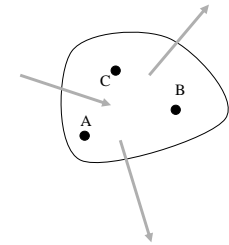lecture
five

CAPSTONE DESIGN

rigid body equilibrium

Equilibrium

- *rigid body*
  - *doesn't deform*
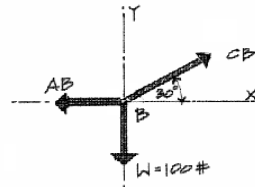 - *coplanar force systems*

- *static:*

$$R_x = \sum F_x = 0$$

$$R_y = \sum F_y = 0$$

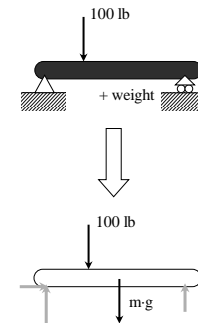$$\boxed{M = \sum M = 0}$$

Free Body Diagram

- *FBD (sketch)*
- *tool to see all forces on a body or a point including*
  - *external forces*
  - *weights*
  - *force reactions*
  - *external moments*
  - *moment reactions*
  - *internal forces*

Free Body Diagram

- *determine body*
- *FREE it from:*
  - *ground*
  - *supports & connections*

- *draw all external forces acting ON the body*
  - *reactions*
 - *applied forces*
  - *gravity*
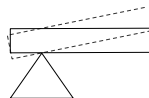
1

Free Body Diagram

- *sketch FBD with relevant geometry*
- *resolve each force into components*
  - *known & unknown <u>angles</u> – name them*
  - *known & unknown <u>forces</u> – name them*
  - *known & unknown <u>moments</u> – name them*
- *are any forces related to other forces?*
- *for the unknowns*
- *write only as many equilibrium equations as needed*
- *solve up to 3 equations*

Free Body Diagram

- *solve equations*
  - *most times 1 unknown easily solved*
  - *plug into other equation(s)*

- *common to have unknowns of*
  - *force magnitudes*
  - *force angles*
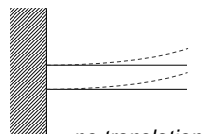 - *moment magnitudes*

Reactions on Rigid Bodies

- *result of applying force*
- *unknown size*
- *connection or support type*
  - *known direction*
  - *related to motion prevented*
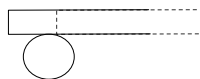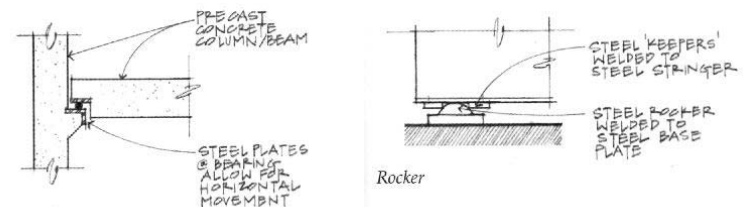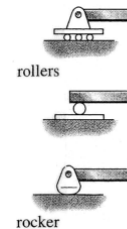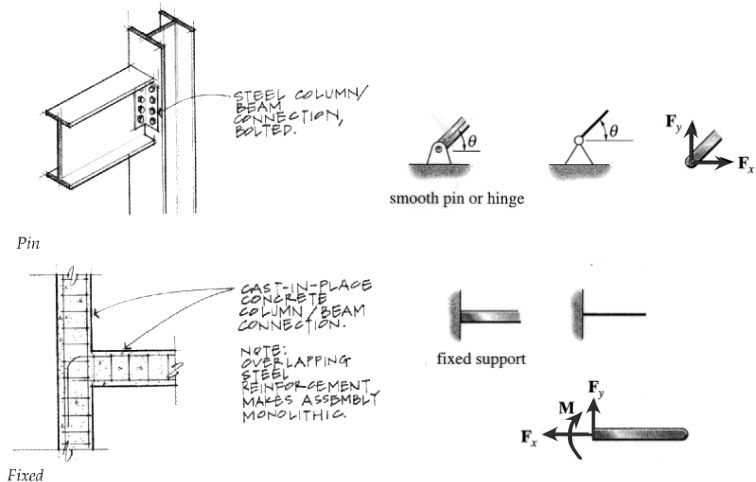

no vertical motion   *no translation*   *no translation no rotation*

Supports and Connections

Supports and Connections

STEEL COLUMN/BEAM CONNECTION, BOLTED.

Pin

smooth pin or hinge

F_y F_x θ

CAST-IN-PLACE CONCRETE COLUMN/BEAM CONNECTION.

NOTE: OVERLAPPING STEEL REINFORCEMENT MAKES ASSEMBLY MONOLITHIC.

fixed support

Fixed

M F_y F_x

FBD Example

- *500 lb known*
- *pin – A_x, A_y*
- *smooth surface – B at 4:3*
- *3 equations*
- *sum moments at*
  - *A?*
  - *B?   (B_x)*
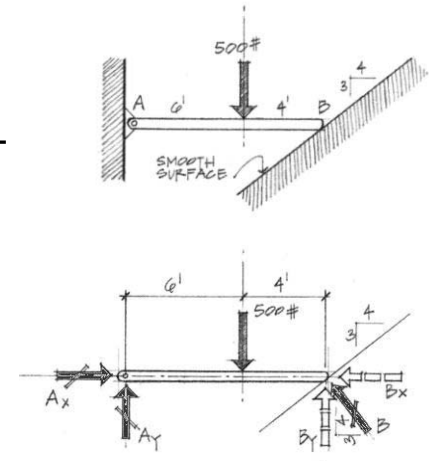
Moment Equations

- *sum moments at intersection where the most forces intersect*
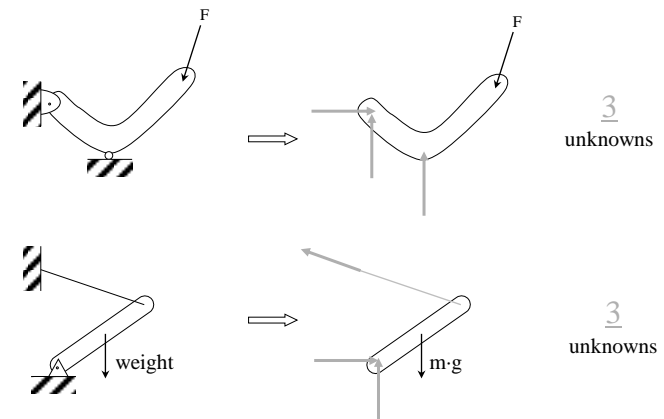- *multiple moment equations may not be useful*
- *combos:*

$$\sum F_x = 0 \qquad \sum F = 0 \qquad \sum M_1 = 0$$
$$\sum F_y = 0 \qquad \sum M_1 = 0 \qquad \sum M_2 = 0$$
$$\sum M_1 = 0 \qquad \sum M_2 = 0 \qquad \sum M_3 = 0$$

Recognizing Reactions

F   F
                      $\underline{3}$
                      unknowns

weight   m·g
                      $\underline{3}$
                      unknowns

Recognizing Reactions

<u>3</u>
unknowns

weight

F_1 F_2

F_1 F_2

not independent

<u>6</u>
unknowns for
2 bodies

weight
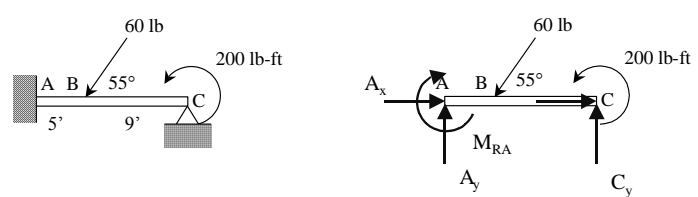
m·g

<u>2</u>
unknowns
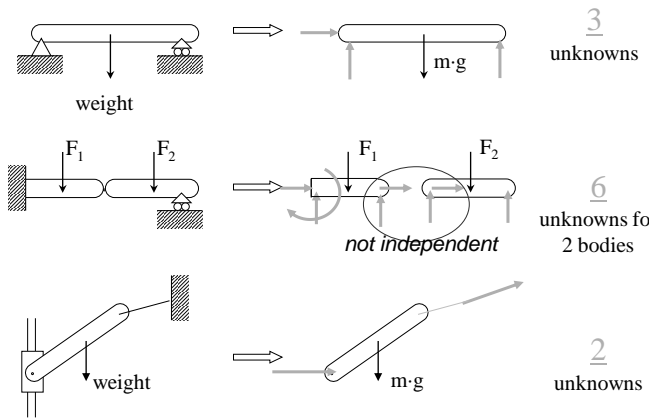
Constraints

- *completely constrained*
  - *doesn't move*
  - *may not be statically determinate*
- *improperly or partially constrained*
  - *has $\leq$ unknowns*
  - *can't solve*
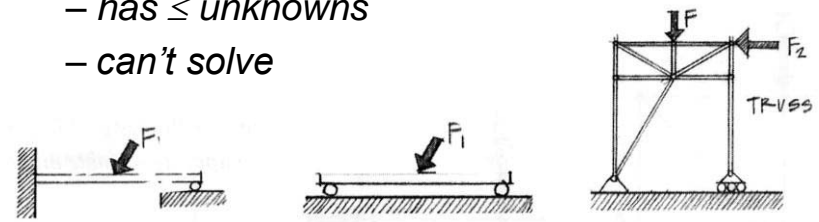
Constraints

- *overconstrained*
  - *won't move*
  - *can't be solved with statics*
  - *statically indeterminate to n^{th} degree*

Partial Constraints
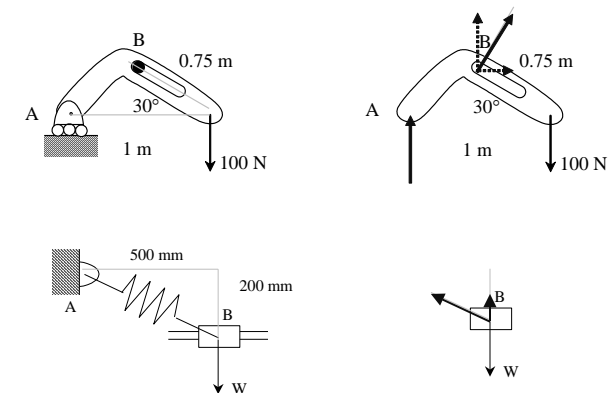
Cable Reactions

- *equilibrium:*
  - *more reactions (4) than equations*
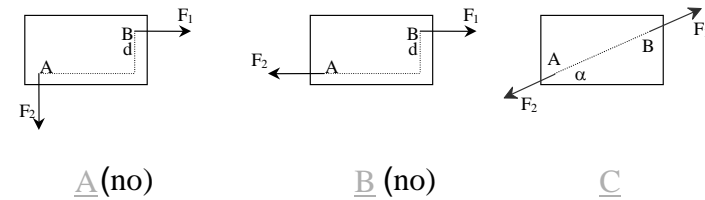 - *but, we have slope relationships*
  - *x component the same everywhere*
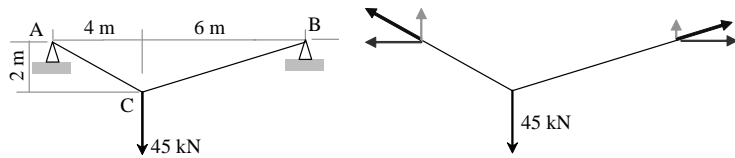
Two Force Rigid Bodies

- *equilibrium:*
  - *forces in line, equal and opposite*

A (no)   B (no)   C

Three Force Rigid Bodies

- *equilibrium:*
  - *concurrent or parallel forces*
    beams!
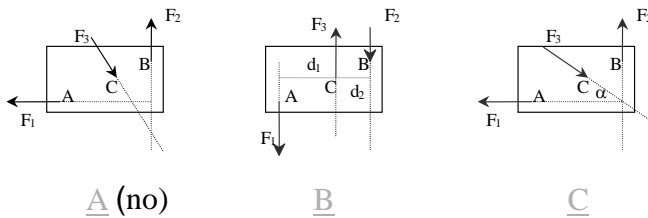
A (no)   B   C

Concentrated Loads

Distributed Loads

Beam Supports

- *statically determinate*

simply supported
(most common)
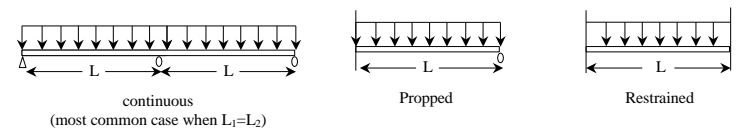
overhang

cantilever

- *statically indeterminate*

continuous
(most common case when $L_1=L_2$)

Propped

Restrained

Equivalent Force Systems

- *replace forces by resultant*
- *place resultant where M = 0*
- *using calculus and area centroids*

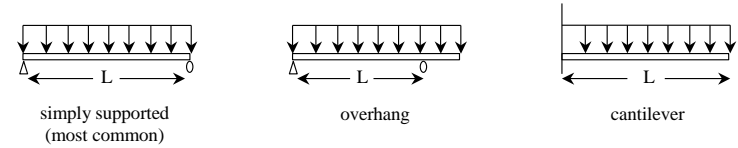$$W = \int_0^L w\,dx = \int dA_{loading} = A_{loading}$$

Load Areas

- *area is width x "height" of load*
- *w is load per unit length*
- *W is total load*
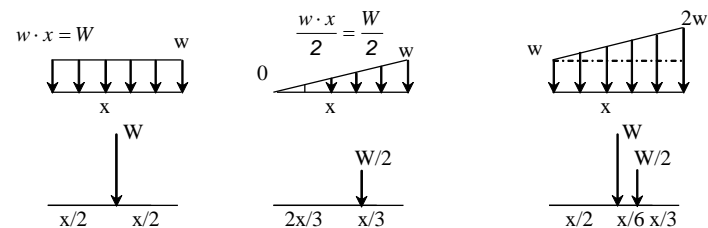
Method of Sections

- *relies on internal forces being in equilibrium on a section*

- *cut to expose <u>3 or less</u> members*

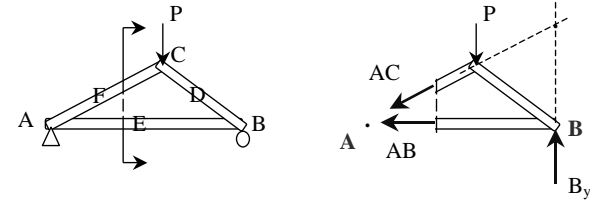- *coplanar forces $\rightarrow \sum M = 0$ too*
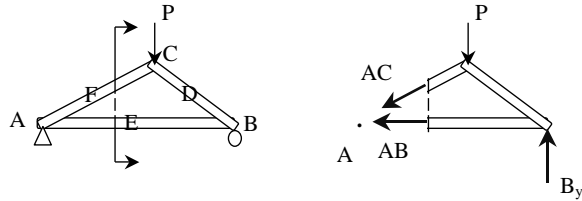
Method of Sections

- *joints on or off the section are good to sum moments*

- *quick for few members*

- *not always obvious where to cut or sum*