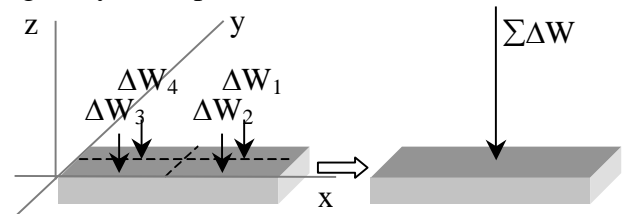


Centers of Gravity - Centroids

Notation:

<p>A = name for area</p> <p>C = designation for channel section</p> <p>= name for centroid</p> <p>F_z = force component in the z direction</p> <p>L = name for length</p> <p>O = name for reference origin</p> <p>Q_x = first moment area about an x axis (using y distances)</p> <p>Q_y = first moment area about an y axis (using x distances)</p> <p>t = name for thickness</p> <p>t_w = thickness of web of wide flange</p> <p>W = name for force due to weight</p> <p>= designation for wide flange section</p> <p>x = horizontal distance</p> <p>\bar{x} = the distance in the x direction from a reference axis to the centroid of a shape</p>	<p>\hat{x} = the distance in the x direction from a reference axis to the centroid of a composite shape</p> <p>y = vertical distance</p> <p>\bar{y} = the distance in the y direction from a reference axis to the centroid of a shape</p> <p>\hat{y} = the distance in the y direction from a reference axis to the centroid of a composite shape</p> <p>z = distance perpendicular to x-y plane</p> <p>\int = symbol for integration</p> <p>Δ = calculus symbol for small quantity</p> <p>γ = density of a material (unit weight)</p> <p>Σ = summation symbol</p>
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- The cross section shape and how it resists bending and twisting is important to understanding beam and column behavior.
- The *center of gravity* is the location of the equivalent force representing the total weight of a body comprised of particles that each have a mass gravity acts upon.



Resultant force: Over a body of constant thickness in x and y

$$\sum F_z = \sum_{i=1}^n \Delta W_i = W \qquad W = \int dW$$

Location: \bar{x} , \bar{y} is the equivalent location of the force W from all ΔW_i 's over all x & y locations (with respect to the moment from each force) from:

$$\sum M_y = \sum_{i=1}^n x_i \Delta W_i = \bar{x} W \qquad \bar{x} W = \int x dW \Rightarrow \bar{x} = \frac{\int x dW}{W} \quad \text{OR} \quad \boxed{\bar{x} = \frac{\sum(x \Delta W)}{W}}$$

$$\sum M_x = \sum_{i=1}^n y_i \Delta W_i = \bar{y} W \qquad \bar{y} W = \int y dW \Rightarrow \bar{y} = \frac{\int y dW}{W} \quad \text{OR} \quad \boxed{\bar{y} = \frac{\sum(y \Delta W)}{W}}$$

- The *centroid of an area* is the average x and y locations of the area particles

For a discrete shape (ΔA_i) of a uniform thickness and material, the weight can be defined as:

$$\Delta W_i = \gamma t \Delta A_i \quad \text{where:}$$

γ is weight per unit **volume** (= specific weight) with units of N/m^3 or lb/ft^3

$t \Delta A_i$ is the volume

So if $W = \gamma A$:

$$\bar{x} \gamma A = \int x \gamma t dA \Rightarrow \bar{x} A = \int x dA \quad \text{OR} \quad \boxed{\bar{x} = \frac{\sum(x \Delta A)}{A}} \quad \text{and similarly} \quad \boxed{\bar{y} = \frac{\sum(y \Delta A)}{A}}$$

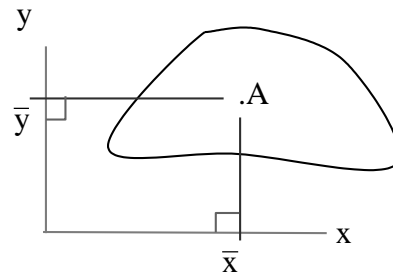
Similarly, for a line with constant cross section, a ($\Delta W_i = \gamma a \Delta L_i$):

$$\bar{x} L = \int x dL \quad \text{OR} \quad \boxed{\bar{x} = \frac{\sum(x \Delta L)}{L}} \quad \text{and} \quad \bar{y} L = \int y dL \quad \text{OR} \quad \boxed{\bar{y} = \frac{\sum(y \Delta L)}{L}}$$

- \bar{x} , \bar{y} **with respect to an x, y coordinate system** is the centroid of an area AND the center of **gravity** for a body of uniform material and thickness.

- The *first moment of the area* is like a force moment: and is the **area** multiplied by the perpendicular distance to an axis.

$$Q_x = \int y dA = \bar{y} A \quad Q_y = \int x dA = \bar{x} A$$



• Centroids of Common Shapes

Centroids of Common Shapes of Areas and Lines

Shape		\bar{x}	\bar{y}	Area
Triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

- Symmetric Areas

- An area is symmetric with respect to a line when every point on one side is mirrored on the other. The line divides the area into equal parts and the centroid will be on that axis.
- An area can be symmetric to a *center point* when every (x,y) point is matched by a (-x,-y) point. It does not necessarily have an axis of symmetry. The center point is the *centroid*.
- If the symmetry line is on an axis, the centroid location is on that axis (value of 0). With double symmetry, the centroid is at the intersection.
- Symmetry can also be defined by areas that match across a line, but are 180° to each other.

Basic Steps

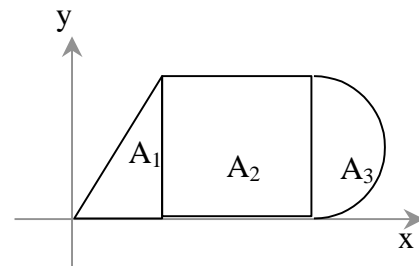
1. Draw a reference origin.
2. Divide the area into basic shapes
3. Label the basic shapes (components)
4. Draw a table with headers of *Component, Area, \bar{x} , $\bar{x}A$, \bar{y} , $\bar{y}A$*
5. Fill in the table value
6. Draw a summation line. Sum all the areas, all the $\bar{x}A$ terms, and all the $\bar{y}A$ terms
7. Calculate \hat{x} and \hat{y}

- Composite Shapes

If we have a shape made up of basic shapes that we know centroid locations for, we can find an “average” centroid of the areas.

$$\hat{x}A = \hat{x} \sum_{i=1}^n A_i = \sum_{i=1}^n \bar{x}_i A_i \quad \hat{y}A = \hat{y} \sum_{i=1}^n A_i = \sum_{i=1}^n \bar{y}_i A_i$$

$$\text{OR} \quad \hat{x} = \frac{\sum \bar{x}A}{A} \quad \hat{y} = \frac{\sum \bar{y}A}{A}$$



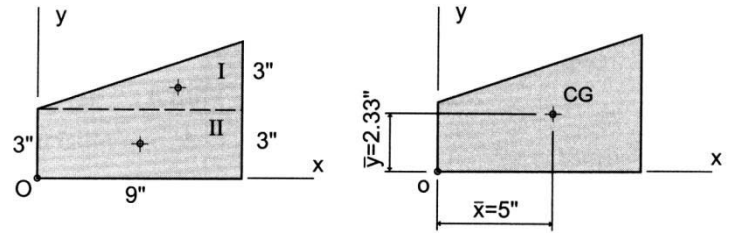
Centroid values can be negative.

Area values can be negative (holes)

Example 1 (pg 243)

Example Problem 7.1: Centroids (Figures 7.5 and 7.6)

Determine the centroidal x and y distances for the composite area shown. Use the lower left corner of the trapezoid as the reference origin.



Component	Area (ΔA) (in. ²)	\bar{x} (in.)	$\bar{x}\Delta A$ (in. ³)	\bar{y} (in.)	$\bar{y}\Delta A$ (in. ³)
<p>(a)</p>	$\frac{9(3)}{2} = 13.5 \text{ in.}^2$	6"	81 in. ³	4"	54 in. ³
<p>(b)</p>	$9(3) = 27 \text{ in.}^2$	4.5"	121.5 in. ³	1.5"	40.5 in. ³
	$A = \sum \Delta A = 40.5 \text{ in.}^2$		$\sum \bar{x}\Delta A = 202.5 \text{ in.}^3$		$\sum \bar{y}\Delta A = 94.5 \text{ in.}^3$

$$\hat{x} = \frac{202.5 \text{ in.}^3}{40.5 \text{ in.}^2} = 5 \text{ in}$$

$$\hat{y} = \frac{94.5 \text{ in.}^3}{40.5 \text{ in.}^2} = 2.33 \text{ in}$$

Example 2 (pg 245)

Example Problem 7.3b (Figure 7.13)

An alternate method that can be employed in solving this problem is referred to as the *negative area method*.

A 6" thick concrete wall panel is precast to the dimensions as shown. Using the lower left corner as the reference origin, determine the center of gravity (centroid) of the panel.

