

ARCHITECTURAL STRUCTURES: FORM, BEHAVIOR, AND DESIGN

ARCH 331

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FALL 2013

lecture
nine

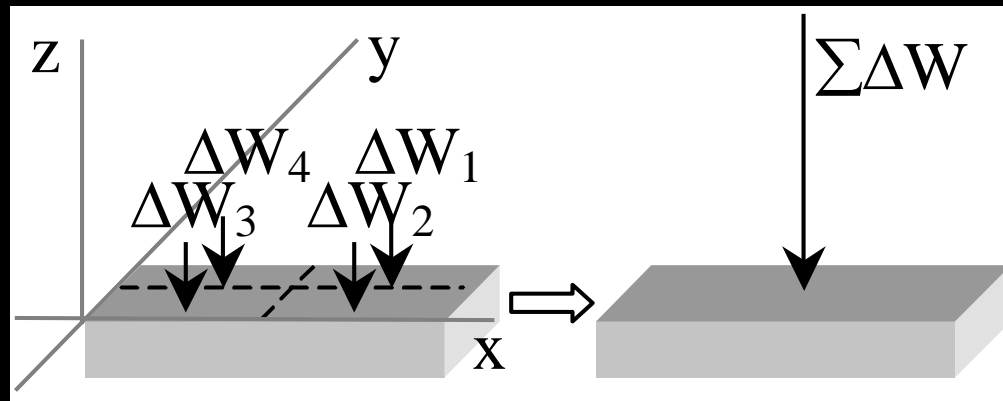
beam sections - geometric properties



Professional Personnel
Professors
Quill-Georgio
Bill Clay Rafter
Dier and Jay Brown
Helen King
Walter and Jane Schwartz
Arthur and Helen Brown
Carroll and Sandra Galtier
Sergiy and Karlyk Shyn
Harold and Doris Stangl
Paul and Doreen Stevens
Klaus and Patti Jones
Peter and Carolyn Smith
Suzanne University School
Langmuir University School
Lyn and Linda Taylor
Early Thompson
Therese Thompson
Therese Thompson
Thompson University School
Gregory and Karen Taylor
Mary and Karen Taylor
John and Karen Taylor
Tom and Karen Taylor
Wendy and Karen Taylor

Center of Gravity

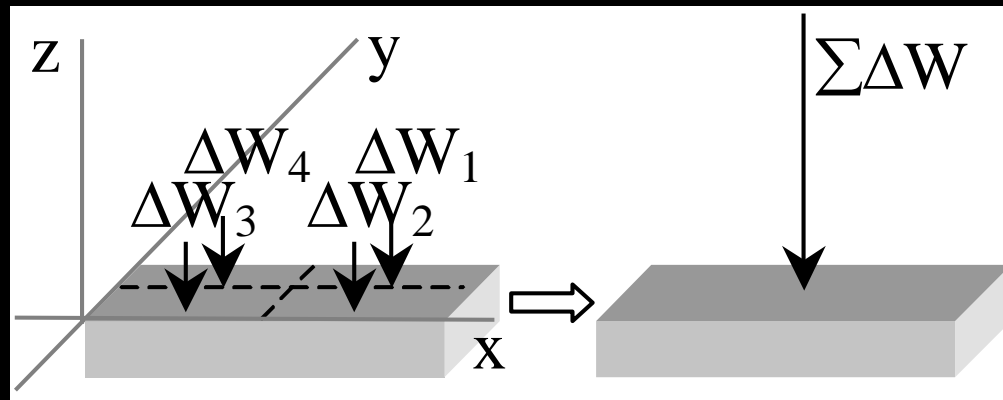
- *location of equivalent weight*
- *determined with calculus*



- *sum element weights* $W = \int dW$

Center of Gravity

- “average” x & y from moment



$$\sum M_y = \sum_{i=1}^n x_i \Delta W_i = \bar{x} W \Rightarrow \bar{x} = \frac{\sum (x \Delta W)}{W}$$

“bar” means average

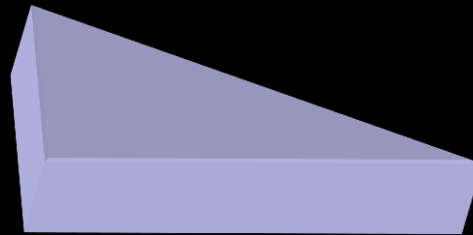
$$\sum M_x = \sum_{i=1}^n y_i \Delta W_i = \bar{y} W \Rightarrow \bar{y} = \frac{\sum (y \Delta W)}{W}$$

Centroid

- “average” x & y of an area
- for a volume of constant thickness
 - $\Delta W = \gamma t \Delta A$ where γ is weight/volume
 - center of gravity = centroid of area

$$\bar{x} = \frac{\sum(x\Delta A)}{A}$$

$$\bar{y} = \frac{\sum(y\Delta A)}{A}$$

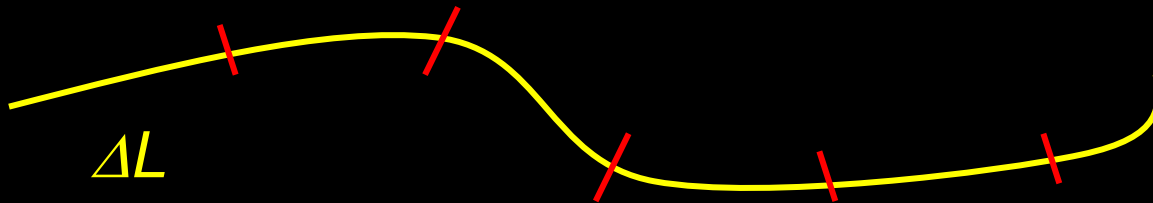


Centroid

- for a line, sum up length

$$\bar{x} = \frac{\sum(x\Delta L)}{L}$$

$$\bar{y} = \frac{\sum(y\Delta L)}{L}$$

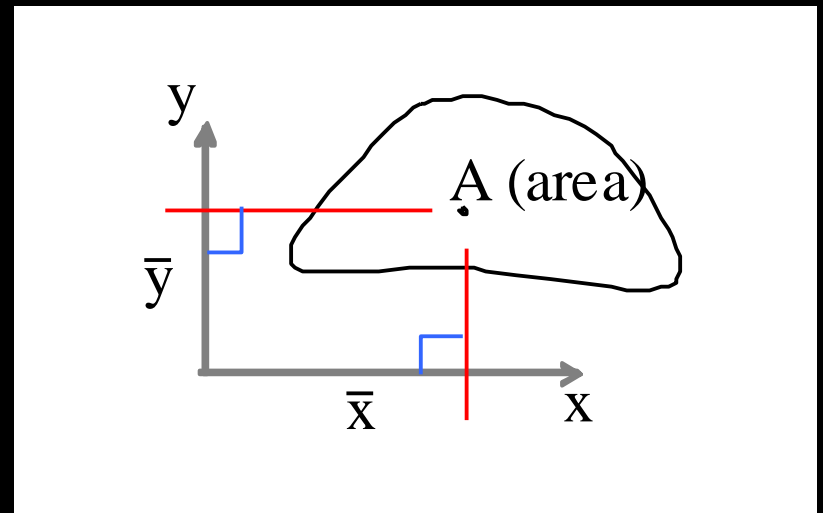


1st Moment Area

- *math concept*
- *the moment of an area about an axis*

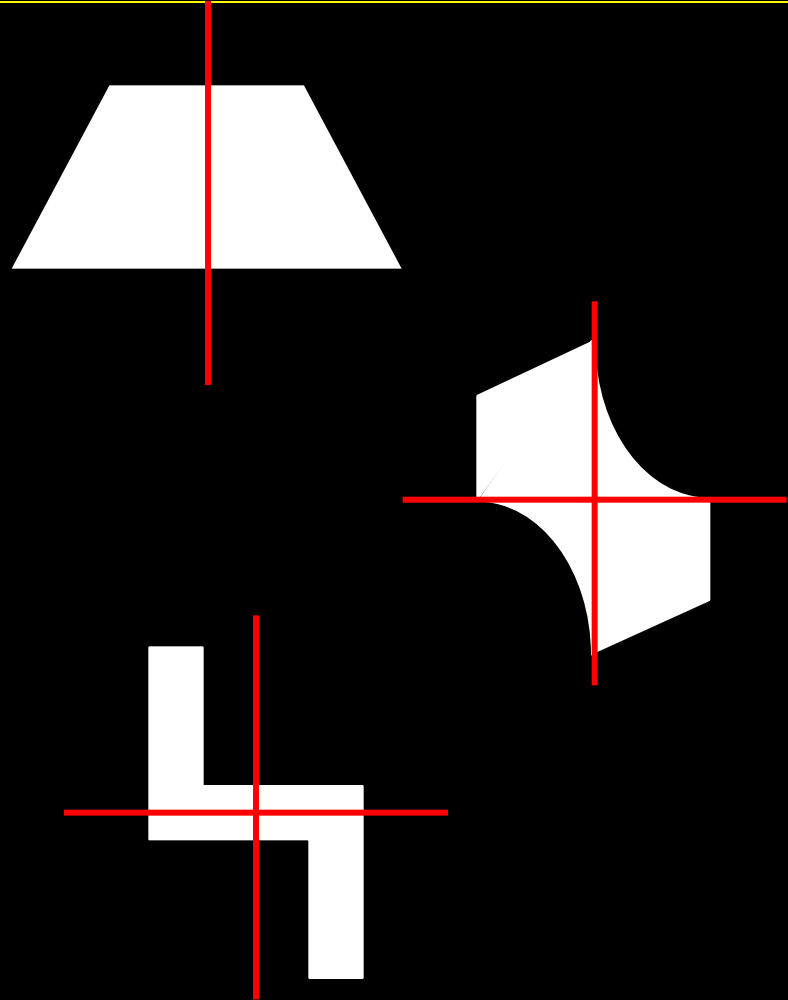
$$Q_x = \bar{y}A$$

$$Q_y = \bar{x}A$$



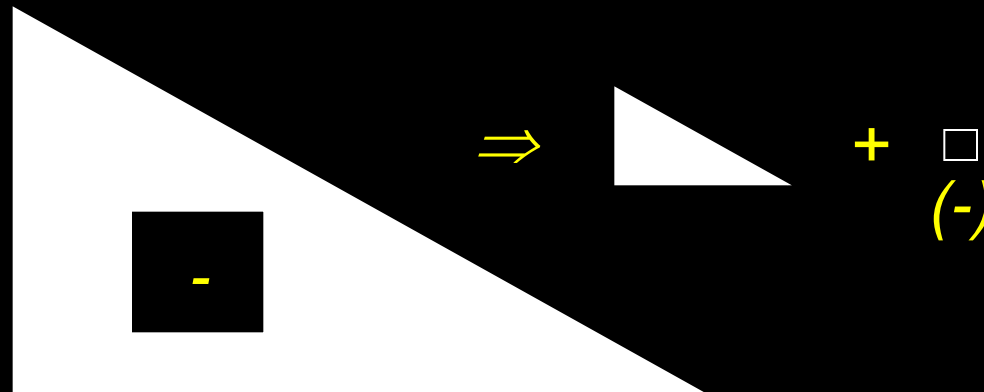
Symmetric Areas

- *symmetric about an axis*
- *symmetric about a center point*
- *mirrored symmetry*



Composite Areas

- *made up of basic shapes*
- *areas can be negative*
- *(centroids can be negative for any area)*



Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate \hat{x} and \hat{y}

Component	Area	\bar{x}	$\bar{x}A$	\bar{y}	$\bar{y}A$
Σ					

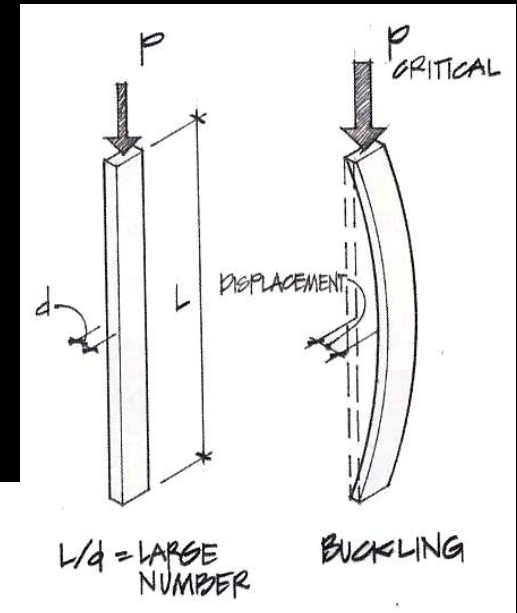
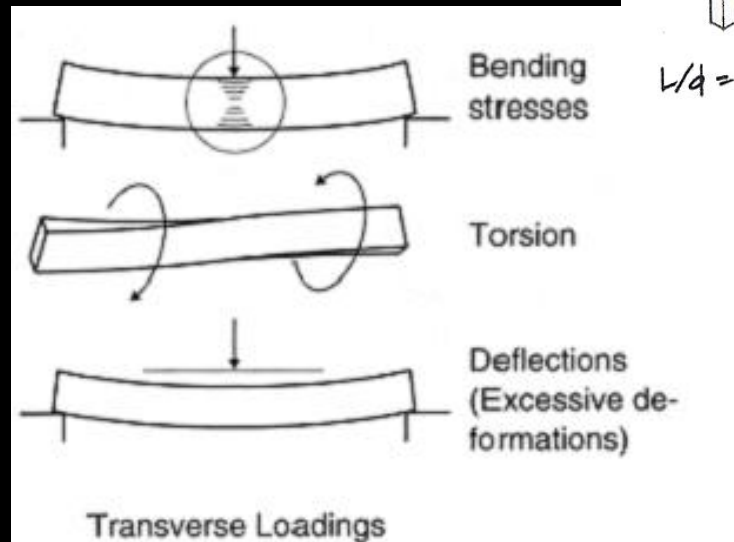
Area Centroids

- *Table 7.1 – pg. 242*

Centroids of Common Shapes of Areas and Lines			
Shape		\bar{x}	\bar{y}
Triangular area		$\frac{b}{3}$	$\frac{h}{3}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Semicircular area		0	$\frac{4r}{3\pi}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$
Parabolic area		0	$\frac{3h}{5}$

Moments of Inertia

- 2nd moment area
 - math concept
 - area \times (distance)²
- need for behavior of
 - beams
 - columns

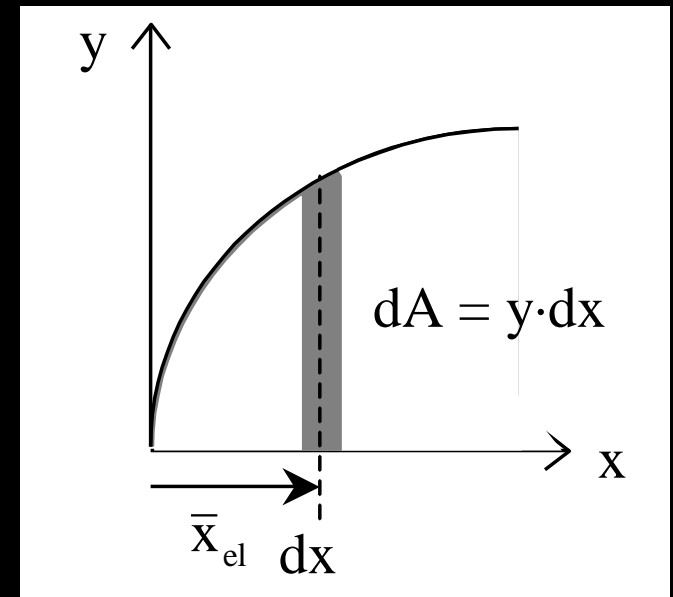


Moment of Inertia

- about any reference axis
- can be negative

$$I_y = \int x^2 dA$$

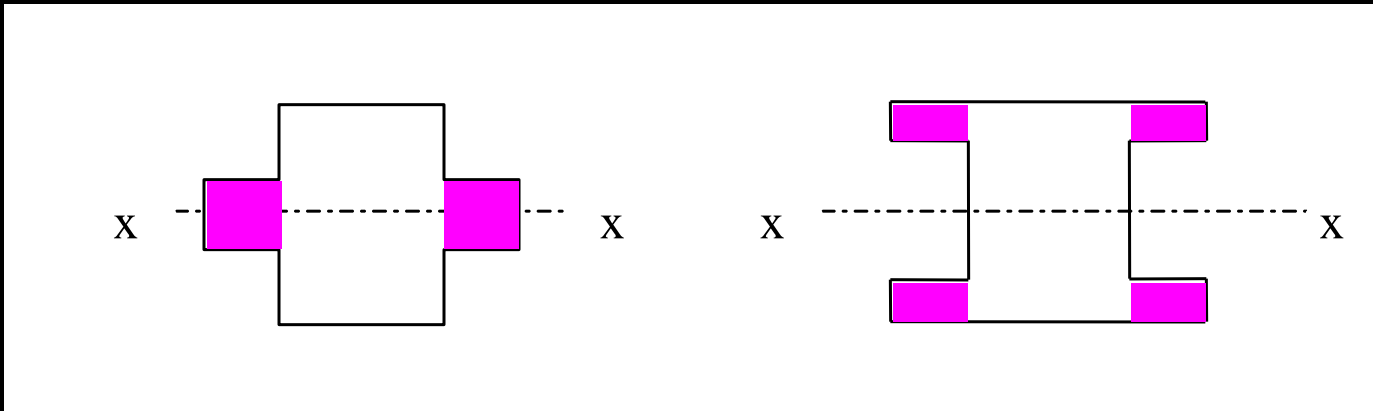
$$I_x = \int y^2 dA$$



- resistance to bending and buckling

Moment of Inertia

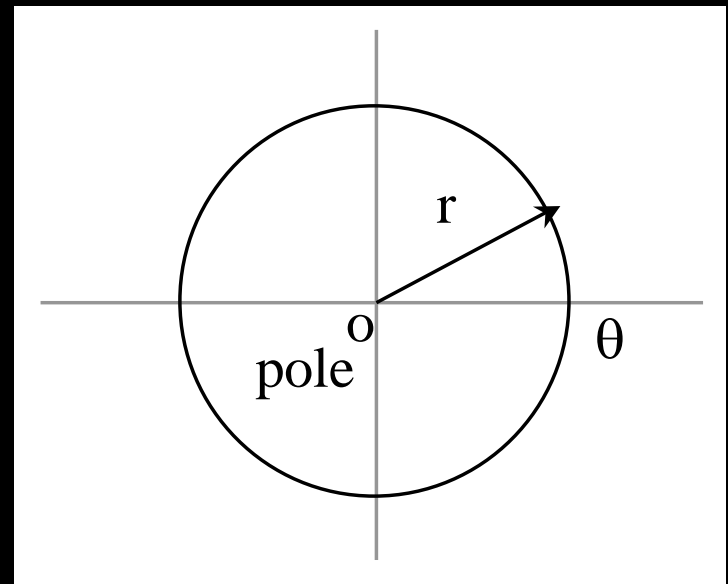
- *same area moved away a distance*
– *larger I*



Polar Moment of Inertia

- *for roundish shapes*
- *uses polar coordinates (r and θ)*
- *resistance to twisting*

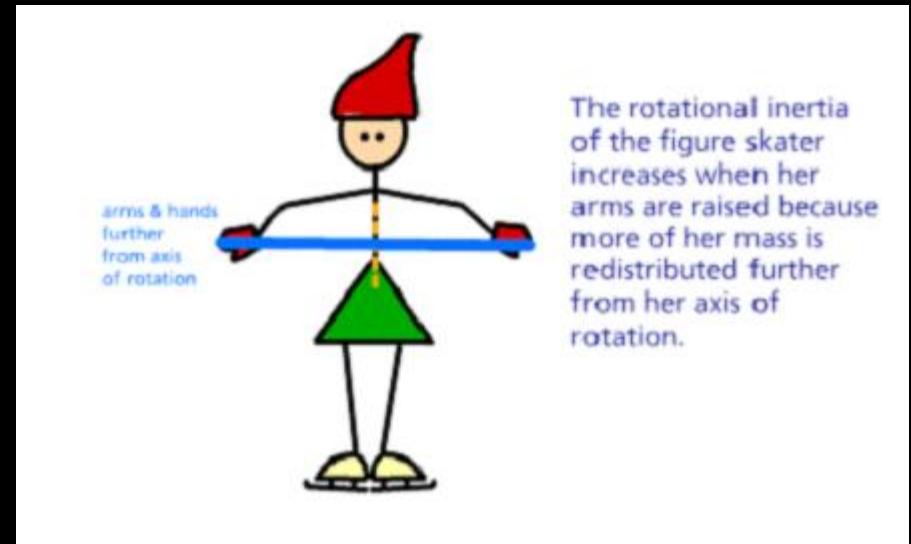
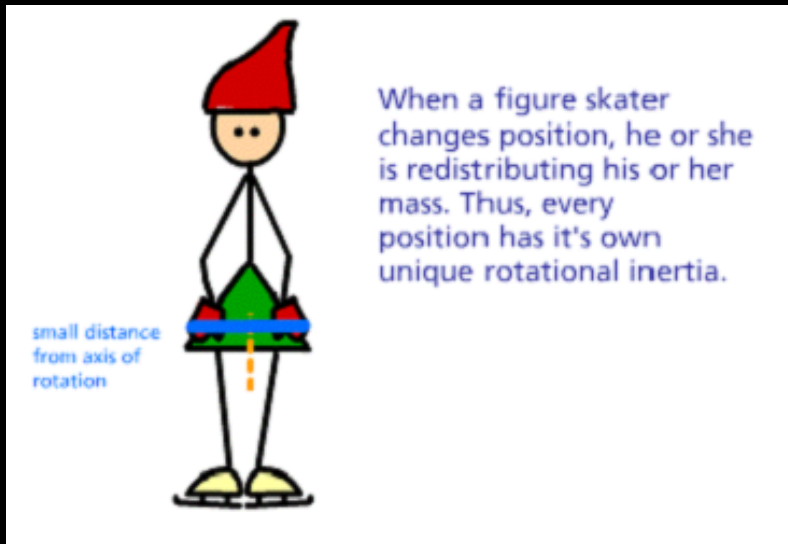
$$J_o = \int r^2 dA$$



Radius of Gyration

- *measure of inertia with respect to area*

$$r_x = \sqrt{\frac{I_x}{A}}$$



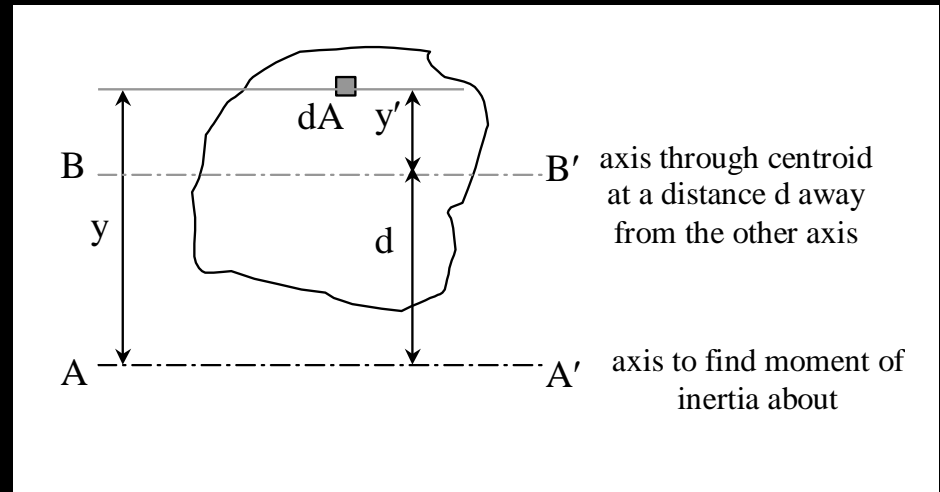
Parallel Axis Theorem

- can find composite I once composite centroid is known (basic shapes)

$$\begin{aligned} I_x &= I_{cx} + Ad_y^2 \\ &= \bar{I}_x + Ad_y^2 \end{aligned}$$

$$I = \sum \bar{I} + \sum Ad^2$$

$$\bar{I} = I - Ad^2$$



Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with A , \bar{x} , $\bar{x}A$, \bar{y} , $\bar{y}A$, \bar{I} 's, d 's, and Ad^2 's
5. Fill in table and get \hat{x} and \hat{y} for composite
6. Sum necessary columns
7. Sum \bar{I} 's and Ad^2 's

$$\begin{pmatrix} d_x \\ d_y \end{pmatrix} = \begin{pmatrix} \hat{x} - \bar{x} \\ \hat{y} - \bar{y} \end{pmatrix}$$

Area Moments of Inertia

- *Table 7.2 – pg. 252: (bars refer to centroid)*

– x, y

– x', y'

– C

Rectangle		$\bar{I}_x = \frac{1}{12}bh^3$ $\bar{I}_y = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_x = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$