ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN

ARCH 331

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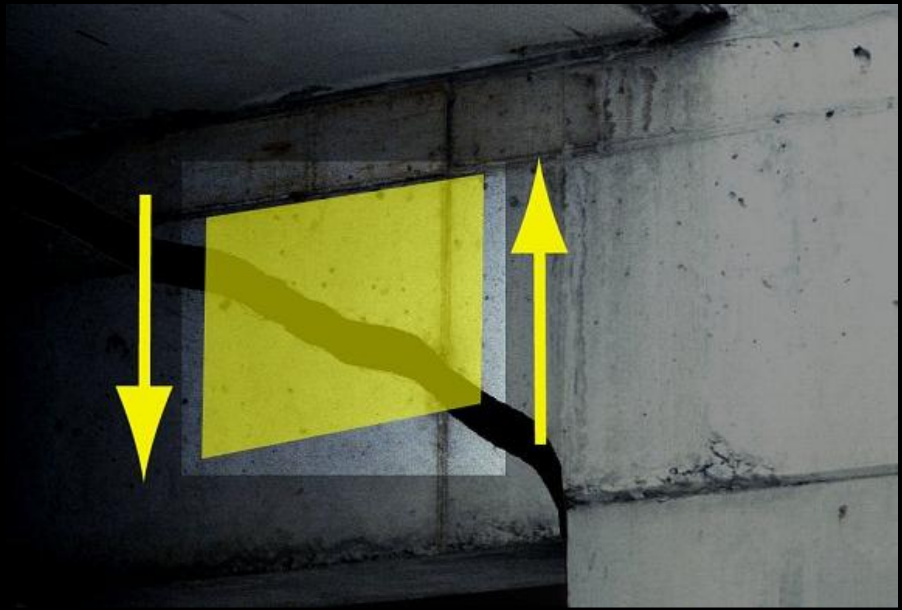
FALL 2013

lecture
twenty four



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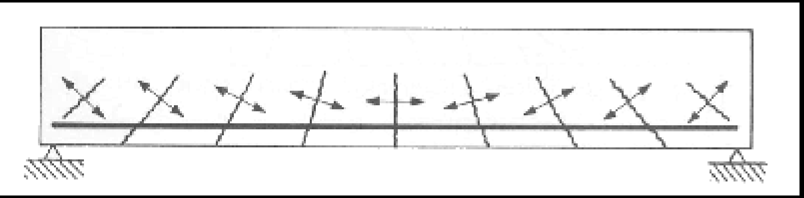
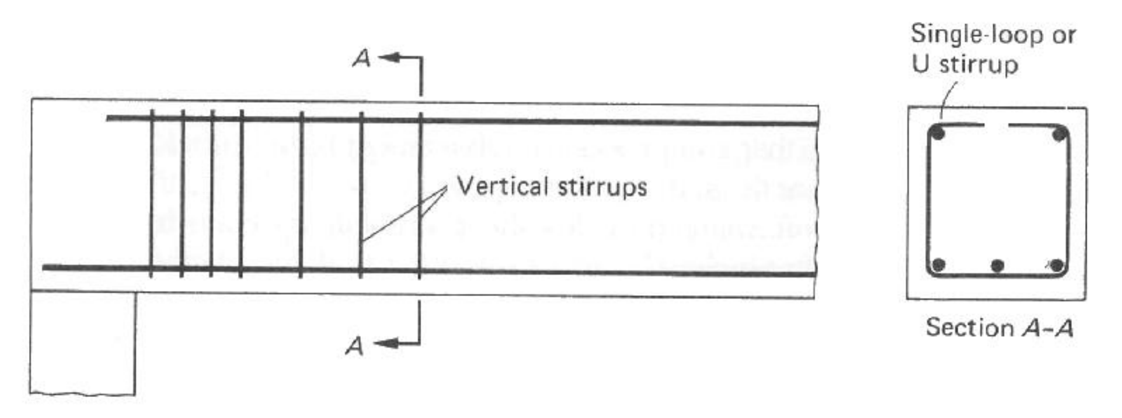
concrete construction: shear & deflection

Shear in Concrete Beams

- flexure combines with shear to form diagonal cracks



- horizontal reinforcement doesn't help
- stirrups = vertical reinforcement



ACI Shear Values

- V_u *is at distance d from face of support*
- *shear capacity:* $V_c = \upsilon_c \times b_w d$
  - *where* b_w *means thickness of* *web* *at n.a.*

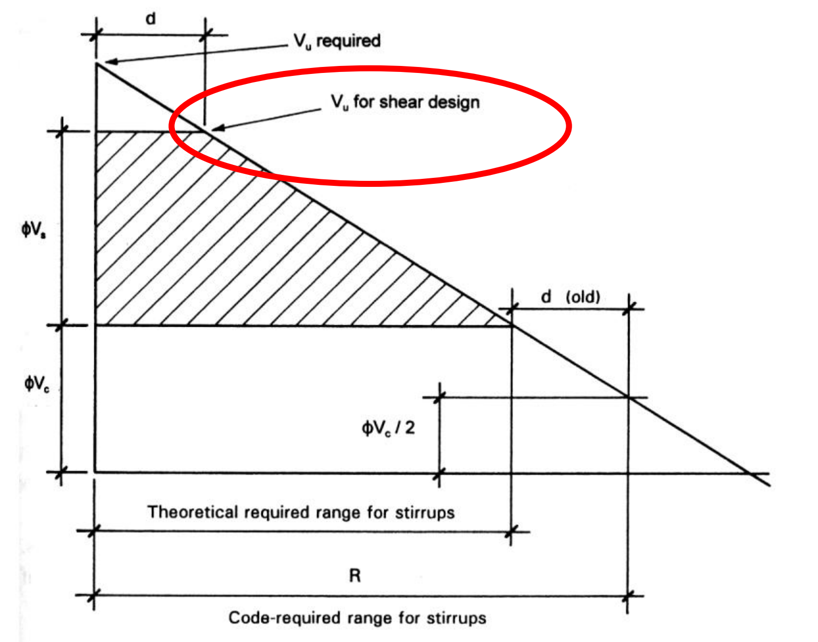


Figure 13.16 Layout for shear stress analysis: ACI Code requirements.

ACI Shear Values

- shear stress (beams)
  - $\upsilon_c = 2\sqrt{f'_c}$

    $$\phi V_c = \phi 2\sqrt{f'_c}\, b_w d$$

ϕ = 0.75 for shear

f'_c is in psi

- shear strength:

$$V_u \le \phi V_c + \phi V_s$$

  - V_s is strength from stirrup reinforcement

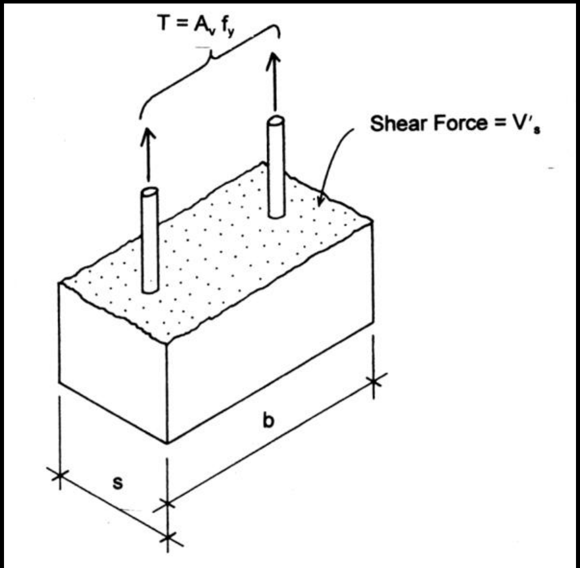


Figure 13.17 Consideration for spacing of a single stirrup.

Stirrup Reinforcement

- shear capacity:

$$V_s = \frac{A_v f_y d}{s}$$

  - A_v = area in all legs of stirrups
  - s = spacing of stirrup

- may need stirrups when concrete has enough strength!

Required Stirrup Reinforcement

- *spacing limits*

Table 3-8 ACI Provisions for Shear Design*

		$V_u \le \frac{\phi V_c}{2}$	$\phi V_c \ge V_u > \frac{\phi V_c}{2}$	$V_u > \phi V_c$
Required area of stirrups, A_v**		none	$\frac{50 b_w s}{f_y}$	$\frac{(V_u - \phi V_c)s}{\phi f_y d}$
Stirrup spacing, s	Required	—	$\frac{A_v f_y}{50 b_w}$	$\frac{\phi A_v f_y d}{V_u - \phi V_c}$
	Recommended Minimum†	—	—	4 in.
	Maximum†† (ACI 11.5.4)	—	$\frac{d}{2}$ or 24 in.	$\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \le \phi 4 \sqrt{f'_c}\, b_w d$
				$\frac{d}{4}$ or 12 in. for $(V_u - \phi V_c) > \phi 4 \sqrt{f'_c}\, b_w d$

*Members subjected to shear and flexure only; $\phi V_c = \phi 2 \sqrt{f'_c}\, b_w d$, $\phi = 0.75$ (ACI 11.3.1.1)

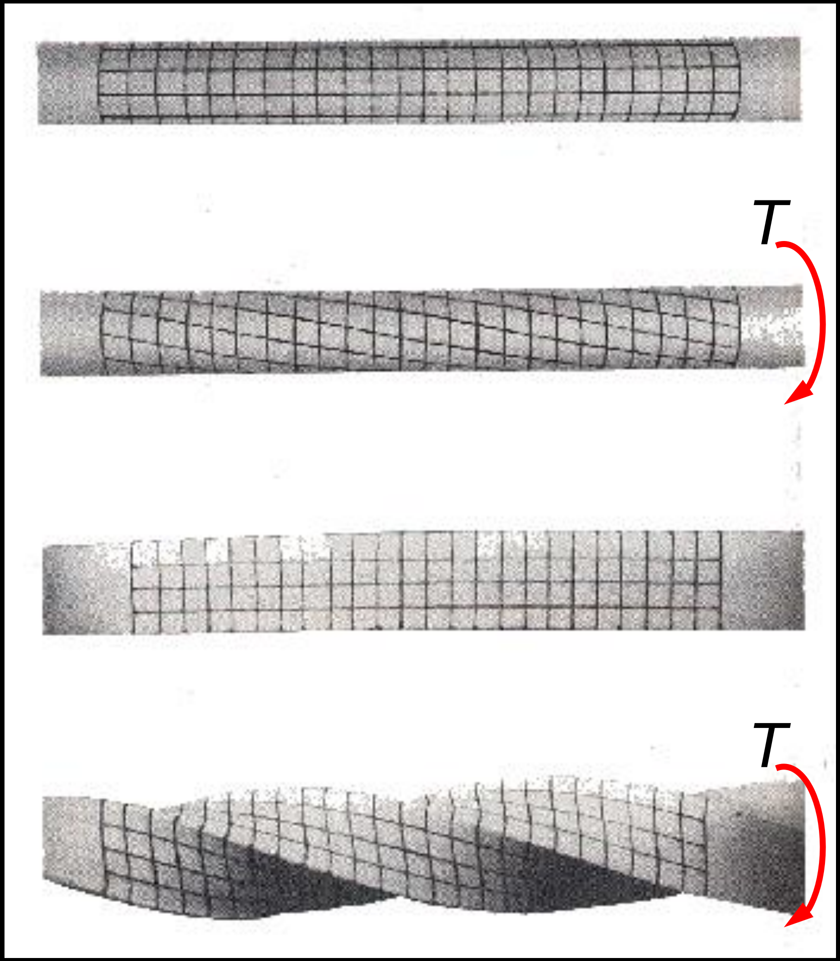
**$A_v = 2 \times A_b$ for U stirrups; $f_y \le 60$ ksi (ACI 11.5.2)

†A practical limit for minimum spacing is d/4

††Maximum spacing based on minimum shear reinforcement ($= A_v f_y / 50 b_w$) must also be considered (ACI 11.5.5.3).

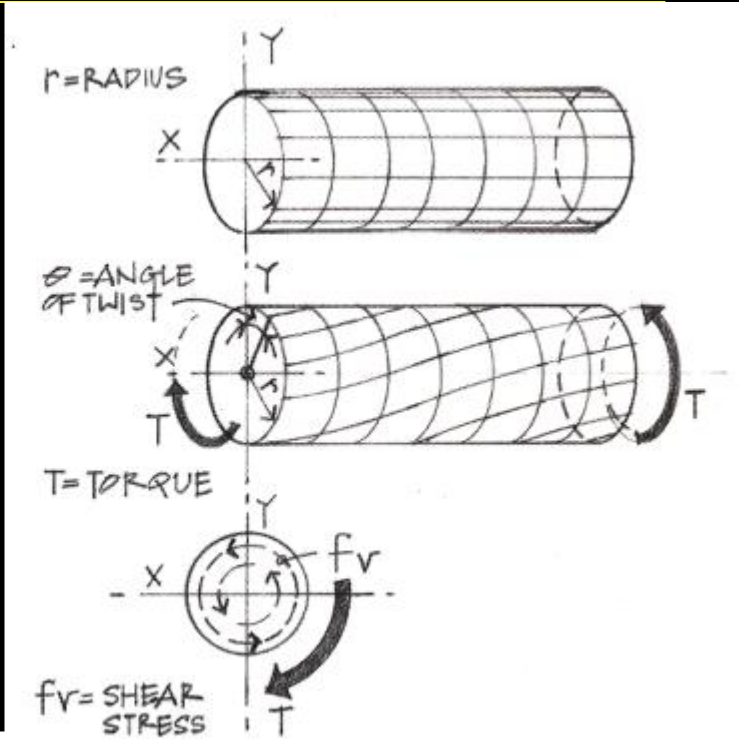
Torsional Stress & Strain

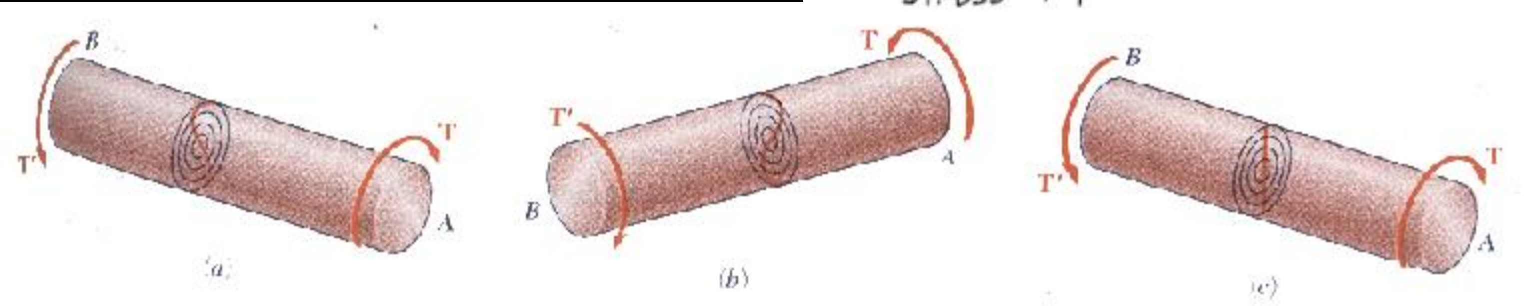
- can see torsional stresses & twisting of axi-symmetrical cross sections
  - torque
  - remain plane
  - undistorted
  - rotates
- not true for square sections....



Shear Stress Distribution

- *depend on the deformation*
- *ϕ = angle of twist*
  - *measure*
- *can prove planar section doesn't distort*



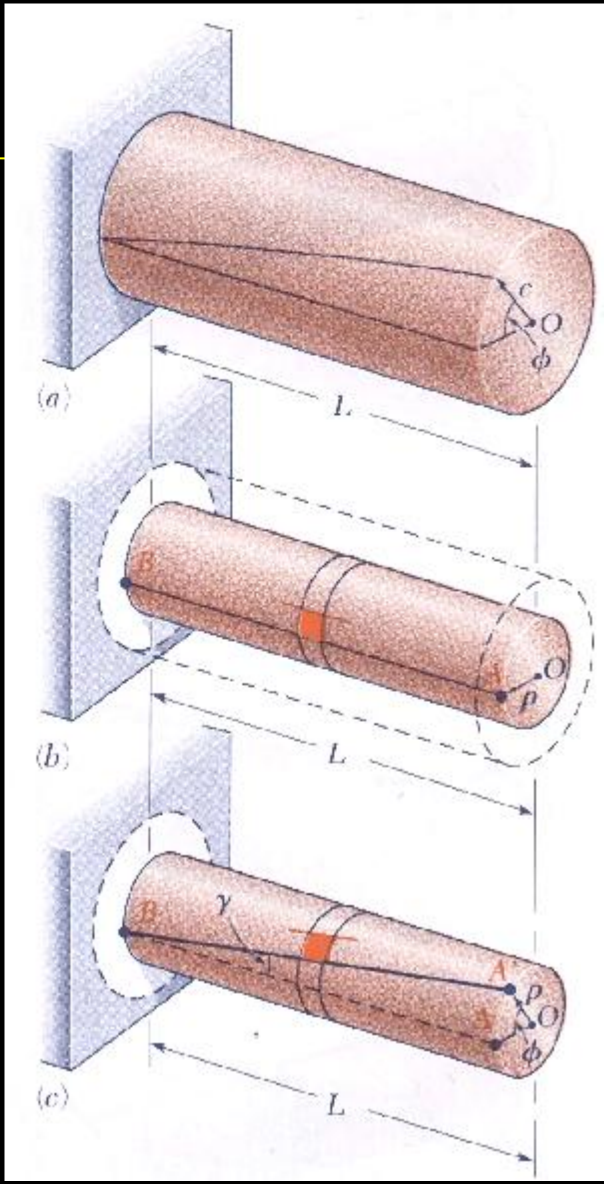


Shearing Strain

- *related to ϕ*

$$\gamma = \frac{\rho\phi}{L}$$

- *ρ is the radial distance from the centroid to the point under strain*
- *shear strain varies linearly along the radius: γ_{max} is at outer diameter*

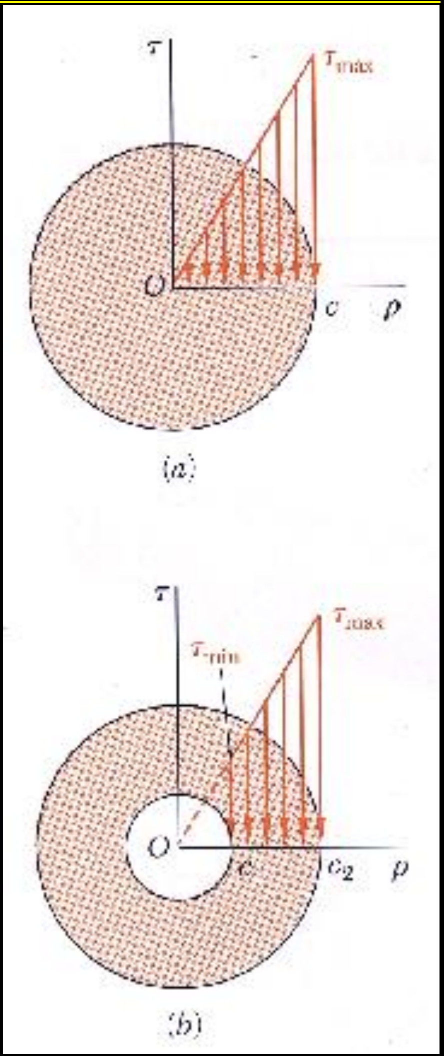


Torsional Stress - Strain

- *know* $f_v = \tau = G \cdot \gamma$ *and* $\gamma = \dfrac{\rho\phi}{L}$
- *so* $\tau = G \cdot \dfrac{\rho\phi}{L}$
- *where G is the Shear Modulus*

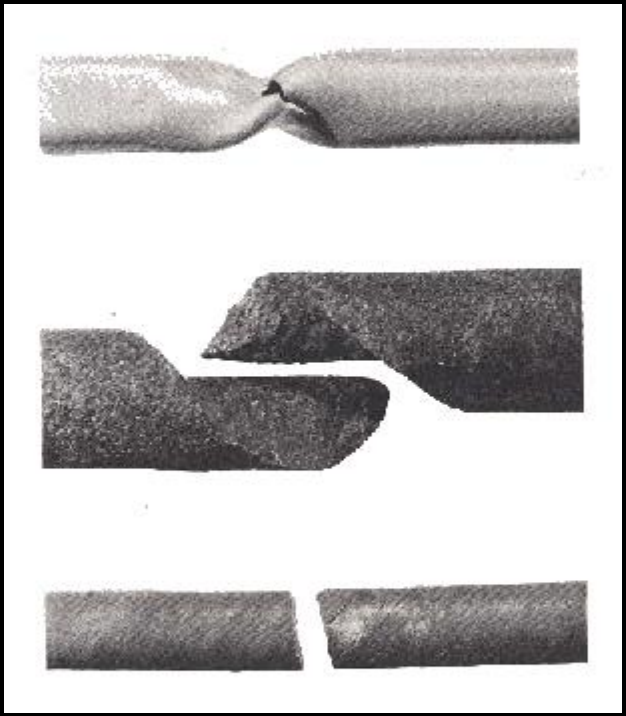
Torsional Stress - Strain

- *from* $$T = \Sigma\tau(\rho)\Delta A$$
- *can derive* $$T = \frac{\tau J}{\rho}$$
  - *where J is the polar moment of inertia*
  - *elastic range* $$\tau = \frac{T\rho}{J}$$



Shear Stress

- τ_{max} *happens at* *outer diameter*



- *combined shear and axial stresses*
  - *maximum shear stress at 45° "twisted" plane*

Shear Strain

- *knowing* $\tau = G \cdot \frac{\rho\phi}{L}$ *and* $\tau = \frac{T\rho}{J}$
- *solve:* $\phi = \frac{TL}{JG}$
- *composite shafts:* $\phi = \sum_i \frac{T_i L_i}{J_i G_i}$

Noncircular Shapes

- torsion depends on J
- plane sections don't remain plane
- τ_{max} is still at outer diameter

$$\tau_{max} = \frac{T}{c_1 ab^2} \qquad \phi = \frac{TL}{c_2 ab^3 G}$$

– where a is longer side (> b)

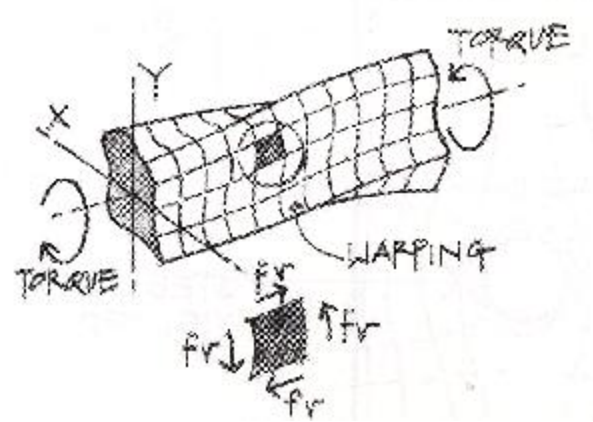
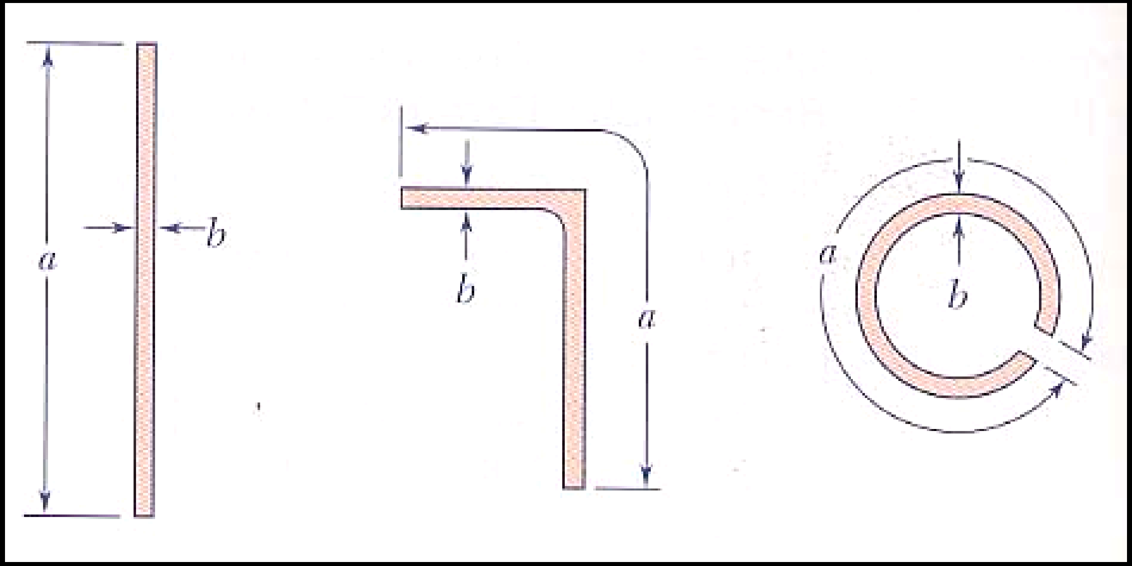


TABLE 3.1. Coefficients for Rectangular Bars in Torsion

a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

Open Thin-Walled Sections

- *with very large a/b ratios:*



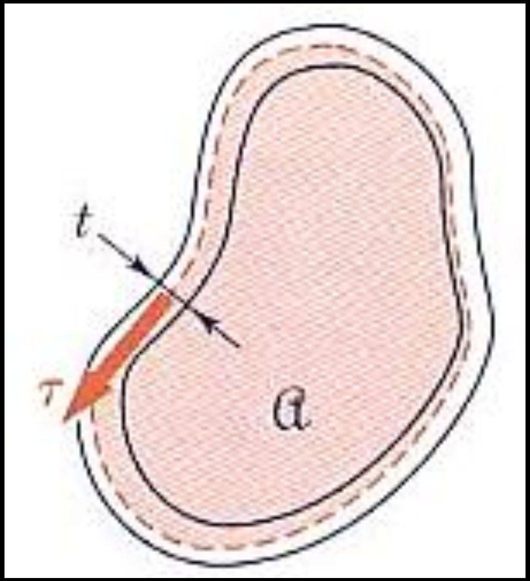
$$\tau_{max} = \frac{T}{\frac{1}{3}ab^2} \qquad \phi = \frac{TL}{\frac{1}{3}ab^3G}$$

Shear Flow in Closed Sections

- q is the internal shear force/unit length

$$\tau = \frac{T}{2t\mathcal{A}}$$

$$\phi = \frac{TL}{4t\mathcal{A}^2}\sum_i \frac{s_i}{t_i}$$

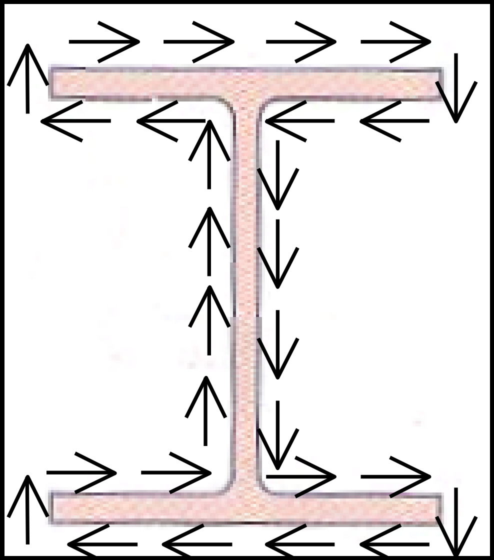


- $\mathcal{A}$ is the area bounded by the centerline
- s_i is the length segment, t_i is the thickness

Shear Flow in Open Sections

- *each segment has proportion of T with respect to torsional rigidity,*

$$\tau_{max} = \frac{Tt_{max}}{\frac{1}{3}\Sigma b_i t_i^3}$$



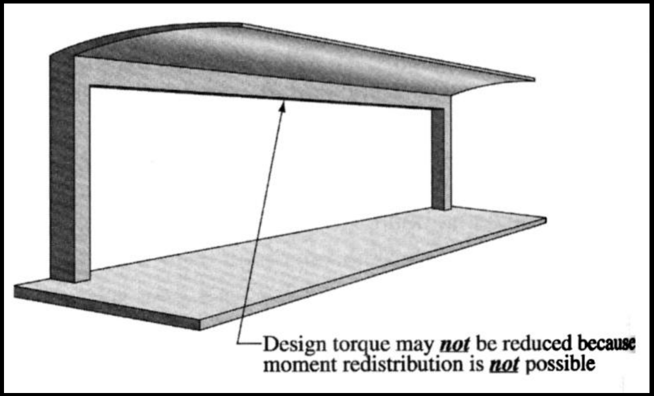
- *total angle of twist:*

$$\phi = \frac{TL}{\frac{1}{3}G\Sigma b_i t_i^3}$$

- *I beams - web is thicker, so τ_{max} is in <u>web</u>*

Torsional Shear Stress

- *twisting moment*
- *and beam shear*



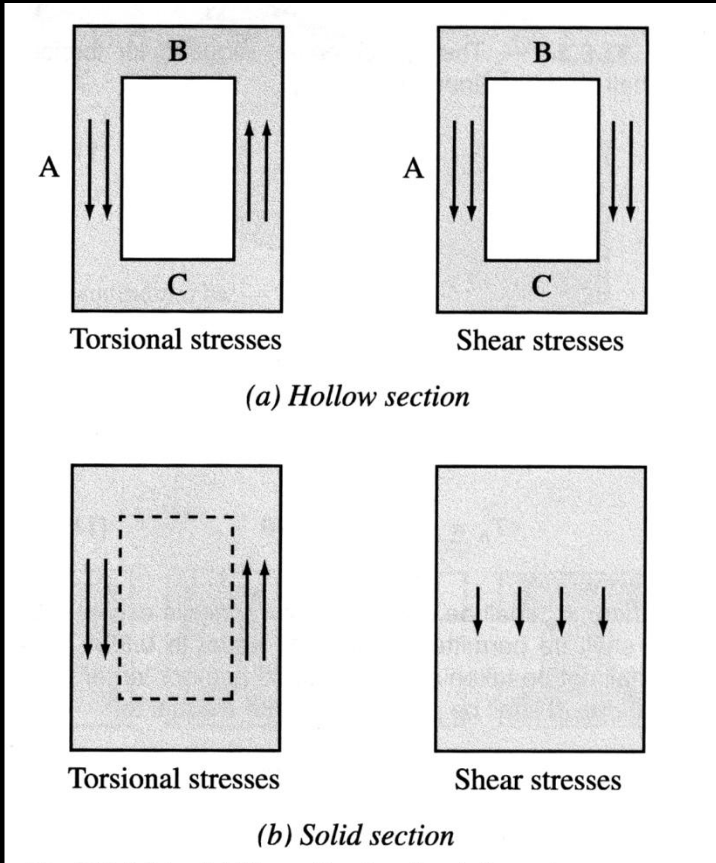


Fig. R11.6.3.1—Addition of torsional and shear stresses

Torsional Shear Reinforcement

- closed stirrups
- more longitudinal reinforcement

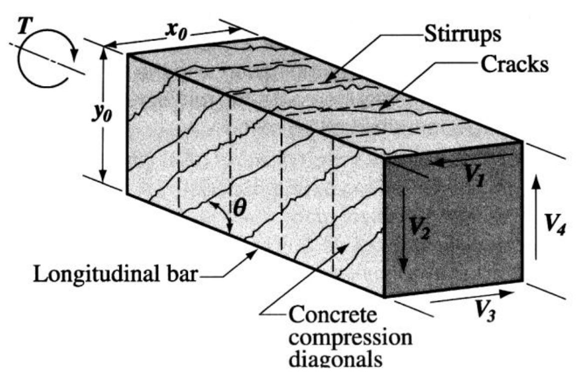


Fig. R11.6.3.6(a)—Space truss analogy

- area enclosed by shear flow

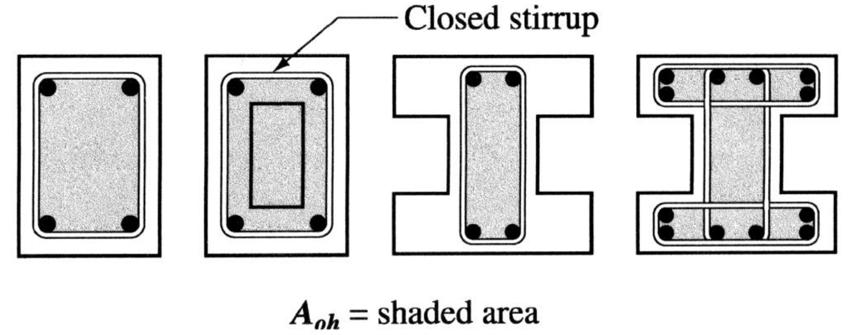
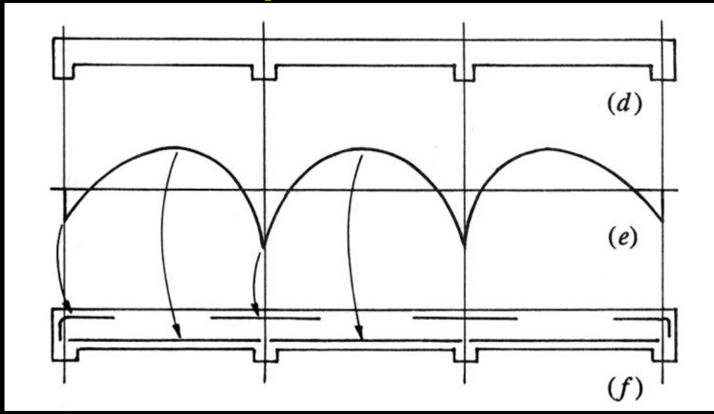


Fig. R11.6.3.6(b)—Definition of A_{oh}

Development Lengths

- *required to allow steel to yield (f_y)*
- *standard hooks*
  - *moment at beam end*



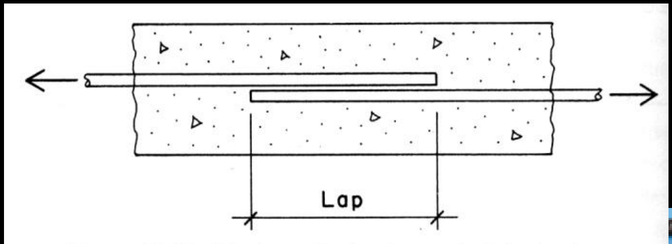
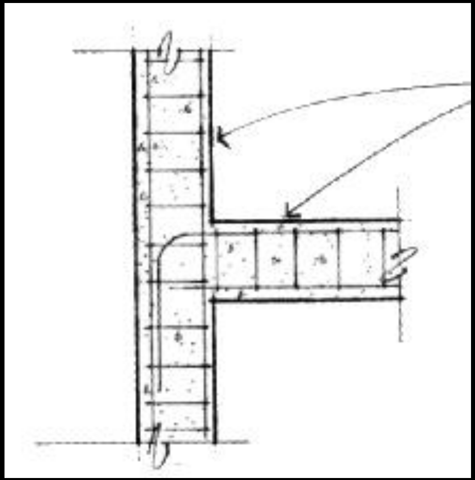


Figure 13.24 The lapped splice for steel reinforcing bars.



- *splices*
  - *lapped*
  - *mechanical connectors*

Development Lengths

- ℓ_d , embedment required <u>both</u> sides
- proper cover, spacing:
  - No. 6 or smaller

$$l_d = \frac{d_b F_y}{25\sqrt{f'_c}}$$

or 12 in. minimum

  - No. 7 or larger

$$l_d = \frac{d_b F_y}{20\sqrt{f'_c}}$$

or 12 in. minimum

Development Lengths

- hooks
  - bend and extension

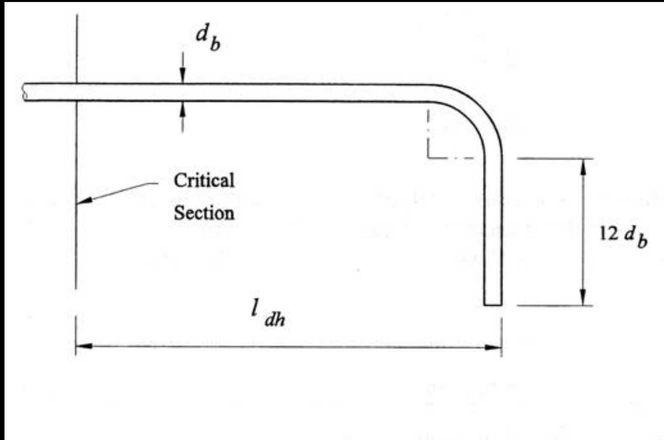


Figure 9-17: Minimum requirements for 90° bar hooks.

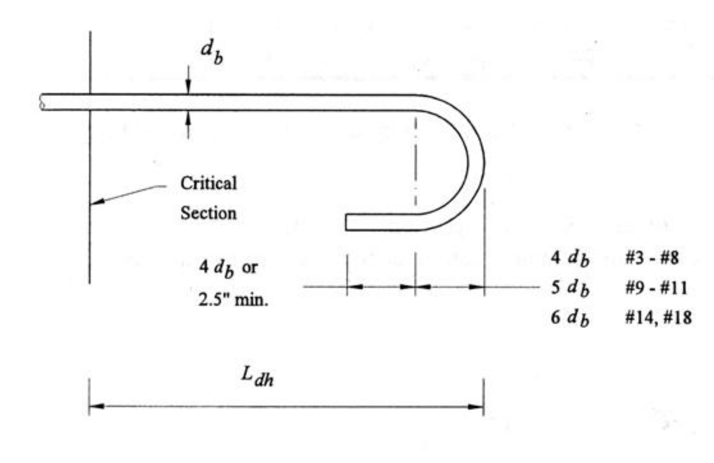


Figure 9-18: Minimum requirements for 180° bar hooks.

- minimum

$$l_{dh} = \frac{1200 d_b}{\sqrt{f'_c}}$$

Development Lengths

- bars in compression

$$l_d = \frac{0.02 d_b F_y}{\sqrt{f'_c}} \le 0.0003 d_b F_y$$

- splices
  - tension minimum is function of l_d and splice classification
  - compression minimum
  - is function of d_b and F_y

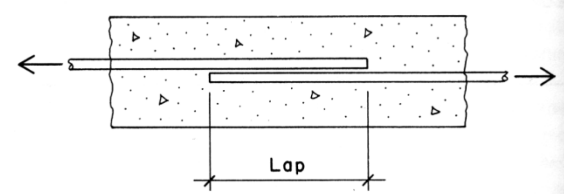


Figure 13.24 The lapped splice for steel reinforcing bars.

Concrete Deflections

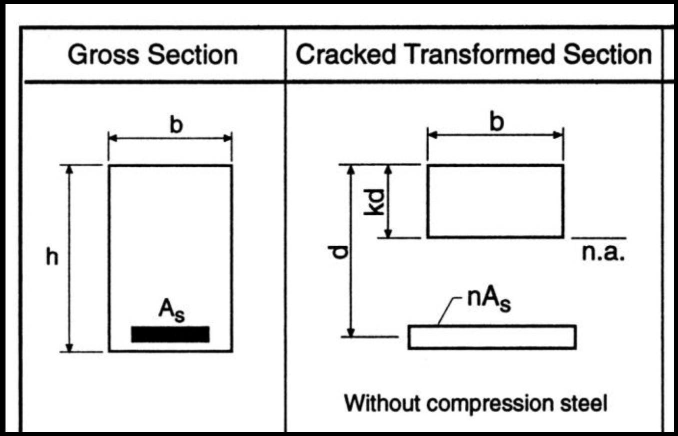
- elastic range
  - I transformed
  - E_c (with f'_c in psi)
    - normal weight concrete (~ 145 lb/ft^3)

$$E_c = 57{,}000\sqrt{f'_c}$$

    - concrete between 90 and 160 lb/ft^3

$$E_c = w_c^{1.5} 33\sqrt{f'_c}$$

- cracked
  - I cracked
  - E adjusted



Deflection Limits

- *relate to whether or not beam supports or is attached to a damageable non-structural element*
- *need to check service live load and long term deflection against these*

L/180	*roof systems (typical) – live*
L/240	*floor systems (typical) – live + long term*
L/360	*supporting plaster – live*
L/480	*supporting masonry – live + long term*