Introduction to Beam Stress Analysis and Preliminary Design

Beam Analysis

When the beam section is already known, beam analysis is used to calculate the maximum stresses. Beam design involves finding a trial section, recognizing that there is more load from the beam weight itself, performing analysis AND comparing stresses to some limits until the section satisfies all criteria.

Analysis Procedure

1. Solve for support forces and draw V & M diagrams to obtain V_{max} and M_{max} *(maximum magnitudes)*
2. Determine the critical section geometry properties:
   - centroid: $\hat{y}$ (*necessary to find the neutral axis, I_x, and to determine c – the distance from the neutral axis to the "extreme" fiber of the cross section)* (Note Set 9.1)
   - moment of inertia about axis of bending: I_x (Note Set 9.2)
   - section modulus S_x *($S_x = I_x/c$)*

   NOTE: if the section is a standard shape, the properties will be pre-determined and available in reference charts.
3. Calculate maximum bending stress using M_{max}: $f_{b-max} = \frac{Mc}{I_x} = \frac{M}{S_x}$
4. Calculate maximum shear stress using V_{max}:

   a. For a rectangular section ONLY: $f_v = \frac{3V}{2A}$

   - A is the area (bh)

   b. For a wide flange section ONLY: $f_v = \frac{V}{A_{web}}$

   - A_{web} is the area determined from the thickness of the web and depth of the W ($t_w d$). These values are available in reference charts.

   c. OTHERWISE: $f_{v-ave} = \frac{VQ}{I_x b}$ where:

   - Q is the first moment area of a section "cut" at the neutral axis. It is the sum of all the basic areas of the section multiplied by **y distances from the neutral axis** for each to their centroids: $Q = \sum A\bar{y}$. $\bar{y}$ *is always measured from the neutral axis as the origin (y=0)*. (Note Set 10.1)
   - b is the thickness of the section "cut" from the real material (voids aren't included).
   - I_x is the moment of inertia about the x axis (neutral axis)

5. If a section is built-up, and the shear force across an interface or the spacing for nails across that interface to resist the shear force is needed, then the form of the shear stress equation becomes:

$$nF_{connector} \geq \frac{VQ_{connected\,area}}{I_x} \cdot p$$

- n is the number of nails or bolts connecting the parts at the interface(s) of interest
- $F_{connector}$ is the shear force per nail or bolt that the connector can resists (capacity)
- $Q_{connected\ area}$ is the first moment of area a section "cut" at the interface(s) of interest to isolate the connected part. It is the sum of all the basic areas of the section multiplied by **y distances from the neutral axis** for each to their centroids: $Q = \sum A\bar{y}$. $\bar{y}$ *is always measured from the neutral axis as the origin (y=0).* (Note Set 10.1)
- p is the "pitch" spacing between connectors along the axis of the beam
- I_x is the moment of inertia about the x axis (neutral axis)

Beam Design

Design implies that the beam section has not yet been determined. Design involves choosing a trial section (preliminary design), then checking at every important computation of stress or deflection that the computed value does not exceed the acceptable limits. A finalized design means the section has been changed because of an unacceptable evaluation, but now meets all criteria.

Preliminary Design Procedure

The intent is to find the most light weight member satisfying the section modulus size.

1. Know F_b (allowable stress) for the material or F_y & F_u for LRFD.
2. Draw V & M, finding M_{max}.
3. Calculate $S_{req'd}$ using M_{max}: $S_{required} \geq \frac{M}{F_b}$
   - This step is equivalent to evaluating if $f_b = \frac{M_{max}}{S_x} \leq F_b$
4. For rectangular beams $S_x = \frac{bh^2}{6}$
   - For steel or timber: use the section charts to find S that will work. And for steel, the design charts show the lightest section within a grouping of similar S's.
   - For any thing else, try a nice value for b, and calculate h or the other way around.