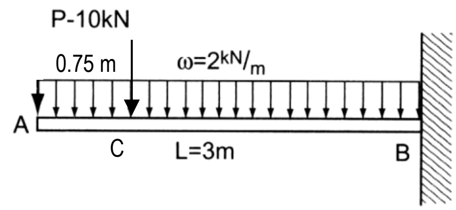


Example 1 (*changed from* pg 284) (*superpositioning*)

Example Problem 8.5 (Semi-Graphical Method)

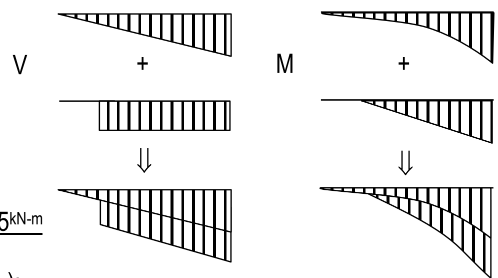
A cantilever beam supports a uniform load of $\omega = 2\,^{kN}/_{m}$ over its entire span, plus a concentrated load of 10 kN at the free end. *Investigate using Beam Diagrams and Formulas.*

SOLUTION:

By examining the support conditions, we are looking for a cantilevered beam from Cases 18 through 23.

There is a case for uniformly distributed load across the span (19) and for a load at any point (21).

For both these cases, it shows that the maximum shear AND maximum moment are located at the fixed end. If we add the values, the shear and diagrams should look like this:



We can find the maximum shear (at B) from $V = P + wl = 10^{kN} + 2^{kN/m}\cdot 3^{m} = \underline{16^{kN}}$

The maximum moment (at B) will be $M = Pb + wl^2/2 = 10^{kN}\cdot 2.25^{m} + 2^{kN/m}(3^{m})^2/2 = \underline{31.5^{kN\text{-}m}}$

The key values for the diagrams can be found with the general equations (V_x and M_x):

$V_{C\leftarrow} = -wx = -2^{kN/m}\cdot 0.25^{m} = -0.5^{kN}$; $V_{C\rightarrow} = -P - wx = -10^{kN} - 2^{kN/m}\cdot 0.25^{m} = -10.5^{kN}$

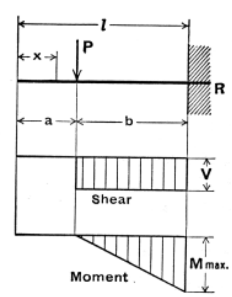
$M_C = -wx^2/2 = 2^{kN/m}(0.25^{m})^2/2 = 0.0625^{kN\text{-}m}$

We can find the maximum deflection by looking at the cases. Both say $\Delta_{max.}$ (at free end), so the values can be added directly. *Superpositioning of values must be at the same x location.* Assume E = 70x10³ MPa and I = 45x10⁶ mm⁴

$$\Delta_{total} = \frac{Pb^2}{6EI}(3l-b) + \frac{wl^4}{8EI} = \frac{10^{kN}(2.25m)^2(10^3\,^{mm}/_{m})^3(10^3\,^{N}/_{kN})}{6(70\times10^3\,MPa)(45\times10^6\,mm^4)}(3\cdot 3m - 2.25m) + \frac{2^{kN/m}(3m)^4(10^3\,^{mm}/_{m})^3(10^3\,^{N}/_{kN})}{8(70\times10^3\,MPa)(45\times10^6\,mm^4)}$$

= 18.08 mm + 6.43 mm = 24.5 mm

21. CANTILEVER BEAM—CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load . . . . = $\frac{8Pb}{l}$

R = V . . . . = P

M max. (at fixed end) . . . . . = Pb

M_x (when x > a) . . . . . = P (x − a)

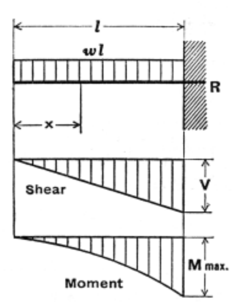
Δmax. (at free end) . . . . . = $\frac{Pb^2}{6EI}(3l - b)$

Δa (at point of load) . . . . = $\frac{Pb^3}{3EI}$

Δx (when x < a) . . . . . = $\frac{Pb^2}{6EI}(3l - 3x - b)$

Δx (when x > a) . . . . . = $\frac{P(l-x)^2}{6EI}(3b - l + x)$

19. CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . . = $4wl$

R = V . . . . = wl

V_x . . . . = wx

M max. (at fixed end) . . . . . = $\frac{wl^2}{2}$

M_x . . . . = $\frac{wx^2}{2}$

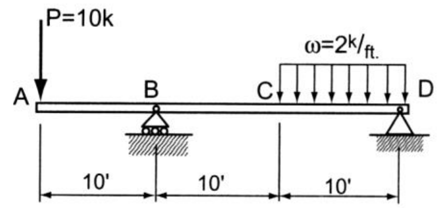
Δmax. (at free end) . . . . . = $\frac{wl^4}{8EI}$

Δx . . . . = $\frac{w}{24EI}(x^4 - 4l^3x + 3l^4)$

Example 2 (pg 275) (*superpositioning*)

Example Problem 8.2(Equilibrium Method)

Draw V and M diagrams for an overhang beam (Figure 8.12) loaded as shown. Determine the critical V_{max} and M_{max} locations and magnitudes *using Beam Diagrams and Formulas.*

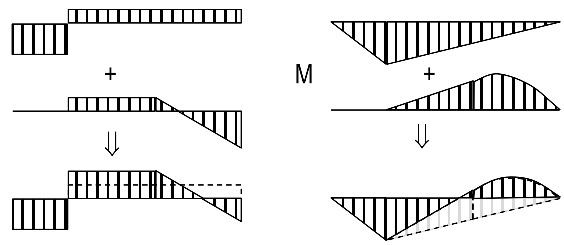


SOLUTION:

By examining the support conditions, we are looking for beam with an overhang on one end from Cases 24 through 28. (Even though the overhang is on the right, and not on the left like our beam, we can still use the information by recognizing that we can mirror the figure about the left end.)

There is a case for a load at the end (26) but none for a load in between the supports. This is because it behaves exactly like a simply supported beam in this instance (no shear or bending on the overhang). The case for this is #5 (reversed again).

If we "flip" the diagrams (both vertically and horizontally) and add the values, the resulting shear and bending moment should look like this:



We still have to find the peak values of shear and the location of the zero shear to find the critical moment values.

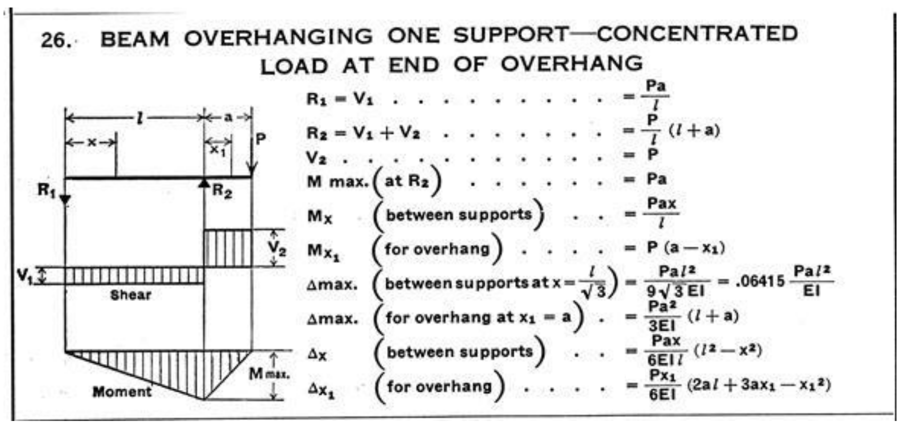
(Notice R_1 is shown *down*.)

$$R_{1(D)} = -\frac{Pa}{l} + \frac{wa}{2l}(2l - a) = -\frac{10^k \cdot 10^{ft}}{20^{ft}} + \frac{2^{k/ft} \cdot 10^{ft}}{2 \cdot 20^{ft}}\left(2 \cdot 20^{ft} - 10^{ft}\right) = 10^k$$

$$R_{2(B)} = 10^{ft} + 2^{k/ft} \cdot 10^{ft} - 10^k = 20^k$$ (from the total downward load – $R_{1(D)}$)

$V_A = -10^k$ $V_B = -10^k + 20^k = 10^k$
$V_D = 10^k - 2^{k/ft}(10ft) = -10^k$
x (from B) = $10^k/2^{k/ft} = 5$ ft
$M_B = -10^k(10^{ft}) = -100^{k\text{-}ft}$
($M_C = -100^{k\text{-}ft} + 10^k(10^{ft}) = 0$)
$M_x = 0 + 10^{k/ft}(5^{ft})/2 = 25^{k\text{-}ft}$

$V_{MAX} = 10^k$ $M_{MAX} = -100^{k\text{-}ft}$



We can calculate the deflection between the supports. (And at the end for case 5 if we derive the slope!) Assume E = 29x10³ ksi and I = 103 in⁴

We'll investigate the maximum between the supports from case 26 (because it isn't obvious where the maximum will be.)

$x = \frac{l}{\sqrt{3}} = \frac{20^{ft}}{\sqrt{3}} = 11.55^{ft}$ (to left of D) and $\Delta_{total} = \Delta_{max.-case\ 26} + \Delta_{x-case\ 5}$ with x (11.55^{ft}) greater than a (10^{ft}):

$$\Delta_{total} = .06415\frac{Pal^2}{EI} + \frac{wa^2(l-x)}{24EIl}\left(4xl - 2x^2 - a^2\right)$$ *Note: Because there is only negative moment, the deflection is actually up!*

$$= -.06415\frac{10^k(10^{ft})(20^{ft})^2(12^{in/ft})^3}{(29 \cdot 10^3\,ksi)(103in^4)} +$$

$$\frac{2^{k/ft}(10^{ft})^2(20^{ft} - 10^{ft})}{24(29 \cdot 10^3\,ksi)(103in^4)(20^{ft})} \times \ldots$$

$$(4 \cdot 11.55^{ft} \cdot 20^{ft} - 2(11.55^{ft})^2 - (10^{ft})^2)(12^{in/ft})$$

= -1.484 in + 1.343 in = -0.141 in (up)