ARCHITECTURAL STRUCTURES: FORM, BEHAVIOR, AND DESIGN

ARCH 331

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FALL 2013

lecture
three

forces and moments

Structural Math

- *quantify environmental loads*
  - *how big is it?*
- *evaluate geometry and angles*
  - *where is it?*
  - *what is the scale?*
  - *what is the size in a particular direction?*
- *quantify what happens in the structure*
  - *how big are the internal forces?*
  - *how big should the beam be?*

Structural Math

- *physics takes observable phenomena and relates the measurement with rules: mathematical relationships*
- *need*
  - *reference frame*
  - *measure of length, mass, time, direction, velocity, acceleration, work, heat, electricity, light*
  - *calculations & geometry*

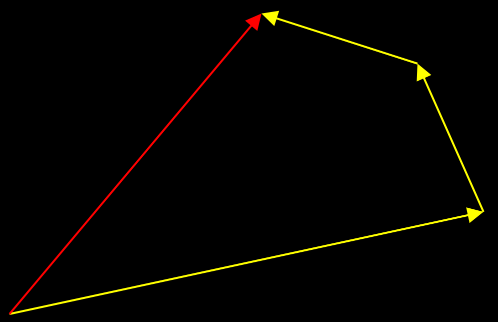
Physics for Structures

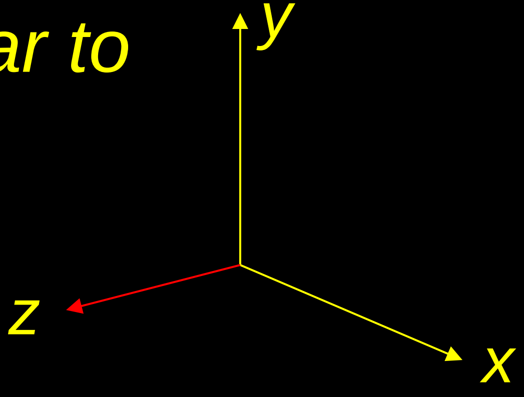
- *measures*
  - *US customary & SI*

Units	*US*	*SI*
Length	*in, ft, mi*	*mm, cm, m*
Volume	*gallon*	*liter*
Mass	*lb mass*	*g, kg*
Force	*lb force*	*N, kN*
Temperature	*F*	*C*

Physics for Structures

- *scalars – any quantity*
- *vectors - quantities with direction*
  - *like displacements*
  - *summation results in the "straight line path" from start to end*
  - *<u>normal</u> vector is perpendicular to something*





Language

- symbols for operations: +,-, /, x
- symbols for relationships: (), =, <, >
- algorithms
  - cancellation
  - factors
  - signs
  - ratios and proportions
  - power of a number
  - conversions, ex. 1X = 10 Y
  - operations on <u>both sides</u> of equality

$$\frac{2}{\cancel{5}}\times\frac{\cancel{5}}{6}=\frac{2}{6}=\frac{\cancel{2}}{\cancel{2}\times 3}=\frac{1}{3}$$

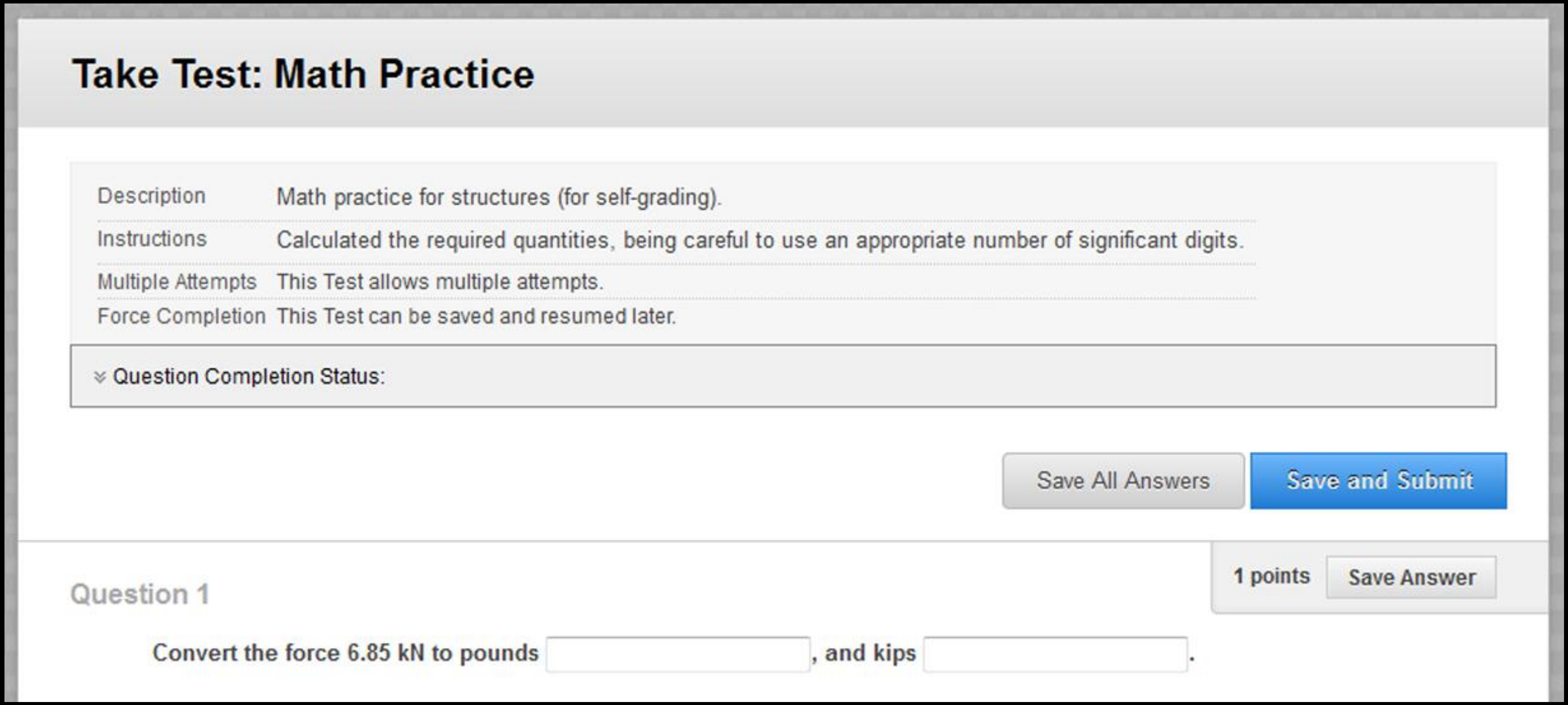
$$\frac{x}{6}=\frac{1}{3}$$

$$10^{3}=1000$$

$$\frac{10Y}{1X}\ or\ \frac{1X}{10Y}=1$$

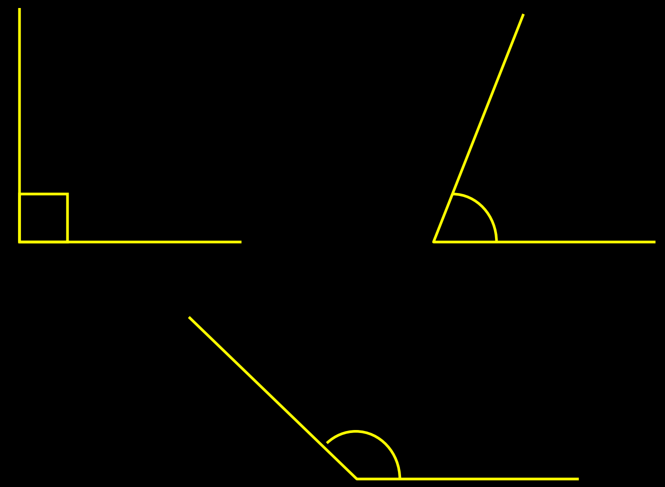
On-line Practice

- *eCampus / Study Aids*

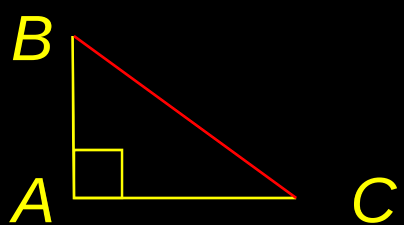


Geometry

- angles
  - – right = 90º
  - – acute < 90º
  - – obtuse > 90º
  - – π = 180º



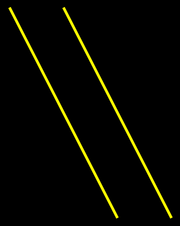
- triangles
  - – area $= \dfrac{b \times h}{2}$
  - – hypotenuse
  - – total of angles = 180º



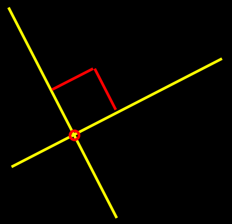
$$AB^2 + AC^2 = BC^2$$

Geometry

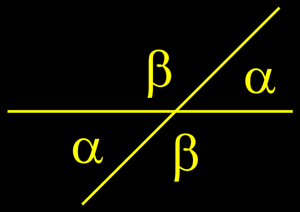
- lines and relation to angles
  – parallel lines can’t intersect



  – perpendicular lines cross at 90º
  – intersection of two lines is a point

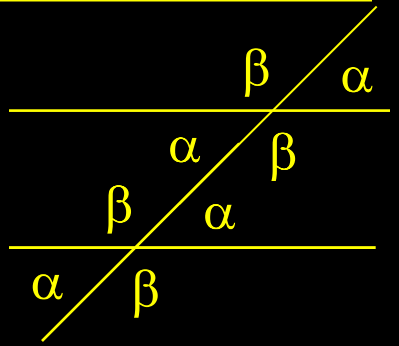


  – opposite angles are equal when two lines cross

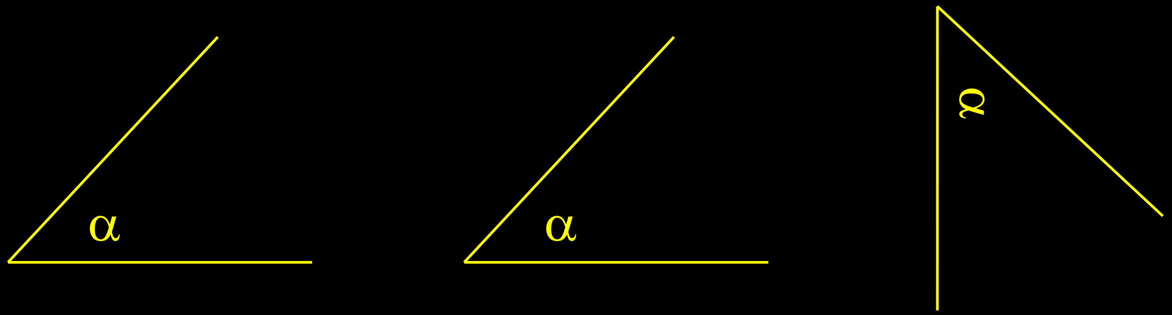


Geometry

– *intersection of a line with parallel lines results in identical angles*

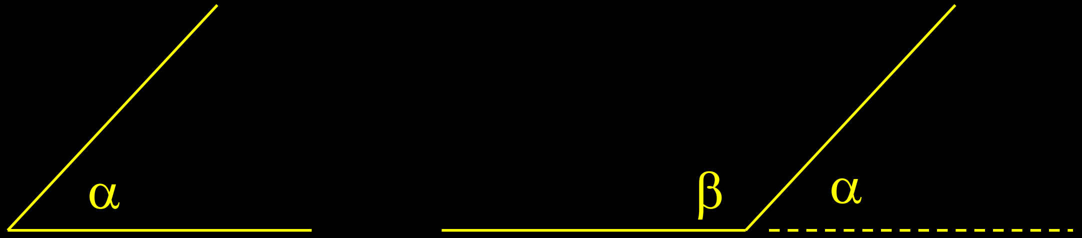


– *two lines intersect in the same way, the angles are identical*



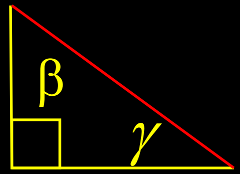
Geometry

– *sides of two angles are parallel and intersect opposite way, the angles are <u>supplementary</u> - the sum is 180°*



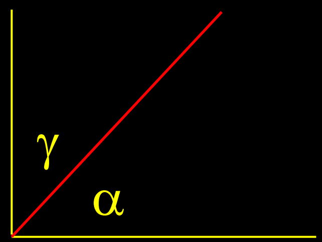
– *two angles that sum to 90° are said to be <u>complimentary</u>*

$$\beta + \gamma = 90°$$



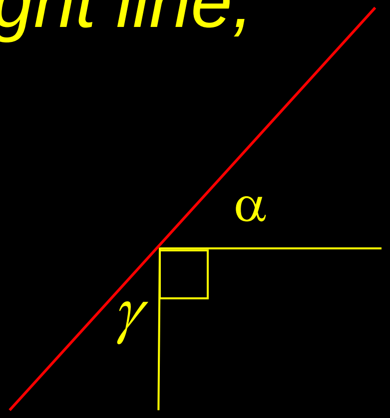
Geometry

- *sides of two angles bisect a right angle (90°), the angles are <u>complimentary</u>*



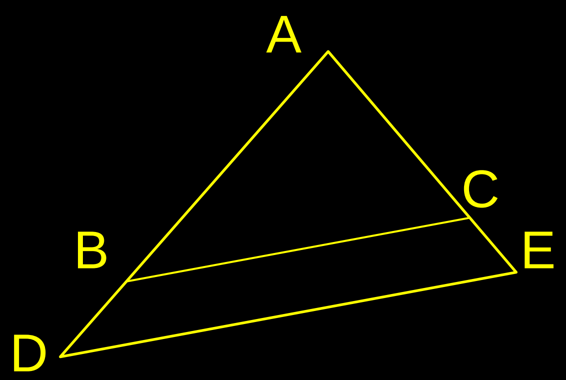
$$\alpha + \gamma = 90°$$

- *right angle bisects a straight line, remaining angles are <u>complimentary</u>*

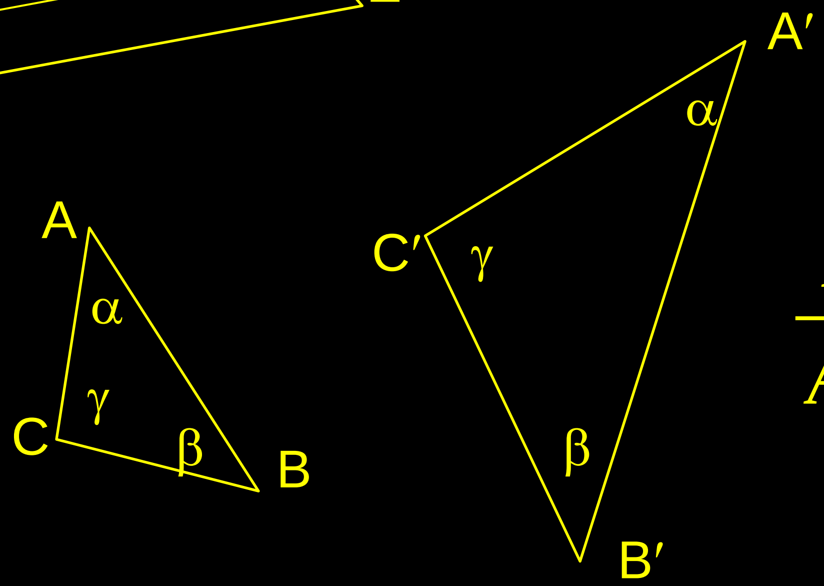


Geometry

– *similar triangles have proportional sides*



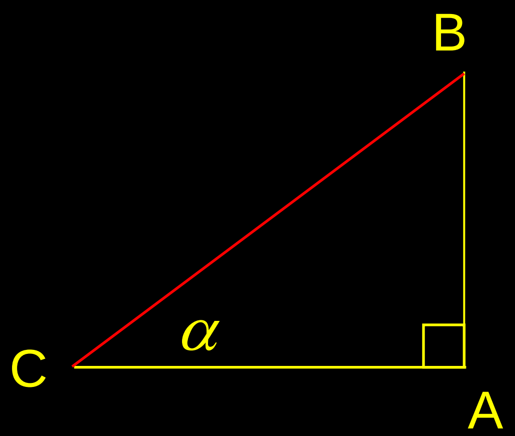
$$\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$$



$$\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$$

Trigonometry

- *for right triangles*



$$\sin = \frac{\textit{opposite side}}{\textit{hypotenuse}} = \sin\alpha = \frac{AB}{CB}$$

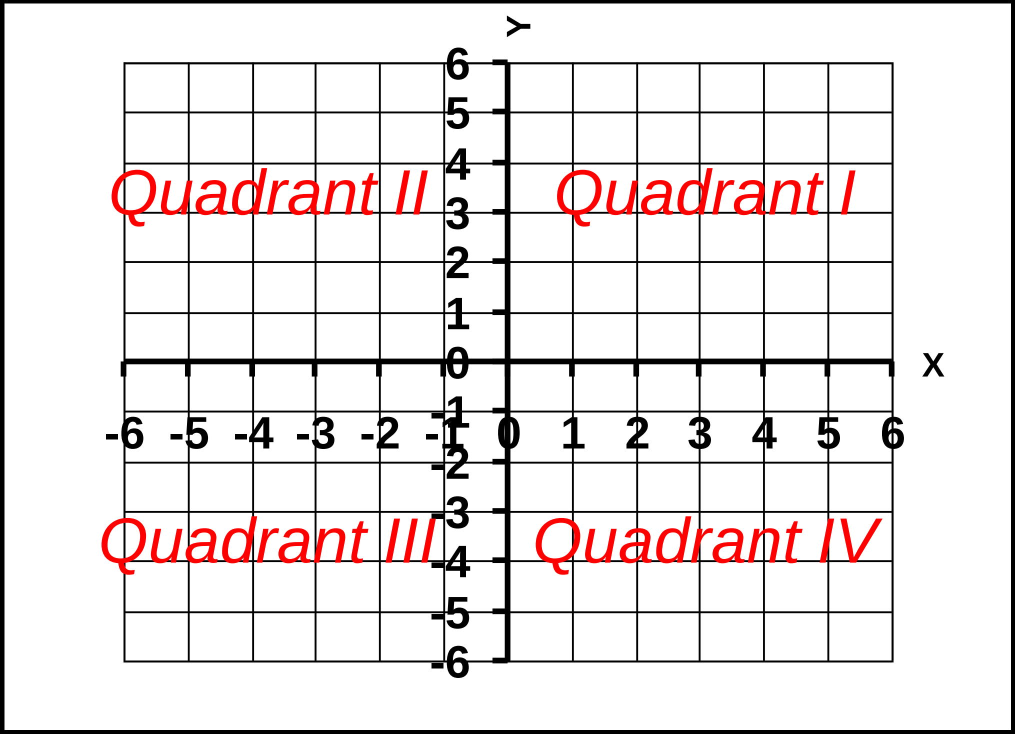
$$\cos = \frac{\textit{adjacent side}}{\textit{hypotenuse}} = \cos\alpha = \frac{AC}{CB}$$

$$\tan = \frac{\textit{opposite side}}{\textit{adjacent side}} = \tan\alpha = \frac{AB}{AC}$$

SOHCAHTOA

Trigonometry

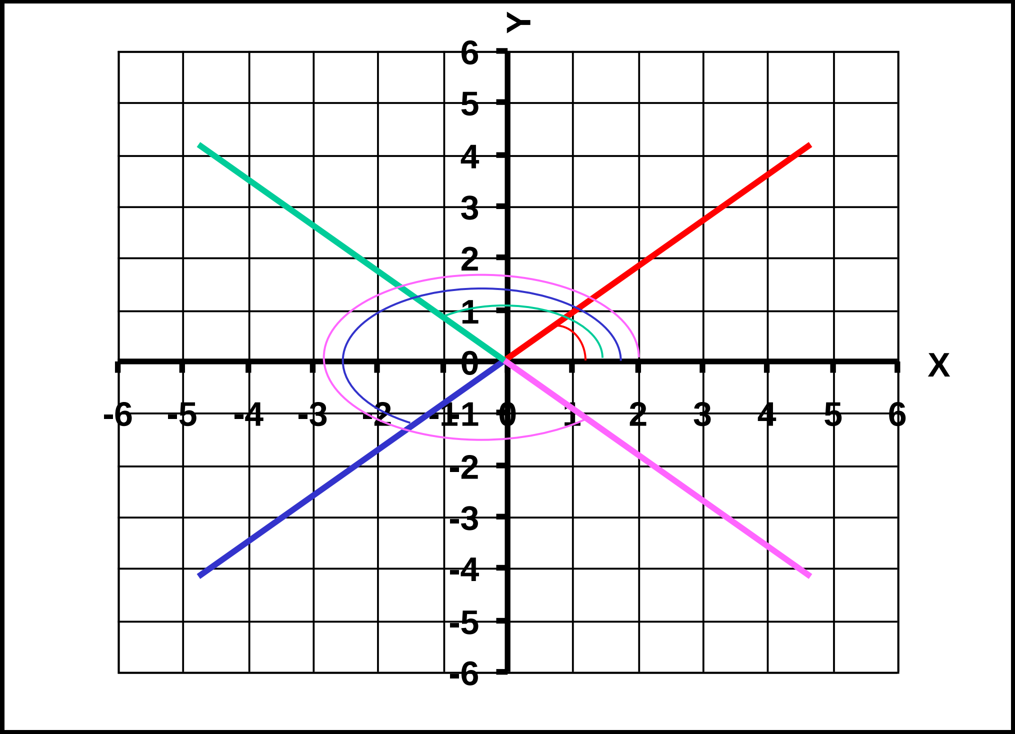
- cartesian coordinate system
  - origin at 0,0
  - coordinates in (x,y) pairs
  - x & y have signs



Trigonometry

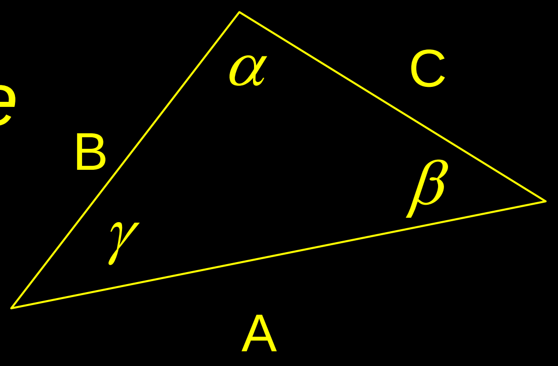
- *for angles starting at <u>positive x</u>*
  - sin *is y side*
  - cos *is x side*

sin<0 for 180-360°
cos<0 for 90-270°
tan<0 for 90-180°
tan<0 for 270-360°



Trigonometry

- *for all triangles*
  - *sides A, B & C are opposite angles α, β & γ*



  - *LAW of SINES*

$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$

  - *LAW of COSINES*

$$A^2 = B^2 + C^2 - 2BC\cos\alpha$$

Algebra

- *equations (something = something)*
- *constants*
  - *real numbers or shown with a, b, c…*
- *unknown terms, variables*
  - *names like R, F, x, y*
- *linear equations*
  - *unknown terms have no exponents*
- *simultaneous equations*
  - *variable set satisfies <u>all equations</u>*

Algebra

- *solving one equation*
  - *only works with one variable*
  - *ex:*
    - *add to both sides*
    - *divide both sides*
    - *get x by itself on a side*

$$2x - 1 = 0$$

$$2x - 1 + 1 = 0 + 1$$

$$2x = 1$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{1}{2}$$

$$x = \tfrac{1}{2}$$

Algebra

- *solving one equations*
  - *only works with one variable*
  - *ex:*

$$2x - 1 = 4x + 5$$

    - *subtract from both sides*

$$2x - 1 - 2x = 4x + 5 - 2x$$

    - *subtract from both sides*

$$-1 - 5 = 2x + 5 - 5$$

    - *divide both sides*

$$\frac{-6}{2} = \frac{-3 \cdot \cancel{2}}{\cancel{2}} = \frac{\cancel{2}x}{\cancel{2}}$$

    - *get x by itself on a side*

$$x = -3$$

Algebra

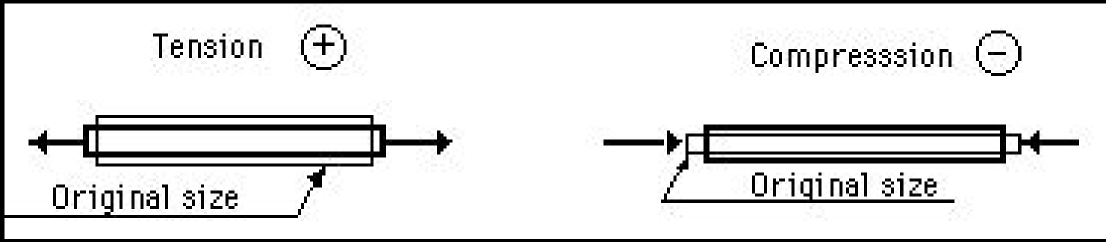
- solving two equation
  - only works with two variables
  - ex:
    - look for term similarity
    - can we add or subtract to eliminate one term?
    - add
    - get x by itself on a side

$$2x + \underline{3y} = 8$$
$$12x - \underline{3y} = 6$$

$$2x + 3y + 12x - 3y = 8 + 6$$
$$14x = 14$$
$$\frac{14x}{14} = \frac{14}{14} = x = 1$$

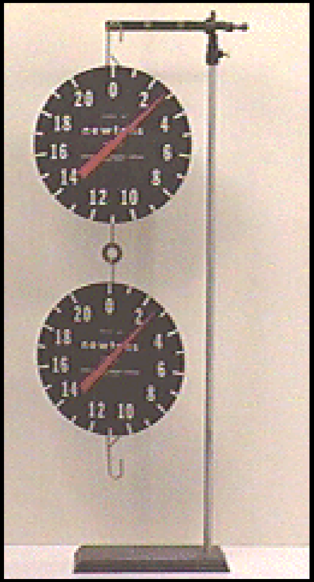
Forces

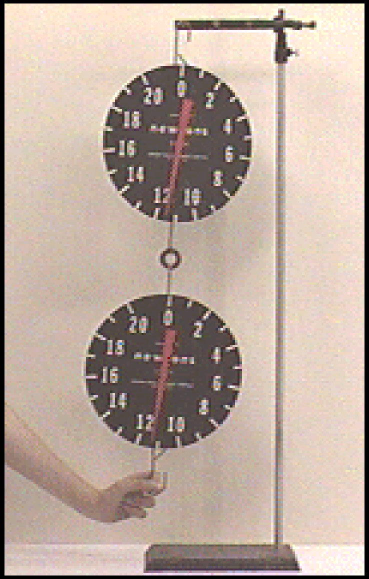
- *statics*
  - *physics of <u>forces</u> and reactions on bodies and systems*
  - *equilibrium (bodies at rest)*
- *forces*
  - *something that exerts on an object:*
    - *motion*
    - *tension*
    - *compression*



Force

- *“action of one body on another that affects the state of motion or rest of the body”*
- *Newton’s 3rd law:*
  - *for every force of action there is an equal and opposite reaction along the same line*

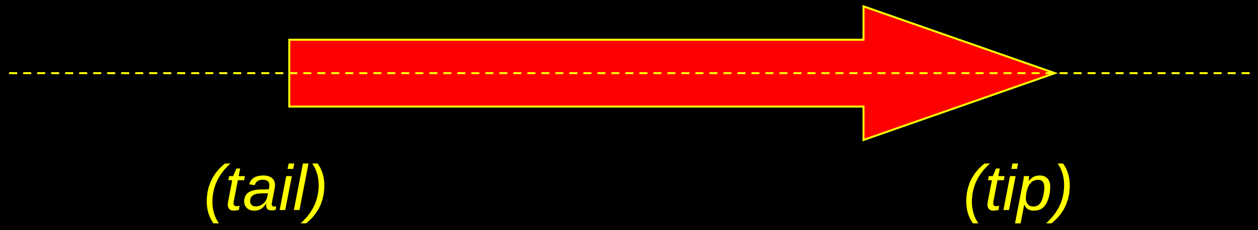




http://www.physics.umd.edu

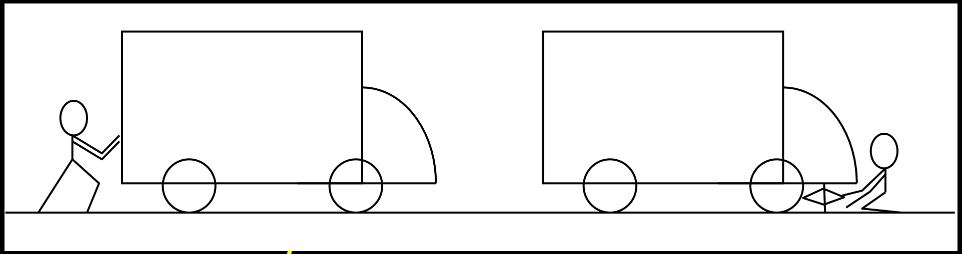
Force Characteristics

- *applied at a point*
- *magnitude*
  - *Imperial units: lb, k (kips)*
  - *SI units: N (newtons), kN*
- *direction*



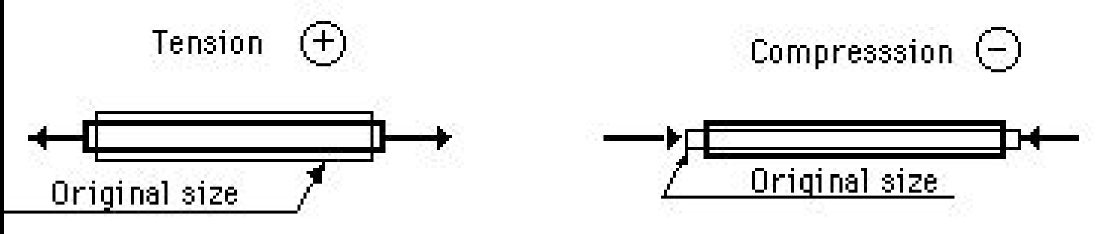
Forces on Rigid Bodies

- for statics, the bodies are ideally rigid
- can translate and rotate



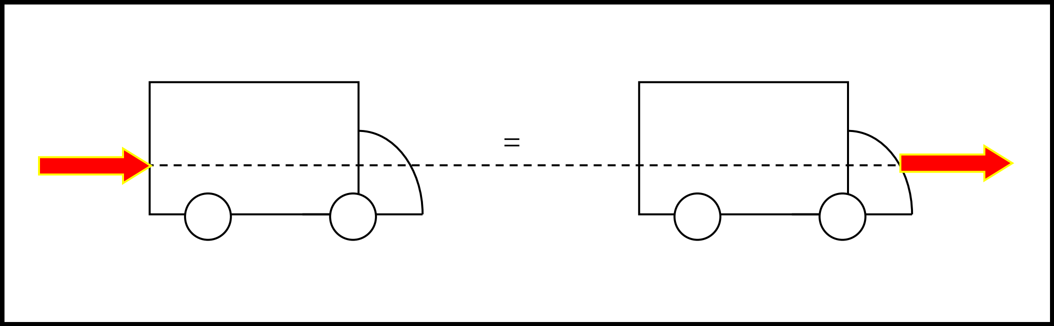
translate rotate

- internal forces are
  - in bodies
  - between bodies (connections)
- external forces act on bodies



Transmissibility

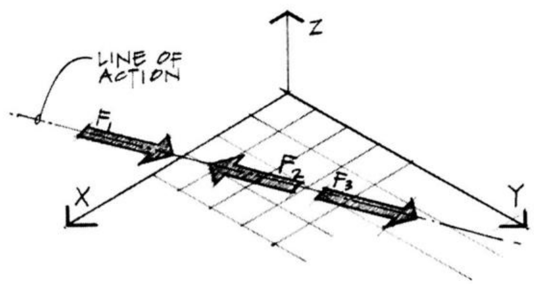
- the force stays on the same line of action
- truck can’t tell the difference



- only valid for EXTERNAL forces

Force System Types

- *collinear*

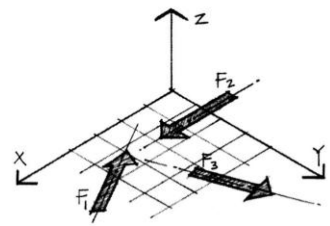


Collinear—*All forces acting along the same straight line.*

Figure 2.17(a) Particle or rigid body.

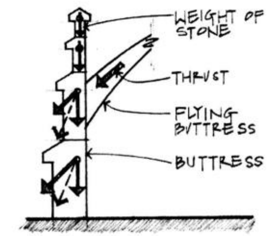
Force System Types

- *coplanar*

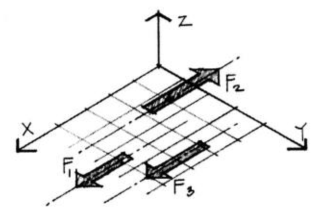


Coplanar—*All forces acting in the same plane.*

Figure 2.17(b) Rigid bodies.

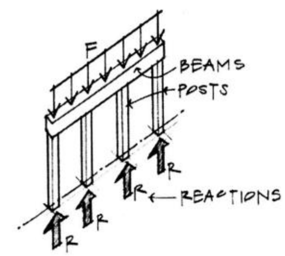


Forces in a buttress system.

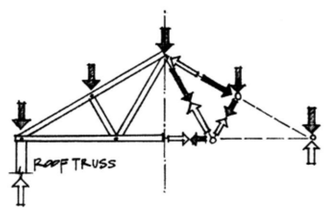


Coplanar, parallel—*All forces are parallel and act in the same plane.*

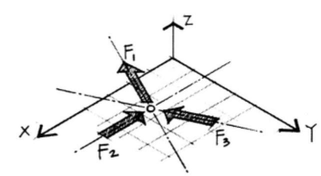
Figure 2.17(c) Rigid bodies.



A beam supported by a series of columns.



Loads applied to a roof truss.

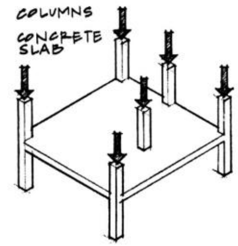


Coplanar, concurrent—*All forces intersect at a common point and lie in the same plane.*

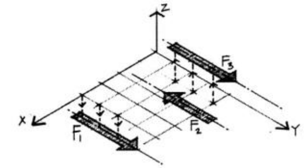
Figure 2.17(d) Particle or rigid body.

Force System Types

- *space*

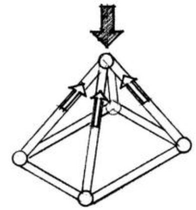


Column loads in a concrete building.

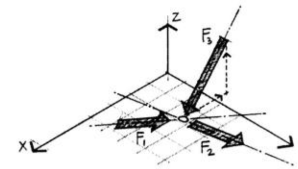


Noncoplanar, parallel—All forces are parallel to each other, but not all lie in the same plane.

Figure 2.17(e) Rigid bodies.

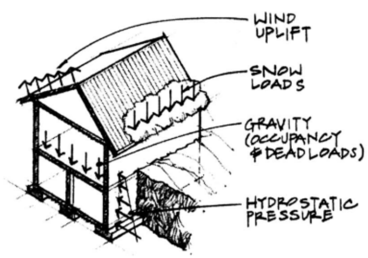


One component of a three-dimensional space frame.

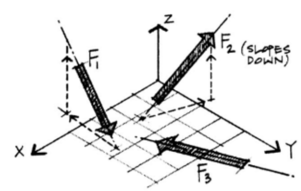


Noncoplanar, concurrent—All forces intersect at a common point but do not all lie in the same plane.

Figure 2.17(f) Particle or rigid bodies.



Array of forces acting simultaneously on a house.

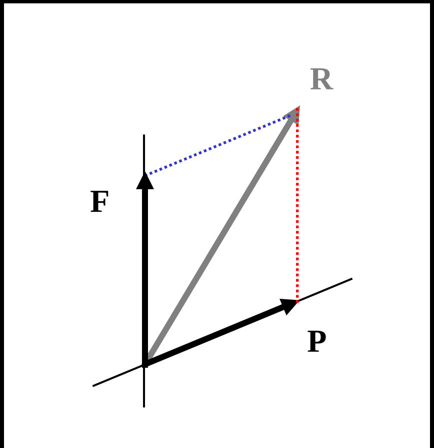


Noncoplanar, nonconcurrent—All forces are skewed.

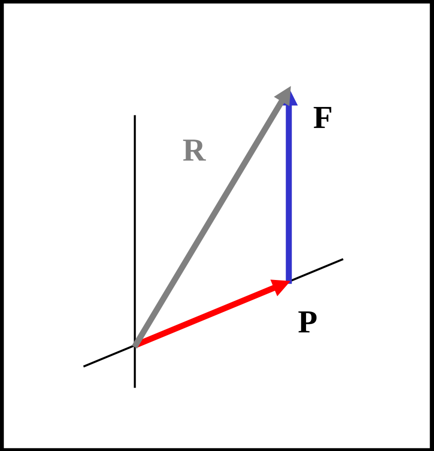
Figure 2.17(g) Rigid bodies.

Adding Vectors

- graphically
  - parallelogram law
    - diagonal
    - long for 3 or more vectors

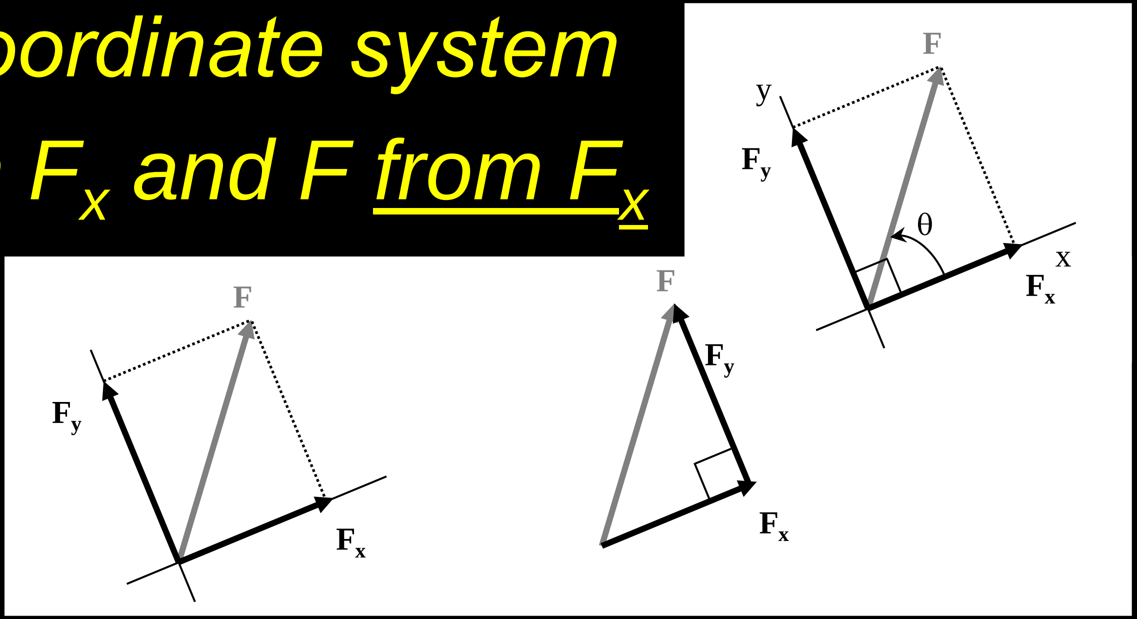


  - tip-to-tail
    - more convenient with lots of vectors



Force Components

- *convenient to resolve into 2 vectors*
- *at right angles*
- *in a "nice" coordinate system*
- *θ is between F_x and F <u>from F_x</u>*



$$F_x = F\cos\theta$$

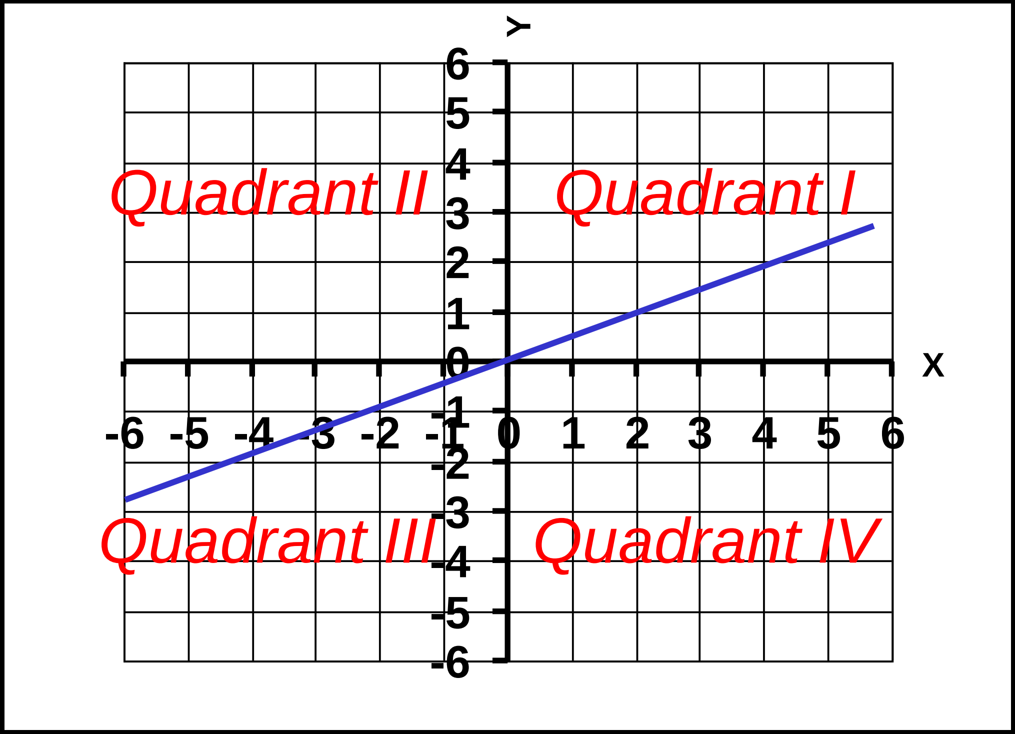
$$F_y = F\sin\theta$$

$$F = \sqrt{F_x^2 + F_y^2}$$

$$\tan\theta = \frac{F_y}{F_x}$$

Trigonometry

- F_x is negative
  - 90° to 270°
- F_y is negative
  - 180° to 360°
- tan is positive
  - quads I & III
- tan is negative
  - quads II & IV

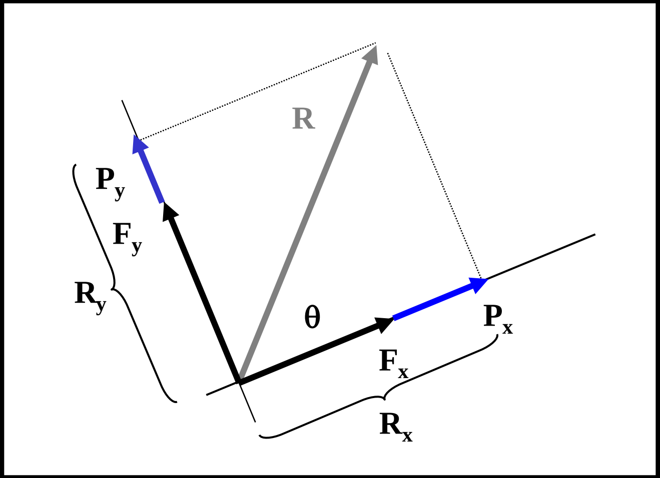


Component Addition

- *find all x components*
- *find all y components*
- *find sum of x components, R_x (resultant)*
- *find sum of y components, R_y*

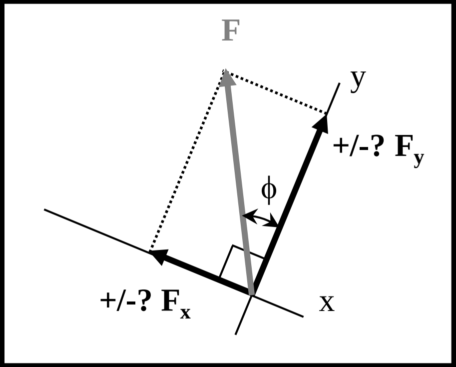
$$R = \sqrt{R_x^2 + R_y^2}$$

$$\tan\theta = \frac{R_y}{R_x}$$



Alternative Trig for Components

- *doesn't relate angle to axis direction*
- ϕ *is "small" angle between F and* *EITHER F_x or F_y*
- *no sign out of calculator!*
- *have to choose RIGHT trig function, resulting direction (sign) and component axis*



Friction

- *resistance to movement*
- *contact surfaces determine μ*
- *proportion of normal force ($\perp$)*
  - *opposite to slide direction*
  - *static > kinetic*

$$F = \mu N$$

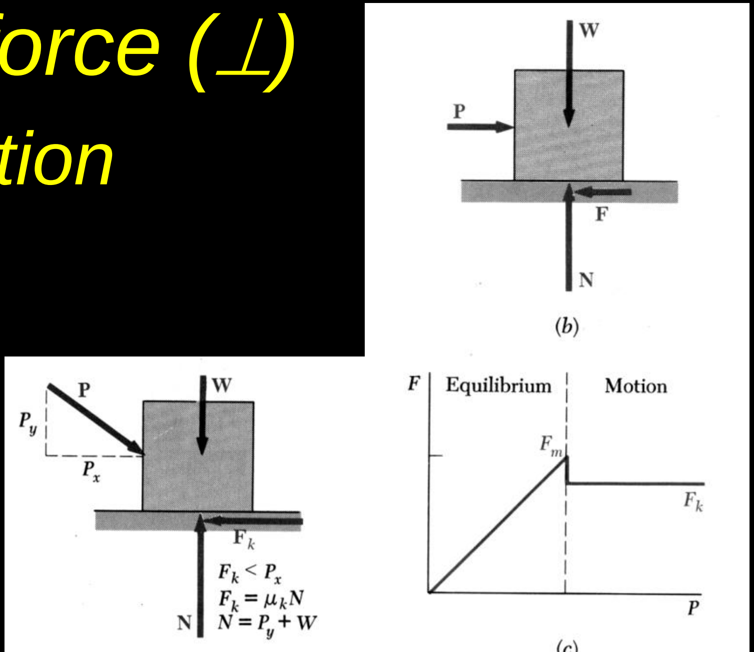


Fig. 8.1

Cables

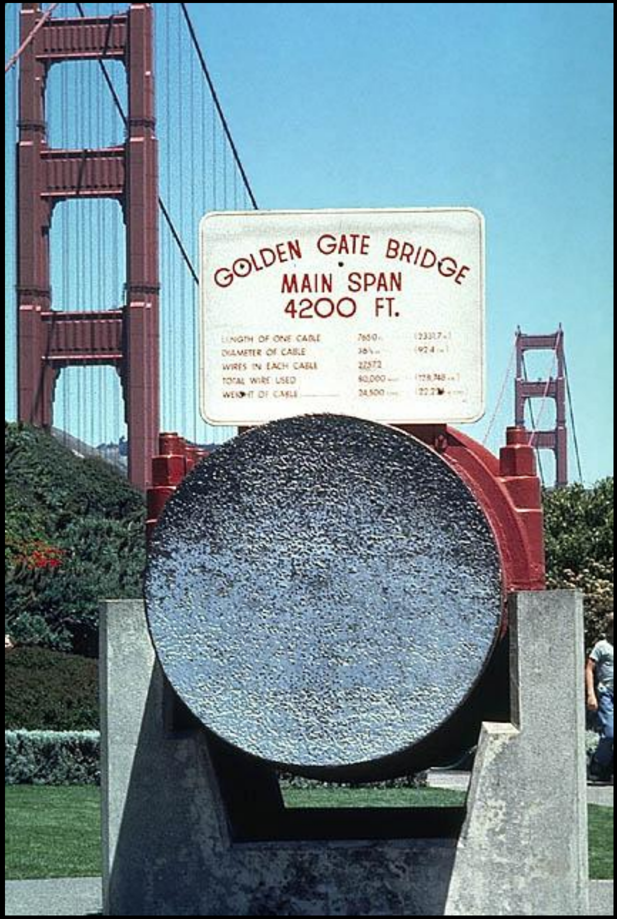
- simple
- uses
  - suspension bridges
  - roof structures
  - transmission lines
  - guy wires, etc.

http:// nisee.berkeley.edu/godden

- have same tension all along
- can't stand compression

Cables Structures

- use high-strength steel
- need
  - towers
  - anchors
- don't want movement



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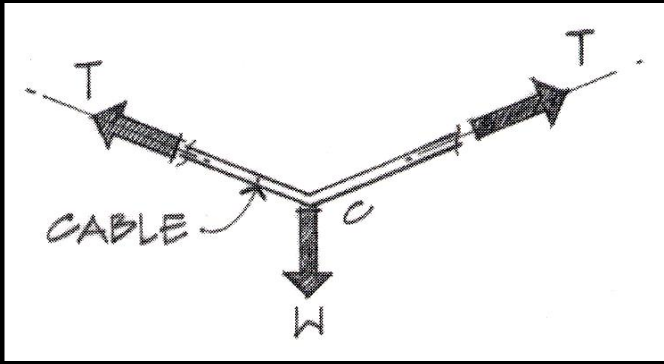
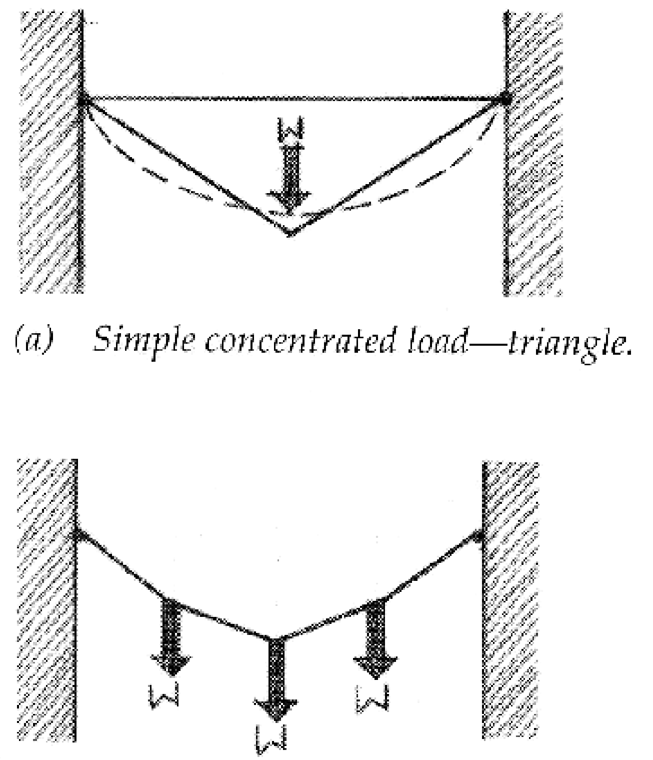
Cable Structures

Cable Loads

- *straight line between forces*
- *with one force*
  - *concurrent*
  - *symmetric*



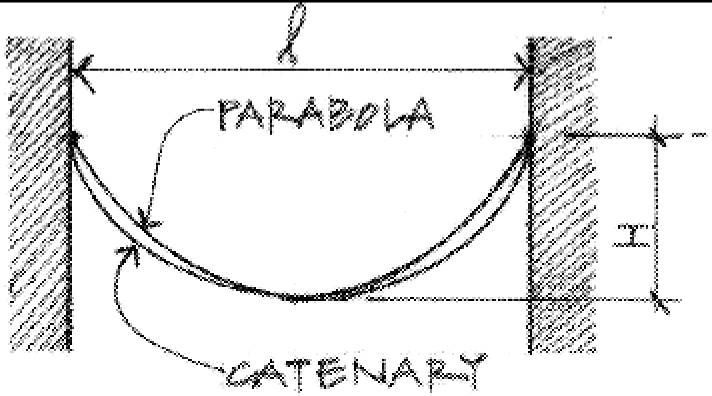


(a) Simple concentrated load—triangle.

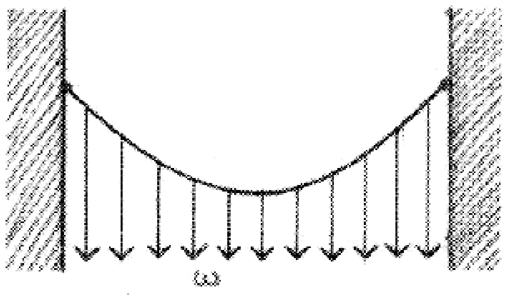
(b) Several concentrated loads—polygon.

Cable Loads

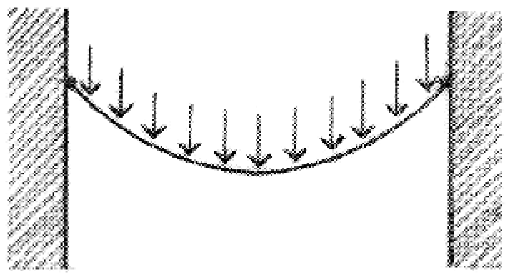
- *shape directly related to the distributed load*



(e) Comparison of a parabolic and a catenary curve.



(c) Uniform loads (horizontally)—parabola.



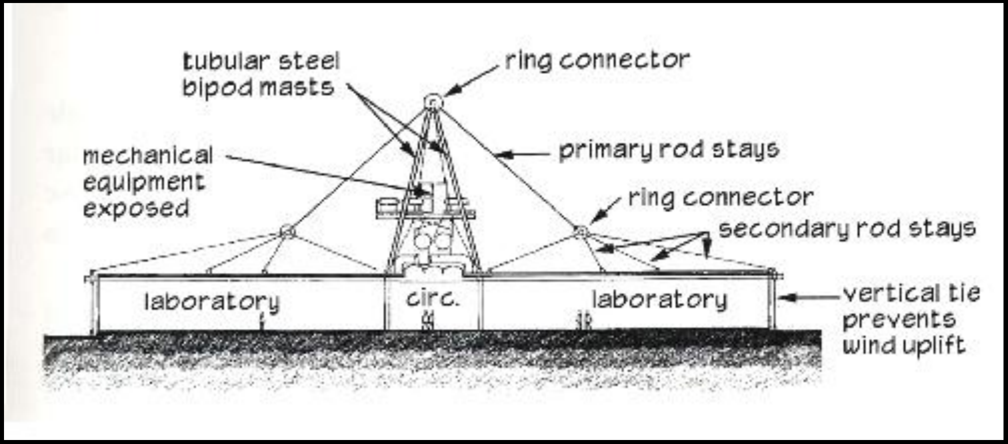
(d) Uniform loads (along the cable length)—catenary.

Cable-Stayed Structures

- *diagonal cables support horizontal spans*
- *typically symmetrical*
- *Patcenter, Rogers 1986*

Patcenter, Rogers 1986

- *column free space*
- *roof suspended*
- *solid steel ties*
- *steel frame supports masts*



Patcenter, Rogers 1986

- *dashes – cables pulling*

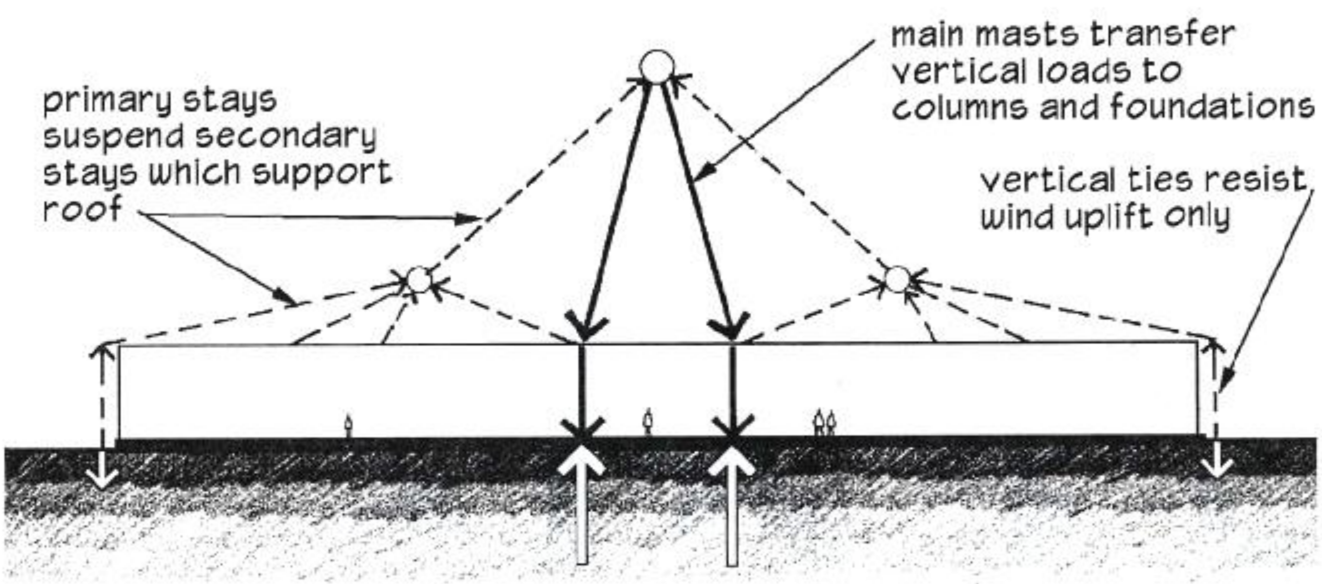
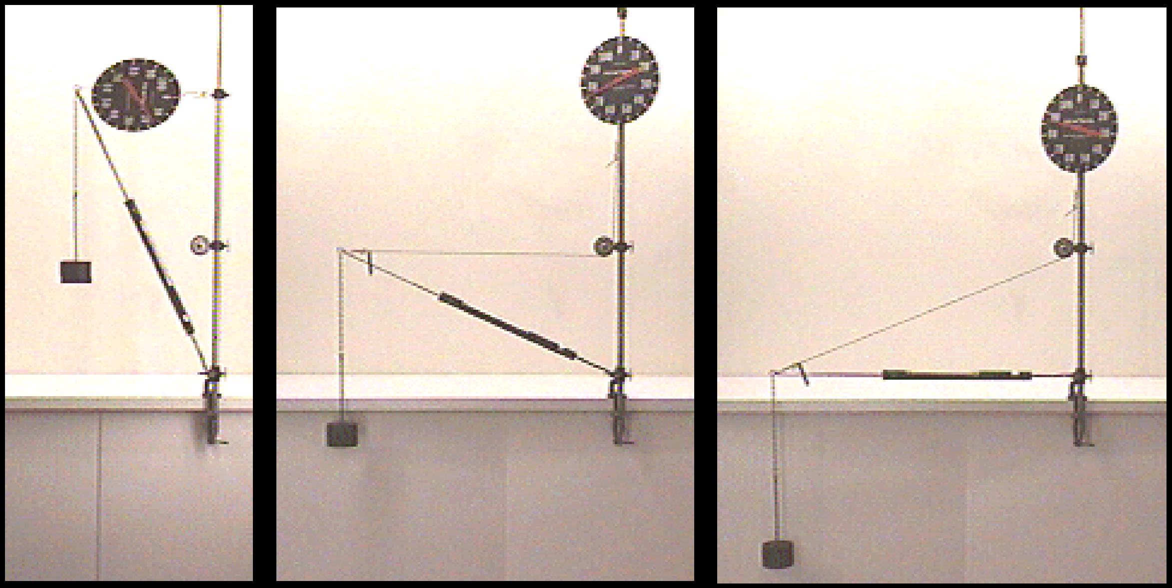


Figure 3.5: Patcenter, load path diagram.

Moments

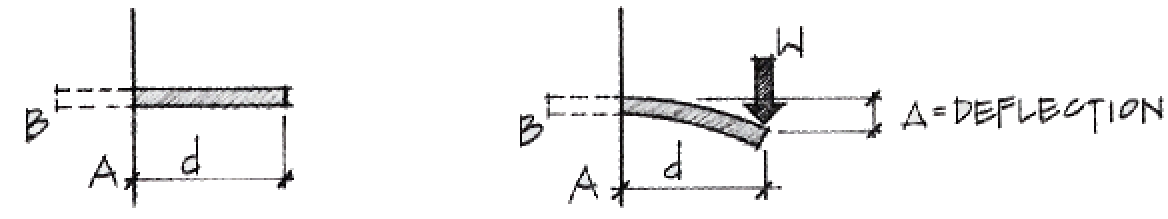
- *forces have the tendency to make a body rotate about an axis*



http://www.physics.umd.edu

  – *same translation but different rotation*

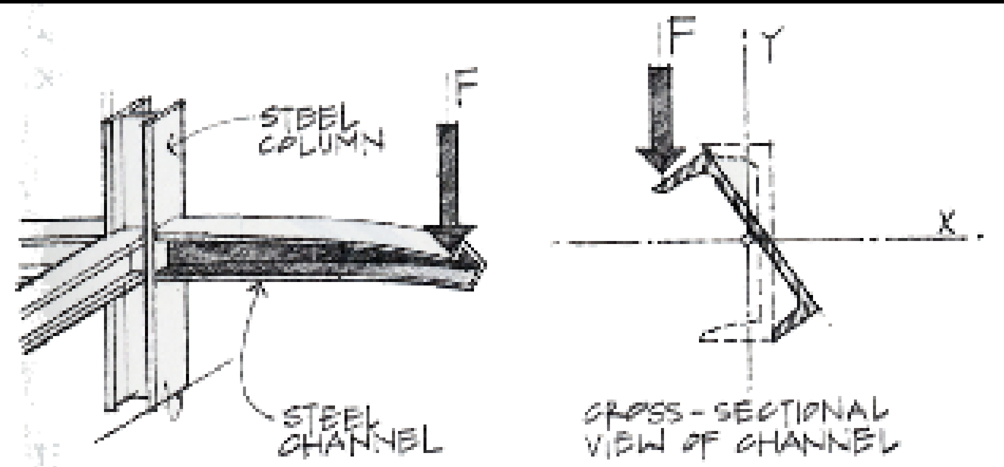
Moments



(a) Unloaded.

(b) Loaded.

Figure 2.33 Moment on a cantilever beam.



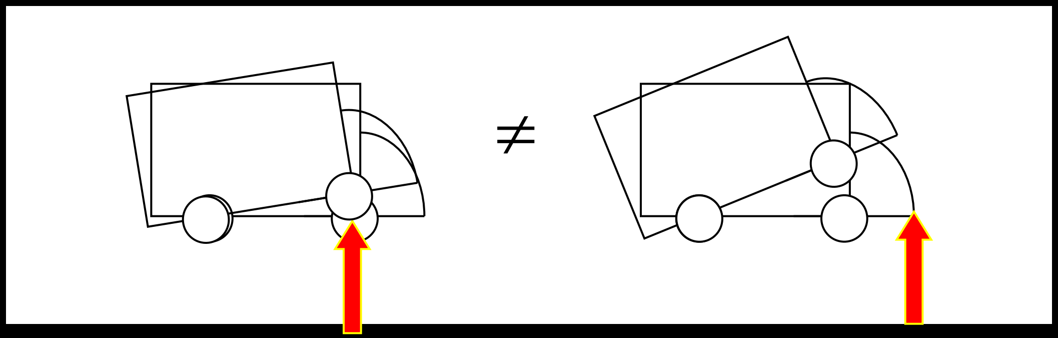
(a)

(b)

Figure 2.34 An example of torsion on a cantilever beam.

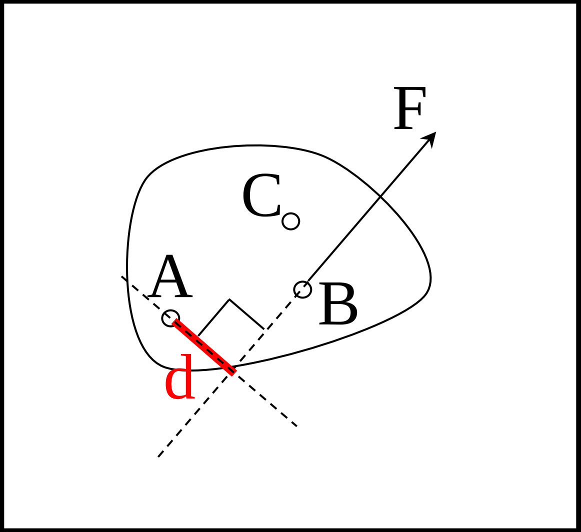
Moments

- a force acting at a different point causes a different moment:



Moments

- defined by magnitude and direction
- units: N·m, k·ft
- direction:
  + ccw (right hand rule)
  - cw
- value found from F and ⊥ distance



$$M = F \cdot d$$

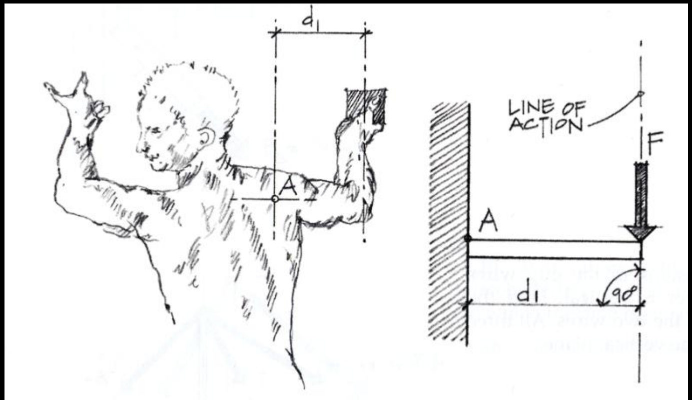
- d also called "lever" or "moment" arm

Moments

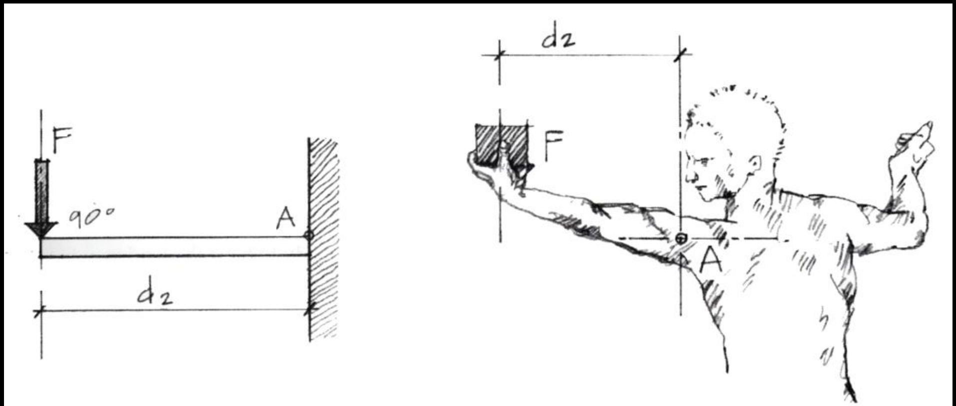
- *with same F:*

$$M_A = F \cdot d_1 < M_A = F \cdot d_2$$

(bigger)



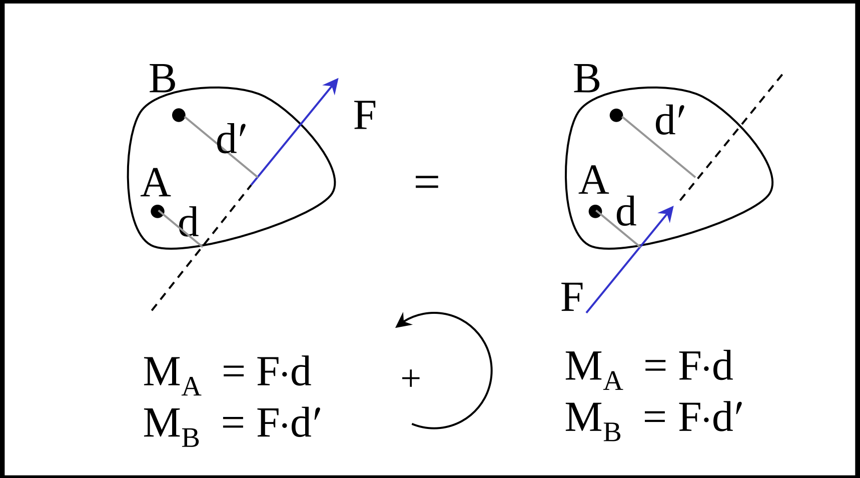
(a) (b)



(a) (b)

Moments

- *additive with sign convention*
- *can still move the force*
  along the line of action

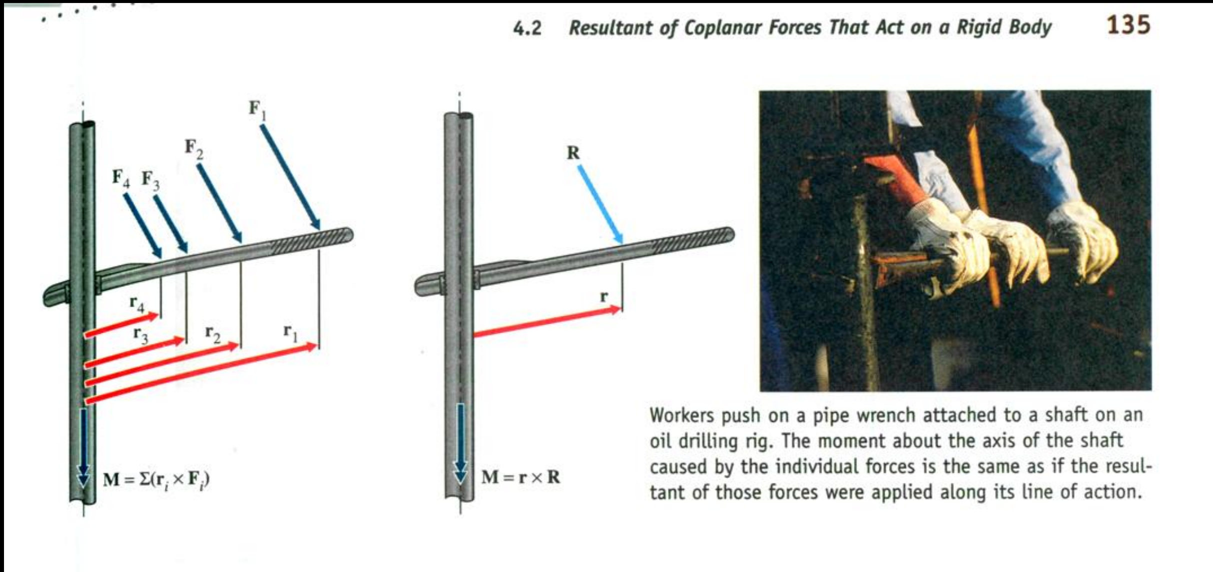


Moments

- *Varignon's Theorem*
  - *resolve a force into components at a point and finding perpendicular distances*
  - *calculate sum of moments*
  - *equivalent to original moment*
- *makes life easier!*
  - *geometry*
  - *when component runs through point, d=0*

Moments of a Force

- moments of a force
  - introduced in Physics as "Torque Acting on a Particle"
  - and used to satisfy rotational equilibrium



Physics and Moments of a Force

- my Physics book:

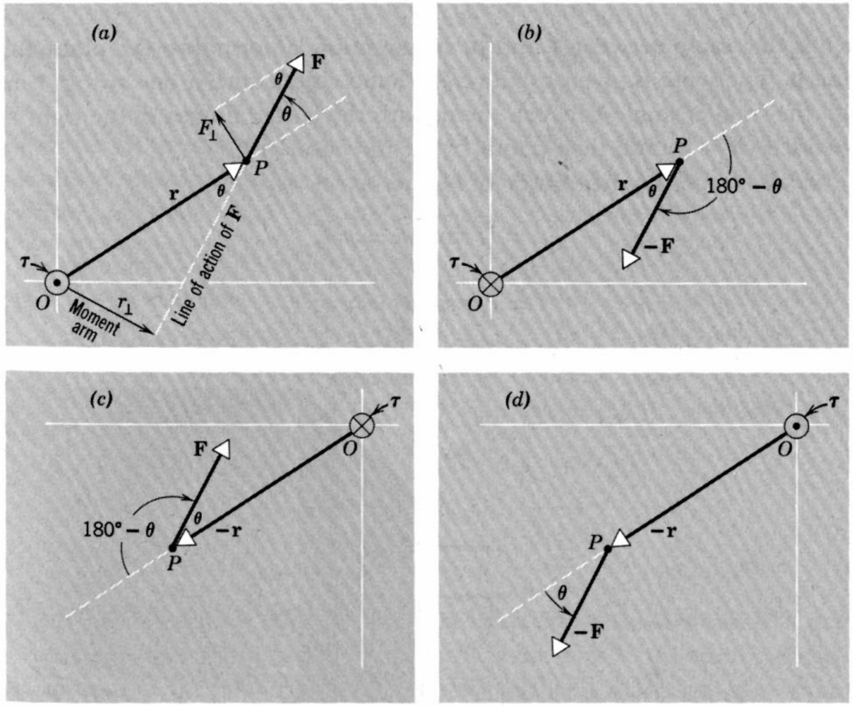
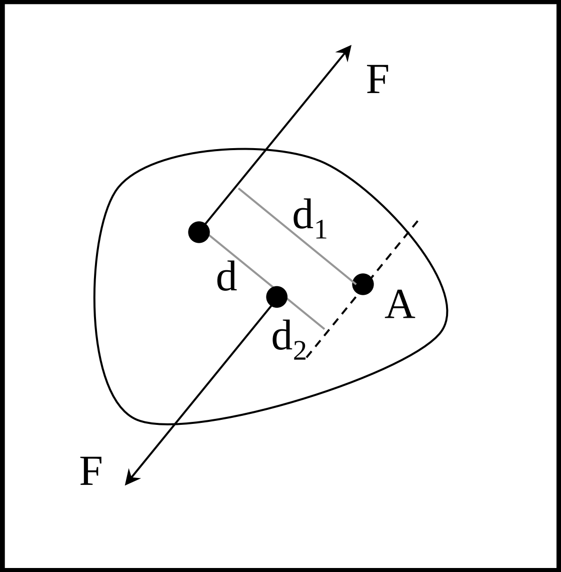


FIGURE 11–2 The plane shown is that defined by **r** and **F** in Fig. 11–1. (*a*) The magnitude of $\boldsymbol{\tau}$ is given by $Fr_\perp$ (Eq. 11–2*b*) or by $rF_\perp$ (Eq. 11–2*c*). (*b*) Reversing **F** reverses the direction of $\boldsymbol{\tau}$. (c) Reversing **r** reverses the direction of $\boldsymbol{\tau}$. (*d*) Reversing **F** and **r** leaves the direction of $\boldsymbol{\tau}$ unchanged. The directions of $\boldsymbol{\tau}$ are represented by ⊙ (perpendicularly out of the figure, the symbol representing the tip of an arrow) and by ⊗ (perpendicularly into the figure, the symbol representing the tail of an arrow).

Moment Couples

- 2 forces
  - same size
  - opposite direction
  - distance d apart
  - cw or ccw



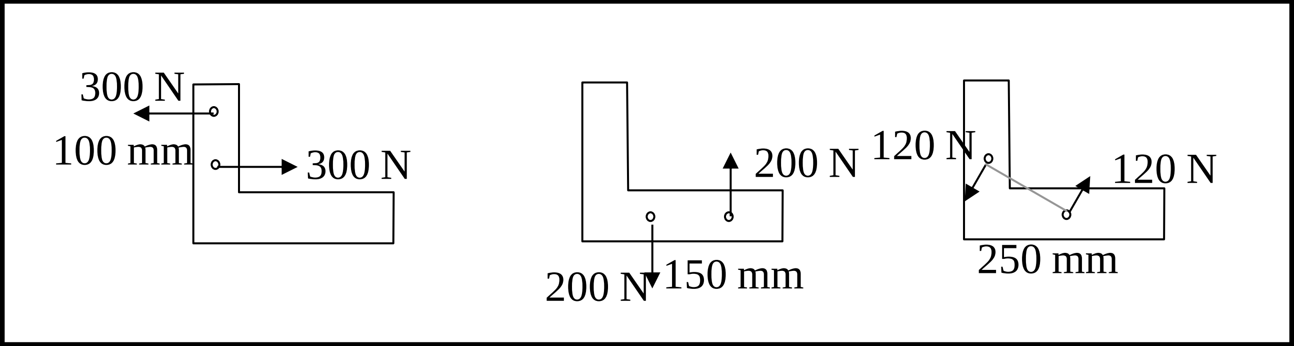
$$M = F \cdot d$$

  - not dependant on point of application

$$M = F \cdot d_1 - F \cdot d_2$$

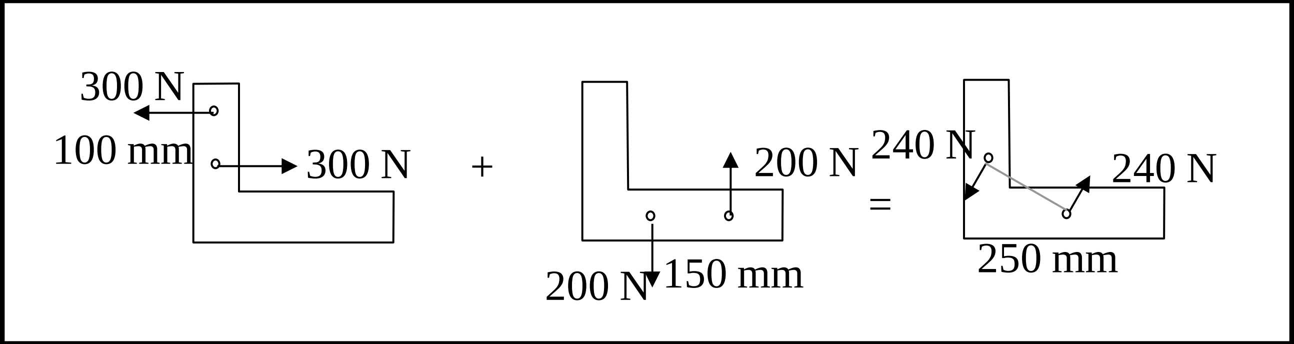
Moment Couples

- *equivalent couples*
  - *same magnitude and direction*
  - *F & d may be different*



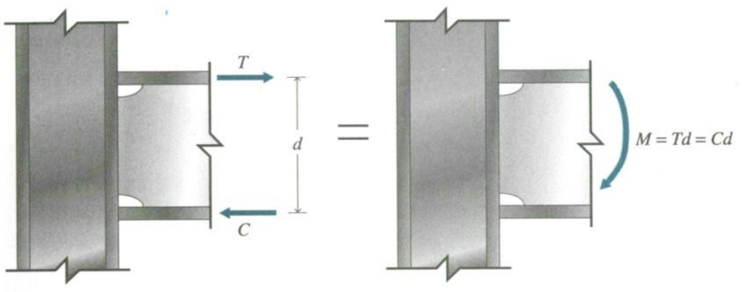
Moment Couples

- *added just like moments caused by one force*
- *can <u>replace</u> two couples with a single couple*

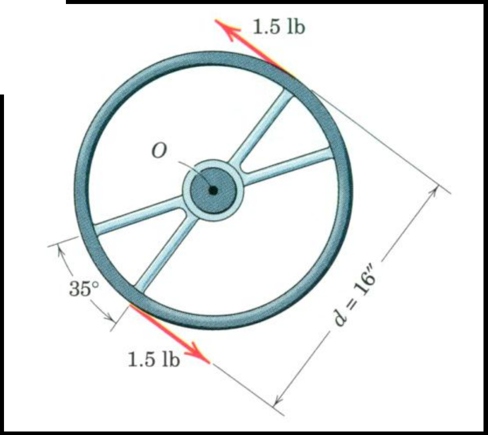


Moment Couples

- *moment couples in structures*



The flanges of a steel beam are welded to the flange of a column. Equal and opposite forces T and C in the beam flanges form a couple with moment M that is transferred into the column.

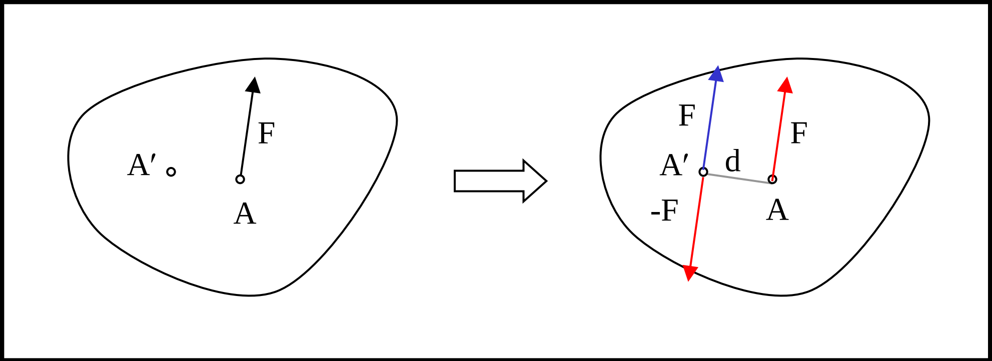


Equivalent Force Systems

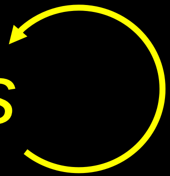
- *two forces at a point is equivalent to the resultant at a point*
- *resultant is equivalent to two components at a point*
- *resultant of equal & opposite forces at a point is zero*
- *put equal & opposite forces at a point (sum to 0)*
- *transmission of a force along action line*

Force-Moment Systems

- single force causing a moment can be replaced by the same force at a different point by providing the moment that force caused

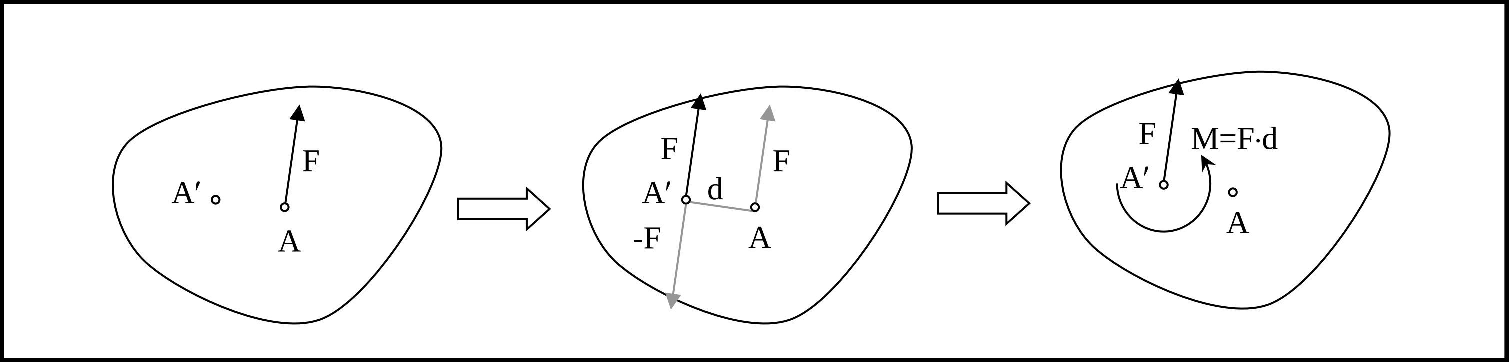


- moments are shown as arched arrows



Force-Moment Systems

- a force-moment pair can be replaced by a force at another point causing the original moment



Parallel Force Systems

- *forces are in the same direction*
- *can find resultant force*
- *need to find <u>location</u> for equivalent moments*