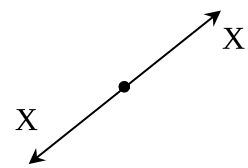
Equilibrium of a Particle & Truss Analysis

Notation:

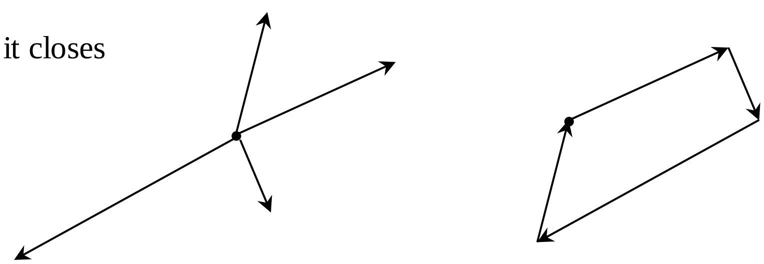
b = number of members in a truss
(C) = shorthand for *compression*
F = name for force vectors, as is X, *and* P
F_{AB} = name of a truss force between joints named A and B, *ex.*
FBD = free body diagram
F_x = force component in the x direction, as is T_x
F_y = force component in the y direction, as is T_y
h = cable sag height
L = span length
n = number of joints in a truss
N = normal force (perpendicular to something)
R = name for resultant vectors
R_x = resultant component in the x direction
R_y = resultant component in the y direction
T = name for a tension force
(T) = shorthand for *tension*
x = x axis direction, or horizontal dimension
y = y axis direction, or vertical dimension
W = name for force due to weight
μ = coefficient of static friction
θ = angle, in a trig equation, ex. $\sin\theta$, that is measured between the x axis and *tail* of a vector
Σ = summation symbol

- EQUILIBRIUM is the state where the resultant of the forces on a particle or a rigid body is *zero*. There will be no rotation or translation. The forces are referred to as balanced.

ex: 2 forces of same size, opposite direction



ex: 4 forces, polygon rule shows that it closes



- Analytically, for a point:

$$R_x = \sum F_x = 0 \qquad R_y = \sum F_y = 0 \text{ (scalar addition)}$$

- NEWTON'S FIRST LAW: If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

<u>Collinear Force System</u>

- *All forces act along the same line.* Only one equilibrium equation is needed: $\sum F_{(in-line)} = 0$
- Equivalently: $R_x = \sum F_x = \mathbf{0}$ and $R_y = \sum F_y = \mathbf{0}$

<u>Concurrent Force System</u>

- *All forces act through the same point.* Only two equilibrium equations are needed:
$$R_x = \sum F_x = \mathbf{0} \text{ and } R_y = \sum F_y = \mathbf{0}$$
- It is ABSOLUTELY NECESSARY to consider all the forces acting on a body (applied directly and indirectly) using a FREE BODY DIAGRAM. Omission of a force would ruin the conditions for equilibrium.
- FREE BODY DIAGRAM (aka FBD): Sketch of a significant isolated particle of a body or structure showing all the forces acting on it. Forces can be from
  - externally applied forces
  - weight of the rigid body
  - reaction forces or constraints
  - forces developed within a section member
- How to solve when there are more than three forces on a free body:
  1. *Resolve all forces into x and y components using known and unknown forces and angles. (Tables are helpful.)*
  2. *Determine if any unknown forces are related to other forces and write an equation.*
  3. *Write the two equilibrium equations (in x and y).*
  4. *Solve the equations simultaneously when there are the same number of equations as unknown quantities. (see math handout)*
- Common problems have unknowns of: 1) **magnitude of force**
  2) **direction of force**

<u>*FREE BODY DIAGRAM STEPS FOR A POINT;*</u>

1. Determine the point of interest. (What point is in equilibrium?)
2. Detach the point from and all other bodies (*"free" it).*

3. Indicate all external forces which include:

    - action <u>on</u> the point by the **supports & connections**

    - action on the point by other bodies

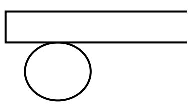
    - the weigh effect (=force) of any body attached to the point (force due to gravity)

4. All forces should be clearly marked with magnitudes and direction. The sense of forces should be those acting *on the point* not <u>from</u> the point.

5. Dimensions/angles should be included for force component computations.

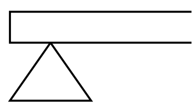
6. Indicate the <u>unknown</u> forces, such as those reactions or constraining forces where the body is supported or connected.

- *Force Reactions* can be categorized by the type of connections or supports. A force reaction is a force with known line of action, or a force of unknown direction. The line of action of the force is directly related to the motion that is prevented.



prevents motion:

up and down



prevents motion:

vertical & horizontal

- The line of action should be indicated on the FBD. The sense of direction is determined by the type of support. (Cables are in tension, etc…) *If the sense isn't obvious, assume a sense.* When the reaction value comes out positive, the assumption was correct. When the reaction value comes out negative, the assumption was *opposite* the actual sense. *DON'T CHANGE THE ARROWS ON YOUR FBD OR SIGNS IN YOUR EQUATIONS.*

- With the 2 equations of equilibrium for a point, <u>there can be no more than 2 unknowns</u>.

Friction

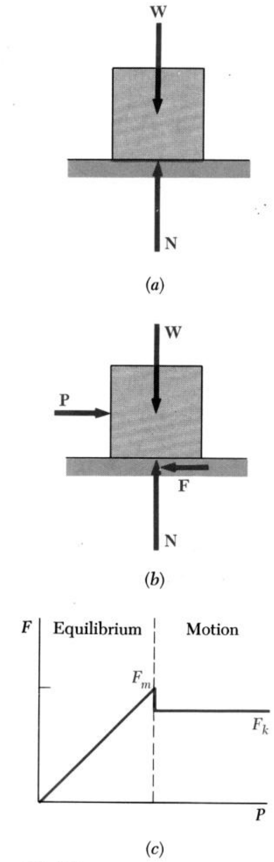
- There will be a force of resistance to movement developed at the contact face between objects when one is made to slide against the other. This is known as *static friction* and is determined from the reactive force, *N*, which is normal to the surface, and a coefficient of friction, μ, which is based on the materials in contact.

$$F = \mu N$$

- If the static friction force is exceeded by the pushing force, there will still be friction, but it is called *kinetic friction*, and it is smaller than static friction, so it is moving.

- The friction resistance is independent of the amount of contact area.

Materials	μ range
Metal on ice	0.03-0.05
Metal on metal	0.15-0.60
Metal on wood	0.20-0.60
Metal on stone	0.30-0.70
Wood on wood	0.30-0.70
Steel on steel	0.75
Stone on stone	0.40-0.70
Rubber on concrete	0.60-0.90
Aluminum on aluminum	1.10-1.70



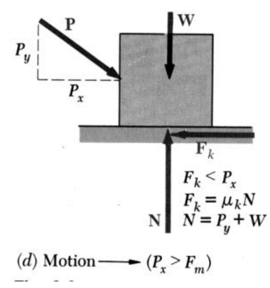


Fig. 8.1

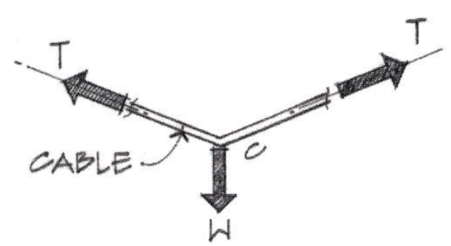
- CABLE STRUCTURES:

  Cables have the same tension all along the length if they are not cut. The force *magnitude* is the same everywhere in the cable even if it changes angles. Cables CANNOT be in compression. (They flex instead.)

  High-strength steel is the most common material used for cable structures because it has a high strength to weight ratio.

  Cables must be supported by vertical supports or towers and must be anchored at the ends. Flexing or unwanted movement should be resisted. (Remember the Tacoma Narrows Bridge?)

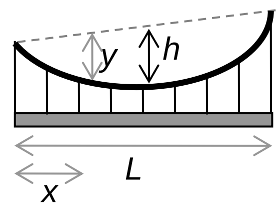
  Cables with a single load have a **concurrent** force system. It will only be in equilibrium if the cable is **symmetric**.



  The forces anywhere in a *straight segment* can be resolved into x and y components of $T_x = T\cos\theta$ and $T_y = T\sin\theta$.

The shape of a cable having a *uniform distributed load* is almost parabolic, which means the geometry and cable length can be found with:

$$y = 4h(Lx - x^2)/L^2$$



where y is the vertical distance from the straight line from cable start to end

h is the vertical sag (maximum y)

x is the distance from one end to the location of y

L is the horizontal span.

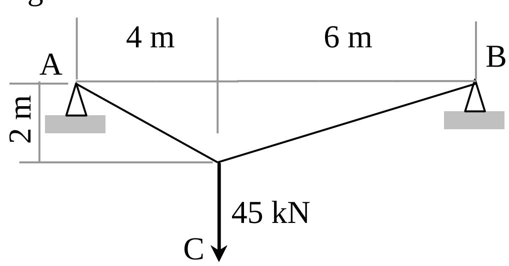
$$L_{total} = L\left(1 + \tfrac{8}{3}\tfrac{h^2}{L^2} - \tfrac{32}{5}\tfrac{h^4}{L^4}\right)$$

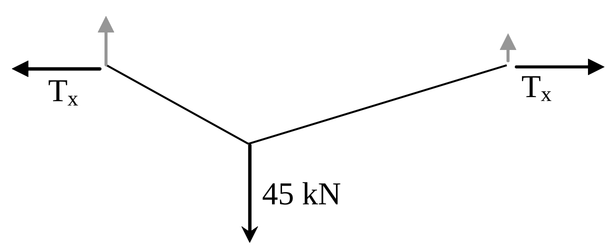
where L_{total} is the total length of parabolic cable

h and L are defined above.

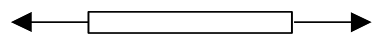
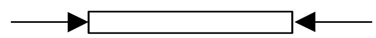
Cables with Several Concentrated Loads or Fixed Geometry

- In order to completely constrain cables, the number of unknown support reactions *will be more* than the available number of equilibrium equations. We can solve because we have additional equations from geometry due to the **slope** of the cable.
- The tension in the cable IS NOT the same everywhere, but the horizontal component in a cable segment WILL BE.





Truss Structures

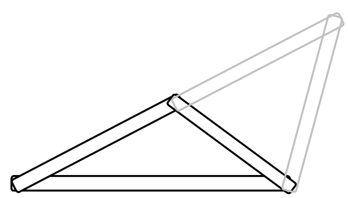
- A truss is made up of straight two-force members connected at its ends. The triangular arrangement produces stable geometry. Loads on a truss are applied at the joints only.
- Joints are pin-type connections (resist translation, not rotation).
- Forces of action and reaction on a joint must be equal and opposite.
- Members in TENSION are being pulled. 
- Members in COMPRESSION are being squeezed. 
- External forces act on the joints.

- Truss configuration:

  Three members form a rigid assembly with **3 (three)** connections.

  To add members and still have a rigid assembly, **2 (two)** more must be added with one connection between.

  For rigidity: b = 2n – 3, where *b* is number of members and *n* is number of joints



Method of Joints

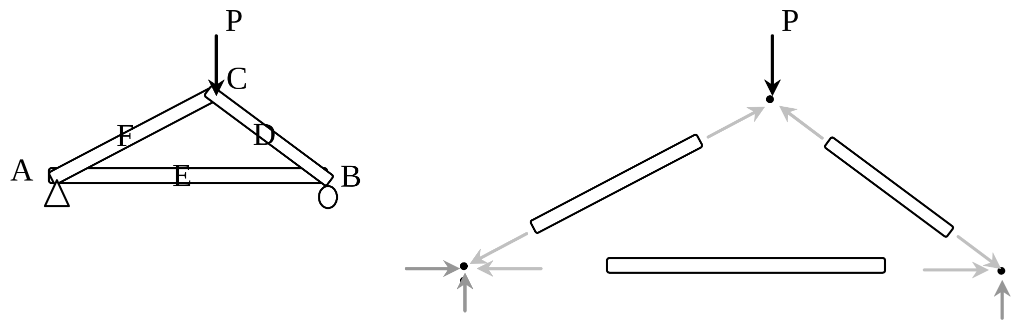
- The method takes advantage of the conditions of equilibrium at each joint.

1. Determine support reaction forces.
2. Draw a FBD of each member AND each joint
3. Identify geometry of angled members
4. Identify zero force members and other special (easy to solve) cases
5. Each pin is in equilibrium ($\sum F_x = 0$ and $\sum F_y = 0$ for a concurrent force system)
6. Total equations = 2n = b+3 (one force per member + 3 support reactions)

Advantages: Can find every member force
Disadvantages: Lots of equations, easy to lose track of forces found.

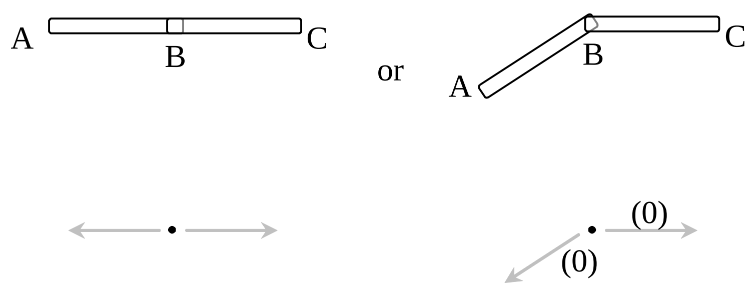
Tools available:

Tip-to-tail method for 3 joint forces must close
Analytically, there will be at most 2 unknowns with 2 equilibrium equations.



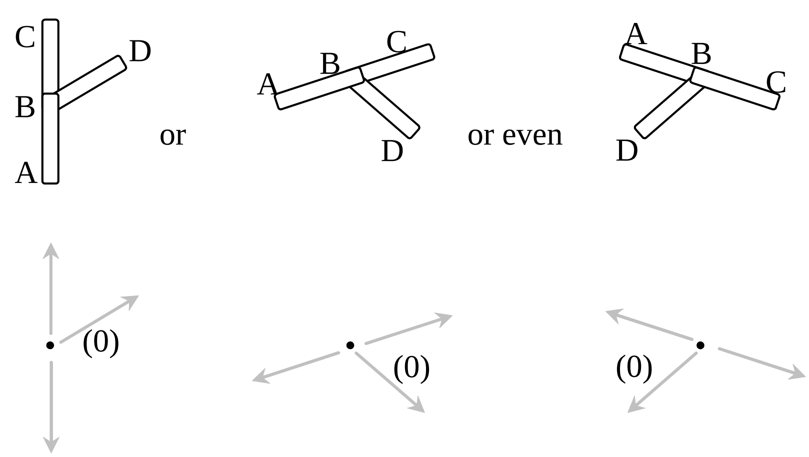
Joint Configurations (special cases to recognize for faster solutions)

Case 1) Two Bodies Connected



F_{AB} has to be **equal (=)** to F_{BC}

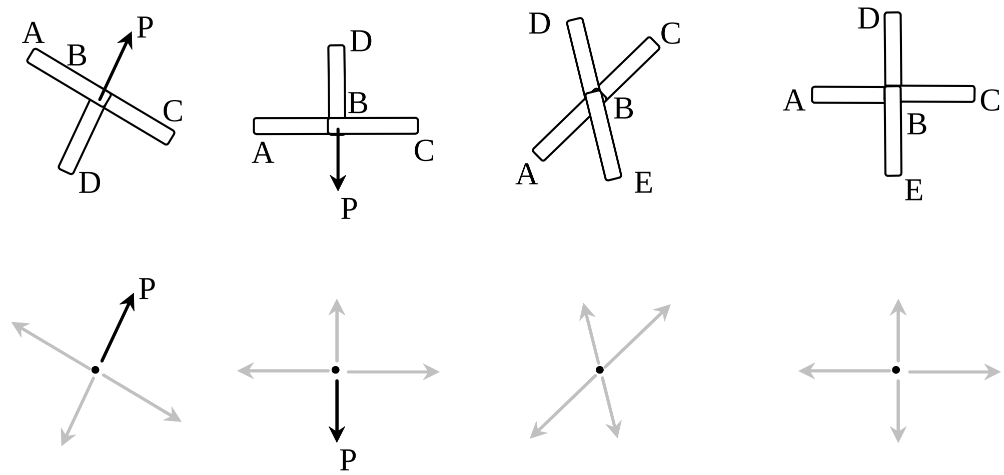
Case 2) Three Bodies Connected with Two Bodies in Line



F_{AB} and F_{BC} have to be equal, and F_{BD} has to be **0 (zero).**

Case 3) Three Bodies Connected and a Force – 2 Bodies aligned & 1 Body and a Force are Aligned

Four Bodies Connected - 2 Bodies Aligned and the Other 2 Bodies Aligned



F_{AB} has to equal F_{BC}, and [F_{BD} has to equal P] or [F_{BD} has to equal F_{BE}]

Graphical Analysis

The method utilizes what we know about force triangles and plotting force magnitudes *to scale*.

1. Draw an accurate form diagram of the truss at a convenient scale with the loads and support reaction forces.

2. Determine the support reaction forces.

3. Working clockwise and from left to right, apply interval notation to the diagram, assigning capital letters to the spaces between external forces and numbers to internal spaces.

4. Construct a load line to a convenient scale of length to force by using the interval notation and working clockwise around the truss from the upper left plotting the lengths of the vertical and horizontal loads.

5. Starting at a left joint where we know there are fewer than three forces, we draw reference lines in the direction of the unknown members so that they intersect. Label the intersection with the number of the internal space.

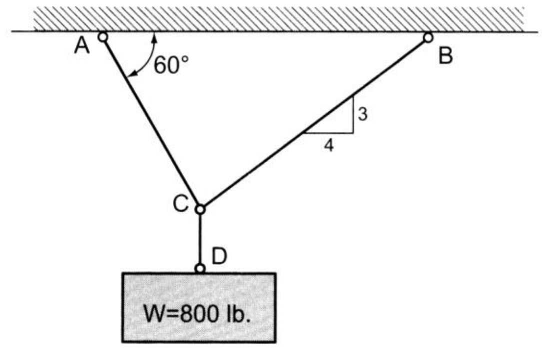
6. Go to the next joint (clockwise and left to right) with two unknown forces and repeat for all joints. The diagram should close.

7. Measure the line segments and apply interval notation to determine their sense: Proceeding clockwise around the joint, follow the notation. The direction toward the joint is compressive. The direction away from the joint is tensile.

Example 1 (pg 49)

Example Problem 3.1: Equilibrium of a Particle

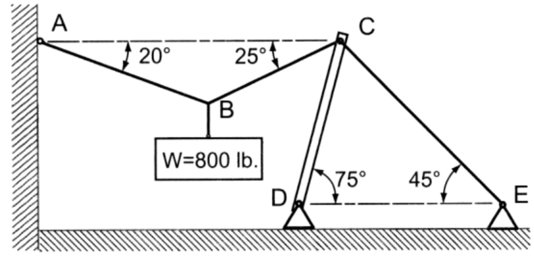
Two cables, shown in Figure 3.8, are used to support a weight W = 800 lb., suspended at concurrent point $\boldsymbol{C}$. Determine the tension developed in cables CA and CB for the system to be in equilibrium. Solve this problem analytically and check the answer graphically.



Example 2 (pg 56)

Example Problem 3.5

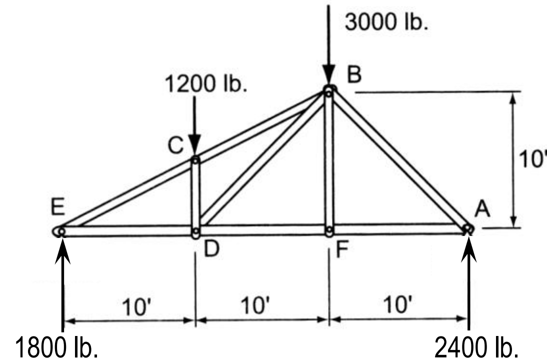
A compound cable system supports a weight $W = 800$ lb. at point ***B***, as shown in Figure 3.18. Cable *BA* is attached to a wall support at ***A*** and concurrent point ***C*** is supported by a compression strut *DC*. Determine all of the cable forces and the compression in strut *DC*.

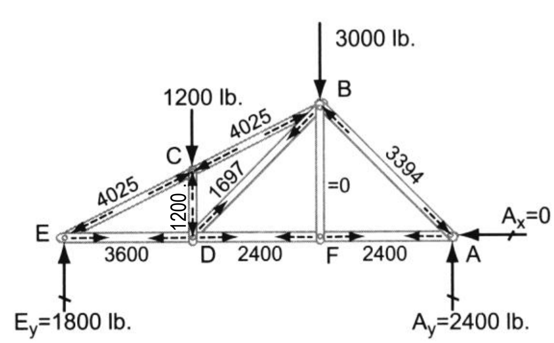


Example 3 (pg 90)

Example Problem 4.1 (Method of Joints)

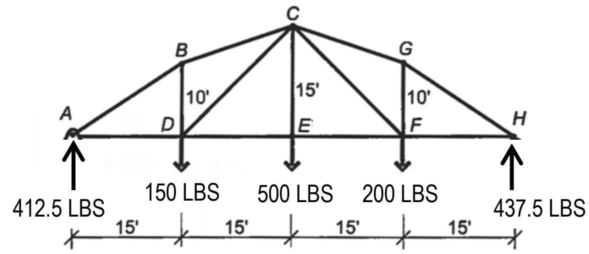
An asymmetrical roof truss, shown in Figure 4.4, supports two vertical roof loads. ~~Determine the support reactions at each end, then,~~ Using the method of joints, solve for all member forces. ~~Summarize the results of all member forces on a FBD (this diagram is referred to as a *force summation diagram*)~~.





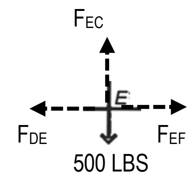
Example 4

Using the method of joint, determine all member forces.



SOLUTION:

Find the joints with 2 (or less unknowns) for FBD's : *A* and *H,* while looking for any special cases like *E,* which has "crossed" members and forces. $F_{DE} = F_{EF}$ and F_{EC} = 500 lb (tension).
(Check off members found:
AB, BD, AD, BC, DC, DE, ~~EC~~, EF, CG, CF, FG, GH, FH)

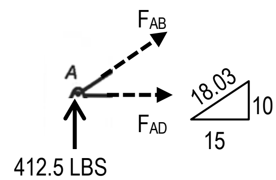


Let's use *A* first (but H is just as acceptable). Draw the point, adding the known force, and draw the unknown member forces ***away*** from the point, assuming tension (shown as dashed). Find the geometry of member AB from the rise of 10 ft and the run of 15 ft. The hypotenuse will be $\sqrt{10^2+15^2}$ = 18.03:

$$\Sigma F_x = F_{AD} + F_{AB}\frac{15}{18.03} = 0$$

$$\Sigma F_y = 412.5^{lb} + F_{AB}\frac{10}{18.03} = 0$$ F_{AB} =(-412.5)*18.03/10 = -743.7 lb (C)

and substituting the (negative) value of F_{AB} into the ΣF_x , F_{AD} = 618.75 lb (T)
(Check off members found: ~~AB~~, BD, ~~AD~~, BC, DC, DE, ~~EC~~, EF, CG, CF, FG, GH, FH)



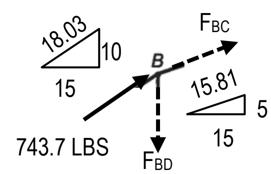
Review which joints have 2 (or less) unknowns: *B* and *H.*
Let's use *B*. Because we know F_{AB} is in ***compression*** we draw the force ***into*** the point. We need the geometry of member BC with a rise of 5 (15-10) and a run of 15 with a hypotenuse of $\sqrt{5^2+15^2}$ = 15.81:

$$\Sigma F_x = 743.7^{lb}\frac{15}{18.03} + F_{BC}\frac{15}{15.81} = 0$$ F_{BC} = -652.1 lb (C)

$$\Sigma F_y = 743.7^{lb}\frac{10}{18.03} + F_{BC}\frac{5}{15.81} - F_{BD} = 0$$ (substituting the *negative* value of F_{BC})

F_{BD} = 206.2 lb (T)

(Check off members found: ~~AB~~, ~~BD~~, ~~AD~~, ~~BC~~, DC, DE, ~~EC~~, EF, CG, CF, EF, FG, GH, FH)

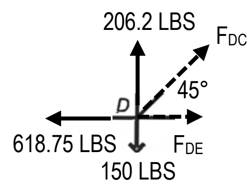


Review which joints have 2 (or less) unknowns: *D* and *H.*
Let's use *D*. Both F_{AD} and F_{BD} are tensile, so we can draw them *away*. The geometry of DE is familiar with the rise the same as the run for an angle of 45°:

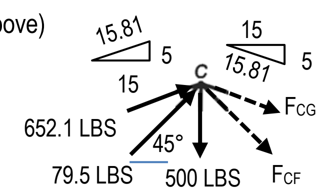
$$\Sigma F_x = -618.75^{lb} + F_{DC}\cos 45° + F_{DE} = 0$$

$$\Sigma F_y = -150^{lb} + 206.2^{lb} + F_{DC}\sin 45° = 0$$ F_{DC} = -79.5 lb (C)

and substituting the (negative) value of F_{DC} into the ΣF_x , F_{DE} = 675.0 lb (T) = F_{EF} (! from above)
(Check off members found: ~~AB~~, ~~BD~~, ~~AD~~, ~~BC~~, ~~DC~~, ~~DE~~, ~~EC~~, ~~EF~~, CG, CF, FG, GH, FH)



Review which joints have 2 (or less) unknowns: *C* and *H.*
Let's use *C*. Draw F_{DC} and F_{BC} as compressive forces. And the geometry has been found due to symmetry, with the angle of F_{CF} a *negative* 45°:



$$F_x = 652.1^{lb}\frac{15}{15.81} + 79.5^{lb}\cos 45° + F_{CF}\cos(-45°) + F_{CG}\frac{15}{15.81} = 0$$

$$\Sigma F_y = 652.1^{lb}\frac{5}{15.81} + 79.5^{lb}\sin 45° - 500^{lb} + F_{CF}\sin(-45°) - F_{CG}\frac{5}{15.81} = 0$$

Solve simultaneously because there isn't an easy way to find one unknown equal to a value multiplied by the other unknown:

$$\Sigma F_x = 674.9^{lb} + 0.707F_{CF} + 0.949F_{CG} = 0$$

$$\Sigma F_y = -237.6^{lb} - 0.707F_{CF} - 0.316F_{CG} = 0$$

add: $$437.5^{lb} + 0F_{CF} + 0.633F_{CG} = 0$$ F_{CG} = -690.8 lb (C) and substituting, F_{CF} = -27.6 lb (C)

(Check off members found: ~~AB~~, ~~BD~~, ~~AD~~, ~~BC~~, ~~DC~~, ~~DE~~, ~~EC~~, ~~EF~~, ~~CG~~, ~~CF~~, FG, GH, FH)

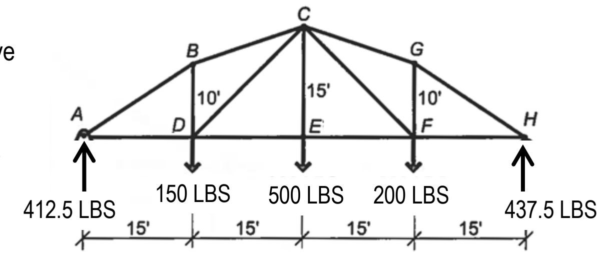
Example 4 (continued)

Review which joints have 2 (or less) unknowns: *G, F* and *H.*
Let's use *F* (because H really looks like *A* mirrored). Draw F_{CF} as compressive and F_{EF} in tension. The angle from for F_{CF} is negative 45°:

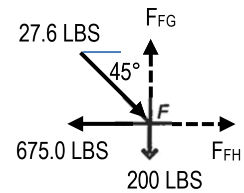
$$\Sigma F_x = -675.0^{lb} + 27.6^{lb}\cos(-45°) + F_{FH} = 0 \qquad F_{FH} = 655.5 \text{ lb (T)}$$

$$\Sigma F_y = 27.6^{lb}\sin(-45°) - 200^{lb} + F_{FG} = 0 \qquad F_{FG} = 219.5 \text{ lb (T)}$$

(Check off members found:
~~AB~~, ~~BD~~, ~~AD~~, ~~BC~~, ~~DC~~, ~~DE~~, ~~EC~~, ~~EF~~, ~~CG~~, ~~CF~~, ~~FG~~, GH, ~~FH~~)



Review which joints have 2 (or less) unknowns; which are *G* and *H.*
Let's use *G* and pretend that we have only found F_{GF} (and not F_{CG}) in order to show a set of equations that use substitution with the algebra. The geometry has been found due to symmetry:



$$\Sigma F_x = -F_{CG}\frac{15}{15.81} + F_{GH}\frac{15}{18.03} = 0 \qquad \text{rearranging: } F_{CG} = F_{GH}\frac{15}{18.03}\cdot\frac{15.81}{15} = F_{GH}\frac{15.81}{18.03}$$

$$\Sigma F_y = F_{CG}\frac{5}{15.81} - F_{GH}\frac{10}{18.03} - 219.5^{lb} = 0$$

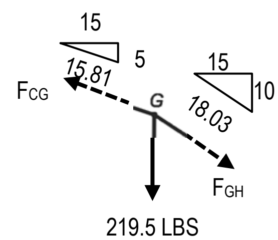
Substituting:

$$\Sigma F_y = (F_{GH}\frac{15.81}{18.03})\frac{5}{15.81} - F_{GH}\frac{10}{18.03} - 219.5^{lb} = 0$$

Simplifying $\quad -0.277F_{GH} = 219.5^{lb} \qquad F_{GH} = -791.6$ lb (C)

and $F_{CG} = -694.1$ lb (C) (*which validates the earlier answer found of 690.8 lb (C) with respect to rounding errors in all fractions and trig functions)*

(Check off members found: ~~AB~~, ~~BD~~, ~~AD~~, ~~BC~~, ~~DC~~, ~~DE~~, ~~EC~~, ~~EF~~, ~~CG~~, ~~CF~~, ~~FG~~, ~~GH~~, ~~FH~~)



(Typically, the last joint left will verify that the joint is in equilibrium with values found, but in this exercise the last joint was used to show the algebraic method of substitution.)