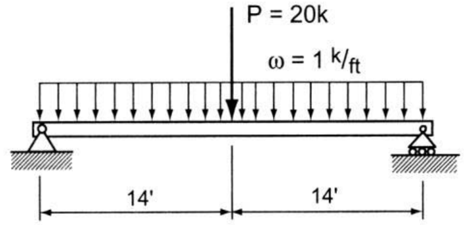


(b)

Sway Frames

Example 1 (pg 330)

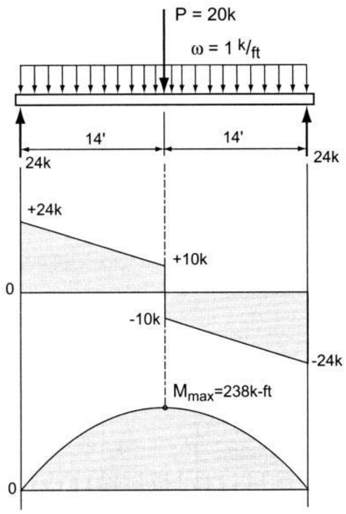
**Hypothetically determine the size of section required when the deflection criteria is NOT met*

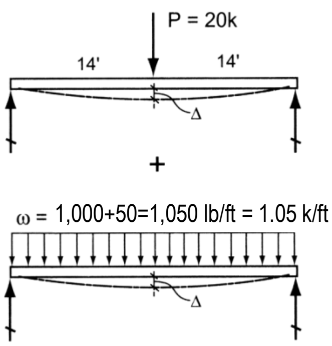


Example Problem 9.16 (Figures 9.76 to 9.78)

A steel beam (A572/50) is loaded as shown. Assuming a deflection requirement of $\Delta_{total} = L/240$ and a depth restriction of 18" nominal, select the most economical section. **(unified ASD)**

$F_b = 30$ ksi; $F_v = 20$ ksi; $E = 30\times10^3$ ksi $F_y = 50$ ksi

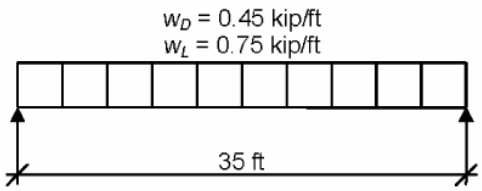




Example 2

Given:

Select an ASTM A992 W-shape beam with a simple span of 35 feet. Limit the member to a maximum nominal depth of 18 in. Limit the live load deflection to $L/360$. The nominal loads are a uniform dead load of 0.45 kip/ft and a uniform live load of 0.75 kip/ft. Assume the beam is continuously braced. Use ASD of the Unified Design method.



Beam Loading & Bracing Diagram
(full lateral support)

Solution:

Material Properties:

ASTM A992 $F_y = 50$ ksi $F_u = 65$ ksi

1. The unbraced length is 0 because it says it is fully braced.
2. Find the maximum shear and moment from unfactored loads: w_a = 0.450 k/ft + 0.750 k/ft = 1.20 k/ft

   V_a = 1.20 k/ft(35 ft)/2 = 21 k

   M_a = 1.20 k/ft(35 ft)^2/8 = 184 k-ft

   If $M_a \leq M_n/\Omega$, the maxmimum moment for design is $M_a\Omega$: M_{max} = 184 k-ft

3. Find $Z_{req'd}$:

   $Z_{req'd} \geq M_{max}/F_b = M_{max}(\Omega)/F_y$ = 184 k-ft(1.67)(12 in/ft)/50 ksi = 73.75 in^3 (F_y is the limit stress when fully braced)

4. Choose a trial section, and also limit the depth to 18 in as instructed:

   W18 x 40 has a plastic section modulus of 78.4 in^3 and is the most light weight (as indicated by the bold text) in Table 9.1

   Include the self weight in the maximum values: $w^*_{a\text{-}adjusted}$ = 1.20 k/ft + 0.04 k/ft

   $V^*_{a\text{-}adjusted}$ = 1.24 k/ft(35 ft)/2 = 21.7 k

   $M^*_{a\text{-}adjusted}$ = 1.24 k/ft(35 ft)^3/8 = 189.9 k

   $Z_{req'd} \geq$ 189.9 k-ft(1.67)(12 in/ft)/50 ksi = 76.11 in^3 And the Z we have (78.4) is larger than the Z we need (76.11), so OK.

6. Evaluate shear (is $V_a \leq V_n/\Omega$): $A_w = dt_w$ so look up section properties for W18 x 40: d = 17.90 in and t_w = 0.315 in

   $V_n/\Omega = 0.6F_{yw}A_w/\Omega$ = 0.6(50 ksi)(17.90 in)(0.315 in)/1.5 = 112.8 k which is much larger than 21.7 k, so OK.

9. Evaluate the deflection with respect to the limit stated of L/360 for the live load. (If we knew the **total** load limit we would check that as well). The moment of inertia for the W18 x 40 is needed. I_x = 612 in^4

   $\Delta_{live\ load\ limit}$ = 35 ft(12 in/ft)/360 = 1.17 in

   $\Delta = 5wL^4/384EI$ = 5(0.75 k/ft)(35 ft)^4(12 in/ft)^3/384(29 x 10^3 ksi)(612 in^4) = 1.42 in! This is TOO BIG (not less than the limit.

   Find the moment of inertia needed:

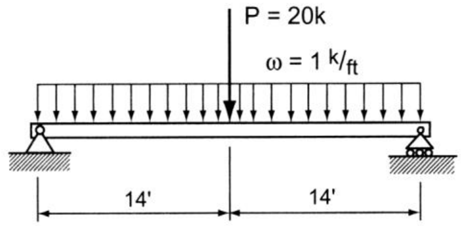
   $I_{req'd} \geq \Delta_{too\ big}(I_{trial})/\Delta_{limit}$ = 1.42 in(612 in^4)/(1.17 in) = 742.8 in^4

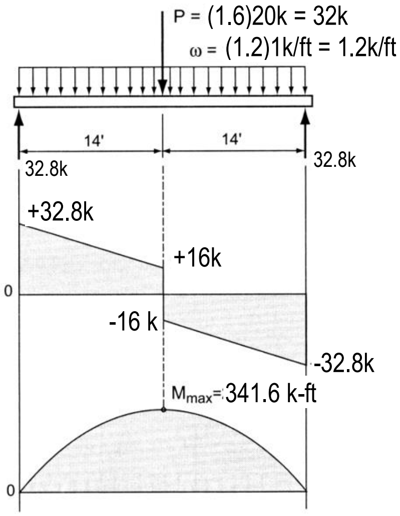
   From Table 9.1, a W16 x 45 is larger (by Z), but not the most light weight (efficient), as is W10 x 68, W14 x 53, W18 x 46, (W21 x 44 is too deep) and W18 x 50 is bolded (efficient). (Now look up I's). (In order: I_x = 586, 394, 541, 712 and 800 in^4)

<u>Choose a W18 x 50</u>

Example 3

For the same beam and loading of Example 1, select the most economical beam using Load and Resistance Factor Design (LRFD) with the 18" depth restriction. Assume the distributed load is dead load, and the point load is live load. $F_y = 50$ ksi and $E = 30\text{x}10^3$ ksi





1. To find $V_{u\text{-}max}$ and $M_{u\text{-}max}$, factor the loads, construct a *new* load diagram, shear diagram and bending moment diagram.

2. To satisfy $M_u \leq \phi_b M_n$, we find $M_n = \dfrac{M_u}{\phi_b} = \dfrac{341.6^{k-ft}}{0.9} = 379.6^{k-ft}$ and

   solve for Z needed: $Z = \dfrac{M_n}{F_y} = \dfrac{379.6^{k-ft}(12\,^{in}/_{ft})}{50ksi} = 91.1in^3$

   Choose a *trial* section from the <u>Listing of W Shapes in Descending Order of Z</u> by selecting the **bold** section at the top of the grouping satisfying our Z and depth requirement – W18 x 50 is the *lightest* with Z = 101 in^3. (W22 x 44 is the lightest without the depth requirement.) Include the additional self weight (dead load) and find the maximum shear and bending moment:

$$V^*_{u-adjusted} = 32.8k + \frac{1.2(50\,^{lb}/_{ft})28\,ft}{2(1000\,^{lb}/_{k})} = 33.64k$$

$$M^*_{u-adjusted} = 341.6^{k-ft} + \frac{1.2(50\,^{lb}/_{ft})(28\,ft)^2}{8(1000\,^{lb}/_{k})} = 347.5^{k-ft}$$

$$Z^*_{req'd} \geq \frac{M_u}{\phi_b F_y} = \frac{347.5^{k-ft}(12\,^{in}/_{ft})}{0.9(50ksi)} = 92.7in^3$$

, so Z (have) of 101 in^3 is greater than the Z (needed).

3. Check the shear capacity to satisfy $V_u \leq \phi_v V_n$: $A_{web} = dt_w$ and d=17.99 in., t_w = 0.355 in. for the W18x50

$$\phi_v V_n = \phi_v 0.6 F_{yw} A_w = 1.0(0.6)50ksi(17.99in)0.355in = 191.6k$$

So 33.64k ≤ 191.6 k <u>OK</u>

4. Calculate the deflection from the ***unfactored*** loads, including the self-weight now because it is known, and satisfy the deflection criteria of $\Delta_{LL} \leq \Delta_{LL\text{-}limit}$ and $\Delta_{total} \leq \Delta_{total\text{-}limit}$. (This is <u>identical</u> to what is done in Example 1.) I_x =800 in^3 for the W18x50

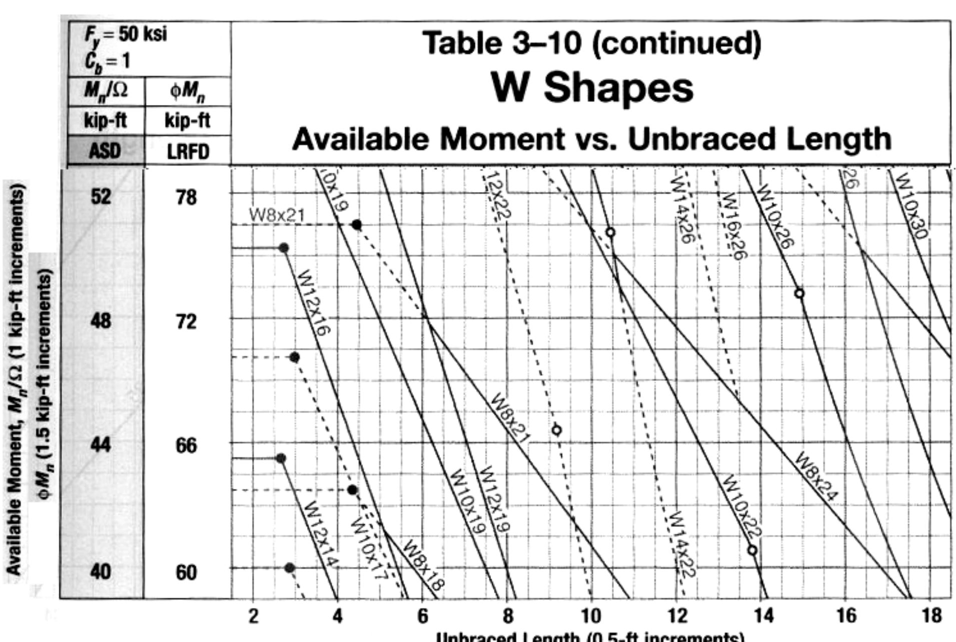
   $\Delta_{total\text{-}limit}$ = L/240 = 1.4 in., say Δ_{LL} = L/360 = 0.93 in

$$\Delta_{total} = \frac{PL^3}{48EI} + \frac{5wL^4}{384EI} = \frac{20k(28\,ft)^3(12\,^{in}/_{ft})^3}{48(30x10^3\,ksi)800in^4} + \frac{5(1.050\,^{k}/_{ft})(28\,ft)^4(12\,^{in}/_{ft})^3}{384(30x10^3\,ksi)800in^4} = 0.658 + 0.605 = 1.26in$$

So 1.26 in. ≤ 1.4 in., and 0.658 in. ≤ 0.93 in. <u>OK</u> $\therefore$ **FINAL SELECTION IS W18x50**

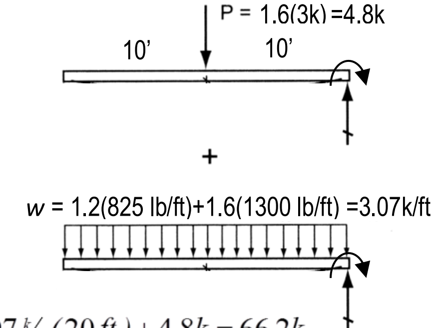
Example 4

A steel beam with a 20 ft span is designed to be simply supported at the ends on columns and to carry a floor system made with open-web steel joists at 4 ft on center. The joists span 28 feet and frame into the beam from *one side only* and have a self weight of 8.5 lb/ft. Use A992 (grade 50) steel and select the most economical wide-flange section for the beam with LRFD design. Floor loads are 50 psf LL and 14.5 psf DL.



Example 5

Select a A992 W shape flexural member (F_y = 50 ksi, F_u = 65 ksi) for a beam with distributed loads of 825 lb/ft (dead) and 1300 lb/ft (live) and a live point load at midspan of 3 k using the Available Moment tables. The beam is simply supported, 20 feet long, and braced at the ends and midpoint only (L_b = 10 ft.) The beam is a roof beam for an institution without plaster ceilings. (LRFD)



SOLUTION:

To use the Available Moment tables, the maximum moment required is plotted against the unbraced length. The first solid line with capacity or unbraced length *above* what is needed is the most economical.

DESIGN LOADS (load factors applied on figure):

$$M_u = \frac{wl^2}{2} + Pb = \frac{3.07\,{}^{k}\!/\!_{ft}(20ft)^2}{2} + 4.8k(10ft) = 662^{k-ft} \quad V_u = wl + P = 3.07\,{}^{k}\!/\!_{ft}(20ft) + 4.8k = 66.2k$$

Plotting 662 k-ft vs. 10 ft lands just on the capacity of the W21x83, but it is dashed (and not the most economical) AND we need to consider the contribution of self weight to the total moment. Choose a *trial* section of W24 x 76. Include the new dead load:

$$M^*_{u-adjusted} = 662^{k-ft} + \frac{1.2(76\,{}^{lb}\!/\!_{ft})(20ft)^2}{2(1000\,{}^{lb}\!/\!_{k})} 680.2^{k-ft} \qquad V^*_{u-adjusted} = 66.2k + 1.2(0.076\,{}^{k}\!/\!_{ft})(20ft) = 68.0k$$

<u>Replot</u> 680.2 k-ft vs. 10ft, which lands *above* the capacity of the W21x83. We can't look up because the chart ends, but we can look for that capacity with a longer unbraced length. This leads us to a **W24 x 84** as the most economical. (With the additional self weight of 84 - 76 lb/ft = 8 lb/ft, the increase in the *factored* moment is only 1.92 k-ft; therefore, it is still OK.)

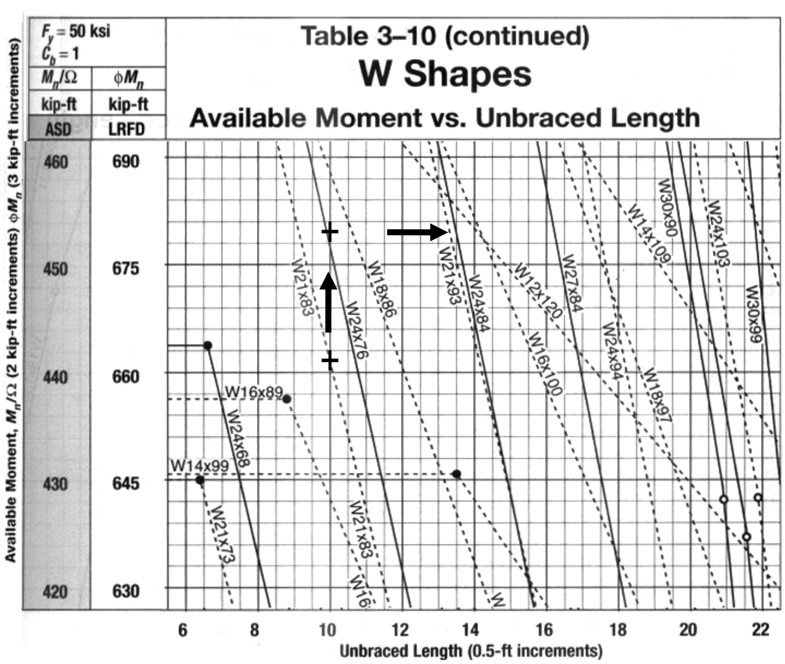
Evaluate the shear capacity:

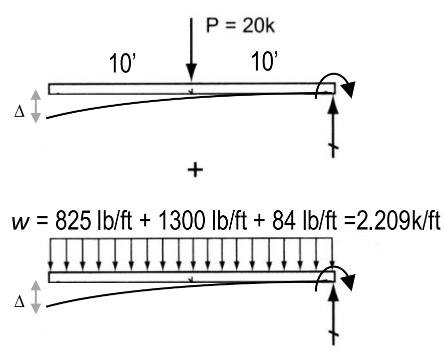
$$\phi_v V_n = \phi_v 0.6F_{yw}A_w = 1.0(0.6)50ksi(24.10in)0.47in = 338.4k$$ so yes, 68 k ≤ 338.4k <u>OK</u>

Evaluate the deflection with respect to the limits of L/240 for live (*unfactored*) load and L/180 for total (*unfactored*) load:

L/240 = 1 in. and L/180 = 1.33 in.

$$\Delta_{total} = \frac{Pb^2(3l-b)}{6EI} + \frac{wL^4}{24EI} = \frac{3k(10ft)^2(3\cdot 20-10ft)(12\,{}^{in}\!/\!_{ft})^3}{6(30x10^3ksi)2370in^4} + \frac{(2.209\,{}^{k}\!/\!_{ft})(20ft)^4(12\,{}^{in}\!/\!_{ft})^3}{24(30x10^3ksi)2370in^4} = 0.06 + 0.36 = 0.42in$$





So, $\Delta_{LL} \leq \Delta_{LL-limit}$ and $\Delta_{total} \leq \Delta_{total-limit}$:

0.06 in. ≤ 1 in. and 0.42 in. ≤ 1.33 in.

(This section is so big to accommodate the large bending moment at the cantilever support that it deflects very little.)

∴ **FINAL SELECTION IS W24x84**

Example 6

Select the most economical joist for the 40 ft grid structure with floors and a flat roof. The roof loads are 10 lb/ft^2 dead load and 20 lb/ft^2 live load. The floor loads are 30 lb/ft^2 dead load 100 lb/ft^2 live load. (Live load deflection limit for the roof is L/240, while the floor is L/360). Use the (LRFD) K and LH series charts provided.

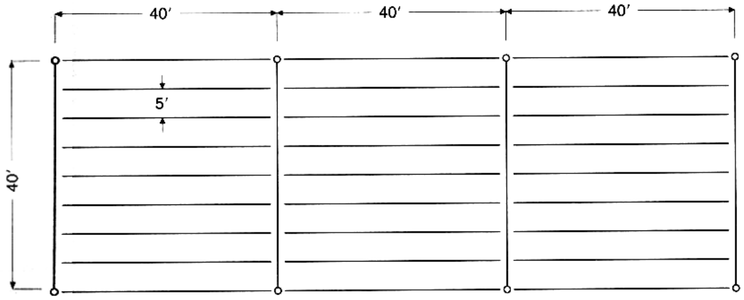


Figure 7.218 Framing plan for joists, girders, and columns on 40 ft × 40 ft grid.

(Top values are maximum total factored load in lb/ft, while the lower (lighter) values are maximum (unfactored) live load for a deflection of L/360)

STANDARD LOAD TABLE FOR OPEN WEB STEEL JOISTS, K-SERIES
Based on a 50 ksi Maximum Yield Strength - Loads Shown in Pounds per Linear Foot (plf)

Joist Designation	18K3	18K4	18K5	18K6	18K7	18K9	18K10	20K3	20K4	20K5	20K6	20K7	20K9	20K10	22K4	22K5	22K6	22K7	22K9	22K10	22K11
Depth (In.)	18	18	18	18	18	18	18	20	20	20	20	20	20	20	22	22	22	22	22	22	22
Approx. Wt. (lbs./ft.)	6.6	7.2	7.7	8.5	9	10.2	11.7	6.7	7.6	8.2	8.9	9.3	10.8	12.2	8	8.8	9.2	9.7	11.3	12.6	13.8
Span (ft.) ↓																					
38								211 74	255 87	286 98	312 106	348 118	418 139	496 164	280 107	316 119	345 130	384 144	462 170	549 200	628 228
39								199 69	241 81	271 90	297 98	330 109	397 129	471 151	267 98	300 110	327 120	364 133	438 157	520 185	595 211
40								190 64	229 75	258 84	282 91	313 101	376 119	447 140	253 91	285 102	310 111	346 123	417 146	495 171	565 195
41															241 85	271 95	295 103	330 114	396 135	471 159	538 181

Joist Designation	24K4	24K5	24K6	24K7	24K8	24K9	24K10	24K12	26K5	26K6	26K7	26K8	26K9	26K10	26K12
Depth (In.)	24	24	24	24	24	24	24	24	26	26	26	26	26	26	26
Approx. Wt. (lbs./ft.)	8.4	9.3	9.7	10.1	11.5	12.0	13.1	16.0	9.8	10.6	10.9	12.1	12.2	13.8	16.6
Span (ft.) ↓															
38	307 128	346 143	378 156	421 172	465 189	507 204	601 240	691 275	376 169	411 184	457 204	505 223	550 241	654 284	691 299
39	292 118	328 132	358 144	399 159	441 174	480 189	570 222	673 261	357 156	390 170	433 188	480 206	522 223	619 262	673 283
40	277 109	312 122	340 133	379 148	420 161	456 175	541 206	657 247	340 145	370 157	412 174	456 191	496 207	589 243	657 269
41	264 101	297 114	324 124	361 137	399 150	435 162	516 191	640 235	322 134	352 146	393 162	433 177	472 192	561 225	640 256

Joist Designation	28K6	28K7	28K8	28K9	28K10	28K12	30K7	30K8	30K9	30K10	30K11	30K12
Depth (In.)	28	28	28	28	28	28	30	30	30	30	30	30
Approx. Wt. (lbs./ft.)	11.4	11.8	12.7	13.0	14.3	17.1	12.3	13.2	13.4	15.0	16.4	17.6
Span (ft.) ↓												
38	444 214	493 237	546 260	594 282	691 325	691 325	531 274	586 300	639 325	691 353	691 353	691 353
39	420 198	469 219	519 240	564 260	670 306	673 308	504 253	556 277	606 300	673 333	673 333	673 333
40	399 183	445 203	492 222	535 241	636 284	657 291	478 234	529 256	576 278	657 315	657 315	657 315
41	379 170	424 189	468 206	510 224	606 263	640 277	454 217	502 238	547 258	640 300	640 300	640 300

Shaded areas indicate the bridging requirements.

Example 6 (continued)

(Top values are maximum total factored load in lb/ft, while the lower (lighter) values are maximum (unfactored) live load for a deflection of L/360)

STANDARD LOAD TABLE FOR LONGSPAN STEEL JOISTS, LH-SERIES
Based on a 50 ksi Maximum Yield Strength - Loads Shown in Pounds per Linear Foot (plf)

Joist Designation	Approx. Wt in Lbs. Per Linear Ft (Joists only)	Depth in inches	SAFE LOAD* in Lbs. Between	CLEAR SPAN IN FEET															
			22-24	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
20LH02	10	20	16950	663	655	646	615	582	547	516	487	460	436	412	393	373	355	337	322
				306	303	298	274	250	228	208	190	174	160	147	136	126	117	108	101
20LH03	11	20	18000	703	694	687	678	651	621	592	558	528	499	474	448	424	403	382	364
				337	333	317	302	280	258	238	218	200	184	169	156	143	133	123	114
20LH04	12	20	22050	861	849	837	792	744	700	660	624	589	558	529	502	477	454	433	412
				428	406	386	352	320	291	265	243	223	205	189	174	161	149	139	129
20LH05	14	20	23700	924	913	903	892	856	816	769	726	687	651	616	585	556	529	504	481
				459	437	416	395	366	337	308	281	258	238	219	202	187	173	161	150
20LH06	15	20	31650	1233	1186	1144	1084	1018	952	894	840	790	745	703	666	631	598	568	541
				606	561	521	477	427	386	351	320	292	267	246	226	209	192	178	165
20LH07	17	20	33750	1317	1267	1221	1179	1140	1066	1000	940	885	834	789	745	706	670	637	606
				647	599	556	518	484	438	398	362	331	303	278	256	236	218	202	187
20LH08	19	20	34800	1362	1309	1263	1219	1177	1140	1083	1030	981	931	882	837	795	754	718	685
				669	619	575	536	500	468	428	395	365	336	309	285	262	242	225	209
20LH09	21	20	38100	1485	1429	1377	1329	1284	1242	1203	1167	1132	1068	1009	954	904	858	816	775
				729	675	626	581	542	507	475	437	399	366	336	309	285	264	244	227
20LH10	23	20	41100	1602	1542	1486	1434	1386	1341	1297	1258	1221	1186	1122	1060	1005	954	906	862
				786	724	673	626	585	545	510	479	448	411	377	346	320	296	274	254
				33	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**
24LH03	11	24	17250	513	508	504	484	460	439	418	400	382	366	351	336	322	310	298	286
				235	226	218	204	188	175	162	152	141	132	124	116	109	102	96	90
24LH04	12	24	21150	628	597	568	540	514	490	468	447	427	409	393	376	361	346	333	321
				288	265	246	227	210	195	182	169	158	148	138	130	122	114	107	101
24LH05	13	24	22650	673	669	660	628	598	570	544	520	496	475	456	436	420	403	387	372
				308	297	285	264	244	226	210	196	182	171	160	150	141	132	124	117
24LH06	16	24	30450	906	868	832	795	756	720	685	655	625	598	571	546	522	501	480	460
				411	382	356	331	306	284	263	245	228	211	197	184	172	161	152	142
24LH07	17	24	33450	997	957	919	882	847	811	774	736	702	669	639	610	583	559	535	514
				452	421	393	367	343	320	297	276	257	239	223	208	195	182	171	161
24LH08	18	24	35700	1060	1015	973	933	895	858	817	780	745	712	682	652	625	600	576	553
				480	447	416	388	362	338	314	292	272	254	238	222	208	196	184	173
24LH09	21	24	42000	1248	1212	1177	1146	1096	1044	994	948	903	861	822	786	751	720	690	661
				562	530	501	460	424	393	363	337	313	292	272	254	238	223	209	196
24LH10	23	24	44400	1323	1284	1248	1213	1182	1152	1105	1053	1002	955	912	873	834	799	766	735
				596	559	528	500	474	439	406	378	351	326	304	285	266	249	234	220
24LH11	25	24	46800	1390	1350	1312	1276	1243	1210	1180	1152	1101	1051	1006	963	924	885	850	816
				624	588	555	525	498	472	449	418	388	361	337	315	294	276	259	243
			33-40	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**
28LH05	13	28	21000	505	484	465	445	429	412	397	382	367	355	342	330	319	309	298	289
				219	205	192	180	169	159	150	142	133	126	119	113	107	102	97	92
28LH06	16	28	27900	672	643	618	592	568	546	525	505	486	469	451	436	421	406	393	379
				289	270	253	238	223	209	197	186	175	166	156	148	140	133	126	120
28LH07	17	28	31500	757	726	696	667	640	615	591	568	547	528	508	490	474	457	442	427
				326	305	285	267	251	236	222	209	197	186	176	166	158	150	142	135
28LH08	18	28	33750	810	775	744	712	684	657	630	604	580	556	535	516	496	478	462	445
				348	325	305	285	268	252	236	222	209	196	185	175	165	156	148	140
28LH09	21	28	41550	1000	958	918	879	844	810	778	748	721	694	669	645	622	601	580	561
				428	400	375	351	329	309	291	274	258	243	228	216	204	193	183	173
28LH10	23	28	45450	1093	1056	1018	976	937	900	864	831	799	769	742	715	690	666	643	622
				466	439	414	388	364	342	322	303	285	269	255	241	228	215	204	193
28LH11	25	28	48750	1170	1143	1104	1066	1023	982	943	907	873	841	810	781	753	727	702	679
				498	475	448	423	397	373	351	331	312	294	278	263	249	236	223	212
28LH12	27	28	53550	1285	1255	1227	1200	1173	1149	1105	1063	1023	984	948	913	880	849	819	790
				545	520	496	476	454	435	408	383	361	340	321	303	285	270	256	243
28LH13	30	28	55800	1342	1311	1281	1252	1224	1198	1173	1149	1126	1083	1041	1002	964	930	897	865
				569	543	518	495	472	452	433	415	396	373	352	332	314	297	281	266

Shaded areas indicate the bridging requirements.

Example 7 (LRFD)

EXAMPLE 5.1 Open-Web Steel Joist Design

A fully exposed roof system for a commercial building, spanning 35 ft, located in Muncie, Indiana, in an urban environment.

IBC specifies a **20 psf snow live load** for Muncie, Indiana, home of Ball State University. Table 1.3 indicates the snow exposure factor: $C_e = 0.9$. Table 1.4 indicates the snow thermal factor: $C_t = 1.0$. Table 1.7 indicates an occupancy importance factor (for Category II): $I_S = 1.0$. Fig. 1.2 indicates the ground snow load: $p_g = 20$ psf

$$P_S = 0.7(0.9)1.0(1.0)20 \text{ psf} = 13.9 \text{ psf}$$

A typical roof construction might consist of:

Membrane roofing	1.0 psf
4 in. average tapered rigid insulation	6.0 psf
Steel deck (2–4 ft span)	1.0 psf
Estimated joist weight:	
35 ft span would be a minimum 18 in. joist	
An average 18 in. joist weight = 9.0 plf	
Spaced @ 4 ft-0 in. o.c. 9.0 plf/4 ft	2.3 psf
Ceiling suspension system	1.0 psf
1/2 in. gypsum ceiling	2.0 psf

Mechanical system estimates should also be included; the heavy sprinkler/drain piping running parallel to a joist or pair of joists is especially critical.

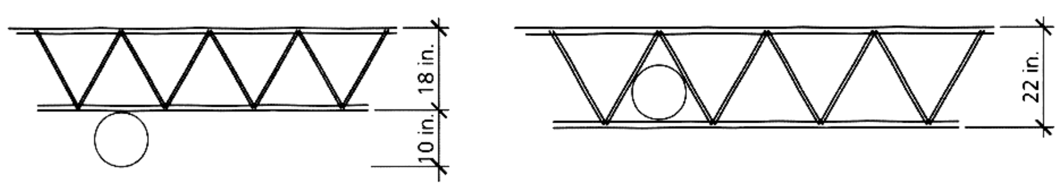
Miscellaneous ductwork/electrical	1.0 psf
Total dead load	14.3 psf × 4 ft o.c. = 57.2 plf
Total live load	13.9 psf X 4 ft o.c. = 55.6 plf

Total factored live snow load + dead load = 1.2(55.6) + 1.6 (57.2) = 158.2 plf

Use joist load tables to select the best section:

At 35 ft, 18K3 joists carry 237 plf TFL and 84 plf LL
LL: deflection controls and the weight is 6.4 plf.

At least on the surface, this is the best choice, but depending upon the need to integrate mechanical systems into the joist space, a 20K3 at 6.5 plf or even a 22K4 at 7.3 plf which is both deeper and heavier than the previous selection may be best:



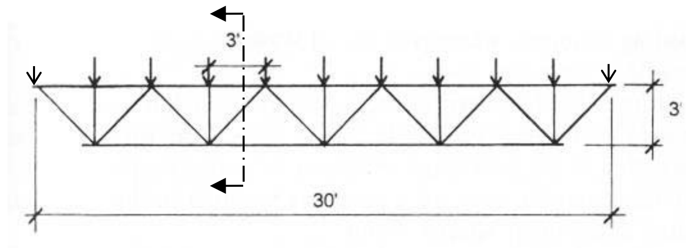
LRFD

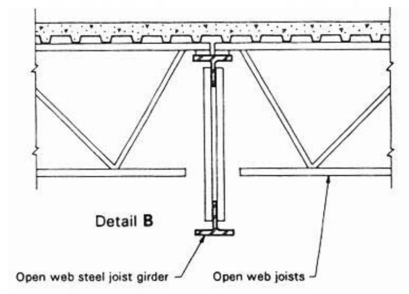
STANDARD LOAD TABLE FOR OPEN WEB STEEL JOISTS, K-SERIES
Based On A 50 ksi Maximum Yield Strength - Loads Shown In Pounds Per Linear Foot (plf)

Joist Designation	18K3	18K4	18K5	18K6	18K7	18K9	18K10	20K3	20K4	20K5	20K6	20K7	20K9	20K10	22K4	22K5	22K6	22K7	22K9	22K10	22K11
Depth (In.)	18	18	18	18	18	18	18	20	20	20	20	20	20	20	22	22	22	22	22	22	22
Approx. Wt. (lbs./ft.)	6.4	7.2	7.7	8.4	8.9	10.1	11.6	6.5	7.2	7.7	8.4	8.9	10.1	11.6	7.3	7.7	8.5	9.0	10.2	11.7	11.9
Span (ft.) ↓																					
34	237	285	321	349	390	468	555	264	318	358	391	435	523	621	352	397	432	481	579	687	774
	84	98	110	120	132	156	184	105	122	137	149	165	195	229	149	167	182	202	239	280	314
35	223	268	303	330	367	441	523	249	300	339	369	411	493	585	331	373	408	454	546	648	741
	77	90	101	110	121	143	168	96	112	126	137	151	179	210	137	153	167	185	219	257	292

Example 8

A floor with multiple bays is to be supported by open-web steel joists spaced at 3 ft. on center and spanning 30 ft. having a dead load of 70 lb/ft^2 and a live load of 100 lb/ft^2. The joists are supported on joist girders spanning 30 ft. with 3 ft.-long panel points (shown). Determine the member forces at the location shown in a horizontal chord and the maximum force in a web member for an interior girder. Use factored loads. *Assume a self weight for the open-web joists of 12 lb/ft, and the self weight for the joist girder of 35 lb/ft.*





Example 9

A floor is to be supported by trusses spaced at 5 ft. on center and spanning 60 ft. having a dead load of 53 lb/ft^2 and a live load of 100 lb/ft^2. With 3 ft.-long panel points, the depth is assumed to be 3 ft with a span-to-depth ratio of 20. With 6 ft.-long panel points, the depth is assumed to be 6 ft with a span-to-depth ratio of 10. Determine the maximum force in a horizontal chord and the maximum force in a web member. Use factored loads. *Assume a self weight of 40 lb/ft.*

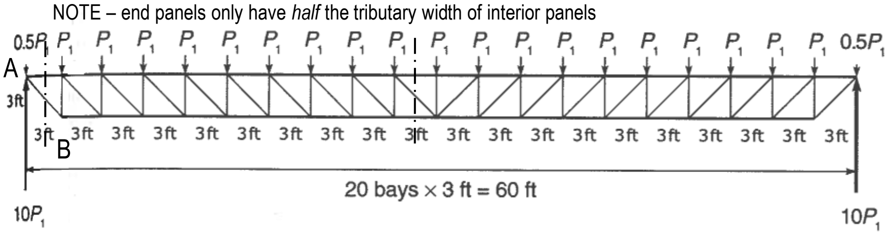
Table 7.2 Computation of Truss Joint Loads

	area loads				*tributary widths*							
	w_{dead}		w_{live}		Node-to-Node Spacing	Truss-to-Truss Spacing	Floor Area per Node A	P_{dead} (= $w_{dead}\cdot$ A)	P_{live} (= $w_{live}\cdot$ A)	Factored Dead Load $1.2\cdot P_{dead}$	Factored Live Load $1.6\cdot P_{live}$	Factored Total Load $1.2\cdot P_{dead}+1.6\cdot P_{live}$
Truss	(#/ft²)	(K/ft²)	(#/ft²)	(K/ft²)	(ft)	(ft)	(ft²)	(K)	(K)	(K)	(K)	(K)
3 ft deep	53	0.053	100	0.100	3	5	15	0.795	1.50	0.954	2.40	3.35 + 0.14 = 3.49
6 ft deep	53	0.053	100	0.100	6	5	30	1.59	3.00	1.908	4.80	6.71 + 0.29 = 7.00

self weight 0.04 k/ft (distributed) 3 $1.2P_{dead} = 1.2w_{dead}\cdot$ *tributary width* = 0.14 K

6 $1.2P_{dead} = 1.2w_{dead}\cdot$ *tributary width* = 0.29 K

NOTE – end panels only have *half* the tributary width of interior panels

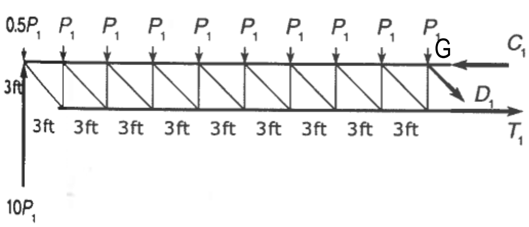


***FBD 1** for 3 ft deep truss*

***FBD 3**:* Maximum web force will be in the end diagonal (just like maximum shear in a beam)

$\Sigma F_y = 10P_1 - 0.5P_1 - F_{AB}\cdot \sin 45^\circ = 0$

$F_{AB} = 9.5P_1/\sin 45^\circ = 9.5(3.49\ k)/0.707 = \underline{46.9\ k}$



***FBD 2** of cut just to the left of midspan*

***FBD 3** of cut just to right of left support*

FBD 2: Maximum chord force (top or bottom) will be at midspan

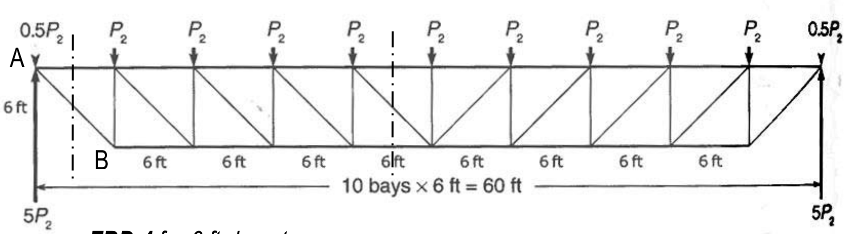
$\Sigma M_G = -9.5P_1(27^{ft}) + P_1(24^{ft}) + P_1(21^{ft}) + P_1(18^{ft}) + P_1(15^{ft}) + P_1(12^{ft}) + P_1(9^{ft}) + P_1(6^{ft}) + P_1(3^{ft}) + T_1(3^{ft}) = 0$

$T_1 = P_1(148.5^{ft})/3^{ft} = (3.49\ k)(49.5) = 172.8\ k$

$\Sigma F_y = 10P_1 - 9.5P_1 - D_1\cdot \sin 45^\circ = 0$

$D_1 = 0.5(3.49\ k)/0.707 = 2.5\ k$ *(minimum near midspan)*

$\Sigma F_x = -C_1 + T_1 + D_1\cdot \cos 45^\circ = 0$ $\underline{C_1 = 174.5\ k}$

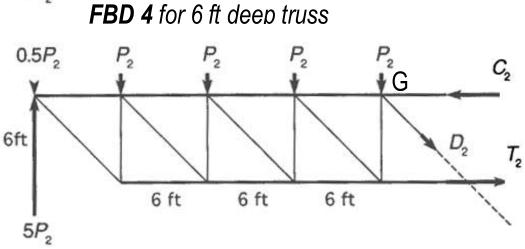


***FBD 4** for 6 ft deep truss*

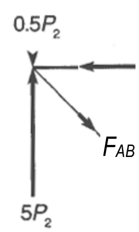
FBD 6: Maximum web force will be in the end diagonal

$\Sigma F_y = 5P_2 - 0.5P_2 - F_{AB}\cdot \sin 45^\circ = 0$

$F_{AB} = 4.5P_2/\sin 45^\circ = 4.5(7\ k)/0.707 = \underline{44.5\ k}$



***FBD 5** of cut just to the left of midspan*



***FBD 6** of cut just to right of left support*

FBD 5: Maximum chord (top or bottom) force will be at midspan

$\Sigma M_G = -4.5P_2(24^{ft}) + P_2(18^{ft}) + P_2(12^{ft}) + P_2(6^{ft}) + T_2(6^{ft}) = 0$

$T_2 = P_2(72^{ft})/6^{ft} = (7\ k)(12) = 84\ k$

$\Sigma F_y = 5P_2 - 4.5P_1 - D_s\cdot \sin 45^\circ = 0$

$D_2 = 0.5(7\ k)/0.707 = 4.9\ k$ *(minimum near midspan)*

$\Sigma F_x = -C_2 + T_2 + D_2\cdot \cos 45^\circ = 0$ $\underline{C_2 = 87.5\ k}$

Example 10 (pg 367) + LRFD

Example Problem 10.10 (Figure 10.41)

A 24-ft.-tall, A572 grade 50, steel column (W14×82) with an F_y = 50 ksi has pins at both ends. Its weak axis is braced at midheight, but the column is free to buckle the full 24 ft. in the strong direction. Determine the safe load capacity for this column. using ASD and LRFD.

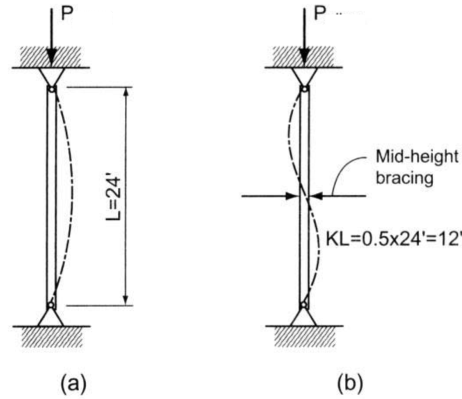


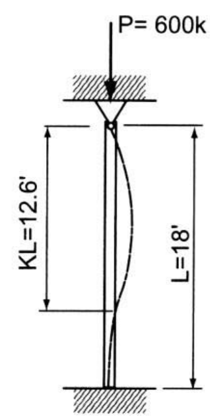
Figure 10.41 (a) Strong axis buckling. (b) Weak axis buckling.

Example 11 (pg 371) + chart method

Example Problem 10.14: Design of Steel Columns (Figure 10.48)

Select the most economical W12 × column 18' in height to support an axial load of 600 kips using A572 grade 50 steel. Assume that the column is hinged at the top but fixed at the base. Use LRFD assuming that the load is a dead load (factor of 1.4)

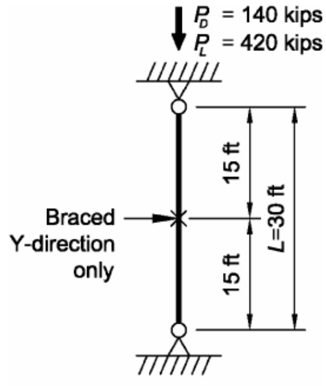
ALSO: Select the W12 column using the Available Strength charts.



Example 12

Given:

Redesign the column from Example E.1a assuming the column is laterally braced about the y-y axis and torsionally braced at the midpoint. Use both ASD and LRFD. F_y = 50 ksi. (Not using Available Strength charts)



Solution:

ASD:

1. P_a = 140 k + 420 k = 560 k

2. The effective length in the weak (y-y) axis is 15 ft, while the effective length in the strong (x-x) axis is 30 ft. (K = 1, KL = 1×30 ft). To find kL/r_x and kL/r_y we can assume or choose values from the wide flange charts. r_y's range from 1 to 3 in., while r_x's range from 3 to 14 inches. Let's try r_y = 2 in and r_x = 9 in. (something in the W21 range, say.)

   $kL/r_y \cong$ 15 ft(12 in/ft)/2 in. = 90 $\Leftarrow$ GOVERNS (is larger)

   $kL/r_x \cong$ 30 ft(12 in/ft)/9 in. = 40

3. Find a section with sufficient area (which then will give us "real" values for r_x and r_y):

   If $P_a \leq P_n/\Omega$, and $P_n = F_{cr} A$, we can find $A \geq P_a\Omega/F_{cr}$ with Ω = 1.67

   The tables provided have ϕF_{cr}, so we can get F_{cr} by dividing by ϕ = 0.9

   ϕF_{cr} for 90 is 24.9 ksi, F_{cr} = 24.9 ksi/0.9 = 27.67 ksi so A ≥ 560 k(1.67)/27.67 ksi = 33.8 in^2

4. Choose a trial section, and find the effective lengths and associated available strength, F_{cr} :

   Looking from the smallest sections, the W14's are the first with a big enough area:

   Try a W14 x 120 (A = 35.3 in^2) with r_y = 3.74 in and r_x = 6.24 in.: kL/r_y = 48.1 and kL/r_x = 57.7 (GOVERNS)

   ϕF_{cr} for 58 is 35.2 ksi, F_{cr} = 39.1 ksi so A ≥ 560 k(1.67)/39.1 ksi = 23.9 in^2

Choose a W14 x 90 (Choosing a W14 x 82 would make kL/r_x = 59.5, and $A_{req'd}$ = 24.3 in^2, which is more than 24.1 in^2!)

LRFD:

1. P_u = 1.2(140 k) + 1.6(420 k) = 840 k

2. The effective length in the weak (y-y) axis is 15 ft, while the effective length in the strong (x-x) axis is 30 ft. (K = 1, KL = 1×30 ft). To find kL/r_x and kL/r_y we can assume or choose values from the wide flange charts. r_y's range from 1 to 3 in., while r_x's range from 3 to 14 inches. Let's try r_y = 2 in and r_x = 9 in. (something in the W21 range, say.)

   $kL/r_y \cong$ 15 ft(12 in/ft)/2 in. = 90 $\Leftarrow$ GOVERNS (is larger)

   $kL/r_x \cong$ 30 ft(12 in/ft)/9 in. = 40

3. Find a section with sufficient area (which then will give us "real" values for r_x and r_y):

   If $P_u \leq \phi P_n$, and $\phi P_n = \phi F_{cr} A$, we can find $A \geq P_u/\phi F_{cr}$ with ϕ = 0.9

   ϕF_{cr} for 90 is 24.9 ksi, so A ≥ 840 k/24.9 ksi = 33.7 in^2

4. Choose a trial section, and find the effective lengths and associated available strength, ϕF_{cr} :

   Looking from the smallest sections, the W14's are the first with a big enough area:

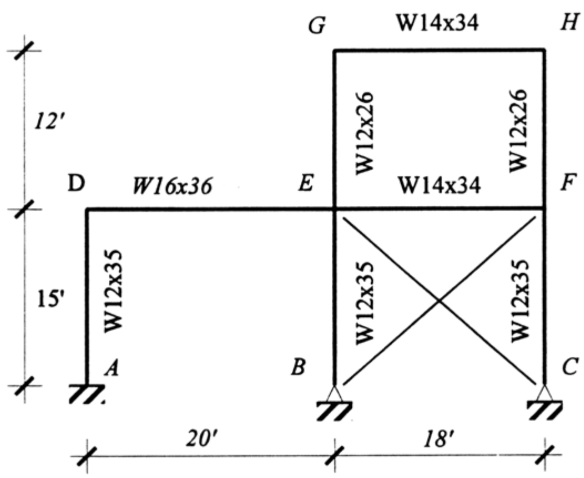
   Try a W14 x 120 (A = 35.3 in^2) with r_y = 3.74 in and r_x = 6.24 in.: kL/r_y = 48.1 and kL/r_x = 57.7 (GOVERNS)

   ϕF_{cr} for 58 is 35.2 ksi, so A ≥ 840 k/35.2 ksi = 23.9 in^2

Choose a W14 x 90 (Choosing a W14 x 82 would make kL/r_x = 59.5, and $A_{req'd}$ = 24.3 in^2, which is more than 24.1 in^2!)

Example 13

Example 6-1: For the building frame shown in Fig. 6-20, determine the effective column length factor, K, the slenderness ratio, KL/r for each column. Assume the columns buckle and the beams bend about their strong axis.



W12x26:

$I_x = 204$ in.^{4}
$r_x = 5.17$ in.^{4}

W12x35:

$I_x = 285$ in.^{4}
$r_x = 5.25$ in.^{4}

W14x34:

$I_x = 340$ in.^{4}

W16x36:

$I_x = 448$ in.^{4}

Figure 6-20: Building frame for Example 6-1.

Solution:

Note: The diagonal bracing prevents sidesway of the first story columns only.

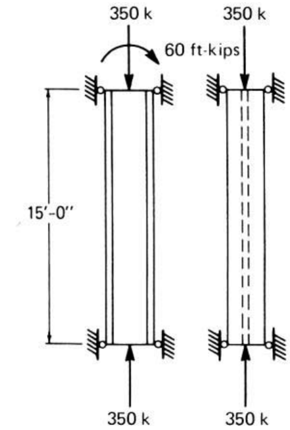
$G_A =$ 1.0 (fixed support) $G_B = G_C = 10.0$ (pinned support)

$$G_D = \frac{\frac{285}{15}}{\frac{448}{20}} = 0.85 \qquad G_E = \frac{\frac{285}{15} + \frac{204}{12}}{\frac{448}{20} + \frac{340}{18}} = 0.87$$

$$G_F = \frac{\frac{285}{15} + \frac{204}{12}}{\frac{340}{18}} = 1.91 \qquad G_G = G_H = \frac{\frac{204}{12}}{\frac{340}{18}} = 0.90$$

Column	G_{Top}	G_{Bot}	K		KL/r
AD	0.85	1.0	0.76	Braced	0.76(15)(12)/5.25 = 26.1
BE	0.87	10.0	0.85	Braced	0.85(15)(12)/5.25 = 29.1
CF	1.91	10.0	0.90	Braced	0.90(15)(12)/5.25 = 30.9
EG	0.90	0.87	1.29	Unbraced	1.29(12)(12)/5.17 = 35.9
FH	0.90	1.91	1.43	Unbraced	1.43(12)(12)/5.17 = 39.8

Table 6-1: Column effective length factors and slenderness ratios for Example 6-1.



Example 14

Investigate the accepatbility of a W16 x 67 used as a beam-column under the unfactored loading shown in the figure. It is A992 steel (F_y = 50 ksi). Assume 25% of the load is dead load with 75% live load.

SOLUTION:

DESIGN LOADS (shown on figure):

Axial load = 1.2(0.25)(350k)+1.6(0.75)(350k)=525k

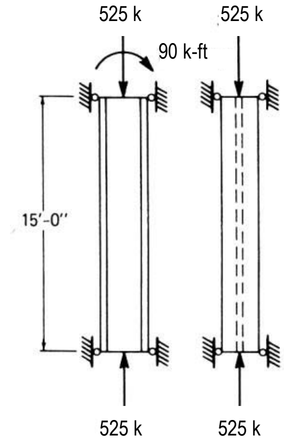
Moment at joint = 1.2(0.25)(60 $^{k\text{-}ft}$) + 1.6(0.75)(60 $^{k\text{-}ft}$) = 90 $^{k\text{-}ft}$

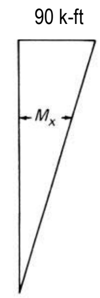
Determine column capacity and fraction to choose the appropriate interaction equation:

$$\frac{kL}{r_x} = \frac{15ft(12^{in}/_{ft})}{6.96in} = 25.9 \quad \text{and} \quad \frac{kL}{r_y} = \frac{15ft(12^{in}/_{ft})}{2.46in} = 73 \quad \text{(governs)}$$

$$P_c = \phi_c P_n = \phi_c F_{cr} A_g = (30.5ksi)19.7in^2 = 600.85k$$

$$\frac{P_r}{P_c} = \frac{525k}{600.85k} = 0.87 > 0.2 \quad \text{so use} \quad \frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \le 1.0$$





There is no bending about the y axis, so that term will not have any values.

Determine the bending moment capacity in the x direction:

The unbraced length to use the full plastic moment (L_p) is listed as 8.69 ft, and we are over that so of we don't want to determine it from formula, we can find the beam in the Available Moment vs. Unbraced Length tables. The value of ϕM_n at L_b =15 ft is 422 k-ft.

Determine the magnification factor when $M_1 = 0$, M_2 = 90 k-ft:

$$C_m = 0.6 - 0.4\frac{M_1}{M_2} = 0.6 - \frac{0^{k-ft}}{90^{k-ft}} = 0.6 \le 1.0 \qquad P_{e1} = \frac{\pi^2 EA}{\left(Kl/_r\right)^2} = \frac{\pi^2(30x10^3 ksi)19.7in^2}{(25.9)^2} = 8{,}695.4k$$

$$B_1 = \frac{C_m}{1-(P_u/P_{e1})} = \frac{0.6}{1-(525k/8695.4k)} = 0.64 \ngeq 1.0 \quad \text{USE 1.0} \qquad M_u = (1)90 \text{ k-ft}$$

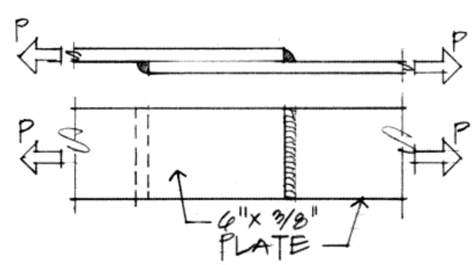
Finally, determine the interaction value:

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.87 + \frac{8}{9}\left(\frac{90^{k-ft}}{422^{k-ft}}\right) = 1.06 \nleq 1.0$$

This is **NOT OK.** (and outside error tolerance). The section should be larger.

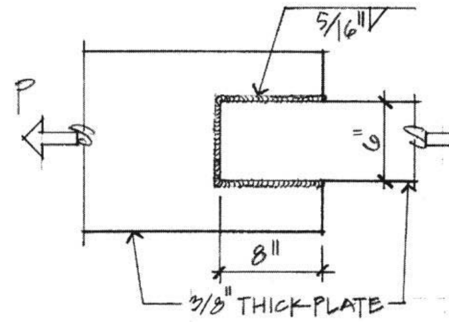
Example 15

10.9 Determine the maximum load carrying capacity of this lap joint., assuming A36 steel with E60XX electrodes.



Example 16

10.7 Determine the capacity of the connection in Figure 10.44 assuming A36 steel with E70XX electrodes.



Solution:

Capacity of weld:

For a 5⁄16″ fillet weld, ϕS = 6.96 k/in

Weld length = 8 in + 6 in + 8 in = 22 in.

Weld capacity = 22″ × 6.96 k/in = 153.1 k

Capacity of plate:

$\phi P_n = \phi F_y A_g \quad \phi = 0.9$

Plate capacity = 0.9 x 36 k/in^2 x 3/8″ x 6″= 72.9 k

∴ Plate capacity governs, P_{allow} = 72.9 k

The weld size used is obviously too strong. What size, then, can the weld be reduced to so that the weld strength is more compatible to the plate capacity? To make the weld capacity ≈ plate capacity:

22″ × (weld capacity per in.) = 72.9 k

Weld capacity per inch = $\dfrac{72.9\text{ k}}{22\text{ in.}}$ = 3.31 k/in.

From Available Strength table, use 3/16″ weld (ϕS = 4.18 k/in.)

Minimum size fillet = 3⁄16″ based on a 3⁄8″ thick plate.

Example 17

10.5 Using the AISC framed beam connection bolt shear in Table 7-1, determine the shear adequacy of the connection shown in Figure 10.28. What thickness and angle length are required? Also determine the bearing capacity of the wide flange sections.

Factored end beam reaction = 90 k.

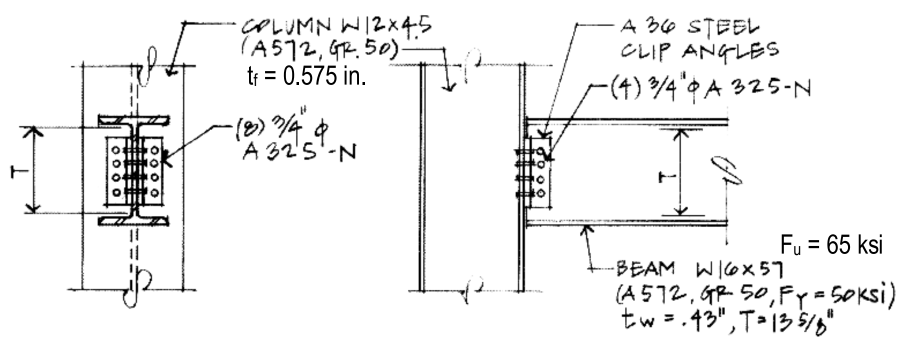
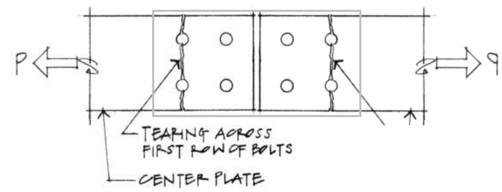


Figure 10.28 Typical beam–column connection.

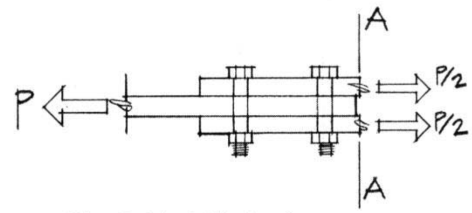
Example 18

10.2 The butt splice shown in Figure 10.22 uses two 8 × ⅜" plates to "sandwich" in the 8 × ½" plates being joined. Four ⅞"ϕ A325-SC bolts are used on both sides of the splice. Assuming A36 steel and standard round holes, determine the allowable capacity of the connection.



SOLUTION:

Shear, bearing and net tension will be checked to determine the critical conditions that governs the capacity of the connection.



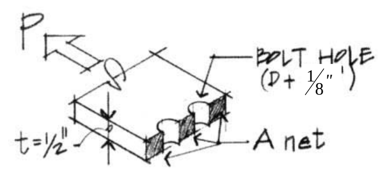
SECTION CUT A-A

Shear: Using the AISC available shear in Table 7-3 (Group A):

ϕR_n = 26.4 k/bolt x 4 bolts = 105.6 k

Bearing: Using the AISC available bearing in Table 7-4:

There are 4 bolts bearing on the center (1/2") plate, while there are 4 bolts bearing on a total width of two sandwich plates (3/4" total). The thinner bearing width will govern. Assume 3 in. spacing (center to center) of bolts. For A36 steel, F_u = 58 ksi.



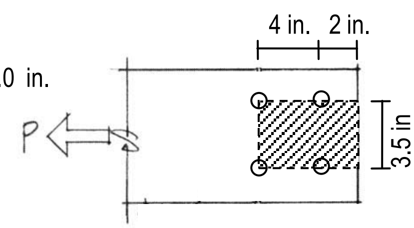
ϕR_n = 91.4 k/bolt/in. x 0.5 in. x 4 bolts = 182.8 k (Table 7-4)

With the edge distance of 2 in., the bearing capacity might be smaller from Table 7-5 which says the distance should be 2 ¼ in for full bearing (and we have 2 in.).

ϕR_n = 79.9 k/bolt/in. x 0.5 in. x 4 bolts = 159.8 k

Tension: The center plate is critical, again, because its thickness is less than the combined thicknesses of the two outer plates. We must consider tension yielding and tension rupture:

$\phi R_n = \phi F_y A_g$ *and* $\phi R_n = \phi F_u A_e$ where $A_e = A_{net}U$



A_g = 8 in. x ½ in. = 4 in^2

The holes are considered 1/8 in. larger than the bolt hole diameter = (7/8 + 1/8) = 1.0 in.

A_n = (8 in. – 2 holes x 1.0 in.) x ½ in. = 3.0 in^2

The whole cross section sees tension, so the shear lag factor U = 1

$\phi F_y A_g$ = 0.9 x 36 ksi x 4 in^2 = 129.6 k

$\phi F_u A_e$ = 0.75 x 58 ksi x (1) x 3.0 in^2 = 130.5 k

The maximum connection capacity (*smallest value*) **so far** is governed by bolt shear: $\underline{\phi R_n = 105.6 \text{ k}}$

Block Shear Rupture: It is possible for the center plate to rip away from the sandwich plates leaving the block (shown hatched) behind:

$\phi R_n = \phi(0.6F_uA_{nv} + U_{bs}F_uA_{nt}) \leq \phi(0.6F_yA_{gv} + U_{bs}F_uA_{nt})$

where A_{nv} is the area resisting shear, A_{nt} is the area resisting tension, A_{gv} is the gross area resisting shear, and U_{bs} = 1 when the tensile stress is uniform.

A_{gv} = 2 x (4 + 2 in.) x ½ in. = 6 in^2

$A_{nv} = A_{gv}$ – 1 ½ holes areas = 6 in^2 – 1.5 x 1 in. x ½ in. = 5.25 in^2

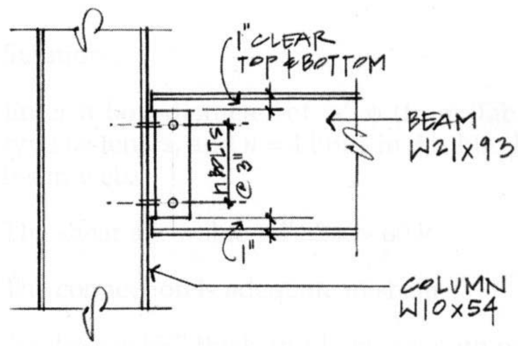
A_{nt} = 3.5 in. x t – 2(½ hole areas) = 3.5 in. x ½ in – 1 x 1 in. x ½ in. = 1.25 in^2

$\phi(0.6F_uA_{nv} + U_{bs}F_uA_{nt})$ = 0.75 x (0.6 x 58 ksi x 5.25 in^2 + 1 x 58 ksi x 1.25 in^2) = 191.4 k

$\phi(0.6F_yA_{gv} + U_{bs}F_uA_{nt})$ = 0.75 x (0.6 x 36 ksi x 6 in^2 + 1 x 58 ksi x 1.25 in^2) = 151.6 k

The maximum connection capacity (*smallest value*) is governed by block shear rupture: $\underline{\phi R_n = 151.6 \text{ k}}$

Example 19



The steel used in the connection and beams is A992 with F_y = 50 ksi, and F_u = 65 ksi. Using A490-N bolt material, determine the maximum capacity of the connection based on shear in the bolts, bearing in all materials and pick the number of bolts and angle length (not staggered). Use A36 steel for the angles.

W21x93: d = 21.62 in, t_w = 0.58 in, t_f = 0.93 in
W10x54: t_f = 0.615 in

SOLUTION:

The maximum length the angles can be depends on how it fits between the top and bottom flange with some clearance allowed for the fillet to the flange, and getting an air wrench in to tighten the bolts. This example uses 1" of clearance:

Available length = beam depth – both flange thicknesses – 1" clearance at top & 1" at bottom

= 21.62 in – 2(0.93 in) – 2(1 in) = 17.76 in.

With the spaced at 3 in. and 1 ¼ in. end lengths (each end), the maximum number of bolts can be determined:

Available length ≥ 1.25 in. + 1.25 in. + 3 in. x (number of bolts – 1)

number of bolts ≤ (17.76 in – 2.5 in. - (-3 in.))/3 in. = 6.1, so 6 bolts.

It is helpful to have the All-bolted Double-Angle Connection Tables 10-1. They are available for ¾", 7/8", and 1" bolt diameters and list angle thicknesses of ¼", 5/16", 3/8", and ½". Increasing the angle thickness is likely to increase the angle strength, although the limit states include shear yielding of the angles, shear rupture of the angles, and block shear rupture of the angles.

For these diameters, the available **shear** (double) from Table 7-1 for 6 bolts is (6)45.1 k/bolt = 270.6 kips, (6)61.3 k/bolt = 367.8 kips, and (6)80.1 k/bolt = 480.6 kips.

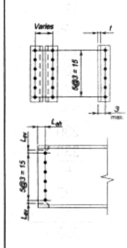
Tables 10-1 (not all provided here) list a bolt and angle available strength of 271 kips for the ¾" bolts, 296 kips for the 7/8" bolts, and 281 kips for the 1" bolts. It appears that increasing the bolt diameter to 1" will not gain additional load. Use 7/8" bolts.

Table 10-1 (continued) All-Bolted Double-Angle Connections — 7/8-in. Bolts

Beam F_y = 50 ksi, F_u = 65 ksi; Angle F_y = 36 ksi, F_u = 58 ksi

Bolt and Angle Available Strength, kips — 6 Rows — W40, 36, 33, 30, 27, 24, 21

Bolt Group	Thread Cond.	Hole Type	1/4 ASD	1/4 LRFD	5/16 ASD	5/16 LRFD	3/8 ASD	3/8 LRFD	1/2 ASD	1/2 LRFD
Group A	N	STD	98.6	148	123	185	148	222	195	292
	X	STD	98.6	148	123	185	148	222	197	296
	SC Class A	STD	98.6	148	106	159	106	159	106	159
		OVS	90.1	135	90.1	135	90.1	135	90.1	135
		SSLT	97.3	146	106	159	106	159	106	159
	SC Class B	STD	98.6	148	123	185	148	222	176	264
		OVS	93.5	140	117	175	140	210	150	225
		SSLT	97.3	146	122	182	146	219	176	264
Group B	N	STD	98.6	148	123	185	148	222	197	296
	X	STD	98.6	148	123	185	148	222	197	296
	SC Class A	STD	98.6	148	123	185	133	199	133	199
		OVS	93.5	140	113	169	113	169	113	169
		SSLT	97.3	146	122	182	133	199	133	199
	SC Class B	STD	98.6	148	123	185	148	222	197	296
		OVS	93.5	140	117	175	140	210	187	281
		SSLT	97.3	146	122	182	146	219	195	292



ϕR_n = 367.8 kips for double shear of 7/8" bolts ϕR_n = 296 kips for limit state in angles

We also need to evaluate **bearing** of bolts on the beam web, and column flange where there are bolt holes. Table 7-4 provides available bearing strength for the material type, bolt diameter, hole type, and spacing per inch of material thicknesses.

a) Bearing for beam web: There are *6* bolt holes through the beam web. This is typically the critical bearing limit value because there are two angle legs that resist bolt bearing and twice as many bolt holes to the column. The material is A992 (F_u = 65 ksi), 0.58" thick, with 7/8" bolt diameters at 3 in. spacing.

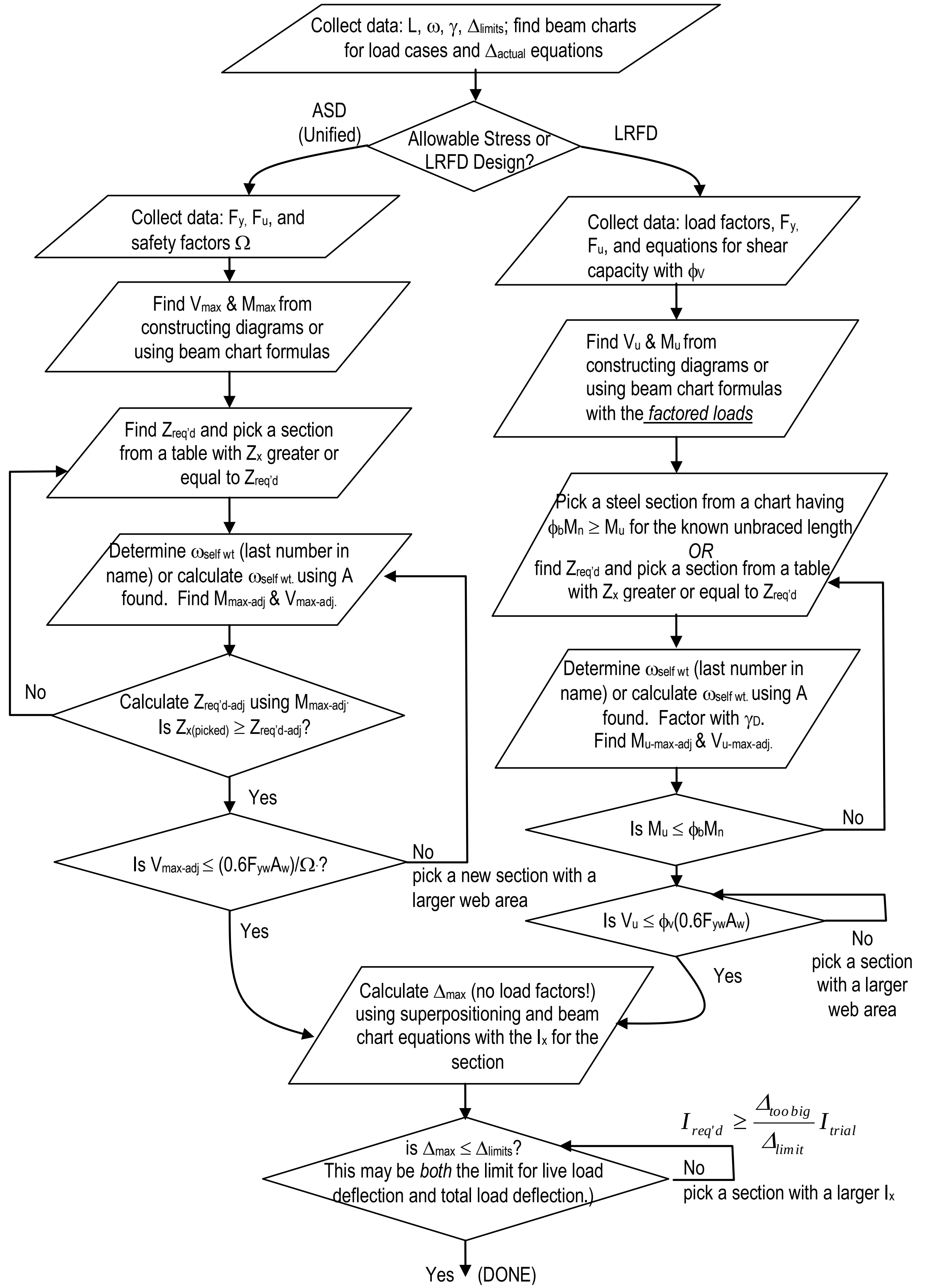
ϕR_n = 6 bolts·(102 k/bolt/inch)·(0.58 in) = 355.0 kips

b) Bearing for column flange: There are 12 bolt holes through the column. The material is A992 (F_u = 65 ksi), 0.615" thick, with 1" bolt diameters.

ϕR_n = 12 bolts·(102 k/bolt/inch)·(0.615 in) = 752.8 kips

Although, the bearing in the beam web is the smallest at 355 kips, with the shear on the bolts even smaller at 324.6 kips, *the maximum capacity for the simple-shear connector is 296 kips* limited by the critical capacity of the angles.

Beam Design Flow Chart



Listing of W Shapes in Descending order of Z_x for Beam Design

Z_x – US (in.^{3})	I_x – US (in.^{4})	Section	I_x – SI (10^6mm.^{4})	Z_x – SI (10^3mm.3)
514	**7450**	**W33X141**	**3100**	**8420**
511	5680	W24X176	2360	8370
509	**7800**	**W36X135**	**3250**	**8340**
500	6680	W30X148	2780	8190
490	4330	W18X211	1800	8030
487	3400	W14X257	1420	7980
481	3110	W12X279	1290	7880
476	4730	W21X182	1970	7800
468	5170	W24X162	2150	7670
467	**6710**	**W33X130**	**2790**	**7650**
464	5660	W27X146	2360	7600
442	3870	W18X192	1610	7240
437	5770	W30X132	2400	7160
436	3010	W14X233	1250	7140
432	4280	W21X166	1780	7080
428	2720	W12X252	1130	7010
418	4580	W24X146	1910	6850
415	**5900**	**W33X118**	**2460**	**6800**
408	5360	W30X124	2230	6690
398	3450	W18X175	1440	6520
395	4760	W27X129	1980	6470
390	2660	W14X211	1110	6390
386	2420	W12X230	1010	6330
378	**4930**	**W30X116**	**2050**	**6190**
373	3630	W21X147	1510	6110
370	4020	W24X131	1670	6060
356	3060	W18X158	1270	5830
355	2400	W14X193	999	5820
348	2140	W12X210	891	5700
346	**4470**	**W30X108**	**1860**	**5670**
343	4080	W27X114	1700	5620
333	3220	W21X132	1340	5460
327	3540	W24X117	1470	5360
322	2750	W18X143	1140	5280
320	2140	W14X176	891	5240
312	**3990**	**W30X99**	**1660**	**5110**
311	1890	W12X190	787	5100
307	2960	W21X122	1230	5030
305	3620	W27X102	1510	5000
290	2460	W18X130	1020	4750
289	3100	W24X104	1290	4740
287	1900	W14X159	791	4700
283	**3610**	**W30X90**	**1500**	**4640**
280	3000	W24X103	1250	4590
279	2670	W21X111	1110	4570
278	3270	W27X94	1360	4560
275	1650	W12X170	687	4510
262	2190	W18X119	912	4290
260	1710	W14X145	712	4260
254	2700	W24X94	1120	4160
253	2420	W21X101	1010	4150
244	**2850**	**W27X84**	**1190**	**4000**
243	1430	W12X152	595	3980
234	1530	W14X132	637	3830
230	1910	W18X106	795	3770
224	**2370**	**W24X84**	**986**	**3670**
221	2070	W21X93	862	3620
214	1240	W12X136	516	3510
212	1380	W14X120	574	3470
211	1750	W18X97	728	3460
200	**2100**	**W24X76**	**874**	**3280**
198	1490	W16X100	620	3240
196	1830	W21X83	762	3210
192	1240	W14X109	516	3150
186	1530	W18X86	637	3050
186	1070	W12X120	445	3050
177	**1830**	**W24X68**	**762**	**2900**
175	1300	W16X89	541	2870
173	1110	W14X99	462	2830
172	1600	W21X73	666	2820
164	933	W12X106	388	2690
163	1330	W18X76	554	2670
160	**1480**	**W21X68**	**616**	**2620**
157	999	W14X90	416	2570
153	**1550**	**W24X62**	**645**	**2510**
150	1110	W16X77	462	2460
147	833	W12X96	347	2410
147	716	W10X112	298	2410
146	1170	W18X71	487	2390

(continued)

Listing of W Shapes in Descending order of Z_x for Beam Design (Continued)

Z_x – US (in.^{3})	I_x – US (in.^{4})	Section	I_x – SI (10^{6}mm.^{4})	Z_x – SI (10^{3}mm.3)
144	**1330**	**W21X62**	**554**	**2360**
139	881	W14X82	367	2280
134	**1350**	**W24X55**	**562**	**2200**
133	1070	W18X65	445	2180
132	740	W12X87	308	2160
130	954	W16X67	397	2130
130	623	W10X100	259	2130
129	1170	W21X57	487	2110
126	**1140**	**W21X55**	**475**	**2060**
126	795	W14X74	331	2060
123	984	W18X60	410	2020
119	662	W12X79	276	1950
115	722	W14X68	301	1880
113	534	W10X88	222	1850
112	**890**	**W18X55**	**370**	**1840**
110	**984**	**W21X50**	**410**	**1800**
108	597	W12X72	248	1770
107	**959**	**W21X48**	**399**	**1750**
105	758	W16X57	316	1720
102	640	W14X61	266	1670
101	800	W18X50	333	1660
97.6	455	W10X77	189	1600
96.8	533	W12X65	222	1590
95.4	**843**	**W21X44**	**351**	**1560**
92.0	659	W16X50	274	1510
90.7	712	W18X46	296	1490
87.1	541	W14X53	225	1430
86.4	475	W12X58	198	1420
85.3	394	W10X68	164	1400
82.3	586	W16X45	244	1350
78.4	**612**	**W18X40**	**255**	**1280**
78.4	484	W14X48	201	1280
77.9	425	W12X53	177	1280
74.6	341	W10X60	142	1220
73.0	**518**	**W16X40**	**216**	**1200**
71.9	391	W12X50	163	1180
70.1	272	W8X67	113	1150
69.6	428	W14X43	178	1140
66.6	303	W10X54	126	1090

Z_x – US (in.^{3})	I_x – US (in.^{4})	Section	I_x – SI (10^{6}mm.^{4})	Z_x – SI (10^{3}mm.3)
66.5	**510**	**W18X35**	**212**	**1090**
64.2	348	W12X45	145	1050
64.0	448	W16X36	186	1050
61.5	385	W14X38	160	1010
60.4	272	W10X49	113	990
59.8	228	W8X58	94.9	980
57.0	307	W12X40	128	934
54.9	248	W10X45	103	900
54.6	**340**	**W14X34**	**142**	**895**
54.0	**375**	**W16X31**	**156**	**885**
51.2	285	W12X35	119	839
49.0	184	W8X48	76.6	803
47.3	**291**	**W14X30**	**121**	**775**
46.8	209	W10X39	87.0	767
44.2	**301**	**W16X26**	**125**	**724**
43.1	238	W12X30	99.1	706
40.2	**245**	**W14X26**	**102**	**659**
39.8	146	W8X40	60.8	652
38.8	171	W10X33	71.2	636
37.2	**204**	**W12X26**	**84.9**	**610**
36.6	170	W10X30	70.8	600
34.7	127	W8X35	52.9	569
33.2	**199**	**W14X22**	**82.8**	**544**
31.3	144	W10X26	59.9	513
30.4	110	W8X31	45.8	498
29.3	**156**	**W12X22**	**64.9**	**480**
27.2	98.0	W8X28	40.8	446
26.0	**118**	**W10X22**	**49.1**	**426**
24.7	**130**	**W12X19**	**54.1**	**405**
23.1	82.7	W8X24	34.4	379
21.6	**96.3**	**W10X19**	**40.1**	**354**
20.4	75.3	W8X21	31.3	334
20.1	**103**	**W12x16**	**42.9**	**329**
18.7	81.9	W10X17	34.1	306
17.4	**88.6**	**W12X14**	**36.9**	**285**
17.0	61.9	W8X18	25.8	279
16.0	68.9	W10X15	28.7	262
13.6	48.0	W8X15	20.0	223
12.6	**53.8**	**W10X12**	**22.4**	**206**
11.4	39.6	W8X13	16.5	187
8.87	**30.8**	**W8X10**	**12.8**	**145**

Available Critical Stress, $\phi_c F_{cr}$, for Compression Members, ksi ($F_y = 36$ ksi and $\phi_c = 0.90$)

KL/r	$\phi_c F_{cr}$	*KL/r*	$\phi_c F_{cr}$	*KL/r*	$\phi_c F_{cr}$	*KL/r*	$\phi_c F_{cr}$	*KL/r*	$\phi_c F_{cr}$
1	32.4	41	29.7	81	22.9	121	15.0	161	8.72
2	32.4	42	29.5	82	22.7	122	14.8	162	8.61
3	32.4	43	29.4	83	22.5	123	14.6	163	8.50
4	32.4	44	29.3	84	22.3	124	14.4	164	8.40
5	32.4	45	29.1	85	22.1	125	14.2	165	8.30
6	32.3	46	29.0	86	22.0	126	14.0	166	8.20
7	32.3	47	28.8	87	21.8	127	13.9	167	8.10
8	32.3	48	28.7	88	21.6	128	13.7	168	8.00
9	32.3	49	28.6	89	21.4	129	13.5	169	7.91
10	32.2	50	28.4	90	21.2	130	13.3	170	7.82
11	32.2	51	28.3	91	21.0	131	13.1	171	7.73
12	32.2	52	28.1	92	20.8	132	12.9	172	7.64
13	32.1	53	27.9	93	20.5	133	12.8	173	7.55
14	32.1	54	27.8	94	20.3	134	12.6	174	7.46
15	32.0	55	27.6	95	20.1	135	12.4	175	7.38
16	32.0	56	27.5	96	19.9	136	12.2	176	7.29
17	31.9	57	27.3	97	19.7	137	12.0	177	7.21
18	31.9	58	27.1	98	19.5	138	11.9	178	7.13
19	31.8	59	27.0	99	19.3	139	11.7	179	7.05
20	31.7	60	26.8	100	19.1	140	11.5	180	6.97
21	31.7	61	26.6	101	18.9	141	11.4	181	6.90
22	31.6	62	26.5	102	18.7	142	11.2	182	6.82
23	31.5	63	26.3	103	18.5	143	11.0	183	6.75
24	31.4	64	26.1	104	18.3	144	10.9	184	6.67
25	31.4	65	25.9	105	18.1	145	10.7	185	6.60
26	31.3	66	25.8	106	17.9	146	10.6	186	6.53
27	31.2	67	25.6	107	17.7	147	10.5	187	6.46
28	31.1	68	25.4	108	17.5	148	10.3	188	6.39
29	31.0	69	25.2	109	17.3	149	10.2	189	6.32
30	30.9	70	25.0	110	17.1	150	10.0	190	6.26
31	30.8	71	24.8	111	16.9	151	9.91	191	6.19
32	30.7	72	24.7	112	16.7	152	9.78	192	6.13
33	30.6	73	24.5	113	16.5	153	9.65	193	6.06
34	30.5	74	24.3	114	16.3	154	9.53	194	6.00
35	30.4	75	24.1	115	16.2	155	9.40	195	5.94
36	30.3	76	23.9	116	16.0	156	9.28	196	5.88
37	30.1	77	23.7	117	15.8	157	9.17	197	5.82
38	30.0	78	23.5	118	15.6	158	9.05	198	5.76
39	29.9	79	23.3	119	15.4	159	8.94	199	5.70
40	29.8	80	23.1	120	15.2	160	8.82	200	5.65

Available Critical Stress, $\phi_c F_{cr}$, for Compression Members, ksi (F_y = 50 ksi and ϕ_c = 0.90)

KL/r	$\phi_c F_{cr}$	KL/r	$\phi_c F_{cr}$	KL/r	$\phi_c F_{cr}$	KL/r	$\phi_c F_{cr}$	KL/r	$\phi_c F_{cr}$
1	45.0	41	39.8	81	27.9	121	15.4	161	8.72
2	45.0	42	39.6	82	27.5	122	15.2	162	8.61
3	45.0	43	39.3	83	27.2	123	14.9	163	8.50
4	44.9	44	39.1	84	26.9	124	14.7	164	8.40
5	44.9	45	38.8	85	26.5	125	14.5	165	8.30
6	44.9	46	38.5	86	26.2	126	14.2	166	8.20
7	44.8	47	38.3	87	25.9	127	14.0	167	8.10
8	44.8	48	38.0	88	25.5	128	13.8	168	8.00
9	44.7	49	37.8	89	25.2	129	13.6	169	7.91
10	44.7	50	37.5	90	24.9	130	13.4	170	7.82
11	44.6	51	37.2	91	24.6	131	13.2	171	7.73
12	44.5	52	36.9	92	24.2	132	13.0	172	7.64
13	44.4	53	36.6	93	23.9	133	12.8	173	7.55
14	44.4	54	36.4	94	23.6	134	12.6	174	7.46
15	44.3	55	36.1	95	23.3	135	12.4	175	7.38
16	44.2	56	35.8	96	22.9	136	12.2	176	7.29
17	44.1	57	35.5	97	22.6	137	12.0	177	7.21
18	43.9	58	35.2	98	22.3	138	11.9	178	7.13
19	43.8	59	34.9	99	22.0	139	11.7	179	7.05
20	43.7	60	34.6	100	21.7	140	11.5	180	6.97
21	43.6	61	34.3	101	21.3	141	11.4	181	6.90
22	43.4	62	34.0	102	21.0	142	11.2	182	6.82
23	43.3	63	33.7	103	20.7	143	11.0	183	6.75
24	43.1	64	33.4	104	20.4	144	10.9	184	6.67
25	43.0	65	33.0	105	20.1	145	10.7	185	6.60
26	42.8	66	32.7	106	19.8	146	10.6	186	6.53
27	42.7	67	32.4	107	19.5	147	10.5	187	6.46
28	42.5	68	32.1	108	19.2	148	10.3	188	6.39
29	42.3	69	31.8	109	18.9	149	10.2	189	6.32
30	42.1	70	31.4	110	18.6	150	10.0	190	6.26
31	41.9	71	31.1	111	18.3	151	9.91	191	6.19
32	41.8	72	30.8	112	18.0	152	9.78	192	6.13
33	41.6	73	30.5	113	17.7	153	9.65	193	6.06
34	41.4	74	30.2	114	17.4	154	9.53	194	6.00
35	41.1	75	29.8	115	17.1	155	9.40	195	5.94
36	40.9	76	29.5	116	16.8	156	9.28	196	5.88
37	40.7	77	29.2	117	16.5	157	9.17	197	5.82
38	40.5	78	28.8	118	16.2	158	9.05	198	5.76
39	40.3	79	28.5	119	16.0	159	8.94	199	5.70
40	40.0	80	28.2	120	15.7	160	8.82	200	5.65

Bolt Strength Tables

Table 7-1
Available Shear Strength of Bolts, kips

Nominal Bolt Diameter, d, in.					5/8		3/4		7/8		1	
Nominal Bolt Area, in.^{2}					0.307		0.442		0.601		0.785	
ASTM Desig.	Thread Cond.	F_{nv}/Ω (ksi)	ϕF_{nv} (ksi)	Loading	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n
		ASD	LRFD		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Group A	N	27.0	40.5	S	8.29	12.4	11.9	17.9	16.2	24.3	21.2	31.8
				D	16.6	24.9	23.9	35.8	32.5	48.7	42.4	63.6
	X	34.0	51.0	S	10.4	15.7	15.0	22.5	20.4	30.7	26.7	40.0
				D	20.9	31.3	30.1	45.1	40.9	61.3	53.4	80.1
Group B	N	34.0	51.0	S	10.4	15.7	15.0	22.5	20.4	30.7	26.7	40.0
				D	20.9	31.3	30.1	45.1	40.9	61.3	53.4	80.1
	X	42.0	63.0	S	12.9	19.3	18.6	27.8	25.2	37.9	33.0	49.5
				D	25.8	38.7	37.1	55.7	50.5	75.7	65.9	98.9
A307	–	13.5	20.3	S	4.14	6.23	5.97	8.97	8.11	12.2	10.6	15.9
				D	8.29	12.5	11.9	17.9	16.2	24.4	21.2	31.9
Nominal Bolt Diameter, d, in.					1 1/8		1 1/4		1 3/8		1 1/2	
Nominal Bolt Area, in.^{2}					0.994		1.23		1.48		1.77	
ASTM Desig.	Thread Cond.	F_{nv}/Ω (ksi)	ϕF_{nv} (ksi)	Loading	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n
		ASD	LRFD		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Group A	N	27.0	40.5	S	26.8	40.3	33.2	49.8	40.0	59.9	47.8	71.7
				D	53.7	80.5	66.4	99.6	79.9	120	95.6	143
	X	34.0	51.0	S	33.8	50.7	41.8	62.7	50.3	75.5	60.2	90.3
				D	67.6	101	83.6	125	101	151	120	181
Group B	N	34.0	51.0	S	33.8	50.7	41.8	62.7	50.3	75.5	60.2	90.3
				D	67.6	101	83.6	125	101	151	120	181
	X	42.0	63.0	S	41.7	62.6	51.7	77.5	62.2	93.2	74.3	112
				D	83.5	125	103	155	124	186	149	223
A307	–	13.5	20.3	S	13.4	20.2	16.6	25.0	20.0	30.0	23.9	35.9
				D	26.8	40.4	33.2	49.9	40.0	60.1	47.8	71.9
ASD	LRFD	For end loaded connections greater than 38 in., see AISC *Specification* Table J3.2 footnote b.										
$\Omega = 2.00$	$\phi = 0.75$											

Table 7-2
Available Tensile Strength of Bolts, kips

Nominal Bolt Diameter, d, in.			5/8		3/4		7/8		1	
Nominal Bolt Area, in.^{2}			0.307		0.442		0.601		0.785	
ASTM Desig.	F_{nt}/Ω (ksi)	ϕF_{nt} (ksi)	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Group A	45.0	67.5	13.8	20.7	19.9	29.8	27.1	40.6	35.3	53.0
Group B	56.5	84.8	17.3	26.0	25.0	37.4	34.0	51.0	44.4	66.6
A307	22.5	33.8	6.90	10.4	9.94	14.9	13.5	20.3	17.7	26.5
Nominal Bolt Diameter, d, in.			1 1/8		1 1/4		1 3/8		1 1/2	
Nominal Bolt Area, in.^{2}			0.994		1.23		1.48		1.77	
ASTM Desig.	F_{nt}/Ω (ksi)	ϕF_{nt} (ksi)	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n	r_n/Ω	ϕr_n
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Group A	45.0	67.5	44.7	67.1	55.2	82.8	66.8	100	79.5	119
Group B	56.5	84.8	56.2	84.2	69.3	104	83.9	126	99.8	150
A307	22.5	33.8	22.4	33.5	27.6	41.4	33.4	50.1	39.8	59.6
ASD	LRFD									
$\Omega = 2.00$	$\phi = 0.75$									

Group A Bolts

A325, A325M
F1858
A354 Grade BC
A449

Table 7-3
Slip-Critical Connections
Available Shear Strength, kips
(Class A Faying Surface, $\mu = 0.30$)

Group A Bolts

Hole Type	Loading	Nominal Bolt Diameter, *d*, in. 5/8 — Min. Pretension 19 kips: r_n/Ω ASD	ϕr_n LRFD	3/4 — 28: r_n/Ω ASD	ϕr_n LRFD	7/8 — 39: r_n/Ω ASD	ϕr_n LRFD	1 — 51: r_n/Ω ASD	ϕr_n LRFD
STD/SSLT	S	4.29	6.44	6.33	9.49	8.81	13.2	11.5	17.3
	D	8.59	12.9	12.7	19.0	17.6	26.4	23.1	34.6
OVS/SSLP	S	3.66	5.47	5.39	8.07	7.51	11.2	9.82	14.7
	D	7.32	10.9	10.8	16.1	15.0	22.5	19.6	29.4
LSL	S	3.01	4.51	4.44	6.64	6.18	9.25	8.08	12.1
	D	6.02	9.02	8.87	13.3	12.4	18.5	16.2	24.2

Hole Type	Loading	Nominal Bolt Diameter, *d*, in. 1 1/8 — Min. Pretension 56 kips: r_n/Ω ASD	ϕr_n LRFD	1 1/4 — 71: r_n/Ω ASD	ϕr_n LRFD	1 3/8 — 85: r_n/Ω ASD	ϕr_n LRFD	1 1/2 — 103: r_n/Ω ASD	ϕr_n LRFD
STD/SSLT	S	12.7	19.0	16.0	24.1	19.2	28.8	23.3	34.9
	D	25.3	38.0	32.1	48.1	38.4	57.6	46.6	69.8
OVS/SSLP	S	10.8	16.1	13.7	20.5	16.4	24.5	19.8	29.7
	D	21.6	32.3	27.4	40.9	32.7	49.0	39.7	59.4
LSL	S	8.87	13.3	11.2	16.8	13.5	20.2	16.3	24.4
	D	17.7	26.6	22.5	33.7	26.9	40.3	32.6	48.9

STD = standard hole
OVS = oversized hole
SSLT = short-slotted hole transverse to the line of force
SSLP = short-slotted hole parallel to the line of force
LSL = long-slotted hole transverse or parallel to the line of force

S = single shear
D = double shear

Hole Type	ASD	LRFD
STD and SSLT	$\Omega = 1.50$	$\phi = 1.00$
OVS and SSLP	$\Omega = 1.76$	$\phi = 0.85$
LSL	$\Omega = 2.14$	$\phi = 0.70$

Note: Slip-critical bolt values assume no more than one filler has been provided or bolts have been added to distribute loads in the fillers.
See AISC *Specification* Sections J3.8 and J5 for provisions when fillers are present.
For Class B faying surfaces, multiply the tabulated available strength by 1.67.

Group B Bolts

A490, A490M
F2280
A354 Grade BD

Table 7-3 (continued)
Slip-Critical Connections
Available Shear Strength, kips
(Class A Faying Surface, $\mu = 0.30$)

Group B Bolts

Hole Type	Loading	Nominal Bolt Diameter, *d*, in. 5/8 — Min. Pretension 24 kips: r_n/Ω ASD	ϕr_n LRFD	3/4 — 35: r_n/Ω ASD	ϕr_n LRFD	7/8 — 49: r_n/Ω ASD	ϕr_n LRFD	1 — 64: r_n/Ω ASD	ϕr_n LRFD
STD/SSLT	S	5.42	8.14	7.91	11.9	11.1	16.6	14.5	21.7
	D	10.8	16.3	15.8	23.7	22.1	33.2	28.9	43.4
OVS/SSLP	S	4.62	6.92	6.74	10.1	9.44	14.1	12.3	18.4
	D	9.25	13.8	13.5	20.2	18.9	28.2	24.7	36.9
LSL	S	3.80	5.70	5.54	8.31	7.76	11.6	10.1	15.2
	D	7.60	11.4	11.1	16.6	15.5	23.3	20.3	30.4

Hole Type	Loading	Nominal Bolt Diameter, *d*, in. 1 1/8 — Min. Pretension 80 kips: r_n/Ω ASD	ϕr_n LRFD	1 1/4 — 102: r_n/Ω ASD	ϕr_n LRFD	1 3/8 — 121: r_n/Ω ASD	ϕr_n LRFD	1 1/2 — 148: r_n/Ω ASD	ϕr_n LRFD
STD/SSLT	S	18.1	27.1	23.1	34.6	27.3	41.0	33.4	50.2
	D	36.2	54.2	46.1	69.2	54.7	82.0	66.9	100
OVS/SSLP	S	15.4	23.1	19.6	29.4	23.3	34.9	28.5	42.6
	D	30.8	46.1	39.3	58.8	46.6	69.7	57.0	85.3
LSL	S	12.7	19.0	16.2	24.2	19.2	28.7	23.4	35.1
	D	25.3	38.0	32.3	48.4	38.3	57.4	46.9	70.2

STD = standard hole
OVS = oversized hole
SSLT = short-slotted hole transverse to the line of force
SSLP = short-slotted hole parallel to the line of force
LSL = long-slotted hole transverse or parallel to the line of force

S = single shear
D = double shear

Hole Type	ASD	LRFD
STD and SSLT	$\Omega = 1.50$	$\phi = 1.00$
OVS and SSLP	$\Omega = 1.76$	$\phi = 0.85$
LSL	$\Omega = 2.14$	$\phi = 0.70$

Note: Slip-critical bolt values assume no more than one filler has been provided or bolts have been added to distribute loads in the fillers.
See AISC *Specification* Sections J3.8 and J5 for provisions when fillers are present.
For Class B faying surfaces, multiply the tabulated available strength by 1.67.

Table 7-4
Available Bearing Strength at Bolt Holes Based on Bolt Spacing
kips/in. thickness

Hole Type	Bolt Spacing, s, in.	F_u, ksi	5/8 r_n/Ω ASD	5/8 ϕr_n LRFD	3/4 r_n/Ω ASD	3/4 ϕr_n LRFD	7/8 r_n/Ω ASD	7/8 ϕr_n LRFD	1 r_n/Ω ASD	1 ϕr_n LRFD
STD SSLT	$2\frac{2}{3} d_b$	58	34.1	51.1	41.3	62.0	48.6	72.9	55.8	83.7
		65	38.2	57.3	46.3	69.5	54.4	81.7	62.6	93.8
	3 in.	58	43.5	65.3	52.2	78.3	60.9	91.4	67.4	101
		65	48.8	73.1	58.5	87.8	68.3	102	75.6	113
SSLP	$2\frac{2}{3} d_b$	58	27.6	41.3	34.8	52.2	42.1	63.1	47.1	70.7
		65	30.9	46.3	39.0	58.5	47.1	70.7	52.8	79.2
	3 in.	58	43.5	65.3	52.2	78.3	60.9	91.4	58.7	88.1
		65	48.8	73.1	58.5	87.8	68.3	102	65.8	98.7
OVS	$2\frac{2}{3} d_b$	58	29.7	44.6	37.0	55.5	44.2	66.3	49.3	74.0
		65	33.3	50.0	41.4	62.2	49.6	74.3	55.3	82.9
	3 in.	58	43.5	65.3	52.2	78.3	60.9	91.4	60.9	91.4
		65	48.8	73.1	58.5	87.8	68.3	102	68.3	102
LSLP	$2\frac{2}{3} d_b$	58	3.62	5.44	4.35	6.53	5.08	7.61	5.80	8.70
		65	4.06	6.09	4.88	7.31	5.69	8.53	6.50	9.75
	3 in.	58	43.5	65.3	39.2	58.7	28.3	42.4	17.4	26.1
		65	48.8	73.1	43.9	65.8	31.7	47.5	19.5	29.3
LSLT	$2\frac{2}{3} d_b$	58	28.4	42.6	34.4	51.7	40.5	60.7	46.5	69.8
		65	31.8	47.7	38.6	57.9	45.4	68.0	52.1	78.2
	3 in.	58	36.3	54.4	43.5	65.3	50.8	76.1	56.2	84.3
		65	40.6	60.9	48.8	73.1	56.9	85.3	63.0	94.5
STD, SSLT, SSLP, OVS, LSLP	$s \geq s_{full}$	58	43.5	65.3	52.2	78.3	60.9	91.4	69.6	104
		65	48.8	73.1	58.5	87.8	68.3	102	78.0	117
LSLT	$s \geq s_{full}$	58	36.3	54.4	43.5	65.3	50.8	76.1	58.0	87.0
		65	40.6	60.9	48.8	73.1	56.9	85.3	65.0	97.5
Spacing for full bearing strength s_{full}[a], in.		STD, SSLT, LSLT	1 15/16		2 5/16		2 11/16		3 1/16	
		OVS	2 1/16		2 7/16		2 13/16		3 1/4	
		SSLP	2 1/8		2 1/2		2 7/8		3 5/16	
		LSLP	2 13/16		3 3/8		3 15/16		4 1/2	
Minimum Spacing[a] = $2\frac{2}{3}d$, in.			1 11/16		2		2 5/16		2 11/16	

STD = standard hole
SSLT = short-slotted hole oriented transverse to the line of force
SSLP = short-slotted hole oriented parallel to the line of force
OVS = oversized hole
LSLP = long-slotted hole oriented parallel to the line of force
LSLT = long-slotted hole oriented transverse to the line of force

ASD	LRFD	
$\Omega = 2.00$	$\phi = 0.75$	Note: Spacing indicated is from the center of the hole or slot to the center of the adjacent hole or slot in the line of force. Hole deformation is considered. When hole deformation is not considered, see AISC *Specification* Section J3.10. [a] Decimal value has been rounded to the nearest sixteenth of an inch.

Table 7-4 (continued)
Available Bearing Strength at Bolt Holes Based on Bolt Spacing
kips/in. thickness

Hole Type	Bolt Spacing, s, in.	F_u, ksi	1 1/8 r_n/Ω ASD	1 1/8 ϕr_n LRFD	1 1/4 r_n/Ω ASD	1 1/4 ϕr_n LRFD	1 3/8 r_n/Ω ASD	1 3/8 ϕr_n LRFD	1 1/2 r_n/Ω ASD	1 1/2 ϕr_n LRFD
STD SSLT	$2\frac{2}{3} d_b$	58	63.1	94.6	70.3	105	77.6	116	84.8	127
		65	70.7	106	78.8	118	86.9	130	95.1	143
	3 in.	58	63.1	94.6	—	—	—	—	—	—
		65	70.7	106	—	—	—	—	—	—
SSLP	$2\frac{2}{3} d_b$	58	52.2	78.3	59.5	89.2	66.7	100	74.0	111
		65	58.5	87.8	66.6	99.9	74.8	112	82.9	124
	3 in.	58	52.2	78.3	—	—	—	—	—	—
		65	58.5	87.8	—	—	—	—	—	—
OVS	$2\frac{2}{3} d_b$	58	54.4	81.6	61.6	92.4	68.9	103	76.1	114
		65	60.9	91.4	69.1	104	77.2	116	85.3	128
	3 in.	58	54.4	81.6	—	—	—	—	—	—
		65	60.9	91.4	—	—	—	—	—	—
LSLP	$2\frac{2}{3} d_b$	58	6.53	9.79	7.25	10.9	7.98	12.0	8.70	13.1
		65	7.31	11.0	8.13	12.2	8.94	13.4	9.75	14.6
	3 in.	58	6.53	9.79	—	—	—	—	—	—
		65	7.31	11.0	—	—	—	—	—	—
LSLT	$2\frac{2}{3} d_b$	58	52.6	78.8	58.6	87.9	64.6	97.0	70.7	106
		65	58.9	88.4	65.7	98.5	72.4	109	79.2	119
	3 in.	58	52.6	78.8	—	—	—	—	—	—
		65	58.9	88.4	—	—	—	—	—	—
STD, SSLT, SSLP, OVS, LSLP	$s \geq s_{full}$	58	78.3	117	87.0	131	95.7	144	104	157
		65	87.8	132	97.5	146	107	161	117	176
LSLT	$s \geq s_{full}$	58	65.3	97.9	72.5	109	79.8	120	87.0	131
		65	73.1	110	81.3	122	89.4	134	97.5	146
Spacing for full bearing strength s_{full}[a], in.		STD, SSLT, LSLT	3 7/16		3 13/16		4 3/16		4 9/16	
		OVS	3 11/16		4 1/16		4 7/16		4 13/16	
		SSLP	3 3/4		4 1/8		4 1/2		4 7/8	
		LSLP	5 1/16		5 5/8		6 3/16		6 3/4	
Minimum Spacing[a] = $2\frac{2}{3}d$, in.			3		3 5/16		3 11/16		4	

STD = standard hole
SSLT = short-slotted hole oriented transverse to the line of force
SSLP = short-slotted hole oriented parallel to the line of force
OVS = oversized hole
LSLP = long-slotted hole oriented parallel to the line of force
LSLT = long-slotted hole oriented transverse to the line of force

ASD	LRFD	
$\Omega = 2.00$	$\phi = 0.75$	— indicates spacing less than minimum spacing required per AISC *Specification* Section J3.3. Note: Spacing indicated is from the center of the hole or slot to the center of the adjacent hole or slot in the line of force. Hole deformation is considered. When hole deformation is not considered, see AISC *Specification* Section J3.10. [a] Decimal value has been rounded to the nearest sixteenth of an inch.

Table 7-5
Available Bearing Strength at Bolt Holes Based on Edge Distance
kips/in. thickness

Hole Type	Edge Distance L_e, in.	F_u, ksi	5/8 r_n/Ω ASD	5/8 ϕr_n LRFD	3/4 r_n/Ω ASD	3/4 ϕr_n LRFD	7/8 r_n/Ω ASD	7/8 ϕr_n LRFD	1 r_n/Ω ASD	1 ϕr_n LRFD
STD SSLT	1 1/4	58	31.5	47.3	29.4	44.0	27.2	40.8	25.0	37.5
		65	35.3	53.0	32.9	49.4	30.5	45.7	28.0	42.0
	2	58	43.5	65.3	52.2	78.3	53.3	79.9	51.1	76.7
		65	48.8	73.1	58.5	87.8	59.7	89.6	57.3	85.9
SSLP	1 1/4	58	28.3	42.4	26.1	39.2	23.9	35.9	20.7	31.0
		65	31.7	47.5	29.3	43.9	26.8	40.2	23.2	34.7
	2	58	43.5	65.3	52.2	78.3	50.0	75.0	46.8	70.1
		65	48.8	73.1	58.5	87.8	56.1	84.1	52.4	78.6
OVS	1 1/4	58	29.4	44.0	27.2	40.8	25.0	37.5	21.8	32.6
		65	32.9	49.4	30.5	45.7	28.0	42.0	24.4	36.6
	2	58	43.5	65.3	52.2	78.3	51.1	76.7	47.9	71.8
		65	48.8	73.1	58.5	87.8	57.3	85.9	53.6	80.4
LSLP	1 1/4	58	16.3	24.5	10.9	16.3	5.44	8.16	—	—
		65	18.3	27.4	12.2	18.3	6.09	9.14	—	—
	2	58	42.4	63.6	37.0	55.5	31.5	47.3	26.1	39.2
		65	47.5	71.3	41.4	62.2	35.3	53.0	29.3	43.9
LSLT	1 1/4	58	26.3	39.4	24.5	36.7	22.7	34.0	20.8	31.3
		65	29.5	44.2	27.4	41.1	25.4	38.1	23.4	35.0
	2	58	36.3	54.4	43.5	65.3	44.4	66.6	42.6	63.9
		65	40.6	60.9	48.8	73.1	49.8	74.6	47.7	71.6
STD, SSLT, SSLP, OVS, LSLP	$L_e \geq L_{e\ full}$	58	43.5	65.3	52.2	78.3	60.9	91.4	69.6	104
		65	48.8	73.1	58.5	87.8	68.3	102	78.0	117
LSLT	$L_e \geq L_{e\ full}$	58	36.3	54.4	43.5	65.3	50.8	76.1	58.0	87.0
		65	40.6	60.9	48.8	73.1	56.9	85.3	65.0	97.5
Edge distance for full bearing strength $L_e \geq L_{e\ full}$[a], in.		STD, SSLT, LSLT	1 5/8		1 15/16		2 1/4		2 9/16	
		OVS	1 11/16		2		2 5/16		2 5/8	
		SSLP	1 11/16		2		2 5/16		2 11/16	
		LSLP	2 1/16		2 7/16		2 7/8		3 1/4	

STD = standard hole
SSLT = short-slotted hole oriented transverse to the line of force
SSLP = short-slotted hole oriented parallel to the line of force
OVS = oversized hole
LSLP = long-slotted hole oriented parallel to the line of force
LSLT = long-slotted hole oriented transverse to the line of force

ASD	LRFD	
$\Omega = 2.00$	$\phi = 0.75$	— indicates spacing less than minimum spacing required per AISC *Specification* Section J3.3. Note: Spacing indicated is from the center of the hole or slot to the center of the adjacent hole or slot in the line of force. Hole deformation is considered. When hole deformation is not considered, see AISC *Specification* Section J3.10. [a] Decimal value has been rounded to the nearest sixteenth of an inch.

Table 7-5 (continued)
Available Bearing Strength at Bolt Holes Based on Edge Distance
kips/in. thickness

Hole Type	Edge Distance L_e, in.	F_u, ksi	1 1/8 r_n/Ω ASD	1 1/8 ϕr_n LRFD	1 1/4 r_n/Ω ASD	1 1/4 ϕr_n LRFD	1 3/8 r_n/Ω ASD	1 3/8 ϕr_n LRFD	1 1/2 r_n/Ω ASD	1 1/2 ϕr_n LRFD
STD SSLT	1 1/4	58	22.8	34.3	20.7	31.0	18.5	27.7	16.3	24.5
		65	25.6	38.4	23.2	34.7	20.7	31.1	18.3	27.4
	2	58	48.9	73.4	46.8	70.1	44.6	66.9	42.4	63.6
		65	54.8	82.3	52.4	78.6	50.0	75.0	47.5	71.3
SSLP	1 1/4	58	17.4	26.1	15.2	22.8	13.1	19.6	10.9	16.3
		65	19.5	29.3	17.1	25.6	14.6	21.9	12.2	18.3
	2	58	43.5	65.3	41.3	62.0	39.2	58.7	37.0	55.5
		65	48.8	73.1	46.3	69.5	43.9	65.8	41.4	62.2
OVS	1 1/4	58	18.5	27.7	16.3	24.5	14.1	21.2	12.0	17.9
		65	20.7	31.1	18.3	27.4	15.8	23.8	13.4	20.1
	2	58	44.6	66.9	42.4	63.6	40.2	60.4	38.1	57.1
		65	50.0	75.0	47.5	71.3	45.1	67.6	42.7	64.0
LSLP	1 1/4	58	—	—	—	—	—	—	—	—
		65	—	—	—	—	—	—	—	—
	2	58	20.7	31.0	15.2	22.8	9.79	14.7	4.35	6.53
		65	23.2	34.7	17.1	25.6	11.0	16.5	4.88	7.31
LSLT	1 1/4	58	19.0	28.5	17.2	25.8	15.4	23.1	13.6	20.4
		65	21.3	32.0	19.3	28.9	17.3	25.9	15.2	22.9
	2	58	40.8	61.2	39.0	58.5	37.2	55.7	35.3	53.0
		65	45.7	68.6	43.7	65.5	41.6	62.5	39.6	59.4
STD, SSLT, SSLP, OVS, LSLP	$L_e \geq L_{e\ full}$	58	78.3	117	87.0	131	95.7	144	104	157
		65	87.8	132	97.5	146	107	161	117	176
LSLT	$L_e \geq L_{e\ full}$	58	65.3	97.9	72.5	109	79.8	120	87.0	131
		65	73.1	110	81.3	122	89.4	134	97.5	146
Edge distance for full bearing strength $L_e \geq L_{e\ full}$[a], in.		STD, SSLT, LSLT	2 7/8		3 3/16		3 1/2		3 13/16	
		OVS	3		3 5/16		3 5/8		3 15/16	
		SSLP	3		3 5/16		3 5/8		3 15/16	
		LSLP	3 11/16		4 1/16		4 1/2		4 7/8	

STD = standard hole
SSLT = short-slotted hole oriented transverse to the line of force
SSLP = short-slotted hole oriented parallel to the line of force
OVS = oversized hole
LSLP = long-slotted hole oriented parallel to the line of force
LSLT = long-slotted hole oriented transverse to the line of force

ASD	LRFD	
$\Omega = 2.00$	$\phi = 0.75$	— indicates spacing less than minimum spacing required per AISC *Specification* Section J3.3. Note: Spacing indicated is from the center of the hole or slot to the center of the adjacent hole or slot in the line of force. Hole deformation is considered. When hole deformation is not considered, see AISC *Specification* Section J3.10. [a] Decimal value has been rounded to the nearest sixteenth of an inch.

Reinforced Concrete Design

Notation:

a = depth of the effective compression block in a concrete beam
A = name for area
A_g = gross area, equal to the total area ignoring any reinforcement
A_s = area of steel reinforcement in concrete beam design
A'_s = area of steel compression reinforcement in concrete beam design
A_{st} = area of steel reinforcement in concrete column design
A_v = area of concrete shear stirrup reinforcement
ACI = American Concrete Institute
b = width, often cross-sectional
b_E = effective width of the flange of a concrete T beam cross section
b_f = width of the flange
b_w = width of the stem (web) of a concrete T beam cross section
cc = shorthand for clear cover
C = name for centroid
= name for a compression force
C_c = compressive force in the compression steel in a doubly reinforced concrete beam
C_s = compressive force in the concrete of a doubly reinforced concrete beam
d = effective depth from the top of a reinforced concrete beam to the centroid of the tensile steel
d' = effective depth from the top of a reinforced concrete beam to the centroid of the compression steel
d_b = bar diameter of a reinforcing bar
D = shorthand for dead load
DL = shorthand for dead load
E = modulus of elasticity or Young's modulus
= shorthand for earthquake load
E_c = modulus of elasticity of concrete
E_s = modulus of elasticity of steel
f = symbol for stress
f_c = compressive stress
f'_c = concrete design compressive stress
f_{pu} = tensile strength of the prestressing reinforcement
f_s = stress in the steel reinforcement for concrete design
f'_s = compressive stress in the compression reinforcement for concrete beam design
f_y = yield stress or strength
F = shorthand for fluid load
F_y = yield strength
G = relative stiffness of columns to beams in a rigid connection, as is Ψ
h = cross-section depth
H = shorthand for lateral pressure load
h_f = depth of a flange in a T section
$I_{transformed}$ = moment of inertia of a multi-material section transformed to one material
k = effective length factor for columns
ℓ_b = length of beam in rigid joint
ℓ_c = length of column in rigid joint
l_d = development length for reinforcing steel
l_{dh} = development length for hooks
l_n = clear span from face of support to face of support in concrete design
L = name for length or span length, as is l
= shorthand for live load
L_r = shorthand for live roof load
LL = shorthand for live load
M_n = nominal flexure strength with the steel reinforcement at the yield stress and concrete at the concrete design strength for reinforced concrete beam design
M_u = maximum moment from factored loads for LRFD beam design
n = modulus of elasticity transformation coefficient for steel to concrete
$n.a.$ = shorthand for neutral axis (N.A.)

pH = chemical alkalinity
P = name for load or axial force vector
P_o = maximum axial force with no concurrent bending moment in a reinforced concrete column
P_n = nominal column load capacity in concrete design
P_u = factored column load calculated from load factors in concrete design
R = shorthand for rain or ice load
R_n = concrete beam design ratio = M_u/bd^2
s = spacing of stirrups in reinforced concrete beams
S = shorthand for snow load
t = name for thickness
T = name for a tension force
= shorthand for thermal load
U = factored design value
V_c = shear force capacity in concrete
V_s = shear force capacity in steel shear stirrups
V_u = shear at a distance of *d* away from the face of support for reinforced concrete beam design
w_c = unit weight of concrete
w_{DL} = load per unit length on a beam from dead load
w_{LL} = load per unit length on a beam from live load
$w_{self\ wt}$ = name for distributed load from self weight of member
w_u = load per unit length on a beam from load factors
W = shorthand for wind load
x = horizontal distance
= distance from the top to the neutral axis of a concrete beam
y = vertical distance
β_1 = coefficient for determining stress block height, *a*, based on concrete strength, f'_c
Δ = elastic beam deflection
ε = strain
ϕ = resistance factor
ϕ_c = resistance factor for compression
γ = density or unit weight
ρ = radius of curvature in beam deflection relationships
= reinforcement ratio in concrete beam design = A_s/bd
$\rho_{balanced}$ = balanced reinforcement ratio in concrete beam design
υ_c = shear strength in concrete design

Reinforced Concrete Design

Structural design standards for reinforced concrete are established by the *Building Code and Commentary (ACI 318-11)* published by the American Concrete Institute International, and uses **ultimate** strength design (also known as ***limit state*** design).

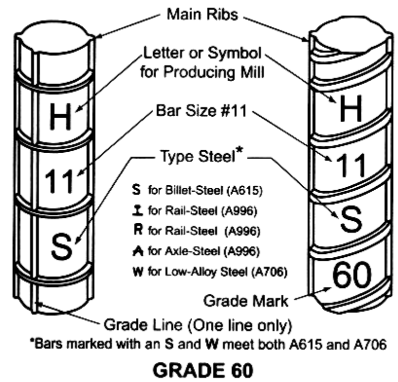
f'_c = concrete compressive design strength at 28 days (units of psi when used in equations)

<u>Materials</u>

Concrete is a mixture of cement, coarse aggregate, fine aggregate, and water. The cement hydrates with the water to form a binder. The result is a hardened mass with "filler" and pores. There are various types of cement for low heat, rapid set, and other properties. Other minerals or cementitious materials (like fly ash) may be added.

ASTM designations are

- Type I: Ordinary portland cement (OPC)
- Type II: Low temperature
- Type III: High early strength
- Type IV: Low-heat of hydration
- Type V: Sulfate resistant



The proper proportions, by volume, of the mix constituents determine strength, which is related to the water to cement ratio (w/c). It also determines other properties, such as workability of fresh concrete. Admixtures, such as retardants, accelerators, or superplasticizers, which aid flow without adding more water, may be added. Vibration may also be used to get the mix to flow into forms and fill completely.

Slump is the measurement of the height loss from a compacted cone of fresh concrete. It can be an indicator of the workability.

Proper mix design is necessary for durability. The pH of fresh cement is enough to prevent reinforcing steel from oxidizing (rusting). If, however, cracks allow corrosive elements in water to penetrate to the steel, a corrosion cell will be created, the steel will rust, expand and cause further cracking. Adequate cover of the steel by the concrete is important.

Deformed reinforcing bars come in grades 40, 60 & 75 (for 40 ksi, 60 ksi and 75 ksi yield strengths). Sizes are given as # of 1/8" up to #8 bars. For #9 and larger, the number is a nominal size (while the actual size is larger).

Reinforced concrete is a composite material, and the average density is considered to be *150* lb/ft^3. It has the properties that it will creep (deformation with long term load) and shrink (a result of hydration) that must be considered.

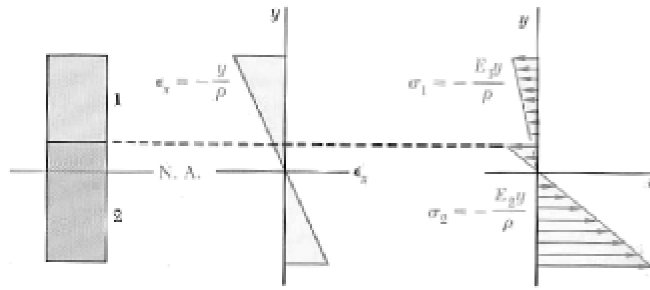
Construction

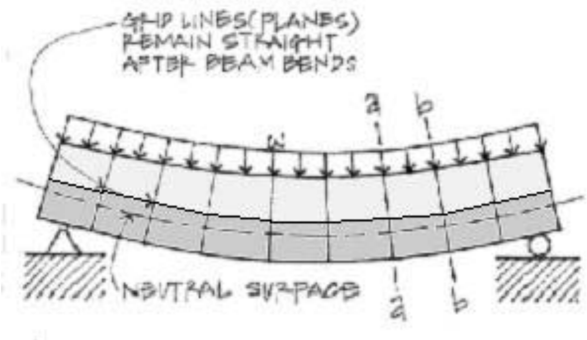
Because fresh concrete is a viscous suspension, it is cast or placed and *not poured*. Formwork must be able to withstand the hydraulic pressure. *Vibration* may be used to get the mix to flow around reinforcing bars or into tight locations, but excess vibration will cause segregation, honeycombing, and excessive *bleed* water which will reduce the water available for hydration and the strength, subsequently.

After casting, the surface must be worked. *Screeding* removes the excess from the top of the forms and gets a rough level. *Floating* is the process of working the aggregate under the surface and to "float" some paste to the surface. *Troweling* takes place when the mix has hydrated to the point of supporting weight and the surface is smoothed further and consolidated. *Curing* is allowing the hydration process to proceed with adequate moisture. Black tarps and curing compounds are commonly used. *Finishing* is the process of adding a texture, commonly by using a broom, after the concrete has begun to set.

Behavior

Plane sections of composite materials can still be assumed to be plane (strain is linear), *but* the stress distribution *is not* the same in both materials because the *modulus of elasticity* is different. (f=E·ε)





$$f_1 = E_1\varepsilon = -\frac{E_1 y}{\rho} \qquad f_2 = E_2\varepsilon = -\frac{E_2 y}{\rho}$$

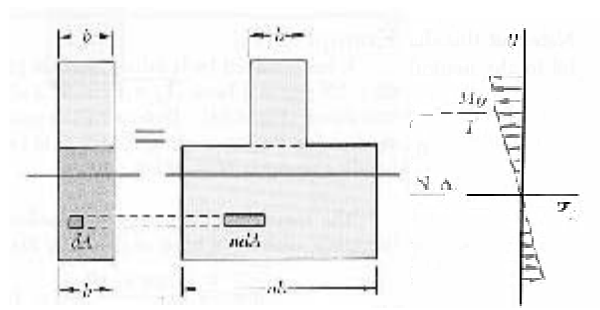
In order to determine the stress, we can define n as the ratio of the elastic moduli:

$$n = \frac{E_2}{E_1}$$

n is used to <u>transform</u> the <u>width</u> of the second material such that it sees the equivalent element stress.

Transformed Section y and I

In order to determine stresses in all types of material in the beam, we transform the materials into a single material, and calculate the location of the neutral axis and modulus of inertia for that material.



ex: When material 1 above is concrete and material 2 is steel

to transform steel into concrete $n = \dfrac{E_2}{E_1} = \dfrac{E_{steel}}{E_{concrete}}$

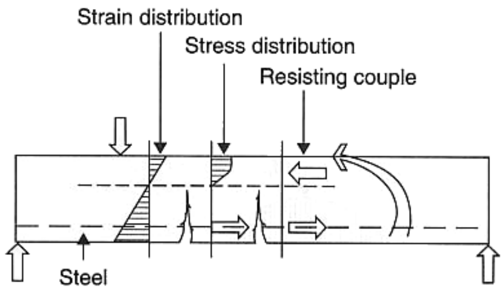
to find the neutral axis of the equivalent concrete member we transform the width of the steel by multiplying by n

to find the moment of inertia of the equivalent concrete member, $I_{transformed}$, use the new geometry resulting from transforming the width of the steel

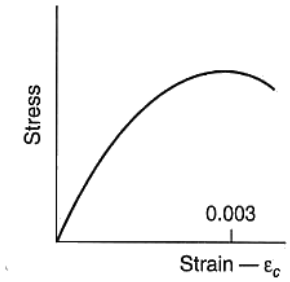
concrete stress: $f_{concrete} = -\dfrac{My}{I_{transformed}}$

steel stress: $f_{steel} = -\dfrac{Myn}{I_{transformed}}$

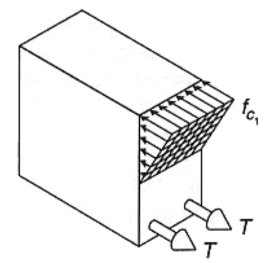
Reinforced Concrete Beam Members



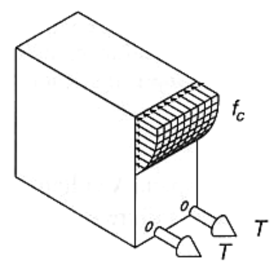
Stresses in the concrete above the neutral axis are compressive and nonlinearly distributed. In the tension zone below the neutral axis, the concrete is assumed to be cracked and the tensile force present to be taken up by reinforcing steel.



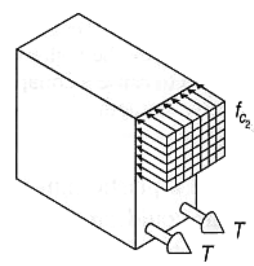
Typical stress-strain curve for concrete.



Working stress analysis. (Concrete stress distribution is assumed to be linear. Service loads are used in calculations.)



Actual stress distribution near ultimate strength (nonlinear).



Ultimate strength analysis. (A rectangular stress block is used to idealize the actual stress distribution. Calculations are based on ultimate loads and failure stresses.)

Ultimate Strength Design for Beams

The ultimate strength design method is similar to LRFD. There is a *nominal* strength that is reduced by a factor ϕ which must exceed the factored design stress. For beams, the concrete only works in compression over a rectangular "stress" block above the n.a. from elastic calculation, and the steel is exposed and reaches the yield stress, F_y

For stress analysis in reinforced concrete beams

- the steel is transformed to concrete
- any concrete in tension is assumed to be cracked and to have no strength
- the steel can be in tension, and is placed in the bottom of a beam that has positive bending moment

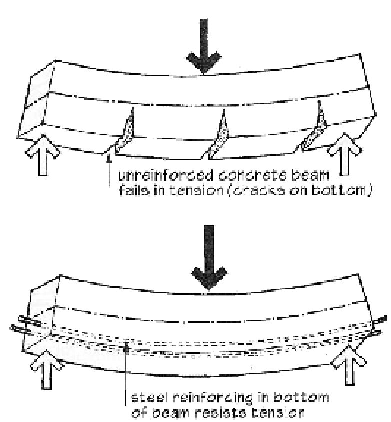
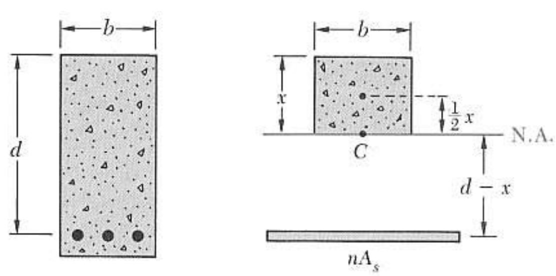
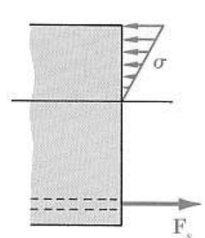


Figure 8.5: Bending in a concrete beam without and with steel reinforcing.





The neutral axis is where there is no stress and no strain. The concrete above the n.a. is in compression. The concrete below the n.a. is considered ineffective. The steel below the n.a. is in tension.

Because the n.a. is defined by the moment areas, we can solve for x knowing that d is the distance from the top of the concrete section to the centroid of the steel:

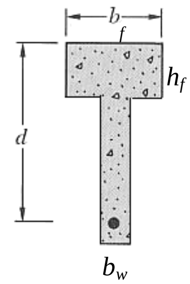
$$bx \cdot \frac{x}{2} - nA_s(d - x) = 0$$

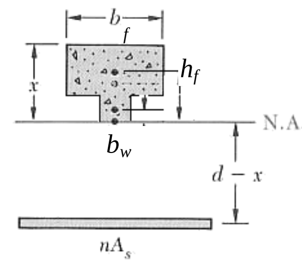
x can be solved for when the equation is rearranged into the generic format with a, b & c in the binomial equation: $ax^2 + bx + c = 0$ by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

T-sections

If the n.a. is *above* the bottom of a flange in a T section, x is found as for a rectangular section.

If the n.a. is *below* the bottom of a flange in a T section, x is found by including the flange and the stem of the web (b_w) in the moment area calculation:





$$b_f h_f \left(x - \frac{h_f}{2} \right) + (x - h_f) b_w \frac{(x - h_f)}{2} - nA_s(d - x) = 0$$

Load Combinations (Alternative values are allowed)

1.4D

1.2D + 1.6L +0.5(L_r or S or R)

1.2D + 1.6(L_r or S or R) + (1.0L or 0.5W)

1.2D + 1.0W +1.0L + 0.5(L_r or S or R)

1.2D + 1.0E + 1.0L + 0.2S

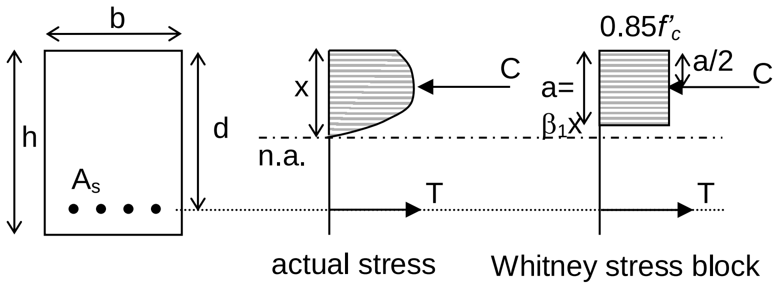
0.9D + 1.0W

0.9D + 1.0E

ASTM STANDARD REINFORCING BARS

Bar size, no.	Nominal diameter, in.	Nominal area, in.^{2}	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.650
18	2.257	4.00	13.600

Internal Equilibrium



C = compression in concrete = stress x area = $0.85 f'_c ba$

T = tension in steel = stress x area = $A_s f_y$

$C = T$ and $M_n = T(d-a/2)$

where f'_c = concrete compression strength
a = height of stress block
β_1 = factor based on f'_c
x = location to the neutral axis
b = width of stress block
f_y = steel yield strength
A_s = area of steel reinforcement
d = effective depth of section
= depth to n.a. of reinforcement

With C=T, $A_s f_y = 0.85 f'_c ba$ so *a* can be determined with $a = \frac{A_s f_y}{0.85 f'_c b}$

Criteria for Beam Design

For flexure design:

$M_u \le \phi M_n$ *ϕ = 0.9 for flexure (when the section is* *<u>tension</u>* *controlled)*
so for design, *M_u can be set to $\phi M_n = \phi T(d-a/2) = \phi A_s f_y (d-a/2)$*

Reinforcement Ratio

The amount of steel reinforcement is *limited.* Too much reinforcement, or **over-reinforcing** will not allow the steel to yield before the concrete crushes and there is a sudden failure. A beam with the proper amount of steel to allow it to yield at failure is said to be **under reinforced**.

The reinforcement ratio is just a fraction: $\rho = \frac{A_s}{bd}$ (or p) and must be less than a value determined with a concrete strain of 0.003 and tensile strain of 0.004 (minimum). When the strain in the reinforcement is 0.005 or greater, the section is **tension controlled**. (For smaller strains the resistance factor reduces to 0.65 – see tied columns - because the stress is less than the yield stress in the steel.) Previous codes limited the amount to $0.75\rho_{balanced}$ where $\rho_{balanced}$ was determined from the amount of steel that would make the concrete start to crush at the exact same time that the steel would yield based on strain.

Flexure Design of Reinforcement

One method is to "wisely" estimate a height of the stress block, *a*, and solve for A_s, and calculate a new value for *a* using M_u.

1. guess *a* (less than n.a.)
2. $A_s = \dfrac{0.85 f'_c ba}{f_y}$
3. solve for *a* from

   setting $M_u = \phi A_s f_y (d - a/2)$:

   $$a = 2\left(d - \frac{M_u}{\phi A_s f_y} \right)$$
4. repeat from 2. until *a* found from step 3 matches *a* used in step 2.

Maximum Reinforcement Ratio ρ for Singly Reinforced Rectangular Beams (tensile strain = 0.005) for which ϕ is permitted to be 0.9

f_y	$f'_c = 3000$ psi, $\beta_1 = 0.85$	$f'_c = 3500$ psi, $\beta_1 = 0.85$	$f'_c = 4000$ psi, $\beta_1 = 0.85$	$f'_c = 5000$ psi, $\beta_1 = 0.80$	$f'_c = 6000$ psi, $\beta_1 = 0.75$
40,000 psi	0.0203	0.0237	0.0271	0.0319	0.0359
50,000 psi	0.0163	0.0190	0.0217	0.0255	0.0287
60,000 psi	0.0135	0.0158	0.0181	0.0213	0.0239
f_y	$f'_c = 20$ MPa, $\beta_1 = 0.85$	$f'_c = 25$ MPa, $\beta_1 = 0.85$	$f'_c = 30$ MPa, $\beta_1 = 0.85$	$f'_c = 35$ MPa, $\beta_1 = 0.81$	$f'_c = 40$ MPa, $\beta_1 = 0.77$
300 MPa	0.0181	0.0226	0.0271	0.0301	0.0327
350 MPa	0.0155	0.0194	0.0232	0.0258	0.0281
400 MPa	0.0135	0.0169	0.0203	0.0226	0.0245
500 MPa	0.0108	0.0135	0.0163	0.0181	0.0196

from Reinforced Concrete, 7th, Wang, Salmon, Pincheira, Wiley & Sons, 2007

Design Chart Method:

1. calculate $R_n = \dfrac{M_n}{bd^2}$
2. find curve for f'_c and f_y to get ρ
3. calculate A_s and *a*, where:

   $A_s = \rho bd$ *and* $a = \dfrac{A_s f_y}{0.85 f'_c b}$

Any method can simplify the size of d using h = 1.1d

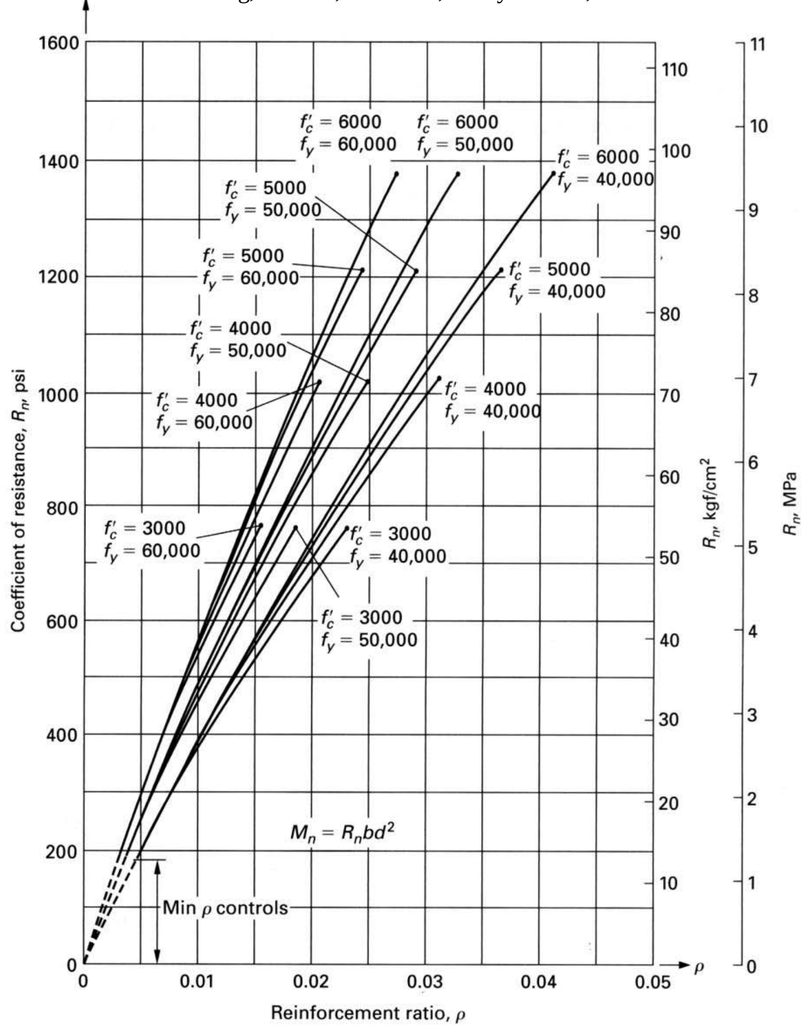


Figure 3.8.1 Strength curves (R_n vs ρ) for singly reinforced rectangular sections. Upper limit of curves is at ρ_{max}. (tensile strain of 0.004)

Maximum Reinforcement

Based on the limiting strain of 0.005 in the steel, *x(or c) = 0.375d* so

$a = \beta_1(0.375d)$ to find $A_{s\text{-max}}$

(β_1 is shown in the table above)

Minimum Reinforcement

Minimum reinforcement is provided even if the concrete can resist the tension. This is a means to control cracking.

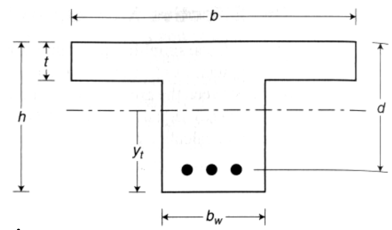
Minimum required: $A_s = \dfrac{3\sqrt{f'_c}}{f_y}(b_w d)$

but not less than: $A_s = \dfrac{200}{f_y}(b_w d)$

where f'_c is in psi. This can be translated to $\rho_{min} = \dfrac{3\sqrt{f'_c}}{f_y}$ but not less than $\dfrac{200}{f_y}$

Cover for Reinforcement

Cover of concrete over/under the reinforcement must be provided to protect the steel from corrosion. For indoor exposure, 1.5 inch is typical for beams and columns, 0.75 inch is typical for slabs, and for concrete cast against soil, 3 inch minimum is required.



Bar Spacing

Minimum bar spacings are specified to allow proper consolidation of concrete around the reinforcement. The minimum spacing is the maximum of 1 in, a bar diameter, or 1.33 times the maximum aggregate size.

T-beams and T-sections (pan joists)

Beams cast with slabs have an effective width, b_E, that sees compression stress in a wide flange beam or joist in a slab system with positive bending.

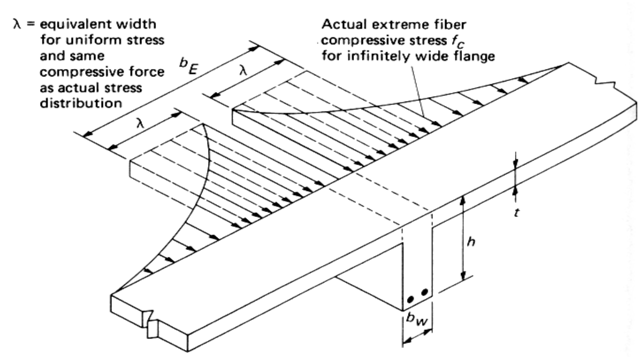


Figure 9.3.1 Actual and equivalent stress distribution over flange width.

For *interior* T-sections, b_E is the smallest of

L/4, b_w + 16t, or center to center of beams

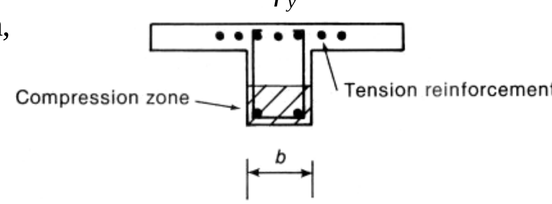
For *exterior* T-sections, b_E is the smallest of

b_w + L/12, b_w + 6t, or b_w + ½(clear distance to next beam)

When the **web** is in tension the minimum reinforcement required is the same as for rectangular sections with the web width (b_w) in place of *b*.

When the **flange** is in tension (negative bending), the minimum reinforcement required is the greater value of $A_s = \frac{6\sqrt{f'_c}}{f_y}(b_w d)$ or $A_s = \frac{3\sqrt{f'_c}}{f_y}(b_f d)$

where f'_c is in psi, b_w is the beam width, and b_f is the effective flange width

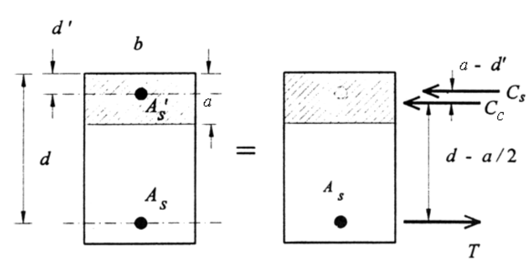


(negative moment).

Compression Reinforcement

If a section is *doubly reinforced,* it means there is steel in the beam seeing compression. The force in the compression steel that *may not be yielding* is

$$C_s = A_s'(f'_s - 0.85f'_c)$$



The total compression that balances the tension is now: $T = C_c + C_s$. And the moment taken about the centroid of the compression stress is $M_n = T(d-a/2)+C_s(a-d')$

where A_s' is the area of compression reinforcement, and *d'* is the effective depth to the centroid of the compression reinforcement

Because the compression steel may not be yielding, the neutral axis *x* must be found from the force equilibrium relationships, and the stress can be found based on strain to see if it has yielded.

Slabs

One way slabs can be designed as "one unit"-wide beams. Because they are thin, control of deflections is important, and minimum depths are specified, as is minimum reinforcement for shrinkage and crack control when not in flexure. Reinforcement is commonly small diameter bars and welded wire fabric. Maximum spacing between bars is also specified for shrinkage and crack control as five times the slab thickness not exceeding 18". For required flexure reinforcement the spacing limit is three times the slab thickness not exceeding 18".

TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE COMPUTED

	Minimum thickness, *h*			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

Notes:
Values given shall be used directly for members with normalweight concrete and Grade 60 reinforcement. For other conditions, the values shall be modified as follows:
a) For lightweight concrete having equilibrium density, w_c, in the range of 90 to 115 lb/ft³, the values shall be multiplied by $(1.65 - 0.005w_c)$ but not less than 1.09.
b) For f_y other than 60,000 psi, the values shall be multiplied by $(0.4 + f_y/100,000)$.

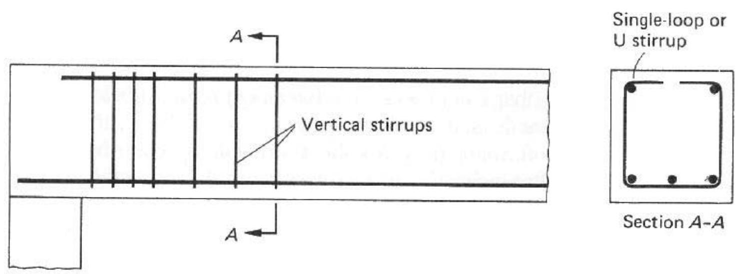
Shrinkage and temperature reinforcement (and minimum for flexure reinforcement):

Minimum for slabs with grade 40 or 50 bars: $\rho = \frac{A_s}{bt} = 0.002$ *or* $A_{s\text{-}min} = 0.002bt$

Minimum for slabs with grade 60 bars: $\rho = \frac{A_s}{bt} = 0.0018$ *or* $A_{s\text{-}min} = 0.0018bt$

Shear Behavior

Horizontal shear stresses occur along with bending stresses to cause tensile stresses where the concrete cracks. Vertical reinforcement is required to bridge the cracks which are called **shear stirrups (**or **stirrups)**.



The maximum shear for design, V_u is the value at a distance of d from the face of the support.

Nominal Shear Strength

The shear force that can be resisted is the shear stress × cross section area: $V_c = \upsilon_c \times b_w d$

The shear stress for beams (one way) $\upsilon_c = 2\sqrt{f'_c}$ so $\phi V_c = \phi 2\sqrt{f'_c}\, b_w d$

where b_w = the beam width or the minimum width of the stem.
ϕ = 0.75 for shear

One-way joists are allowed an increase of 10% V_c if the joists are closely spaced.

Stirrups are necessary for strength (as well as crack control): $V_s = \frac{A_v f_y d}{s} \le 8\sqrt{f'_c}\, b_w d$ *(max)*

where A_v = area of all vertical legs of stirrup
s = spacing of stirrups
d = effective depth

For shear design:

$$V_U \le \phi V_C + \phi V_S \quad \phi = 0.75 \text{ for shear}$$

Spacing Requirements

Stirrups are required when V_u is greater than $\dfrac{\phi V_c}{2}$

Table 3-8 ACI Provisions for Shear Design*

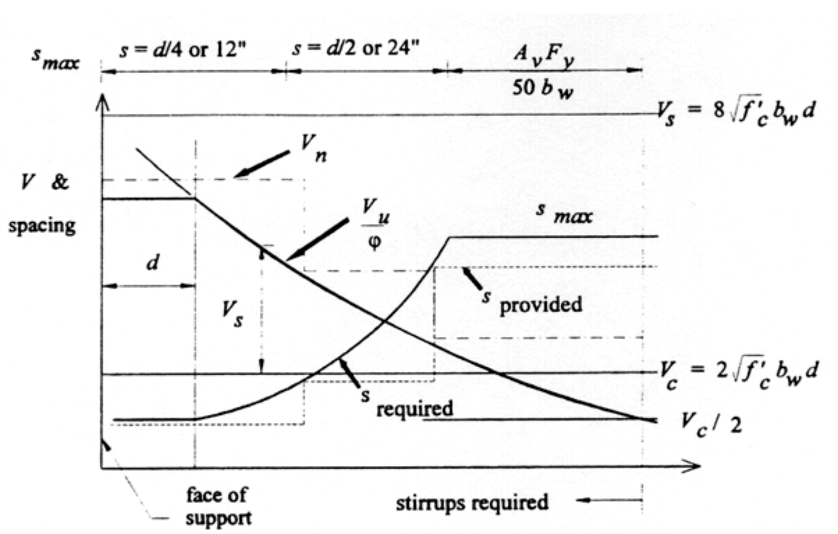
		$V_u \le \frac{\phi V_c}{2}$	$\phi V_c \ge V_u > \frac{\phi V_c}{2}$	$V_u > \phi V_c$
Required area of stirrups, A_v**		none	$\frac{50 b_w s}{f_y}$	$\frac{(V_u - \phi V_c)s}{\phi f_y d}$
Stirrup spacing, s	Required	—	$\frac{A_v f_y}{50 b_w}$	$\frac{\phi A_v f_y d}{V_u - \phi V_c}$
	Recommended Minimum†	—	—	4 in.
	Maximum†† (ACI 11.5.4)	—	$\frac{d}{2}$ or 24 in.	$\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \le \phi 4\sqrt{f'_c}\, b_w d$
				$\frac{d}{4}$ or 12 in. for $(V_u - \phi V_c) > \phi 4\sqrt{f'_c}\, b_w d$

*Members subjected to shear and flexure only; $\phi V_c = \phi 2\sqrt{f'_c}\, b_w d$, $\phi = 0.75$ (ACI 11.3.1.1)
**$A_v = 2 \times A_b$ for U stirrups; $f_y \le 60$ ksi (ACI 11.5.2)
†A practical limit for minimum spacing is d/4
††Maximum spacing based on minimum shear reinforcement (= $A_v f_y / 50 b_w$) must also be considered (ACI 11.5.5.3).

Economical spacing of stirrups is considered to be greater than d/4. Common spacings of d/4, d/3 and d/2 are used to determine the values of ϕV_s at which the spacings can be increased.

$$\phi V_s = \frac{\phi A_v f_y d}{s}$$

This figure shows the size of V_n provided by $V_c + V_s$ (long dashes) exceeds V_u/ϕ in a step-wise function, while the spacing provided (short dashes) is at or less than the required s (limited by the maximum allowed). *(Note that the maximum shear permitted from the stirrups is* $8\sqrt{f'_c}\, b_w d$ *)*



The minimum recommended spacing for the first stirrup is 2 inches from the face of the support.

Torsional Shear Reinforcement

On occasion beam members will see twist along the axis caused by an eccentric shape supporting a load, like on an L-shaped spandrel (edge) beam. The torsion results in shearing stresses, and closed stirrups may be needed to resist the stress that the concrete cannot resist.

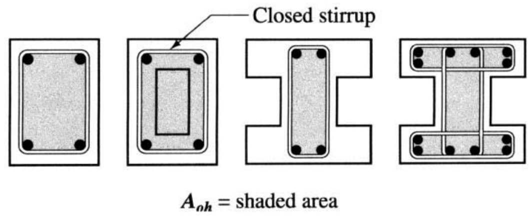


Fig. R11.6.3.6(b)—Definition of $\mathbf{A_{oh}}$

Development Length for Reinforcement

Because the design is based on the reinforcement attaining the yield stress, the reinforcement needs to be properly bonded to the concrete for a finite length (*both sides*) so it won't slip. This is referred to as the development length, l_d. Providing sufficient length to anchor bars that need to reach the yield stress near the end of connections are also specified by hook lengths. *Detailing reinforcement is a tedious job.* Splices are also necessary to extend the length of reinforcement that come in standard lengths. The equations are not provided here.

Development Length in Tension

With the proper bar to bar spacing and cover, the common development length equations are:

#6 bars and smaller: $l_d = \dfrac{d_b F_y}{25\sqrt{f'_c}}$ or 12 in. minimum

#7 bars and larger: $l_d = \dfrac{d_b F_y}{20\sqrt{f'_c}}$ or 12 in. minimum

Development Length in Compression

$$l_d = \frac{0.02 d_b F_y}{\sqrt{f'_c}} \le 0.0003 d_b F_y$$

Hook Bends and Extensions

The minimum hook length is $l_{dh} = \dfrac{1200 d_b}{\sqrt{f'_c}}$

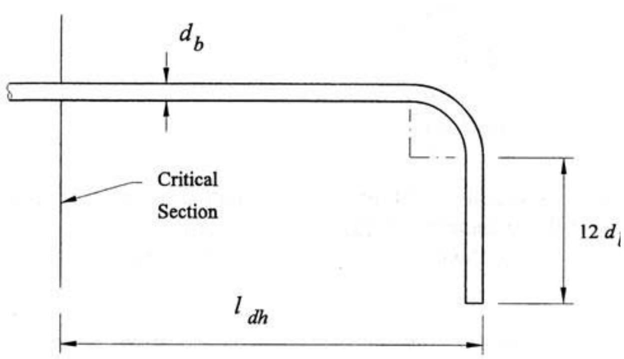


Figure 9-17: Minimum requirements for 90°bar hooks.

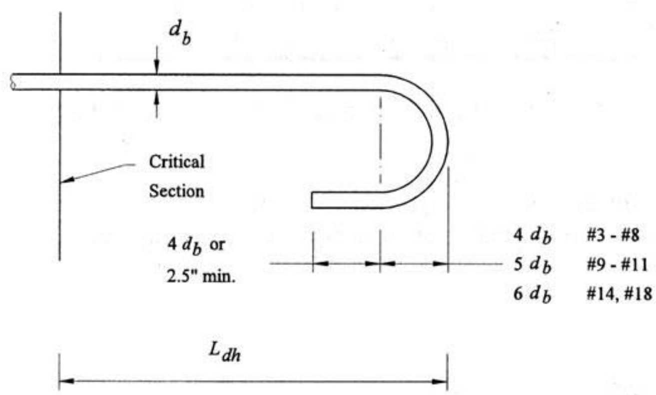


Figure 9-18: Minimum requirements for 180° bar hooks.

Modulus of Elasticity & Deflection

E_c for deflection calculations can be used with the transformed section modulus in the elastic range. After that, the cracked section modulus is calculated and E_c is adjusted.

Code values:

$$E_c = 57{,}000\sqrt{f'_c} \textit{ (normal weight)} \qquad E_c = w_c^{1.5} 33\sqrt{f'_c}, \; w_c = 90 \text{ lb/ft}^3 - 160 \text{ lb/ft}^3$$

Deflections of beams and one-way slabs need not be computed if the overall member thickness meets the minimum specified by the code, and are shown in Table 9.5(a) (see *Slabs*).

Criteria for Flat Slab & Plate System Design

Systems with slabs and supporting beams, joists or columns typically have multiple bays. The horizontal elements can act as one-way or two-way systems. Most often the flexure resisting elements are continuous, having positive and negative bending moments. These moment and shear values can be found using beam tables, or from code specified approximate design factors. Flat slab two-way systems have drop panels (for shear), while flat plates do not.

Criteria for Column Design

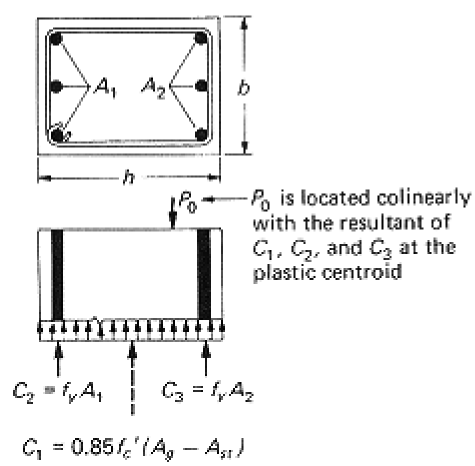
(American Concrete Institute) ACI 318-02 Code and Commentary:

$P_u \le \phi_c P_n$ where

P_u is a factored load
ϕ is a resistance factor
P_n is the nominal load capacity (strength)

Load combinations, ex: 1.4D (D is dead load)
1.2D + 1.6L (L is live load)

For compression, $\phi_c = 0.75$ and $P_n = 0.85P_o$ for spirally reinforced, $\phi_c = 0.65$ and $P_n = 0.8P_o$ for tied columns where $P_o = 0.85 f'_c (A_g - A_{st}) + f_y A_{st}$ and P_o is the name of the maximum axial force with no concurrent bending moment.



Columns which have reinforcement ratios, $\rho_g = \frac{A_{st}}{A_g}$, in the range of 1% to 2% will usually be the most economical, with 1% as a minimum and 8% as a maximum by code.

Bars are symmetrically placed, typically.

Spiral ties are harder to construct.

Columns with Bending (Beam-Columns)

Concrete columns rarely see only axial force and must be designed for the combined effects of axial load and bending moment. The **interaction** diagram shows the reduction in axial load a column can carry with a bending moment.

Design aids commonly present the interaction diagrams in the form of load vs. equivalent eccentricity for standard column sizes and bars used.

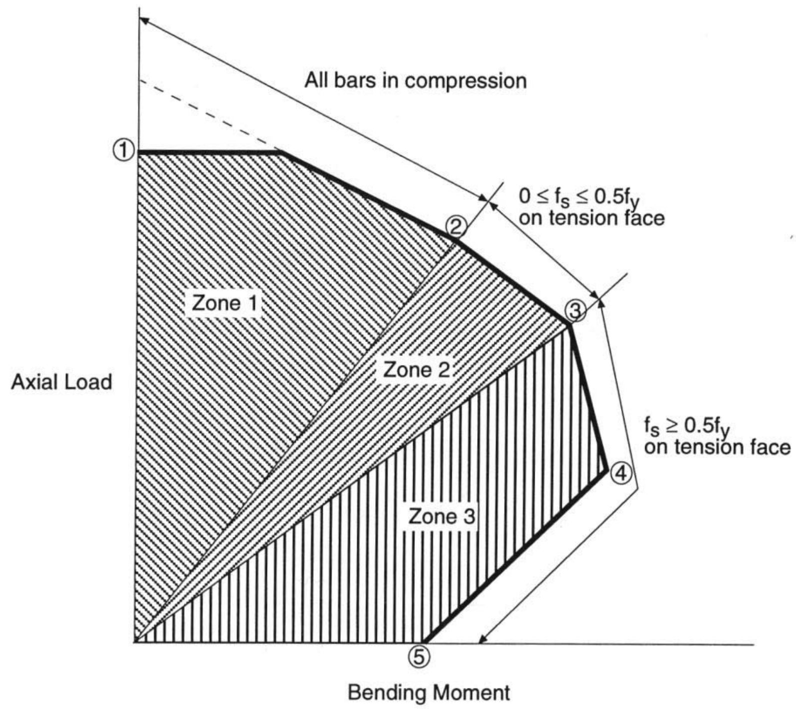


Figure 5-3 Transition Stages on Interaction Diagram

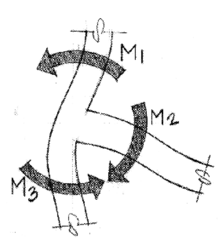
Rigid Frames

Monolithically cast frames with beams and column elements will have members with shear, bending and axial loads. Because the joints can rotate, the effective length must be determined from methods like that presented in the handout on Rigid Frames. The charts for evaluating k for non-sway and sway frames can be found in the ACI code.

Frame Columns

Because joints can rotate in frames, the effective length of the column in a frame is harder to determine. The stiffness (EI/L) of each member in a joint determines how rigid or flexible it is. To find k, the relative stiffness, G or Ψ, must be found for both ends, plotted on the alignment charts, and connected by a line for braced and unbraced fames.

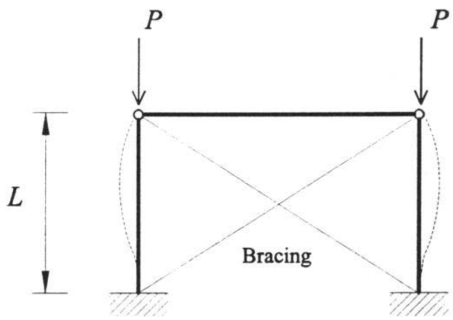
$$G = \Psi = \frac{\Sigma EI/l_c}{\Sigma EI/l_b}$$



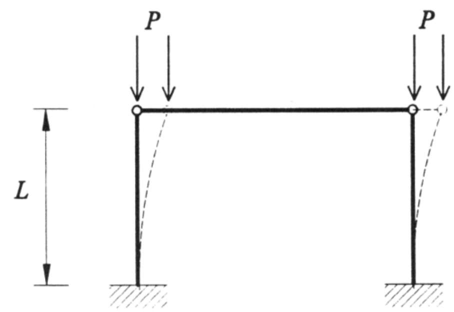
where

E = modulus of elasticity for a member
I = moment of inertia of for a member
l_c = length of the column from center to center
l_b = length of the beam from center to center

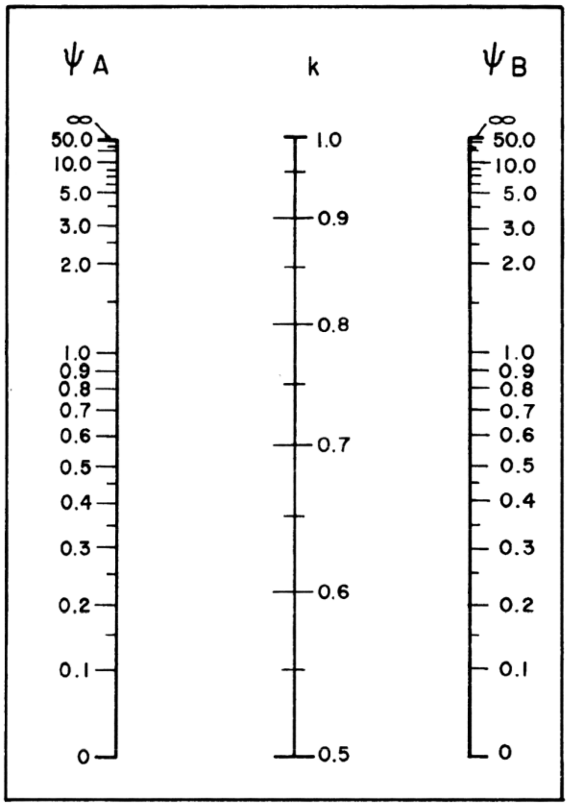
- For pinned connections we typically use a value of 10 for Ψ.
- For fixed connections we typically use a value of 1 for Ψ.



Braced – non-sway frame

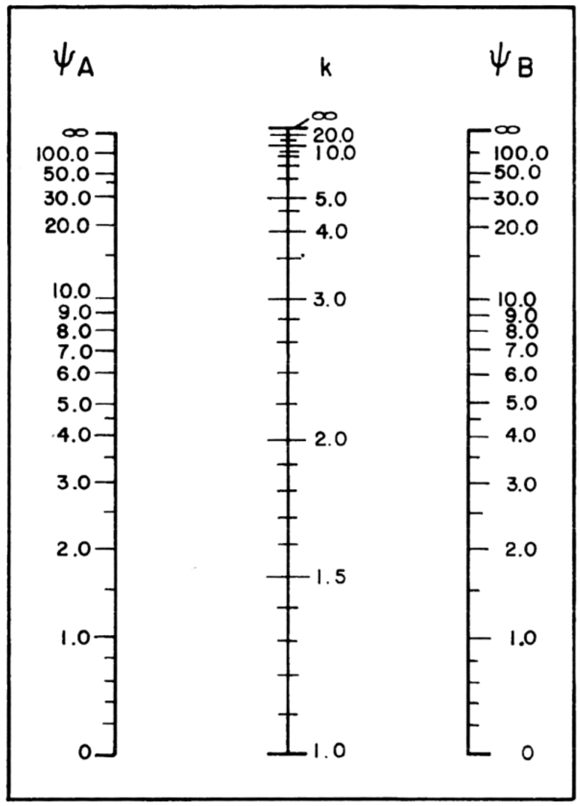


Unbraced – sway frame



(a)

Nonsway Frames

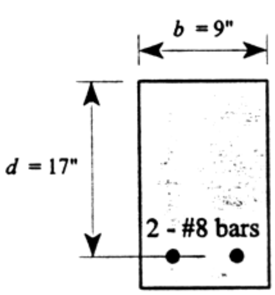


(b)

Sway Frames

Example 1

Determine the design moment capacity for the reinforced concrete cross section shown Assume f'_c = 3000 psi and Grade 60 reinforcing steel.



Example 2

(a) Determine the ultimate moment capacity of a beam with dimensions $b = 10$ in. and $d_{effective} = 15$ in. and that has three No. 9 bars (3.0 in.^{2}) of tension-reinforcing steel. Assume that $h = 18$ in., $F_y = 40$ ksi, and $f'_c = 5$ *ksi*. (b) Assume also that the section is used as a cantilever beam 10 ft long, where the service loads are dead load = 400 lb/ft and live load = 300 lb/ft. Is the beam adequate in bending? Calculate the ultimate moment capacity of the beam first.

Solution:

(a) $a = A_sF_y/0.85f'_cb = (3)(40{,}000)/(0.85)(5000)(10) = 2.82$ in.

$\phi M_n = \phi A_sF_y[d - a/2] = 0.9(3)(40{,}000)[15 - (2.82)/(2)] = 1{,}466{,}640$ in.-lb

Check for overreinforcement, $c = 0.375 \cdot 15 = 5.625$. Depth of stress block $a = 0.80 \cdot 5.625$ in. = 4.5 in. $A_{s,max} = (0.85)(5ksi)(4.5in.)(10in.)/(40ksi) = 4.78$ in.^{2} The beam is not over reinforced Check for minimum steel: $A_{s,min} = \dfrac{3\sqrt{f'_c}}{F_y}bd$ $= 0.80\ in^2$, so beam is sufficiently reinforced.

(b) $U = 1.2D + 1.6L = 1.2(400) + 1.6(300) = 960$ lb/ft

$M_u = w_uL^2/2 = (960)(10^2)/2 = 48{,}000$ ft-lb = 576,000 in.-lb

Since $M_u = 576{,}000_{\text{lb-in}} < \phi M_n = 1{,}466{,}640_{\text{lb-in}}$, the beam is adequate in bending.

EXAMPLE

Determine the ultimate moment capacity of a beam of dimensions $b = 250$ mm and $d = 350$ mm and that has 300 mm^2 of reinforcing steel. Assume that $F_y = 400$ MPa and $f'_c = 25$ MPa.

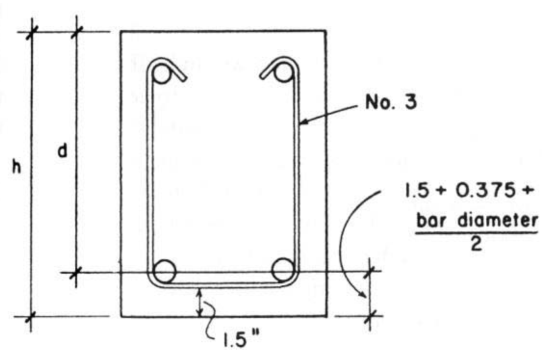
Solution:

$$a = \frac{A_sF_y}{0.85f'_cb} = \frac{(300)(400)}{(0.85)(25)(250)} = 22.6 \text{ mm}$$

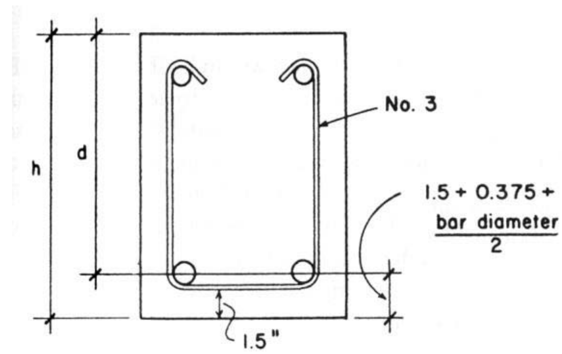
$$\phi M_n = \phi A_sF_y\left(d - \frac{a}{2}\right) = 0.9(300)(400)\left(350 - \frac{22.6}{2}\right) = 36.5 \text{ kN} \cdot \text{m}$$

Example 3

Example 1. The service load bending moments on a beam are 58 kip-ft [78.6 kN-m] for dead load and 38 kip-ft [51.5 kN-m] for live load. The beam is 10 in. [254 mm] wide, f'_c is 3000 psi [27.6 MPa], and f_y is 60 ksi [414 MPa]. Determine the depth of the beam and the tensile reinforcing required.



Example 3 (continued)



Example 4

A simply supported beam 20 ft long carries a service dead load of 300 lb/ft and a live load of 500 lb/ft. Design an appropriate beam (for flexure only). Use grade 40 steel and concrete strength of 5000 psi.

SOLUTION:

Find the design moment, M_u, from the factored load combination of 1.2D + 1.6L. It is good practice to guess a beam size to include self weight in the dead load, because "service" means dead load of everything except the beam itself.

Guess a size of 10 in x 12 in. Self weight for normal weight concrete is the density of 150 lb/ft^3 multiplied by the cross section area: self weight = $150\,{}^{lb}\!/_{ft^3}(10in)(12in)\cdot(\frac{1ft}{12in})^2$ = 125 lb/ft

w_u = 1.2(300 lb/ft + 125 lb/ft) + 1.6(500 lb/ft) = 1310 lb/ft

The maximum moment for a simply supported beam is $\frac{wl^2}{8}$: $M_u = \frac{w_u l^2}{8} = \frac{1310\,{}^{lb}\!/_{ft}(20ft)^2}{8}$ 65,500 lb-ft

M_n required = M_u/ϕ = $\frac{65{,}500^{lb-ft}}{0.9}$ = 72,778 lb-ft

To use the design chart aid, find $R_n = \frac{M_n}{bd^2}$, estimating that d is about 1.75 inches less than h:

d = 12in – 1.75 in – ~~(0.375)~~ = 10.25 in (NOTE: If there are stirrups, you must also subtract the diameter of the stirrup bar.)

$R_n = \frac{72{,}778^{lb-ft}}{(10in)(10.25in)^2}\cdot(12\,{}^{in}\!/_{ft})$ = 831 psi

ρ corresponds to approximately 0.023 (which is less than that for 0.005 strain of 0.0319) , so the estimated area required, A_s, can be found:

$A_s = \rho bd$ = (0.023)(10in)(10.25in) = 2.36 in^2

The number of bars for this area can be found from handy charts.

(Whether the number of bars actually fit for the width with cover and space between bars <u>must also be considered</u>. If you are at ρ_{max} <u>do not</u> choose an area bigger than the maximum!)

Try A_s = 2.37 in^2 from 3#8 bars

d = 12 in – 1.5 in (cover) – ½ (8/8in diameter bar) = 10 in

Check ρ = 2.37 in^2/(10 in)(10 in) = 0.0237 which is less than $\rho_{max-0.005}$ = 0.0319 OK (We cannot have an over reinforced beam!!)

Find the moment capacity of the beam as designed, ϕM_n

$a = A_s f_y/0.85f'_c b$ = 2.37 in^2 (40 ksi)/[0.85(5 ksi)10 in] = 2.23 in

$\phi M_n = \phi A_s f_y(d-a/2) = 0.9(2.37in^2)(40ksi)(10in - \frac{2.23in}{2})\cdot(\frac{1}{12\,{}^{in}\!/_{ft}}) = 63.2$ k-ft $\not>$ 65.5 k-ft needed (not OK)

So, we can increase d to 13 in, and ϕM_n = 70.3 k-ft (OK). Or increase A_s to 2 # 10's (2.54 in^2), for a = 2.39 in and ϕM_n of 67.1 k-ft (OK). <u>*Don't exceed ρ_{max} or $\rho_{max-0.005}$ if you want to use ϕ=0.9*</u>

Example 5

A simply supported beam 20 ft long carries a service dead load of 425 lb/ft (including self weight) and a live load of 500 lb/ft. Design an appropriate beam (for flexure only). Use grade 40 steel and concrete strength of 5000 psi.

SOLUTION:

Find the design moment, M_u, from the factored load combination of 1.2D + 1.6L. *If self weight is not included in the service loads,* you need to guess a beam size to include self weight in the dead load, because "service" means dead load of everything except the beam itself.

w_u = 1.2(425 lb/ft) + 1.6(500 lb/ft) = 1310 lb/ft

The maximum moment for a simply supported beam is $\frac{wl^2}{8}$: $M_u = \frac{w_u l^2}{8} = \frac{1310\,^{lb}/_{ft}(20\,ft)^2}{8}$ 65,500 lb-ft

M_n required = $M_u/\phi = \frac{65{,}500^{lb-ft}}{0.9}$ = 72,778 lb-ft

To use the design chart aid, we can find $R_n = \frac{M_n}{bd^2}$**,** and estimate that h is roughly 1.5-2 times the size of b, and h = 1.1d (rule of thumb): d = h/1.1 = (2b)/1.1, so $d \approx 1.8b$ or $b \approx 0.55d$.

We can find R_n at the maximum reinforcement ratio for our materials, keeping in mind ρ_{max} at a strain = 0.005 is 0.0319 off of the chart at about 1070 psi, with ρ_{max} = 0.037. Let's substitute b for a function of d:

R_n = 1070 psi = $\frac{72{,}778^{lb-ft}}{(0.55d)(d)^2}\cdot(12\,^{in}/_{ft})$ Rearranging and solving for d = 11.4 inches

That would make b a little over 6 inches, which is impractical. 10 in is commonly the smallest width.

So if h is commonly 1.5 to 2 times the width, b, h ranges from 14 to 20 inches. (10x1.5=15 and 10x2 = 20)

Choosing a depth of 14 inches, d ≅ 14 - 1.5 (clear cover) - ½(1" diameter bar guess) -3/8 in (stirrup diameter) = 11.625 in.

Now calculating an updated $R_n = \frac{72{,}778^{lb-ft}}{(10in)(11625in)^2}\cdot(12\,^{in}/_{ft}) = 646.2psi$

ρ now is 0.020 (under the limit at 0.005 strain of 0.0319), so the estimated area required, A_s, can be found:

$A_s = \rho bd$ = (0.020)(10in)(11.625in) = 1.98 in^2

The number of bars for this area can be found from handy charts.
(Whether the number of bars actually fit for the width with cover and space between bars <u>must also be considered</u>. If you are at $\rho_{max-0.005}$ <u>do not</u> choose an area bigger than the maximum!)

Try A_s = 2.37 in^2 from 3#8 bars. (or 2.0 in^2 from 2 #9 bars. 4#7 bars don't fit...)

d(actually) = 14 in. – 1.5 in (cover) – ½ (8/8 in bar diameter) – 3/8 in. (stirrup diameter) = 11.625 in.

Check ρ = 2.37 in^2/(10 in)(11.625 in) = 0.0203 which is less than $\rho_{max-0.005}$ = 0.0319 OK (We cannot have an over reinforced beam!!)

Find the moment capacity of the beam as designed, ϕM_n

$a = A_s f_y/0.85f'_c b$ = 2.37 in^2 (40 ksi)/[0.85(5 ksi)10 in] = 2.23 in

$\phi M_n = \phi A_s f_y(d-a/2) = 0.9(2.37in^2)(40ksi)(11.625in-\frac{2.23in}{2})\cdot(\frac{1}{12\,^{in}/_{ft}}) =$ 74.7 k-ft > 65.5 k-ft needed

OK! <u>Note</u>: *If the section doesn't work, you need to increase d or A_s as long as you don't exceed $\rho_{max-0.005}$*

Example 6

A simply supported beam 25 ft long carries a service dead load of 2 k/ft, an estimated self weight of 500 lb/ft and a live load of 3 k/ft. Design an appropriate beam (for flexure only). Use grade 60 steel and concrete strength of 3000 psi.

SOLUTION:

Find the design moment, M_u, from the factored load combination of 1.2D + 1.6L. If self weight is estimated, and the selected size has a larger self weight, the design moment must be adjusted for the extra load.

w_u = 1.2(2 k/ft + 0.5 k/ft) + 1.6(3 k/ft) = 7.8 k/ft So, $M_u = \frac{w_u l^2}{8} = \frac{7.8\,{}^{k}\!/_{ft}(25\,ft)^2}{8}$ 609.4 k-ft

M_n required = M_u/ϕ = $\frac{609.4^{k-ft}}{0.9}$ = 677.1 k-ft

To use the design chart aid, we can find $R_n = \frac{M_n}{bd^2}$, and estimate that h is roughly 1.5-2 times the size of b, and h = 1.1d (rule of thumb): d = h/1.1 = (2b)/1.1, so d ≈ 1.8b or b ≈ 0.55d.

We can find R_n at the maximum reinforcement ratio for our materials off of the chart at about 700 psi with $\rho_{max\text{-}0.005}$ = 0.0135. Let's substitute b for a function of d:

R_n = 700 psi = $\frac{677.1^{k-ft}(1000^{lb/k})}{(0.55d)(d)^2}\cdot(12\,{}^{in}\!/_{ft})$ Rearranging and solving for d = 27.6 inches

That would make b 15.2 in. (from 0.55d). Let's try 15. So,

h ≅ d + 1.5 (clear cover) +½(1" diameter bar guess) +3/8 in (stirrup diameter) = 27.6 +2.375 = 29.975 in.

Choosing a depth of 30 inches, d ≅ 30 - 1.5 (clear cover) - ½(1" diameter bar guess) -3/8 in (stirrup diameter) = 27.625 in.

Now calculating an updated R_n = $\frac{677{,}100^{lb-ft}}{(15in)(27.625in)^2}\cdot(12\,{}^{in}\!/_{ft}) = 710psi$ This is larger than R_n for the 0.005 strain limit!

We can't just use $\rho_{max\text{-}.005}$. The way to reduce R_n is to increase b or d or both. Let's try increasing h to 31 in., then R_n = 661 psi with d = 28.625 in.. That puts us under $\rho_{max\text{-}0.005}$. We'd have to remember to keep UNDER the area of steel calculated, which is hard to do.

From the chart, ρ ≈ 0.013, less than the $\rho_{max\text{-}0.005}$ of 0.0135, so the estimated area required, A_s, can be found:
$A_s = \rho bd$ = (0.013)(15in)(29.625in) = 5.8 in^2

The number of bars for this area can be found from handy charts. Our charts say there can be 3 – 6 bars that fit when ¾" aggregate is used. We'll assume 1 inch spacing between bars. The actual limit is the maximum of 1 in, the bar diameter or 1.33 times the maximum aggregate size.

Try A_s = 6.0 in^2 from 6#9 bars. Check the width: 15 – 3 (1.5 in cover each side) – 0.75 (two #3 stirrup legs) – 6*1.128 – 5*1.128 in. = -1.16 in NOT OK.

Try A_s = 5.08 in^2 from 4#10 bars. Check the width: 15 – 3 (1.5 in cover each side) – 0.75 (two #3 stirrup legs) – 4*1.27 – 3*1.27 in. = 2.36 OK.

d(actually) = 31 in. – 1.5 in (cover) – ½ (1.27 in bar diameter) – 3/8 in. (stirrup diameter) = 28.49 in.

Find the moment capacity of the beam as designed, ϕM_n

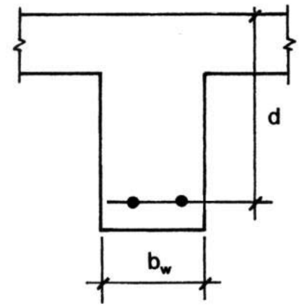
$a = A_s f_y/0.85f'_c b$ = 5.08 in^2 (60 ksi)/[0.85(3 ksi)15 in] = 8.0 in

$\phi M_n = \phi A_s f_y(d-a/2) = 0.9(5.08in^2)(60ksi)(28.49in-\frac{8.0in}{2})\cdot(\frac{1}{12\,{}^{in}\!/_{ft}}) =$ 559.8 k-ft < 609 k-ft needed!! (NO GOOD)

More steel isn't likely to increase the capacity much unless we are close. It looks like we need more steel **and** lever arm. Try h = 32 in. AND b = 16 in., then M_u^* (with the added self weight of 33.3 lb/ft) = 680.2 k-ft, ρ ≈ 0.012, As = 0.012(16in)(29.42in)=5.66 in^2. 6#9's won't fit, but 4#11's will: ρ = 0.0132 ✓, a = 9.18 in, and ϕM_n = 697.2 k-ft which is finally larger than 680.2 k-ft **OK**

Example 7

Example 3. A T-section is to be used for a beam to resist positive moment. The following data are given: beam span is 18 ft [5.49 m], beams are 9 ft [2.74 m] center to center, slab thickness is 4 in. [0.102 m], beam stem dimensions are b_w = 15 in. [0.381 m] and d = 22 in. [0.559 m], f'_c = 4 ksi [27.6 MPa], f_y = 60 ksi [414 MPa]. Find the required area of steel and select the reinforcing bars for a dead load moment of 125 kip-ft [170 kN-m] plus a live load moment of 100 kip-ft [136 kN-m].



Example 8

Design a T-beam for a floor with a 4 in slab supported by 22-ft-span-length beams cast monolithically with the slab. The beams are 8 ft on center and have a web width of 12 in. and a total depth of 22 in.; f'_c = 3000 psi and f_y = 60 ksi. Service loads are 125 psf and 200 psf dead load which does not include the weight of the floor system.

SOLUTION:

1. Establish the design moment:

$$\text{slab weight} = \frac{96(4)}{144}(0.150) = 0.400 \text{ kip/ft}$$

$$\text{stem weight} = \frac{12(18)}{144}(0.150) = 0.225$$

$$\text{total} = 0.625 \text{ kip/ft}$$

$$\text{service DL} = 8(0.200) = 1.60 \text{ kips/ft}$$

$$\text{service LL} = 8(0.125) = 1.00 \text{ kip/ft}$$

Calculate the factored load and moment:

$$w_u = 1.2(0.625 + 1.60) + 1.6(1.00) = 4.27 \text{ kip/ft}$$

$$M_u = \frac{w_u \ell^2}{8} = \frac{4.27(22)^2}{8} = 258 \text{ ft-kips}$$

2. Assume an effective depth $d = h - 3$ in.:

$$d = 22 - 3 = 19 \text{ in.}$$

3. Determine the effective flange width:

$$\tfrac{1}{4} \text{ span length} = 0.25(22)(12) = 66 \text{ in.}$$

$$b_w + 16h_f = 12 + 16(4) = 76 \text{ in.}$$

$$\text{beam spacing} = 96 \text{ in.}$$

Use an effective flange width b = 66 in.

4. Determine whether the beam behaves as a true T-beam or as a rectangular beam by computing the practical moment strength ϕM_{nf} with the full effective flange assumed to be in compression. This assumes that the bottom of the compressive stress block coincides with the bottom of the flange, as shown in Figure 3-10. Thus

$$\phi M_{nf} = \phi(0.85 f'_c) b h_f \left(d - \frac{h_f}{2}\right)$$

$$= 0.9(0.85)(3)(66)\frac{4(19 - 4/2)}{12} = 858 \text{ ft-kips}$$

5. Since 858 ft-kips > 258 ft-kips, the total effective flange need not be completely utilized in compression (i.e., $a < h_f$), and the T-beam behaves as a wide rectangular beam with a width b of 66 in.

6. Design as a rectangular beam with b and d as known values (see Section 2-15):

$$\text{required } R_n = \frac{M_u}{\phi b d^2} = \frac{258(12)}{0.9(66)(19)^2} = 0.1444 \text{ ksi}$$

7. From Table A-8, select the required steel ratio to provide a R_n of 0.1444 ksi

required ρ = 0.0024

8. Calculate the required steel area:

$$\text{required } A_s = \rho b d$$

$$= 0.0024(66)(19) = 3.01 \text{ in.}^2$$

9. Select the steel bars. Use 3#9 (A_s = 3.00 in.^{2})

minimum b_w = 7.125 in (O.K.)

Check the effective depth d:

$$d = 22 - 1.5 - 0.38 - \frac{1.125}{2} = 19.56 \text{ in.}$$

19.49 in. > 19 in. (O.K.)

10. Check $A_{s,min}$. From Table A-5:

$$A_{s,min} = 0.0033 b_w d$$

$$= 0.0033(12)(19) = 0.75 \text{ in.}^2$$

0.75 in.^{2} < 3.00 in^2

11. Check $A_{s,max}$:

$$A_{s,max} = 0.0135(66)(19)$$

$$= 16.93 \text{ in.}^2 > 3.00 \text{ in.}^2$$ (O.K)

12. Verify the moment capacity:
(Is $M_u \le \phi M_n$)

a = (3.00)(60)/[0.85(3)(66)] = 1.07 in.

$$\phi M_n = 0.9(3.00)(60)(19.56 - \frac{1.07}{2})\,\tfrac{1}{12}$$

= 256.9.1 ft-kips (Not O.K)

Choose more steel, A_s = 3.16 in^2 from 4-#8's

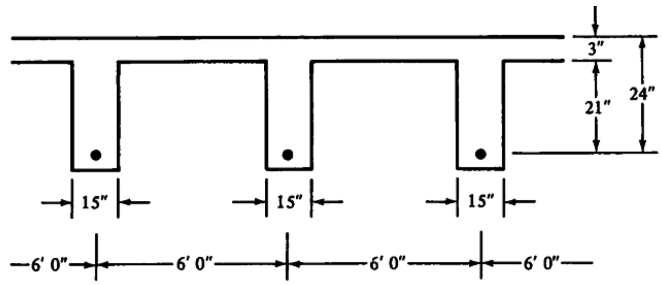
d = 19.62 in, a = 1.13 in

ϕM_n = 271.0 ft-kips, which is OK

13. Sketch the design

Example 9

Design a T-beam for the floor system shown for which b_w and d are given. M_D = 200 ft-k, M_L = 425 ft-k, f'_c = 3000 psi and f_y = 60 ksi, and simple span = 18 ft.



SOLUTION

Effective Flange Width

(a) $\frac{1}{4} \times 18' = 4'6'' = \underline{\underline{54''}}$

(b) $15'' + (2)(8)(3) = 63''$

(c) $6'0'' = 72''$

Moments Assuming $\phi = 0.90$

$$M_u = (1.2)(200) + (1.6)(425) = 920 \text{ ft-k}$$

$$M_n = \frac{M_u}{0.90} = \frac{920}{0.90} = 1022 \text{ ft-k}$$

First assume $a \leq h_f$ (which is very often the case). Then the design would proceed like that of a rectangular beam with a width equal to the effective width of the T beam flange.

$$\frac{M_u}{\phi bd^2} = \frac{920(12{,}000)}{(0.9)(54)(24)^2} = 394.4 \text{ psi}$$

from Table A.12, $\rho = 0.0072$

$$a = \frac{\rho f_y d}{0.85 f'_c} = \frac{0.0072(60)(24)}{(0.85)(3)} = 4.06 \text{ in.} > h_f = 3 \text{ in.}$$

The beams acts like a T beam, not a rectangular beam, and the values for ρ and a above are not correct. If the value of a had been $\leq h_f$, the value of A_s would have been simply $\rho bd = 0.0072(54)(24) = 9.33 \text{ in}^2$. Now break the beam up into two parts (Figure 5.7) and design it as a T beam.

Assuming $\phi = 0.90$

$$A_{sf} = \frac{(0.85)(3)(54 - 15)(3)}{60} = 4.97 \text{ in.}^2$$

$$M_{uf} = (0.9)(4.97)(60)(24 - \tfrac{3}{2}) = 6039 \text{ in.-k} = 503 \text{ ft-k}$$

$$M_{uw} = 920 - 503 = 417 \text{ ft-k}$$

Designing a rectangular beam with $b_w = 15$ in. and $d = 24$ in. to resist 417 ft-k

$$\frac{M_{uw}}{\phi b_w d^2} = \frac{(12)(417)(1000)}{(0.9)(15)(24)^2} = 643.5$$

$$\rho_w = 0.0126 \text{ from Appendix Table A.12}$$

$$A_{sw} = (0.0126)(15)(24) = 4.54 \text{ in.}^2$$

$$A_s = 4.97 + 4.54 = \underline{\underline{9.51 \text{ in.}^2}}$$

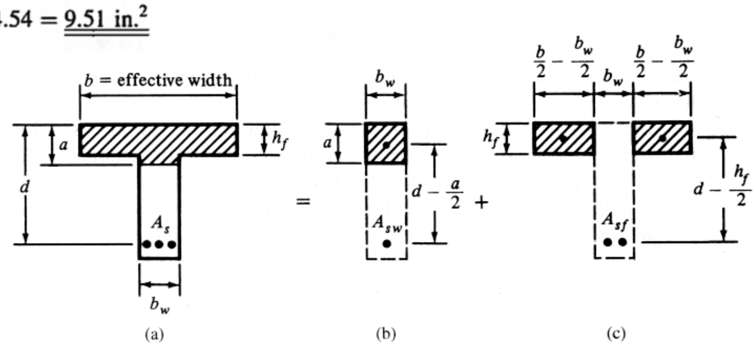
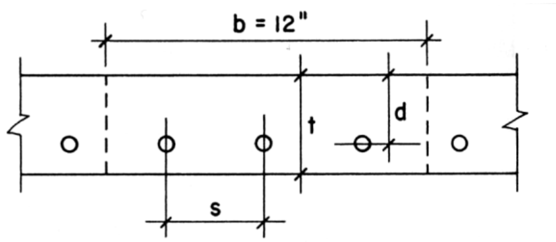


Figure 5.7 Separation of T beam into rectangular parts.

Example 10



Example 6. A one-way solid concrete slab is to be used for a simple span of 14 ft [4.27 m]. In addition to its own weight, the slab carries a superimposed dead load of 30 psf [1.44 kPa] plus a live load of 100 psf [4.79 kPa]. Using f'_c = 3 ksi [20.7 MPa] and f_y = 40 ksi [276 MPa], design the slab for minimum overall thickness.

TABLE 13.6 Areas Provided By Spaced Reinforcement

Bar Spacing (in.)	Area Provided (in.²/ft width)								
	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11
3	0.44	0.80	1.24	1.76	2.40	3.16	4.00		
3.5	0.38	0.69	1.06	1.51	2.06	2.71	3.43	4.35	
4	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4.5	0.29	0.53	0.83	1.17	1.60	2.11	2.67	3.39	4.16
5	0.26	0.48	0.74	1.06	1.44	1.89	2.40	3.05	3.74
5.5	0.24	0.44	0.68	0.96	1.31	1.72	2.18	2.77	3.40
6	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
7	0.19	0.34	0.53	0.75	1.03	1.35	1.71	2.18	2.67
8	0.16	0.30	0.46	0.66	0.90	1.18	1.50	1.90	2.34
9	0.15	0.27	0.41	0.59	0.80	1.05	1.33	1.69	2.08
10	0.13	0.24	0.37	0.53	0.72	0.95	1.20	1.52	1.87
11	0.12	0.22	0.34	0.48	0.65	0.86	1.09	1.38	1.70
12	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
13	0.10	0.18	0.29	0.40	0.55	0.73	0.92	1.17	1.44
14	0.09	0.17	0.27	0.38	0.51	0.68	0.86	1.09	1.34
15	0.09	0.16	0.25	0.35	0.48	0.63	0.80	1.01	1.25
16	0.08	0.15	0.23	0.33	0.45	0.59	0.75	0.95	1.17
18	0.07	0.13	0.21	0.29	0.40	0.53	0.67	0.85	1.04
24	0.05	0.10	0.15	0.22	0.30	0.39	0.50	0.63	0.78

Example 11

nple 2-9

Design a simple-span one-way slab to carry a uniformly distributed live load of 400 psf. The span is 10 ft (center to center of supports). Use $f'_c = 4000$ psi and $f_y = 60{,}000$ psi. Select the thickness to be not less than the ACI minimum thickness requirement.

Solution:

Determine the required minimum h and use this to estimate the slab dead weight.

1. From ACI Table 9.5(a), for a simply supported, solid, one-way slab,

$$\text{minimum } h = \frac{\ell}{20} = \frac{10(12)}{20} = 6.0 \text{ in.}$$

Try $h = 6$ in. and design a 12-in.-wide segment.

2. Determine the slab weight dead load:

$$\frac{6(12)}{144}(0.150) = 0.075 \text{ kip/ft}$$

The total design load is

$$w_u = 1.2w_{DL} + 1.6w_{LL}$$
$$= 1.2(0.075) + 1.6(0.400)$$
$$= 0.730 \text{ kip/ft}$$

3. Determine the design moment:

$$M_u = \frac{w_u \ell^2}{8} = \frac{0.73(10)^2}{8} = 9.125 \text{ ft-kips}$$

4. Establish the approximate d. Assuming No. 6 bars and minimum concrete cover on the bars of ¾ in.,

$$\text{assumed } d = 6.0 - 0.75 - 0.375 = 4.88 \text{ in.}$$

5. Determine the required R_n:

$$\text{required } R_n = \frac{M_u}{\phi b d^2}$$
$$= \frac{9.125(12)}{0.9(12)(4.88)^2} = 0.4257 \text{ ksi}$$

6. From Table A-10, for a required $R_n = 0.4257$, the required $\rho = 0.0077$. (Note that the required ρ selected is the next *higher* value from Table A-10.) Thus

$$\rho_{max} = 0.0181 > 0.0077 \qquad \text{(O.K.)}$$

Use $\rho = 0.0077$.

7. $$\text{required } A_s = \rho b d = 0.0077(12)(4.88) = 0.45 \text{ in.}^2\text{/ft}$$

8. Select the main steel (from Table A-4). Select No. 5 bars at 7½ in. o.c. ($A_s = 0.50$ in.^{2}). The assumption on bar size was satisfactory. The code requirements for maximum spacing have been discussed in Section 2-13. Minimum spacing of bars in slabs, practically, should not be less than 4 in., although the ACI Code allows bars to be placed closer together, as discussed in Example 2-7. Check the maximum spacing (ACI Code, Section 7.6.5):

$$\text{maximum spacing} = 3h \quad \text{or} \quad 18 \text{ in.}$$
$$3h = 3(6) = 18 \text{ in.}$$
$$7\tfrac{1}{2} \text{ in.} < 18 \text{ in.} \qquad \text{(O.K.)}$$

Therefore use No. 5 bars at 7½ in. o.c.

9. Select shrinkage and temperature reinforcement (ACI Code, Section 7.12):

$$\text{required } A_s = 0.0018bh$$
$$= 0.0018(12)(6) = 0.13 \text{ in.}^2\text{/ft}$$

Select No. 3 bars at 10 in. o.c. ($A_s = 0.13$ in.^{2}) or No. 4 bars at 18 in. o.c. ($A_s = 0.13$ in.^{2}):

$$\text{maximum spacing} = 5h \quad \text{or} \quad 18 \text{ in.}$$

Use No. 3 bars at 10 in. o.c.

10. The main steel area must exceed the area required for shrinkage and temperature steel (ACI Code, Section 10.5.4):

$$0.50 \text{ in.}^2 > 0.13 \text{ in.}^2 \qquad \text{(O.K.)}$$

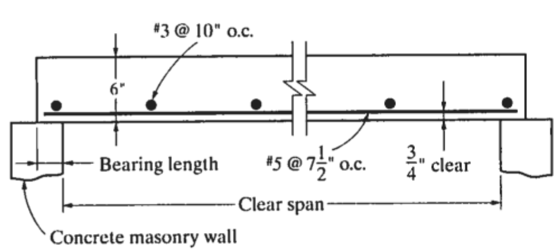
11. Verify the moment capacity:
(Is $M_u \le \phi M_n$)

$$a = \frac{(0.50)(60)}{0.85(4)(12)} = 0.74 in$$

$$\phi M_n = 0.9(0.50)(60)(5.0625 - 0.74/2)\, 1/12$$

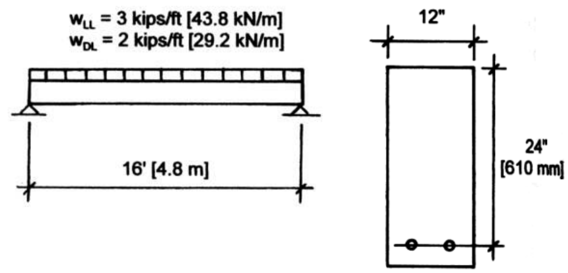
$$= 10.6 \text{ ft-kips} \qquad \text{OK)}$$

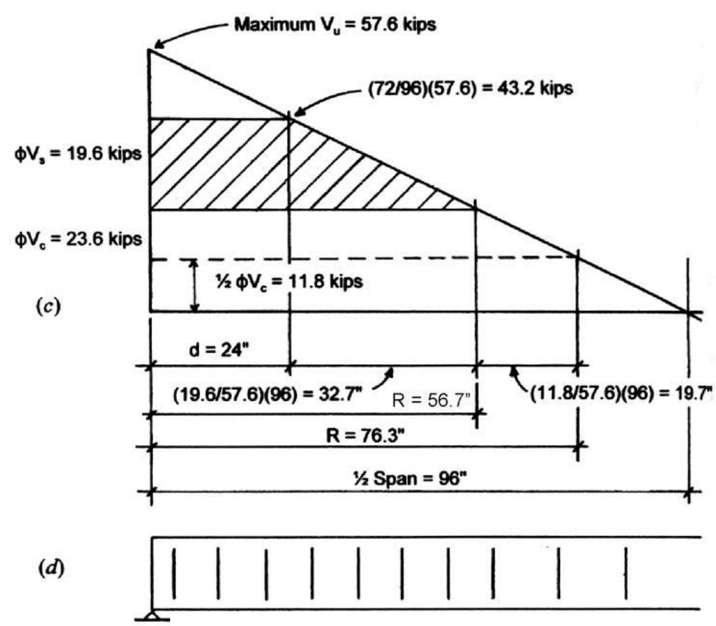
12. A design sketch is drawn:



Example 12

Example 7. Design the required shear reinforcement for the simple beam shown in Figure 13.18. Use $f'_c = 3$ ksi [20.7 MPa] and $f_y = 40$ ksi [276 MPa] and single U-shaped stirrups.





Example 13

For the simply supported concrete beam shown in Figure 5-61, determine the stirrup spacing (if required) using No. 3 U stirrups of Grade 60 (f_y = 60 ksi). Assume f'_c = 3000 psi.

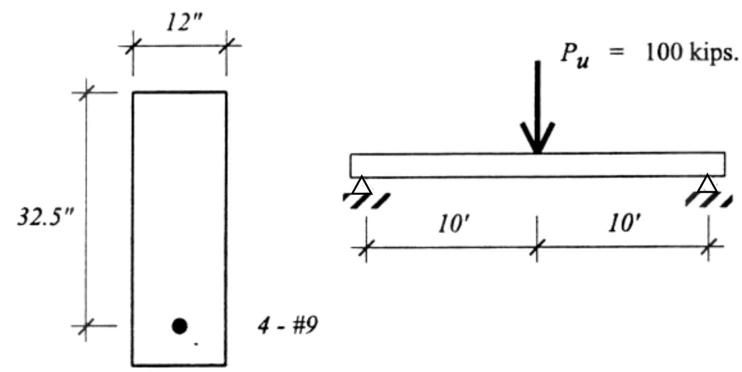


Figure 5-61: Simply supported concrete beam for Example 5-15.

f'_c = 3000 psi. For #3 bars, A_s = 0.11 in.^{2},
F_y = 60 ksi. with 2 legs, then A_v = 0.22 in.^{2}

Solution:

V_u = 50 kips (neglecting weight of the beam)

$$\phi V_c = \phi\, 2\sqrt{f'_c}\, b_w d$$

$$= \frac{(0.75)2\sqrt{3000}(12)(32.5)}{1000} = 32.0 \text{ kips} < V_u \quad \therefore \text{Need Stirrups}$$

Note: If $V_u = \frac{1}{2}\varphi V_c$, minimum stirrups would still be required.

$$V_u \le \phi V_c + \phi V_s$$

$$\therefore \phi V_s = V_u - \phi V_c = 50 - 32.0 = 18.0 \text{ kips} \qquad (< \phi 4\sqrt{f'_c}\, b_w d = 64.1 \text{ kips})$$

$$s_{req'd} \le \frac{\phi A_v F_y d}{\phi V_s} = \frac{(0.75)(0.22in^2)(60ksi)(32.5in)}{18.0k}$$

$$= 17.875 \text{ in.}$$

$$s_{max} = \frac{d}{2} = \frac{32.5}{2} = 16.2 \text{ in.} \quad \text{☚ controls}$$

$$= 24 \text{ in.}$$

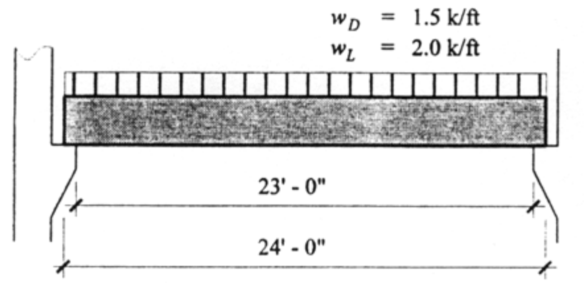
$$s_{req'd} \le \frac{A_v F_y}{50 b_w} = \frac{(0.22)(60{,}000)}{50(12)} = 22.0 \text{ in.}$$, but 16" (d/2) would be the maximum as well.

when $\phi V_c > V_u > \frac{\phi V_c}{2}$

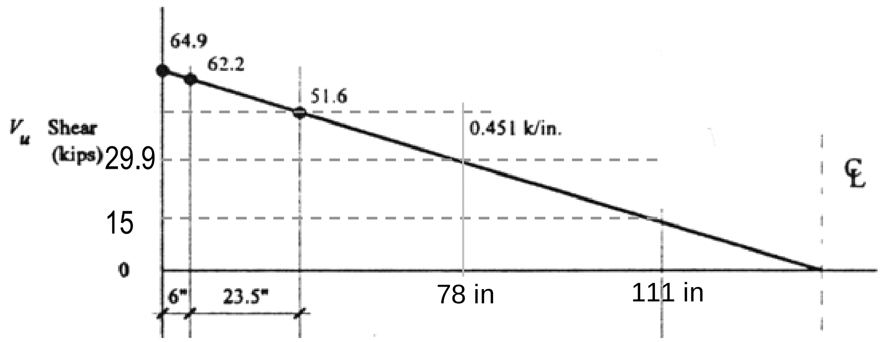
$\therefore$ Use #3 U @ 16" max spacing

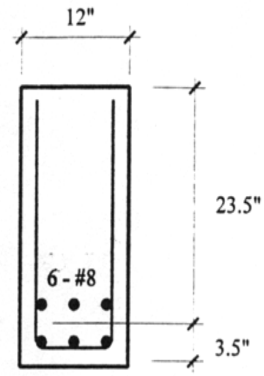
Example 14

Design the shear reinforcement for the simply supported reinforced concrete beam shown with a dead load of 1.5 k/ft and a live load of 2.0 k/ft. Use 5000 psi concrete and Grade 60 steel. Assume that the point of reaction is at the end of the beam.



SOLUTION:





Shear diagram:

Find self weight = 1 ft x (27/12 ft) x 150 lb/ft^3 = 338 lb/ft = 0.338 k/ft

w_u = 1.2 (1.5 k/ft + 0.338 k/ft) + 1.6 (2 k/ft) = 5.41 k/ft (= 0.451 k/in)

$V_{u\ (max)}$ is at the ends = $w_uL/2$ = 5.41 k/ft (24 ft)/2 = 64.9 k

$V_{u\ (support)}$ = $V_{u\ (max)}$ – w_u(distance) = 64.9 k – 5.4 1k/ft (6/12 ft) = 62.2 k

V_u for design is d away from the support = $V_{u\ (support)}$ – w_u(d) = 62.2 k – 5.41 k/ft (23.5/12 ft) = 51.6 k

Concrete capacity:

We need to see if the concrete needs stirrups for strength or by requirement because $V_u \le \phi V_c + \phi V_s$ (design requirement)

$\phi V_c = \phi 2\sqrt{f'_c}\, b_w d$ = 0.75 (2) $\sqrt{5000}$ psi (12 in) (23.5 in) = 299106 lb = 29.9 kips (< 51.6 k!)

Locating end points:

29.9 k = 64.9k – 0.451 k/in x (a)
a = 78 in
15 k = 64.9k – 0.451 k/in x (b)
b = 111 in.

Stirrup design and spacing

We need stirrups: $A_v = V_s s/f_y d$

$\phi V_s \ge V_u - \phi V_c$ = 51.6 k – 29.9 k = 21.7 k

Spacing requirements are in Table 3-8 and depend on $\phi V_c/2$ = 15.0 k and $2\phi V_c$ = 59.8 k

2 legs for a #3 is 0.22 in^2, so $s_{req'd} \le \phi A_v f_y d/\phi V_s$ = 0.75(0.22 in^2)(60 ksi)(23.5 in)/21.7 k = 10.72 in Use s = 10"

our maximum falls into the d/2 or 24", so d/2 governs with 11.75 in Our 10" is ok.

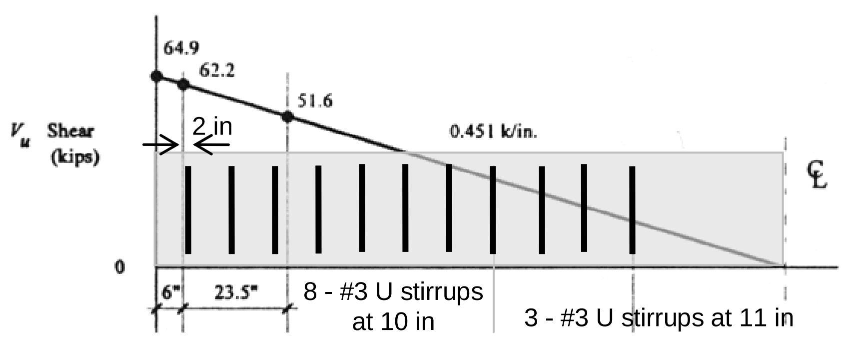
This spacing is valid until $V_u = \phi V_c$ and that happens at (64.9 k – 29.9 k)/0.451 k/in = 78 in

We can put the first stirrup at a minimum of 2 in fr support face, so we need 10" spaces for (78 – 2 - 7 even (8 stirrups altogether ending at 78 in)

After 78" we can change the spacing to the requir more than the maximum of d/2 = 11.75 in ≤ 24in);

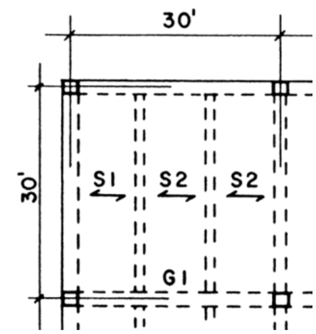
$s = A_v f_y / 50 b_w$ = 0.22 in^2 (60,000 psi)/50 (1

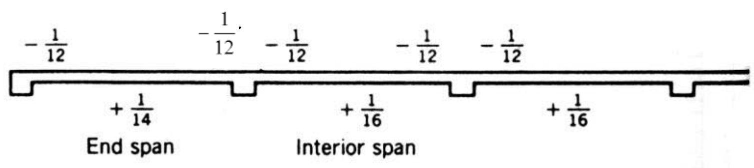
We need to continue to 111 in, so (111 – 78 in)/ even

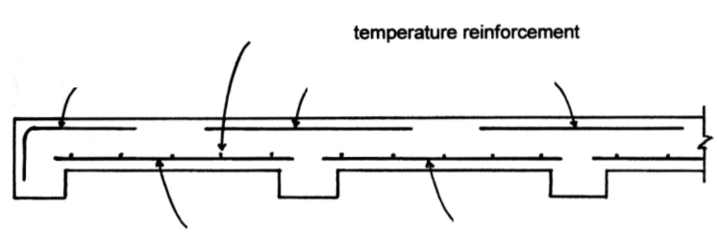


Example 15

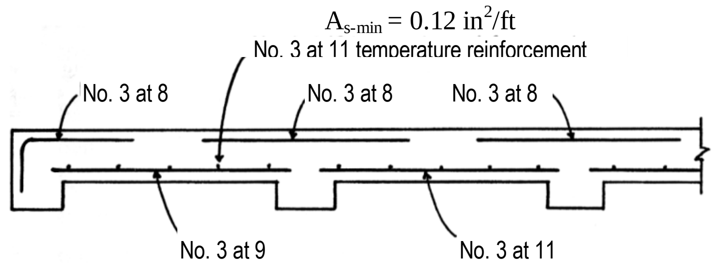
Example 1. A solid one-way slab is to be used for a framing system similar to that shown in Figure 14.1. Column spacing is 30 ft. with evenly spaced beams occurring at 10 ft. center to center. Superimposed loads on the structure (floor live load plus other construction dead load) are a dead load of 38 psf [1.82 kPa] and a live load of 100 psf [4.79 kPa]. Use $f'_c = 3$ ksi [20.7 MPa] and $f_y = 40$ ksi [275 MPa]. Determine the thickness for the slab and select its reinforcement.







Example 15 (continued)

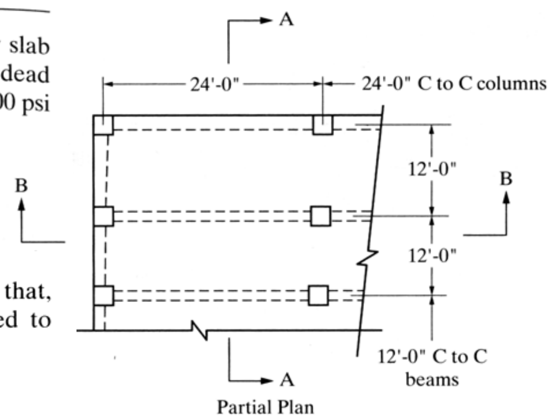


Example 16

Example 6-1

The floor system shown in Figure 6-4 consists of a continuous one-way slab supported by continuous beams. The service loads on the floor are 25 psf dead load (does not include weight of slab) and 250 psf live load. Use $f'_c = 3000$ psi (normal-weight concrete) and $f_y = 60{,}000$ psi. The bars are uncoated.

Design the continuous one-way floor slab.



Solution:

The primary difference in this design from previous flexural designs is that, because of continuity, the ACI coefficients and equations will be used to determine design shears and moments.

A. Continuous one-way floor slab

1. Determine the slab thickness. The slab will be designed to satisfy the ACI minimum thickness requirements from Table 9.5(a) of the code and this thickness will be used to estimate slab weight.

With both ends continuous,

$$\text{minimum } h = \frac{1}{28}\ell_n = \frac{1}{28}(11)(12) = 4.71 \text{ in.}$$

With one end continuous,

$$\text{minimum } h = \frac{1}{24}\ell_n = \frac{1}{24}(11)(12) = 5.5 \text{ in.}$$

Try a 5½-in.-thick slab. Design a 12-in.-wide segment (b = 12 in.).

2. Determine the load:

$$\text{slab dead load} = \frac{5.5}{12}(150) = 68.8 \text{ psf}$$

$$\text{total dead load} = 25.0 + 68.8 = 93.8 \text{ psf}$$

$$w_u = 1.2w_{DL} + 1.6w_{LL} = 1.2(93.8) + 1.6(250) = 112.6 + 400.0 = 516.2 \text{ psf} \quad \text{(design load)}$$

Because we are designing a slab segment that is 12 in. wide, the foregoing loading is the same as 512.6 lb/ft or 0.513 kip/ft.

Example 16 (continued)

3. Determine the moments and shears. Moments are determined using the ACI moment equations. Refer to Figures 6-1 and 6-4. Thus

$$+M_u = \frac{1}{14}w_u\ell_n^2 = \frac{1}{14}(0.513)(11)^2 = 4.43 \text{ ft-kips} \quad \textit{(end span)}$$

$$+M_u = \frac{1}{16}w_u\ell_n^2 = \frac{1}{16}(0.513)(11)^2 = 3.88 \text{ ft-kips} \quad \textit{(interior span)}$$

$$-M_u = \frac{1}{10}w_u\ell_n^2 = \frac{1}{10}(0.513)(11)^2 = 6.20 \text{ ft-kips} \quad \textit{(end span - first interior support)}$$

$$-M_u = \frac{1}{11}w_u\ell_n^2 = \frac{1}{11}(0.513)(11)^2 = 5.64 \text{ ft-kips} \quad \textit{(interior span – both supports)}$$

$$-M_u = \frac{1}{24}w_u\ell_n^2 = \frac{1}{24}(0.513)(11)^2 = 2.58 \text{ ft-kips} \quad \textit{(end span – exterior support)}$$

Similarly, the shears are determined using the ACI shear equations. In the end span at the face of the first interior support,

$$V_u = 1.15\frac{w_u\ell_n}{2} = 1.15(0.513)\left(\frac{11}{2}\right) = 3.24 \text{ kips} \quad \textit{(end span – first interior support)}$$

whereas at all other supports,

$$V_u = \frac{w_u\ell_n}{2} = (0.513)\left(\frac{11}{2}\right) = 2.82 \text{ kips}$$

4. Design the slab. Assume #4 bars for main steel with ¾ in. cover: d = 5.5 – 0.75 – ½(0.5) = 4.5 in.

5. Design the steel. (All moments must be considered.) For example, the negative moment in the end span at the first interior support:

$$R_n = \frac{M_u}{\phi bd^2} = \frac{6.20(12)(1000)}{0.9(12)(4.5)^2} = 340^{\,ft-kips} \quad \text{so } \rho \cong 0.006$$

$A_s = \rho bd$ = 0.006(12)(4.5) = 0.325 in^2 per ft. width of slab ∴Use #4 at 7 in. (16.5 in. max. spacing)

The minimum reinforcement required for flexure is the same as the shrinkage and temperature steel.

(Verify the moment capacity is achieved: a 0.67 in. and ϕM_n = 6.38 ft-kips > 6.20 ft-kips)

For grade 60 the minimum for shrinkage and temperature steel is:

$A_{s\text{-}min} = 0.0018bt$ = 0.0018 (12)(5.5) = 0.12 in^2 per ft. width of slab ∴Use #3 at 11 in. (18 in. max spacing)

6. Check the shear strength.

$$\phi V_c = \phi 2\sqrt{f'_c}bd = 0.75(2)\sqrt{3000}(12)(4.5) = 4436.6lb = 4.44 \text{ kips}$$

$V_u \le \phi V_c$ Therefore the thickness is O.K.

7. Development length for the flexure reinforcement is required. (Hooks are required at the spandrel beam.) For example, #6 bars:

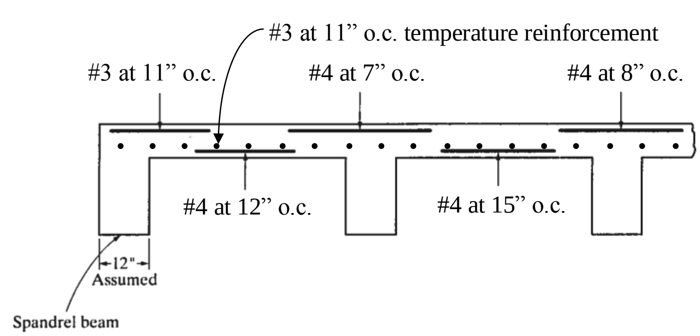
$$l_d = \frac{d_bF_y}{25\sqrt{f'_c}} \quad \textit{or 12 in. minimum}$$

With grade 40 steel and 3000 psi concrete:

$$l_d = \frac{\tfrac{6}{8}in(40{,}000\,psi)}{25\sqrt{3000}\,psi} = 21.9in$$

(which is larger than 12 in.)

8. Sketch:



Example 17

A building is supported on a grid of columns that is spaced at 30 ft on center in both the north-south and east-west directions. Hollow core planks with a 2 in. topping span 30 ft in the east-west direction and are supported on precast L and inverted T beams. Size the hollow core planks assuming a live load of 100 lb/ft^2. Choose the shallowest plank with the least reinforcement that will span the 30 ft while supporting the live load.

SOLUTION:

The shallowest that works is an 8 in. deep hollow core plank.

The one with the least reinforcing has a strand pattern of 68-S, which contains 6 strands of diameter 8/16 in. = ½ in. The S indicates that the strands are straight. The plank supports a superimposed service load of 124 lb/ft^2 at a span of 30 ft with an estimated camber at erection of 0.8 in. and an estimated long-time camber of 0.2 in.

The weight of the plank is 81 lb/ft^2.

Strand Pattern Designation
76-S

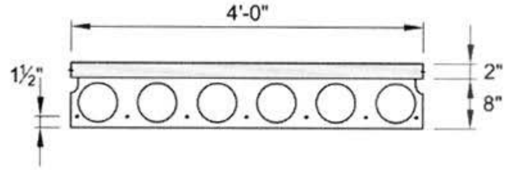
S = straight
Diameter of strand in 16ths
No. of Strand (7)

Safe loads shown include dead load of 10 psf for untopped members and 15 psf for topped members. Remainder is live load. Long-time cambers include superimposed dead load but do not include live load.

Capacity of sections of other configurations are similar. For precise values, see local hollow-core manufacturer.

Key
458 – Safe superimposed service load, psf
0.1 – Estimated camber at erection, in.
0.2 – Estimated long-time camber, in.

HOLLOW-CORE
4'-0" x 8"
Normal Weight Concrete



$f'_c = 5{,}000$ psi
$f_{pu} = 270{,}000$ psi

Section Properties

		Untopped		Topped	
A	=	215	in.^{2}	311	in.^{2}
I	=	1,666	in.^{4}	3,071	in.^{4}
y_b	=	4.00	in.	5.29	in.
y_t	=	4.00	in.	4.71	in.
S_b	=	417	in.^{3}	581	in.^{3}
S_t	=	417	in.^{3}	652	in.^{3}
wt	=	224	plf	324	plf
DL	=	56	psf	81	psf
V/S	=	1.92	in.		

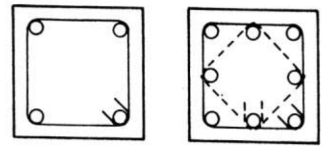
4HC8 + 2

Table of safe superimposed service load (psf) and cambers (in.) **2 in. Normal Weight Topping**

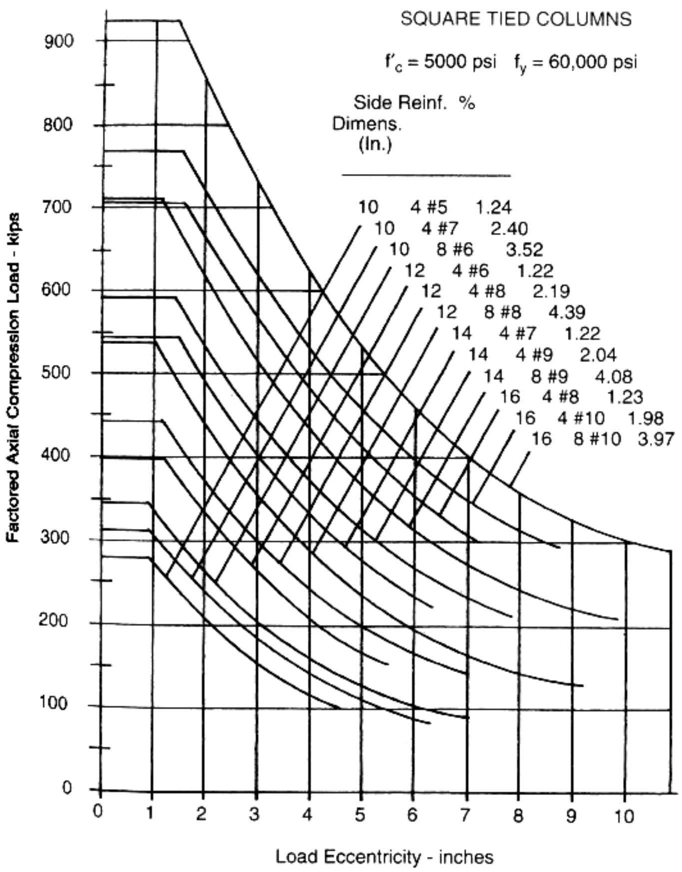
Strand Designation Code	Span, ft																											
	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
66-S	489	445	394	340	294	256	224	197	173	153	135	119	105	93	82	68	56	45	36	26								
	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.2	0.2	0.2	0.2	0.1	0.0	–0.0	–0.1	–0.2	–0.3								
	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.1	0.1	0.0	–0.1	–0.2	–0.3	–0.4	–0.6	–0.7	–0.9	–1.2	–1.4								
76-S	498	457	420	387	347	304	267	235	208	184	164	146	130	116	103	88	74	62	51	41	31							
	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.3	0.4	0.4	0.4	0.3	0.3	0.3	0.3	0.2	0.2	0.1	–0.0	–0.1	–0.2							
	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.3	0.2	0.2	0.2	0.1	0.0	–0.1	–0.2	–0.4	–0.5	–0.7	–0.9	–1.2	–1.4							
58-S	492	451	414	384	357	333	310	293	274	245	219	196	177	159	143	126	110	95	82	70	59	49	40	32				
	0.3	0.3	0.3	0.4	0.4	0.5	0.5	0.5	0.5	0.6	0.6	0.6	0.6	0.6	0.6	0.5	0.5	0.5	0.1	0.3	0.2	0.1	0.0	–0.1				
	0.3	0.3	0.4	0.4	0.4	0.4	0.5	0.5	0.5	0.5	0.4	0.3	0.3	0.3	0.2	0.1	–0.1	–0.2	–0.4	–0.6	–0.9	–1.2	–1.5	–1.8				
68-S		463	426	393	366	342	319	299	282	267	251	239	216	195	177	158	140	124	110	97	84	73	62	53	44	36	28	
		0.4	0.4	0.5	0.5	0.6	0.6	0.7	0.7	0.7	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.6	0.5	0.4	0.2	0.1	–0.1	
		0.4	0.5	0.5	0.6	0.6	0.6	0.6	0.7	0.7	0.7	0.6	0.6	0.6	0.5	0.4	0.3	0.2	0.0	–0.2	–0.4	–0.6	–0.9	–1.2	–1.6	–2.0	–2.4	
78-S		472	435	402	375	348	325	305	288	273	257	245	232	220	207	186	167	149	133	119	106	94	83	73	64	55	46	38
		0.5	0.5	0.6	0.6	0.7	0.7	0.8	0.9	0.9	1.0	1.0	1.0	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.0	0.9	0.9	0.7	0.6	0.5	0.3
		0.5	0.6	0.6	0.7	0.7	0.8	0.8	0.8	0.9	0.9	0.9	0.9	0.8	0.8	0.7	0.7	0.6	0.4	0.3	0.1	–0.1	–0.3	–0.6	–0.9	–1.3	–1.7	–2.2

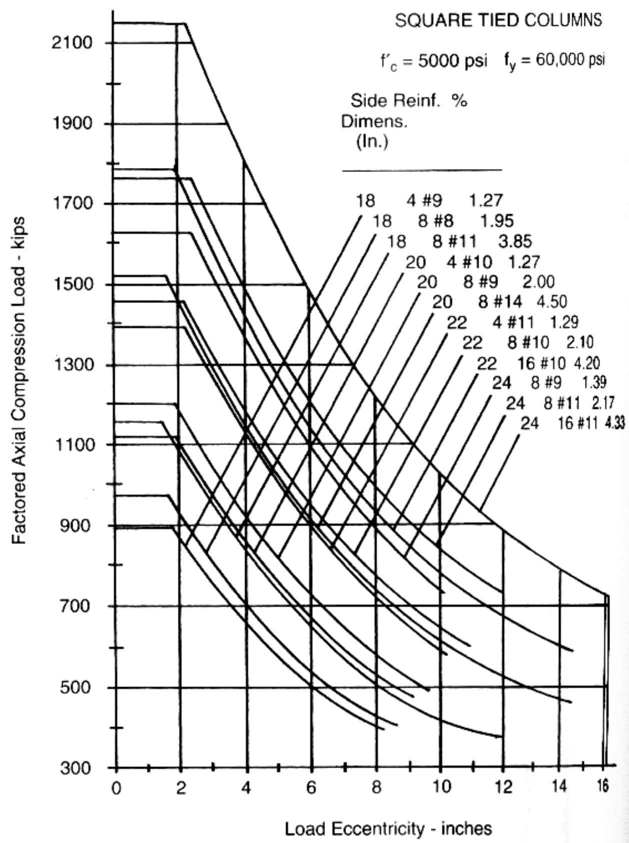
Figure 6.88 Allowed load on 4 ft-wide, 8 in.-deep hollow-core planks (HCPs). (Copyright Prestressed/Precast Concrete Institute (PCI). Reprinted with permission. All rights reserved.)

Example 18



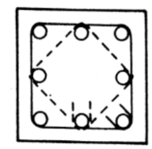
Example 1. A square tied column with $f'_c = 5$ ksi and steel with $f_y = 60$ ksi sustains an axial compression load of 150 kips dead load and 250 kips live load with no computed bending moment. Find the minimum practical column size if reinforcing is a maximum of 4% and the maximum size if reinforcing is a minimum of 1%. Also, design for e = 6 in.





Example 19

Determine the capacity of a 16" x 16" column with 8- #10 bars, tied. Grade 40 steel and 4000 psi concrete.



SOLUTION:

Find ϕP_n, with ϕ=0.65 and $P_n = 0.80P_o$ for tied columns and

$$P_o = 0.85 f'_c(A_g - A_{st}) + f_y A_{st}$$

Steel area (found from reinforcing bar table for the bar size):

A_{st} = 8 bars × (1.27 in^2) = 10.16 in^2

Concrete area (gross):

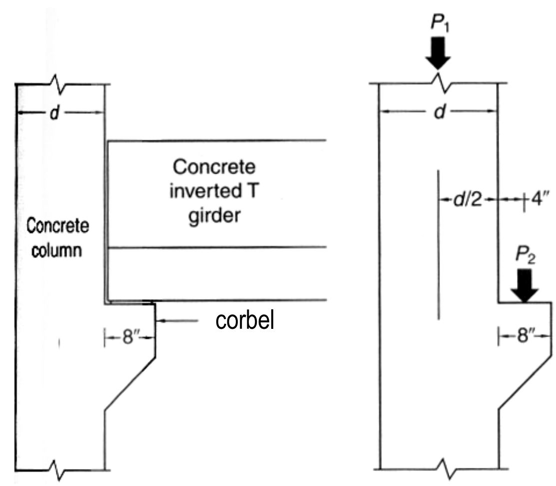
A_g = 16 in × 16 in = 256 in^2

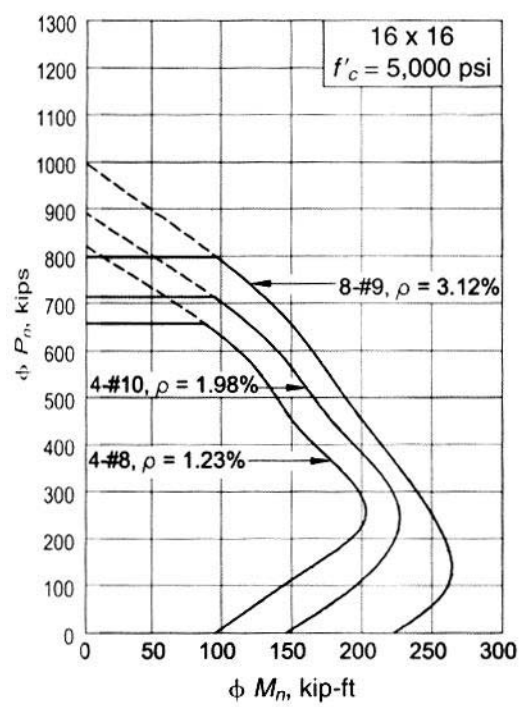
Grade 40 reinforcement has f_y = 40,000 psi and f'_c = 4000psi

ϕP_n = (0.65)(0.80)[0.85(4000 psi)(256 in^2 – 10.16 in^2) + (40,000 psi)(10.16 in^2)] = 646,026 lb = 646 kips

Example 20

16" x 16" precast reinforced columns support inverted T girders on corbels as shown. The unfactored loads on the corbel are 81 k dead, and 72 k live. The unfactored loads on the column are 170 k dead and 150 k live. Determine the reinforcement required using the interaction diagram provided. Assume that half the moment is resisted by the column above the corbel and the other half is resisted by the column below. Use grade 50 steel and 5000 psi concrete.





Example 21

EXAMPLE 5–4

Design a short square tied column to carry an axial dead load of 300 kip and a live load of 200 kip. Assume that the applied moments on the column are negligible. Use $f'_c = 4{,}000$ psi and $f_y = 60{,}000$ psi.

Solution

Step 1 The factored load, P_u, is:

$$P_u = 1.2P_D + 1.6P_L$$
$$P_u = 1.2(300) + 1.6(200)$$
$$P_u = 680 \text{ kip}$$

Assume $\rho_g = 0.03$.

Step 2 The required area of the column, A_g, is:

$$A_g = \frac{P_u}{0.8\phi[0.85f'_c(1-\rho_g) + f_y\rho_g]}$$
$$A_g = \frac{680}{0.80\,(0.65)[0.85\,(4)(1-0.03) + 60(0.03)]}$$
$$A_g = 257 \text{ in}^2$$

Step 3 For a square column, the size, h, is:

$$h = \sqrt{A_g} = \sqrt{257}$$
$$\therefore h = 16.0 \text{ in.}$$

Try a 16 in. × 16 in. column:

$$A_g = (16)(16) = 256 \text{ in}^2$$

Step 4 The required amount of steel, A_{st}, is:

$$A_{st} = \frac{P_u - 0.8\phi(0.85f'_c A_g)}{0.8\phi(f_y - 0.85f'_c)}$$
$$A_{st} = \frac{680 - 0.8 \times 0.65(0.85 \times 4 \times 256)}{0.8 \times 0.65(60 - 0.85 \times 4)} = 7.73 \text{ in}^2$$

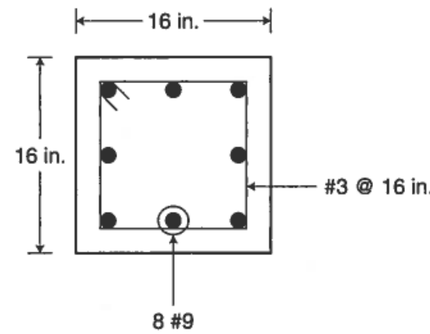
Step 5 Select the size and number of bars. For a square column with bars uniformly distributed along the edges, we keep the number of bars as multiples of four. Using Table A2–9, 8 #9 bars ($A_s = 8 \text{ in}^2$) are selected.

From Table A5–1 ⟶ Maximum of 12 #9 bars ∴ ok

Step 6 Because the longitudinal bars are #9, select #3 bars for the ties. The maximum spacing of the ties (s_{max}) is:

$$s_{max} = \min\{16d_b, 48d_t, b_{min}\}$$
$$s_{max} = \min\{16(1.128), 48(3/8), 16\}$$
$$s_{max} = \min\{18.0, 18.0, 16.0\}$$
$$\therefore s_{max} = 16 \text{ in.}$$

The selected ties are #3 @ 16 in.



Example 22

Design a 10 ft long circular spiral column for a braced system to support the service dead and live loads of 300 k and 460 k, respectively, and the service dead and live moments of 100 ft-k each. The moment at one end is zero. Use $f'_c = 4{,}000$ psi and $f_y = 60{,}000$ psi.

Solution

1. $P_u = 1.2(300) + 1.6(460) = 1096$ k
   $M_u = 1.2(100) + 1.6(100) = 280$ ft-k
2. Assume $\rho_g = 0.01$, from Equation 16.10:

$$A_g = \frac{P_u}{0.60[0.85f'_c(1-\rho_g) + f_y\rho_g]}$$

$$= \frac{1096}{0.60[0.85(4)(1-0.01) + 60(0.01)]}$$

$$= 460.58 \text{ in.}^2$$

$$\frac{\pi h^2}{4} = 460.58$$

or $h = 24.22$ in.
Use $h = 24$ in., $A_g = 452$ in.^{2}

3. Assume #9 size of bar and 3/8 in. spiral
   center-to-center distance
   = 24 − 2(cover) − 2(spiral diameter) − 1 (bar diameter)
   = 24 − 2(1.5) − 2(3/8) − 1.128 = 19.12 in.

ACI 7.7: Concrete exposed to earth or weather:
No. 6 through No. 18 bars....... 2 in. minimum

$$\gamma = \frac{19.12}{24} = 0.8$$

Use the interaction diagram Appendix D.21

4. $K_n = \dfrac{P_u}{\phi f'_c A_g} = \dfrac{1096}{(0.75)(4)(452)} = 0.808$

   $R_n = \dfrac{M_u}{\phi f'_c A_g h} = \dfrac{3360}{(0.75)(4)(452)(24)} = 0.103$

5. At the intersection point of K_n and R_n, $\rho_g = 0.02$
6. The point is above the strain line = 1, hence $\phi = 0.75$ **OK**
7. $A_{st} = (0.02)(452) = 9.04$ in.^{2}
   From Appendix D.2, select 12 bars of #8, $A_{st} = 9.48$ in.^{2}
   From Appendix D.14 for a core diameter of 24 − 3 = 21 in. 17 bars of #8 can be arranged in a row
8. Selection of spirals
   From Appendix D.13, size = 3/8 in.
   pitch = 2¼ in.
   Clear distance = 2.25 − 3/8 = 1.875 > 1 in. **OK**
9. $K = 1$, $l = 10 \times 12 = 120$ in., $r = 0.25(24) = 6$ in.

$$\frac{Kl}{r} = \frac{1(120)}{6} = 20$$

$$\left(\frac{M_1}{M_2}\right) = 0$$

$$34 - 12\left(\frac{M_1}{M_2}\right) = 34$$

ACI 10.12: In nonsway frames it shall be permitted to ignore slenderness effects for compression members that satisfy: $\frac{kl_u}{r} \le 34 - 12\left(M_1/M_2\right)$

since $(Kl/r) < 34$, short column.

Factored Moment Resistance of Concrete Beams, ϕM_n (k-ft) with f'_c = 4 ksi, f_y = 60 ksi[a]

	Approximate Values for a/d		
	0.1	0.2	0.3
	Approximate Values for ρ		
b x d (in)	0.0057	0.01133	0.017
10 x 14	2 #6	2 #8	3 #8
	53	90	127
10 x 18	3 #5	2 #9	3 #9
	72	146	207
10 x 22	2 #7	3 #8	(3 #10)
	113	211	321
12 x 16	2 #7	3 #8	4 #8
	82	154	193
12 x 20	2 #8	3 #9	4 #9
	135	243	306
12 x 24	2 #8	3 #9	(4 #10)
	162	292	466
15 x 20	3 #7	4 #8	5 #9
	154	256	383
15 x 25	3 #8	4 #9	4 #11
	253	405	597
15 x 30	3 #8	5 #9	(5 #11)
	304	608	895
18 x 24	3 #8	5 #9	6 #10
	243	486	700
18 x 30	3 #9	6 #9	(6 #11)
	385	729	1074
18 x 36	3 #10	6 #10	(7 #11)
	586	1111	1504
20 x 30	3 # 10	7 # 9	6 # 11
	489	851	1074
20 x 35	4 #9	5 #11	(7 #11)
	599	1106	1462
20 x 40	6 #8	6 #11	(9 #11)
	811	1516	2148
24 x 32	6 #8	7 #10	(8 #11)
	648	1152	1528
24 x 40	6 #9	7 #11	(10 #11)
	1026	1769	2387
24 x 48	5 #10	(8 #11)	(13 #11)
	1303	2426	3723

[a]Table yields values of factored moment resistance in kip-ft with reinforcement indicated. Reinforcement choices shown in parentheses require greater width of beam or use of two stack layers of bars. (*Adapted and corrected from Simplified Engineering for Architects and Builders, 11th ed, Ambrose and Tripeny, 2010.*

Column Interaction Diagrams

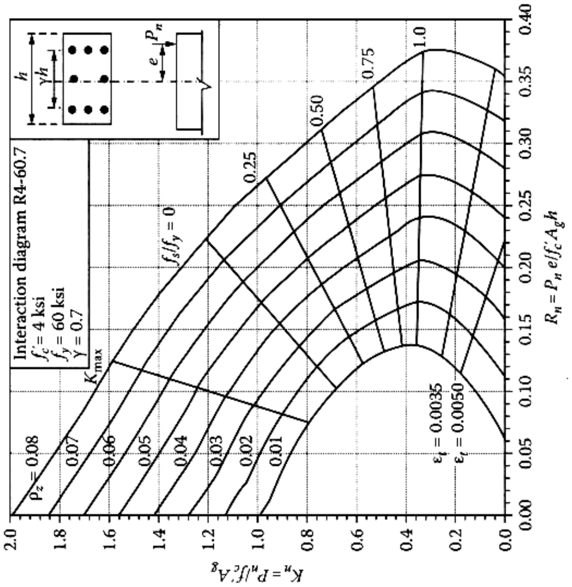


FIGURE D.17 Column interaction diagram for tied column with bars on all faces. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

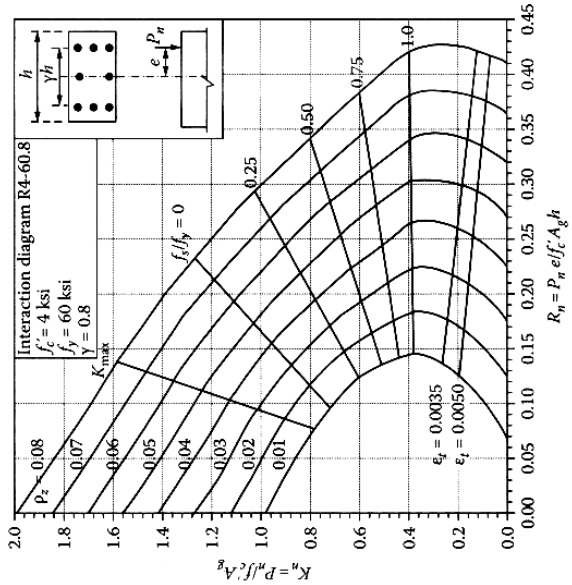


FIGURE D.18 Column interaction diagram for tied column with bars on all faces. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

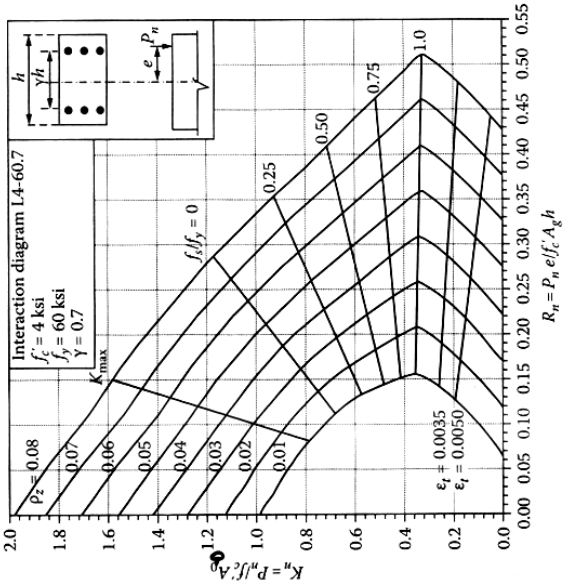


FIGURE D.15 Column interaction diagram for tied column with bars on end faces only. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

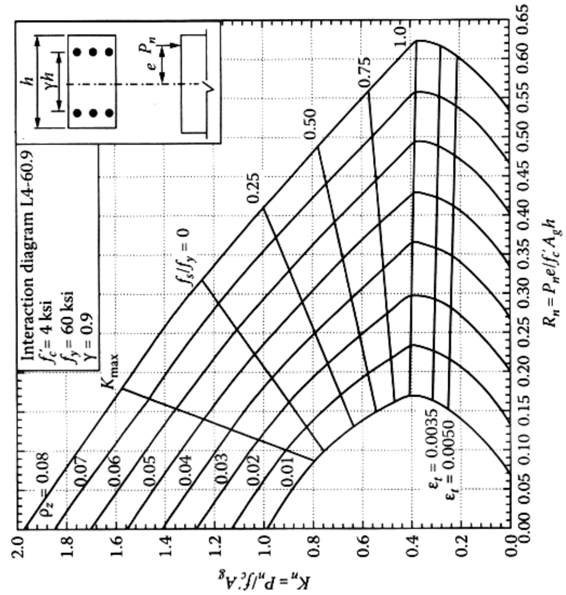


FIGURE D.16 Column interaction diagram for tied column with bars on end faces only. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

Column Interaction Diagrams

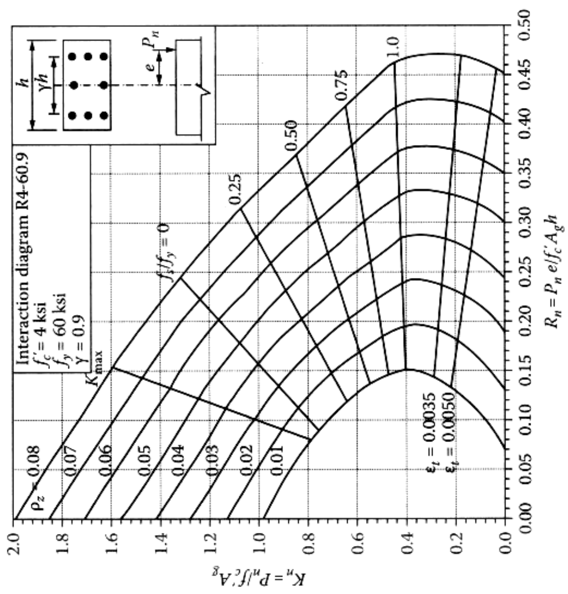


FIGURE D.19 Column interaction diagram for tied column with bars on all faces. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

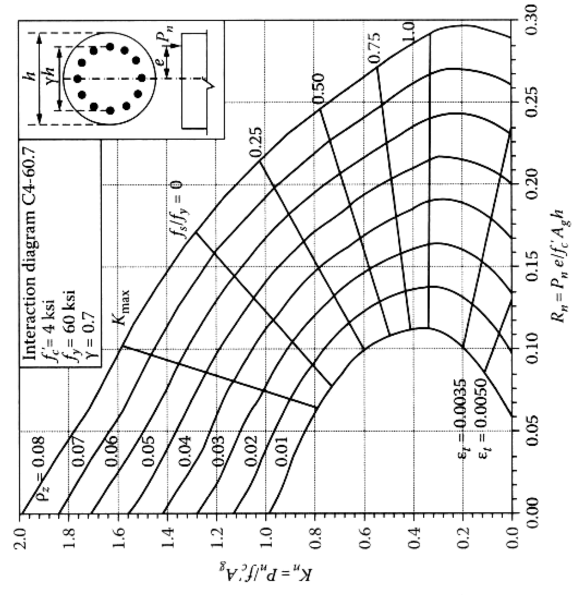


FIGURE D.20 Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

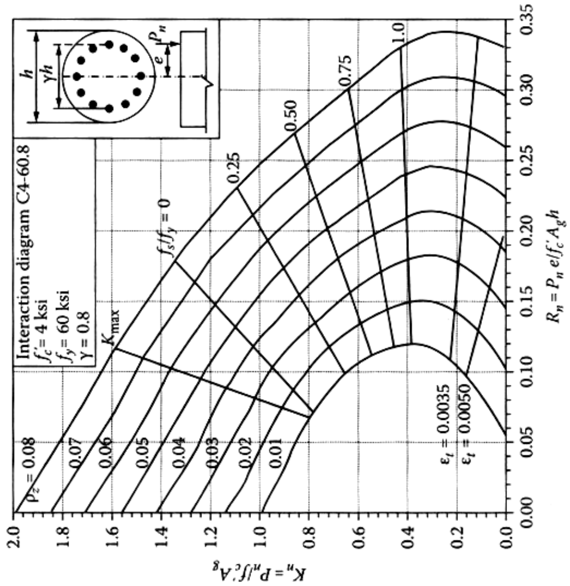


FIGURE D.21 Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

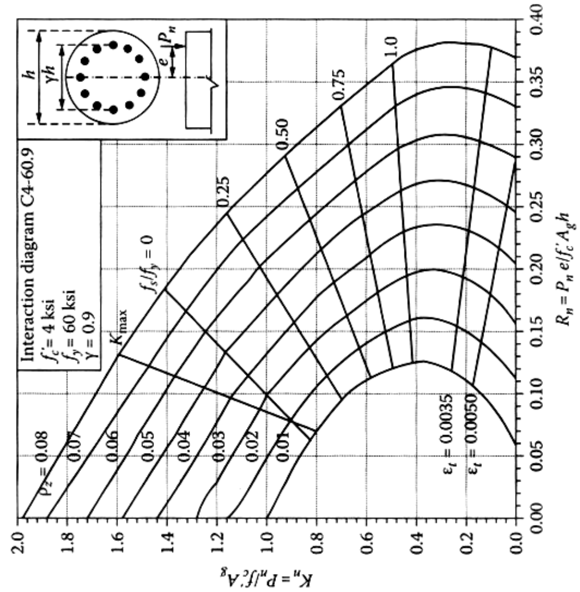
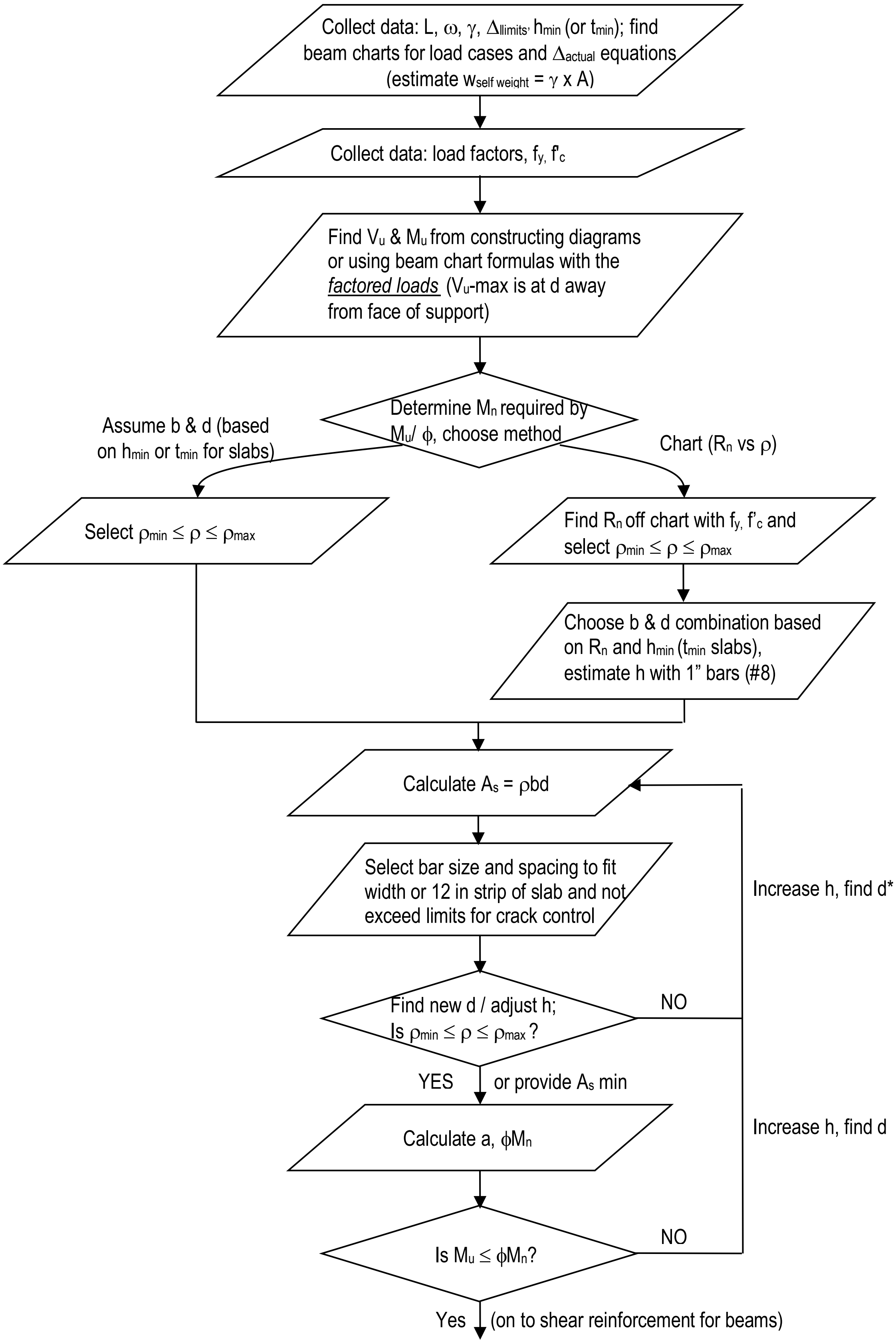
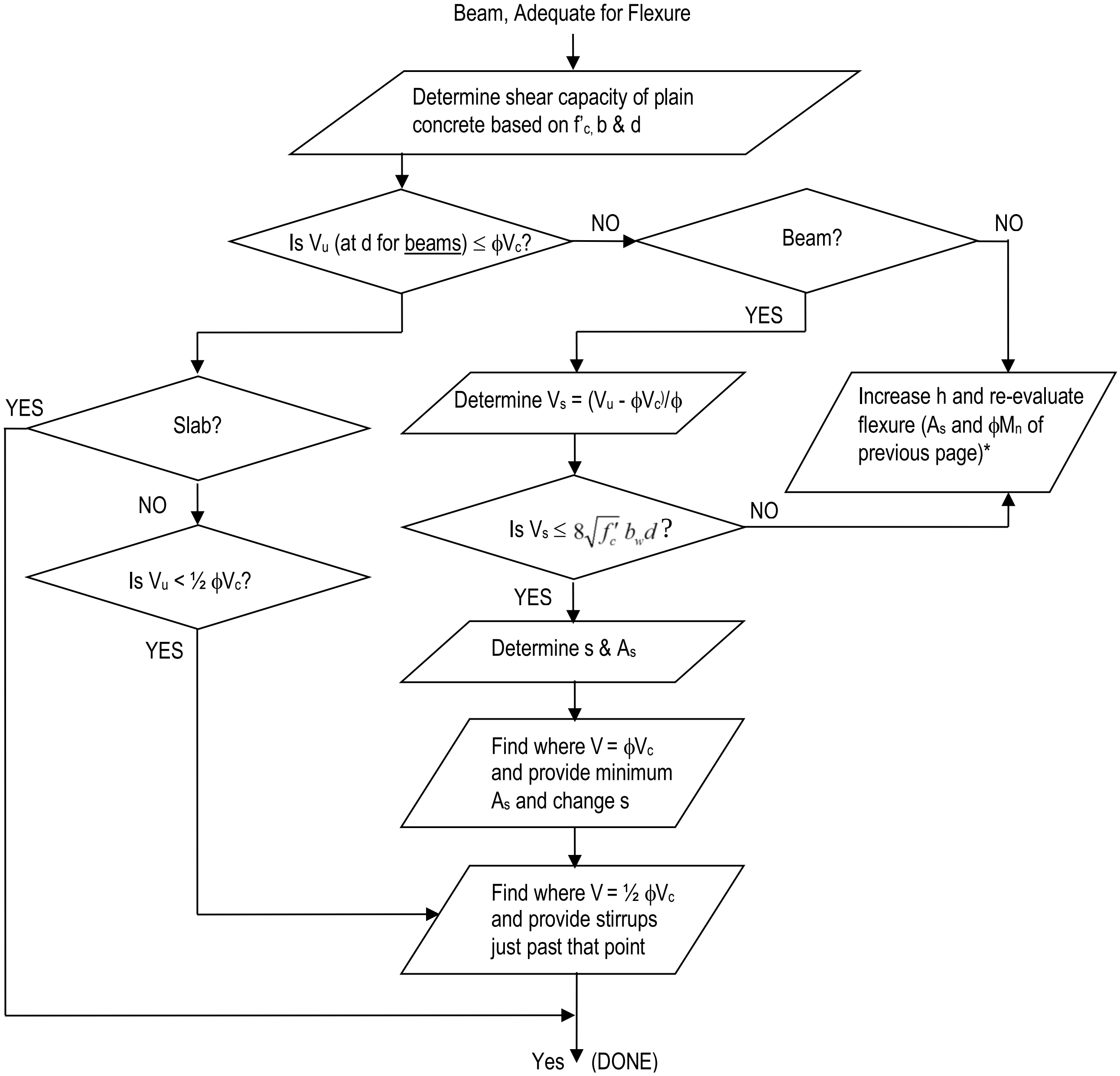


FIGURE D.22 Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

Beam / One-Way Slab Design Flow Chart



Beam / One-Way Slab Design Flow Chart - continued



APPENDIX E — STEEL REINFORCEMENT INFORMATION

As an aid to users of the ACI Building Code, information on sizes, areas, and weights of various steel reinforcement is presented.

ASTM STANDARD REINFORCING BARS

Bar size, no.	Nominal diameter, in.	Nominal area, in.^{2}	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.650
18	2.257	4.00	13.600

ASTM STANDARD PRESTRESSING TENDONS

Type*	Nominal diameter, in.	Nominal area, in.^{2}	Nominal weight, lb/ft
Seven-wire strand (Grade 250)	1/4 (0.250)	0.036	0.122
	5/16 (0.313)	0.058	0.197
	3/8 (0.375)	0.080	0.272
	7/16 (0.438)	0.108	0.367
	1/2 (0.500)	0.144	0.490
	(0.600)	0.216	0.737
Seven-wire strand (Grade 270)	3/8 (0.375)	0.085	0.290
	7/16 (0.438)	0.115	0.390
	1/2 (0.500)	0.153	0.520
	(0.600)	0.217	0.740
Prestressing wire	0.192	0.029	0.098
	0.196	0.030	0.100
	0.250	0.049	0.170
	0.276	0.060	0.200
Prestressing bars (plain)	3/4	0.44	1.50
	7/8	0.60	2.04
	1	0.78	2.67
	1-1/8	0.99	3.38
	1-1/4	1.23	4.17
	1-3/8	1.48	5.05
Prestressing bars (deformed)	5/8	0.28	0.98
	3/4	0.42	1.49
	1	0.85	3.01
	1-1/4	1.25	4.39
	1-3/8	1.58	5.56

* Availability of some tendon sizes should be investigated in advance.

ASTM STANDARD WIRE REINFORCEMENT

W & D size		Nominal	Nominal	Nominal	Area, in.^{2}/ft of width for various spacings						
					Center-to-center spacing, in.						
Plain	Deformed	diameter, in.	area, in.^{2}	weight, lb/ft	2	3	4	6	8	10	12
W31	D31	0.628	0.310	1.054	1.86	1.24	0.93	0.62	0.465	0.372	0.31
W30	D30	0.618	0.300	1.020	1.80	1.20	0.90	0.60	0.45	0.366	0.30
W28	D28	0.597	0.280	0.952	1.68	1.12	0.84	0.56	0.42	0.336	0.28
W26	D26	0.575	0.260	0.934	1.56	1.04	0.78	0.52	0.39	0.312	0.26
W24	D24	0.553	0.240	0.816	1.44	0.96	0.72	0.48	0.36	0.288	0.24
W22	D22	0.529	0.220	0.748	1.32	0.88	0.66	0.44	0.33	0.264	0.22
W20	D20	0.504	0.200	0.680	1.20	0.80	0.60	0.40	0.30	0.24	0.20
W18	D18	0.478	0.180	0.612	1.08	0.72	0.54	0.36	0.27	0.216	0.18
W16	D16	0.451	0.160	0.544	0.96	0.64	0.48	0.32	0.24	0.192	0.16
W14	D14	0.422	0.140	0.476	0.84	0.56	0.42	0.28	0.21	0.168	0.14
W12	D12	0.390	0.120	0.408	0.72	0.48	0.36	0.24	0.18	0.144	0.12
W11	D11	0.374	0.110	0.374	0.66	0.44	0.33	0.22	0.165	0.132	0.11
W10.5		0.366	0.105	0.357	0.63	0.42	0.315	0.21	0.157	0.126	0.105
W10	D10	0.356	0.100	0.340	0.60	0.40	0.30	0.20	0.15	0.12	0.10
W9.5		0.348	0.095	0.323	0.57	0.38	0.285	0.19	0.142	0.114	0.095
W9	D9	0.338	0.090	0.306	0.54	0.36	0.27	0.18	0.135	0.108	0.09
W8.5		0.329	0.085	0.289	0.51	0.34	0.255	0.17	0.127	0.102	0.085
W8	D8	0.319	0.080	0.272	0.48	0.32	0.24	0.16	0.12	0.096	0.08
W7.5		0.309	0.075	0.255	0.45	0.30	0.225	0.15	0.112	0.09	0.075
W7	D7	0.298	0.070	0.238	0.42	0.28	0.21	0.14	0.105	0.084	0.07
W6.5		0.288	0.065	0.221	0.39	0.26	0.195	0.13	0.097	0.078	0.065
W6	D6	0.276	0.060	0.204	0.36	0.24	0.18	0.12	0.09	0.072	0.06
W5.5		0.264	0.055	0.187	0.33	0.22	0.165	0.11	0.082	0.066	0.055
W5	D5	0.252	0.050	0.170	0.30	0.20	0.15	0.10	0.075	0.06	0.05
W4.5		0.240	0.045	0.153	0.27	0.18	0.135	0.09	0.067	0.054	0.045
W4	D4	0.225	0.040	0.136	0.24	0.16	0.12	0.08	0.06	0.048	0.04
W3.5		0.211	0.035	0.119	0.21	0.14	0.105	0.07	0.052	0.042	0.035
W3		0.195	0.030	0.102	0.18	0.12	0.09	0.06	0.045	0.036	0.03
W2.9		0.192	0.029	0.098	0.174	0.116	0.087	0.058	0.043	0.035	0.029
W2.5		0.178	0.025	0.085	0.15	0.10	0.075	0.05	0.037	0.03	0.025
W2		0.159	0.020	0.068	0.12	0.08	0.06	0.04	0.03	0.024	0.02
W1.4		0.135	0.014	0.049	0.084	0.056	0.042	0.028	0.021	0.017	0.014

STEEL REINFORCEMENT INFORMATION

Table 3.7.1
Total Areas for Various Numbers of Reinforcing Bars

Bar Size	Nominal Diameter (in.)	Weight (lb/ft)	Number of Bars: 1	2	3	4	5	6	7	8	9	10
#3	0.375	0.376	0.11	0.22	0.33	0.44	0.55	0.66	0.77	0.88	0.99	1.10
#4	0.500	0.668	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
#5	0.625	1.043	0.31	0.62	0.93	1.24	1.55	1.86	2.17	2.48	2.79	3.10
#6	0.750	1.502	0.44	0.88	1.32	1.76	2.20	2.64	3.08	3.52	3.96	4.40
#7	0.875	2.044	0.60	1.20	1.80	2.40	3.00	3.60	4.20	4.80	5.40	6.00
#8	1.000	2.670	0.79	1.58	2.37	3.16	3.95	4.74	5.53	6.32	7.11	7.90
#9	1.128	3.400	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
#10	1.270	4.303	1.27	2.54	3.81	5.08	6.35	7.62	8.89	10.16	11.43	12.70
#11	1.410	5.313	1.56	3.12	4.68	6.24	7.80	9.36	10.92	12.48	14.04	15.60
#14[a]	1.693	7.65	2.25	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50
#18[a]	2.257	13.60	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00

[a] #14 and #18 bars are used primarily as column reinforcement and are rarely used in beams.

Table 3-7 Areas of Bars per Foot Width of Slab—A_s (in.^{2}/ft)

Bar size	Bar spacing (in.): 6	7	8	9	10	11	12	13	14	15	16	17	18
#3	0.22	0.19	0.17	0.15	0.13	0.12	0.11	0.10	0.09	0.09	0.08	0.08	0.07
#4	0.40	0.34	0.30	0.27	0.24	0.22	0.20	0.18	0.17	0.16	0.15	0.14	0.13
#5	0.62	0.53	0.46	0.41	0.37	0.34	0.31	0.29	0.27	0.25	0.23	0.22	0.21
#6	0.88	0.75	0.66	0.59	0.53	0.48	0.44	0.41	0.38	0.35	0.33	0.31	0.29
#7	1.20	1.03	0.90	0.80	0.72	0.65	0.60	0.55	0.51	0.48	0.45	0.42	0.40
#8	1.58	1.35	1.18	1.05	0.95	0.86	0.79	0.73	0.68	0.63	0.59	0.56	0.53
#9	2.00	1.71	1.50	1.33	1.20	1.09	1.00	0.92	0.86	0.80	0.75	0.71	0.67
#10	2.54	2.18	1.91	1.69	1.52	1.39	1.27	1.17	1.09	1.02	0.95	0.90	0.85
#11	3.12	2.67	2.34	2.08	1.87	1.70	1.56	1.44	1.34	1.25	1.17	1.10	1.04

Table 3-4 Maximum Bar Spacing in One-Way Slabs for Crack Control (in.)*

Bar Size	Exterior Exposure (z = 129 kips/in.) Cover (in.)				Interior Exposure (z = 156 kips/in.) Cover (in.)			
	3/4	1	1-1/2	2	3/4	1	1-1/2	2
#4	--	14.7	7.5	4.5	--	--	13.3	8.0
#5	--	13.4	7.0	4.3	--	--	12.4	7.6
#6	--	12.2	6.5	4.1	--	--	11.6	7.2
#7	16.3	11.1	6.1	3.9	--	--	10.8	6.8
#8	14.7	10.2	5.8	3.7	--	--	10.2	6.5
#9	13.3	9.4	5.4	3.5	--	16.6	9.6	6.2
#10	12.0	8.6	5.0	3.3	--	15.2	8.9	5.9
#11	10.9	7.9	4.7	3.1	--	14.0	8.4	5.6

*Valid for $f_s = 0.6f_y$ = 36 ksi, and single layer of reinforcement. Spacing should not exceed 3 times slab thickness nor 18 in. (ACI 7.6.5). No value indicates spacing greater than 18 in.

Table 3-3 Maximum Number of Bars in a Single Layer

Bar Size	Maximum size coarse aggregate—3/4 in. Beam width, b_w (in.)										
	10	12	14	16	18	20	22	24	26	28	30
#5	3	5	6	7	8	10	11	12	13	15	16
#6	3	4	6	7	8	9	10	11	12	14	15
#7	3	4	5	6	7	8	9	10	11	12	13
#8	3	4	5	6	7	8	9	10	11	12	13
#9	2	3	4	5	6	7	8	8	9	10	11
#10	2	3	4	4	5	6	7	8	8	9	10
#11	2	3	3	4	5	5	6	7	8	8	9

Bar Size	Maximum size coarse aggregate—1 in. Beam width, b_w (in.)										
	10	12	14	16	18	20	22	24	26	28	30
#5	3	4	5	6	7	8	9	10	11	12	13
#6	3	4	5	6	7	8	9	10	10	11	12
#7	2	3	4	5	6	7	8	9	10	10	11
#8	2	3	4	5	6	7	7	8	9	10	11
#9	2	3	4	5	5	6	7	8	9	9	10
#10	2	3	4	4	5	6	7	7	8	9	10
#11	2	3	3	4	5	5	6	7	8	8	9

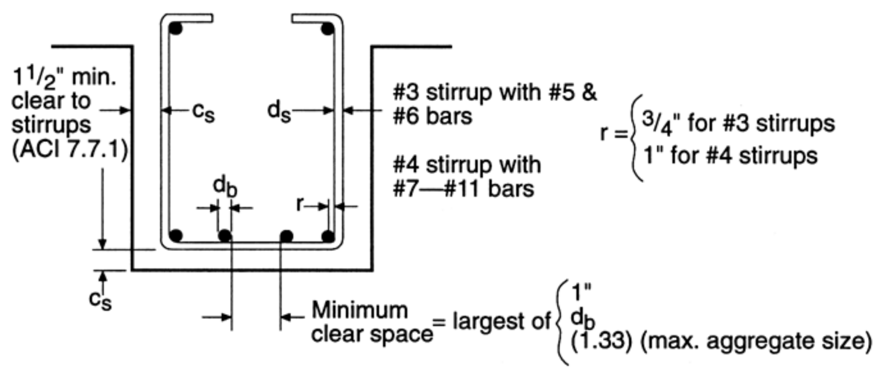


Figure 3-2 Cover and Spacing Requirements for Tables 3-2 and 3-3

Table 3-2 Minimum Number of Bars in a Single Layer (ACI 10.6)*

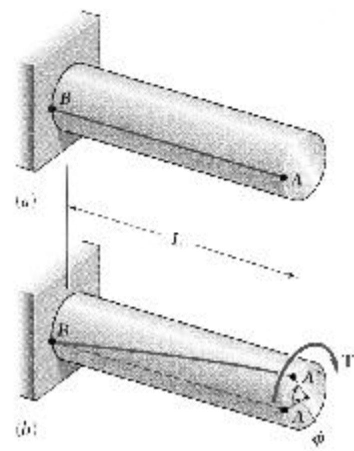
Bar Size	INTERIOR EXPOSURE (z = 175 kips/in.) Beam width, b_w (in.)										
	10	12	14	16	18	20	22	24	26	28	30
#5	1	2	2	2	2	2	3	3	3	3	3
#6	1	2	2	2	2	2	3	3	3	3	3
#7	2	2	2	2	2	3	3	3	3	3	4
#8	2	2	2	2	2	3	3	3	3	4	4
#9	2	2	2	2	3	3	3	3	3	4	4
#10	2	2	2	2	3	3	3	3	4	4	4
#11	2	2	2	3	3	3	3	4	4	4	4

Torsion

Notation:

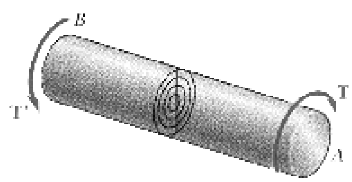
a = name for width dimension
$\mathcal{a}$ = area bounded by the centerline of a thin walled section subjected to torsion
b = name for height dimension
c = radial distance to shear stress location
c_i = inner radial distance to shear stress location
c_o = outer radial distance to shear stress location
c_1 = coefficient for shear stress for a rectangular bar in torsion
c_2 = coefficient for shear twist for a rectangular bar in torsion
G = shear modulus
J = polar moment of inertia
L = length
s = length of a segment of a thin walled section
t = name for thickness
T = torque (axial moment)
ϕ = angle of twist
π = pi (3.1415 radians or 180°)
ρ = radial distance
τ = engineering symbol for shearing stress
Σ = summation symbol

Deformation in Torsionally Loaded Members



Axi-symmetric cross sections subjected to axial moment or **torque** will remain plane and undistorted.

At a section, internal torque (resisiting applied torque) is made up of shear forces parallel to the area and in the direction of the torque. The distribution of the shearing stresses depends on the angle of twist, ϕ. The cross section remains plane and undistored.



Shearing Strain

Shearing strain is the angle change of a straight line segment along the axis.

$$\gamma = \frac{\rho\phi}{L}$$

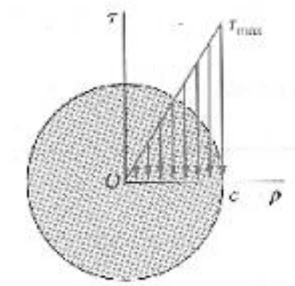
where

ρ is the radial distance from the centroid to the point under strain.

The maximum strain is at the surface, a distance c from the centroid: $\gamma_{max} = \frac{c\phi}{L}$

G is the Shear Modulus or Modulus of Rigidity: $\tau = G \cdot \gamma$

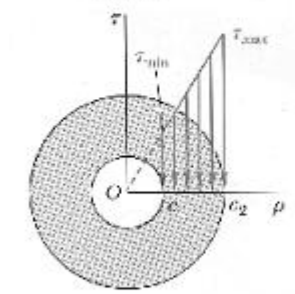
Shearing Strain and Stress



In the linear elastic range: the torque is the summation of torsion stresses over the area:

$$T = \frac{\tau J}{\rho} \qquad \text{gives:} \qquad \tau = \frac{T\rho}{J}$$

Maximum torsional stress, τ_{max}, occurs at the **outer diameter** (or **perimeter**).



Polar Moment of Inertia

For axi-symmetric shapes, there is only one value for polar moment of inertia, J, determined by the radius, c:

solid section: $$J = \frac{\pi c^4}{2}$$ hollow section: $$J = \frac{\pi\left(c_o^{\,4} - c_i^{\,4}\right)}{2}$$

Combined Torsion and Axial Loading

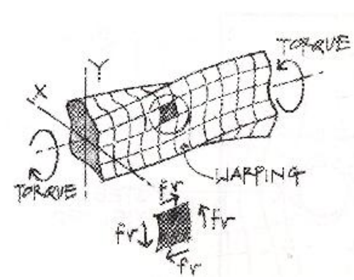
Just as with combined axial load and shear, combined torsion and axial loading result in maximum shear stress at a 45° oblique "plane" of twist.

Shearing Strain

In the linear elastic range: $$\phi = \frac{TL}{JG}$$ and for composite shafts: $$\phi = \sum_i \frac{T_i L_i}{J_i G_i}$$

Torsion in Noncircular Shapes

J is no longer the same along the lateral axes. Plane sections do not remain plane, but distort. τ_{max} is still at the furthest distance away from the centroid. For rectangular shapes:



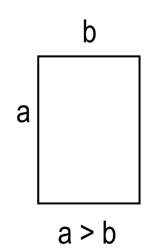
$$\tau_{max} = \frac{T}{c_1 ab^2} \qquad \phi = \frac{TL}{c_2 ab^3 G}$$

For a/b > 5:

$$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\, b/a\right)$$

TABLE 3.1. Coefficients for Rectangular Bars in Torsion

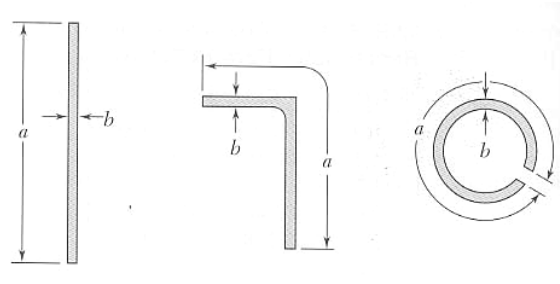
a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333



Open Sections

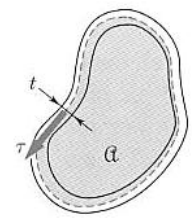
For long narrow shapes where a/b is very large ($a/b \rightarrow \infty$) $c_1 = c_2 = 1/3$ and:

$$\tau_{max} = \frac{T}{\frac{1}{3}ab^2} \qquad \phi = \frac{TL}{\frac{1}{3}ab^3G}$$



Shear Flow of Closed Thin Walled Sections

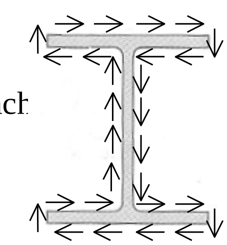
q is the internal shearing force per unit length, and is constant on a cross section even though the thickness of the wall may very. $\mathcal{A}$ is the area bounded by the centerline of the wall section; s_i, is a length segment of the wall and t_i is the corresponding thickness of the length segment.



$$\tau = \frac{T}{2t\mathcal{A}} \qquad \phi = \frac{TL}{4t\mathcal{A}^2}\sum_i \frac{s_i}{t_i}$$

Shear Flow in Open Sections

The shear flow must wrap around at all edges, and the total torque is distributed among the areas making up the cross section in proportion to the torsional rigidity of each rectangle ($ab^2/3$). The total angle of twist is the sum of the ϕ values from each rectangle. t_i is the thickness of each rectangle and b_i is the length of each rectangle.

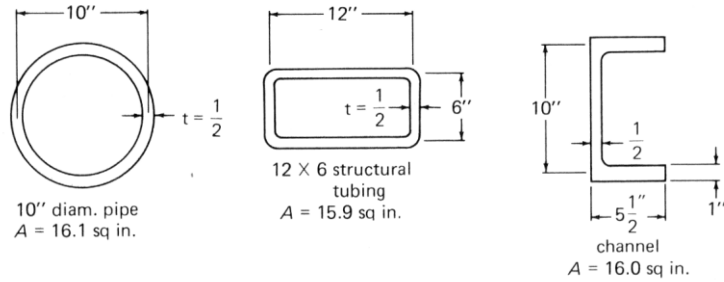


$$\tau_{max} = \frac{Tt_{max}}{\frac{1}{3}\Sigma b_i t_i^3} \qquad \phi = \frac{TL}{\frac{1}{3}G\Sigma b_i t_i^3}$$

<u>Example 1</u>

Example 8.9.1

Compare the torsional resisting moment T and the torsional constant J for the sections of Fig. 8.9.4 all having about the same cross-sectional area. The maximum shear stress τ is 14 ksi.



SOLUTION

(a) Circular thin-wall section.

$$T = \frac{\tau J}{\rho} = \frac{(14ksi)(393.7in^4)}{5.25in} \cdot \frac{1\,ft}{12in} = 87.5k - ft$$

$$J = \frac{\pi(c_o^4 - c_i^4)}{2} = \frac{\pi((5.25in)^4 - (4.75in)^4)}{2} = 393.7in^4$$

(b) Rectangular box section. $\tau = \dfrac{T}{2t\mathcal{A}}$

$$T = \tau 2t\mathcal{A} = (14ksi)2(0.5in)(72in^2) \cdot \frac{1\,ft}{12in} = 84k - ft$$

$$\mathcal{A} \approx (12in)(6in) = 72in^2$$

(c) Channel section. Since for this open section,

$$\tau_{max} = \frac{Tt_{max}}{\frac{1}{3}\sum b_i t_i^3} = \frac{Tt}{J} \qquad T = \frac{\tau J}{t_{max}} \frac{(14ksi)(4.08in^4)}{1in} \cdot \frac{1\,ft}{12in} = 4.8k - ft$$

the maximum shear stress will be in the flange. Also,

$$J = \sum \frac{bt^3}{3} \qquad J = \tfrac{1}{3}\left[10in(0.5in)^3 + (5.5in)(1in)^3 + (5.5in)(1in)^3\right] = 4.08in^4$$

One-Way Frame Analysis
Simplified Design, 3rd ed., PCA 2004

Notation:

D = shorthand for dead load
l_n = clear span from face of support to face of support in concrete design
L = shorthand for live load
w_d = load per unit length on a beam from dead load
w_ℓ = load per unit length on a beam from live load
w_u = load per unit length on a beam from load factors

2.3 FRAME ANALYSIS BY COEFFICIENTS

The ACI Code provides a simplified method of analysis for both one-way construction (ACI 8.3.3) and two-way construction (ACI 13.6). Both simplified methods yield moments and shears based on coefficients. Each method will give satisfactory results within the span and loading limitations stated in Chapter 1. The direct design method for two-way slabs is discussed in Chapter 4.

2.3.1 Continuous Beams and One-Way Slabs

When beams and one-way slabs are part of a frame or continuous construction, ACI 8.3.3 provides approximate moment and shear coefficients for gravity load analysis. The approximate coefficients may be used as long as all of the conditions illustrated in Fig. 2-2 are satisfied: (1) There must be two or more spans, approximately equal in length, with the longer of two adjacent spans not exceeding the shorter by more than 20 percent; (2) loads must be uniformly distributed, with the service live load not more than 3 times the dead load ($L/D \leq 3$); and (3) members must have uniform cross section throughout the span. Also, no redistribution of moments is permitted (ACI 8.4). The moment coefficients defined in ACI 8.3.3 are shown in Figs. 2-3 through 2-6. In all cases, the shear in end span members at the interior support is taken equal to $1.15w_u\ell_n/2$. The shear at all other supports is $w_u/2$ (see Fig. 2-7). $w_u\ell_n$ is the combined factored load for dead and live loads, $w_u = 1.2w_d + 1.6w_\ell$. For beams, w_u is the uniformly distributed load per unit length of beam (plf), and the coefficients yield total moments and shears on the beam. For one-way slabs, w_u is the uniformly distributed load per unit area of slab (psf), and the moments and shears are for slab strips one foot in width. The span length ℓ_n is defined as the clear span of the beam or slab. For negative moment at a support with unequal adjacent spans, ℓ_n is the average of the adjacent clear spans. Support moments and shears are at the faces of supports.

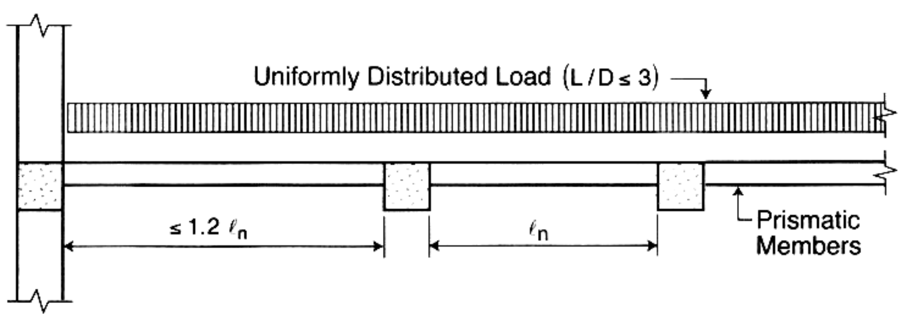


Figure 2-2 Conditions for Analysis by Coefficients (ACI 8.3.3)

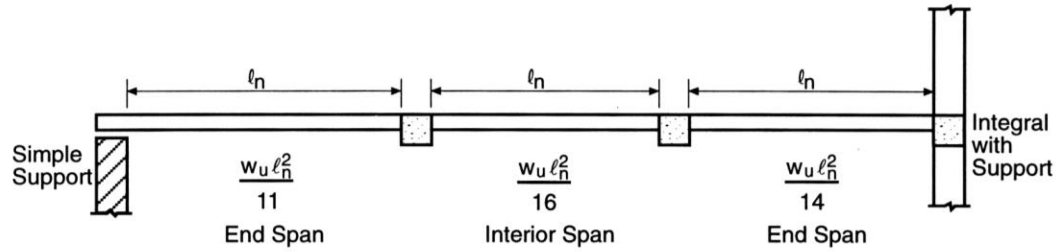


Figure 2-3 Positive Moments—All Cases

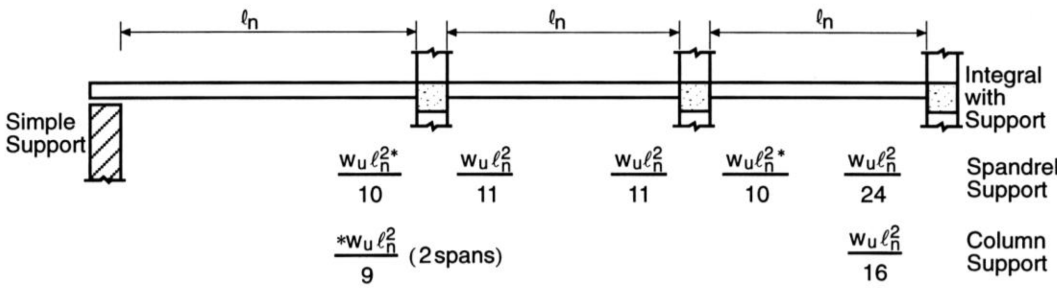


Figure 2-4 Negative Moments—Beams and Slabs

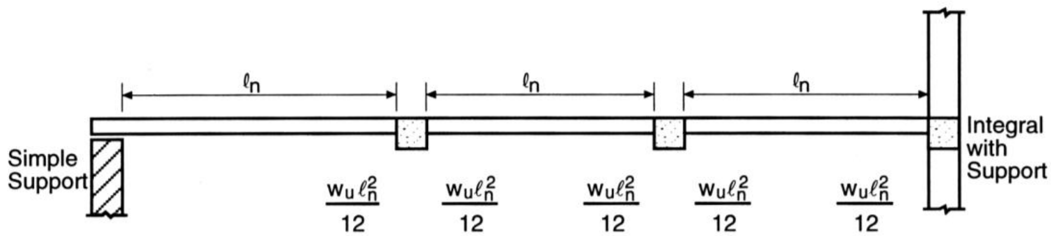


Figure 2-5 Negative Moments—Slabs with spans ≤ 10 ft

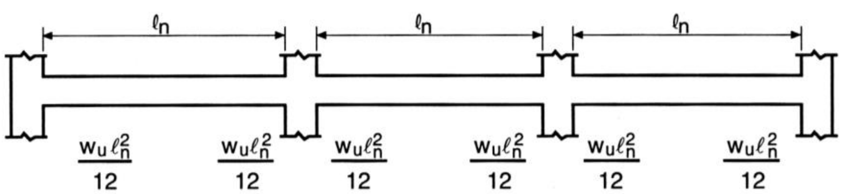


Figure 2-6 Negative Moments—Beams with Stiff Columns ($\Sigma K_c/\Sigma K_b > 8$)

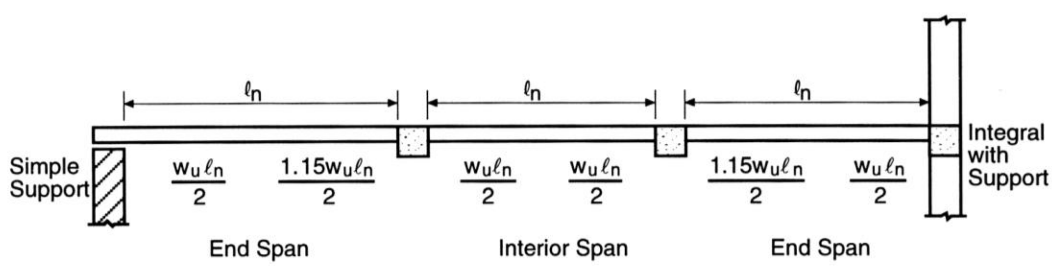


Figure 2-7 End Shears—All Cases

Thickness and Cover Requirements for Fire Protection
Simplified Design, PCA 1993

Table 10-1 Minimum Thickness for Floor and Roof Slabs and Cast-In-Place Walls, in. (Load Bearing and Nonload-Bearing)

	Fire resistance rating				
Concrete type	1 hr.	1 1/2 hr.	2 hr.	3 hr.	4 hr.
Siliceous aggregate	3.5	4.3	5.0	6.2	7.0
Carbonate aggregate	3.2	4.0	4.6	5.7	6.6
Sand-lightweight	2.7	3.3	3.8	4.6	5.4
Lightweight	2.5	3.1	3.6	4.4	5.1

Table 10-2 Minimum Concrete Column Dimensions, in.

	Fire resistance rating				
Concrete type	1 hr.	1 1/2 hr.	2 hr.	3 hr.	4 hr.
Siliceous aggregate	8	8	10	12	14
Carbonate aggregate	8	8	10	12	12
Sand-lightweight	8	8	9	10.5	12

Table 10-3 Minimum Cover for Reinforced Concrete Floor or Roof Slabs, in.

	Restrained Slabs*				Unrestrained Slabs*			
	Fire resistance rating				Fire resistance rating			
Concrete type	1 hr.	1 1/2 hr.	2 hr.	3 hr.	1 hr.	1 1/2 hr.	2 hr.	3 hr.
Siliceous aggregate	3/4	3/4	3/4	3/4	3/4	3/4	1	1 1/4
Carbonate aggregate	3/4	3/4	3/4	3/4	3/4	3/4	3/4	1 1/4
Sand-lightweight or lightweight	3/4	3/4	3/4	3/4	3/4	3/4	3/4	1 1/4

*See Table 10-5

Table 10-4 Minimum Cover to Main Reinforcing Bars in Reinforced Concrete Beams, in. (Applicable to All Types of Structural Concrete)

		Fire resistance rating				
Restrained or unrestrained*	Beam width, in.**	1 hr.	1 1/2 hr.	2 hr.	3 hr.	4 hr.
Restrained	5	3/4	3/4	3/4	1	1 1/4
Restrained	7	3/4	3/4	3/4	3/4	3/4
Restrained	≥ 10	3/4	3/4	3/4	3/4	3/4
Unrestrained	5	3/4	1	1 1/4	—	—
Unrestrained	7	3/4	3/4	3/4	1 3/4	3
Unrestrained	≥ 10	3/4	3/4	3/4	1	1 3/4

*See Table 10-5

**For beam widths between the tabulated values, the minimum cover can be determined by interpolation.

Table 10-6 Minimum Cover for Reinforced Concrete Columns, in.

	Fire resistance rating				
Concrete type	1 hr.	1 1/2 hr.	2 hr.	3 hr.	4 hr.
Siliceous aggregate	1 1/2	1 1/2	1 1/2	1 1/2	2
Carbonate aggregate	1 1/2	1 1/2	1 1/2	1 1/2	1 1/2
Sand-lightweight	1 1/2	1 1/2	1 1/2	1 1/2	1 1/2

Openings in Concrete Slab Systems
from Notes on ACI 318-99, Portland Cement Association, 1999

11.12.5 Openings in Slabs

The effect of openings (vertical holes through slabs) on the shear strength of slabs must be investigated when the openings are within the column strip areas of slabs or within middle strip areas when the openings are closer than 10 times the slab thickness (10h) from a column. A reduction in shear strength is made by considering as ineffective that portion of the critical section b_o which is enclosed by straight lines projecting from the column centroid to the edges of the opening. Ineffective portions of critical sections b_o are illustrated in Fig. 18-10. For slabs with shear reinforcement, the ineffective portion of the perimeter b_o is one-half of that without shear reinforcement. The one-half factor is interpreted to apply equally to shearhead reinforcement and bar or wire reinforcement.

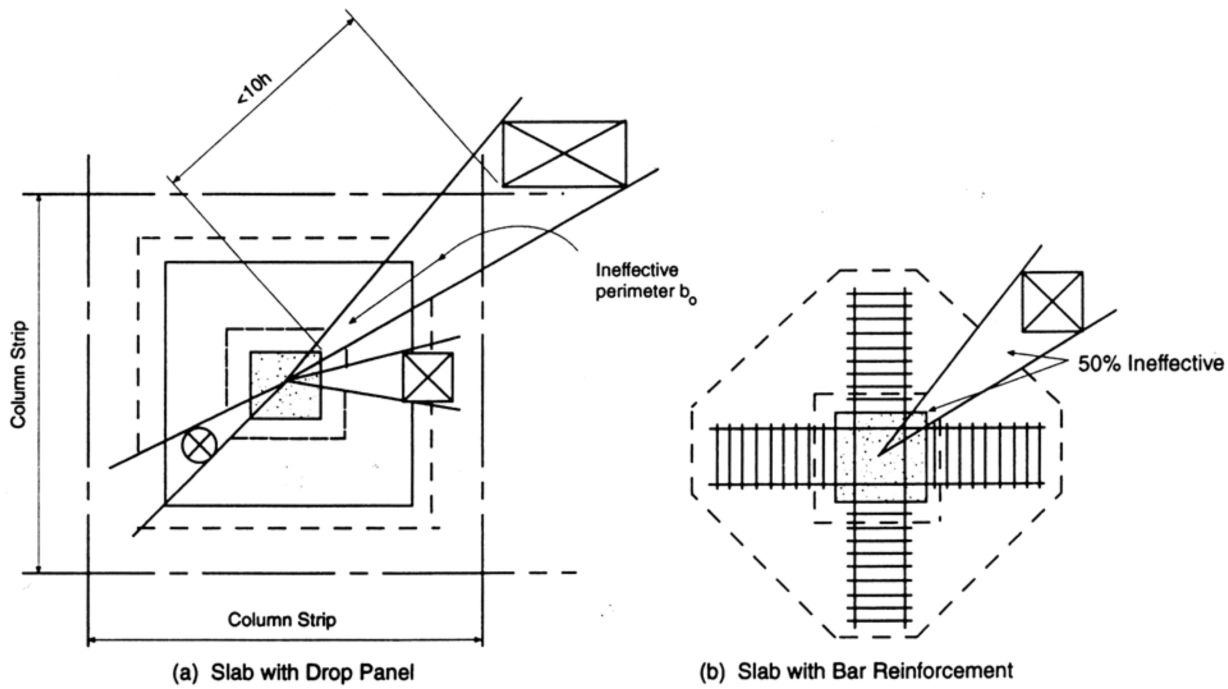


Figure 18-10 Effect of Slab Openings on Shear Strength

13.4 OPENINGS IN SLAB SYSTEMS

Openings of any size are permitted in slab systems without beams if special analysis indicates that both strength and serviceability of the slab system, considering the effects of the opening, are satisfied. Without special analysis, openings up to a certain size are permitted as illustrated in Fig. 18-11. The size of openings located within intersecting middle strip areas is unlimited. Within the area of the slab common to intersecting column strips, size of openings is the most restrictive, due to their effect on slab shear strength or load transfer near slab-column connections. See discussion on effect of slab openings on shear strength (11.12.5) and Fig. 18-10. Without special analysis, size of openings within intersecting column strips is limited to one-sixteenth of the slab span length in either direction $\left(1/8\ (\ell/2) = \ell/16\right)$. Within the slab area common to one column and one middle strip, opening size is limited to one-eighth the span length in either direction $\left(1/4\ (\ell/2) = \ell/8\right)$.

The total amount of reinforcement required for the panel without openings, in both directions, must be maintained; reinforcement interrupted by any opening must be replaced, one-half on each side of the opening.

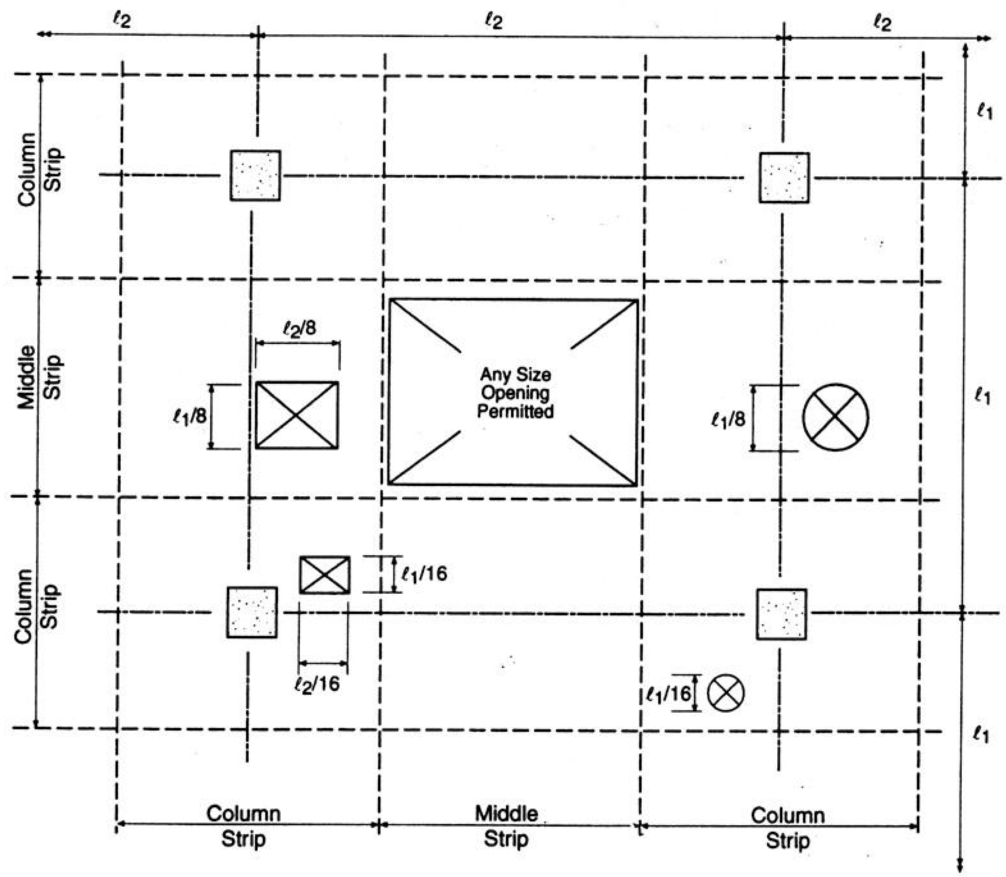


Figure 18-11 Openings in Slab Systems without Beams

Foundation Design

Notation:

a = name for width dimension
A = name for area
b = width of retaining wall stem at base
= width resisting shear stress
b_o = perimeter length for two-way shear in concrete footing design
B = spread footing or retaining wall base dimension in concrete design
cc = shorthand for clear cover
d = effective depth from the top of a reinforced concrete member to the centroid of the tensile steel
= name for diameter
e = eccentric distance of application of a force (P) from the centroid of a cross section
f = symbol for stress
f'_c = concrete design compressive stress
$F_{horizontal-resisting}$ = total force resisting horizontal sliding
$F_{sliding}$ = total sliding force
F_x = force in the x direction
$F.S.$ = shorthand for factor of safety
h_f = height of a concrete spread footing
H = height of retaining wall
H_A = horizontal force due to active soil pressure
l_d = development length for reinforcing steel
L = name for length or span length
M = moment due to a force
M_n = nominal flexure strength with the steel reinforcement at the yield stress and concrete at the concrete design strength for reinforced concrete beam design
$M_{overturning}$ = total overturning moment
$M_{resisting}$ = total moment resisting overturning about a point
M_u = maximum moment from factored loads for LRFD beam design
n = name for number
N = name for normal force to a surface
o = point of overturning of a retaining wall, commonly at the "toe"
p = pressure
p_A = active soil pressure
P = name for axial force vector
= force due to a pressure
P_D = dead load axial force
P_L = live load axial force
P_u = factored axial force
q = soil bearing pressure
q_a = allowable soil bearing stress in allowable stress design, as is $q_{allowable}$
q_g = gross soil bearing pressure
q_{net} = net allowed soil bearing pressure, as is q_n
q_u = ultimate soil bearing strength in allowable stress design
= factored soil bearing capacity in concrete footing design from load factors, as is q_{nu}
R = name for reaction force vector
SF = shorthand for factor of safety
t = thickness of retaining wall stem at top
T = name of a tension force
V = name for volume
V_c = shear force capacity in concrete
V_u = factored shear for reinforced concrete design
w = name for width
w_u = load per unit length on a beam from load factors
W = name for force due to weight
x = horizontal distance
$\bar{y}$ = the distance in the y direction from a reference axis to the centroid of a shape
ϕ = resistance factor
γ_c = density or unit weight of concrete
γ_s = density or unit weight of soil
π = pi (3.1415 radians or 180°)
ρ = reinforcement ratio in concrete beam design = A_s/bd
μ = coefficient of static friction

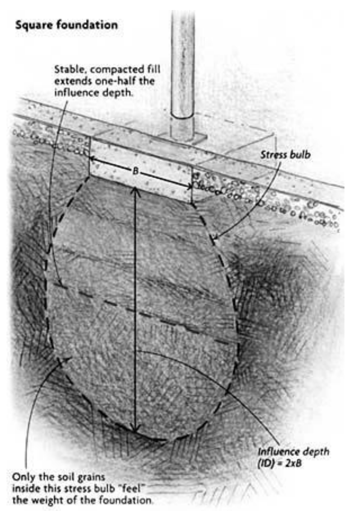
Foundations

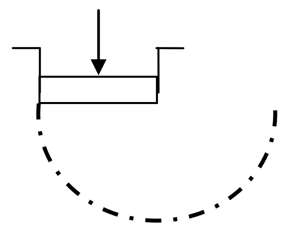
A foundation is defined as the engineered interface between the earth and the structure it supports that transmits the loads to the soil or rock. The design differs from structural design in that the choices in material and framing system are not available, and quality of materials cannot be assured. Foundation design is dependent on geology and climate of the site.

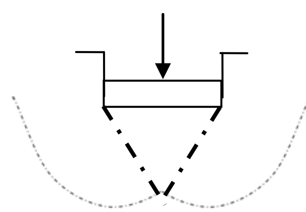
Soil Mechanics

Soil is another building material and the properties, just like the ones necessary for steel and concrete and wood, must be known before designing. In addition, soil has other properties due to massing of the material, how soil particles pack or slide against each other, and how water affects the behavior. The important properties are

- specific weight (density)
- allowable soil pressure
- factored net soil pressure – allowable soil pressure less surcharge with a factor of safety
- shear resistance
- backfill pressure
- cohesion & friction of soil
- effect of water
- settlement
- rock fracture behavior




slip zone


punched wedge

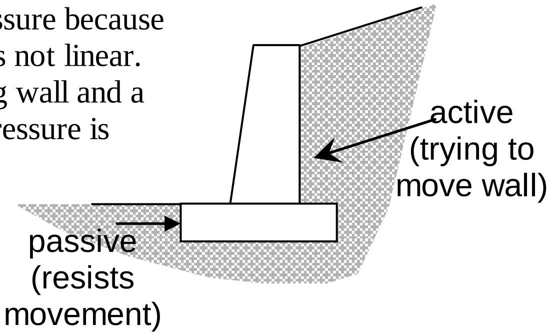
Structural Strength and Serviceability

There are significant serviceability considerations with soil. Soils can settle considerably under foundation loads, which can lead to redistribution of moments in continuous slabs or beams, increases in stresses and cracking. Excessive loads can cause the soil to fail in bearing and in shear. The presence of water can cause soils to swell or shrink and freeze and thaw, which causes heaving. Fissures or fault lines can cause seismic instabilities.

A geotechnical engineer or engineering service can use tests on soil bearings from the site to determine the ultimate bearing capacity, q_u. Allowable stress design is utilized for soils because of the variability do determine the allowable bearing capacity, $q_a = q_u$/(safety factor).

Values of q_a range from 3000 – 4000 psi for most soils, while clay type soils have lower capacities and sandy soils to rock have much higher capacities.

Soil acts somewhat like water, in that it exerts a lateral pressure because of the weight of the material above it, but the relationship is not linear. Soil can have an <u>active</u> pressure from soil behind a retaining wall and a <u>passive</u> pressure from soil in front of the footing. Active pressure is typically greater than passive pressure.



<u>Foundation Materials</u>

Typical foundation materials include:

- plain concrete
- reinforced concrete
- steel
- wood
- composites, ie. steel tubing filled with concrete

Table 7-1 Average Bearing Capacities of Various Foundation Beds

Soil	Bearing Capacity, q_a (ksf)
Alluvial soil	≤ 1
Soft clay	2
Firm clay	4
Wet sand	4
Sand and clay mixed	4
Fine dry sand (compact)	6
Hard clay	8
Coarse dry sand (compact)	8
Sand and gravel mixed (compact)	10
Gravel (compact)	12
Soft rock	16
Hard pan or hard shale	20
Medium rock	30
Hard rock	80

Foundation Design

<u>Generalized Design Steps</u>

Design of foundations with variable conditions and variable types of foundation structures will be different, but there are steps that are typical to every design, including:

1. Calculate loads from structure, surcharge, active & passive pressures, etc.
2. Characterize soil – hire a firm to conduct soil tests and produce a report that includes soil material properties
3. Determine footing location and depth – shallow footings are less expensive, but the variability of the soil from the geotechnical report will drive choices
4. Evaluate soil bearing capacity – the factor of safety is considered here
5. Determine footing size – these calculations are based on working loads and the allowable soil pressure
6. Calculate contact pressure and check stability
7. Estimate settlements
8. Design the footing structure – design for the material based on applicable structural design codes which may use allowable stress design, LRFD or limit state design (concrete).

Shallow Foundation Types

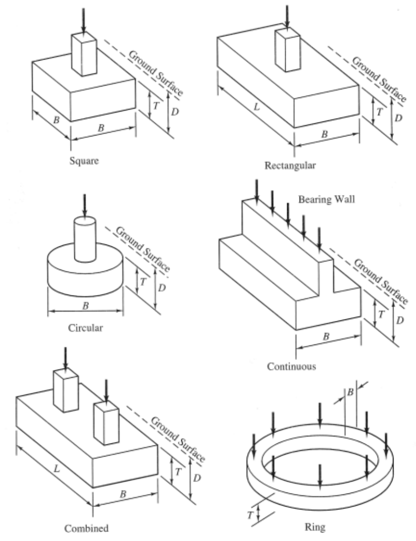


Figure 5.1 Spread footing shapes and dimensions.

Considered simple and cost effective because little soil is removed or disturbed.

Spread footing – A single column bears on a square or rectangular pad to distribute the load over a bigger area.

Wall footing – A continuous wall bears on a wide pad to distribute the load.

Eccentric footing – A spread or wall footing that also must resist a moment in addition to the axial column load.

Combined footing – Multiple columns (typically two) bear on a rectangular or trapezoidal shaped footing.

Unsymmetrical footing – A footing with a shape that does not evenly distribute bearing pressure from column loads and moments. It typically involves a hole or a non-rectangular shape influenced by a boundary or property line.

Strap footing – A combined footing consisting of two spread footings with a beam or strap connecting the slabs. The purpose of this is to limit differential settlements.

Mat foundation – A slab that supports multiple columns. The mat can be stiffened with a grid or grade beams. It is typically used when the soil capacity is very low.

Deep Foundation Types

Considerable material and excavation is required, increasing cost and effort.

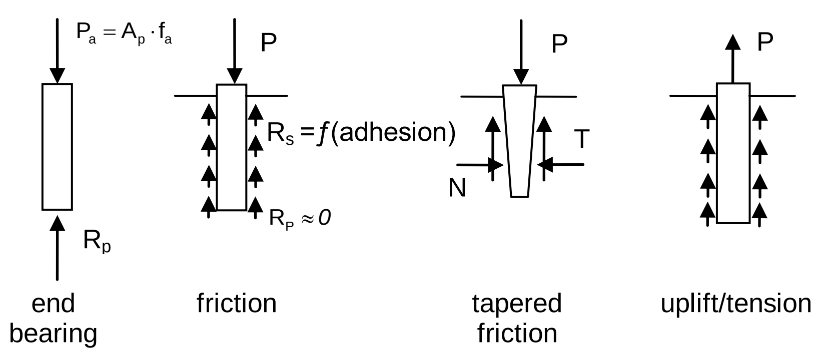
Retaining Walls – A wall that retains soil or other materials, and must resist sliding and overturning. Can have counterforts, buttresses or keys.

Basement Walls – A wall that encloses a basement space, typically next to a floor slab, and that may be restrained at the top by a floor slab.

Piles – Next choice when spread footings or mats won't work, piles are used to distribute loads by end bearing to strong soil or friction to low strength soils. Can be used to resist uplift, a moment causing overturning, or to compact soils. Also useful when used in combination to control settlements of mats or slabs.

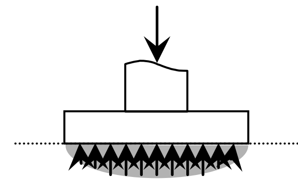
Drilled Piers – Soil is removed to the shape of the pier and concrete is added.

Caissons –Water and possibly wet soil is held back or excavated while the footing is constructed or dropped into place.

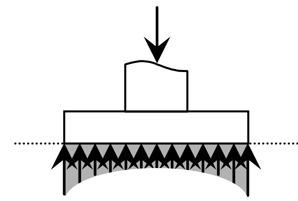


Pile Types

Loads and Stresses

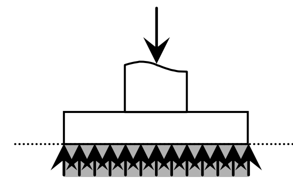

RIGID footing on sand

Bearing loads must be distributed to the soil materials, but because of their variability and the stiffness of the footing pad, the resulting stress, or soil pressure, is not necessarily uniform. But we assume it is for design because dealing with the complexity isn't worth the time or effort.


RIGID footing on clay

The increase in weight when replacing soil with concrete is called the overburden. Overburden may also be the result of adding additional soil to the top of the excavation for a retaining wall. It is extra *uniformly distributed load* that is considered by reducing the allowable soil pressure (instead of increasing the loads), resulting in a net allowable soil pressure, q_{net}:

$$q_{net} = q_{allowable} - h_f(\gamma_c - \gamma_s)$$

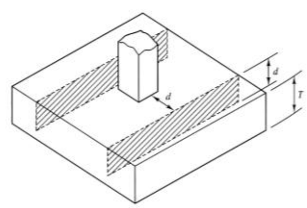

IDEAL stress

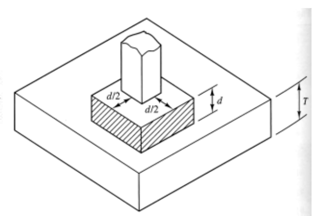
In order to design the footing size, the actual stress P/A must be less than or equal to the allowable pressure:

$$\frac{P}{A} \le q_{net}$$

Design Stresses

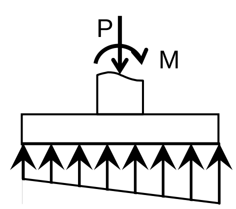
The result of a uniform pressure on the underside of a footing is identical to a distributed load on a slab over a column when looked at *upside down*. The footing slab must resist bending, one-way shear and two-way shear (punching).

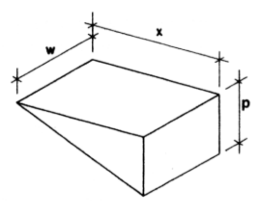

one-way shear

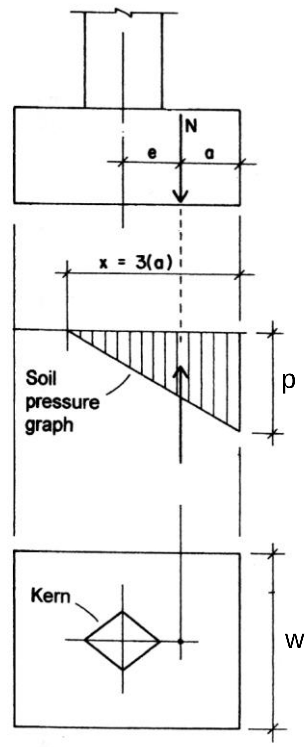

two-way shear

Stresses with Eccentric Loading

Combined axial and bending stresses increase the pressure on one edge or corner of a footing. We assume again a linear distribution based on a constant relationship to settling. If the pressure combination is in tension, this effectively means the contact is gone between soil and footing and the pressure is really zero. To avoid zero pressure, the eccentricity must stay within the kern. The maximum pressure must not exceed the net allowable soil pressure.







If the contact is gone, the maximum pressure can be determined knowing that the volume of the *pressure wedge* has to equal the column load, and the centroid of the *pressure wedge* coincides with the effective eccentricity.

Wedge volume is $V = \frac{wpx}{2}$ where *w* is the width, *p* is the soil pressure, and *x* is the wedge length (3*a*), so $p = \frac{2P}{wx} or \frac{2N}{wx}$ (and $e = \frac{M}{P} or \frac{M}{N}$ and *a*=½ *width - e)*

Overturning is considered in design such that the resisting moment from the soil pressure (equivalent force at load centroid) is greater than the overturning moment, M, by a factor of safety of at least 1.5

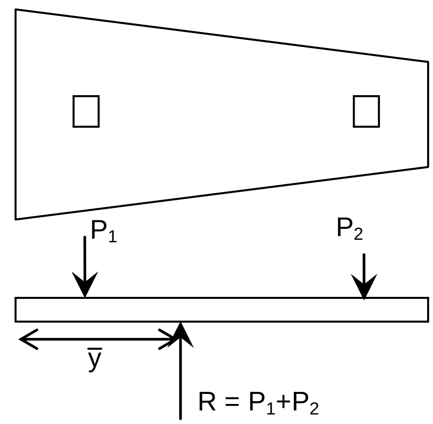
$$SF = \frac{M_{resist}}{M_{overturning}} \geq 1.5$$

where

M_{resist} = average resultant soil pressure x width x location of load centroid with respect to column centroid

$M_{overturning}$ = P x e

Combined Footings

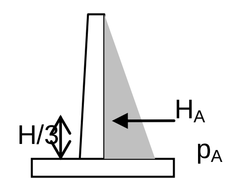


The design of combined footing requires that the centroid of the area be as close as possible to the resultant of the two column loads for uniform pressure and settling.

Retaining Walls

The design of retaining walls must consider overturning, settlement, sliding and bearing pressure. The water in the retained soil can significantly affect the loading and the active pressure of the soil. The lateral force acting at a height of H/3 is determined from the active pressure, p_A, (in force/cubic area) as:

$$H_A = \frac{p_A H^2}{2}$$



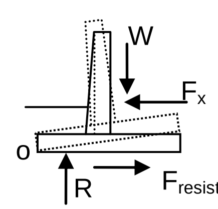
Overturning is considered the same as for eccentric footings:

$$SF = \frac{M_{resist}}{M_{overturning}} \geq 1.5 - 2$$

where

M_{resist} = summation of moments about "o" to resist rotation, typically including the moment due to the weight of the stem and base and the moment due to the passive pressure.

$M_{overturning}$ = moment due to the active pressure about "o".



Sliding must also be avoided:

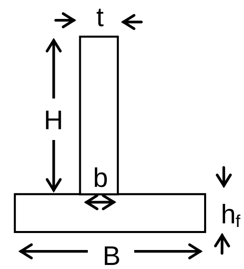
$$SF = \frac{F_{horizontal\text{-}resist}}{F_{sliding}} \geq 1.25 - 2$$

where:

$F_{horizontal\text{-}resist}$ = summation of forces to resist sliding, typically including the force from the passive pressure and friction (F=μ·N where μ is a constant for the materials in contact and N is the normal force to the ground acting down and shown as R).

$F_{sliding}$ = sliding force as a result of active pressure.

For sizing, some rules of thumbs are:



- footing size, B
- reinforced concrete, B ≈ 2/5 - 2/3 wall height (H)
- footing thickness, h_f ≈ 1/12 - 1/8 footing size (B)
- base of stem, b ≈ 1/10 - 1/12 wall height ($H+h_f$)
- top of stem, t ≥ 12 inches

Example 1

Example 2. Design a square column footing for the following data:

Soil density = 100 lb/ft^3, Concrete density = 150 lb/ft^3

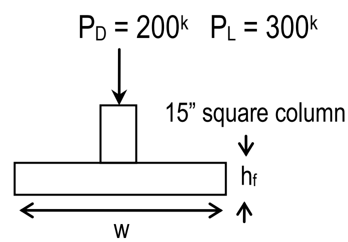
Column load = 200 kips [890 kN] dead load and 300 kips [1334 kN] live load

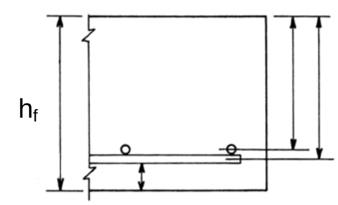
Column size = 15 in. [380 mm] square

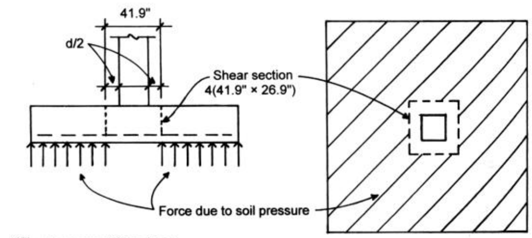
Maximum allowable soil pressure = 4000 psf [200 kPa]

Concrete design strength = 3000 psi [21 MPa]

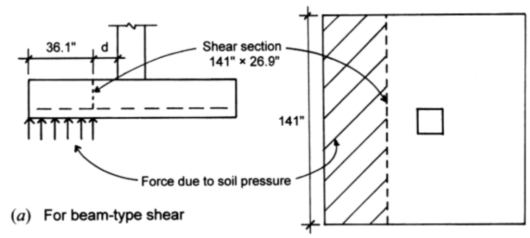
Yield stress of steel reinforcement = 40 ksi [280 MPa]



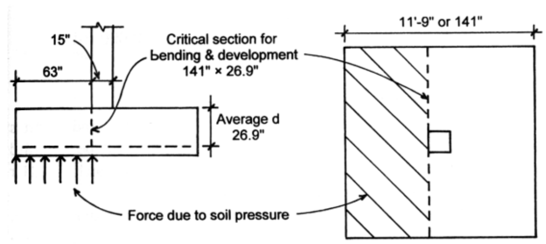




(b) For punching shear

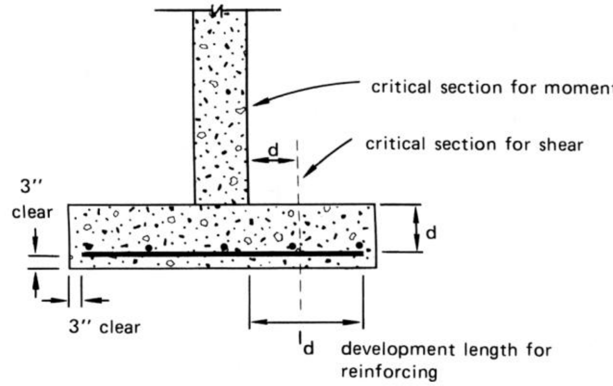


(a) For beam-type shear



Example 2

For the 16 in. thick 8.5 ft. square reinforced concrete footing carrying 150 kips dead load and 100 kips live load on a 24 in. square column, determine if the footing thickness is adequate for 4000 psi . A 3 in. cover is required with concrete in contact with soil.
Also determine the moment for reinforced concrete design.



SOLUTION:

1. Find design soil pressure: $q_u = \dfrac{P_u}{A}$

   P_u = 1.2D + 1.6L = 1.2 (150 k) + 1.6 (100 k) = 340 k

   $q_u = \dfrac{340k}{(8.5\,ft)^2}$ = 4.71 k/ft^2

2. Evaluate one-way shear at d away from column face (Is $V_u < \phi V_c$?)

   d = h_f – c.c. – distance to bar intersection

   presuming #8 bars:

   d = 16 in. – 3 in. (soil exposure) - 1 in. x (1 layer of #8's) = 12 in.

   V_u = total shear = q_u (edge area)

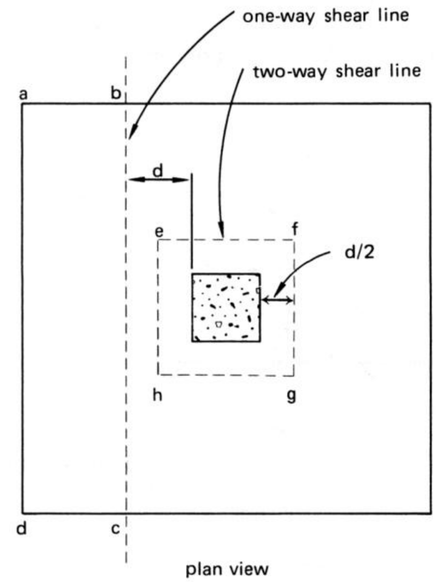
   V_u on a 1 ft strip = q_u (edge distance) (1 ft)

   V_u = 4.71 k/ft^2 [(8.5 ft – 2 ft)/2 – (12 in.)(1 ft/12 in.)] (1 ft) = 10.6 k

   ϕV_n = one-way shear resistance = $\phi 2\sqrt{f'_c}$ bd

   for a one foot strip, b = 12 in.

   $\phi V_c = 0.75(2\sqrt{4000}$ psi)(12 in.)(12 in.) = 13.7 k > 10.6 k OK



3. Evaluate two-way shear at d/2 away from column face (Is $V_u < \phi V_c$?)

   b_o = perimeter = 4 (24 in. + 12 in.) = 4 (36 in.) = 144 in

   V_u = total shear on area outside perimeter = P_u – q_u (punch area)

   V_u = 340 k – (4.71 k/ft^2)(36 in.)^2(1 ft/12 in.)^2 = 297.6 kips

   ϕV_n = two-way shear resistance = $\phi 4\sqrt{f'_c}\, b_o d = 0.75(4\sqrt{4000}$ psi)(144 in.)(12 in.) = 327.9 k > 297.6 k OK

4. Design for bending at column face

   $M_u = w_u L^2/2$ for a cantilever. L = (8.5 ft – 2 ft)/2 = 3.25 ft, and w_u for a 1 ft strip = q_u (1 ft)

   M_u = 4.71 k/ft^2(1 ft)(3.25 ft)^2/2 = 24.9 k-ft (per ft of width)

   To complete the reinforcement design, use b =12 in. and trial d = 12 in., choose ρ, determine A_s, find if $\phi M_n > M_u$.....

5. Check transfer of load from column to footing:

   $\phi P_n = \phi 0.85 f'_c ? A_1 \sqrt{A_2/A_1} \le \phi 0.85 f_c' 2 A_1$ = 0.65(0.85)(4000psi)(2)(12 in.)(12 in.) = 636.5 k > 340 k OK

Example 3

Example 8-1: Evaluate the suitability of a 4-ft square footing supporting a 1-ft square column (P_D = 75 kips and P_L = 25 kips) for an allowable soil pressure of 7 k/ft² using a) gross soil pressure, b) net soil pressure. The bottom of the one-foot thick footing is set at 5 ft below grade. The unit weight of soil is given as 125 pcf.

a) gross soil pressure, q_g:

- footing weight: (4)(4)(1)(0.150)	=	2.4
- column weight: (1)(1)(4)(0.150)	=	0.6
- soil weight: (4)(16-1)(0.125)	=	7.5
- service loads: 75 + 25	=	100.0
Total		110.5 kips

$$q_g = \frac{P}{A} = \frac{110.5}{16} = 6.9 \text{ kips/ft}^2 < 7 \text{ kips/ft}^2 \text{ O.K.}$$

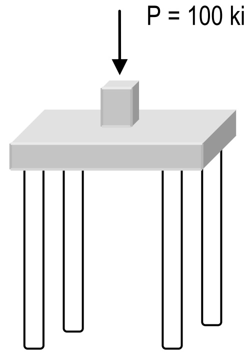
b) net soil pressure, q_n:

$$q_n = \frac{100}{16} = 6.25 \text{ kips/ft}^2 < q_n = 7 - 1(0.150\text{-}0.125) = 6.975 \text{ kips/ft}^2 \text{ O.K.}$$

c) $$q_{nu} = \frac{1.2(75) + 1.6(25)}{16} = 8.13 \text{ kips/ft}^2$$

Example 4

Determine the depth required for the group of 4 friction piles having 12 in. diameters if the column load is 100 kips and the frictional resistance is 400 lbs/ft².



SOLUTION:
The downward load is resisted by a friction force. Friction is determined by multiplying the friction resistance (a stress) by the area: $F = fA_{SKIN}$

The area of n cylinders is: $A_{SKIN} = n(2\pi \frac{d}{2} L)$

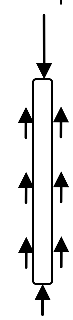
Our solution is to set P ≤ F and solve for length:

$$100k \le 400 \, {}^{lb}/_{ft^2} (4^{piles})(2\pi)(\frac{12in}{2})L \cdot (\frac{1ft}{12in}) \cdot (\frac{1k}{1000lb})$$

$$L \ge 19.9 \, {}^{ft}/_{pile}$$

Example 5

Determine the depth required for the friction and bearing pile having a 36 in. diameter if the column load is 300 kips, the frictional resistance is 600 lbs/ft² and the end bearing pressure allowed is 8000 psf.



SOLUTION:
The downward load is resisted by a friction force and a bearing force, which can be determined from multiplying the bearing pressure by the area in contact: $F = fA_{SKIN} + qA_{TIP}$

The area of a circle is: $A_{TIP} = \pi \frac{d^2}{4}$

Our solution is to set P ≤ F and solve for length:

$$300k \le 600 \, {}^{lb}/_{ft^2} \, 2\pi(\frac{36in}{2})L \cdot (\frac{1ft}{12in}) \cdot (\frac{1k}{1000lb}) + 8000 \, {}^{lb}/_{ft^2} \, \pi \frac{(36in)^2}{4} \cdot (\frac{1ft}{12in})^2 \cdot (\frac{1k}{1000lb})$$

$$L \ge 43.1 ft$$

Design of Isolated Square and Rectangular Footings (ACI 318-02)

Notation:

a = equivalent square column size in spread footing design
= depth of the effective compression block in a concrete beam
A_g = gross area, equal to the total area ignoring any reinforcement
A_{req} = area required to satisfy allowable stress
A_s = area of steel reinforcement in concrete design
A_1 = area of column in spread footing design
A_2 = projected bearing area of column load in spread footing design
b = rectangular column dimension in concrete footing design
= width, often cross-sectional
b_f = width of the flange of a steel or cross section
b_o = perimeter length for two-way shear in concrete footing design
B = spread footing dimension in concrete design
= dimension of a steel base plate for concrete footing design
B_s = width within the longer dimension of a rectangular spread footing that reinforcement must be concentrated within for concrete design
c = rectangular column dimension in concrete footing design
C = dimension of a steel base plate for concrete footing design
d = effective depth from the top of a reinforced concrete member to the centroid of the tensile steel
d_b = bar diameter of a reinforcing bar
d_f = depth of a steel column flange (wide flange section)
f'_c = concrete design compressive stress
f_y = yield stress or strength
h_f = height of a concrete spread footing
l_d = development length for reinforcing steel
l_{dc} = development length for column
l_s = lap splice length in concrete design
L = name for length or span length
L_m = projected length for bending in concrete footing design
L' = length of the one-way shear area in concrete footing design
M_n = nominal flexure strength with the steel reinforcement at the yield stress and concrete at the concrete design strength for reinforced concrete flexure design
M_u = maximum moment from factored loads for LRFD beam design
P = name for axial force vector
P_{dowels} = nominal capacity of dowels from concrete column to footing in concrete design
P_D = dead load axial force
P_L = live load axial force
P_n = nominal column or bearing load capacity in concrete design
P_u = factored axial force
$q_{allowable}$ = allowable soil bearing stress in allowable stress design
q_{net} = net allowed soil bearing pressure
q_u = factored soil bearing capacity in concrete footing design from load factors
V_c = shear force capacity in concrete
V_n = nominal shear force capacity
V_{u1} = maximum one-way shear from factored loads for LRFD beam design
V_{u2} = maximum two-way shear from factored loads for LRFD beam design
β_c = ratio of long side to short side of the column in concrete footing design
ϕ = resistance factor
γ_c = density or unit weight of concrete
γ_s = density or unit weight of soil
ρ = reinforcement ratio in concrete beam design = A_s/bd
υ_c = shear strength in concrete design

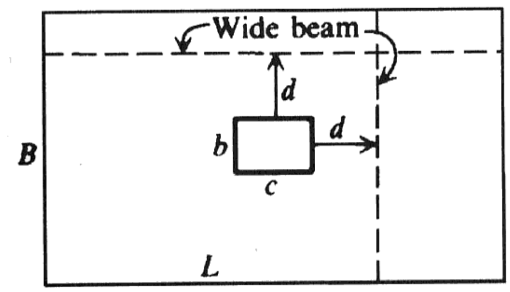
NOTE: This procedure assumes that the footing is concentrically loaded and carries no moment so that the soil pressure may be assumed to be uniformly distributed on the base.

1) Find service dead and live column loads:

P_D = Service dead load from column

P_L = Service live load from column

$P = P_D + P_L$ (typically – see **ACI 9.2**)



2) Find design (factored) column load, Pu:

$P_U = 1.2P_D + 1.6P_L$

3) Find an approximate footing depth, h_f

$h_f = d + 4"$ and is usually in multiples of 2, 4 or 6 inches.

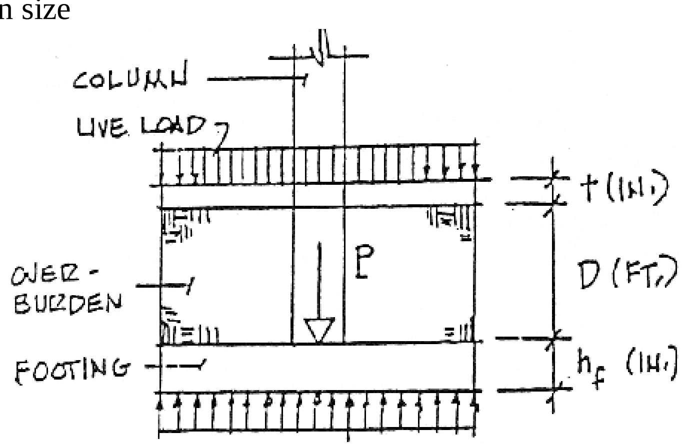
a) For rectangular columns $4d^2 + 2(b+c)d = \dfrac{P_u}{\phi \upsilon_c}$

b) For round columns $d^2 + ad = \dfrac{P_u}{\phi \upsilon_c}$ $a = \sqrt{\dfrac{\pi d^2}{4}}$

where: a is the equivalent square column size

$\upsilon_c = 4\sqrt{f'_c}$ for two-way shear

$\phi = 0.75$ for shear



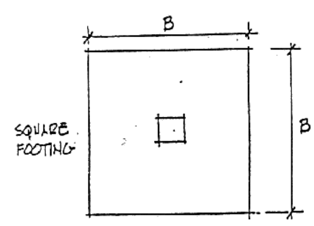
4) Find net allowable soil pressure, q_{net}:

By neglecting the weight of any additional top soil added, the net allowable soil pressure takes into account the change in weight when soil is removed and replaced by concrete:

$$q_{net} = q_{allowable} - h_f(\gamma_c - \gamma_s)$$

where γ_c is the unit weight of concrete (typically 150 lb/ft^3) and γ_s is the unit weight of the displaced soil

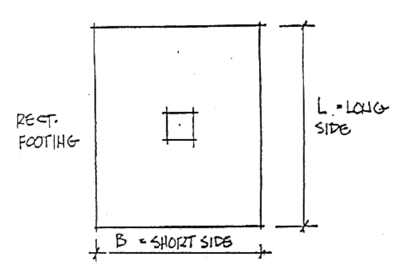
5) Find required area of footing base and establish length and width:



$$A_{req} \geq \frac{P}{q_{net}}$$

For square footings choose $B \geq \sqrt{A_{req}}$

For rectangular footings choose $B \times L \geq A_{req}$



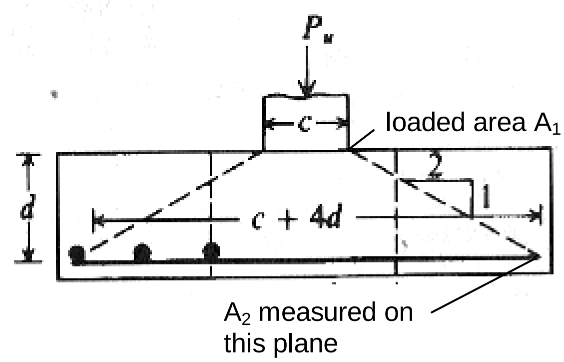
6) <u>Check transfer of load from column to footing:</u> **ACI 15.8**

a) Find load transferred by bearing on concrete in column: **ACI 10.17**

basic: $\phi P_n = \phi 0.85 f'_c A_1$ where $\phi = 0.65$ and A_1 is the area of the column

with confinement: $\phi P_n = \phi 0.85 f'_c A_1 \sqrt{\frac{A_2}{A_1}}$ where $\sqrt{\frac{A_2}{A_1}}$ cannot exceed 2.

IF the column concrete strength is lower than the footing, calculate ϕP_n for the column too.



b) Find load to be transferred by dowels:

$\phi P_{dowels} = P_u - \phi P_n$

IF $\phi P_n \geq P_u$ only nominal dowels are required.

c) Find required area of dowels and choose bars

Req. dowel $A_s = \frac{\phi P_{dowels}}{\phi f_y}$ where $\phi = 0.65$ and f_y is the reinforcement grade

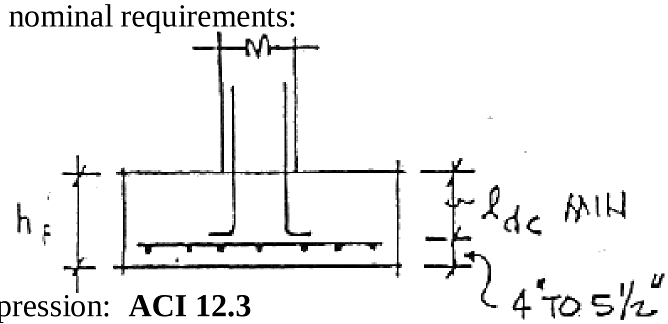
Choose dowels to satisfy the required area and nominal requirements:

i) Minimum of 4 bars

ii) Minimum $A_s = 0.005 A_g$ **ACI 15.8.2.1**

where A_g is the gross column area

iii) 4 - #5 bars



d) Check dowel embedment into footing for compression: **ACI 12.3**

$$l_{dc} = \frac{0.02 f_y d_b}{\sqrt{f'_c}}$$ but not less than $0.0003 f_y d_b$ or 8" where d_b is the bar diameter

NOTE: The footing must be deep enough to accept l_{dc}. Hooks are not considered effective in compression and are only used to support dowels during construction.

e) Find length of lapped splices of dowels with column bars: **ACI 12.16**

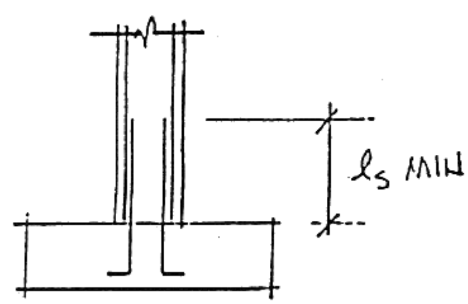
l_s is the largest of:

i) larger of l_{dc} or $0.0005 f_y d_b$ (f_y of grade 60 or less)

of smaller bar $(0.0009 f_y - 24) d_b$ (f_y over grade 60)

ii) l_{dc} of larger bar

iii) not less than 12"



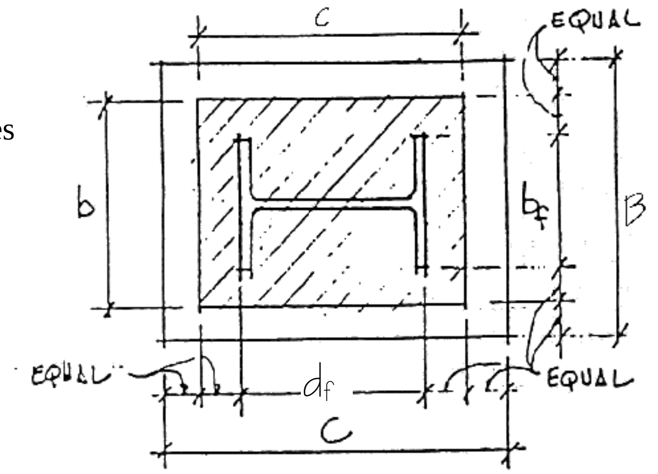
See **ACI 12.17.2** for possible reduction in l_s

7) Check two-way (slab) shear:

a) Find dimensions of loaded area:

i) For concrete columns, the area coincides with the column area, if rectangular, or equivalent square area if circular (see 3)b))

ii) For steel columns an equivalent loaded area whose boundaries are halfway between the faces of the steel column and the edges of the steel base plate is used: **ACI 15.4.2c**.



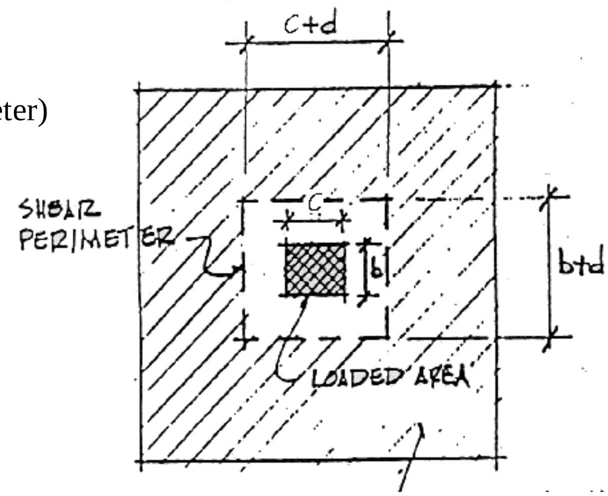
$$b = b_f + \frac{(B - b_f)}{2}$$ where b_f is the width of column flange and B is base plate side

$$c = d_f + \frac{(C - d_f)}{2}$$ where d_f is the depth of column flange and C is base plate side

b) Find shear perimeter: **ACI 11.12.1.2**

Shear perimeter is located at a distance of $d/2$ outside boundaries of loaded area and length is $b_o = 2(c+d) + 2(b+d)$

(average d = h_f – 3 in. cover – 1 assumed bar diameter)



c) Find factored net soil pressure, q_u:

$$q_u = \frac{P_u}{B^2} \; or \; \frac{P_u}{B \times L}$$

d) Find total shear force for two-way shear, V_{u2}:

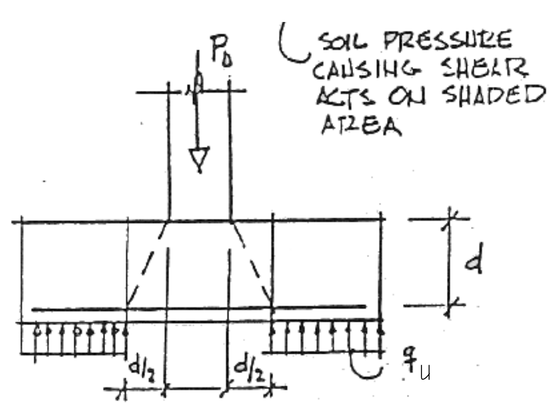
$$V_{u2} = P_u - q_u(c+d)(b+d)$$

e) Compare V_{u2} to two-way capacity, ϕV_n:

$$V_{u2} \le \phi\left(2 + \frac{4}{\beta_c}\right)\sqrt{f'_c}\,b_o d \le \phi 4\sqrt{f'_c}\,b_o d$$ **ACI 11.12.2.1**

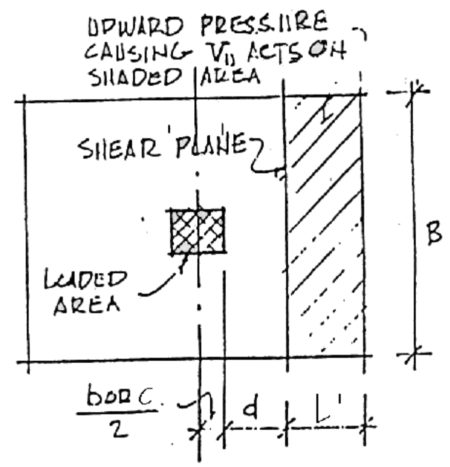
where ϕ = 0.75 and β_c is the ratio of long side to short side of the column

NOTE: This should be acceptable because the initial footing size was chosen on the basis of two-way shear limiting. If it is not acceptable, increase h_f and repeat steps starting at b).



8) Check one-way (beam) shear:

The critical section for one-way shear extends across the width of the footing at a distance *d* from the face of the loaded area (see 7)a) for loaded area). The footing is treated as a cantilevered beam. **ACI 11.12.1.1**



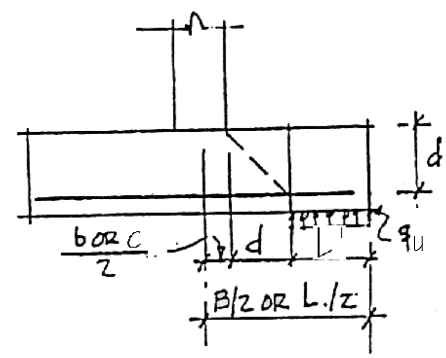
a) Find projection, *L'*:

i) For square footing:

$L' = \frac{B}{2} - (d + b/2)$ where b is the smaller dim. of the loaded area

ii) For rectangular footings:

$L' = \frac{L}{2} - (d + \bullet/2)$ where • is the dim. parallel to the long side of the footing



b) Find total shear force on critical section, V_{u1}:

$$V_{u1} = BL'q_u$$

c) Compare V_{u1} to one-way capacity, ϕV_n:

$V_{u1} \le \phi 2\sqrt{f'_c}Bd$ **ACI 11.12.3.1** where $\phi = 0.75$

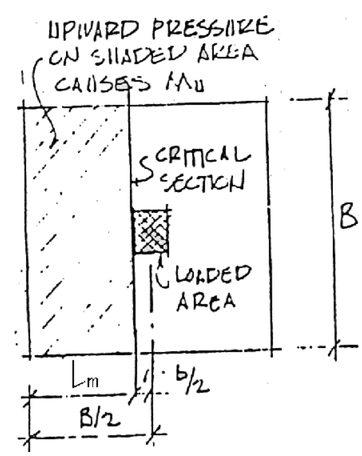
NOTE: If it is not acceptable, increase h_f .

9) Check for bending stress and design reinforcement:

Square footings may be designed for moment in one direction and the same reinforcing used in the other direction. For rectangular footings the moment and reinforcing must be calculated separately in each direction. The critical section for moment extends across the width of the footing at the face of the loaded area. **ACI 15.4.1, 15.4.2**.

a) Find projection, L_m:

$L_m = \frac{B}{2} - \frac{\bullet}{2}$ where • is the smaller dim. of column for a square footing. For a rectangular footing, use the value perpendicular to the critical section.

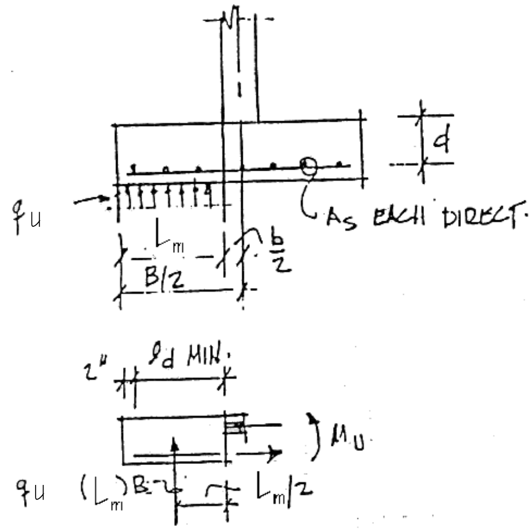


b) Find total moment, M_u, on critical section:

$M_u = q_u \frac{BL_m^2}{2}$ (find both ways for a rectangular footing)

c) Find required A_s:

$R_n = \frac{M_n}{bd^2} = \frac{M_u}{\phi bd^2}$, where $\phi = 0.9$, and ρ can be found from Figure 3.8.1 of Wang & Salmon.



or:

i) guess a

ii) $A_s = \frac{0.85 f'_c ba}{f_y}$

iii) solve for $a = 2\left(d - \frac{M_u}{\phi A_s f_y}\right)$

iv) repeat from ii) until a converges, solve for A_s

Minimum A_s

= 0.0018bh Grade 60 for temperature and shrinkage control

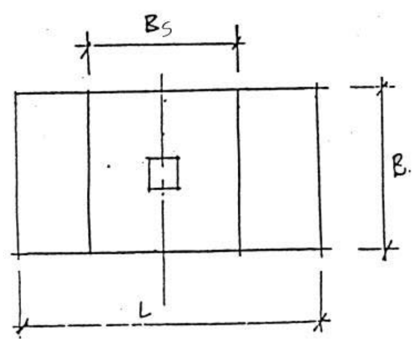
= 0.002bh Grade 40 or 50

ACI 10.5.4 specifies the requirements of **7.12** must be met, and max. spacing of 18”

d) Choose bars:

For square footings use the same size and number of bars uniformly spaced in each direction (**ACI 15.4.3**). Note that required A_s must be furnished in each direction.

For rectangular footings bars in long direction should be uniformly spaced. In the short direction bars should be distributed as follows (**ACI 15.4.4**):



i) In a band of width B_s centered on column:

$$\#\text{ bars } = \frac{2}{L/B + 1} \cdot (\# bars\ in\ B) \quad \text{(integer)}$$

ii) Remaining bars in short direction should be uniformly spaced in outer portions of footing.

e) Check development length:

Find required development length, l_d, in tension from handout or from equations in **ACI 12.2**. l_d must be less than (L_m – 2”) (end cover). If not possible, use more bars of smaller diameter.

Masonry Design

Notation:

A = name for area
A_n = net area, equal to the gross area subtracting any reinforcement
A_{nv} = net shear area of masonry
A_s = area of steel reinforcement in masonry design
A_{st} = area of steel reinforcement in masonry column design
ACI = American Concrete Institute
ASCE = American Society of Civil Engineers
b = width, often cross-sectional
C = name for a compression force
C_m = compression force in the masonry for masonry design
CMU = shorthand for concrete masonry unit
d = effective depth from the top of a reinforced masonry beam to the centroid of the tensile steel
e = eccentric distance of application of a force (P) from the centroid of a cross section
f_a = axial stress
f_b = bending stress
f_m = calculated compressive stress in masonry
f'_m = masonry design compressive stress
f_s = stress in the steel reinforcement for masonry design
f_v = shear stress
F_a = allowable axial stress
F_b = allowable bending stress
F_s = allowable tensile stress in reinforcement for masonry design
F_t = allowable tensile stress
F_v = allowable shear stress
F_{vm} = allowable shear stress of the masonry
F_{vs} = allowable shear stress of the shear reinforcement
h = name for height
= effective height of a wall or column
I_x = moment of inertia with respect to an x-axis
j = multiplier by effective depth of masonry section for moment arm, jd
k = multiplier by effective depth of masonry section for neutral axis, kd
L = name for length or span length
M = internal bending moment
= type of masonry mortar
M_m = moment capacity of a reinforced masonry beam governed by steel stress
M_s = moment capacity of a reinforced masonry beam governed by masonry stress
MSJC = Masonry Structural Joint Council
n = modulus of elasticity transformation coefficient for steel to masonry
n.a. = shorthand for neutral axis (N.A.)
N = type of masonry mortar
NCMA = National Concrete Masonry Association
O = type of masonry mortar
P = name for axial force vector
P_a = allowable axial load in columns
r = radius of gyration
S = section modulus
= type of masonry mortar
S_x = section modulus with respect to an x-axis
t = name for thickness
T = name for a tension force
T_s = tension force in the steel reinforcement for masonry design
TMS = The Masonry Society
w = name for distributed load
β_1 = coefficient for determining stress block height, c, in masonry LRFD design
ε_m = strain in the masonry
ε_s = strain in the steel
ρ = reinforcement ratio in masonry design

Reinforced Masonry Design

Structural design standards for reinforced masonry are established by the *Masonry Standards Joint Committee* consisting of ACI, ASCE and The Masonry Society (TMS), and presents allowable stress design as well as limit state (strength) design.

Materials

f'_m = masonry prism compressive strength from testing

Reinforcing steel grades are the same as those used for reinforced concrete beams.

Units can be brick, concrete or stone.

Mortar consists of masonry cement, lime, sand, and water. Grades are named from the word MASONWORK, with average strengths of 2500psi, 1800 psi, 750 psi, 350 psi, and 75 psi, respectively.

Grout is a flowable mortar, usually with a high amount of water to cement material. It is used to fill voids and bond reinforcement.

Allowable Stress Design

For unreinforced masonry, like masonry walls, tension stresses are allowed in flexure. Masonry walls typically see compression stresses too.

For reinforced masonry, the steel is presumed to resist *all* tensile stresses and the tension in the masonry is ignored.

Factors of Safety are applied to the limit stresses for allowable stress values:

bending (unreinforced)	$F_b = 1/3\, f'_m$
bending (reinforced)	$F_b = 0.45\, f'_m$
bending (tension/unreinforced)	table 2.2.3.2
beam shear (unreinforced for flexure)	$F_v = 1.5\sqrt{f'_m} \leq 120$ psi
beam shear (reinforced) – $M/(Vd) \leq 0.25$	$F_v = 3.0\sqrt{f'_m}$
beam shear (reinforced) – $M/(Vd) \geq 1.0$	$F_v = 2.0\sqrt{f'_m}$
Grades 40 or 50 reinforcement	F_s = 20 ksi
Grades 60 reinforcement	F_s = 32 ksi
Wire joint reinforcement	F_s = 30 ksi

where f''_m = specified compressive strength of masonry

Internal Equilibrium for Bending

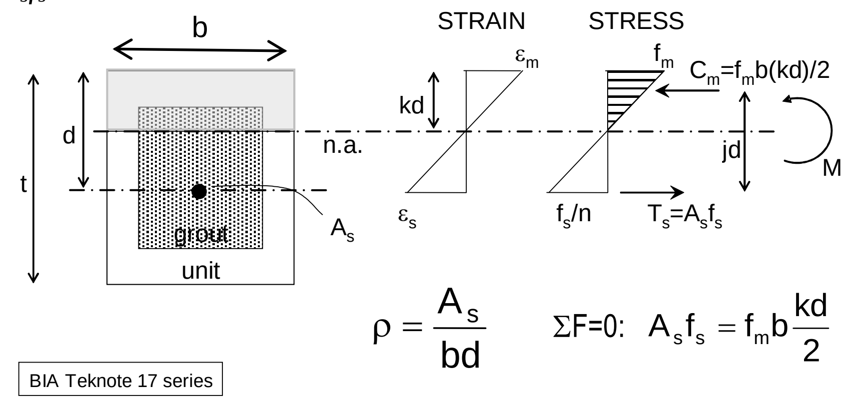
C_m = compression in masonry = stress x area = $f_m \frac{b(kd)}{2}$

T_s = tension in steel = stress x area = $A_s f_s$

$C_m = T_s$ and •

$$M_m = T_s(d\text{-}kd/3) = T_s(jd)$$

$$M_s = C_m(jd)$$



where

- f_m = compressive stress in the masonry from flexure
- f_s = tensile stress in the steel reinforcement
- kd = the height to the neutral axis
- b = width of stress area
- d = effective depth of section = depth to n.a. of reinforcement
- jd = moment arm from tension force to compression force
- A_s = area of steel
- n = E_s/E_m used to transform steel to equivalent area of masonry for elastic stresses
- ρ = reinforcement ratio

Criteria for Beam Design

For flexure design:

$$M_m = f_m b\frac{kd}{2}jd = 0.5 f_m bd^2 jk \text{ or } M_s = A_s f_s jd = \rho bd^2 jf_s$$

The design is adequate when $f_b \le F_b$ in the masonry and $f_s \le F_s$ .in the steel.

Shear stress is determined by $f_v = V/A_{nv}$ where A_{nv} is net shear area. Shear strength is determined from the shear capacity of the masonry and the stirrups: $F_v = F_{vm} + F_{vs}$. Stirrup spacings are limited to d/2 but not to exceed 48 in.

where:

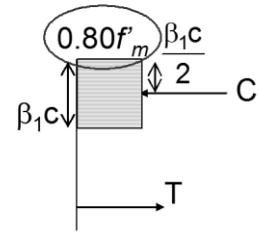
$$F_{vm} = \frac{1}{2}\left[\left(4.0 - 1.75\left(\frac{M}{Vd}\right)\right)\sqrt{f'_m}\right] + 0.25\frac{P}{A_n}$$ where M/(Vd) is positive and cannot exceed 1.0

$$F_{vs} = 0.5\left(\frac{A_v F_s d}{A_{nv} s}\right)$$

($F_v = 3.0\sqrt{f'_m}$ when M/(Vd) ≥ 0.25)

($F_v = 2.0\sqrt{f'_m}$ when M(Vd) ≥ 1.0.) Values can be linearly interpolated.

Load and Resistance Factor Design



The design methodology is similar to reinforced concrete ultimate strength design. It is useful with high shear values and for seismic design. The limiting masonry strength is $0.80f'_m$.

Criteria for Column Design

(Masonry Joint Code Committee) Building Code Requirements and Commentary for Masonry Structures define a column as having b/t < 3 and h/t > 4.

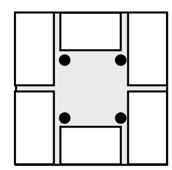
where

b = width of the "wall"
t = thickness of the "wall"
h = height of the "wall"

A slender column has a minimum dimension of 8" on one side and h/t ≤ 25.

Columns must be reinforced, and have ties. A minimum eccentricity (causing bending) of 0.1 times the side dimension is required.

Allowable Axial Load for Reinforced Masonry



$$P_a = \left[0.25 f'_m A_n + 0.65 A_{st} F_s\right]\left[1 - \left(\frac{h}{140r}\right)^2\right] \quad \text{for } h/t \le 99$$

$$P_a = \left[0.25 f'_m A_n + 0.65 A_{st} F_s\right]\left(\frac{70r}{h}\right)^2 \quad \text{for } h/t > 99$$

Allowable Axial Stresses for Unreinforced Masonry

$$F_a = 0.25 f'_m\left[1 - \left(\frac{h}{140r}\right)^2\right] \quad \text{for } h/t \le 99$$

$$F_a = 0.25 f'_m\left(\frac{70r}{h}\right)^2 \quad \text{for } h/t > 99$$

where

h = effective length
r = radius of gyration
A_n = effective (or net) area of masonry
A_{st} = area of steel reinforcement
f'_m = specified masonry compressive strength
F_s = allowable compressive stress in column reinforcement with lateral confinement.

Combined Stresses

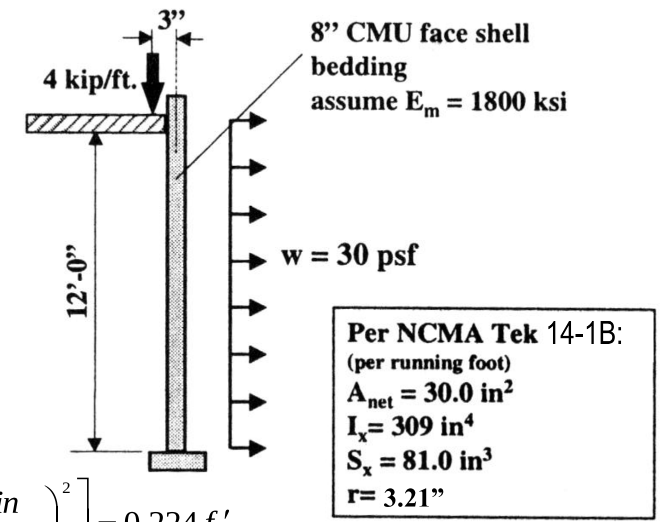
When maximum moment occurs somewhere other than at the end of the column or wall, a "virtual" eccentricity can be determined from $e = M/P$.

Masonry Columns and Walls

There are no modification factors, but in addition to satisfying $\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0$, the tensile stress cannot exceed the allowable: $f_b - f_a \leq F_t$ or the compressive stress exceed allowable for reinforced masonry: $f_a + f_b \leq F_b$ provided $f_a \leq F_a$.

Example 1

Determine if the unreinforced CMU wall can sustain its loads with the wind. Specify a mortar type and unit strength per MSJC.



$$\frac{f_a}{F_a}+\frac{f_b}{F_b}\le 1.0 \qquad F_b = \tfrac{1}{3} f'_m \qquad f_b=\frac{M}{S} \qquad f_a=\frac{P}{A}$$

$$F_a = 0.25 f'_m\left[1-\left(\frac{h}{140r}\right)^2\right] \; for\; \frac{h}{r}\le 99$$

$$F_a = 0.25 f'_m\left(\frac{70r}{h}\right)^2 \; for\; \frac{h}{r}> 99$$

$$\frac{h}{r}=\frac{12\,ft(12in)}{3.21in}=44.9 \; so\; F_a = 0.25 f'_m\left[1-\left(\frac{12\cdot 12in}{140\cdot 3.21in}\right)^2\right]=0.224 f'_m$$

$$f_a=\frac{4k(1000\, {}^{lb}\!/_{k})}{30in^2}=133psi$$

Case "A" with wind

at midheight of wall : (1 ft·kips/ft²) (ft) (in/ft)

$$M=\frac{Pe}{2}+\frac{wh^2}{8}=\frac{4\,kip\;x\;3''}{2}+\left[\frac{(0.030)(12)^2}{8}\right]x12=12.5\;kip-in.$$

$$f_b=\frac{12{,}500\;lb-in}{81.0\;in^3}=154\;psi \qquad f_b \le 1/3f'_m$$

tension criterion : $\quad f'_m \ge 154/(1/3) = 462$ psi

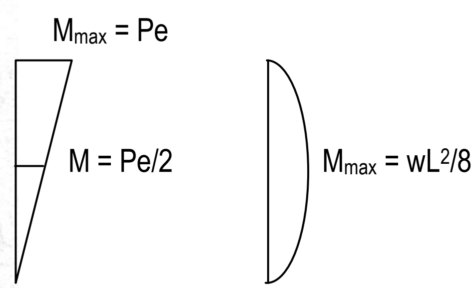
$$-f_a+f_b=F_t$$

$$-133\;psi+154\;psi=21\;psi$$

$$F_{t\;req'd}=21\;psi$$

compression criterion :

$$\frac{f_a}{F_a}+\frac{f_b}{F_b}<1 \qquad \frac{133}{0.174f'_m}+\frac{154}{0.333f'_m}=1; \qquad f'_m=1056\;psi$$



Moment distribution from eccentricity

Moment distribution from distributed wind load

Case "B" without wind

at top of wall : $M = Pe = 12.0$ kip - in.

$f_b = 12{,}000$ lb - in/81 in³ $= 148$ psi

tension criterion : $-f_a + f_b = F_t$

-133 psi $+ 148$ psi $= 15$ psi $\quad F_{t\;req'd} = 15$ psi

Per MSJC Table 2.2.3.2, use PCL Type N mortar F_t = 25 psi

compression criterion : $\dfrac{f_a}{F_a}+\dfrac{f_b}{F_b}\le 1.0$

$$\frac{133}{0.224\,f'_m}+\frac{148}{0.333\,f'_m}=1.00 \qquad f'_m=1038\;psi$$

f'ₘ = 1056 psi (governs)

Table 2.2.3.2 — Allowable flexural tensile stresses for clay and concrete masonry, psi (kPa)

Direction of flexural tensile stress and masonry type	Mortar types			
	Portland cement/lime or mortar cement		Masonry cement or air entrained portland cement/lime	
	M or S	N	M or S	N
Normal to bed joints				
Solid units	53 (366)	40 (276)	32 (221)	20 (138)
Hollow units[1]				
Ungrouted	33 (228)	25 (172)	20 (138)	12 (83)
Fully grouted	86 (593)	84 (579)	81 (559)	77 (531)
Parallel to bed joints in running bond				
Solid units	106 (731)	80 (552)	64 (441)	40 (276)
Hollow units				
Ungrouted and partially grouted	66 (455)	50 (345)	40 (276)	25 (172)
Fully grouted	106 (731)	80 (552)	64 (441)	40 (276)
Parallel to bed joints in masonry not laid in running bond				
Continuous grout section parallel to bed joints	133 (917)	133 (917)	133 (917)	133 (917)
Other	0 (0)	0 (0)	0 (0)	0 (0)

1 For partially grouted masonry, allowable stresses shall be determined on the basis of linear interpolation between fully grouted hollow units and ungrouted hollow units based on amount (percentage) of grouting.

Technical Notes on Brick Construction

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REINFORCED BRICK MASONRY BEAMS

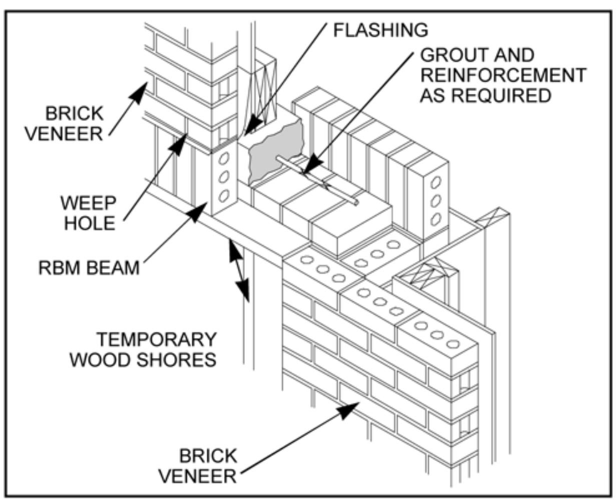
Abstract: Reinforced brick masonry (RBM) beams are an efficient and attractive means of spanning building openings. The addition of steel reinforcement and grout permits brick masonry to span considerable distances while maintaining continuity of the building facade. Attractive brick soffits and elimination of steel support members are two of the advantages of reinforced brick masonry beams. This *Technical Notes* addresses the design of reinforced brick masonry beams. Building code requirements are reviewed and design aids are provided to simplify the design process. Illustrations indicate the proper detailing and typical construction of reinforced brick masonry beams.

Key Words: beam, deflection, girder, lintel, reinforced brick masonry, reinforcement.

INTRODUCTION

Reinforced brick masonry (RBM) beams are widely used as flexural members. Common applications of RBM beams include girders supporting floor and roof systems, and arches and lintels spanning openings for windows and doors. Girder is the term applied to a large beam with a long span that usually supports smaller framing members. A lintel is a beam over a wall opening, typically simply supported with no framing members. The main advantage of RBM beams is that the structural element and the architectural finish are one and the same. In some cases, however, they provide economical solutions without considering the savings due to a built-in finish. They are often built as an integral part of a masonry wall as illustrated in Figure 1. RBM beams are designed to carry all superimposed loads, including that portion of the wall weight above supported by the beam. While steel lintels are more common, RBM beams provide distinct advantages over steel lintels. Among the advantages are:

1. More efficient use of materials. The masonry serves as a structural element with a relatively small amount of steel reinforcement added.
2. Elimination of differential movement. This movement is often the cause of cracks in masonry.
3. Inherent fire resistance.
4. Reduced maintenance. Periodic painting of exposed steel is eliminated.
5. Lower cost.



Typical RBM Beam in Brick Veneer Wall
FIG. 1

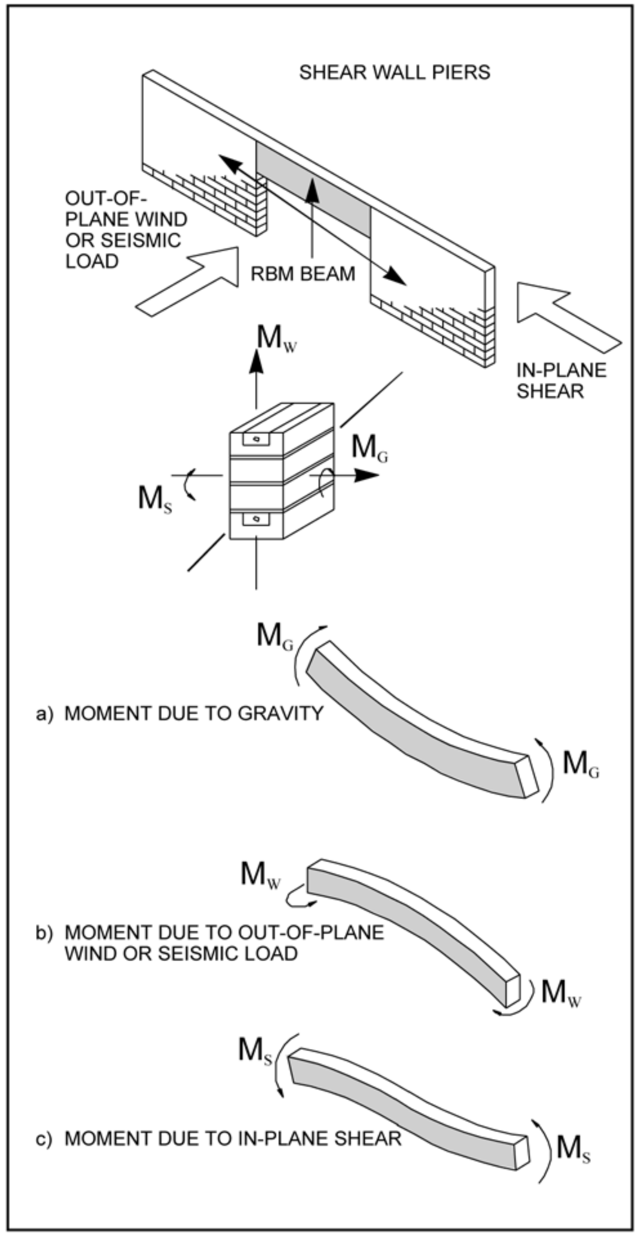
This *Technical Notes* provides a review of the design of RBM beams. Factors influencing design and performance are reviewed. Design recommendations and aids are provided and their use illustrated with an example. For additional information about RBM beams and design calculations, refer to the *Masonry Designers' Guide* (MDG) [2]. The MDG also provides an extensive review of the requirements of the *Building Code Requirements for Masonry Structures* (ACI 530/ASCE 5/TMS 402-95)[1], hereafter termed the MSJC Code. Other *Technical Notes* in this series provide the history of RBM, material and construction requirements, and design of other RBM elements.

This *Technical Notes* does not address the design of deep beams (wall beams) or bond beams. A deep beam is one with a depth-to-span ratio exceeding 0.8. Assumptions made in this *Technical Notes* regarding the distribution of stress in beams under flexure and the loading conditions do not apply to deep beams. Bond beams are formed by placing horizontal reinforcement in a wall without an opening underneath.

NOTATION

Following are notations used in the text, figures, and table in this *Technical Notes.*

A_v Area of shear reinforcement, in.^{2} (mm^2)
b Length of bearing plate, ft (m)
d Effective depth of beam, in. (mm)
d_b Nominal diameter of reinforcement, in. (mm)
F_s Allowable steel stress, psi (MPa)
f'_m Specified compressive strength of masonry, psi (MPa)
H Height of beam, in. (mm)
l_d Embedment length of reinforcement, in. (mm)
M_G Design moment due to gravity loads, in.-lb (N-m)
M_s Design moment due to in-plane shear, in.-lb (N-m)
M_w Design moment due to out-of-plane wind or seismic load, in.-lb (N-m)
P Design concentrated load, lb (kg)
s Spacing of shear reinforcement, in. (mm)
V Design shear force, lb (kg)
W Width of beam, in. (mm)
w_p Design uniform distributed load, lb/ft (kg/m)
y Distance from top of beam to bearing plate, ft (m)



Moments on RBM Beam
FIG. 2

DETERMINATION OF LOADING

The basic concept of a beam is as a pure flexural member. A flexural member spans an opening and transfers vertical gravity loads to its supports, as illustrated in Fig. 2(a). RBM beams act in this manner to support their own weight and other applied gravity loads. However, it is also common for RBM beams to be part of a masonry wall. As such, RBM beams are often subjected to out-of-plane wind and seismic forces, as depicted in Fig. 2(b). This causes bending of the RBM beam in the out-of-plane direction, which is often about the weak axis of the beam. In addition, reinforced masonry walls may be shear-resisting members, or "shear walls", which are part of the lateral load-resisting system of a building. In such a structural system, RBM beams may be used as connections between shear walls or piers, as illustrated in Fig. 2(c). Such beams are called coupling beams because they "couple" the shear walls or piers. If the relative sizes of the two piers being coupled are similar, the RBM beam is subject to considerable load when an in-plane shear force is applied to the wall. This is why damage to masonry shear walls is often concentrated at coupling beams following an earthquake or high-wind event.

The designer should consider all aspects of loading for an RBM beam. It is difficult to predict the loading condition that will produce the critical design condition. For example, a RBM beam that is part of a wall will be subject to a combination of gravity loads and out-of-plane wind or seismic loads. Many factors influence the loading conditions for RBM beams.

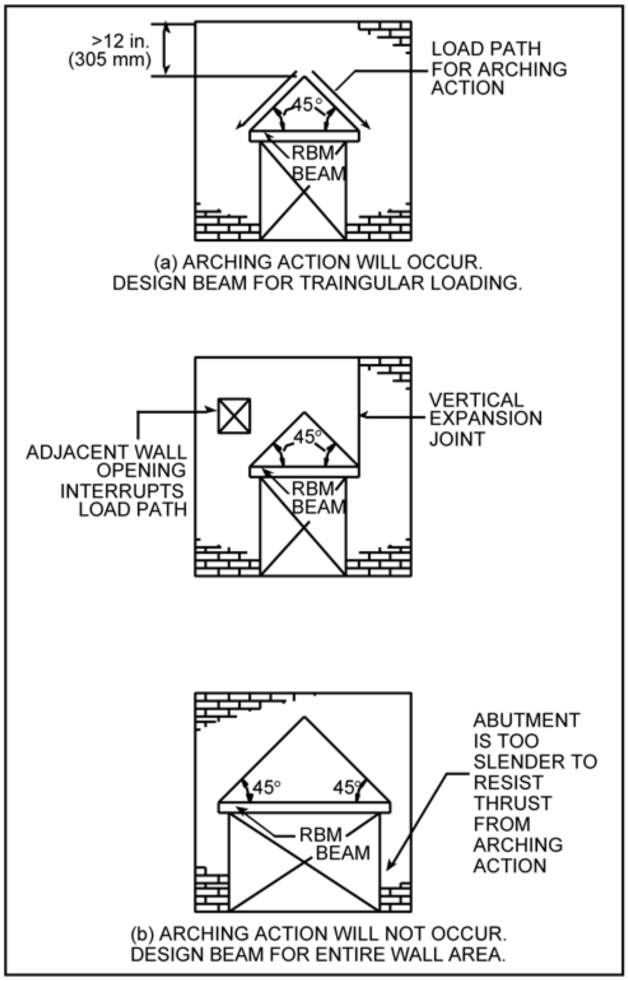
Arching Action

Arching action is a property of all masonry walls which are laid in an overlapping bond pattern. Brick masonry will span, in a step-like manner similar to a corbel, over a wall opening when laid in running bond pattern. Vertical gravity loads above the openings are transferred to the wall elements on each side. This is the reason why sizable holes can be created in masonry walls without causing collapse. Arching action will occur provided that the following conditions are met:

1. An overlapping bond pattern is used in the masonry surrounding the opening.
2. The masonry above the apex of a 45 degree isosceles triangle above the beam exceeds 12 in. (300 mm).
3. There are no movement joints or adjacent wall openings that hinder the load path of arching action.
4. The abutments are sufficiently strong and rigid to resist the horizontal thrust due to arching action.

These concepts are illustrated in Fig. 3.

Provided arching action occurs, the self weight of masonry wall carried by the beam may be safely as-



Conditions for Arching Action
FIG. 3

sumed as the weight within a triangular area above the beam formed by 45 degree angles, as shown in Fig. 3. The self weight of the wall must be added to the live and dead loads of floors and roofs which bear on the wall above the opening. If a stack bond pattern is used, the full area of brick masonry above the wall opening should be considered in the RBM beam design with no assumption of arching action.

Concentrated Loads

Loads from beams, girders, trusses and other concentrated loads that frame into the wall must be applied to the RBM beam in the appropriate manner. Concentrated loads may be assumed to be distributed over a wall length equal to the base of a trapezoid whose top is at the point of load application and whose sides make an angle of 60 degrees with the horizontal. In Fig. 4, the portion of the concentrated load carried by the beam is distributed over the length indicated as a uniform load. The distributed load, w_p, on the RBM beam is computed by the following equation:

$$w_p = P/(b + 2y\tan 30) \qquad \text{Eq. 1}$$

where:

w_p= design uniform distributed load, 1b/ft (kg/m)
P = design concentrated load, 1b (kg)
b = length of bearing plate, ft (m)
y = distance from top of beam to bearing plate, ft (m)

This is approximately 0.866 times P divided by y. Because the apex of the 45 degree triangle is above the top of the wall in this example, the RBM beam should be designed assuming no arching action occurs.

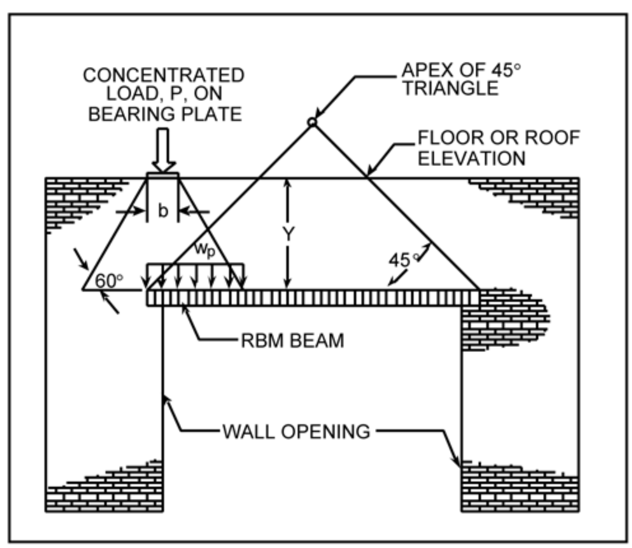
The designer should check the stress condition at bearing points for RBM beams. This applies to loads on the beam and to the beam's reaction on the wall. The MSJC Code limits the bearing stress to 0.25 f'_m, where f'_m is the specified compressive strength of masonry. A rule-of-thumb recommended for many years is to provide a minimum of 4 in. (100 mm) of bearing length for masonry beams. The masonry directly beneath a bearing point should be constructed with solid brick or with solidly grouted hollow brick. Concentrated loads should not bear directly on ungrouted hollow brick masonry because of the potential for localized cracking or crushing of the face shells.

Construction Loads

When designing a RBM beam that is prefabricated or built on the ground and lifted into place, it is important to consider the loads during transport and handling. To address these loads, the beam may require reinforcement at both the top and bottom of the beam. Beams built in place are constructed on shores. These must be designed for the dead weight of the beam plus any superimposed load prior to adequate curing of the reinforced brickwork.

Movement Joints

Movement joints are a necessity in masonry walls to accommodate differential movement and avoid cracking. It is common to place vertical expansion joints at or near the jamb of wall openings. In RBM buildings there is a reduced need for expansion joints and such joints may be spaced farther apart. Refer to *Technical Notes* 18 Series for a discussion of the placement of movement joints. The presence of a movement joint



Loads on RBM Beam
FIG. 4

near a RBM beam will influence the loads and support conditions for the beam. For example, a simple support condition should be assumed since arching action will not occur if a movement joint is at or near the jamb of the opening. Furthermore, the beam will not act as a coupling beam between shear walls. This is, in fact, one means of simplifying the design and function of a RBM beam by eliminating loads due to in-plane shear.

DESIGN OF RBM BEAMS

RBM beam design should not be relegated to "rule-of-thumb" methods or arbitrary selection of beam configuration and steel reinforcement. In any beam design, a careful analysis of the loads to be carried and a calculation of the resultant stresses should be incorporated to provide adequate strength and to prevent excessive cracking and deflection.

In addition to adequate strength, it is preferred that beams exhibit ductile behavior when overloaded. If the beam is overloaded, it should deform (deflect) a considerable amount prior to collapse. Deformation allows redistribution of loads to other members and provides visual indication that the beam is overloaded. Some building codes stipulate a maximum reinforcement ratio for RBM beams for this purpose.

Another aspect is the relation between the RBM beam's strength and its cracking moment. Failure of unreinforced masonry in flexure is brittle, exhibiting sudden cracking and often collapse. Consequently, a reinforced beam should provide a moment strength in excess of its cracking moment. The amount of this overstrength is somewhat arbitrary, but a factor of 1.3 is required by the *Uniform Building Code*[3]. This means that the moment strength of a cracked-section, RBM beam should exceed 1.3 times the cracking moment of the beam. This is not a requirement of the MSJC Code, but is considered good engineering practice.

Beam Sizing

In the design of an RBM beam, the required cross-sectional area of masonry is based primarily on the maximum bending moment. However, there are other factors to consider when sizing an RBM beam. For example, it is often desirable to have the width of the RBM beam coincide with the specified wall thickness. RBM beams are sometimes formed with special U-shaped, hollow brick for this reason. These brick may be manufactured specially for this purpose or they may be cut from full-size units at the site. Manufactured special shapes may not be readily available in many localities, so it is best to contact the brick manufacturer as early as possible before proceeding with a design based on their use. The beam's depth will be determined by the appropriate number of courses of masonry units present. The beam's depth should be taken as only those courses of solid brick or that are solidly grouted. The beam's depth may be limited by the height of the wall above an opening. In such cases, compression steel may be necessary when sufficient masonry area is not provided.

Lateral Bracing

With short spans and relatively deep beams, there is little likelihood of excessive cracking, deflection or rotation. This may not be the case, however, for beams that are relatively long span, shallow or highly loaded. Such beams may be vulnerable to lateral torsional buckling. The designer should consider the lateral bracing conditions to ensure that the beam is laterally braced. The MSJC Code requires that the compression face of beams be laterally supported at a maximum spacing of 32 times the beam thickness. A brick veneer wall is laterally braced by wall ties to the backup system. A RBM beam that is part of a load-bearing wall system may not be laterally braced along its span length. In addition, movement joints at the jambs of a wall opening may result in a lack of lateral bracing for the beam at its supports. In such cases, attachment of the wall to the floor or roof diaphragm is the common means of providing lateral bracing for the beam.

RBM Arches

Design of RBM arches should begin with an analysis assuming the arch is unreinforced, in accordance with *Technical Notes* 31A or the ARCH computer program available from the Brick Industry Association. Such an analysis will indicate the locations of highest moment and shear, and the horizontal thrust at the abutments. Should the analysis so indicate, the arch should be designed as a reinforced beam. Further, if the conditions shown in Fig. 3 are not met, or if movement joints are provided at the abutments so that the arch may spread under load, the arch should be designed as if it were a straight, simply supported beam as a conservative measure. Alternately, a finite element analysis of the arch may be conducted to determine design moment, shear, and thrust values.

RBM arches cause both a vertical bearing stress and a horizontal thrust on their abutments. The designer has the option of resisting the horizontal thrust of the arch by the abutments or providing room for movement as the RBM arch deforms under load. Judicious placement of vertical expansion joints and flashing will permit horizontal movement and simplify the arch design. This is recommended for longer span arches because providing adequate thrust resistance is difficult and movement joint spacing is limited. In this case, it is very important to provide adequate bearing at the abutments.

STEEL REINFORCEMENT AND TIES

The quantity of reinforcement required for an RBM beam is typically determined by the applied loads. However, the applicable building code may prescribe a minimum amount of reinforcement and this may dictate the amount of reinforcement required in a RBM beam. For example, all building codes now stipulate a minimum amount of reinforcement for masonry members in areas prone to earthquakes. Some building codes require that reinforcement in masonry coupling beams be uniformly distributed throughout the beam's height. This may require additional reinforcement and grouting of the masonry above wall openings in RBM beams.

Bond and Hooks

Typically, reinforcement is inserted in masonry beams to resist tension. The tension must be transferred from the masonry to the reinforcement. This is achieved through adequate bond between the steel reinforcement and the masonry. The bond stress along the length of the reinforcement should not exceed an allowable bond stress of 160 psi (1.1 MPa), according to the MSJC Code Commentary. A minimum embedment length must be provided in order to not exceed this bond stress. Consequently, the MSJC Code stipulates a required bond length for reinforcement in tension, called the minimum embedment length. The minimum embedment length is computed by the following equation:

$$l_d = 0.0015 d_b F_s \qquad \text{Eq. 2}$$

where:

l_d = embedment length of reinforcement, in. (mm)
d_b = nominal diameter of reinforcement, in. (mm)
F_s = allowable steel stress, psi (MPa)

Table 1 provides the minimum development lengths for various bar and wire sizes, based on Grade 60 ksi (414 MPa) reinforcing bars and 70 ksi (483 MPa) steel wire.

The ends of reinforcing bars and wires may require a standard hook to properly secure the reinforcement and to achieve its strength. In simply-supported beams, the peak moment is often at midspan. For this case, the reinforcement in RBM beams can likely be developed by the bond between the bar or wire and the surrounding masonry with no need for hooks at the ends of the beam. However, a cantilever RBM beam may require a hook at the support end. In addition, shear reinforcement should always be terminated with a hook. Standard hooks for principal reinforcement may be either a 90 degree or 180 degree turn. Often, the designated space for grout and reinforcement in RBM beams is very small. It can be difficult for a contractor to execute a reinforcement detail properly. Consider that a 180 degree hook doubles the number of bars at a given cross section. The designer should always consider the reinforcement placement, tolerances, and cover restrictions stated in the building codes. *Technical Notes* 17A Revised provides further information on bar sizes, placement requirements and construction tolerances.

TABLE 1

Minimum Development Lengths

Reinforcement		Minimum Development Length, l_d
Type	**No., in. (mm)**	**in. (mm)**
Bars 60 ksi (414 MPa)	3, 0.38 (09.5)	13.5 (343)
	4, 0.50 (12.7)	18.0 (457)
	5, 0.63 (15.9)	22.5 (572)
	6, 0.75 (19.1)	27.0 (686)
	7, 0.88 (22.2)	31.5 (800)
	8, 1.00 (25.4)	36.0 (914)
	9, 1.13 (28.7)	40.6 (1030)
	10, 1.27 (32.3)	45.7 (1160)
	11, 1.41 (35.8)	50.8 (1290)
Wires 70 ksi (483 MPa)	W1.1, 11 Gage (3.1)	min. 6 (152) governs
	W1.7, 9 Gage (3.8)	6.7 (170)
	W2.1, 8 Gage (4.1)	7.3 (185)
	W2.8, 0.188 (4.8)	8.3 (214)
	W4.9, 0.256 (6.4)	11.3 (286)

Shear Reinforcement

Where shear reinforcement is required, it should be spaced so that every potential crack is crossed by shear reinforcement. Shear cracks are assumed to be oriented at a 45 degree angle to the longitudinal axis of the RBM beam. This restricts the spacing of shear reinforcement to one-half the beam's effective depth, d. The spacing of shear reinforcement may be computed by the following equation:

$$s = A_v F_s d / V \qquad \text{Eq. 3}$$

where:

s = spacing of shear reinforcement, in. (mm)
A_v = area of shear reinforcement, in.^{2} (mm^2)
F_s = allowable stress for shear reinforcement, psi (MPa)
d = effective depth of beam, in. (mm)
V = design shear force, 1b (kg)

When shear reinforcement is required, it should be designed to resist the entire shear force. Shear reinforcement should always be placed parallel to the shear force. For RBM beams the shear reinforcement should be placed vertically. It can be difficult to provide shear reinforcement in RBM beams due to the limited size of grout spaces. This is especially the case with hollow brick units 6 in. (150 mm) or less in thickness and grout spaces between wythes less than approximately 2 in. (50 mm) in width. Consequently, it may be advantageous to increase the beam's depth so that shear reinforcement is not necessary. In fact, this is often the method used by designers to determine the minimum depth of a RBM beam required for a given loading.

Ties

There are two instances when it may be necessary to include ties in reinforced brick beams. These instances occur only when the beam is formed by grouting between wythes. If the beam has sufficient depth, ties may be required between the wythes. The grout exerts a hydrostatic pressure that must be resisted during construction. The MSJC requires wall ties between wythes as follows:

Wire size W1.7 (3.8 mm), one tie per 2⅔ ft^2 (0.25 m^2)
Wire size W2.8 (4.8 mm), one tie per 4½ ft^2 (0.42 m^2)
Maximum spacing of 36 in. (914 mm) horizontally and 24 in. (610 mm) vertically
Rectangular or Z ties may be used.

In beams that form deep soffits (large beam widths) it may be advisable to tie the soffit brickwork to the grout. Although the grout does bond to the brick, the metal ties should provide additional capacity and safety. Such ties are placed in the mortar joint and extend into the grout.

DEFLECTION

Deflection of RBM beams is considered a serviceability issue. Excessive deflection might cause damage to interior finishes, functional problems with doors or windows, and cracking of masonry supported by the beam. The MSJC Code requires that the deflection of RBM beams that support unreinforced or empirically-designed masonry should not exceed the lesser of 0.3 in. (7.6 mm) or span length divided by 600. Deflection of RBM beams may be computed based on uncracked or cracked section properties. Use of uncracked sections results in underestimating the deflection. Deflection based on cracked sections only are over-estimated and are more difficult to calculate. Use of uncracked section is recommended.

Creep is a time-dependent property of brick masonry that will cause the deflection of RBM beams to increase over time. An accurate formula for the estimation of long-term deflections of RBM beams due to creep, that is applicable for all cases and easy to use, does not currently exist. A rule-of-thumb is that the long-term deflection of RBM beams due to creep will be approximately 50 percent greater than their instantaneous deflection. This means that a beam that deflects 1.0 in. (25 mm) when it is fully loaded will creep over time such that its final deflection will be approximately 1.5 in. (38 mm).

DESIGN CURVES

Maximum efficiency and safety dictate the need for a rational design of all RBM beams according to the applicable building code. However, it is often helpful for the designer to have design aids that can be used to quickly develop a preliminary beam design. The design curves in Figs. 5-9 are provided for that purpose. The size and configuration of masonry and quantity of reinforcement can be quickly determined from these curves based on the span of the beam and the uniform gravity load supported by the beam, including the beam's self-weight. The curves are based on the following assumptions:

1. Compressive strength of masonry is not less than 2000 psi (14 MPa). For most brick masonry, this value will be exceeded. This value was chosen so that beam capacity was not limited by the masonry's compressive strength.
2. Elastic modulus of masonry is not less than 1600 ksi (11030 MPa).
3. The beam is simply supported and subject to uniform gravity loads only.
4. No compression or shear reinforcement is provided.
5. Deflection is calculated on uncracked section properties. The deflection limit of span length divided by 600 does not govern for span lengths less than 14 ft. (4.3 m).

The effective depth, d, reflected in the design curves is based on the beam height, H, minus a value for masonry cover. The cover value is based on a reasonable approximation of brick, mortar and grout cover on the underside of reinforcement for the beams shown. The actual effective depth should always be checked for each particular RBM beam configuration.

DESIGN EXAMPLE

To illustrate the use of the Design Curves, consider the following example. A RBM beam is to span over a garage door with a clear span of 9 ft (2.7 m). The beam supports its own weight and the weight of the brick masonry wall above the beam, so that the uniform load on the beam is 250 lbs/ft (372 kg/m) of span. The RBM beam and the wall above the beam are nominal 6 in. (150 mm) wide and constructed with hollow brick. Determine the beam depth and reinforcement required for these conditions. From Figs. 5(b) and 5(e), one concludes that a 4 in. (100 mm) or 8 in. (200 mm) high by 6 in. (150 mm) wide RBM beam is not adequate for the given span and loading. Therefore, the applicable Design Curve is Fig. 6(b), which is for a full unit depth, RBM beam. For the given conditions, a minimum depth of 12 in. (300 mm) and one No. 4 bar are required. At this point, any deflection criteria should be considered and may require a greater beam depth.

SUMMARY

RBM beams are an attractive and efficient means of spanning openings. Attention to detailing of reinforcement and proper design are the key aspects addressed in this *Technical Notes*. The most common RBM beam configurations are shown with consideration of the inter-connection of beam and wall elements. Design curves provided in this *Technical Notes* can be used to develop preliminary beam designs for many different applications and loading conditions.

The information and suggestions contained in this *Technical Notes* are based on the available data and the experience of the engineering staff of the Brick Industry Association. The information contained herein must be used in conjunction with good technical judgment and a basic understanding of the properties of brick masonry. Final decisions on the use of the information contained in this *Technical Notes* are not within the purview of the Brick Industry Association and must rest with the project architect, engineer and owner.

REFERENCES

1. *Building Code Requirements for Masonry Structures* (ACI 530/ASCE 5/TMS 402-95), American Society of Civil Engineers, Reston, VA, 1996.
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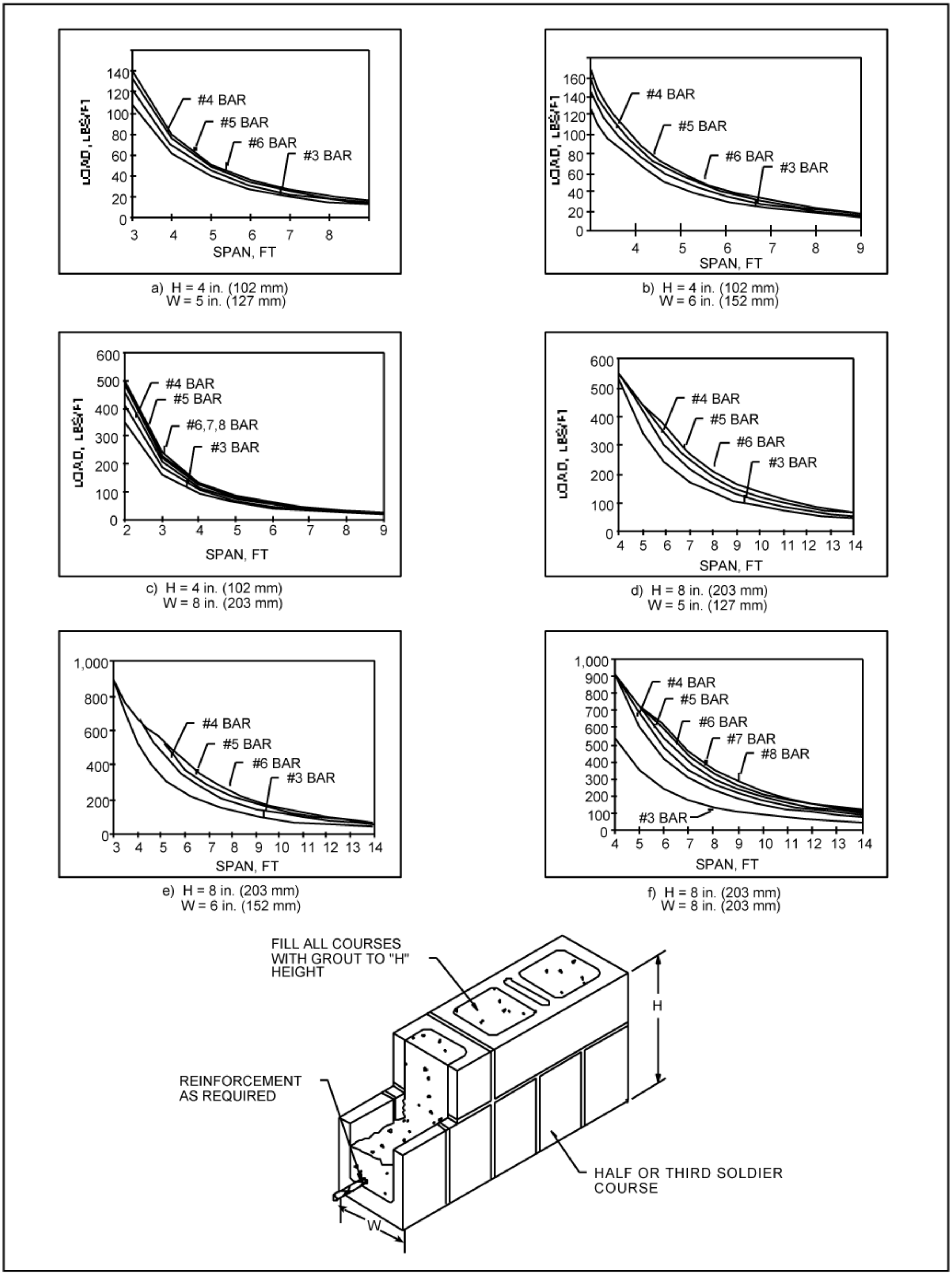
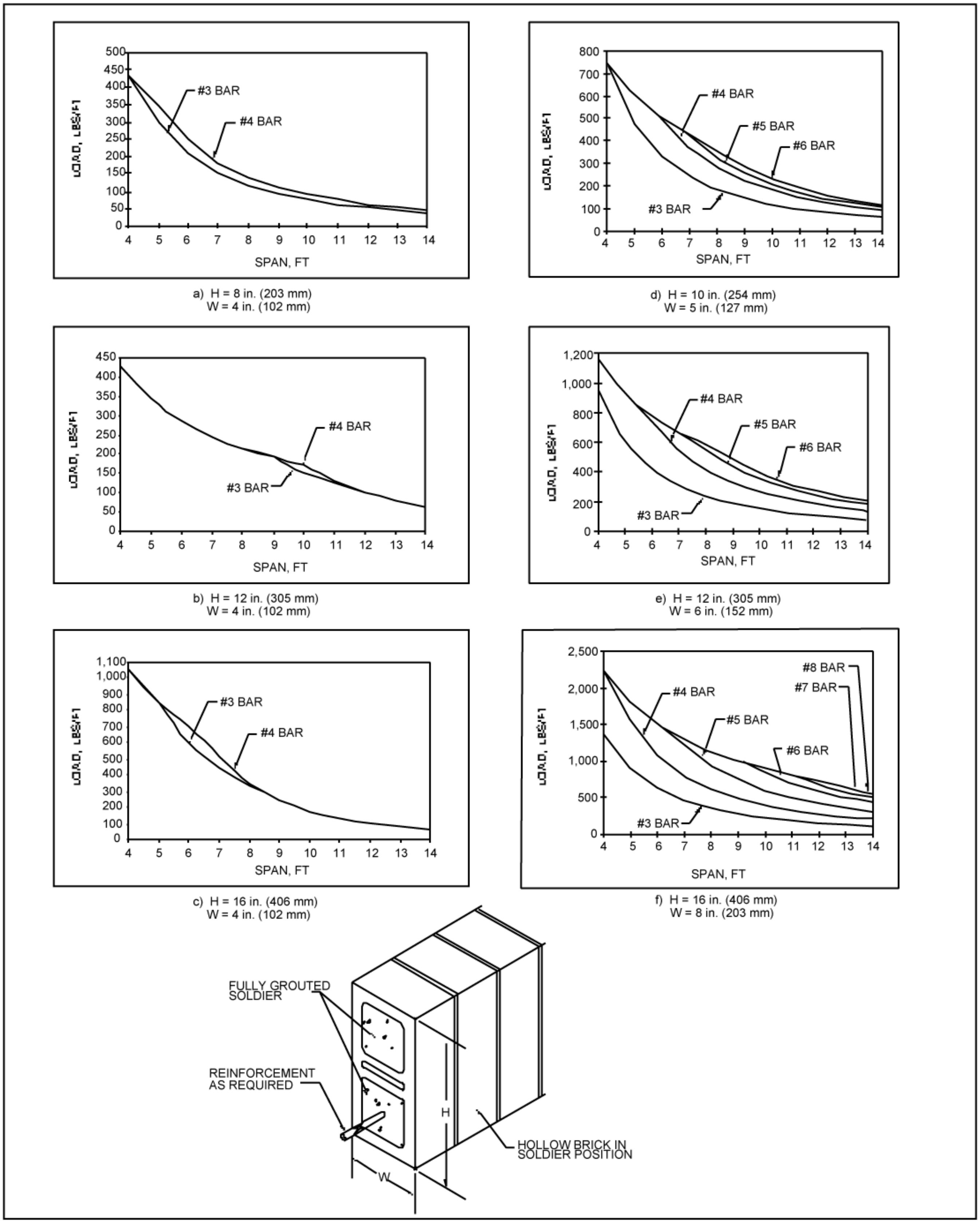
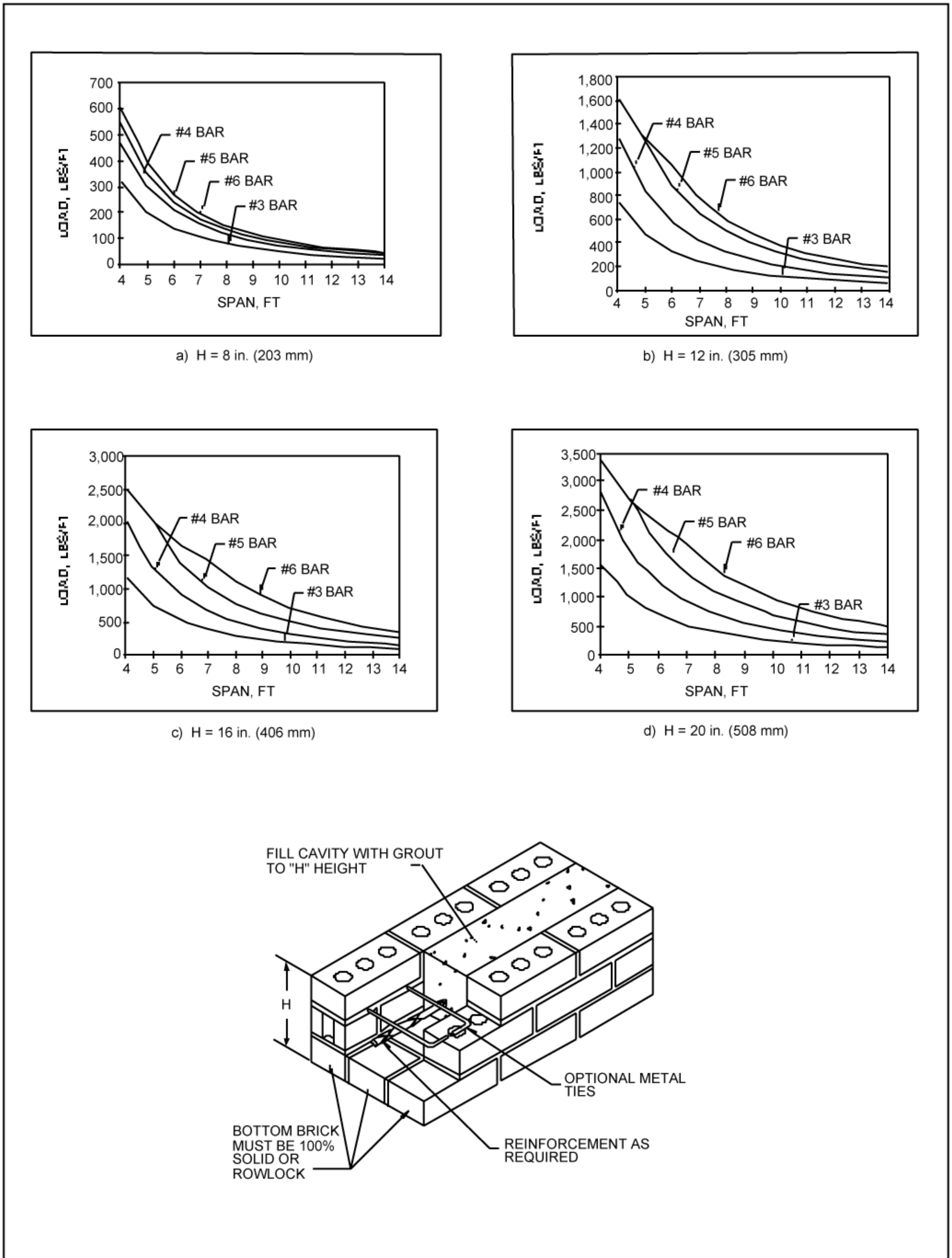


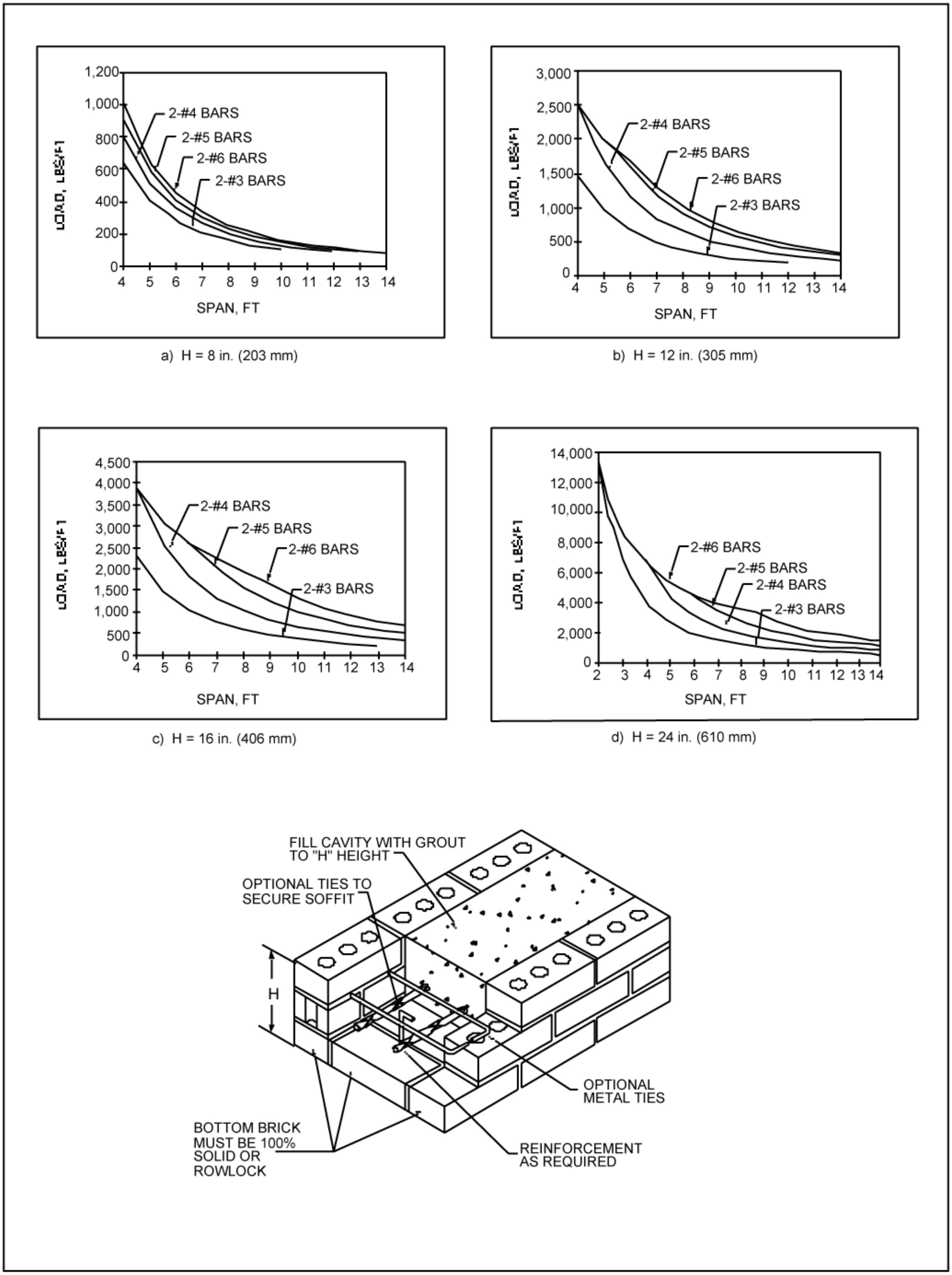
Design Curves for Partial Soldier Course Beams
FIG. 5



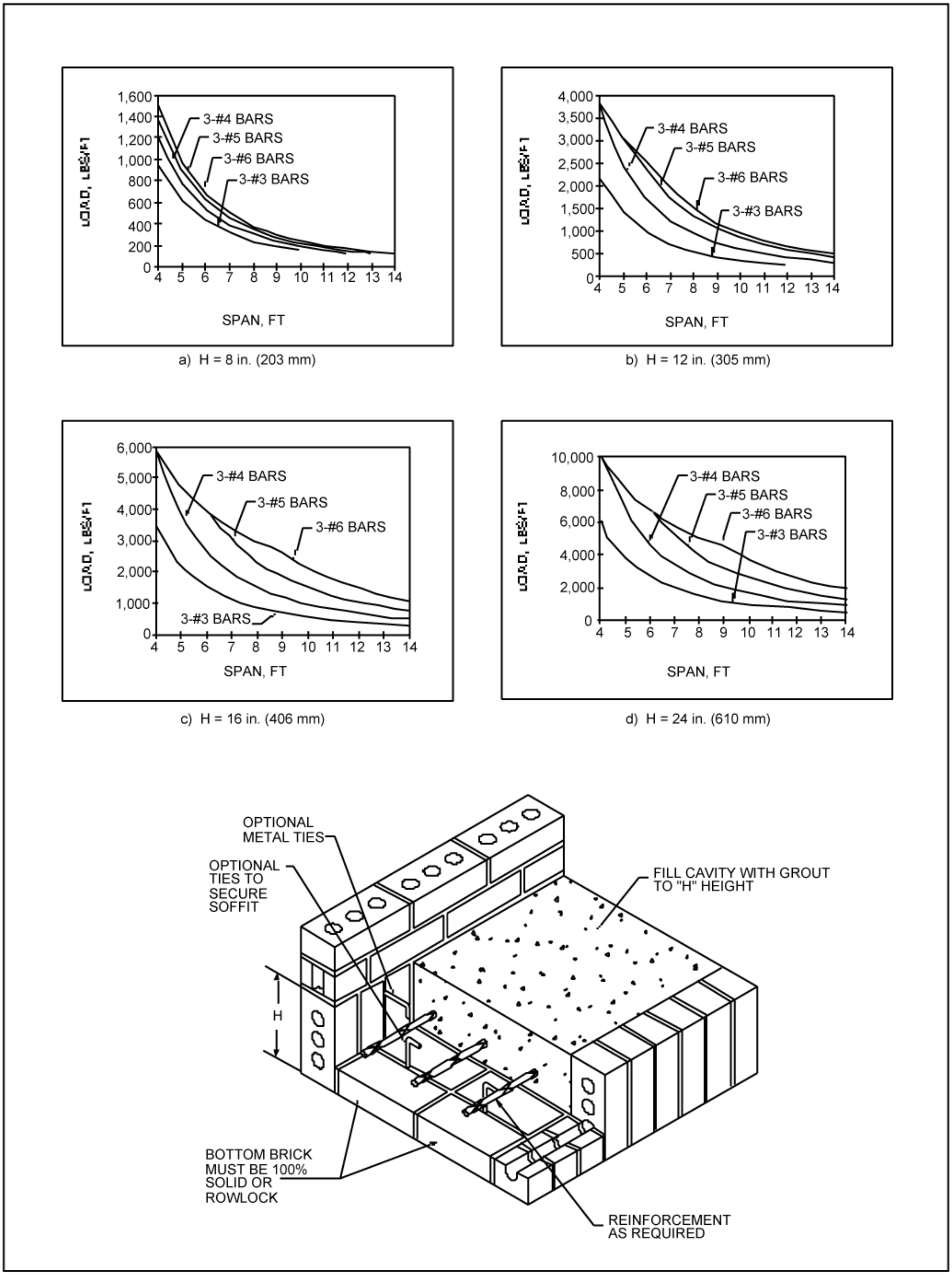
Design Curves for Soldier Course Beams
FIG. 6



Design Curves for 12 in. (305 mm) Wide Beams
FIG. 7



Design Curves for 16 in. (406 mm) Wide Beams
FIG. 8



Design Curves for 24 in. (610 mm) Wide Beams
FIG. 9

Excerpts from NCMA TEK Manual for Concrete Masonry Design and Construction

Section Properties (14-1B 2007)

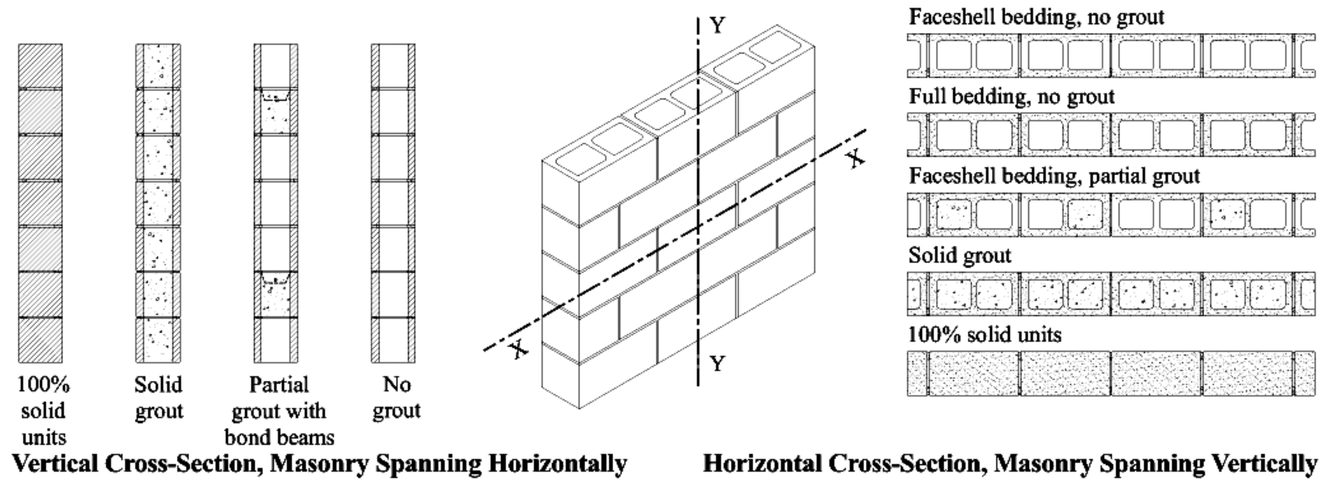


Figure 3—Horizontal and Vertical Cross-Sections

Table for Horizontal Cross Sections (net)

Units	Grouted Spacing	Mortar Bedding	A in^2/ft (10^3mm^2/m)	I_x in^4/ft (10^6 mm^4/m)	S_x in^3/ft (10^6 mm^3/m)	r in (mm)
4 Inch Single Wythe Walls, ¾ in. Face Shells (standard)						
Hollow	No grout	Faceshell	18.0 (38.1)	38.0 (51.9)	21.0 (1.13)	1.45 (36.9)
Hollow	No grout	Full	21.6 (45.7)	39.4 (53.8)	21.7 (1.17)	1.35 (34.3)
100 % solid/grouted		Full	43.5 (92.1)	47.4 (64.7)	26.3 (1.41)	1.04 (26.5)
6 Inch Single Wythe Walls, 1 in. Face Shells (standard)						
Hollow	No grout	Faceshell	24.0 (50.8)	130.3 (178)	46.3 (2.49)	2.33 (59.2)
Hollow	None	Full	32.2 (68.1)	139.3 (190)	49.5 (2.66)	2.08 (52.9)
100% Solid/grouted		Full	67.5 (143)	176.9 (242)	63.3 (3.40)	1.62 (41.1)
Hollow	16" o. c.	Faceshell	46.6 (98.6)	158.1 (216)	55.1 (2.96)	1.79 (45.5)
Hollow	24" o. c.	Faceshell	39.1 (82.7)	151.8 (207)	52.2 (2.81)	1.87 (47.4)
Hollow	32" o. c.	Faceshell	35.3 (74.7)	148.7 (203)	50.7 (2.73)	1.91 (48.5)
Hollow	40" o. c.	Faceshell	33.0 (69.9)	146.8 (200)	49.9 (2.68)	1.94 (49.3)
Hollow	48" o. c.	Faceshell	31.5 (66.7)	145.5 (199)	49.3 (2.65)	1.96 (49.8)
Hollow	72" o. c.	Faceshell	29.0 (61.45)	143.5 (196)	51.0 (2.74)	2.00 (50.8)
Hollow	96" o. c.	Faceshell	27.8 (58.8)	142.4 (194)	50.6 (2.72)	2.02 (51.3)
Hollow	122" o. c.	Faceshell	27.0 (57.1)	141.8 (194)	50.4 (2.71)	2.03 (51.5)
8 Inch Single Wythe Walls, 1 ¼ in. Face Shells (standard)						
Hollow	No grout	Faceshell	30.0 (63.5)	308.7 (422)	81.0 (4.35)	3.21 (81.5)
Hollow	No grout	Full	41.5 (87.9)	334.0 (456)	87.6 (4.71)	2.84 (72.0)
100% solid/grouted		Full	91.5 (194)	440.2 (601)	116.3 (6.25)	2.19 (55.7)
Hollow	16" o. c.	Faceshell	62.0 (131)	387.1 (529)	99.3 (5.34)	2.43 (61.6)
Hollow	24" o. c.	Faceshell	51.3 (109)	369.4 (504)	93.2 (5.01)	2.53 (64.3)
Hollow	32" o. c.	Faceshell	46.0 (97.3)	360.5 (492)	90.1 (4.85)	2.59 (65.8)
Hollow	40" o. c.	Faceshell	42.8 (90.6)	355.2 (485)	88.3 (4.75)	2.63 (66.9)
Hollow	48" o. c.	Faceshell	40.7 (86.0)	351.7 (480)	87.1 (4.68)	2.66 (67.6)
Hollow	72" o. c.	Faceshell	37.1 (78.5)	345.8 (472)	85.0 (4.57)	2.71 (69.0)
Hollow	92" o. c.	Faceshell	35.3 (74.7)	342.8 (468)	89.9 (4.83)	2.74 (69.6)
Hollow	120" o. c.	Faceshell	34.3 (72.6)	341.0 (466)	89.5 (4.81)	2.76 (70.1)

Units	Grouted Cores	Mortar Bedding	A in^2/ft $(10^3 mm^2/m)$	I_x in^4/ft $(10^6\ mm^4/m)$	S_x in^3/ft $(10^6\ mm^3/m)$	r in (mm)
10 Inch Single Wythe Walls, 1 ¼ in. Face Shells (standard)						
Hollow	No grout	Faceshell	30.0 (63.5)	530.0 (724)	110.1 (5.92)	4.20 (107)
Hollow	No grout	Full	48.0 (102)	606.3 (828)	126.0 (6.77)	3.55 (90.2)
100% solid/grouted		Full	115.5 (244)	891.7 (1218)	185.3 (9.96)	2.78 (70.6)
Hollow	16" o. c.	Faceshell	74.8 (158)	744.7 (1017)	154.7 (8.32)	3.04 (77.2)
Hollow	24" o. c.	Faceshell	59.8 (127)	698.6 (954)	145.2 (7.81)	3.16 (80.3)
Hollow	32" o. c.	Faceshell	52.4 (111)	675.5 (923)	140.4 (7.55)	3.24 (82.3)
Hollow	40" o. c.	Faceshell	47.9 (101)	661.6 (904)	137.5 (7.39)	3.29 (83.6)
Hollow	48" o. c.	Faceshell	44.9 (95.0)	652.4 (891)	135.6 (7.29)	3.33 (84.6)
Hollow	72" o. c.	Faceshell	39.9 (84.5)	637.0 (870)	132.4 (7.12)	3.39 (86.1)
Hollow	96" o. c.	Faceshell	37.5 (79.4)	629.3 (859)	130.8 (7.03)	3.43 (87.1)
Hollow	120" o. c.	Faceshell	36.0 (76.2)	624.7 (853)	129.8 (6.98)	3.45 (87.6)
12 Inch Single Wythe Walls. 1 ¼ in. Face Shells (standard)						
Hollow	No grout	Faceshell	30.0 (63.5)	811.2 (1108)	139.6 (7.50)	5.20 (132)
Hollow	No grout	Full	53.1 (112)	971.5 (1327)	167.1 (8.98)	4.28 (109)
100% solid/grouted		Full	139.5 (295)	1571.0 (2145)	270.3 (14.5)	3.36 (85.3)
Hollow	16" o. c.	Faceshell	87.3 (185)	1262.3 (1724)	217.2(11.7)	3.64 (92.5)
Hollow	24" o. c.	Faceshell	68.2 (144)	1165.4 (1591)	200.5 (10.7)	3.79 (96.3)
Hollow	32" o. c.	Faceshell	58.7 (124)	1116.9 (1525)	192.2 (10.3)	3.88 (98.6)
Hollow	40" o. c.	Faceshell	52.9 (112)	1087.8 (1486)	187.2 (10.1)	3.95 (100)
Hollow	48" o. c.	Faceshell	49.1 (104)	1068.4 (1459)	183.8 (9.88)	3.99 (101)
Hollow	72" o. c.	Faceshell	42.7 (90.4)	1036.1 (1415)	178.3 (9.59)	4.07 (103)
Hollow	96" o. c.	Faceshell	39.6 (83.8)	1020.0 (1393)	175.5 (9.44)	4.12 (105)
Hollow	120" o. c.	Faceshell	37.6 (79.6)	1010.3 (1380)	173.8 (9.34)	4.15 (105)
14 Inch Single Wythe Walls. 1 ¼ in. Face Shells (standard)						
Hollow	No grout	Faceshell	30.0 (63.5)	1152.5 (1574)	169.2 (9.09)	6.20 (157)
Hollow	No grout	Full	58.2 (123)	1442.9 (1970)	211.8 (11.4)	4.98 (126)
100% solid/grouted		Full	163.5 (346)	2529.4 (3454)	371.3 (20.0)	3.93 (99.8)
Hollow	16" o. c.	Faceshell	99.9 (211)	1970.0 (2690)	289.2(15.5)	4.25 (108)
Hollow	24" o. c.	Faceshell	76.6 (162)	1794.3 (2450)	263.4 (14.2)	4.41 (112)
Hollow	32" o. c.	Faceshell	64.9 (137)	1706.4 (2330)	250.5 (13.5)	4.51 (115)
Hollow	40" o. c.	Faceshell	58.0 (123)	1653.7 (2258)	242.8 (13.0)	4.59 (117)
Hollow	48" o. c.	Faceshell	53.3 (113)	1618.6 (2210)	237.6 (12.8)	4.64 (118)
Hollow	72" o. c.	Faceshell	45.5 (96.3)	1560.0 (2130)	229.0 (12.3)	4.74 (120)
Hollow	96" o. c.	Faceshell	41.6 (88.1)	1530.7 (2090)	224.7 (12.1)	4.79 (122)
Hollow	120" o. c.	Faceshell	39.3 (83.2)	1513.2 (2067)	221.1 (11.9)	4.83 (123)
16 Inch Single Wythe Walls. 1 ¼ in. Face Shells (standard)						
Hollow	No grout	Faceshell	30.0 (63.5)	1553.7 (2122)	198.9 (10.2)	7.20 (183)
Hollow	No grout	Full	63.2 (134)	2030.6 (2773)	259.9 (13.9)	5.67 (144)
100% solid/grouted		Full	187.5 (397)	3814.7 (5209)	488.3 (26.3)	4.51 (115)
Hollow	16" o. c.	Faceshell	112.4 (238)	2896.2 (3955)	370.7(19.9)	4.84 (123)
Hollow	24" o. c.	Faceshell	85.0 (180)	2607.7 (3561)	333.8 (17.9)	5.02 (127)
Hollow	32" o. c.	Faceshell	71.2 (151)	2463.4 (3364)	315.3 (17.0)	5.14 (131)
Hollow	40" o. c.	Faceshell	63.0 (133)	2376.9 (3246)	304.2 (16.4)	5.22 (133)
Hollow	48" o. c.	Faceshell	57.5 (122)	2319.1 (3167)	296.9 (16.0)	5.28 (134)
Hollow	72" o. c.	Faceshell	48.3 (102)	2223.0 (3036)	284.5 (15.3)	5.39 (137)
Hollow	96" o. c.	Faceshell	43.7 (92.5)	2174.9 (3970)	278.4 (15.0)	5.45 (138)
Hollow	120" o. c.	Faceshell	41.0 (86.8)	2146.0 (2931)	274.7 (14.8)	5.49 (139)

Allowable Stresses for Unreinforced Concrete Masonry (14-7C 2012)

Compression

Axial ....$F_a = 1/4\, f'_m[1-(h/140r)^2]$, where h/r ≥99

....$F_a = 1/4\, f'_m\,(70r/h)^2$, where h/r > 99

Flexural ....$F_b = 1/3\, f'_m$

Shear

where $f_v = \dfrac{VQ}{I_n b}$

$1.5\sqrt{f'_m} \le 120$ psi

Table 1—Allowable Flexural Tensile Stresses, psi (kPa) (ref. 1a)

Direction of flexural tensile stress and masonry type	Mortar types			
	Portland cement/ lime or mortar cement		Masonry cement or air-entrained portland cement/lime	
	M or S	N	M or S	N
Normal to bed joints:				
Solid units	53 (366)	40 (276)	32 (221)	20 (138)
Hollow units[A]				
Ungrouted	33 (228)	25 (172)	20 (138)	12 (83)
Fully grouted	86 (593)	84 (579)	81 (559)	77 (531)
Parallel to bed joints in running bond:				
Solid units	106 (731)	80 (552)	64 (441)	40 (276)
Hollow units				
Ungrouted & partially grouted	66 (455)	50 (345)	40 (276)	25 (172)
Fully grouted	106 (731)	80 (552)	64 (441)	40 (276)
Parallel to bed joints in masonry not laid in running bond:				
Continuous grout section parallel to bed joints	133 (917)	133 (917)	133 (917)	133 (917)
Other	0 (0)	0 (0)	0 (0)	0 (0)

[A] For partially grouted masonry, allowable stresses are determined on the basis of linear interpolation between fully grouted hollow units and ungrouted hollow units based on amount (percentage) of grouting.

Allowable Stresses for Reinforced Concrete Masonry (14-7C 2012)

Compression

Axial .... $P_a = \left[0.25 f'_m A_n + 0.65 A_{st} F_s\right]\left[1-\left(\dfrac{h}{140r}\right)^2\right]$, where h/r ≥99

.... $P_a = \left[0.25 f'_m A_n + 0.65 A_{st} F_s\right]\left(\dfrac{70r}{h}\right)^2$, where h/r > 99

Flexural ....$F_b = 0.45\, f'_m$

Shear

where $f_v = \dfrac{V}{A_{nv}}$ and $F_v = F_{vm} + F_{vs}$

$M/Vd \leq 0.25$..................$F_v = 3\sqrt{f'_m}$

$M/Vd \geq 1.0$....................$F_v = 2\sqrt{f'_m}$

M/Vd falls between........may be linearly interpolated

and

$$F_{vm} = \frac{1}{2}\left[\left(4.0 - 1.75\left(\frac{M}{Vd}\right)\right)\sqrt{f'_m}\right] + 0.25\frac{P}{A_n}$$

$$F_{vs} = 0.5\left(\frac{A_v F_s d}{A_n s}\right)$$

Steel Reinforcement

Tension

Grade 40...................... F_s = 20,000 psi (137.9 MPa)
Grade 60...................... F_s = 32,000 psi (220.7 MPa)
Joint reinforcement..... F_s = 30,000 psi (206.9 MPa)

NOTATIONS

A_n net cross-sectional area of masonry, in.^{2} (mm^2)
A_{nv} net shear area, in.^{2} (mm^2)
A_v cross-sectional area of shear reinforcement, in.^{2} (mm^2)
b width of section, in. (mm)
d distance from extreme compression fiber to centroid of tension reinforcement, in. (mm)
F_a allowable compressive stress due to axial load only, psi (MPa)
F_b allowable compressive stress due to flexure only, psi (MPa)
F_s allowable tensile or compressive stress in reinforcement, psi (MPa)
F_v allowable shear stress in masonry, psi (MPa)
F_{vm} allowable shear resisted by the masonry, psi (MPa)
F_{vx} allowable shear resisted by the shear reinforcement, psi (MPa)
f'_m specified compressive strength of masonry, psi (MPa)
f_s calculated shear stress in the masonry, psi (MPa)
h effective height of column, wall, or pilaster, in. (mm)
I_n moment of inertia of net cross-sectional area of a member, in.^{4} (mm^4)
M maximum moment occurring simultaneously with design shear force, V, at section under consideration, in.-lb (N.m)
P axial compression load, lb (N)
P_a allowable axial compressive force in a reinforced member, lb (N)
Q first moment of inertia about the neutral axis of an area between the extreme fiber and the plane at which the shear stress is being calculated, in.^{3}(mm^3)
r radius of gyration, in. (mm)
s spacing of shear reinforcement, in. (mm)
V design shear force, lb (N)

Table of Reference Figures and Charts